S-matrix theory with near-threshold states & $\bar{D}N$ interaction from HALQCD

HHIQCD 2024 @ YITP, October 29, 2024

Wren Yamada (Rento Yamada)

RIKEN iTHEMS (HALQCD Collaboration)

Content

- General behavior of spectral function and time evolution in the Presence of Near-threshold states

  W.Yamada, O.Morimatsu, Phys.Rev. C 102, 055201

  W.Yamada, O.Morimatsu, T.Sato, K.Yazaki, Phys.Rev. D 108, L071502

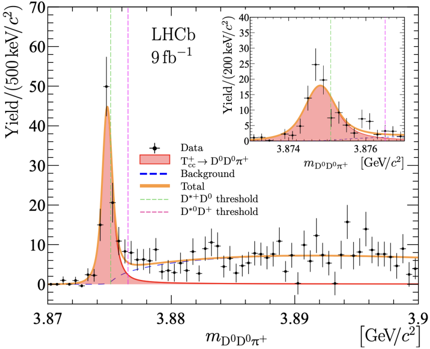
- Preliminary results of the $\bar{D}N$ interaction from HALQCD

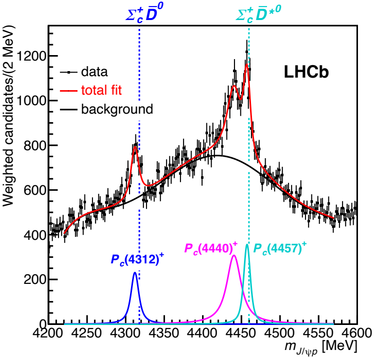
  HALQCD collab.

Spectral Function and Time Evolution in the Presence of Near-threshold states

Introduction

- Many candidates of exotic hadrons ($qq\bar{q}\bar{q}$, $qqqq\bar{q}$, $\cdots$) observed in the vicinity of the hadronic 2-body thresholds e.g. $X(3872)$, T_{cc}, P_c





R. Aaij et al. (LHCb Collaboration), arXiv:2109.01038

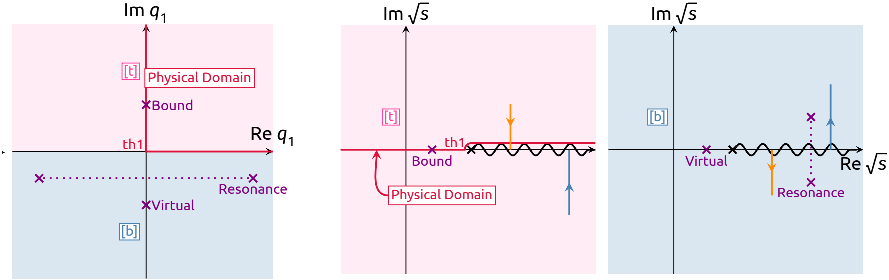
R. Aaij et al. (LHCb Collaboration), Phys.Rev.Lett. 115, 072001 (2015)

- Determination of the pole position is important for a better understanding of the nature of such states

Single-channel 2-body Scattering

- Descrete states (e.g. Bound state, Virtual state, Resonance state)
  Solutions of wave functions under the out-going-wave boudary condition
  A. F. J. Siegert, Phys. Rev. 56, 750 (1939)

Descrete States $\leftrightarrow$ Poles of the S-matrix



In the following assume that:

- Analytic continuation of the S-matrix is possible to complex momenta
- LH-cut contributions from t-channel, u-channel negligible

Single-channel 2-body Scattering

S-matrix: meromorphic function of CM momentum k

→ Mittag-Leffler theorem

- Mittag-Leffler Expansion (ML Expansion)

  J. Humblet, L.Rosenfeld, Nucl. Physics 26 (1961)

  D. Ramírez Jiménez, N. Kelkar, Annals of Physics 396, 18 (2018)

$$\mathcal{A}(k) = \sum_n \left[\frac{r_n}{k - k_n} - \frac{r_n^*}{k + k_n^*} \right] + \text{[subtractions (entire function)]}$$

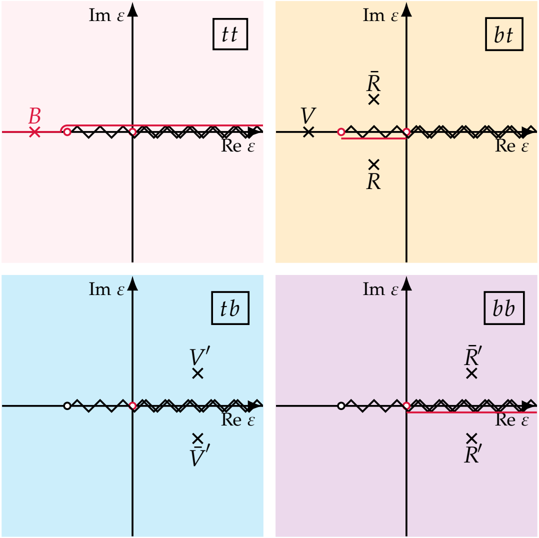
c.f.

$$\mathcal{A}(s) = \frac{1}{2i\pi} \int_{s_0}^{\infty} ds' \frac{\text{disc}\mathcal{A}(s')}{s' - s - i\epsilon} + \text{[subtractions (entire function)]}$$

→ Extension to coupled-channels

$\mathbb{CP}^1$ Representation of the 2-channel S-matrix

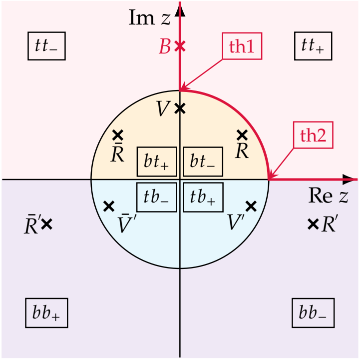
E-plane | 4-sheeted Riemann Surface (RS)



$\mathbb{CP}^1$ Representation of the 2-channel S-matrix

z-plane M. Kato, Annals of Physics 31, 130 (1965)

$$z = \frac{1}{\Delta}(k_1 + k_2), \quad k_i = (\epsilon - \epsilon_i)^{1/2}, \quad \Delta = (\epsilon_2^2 - \epsilon_1^2)^{1/2}$$



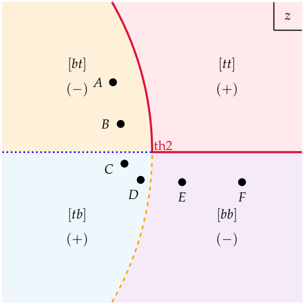
- 2-channel Mittag-Leffler Expansion W. Yamada, O. Morimatsu, Phys. Rev. C 102, 055201 (2020)

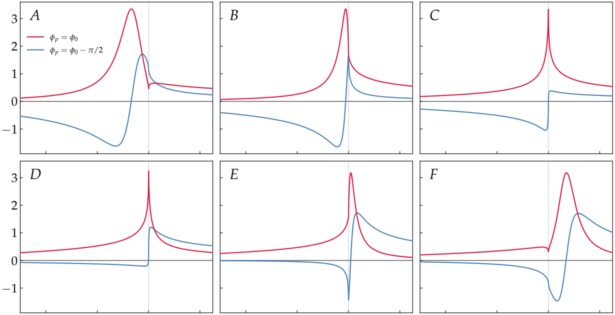
$$\mathcal{A}(z) = \sum_i \left[\frac{r_n}{z - z_n} - \frac{r_n^*}{z + z_n^*} \right] + \text{(subtraction)}$$

Spectral decomposition invariant under: $Aut(\mathbb{CP}^1) \simeq PGL(2, \mathbb{C})$

Spectral Function: Lineshapes

Lineshapes of the spectral function in the presence a near-threshold pole (near upper threshold)





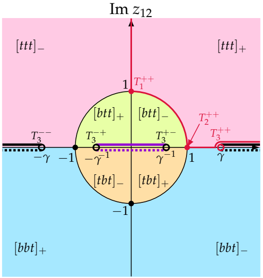
	A	B	C	D	E	F
z_p	$0.869 + 0.233i$	$0.895 + 0.094i$	$0.908 - 0.038i$	$0.962 - 0.092i$	$1.100 - 0.100i$	$1.300 - 0.100i$
e_p	$0.943 - 0.053i$	$1.000 - 0.022i$	$1.007 + 0.008i$	$0.992 + 0.007i$	$1.002 - 0.018i$	$1.065 - 0.043i$
e_0	0.933	0.989	1	1	1.01	1.065
γ_0	0.100	0.100	0.100	0.100	0.100	0.100
Sheet	$[bt]$	$[bt]$	$[tb]$	$[tb]$	$[bb]$	$[bb]$

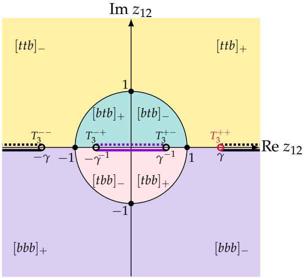
- Peak position ≈ Closest Physical Point on z ($\neq \text{Re}\, E_R$)
- Width of Structure ∝ Minimal Distance from Physical Domain on z ($\neq |\text{Im}\, E_R|$)

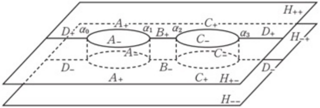
$\mathbb{C}/\mathbb{Z}^2$ Representation of the 3-channel S-matrix

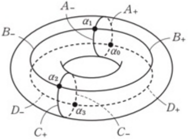
2-sheeted z_{12}-plane (z-plane using channel mass ϵ_1, ϵ_2)

$$q_1 = \frac{\Delta_{12}}{2}\left[z_{12} + 1/z_{12}\right], \quad q_2 = \frac{\Delta_{12}}{2}\left[z_{12} - 1/z_{12}\right], \quad q_3 = \frac{\Delta_{12}}{2z_{12}}\underbrace{\sqrt{(1-z_{12}^2\gamma^2)(1-z_{12}^2/\gamma^2)}}_{\text{sq.root cut } z_{12}=\pm\gamma,\pm 1/\gamma}, \quad \left(\gamma = \frac{\sqrt{\epsilon_3^2-\epsilon_1^2}+\sqrt{\epsilon_3^2-\epsilon_2^2}}{\Delta_{12}}\right)$$









W.Y. O.M. T.S. arXiv:2203.17069 [hep-ph], Fig.1

(楕円積分と楕円関数 おとぎの国の歩き方)

- 3-channel ML Expansion — W. Yamada, O. Morimatsu, T. Sato, PRL 129, 192001 (2022)

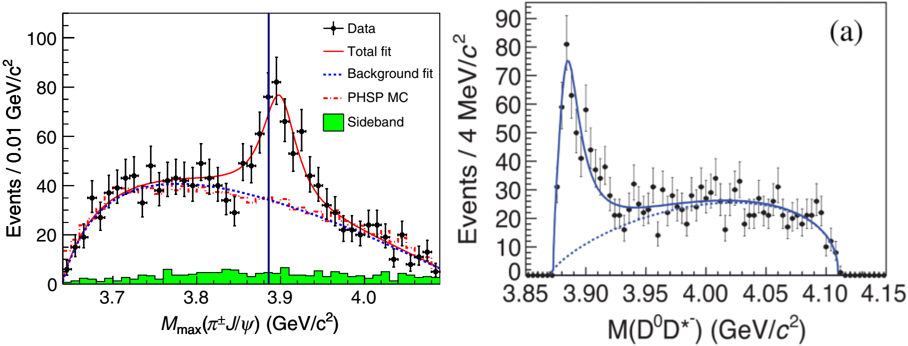
$$\mathcal{A}(z) = \sum_{z_i\in\Lambda^*} \boxed{r_i\left[\zeta(z-z_i)+\zeta(z_i)\right]} \qquad z[\tau] = \frac{1}{4K(k)}\int_0^{\gamma/z_{12}} \frac{d\xi}{\sqrt{1-\xi^2}\sqrt{1-k^2\xi^2}}$$

Pole term

M. Ablikim et al., Phys. Rev. Lett. 110, 252001 (BESIII)

M. Ablikim et al., Phys. Rev. Lett. 112, 022001 (BESIII)

Z. Q. Liu et al., Phys. Rev. Lett. 110, 252002 (Belle)

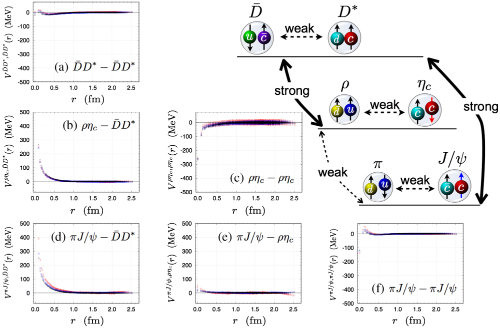


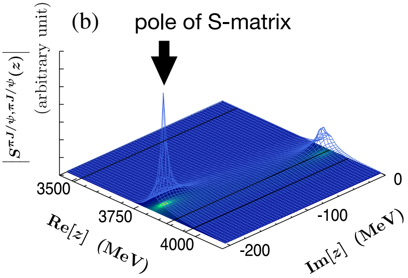
Enhancement at $D\bar{D}^*$ threshold

- Resonance?
- Threshold cusp?

Y. Ikeda et.al. (HALQCD collab.), Phys. Rev. Lett. 117, 242001 (2016)

$\pi J/\psi$-$\rho\eta_c$-$D\bar{D}^*$, s-wave interactions, (2+1)-flavor, m_π =410-700 MeV





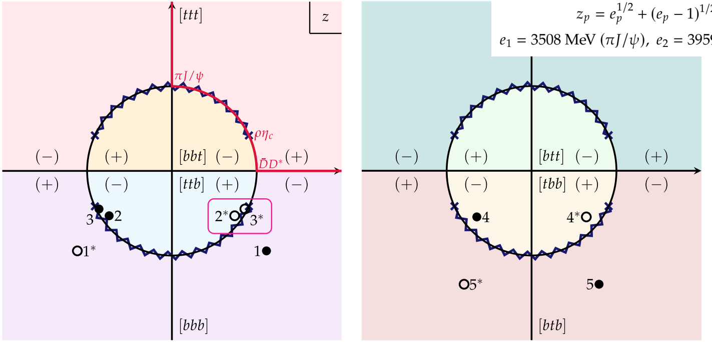
Case	[ttb]	[tbb]	[btb]
I	−146(112)(108) − i 38(148)(32) −93(55)(21) − i 9(25)(7)	−177(116)(61) − i 175(30)(22)	−369(129)(102) − i 207(61)(20)
II	−102(84)(45) − i 14(11)(7) −59(67)(11) − i 3(12)(1)	−141(92)(64) − i 151(149)(132)	−322(141)(111) − i 114(96)(75)
III	−100(48)(29) −i 7(37)(17) −53(30)(5) −i 2(11)(3)	−127(52)(43) −i 199(44)(28)	−356(108)(28) −i 277(138)(95)

$Z_c(3900)$

Pole positions from the HALQCD result ($m_\pi = 410$ MeV)

$\mathbb{CP}^1$ plane of channels $\pi J/\psi$ and $D\bar{D}^*$

$$e_p = (s_p - e_1^2)/(e_2^2 - e_1^2),$$
$$z_p = e_p^{1/2} + (e_p - 1)^{1/2}.$$
$$e_1 = 3508 \text{ MeV } (\pi J/\psi),\ e_2 = 3959 \text{ MeV } (\bar{D}D^*)$$



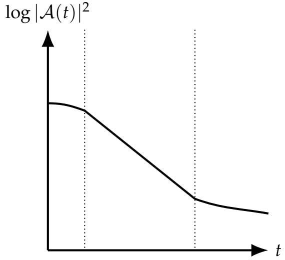
- Pole positions and residues calculated by solving the Lippmann-Scheinger equation with the HAL potential → residues of pole 2* and 3* dominant on the $\mathbb{C}/\mathbb{Z}^2$-plane of $\pi J/\psi$-$\rho\eta_c$-$D\bar{D}^*$
- $Z_c(3900)$ is a threshold cusp enhanced by poles on $[ttb]_+$
- Global coupled-channel analysis of $e^+e^- \to c\bar{c}$ by S. Nakamura et al. obtain a pole near HALQCD results (Private communication)

Survival amplitude: General behavior

Survival amplitude

$$\mathcal{A}(t) = \langle \psi(0)|\psi(t)\rangle = \sum_B |\langle \psi(0)|B\rangle|^2 e^{-iE_B t} - \frac{1}{2\pi i}\int_0^\infty dE\, e^{-iEt}\,\mathrm{disc}\,\mathcal{G}(E)$$

- Small t: Non-exponential decay due to time-reversal invariance
- Intermidiate t: Exponential decay
- Large t ($\Gamma t \gg 1$): Only contribution from the end point (threshold)



- Short time: Quadratic decay ($t \sim 1/|E_R|$) Quantum Zeno Effect
- Intermidiate time: Exponential decay
- Large time: Inverse power decay

  L. A. Khalfin, Sov. Phys. JETP 6, 1053 (1958)

c.f. imaginary-time correlations

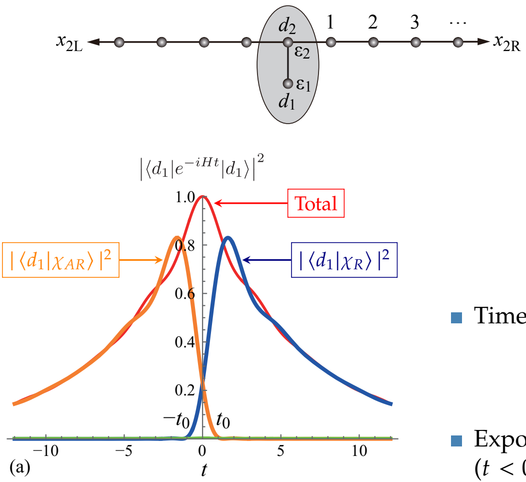
L.Maiani M.Testa Phys.Lett.B 245 (1990) 585-590

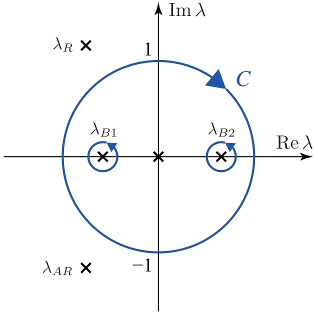
$$\langle \pi(t,\vec{q}=0)\pi(t,\vec{q}=0)\mathcal{J}(0)\rangle \to \frac{Z_\pi}{(2M_\pi)^2} e^{-2M_\pi t} f(4M_\pi^2)\left[1 - a\sqrt{\frac{M_\pi}{4\pi t}} + \cdots\right]$$

Pole expansion of survival amplitude: single-channel

G. Ordonez and N. Hatano J. Phys. A 50, 40, 405304 (2017)

- 1D tight-binding model consisting of a quantum dot connected to two semi-infinite leads: *electron hopping from site to site*





- Time-reversal symmetry

$$|\langle d_1|\chi_R(t)\rangle + \langle d_1|\chi_{AR}(t)\rangle|^2$$
$$= |\langle d_1|\chi_R(-t)\rangle + \langle d_1|\chi_{AR}(-t)\rangle|^2$$

- Exponential decay in intermidiate $t > 0$ ($t < 0$) region dominated by resonance (anti-resonance) contribution

“Survival Amplitude” in coupled-channels

Extension of J. Phys. A 50, 40, 405304 (2017) to coupled-channel systems

Pole Expansion of “Survival Amplitude” (2-channel)

PRD 108, L071502(2023) W. Yamada, O. Morimatsu, T. Sato, K. Yazaki

Unstable state $|d_1\rangle$

$$\mathcal{A}(t) = \langle d_1|e^{-iHt}|d_1\rangle = \sum_B |\langle d_1|\phi_B\rangle|e^{-iE_Bt} + \boxed{\sum_n r_n \mathcal{A}_n(t;z_n)}$$

$$\mathcal{A}_n(t;z_n) = \frac{i}{4\pi}\left[\left(1-\frac{1}{z_n^2}\right)I(t;\varepsilon_n) + \left(1+\frac{1}{z_n^2}\right)e^{-it}I(t;\varepsilon_n-1) + \frac{2i}{z_n}J(t;\varepsilon_n)\right]$$

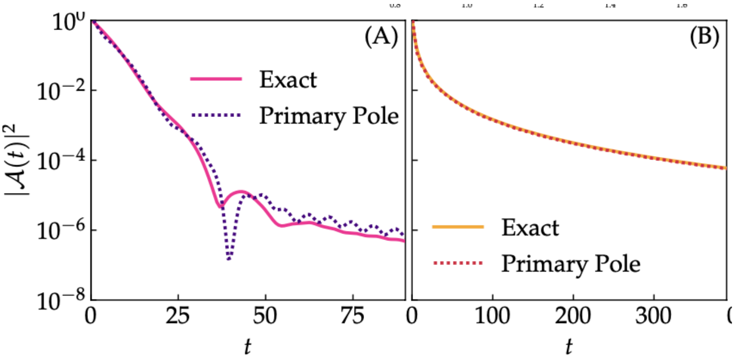
$$I(t;\varepsilon_n) = \sqrt{\frac{\pi}{it}} - i\pi\sqrt{\varepsilon_n}e^{-it\varepsilon_n}\mathrm{erfc}(i\sqrt{it\varepsilon_n}), \quad J(t;\varepsilon_n) = \int_0^1 d\varepsilon\,\frac{\sqrt{\varepsilon}\sqrt{1-\varepsilon}}{\varepsilon-\varepsilon_n}e^{-it\varepsilon}$$

- $I(t;\varepsilon_n)$ matches the analytic expression for the single-channel case
  Contributions from each channel (first term, second term)
- Interference term: $J(t;\varepsilon_n)$ involving both channels

"Survival Amplitude" in coupled-channels

Survival amplitude: Toy model

$$\hat{\mathcal{V}} = g_1 \int \frac{d\vec{q}_1}{(2\pi)^3} v(q_1) \Big[|\vec{q}_1\rangle \langle d| + |d\rangle \langle \vec{q}_1| \Big] + g_2 \int \frac{d\vec{q}_2}{(2\pi)^3} v(q_2) \Big[|\vec{q}_2\rangle \langle d| + |d\rangle \langle \vec{q}_2| \Big]$$



- case[A] Resonance pole: Exponential decay → inverse-power decay
- case[B] Enhanced threshold cusp: Non-exponential decay in all time regions

Summary

- Analytic structure of the S-matrix
  - 2-channel $\mathbb{CP}^1$
  - 3-channel $\mathbb{C}/\mathbb{Z}^2$
- General behavior of the spectral function
  - Resonance peaks: $[(b)bt]_-$, $[(b)bb]_-$, $[btt]_-$
  - Enhanced threshold cusps: $[(t)tb]_+$, $[tbt]_+$
- Survival amplitude
  - Resonance: Exponential decay $\to$ inverse-power decay
  - Enhanced threshold cusp: Non-exponential decay in all time regions

$\bar{D}N$ interaction from HALQCD

Introduction

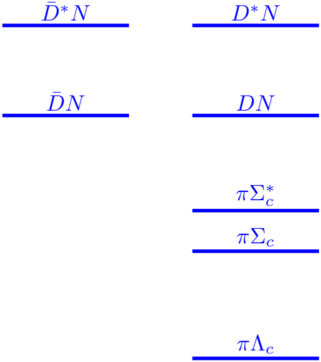
Studies on the $\bar{D}N$ system

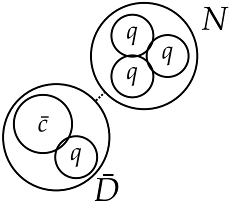
- Exotic channel: $\bar{c}q + qqq$

  No $q\bar{q}$ annihilation of constituent quarks

  Bound state → Pentaquark
- Possibility of Charmed nuclei

  Approximate degenarate states from HQSS
- in-medium effects to the $\bar{D}$-meson





- No open channels (↔ DN system has lower open channels e.g. $\pi\Lambda_c$, $\pi\Sigma_c^{(*)}$)
- No $q\bar{q}$ annihilation suited for Lattice simulations

  ($q\bar{q}$ annihilation → large computational cost, Murakami-san's Talk 10.28)

Introduction

- Experimental constraints insufficient (almost none)

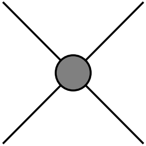
  Results of various theoretical models

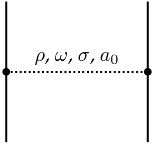
Table from Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP 96, 88 (2017)

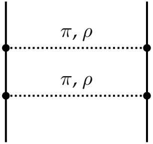
Model	$a^{I=0}_{\bar{D}N}$	$a^{I=1}_{\bar{D}N}$	$a_{\bar{D}}$
SU(4) contact [185]	−0.16	−0.26	−0.24
Meson exchange [194]	0.07	−0.45	−0.32
Pion exchange [192]	−4.38	−0.07	−1.15
Chiral quark model [219]	0.03–0.16	0.20–0.25	0.16–0.23

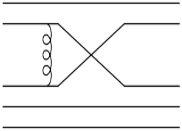
Model	$\bar{D}N(I=0)$	$\bar{D}N(I=1)$
SU(4) contact [185]	None	None
SU(8) contact [193]	2805	None
Meson exchange [194]	None	None
Pion exchange [192]	2804	None
Chiral quark model [195]	None	None

[185] J.Hofmann, M.F.M.Lutz, Nucl.Phys. A763 (2005) 90-139
[192] Y. Yamaguchi, S. Ohkoda, S. Yasui, A. Hosaka, Phys.Rev. D 84 (2011) 014032
[193] D.Gamermann, C.Garcia-Recio, J.Nieves, L.L.Salcedo, L.Tolos, Phys.Rev. D81 (2010) 094016
[194] J.Haidenbauer, G.Krein, U.-G.Meissner, A.Sibirtsev, Eur.Phys.J. A33 (2007) 107-117
[195] T.F.Carames, A.Valcarce, Phys.Rev. D85 (2012) 094017
[219] C.E. Fontoura, G. Krein, V.E. Vizcarra, Phys.Rev. C 87 (2) (2013) 025206







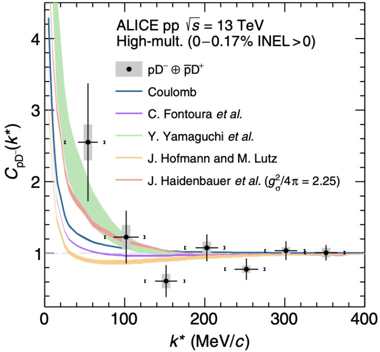


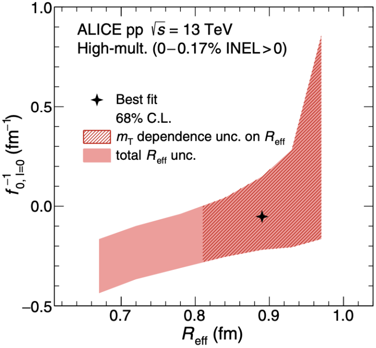
Introduction

First experimental study of the two-body scattering of $\bar{D}N$

S. Acharya et al. ALICE collab. Phys.Rev.D 106 (2022) 5, 052010

- $D^- p$ correlation from pp collision
- Assuming negligible interaction in the $I = 1$ channel





- Attractive $\bar{D}N$ strong interaction + coulomb
- inverse scattering length a_0^{-1}: $-0.4 \sim +0.9$ fm
  $\rightarrow$ Existance of bound state or shallow virtual state

Objective

Objective

Study the $\bar{D}N$ interaction on the Lattice by the HALQCD method

- configs at physical point ($m_\pi \simeq 137$ MeV)

HALQCD method

Scattering information (Phase shift) from correlation function:

$$\mathcal{C}(t,\vec{r}) = \sum_{\vec{x}} \langle \mathcal{O}_1(t,\vec{x}+\vec{r})\mathcal{O}_2(t,\vec{x})\bar{\mathcal{J}}(0)\rangle$$
$$= \sum_{n} A_n \psi(\vec{r};E_n) e^{-E_n t}$$

- Lüscher's finite volume method

  M. Lüscher, Nucl.Phys.B 354 531-578 (1991)

  Energy spectra in finite volume $\{E_n\}$ → quatization condition (e.g. Lüscher's formula) → Phase shift

- HALQCD method

  N. Ishii, S. Aoki, and T. Hatsuda PRL 99, 022001 (2007)

  Temporal + spacial info. $\psi(\vec{r};E_n)$ → Potential → Phase shift

HALQCD method

HALQCD

$$\mathcal{C}(t,\vec{r}) = \sum_{\vec{x}} \langle \mathcal{O}_1(t,\vec{x}+\vec{r})\mathcal{O}_2(t,\vec{x})\bar{\mathcal{J}}(0)\rangle$$

$$= \sum_n A_n \psi(\vec{r};E_n) e^{-E_n t}$$

$$\psi(\vec{r}) \to \frac{\sin(kr - l\pi/2 + \delta(k))}{kr} e^{i\delta(k)} \quad (r \gg R)$$

$$(\nabla^2 + k^2)\psi(\vec{r};E_n) = \int d\vec{r}' V(\vec{r},\vec{r}')\psi(\vec{r}';E_n)$$

- $V(\vec{r},\vec{r}')$ is a potential that produces the correct phase shift of the QCD S-matrix

Time-dependent method

Ishii et al. (HAL QCD), PLB712, 437(2012)

$$R(t,\vec{r}) = \frac{\mathcal{C}(t,\vec{r})}{\sqrt{Z_1 Z_2} e^{-(m_1+m_2)t}}$$

$$\left[\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right]R(t,\vec{r}) = \int d\vec{r}' V(\vec{r},\vec{r}')R(t,\vec{r}')$$

- Excited-state contamination surpressed

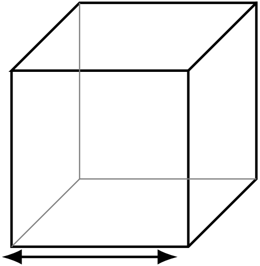
Setup

- configuation: F-conf

$$a = 0.084372(54)\ [\text{fm}]$$
$$L = 96, \quad V = L^3 \times 96$$

- Iwasaki gauge action ($\beta = 1.82$)
- 2+1 flavor
- $\mathcal{O}$(a)-improved Wilson quark action
- slightly heavy c-quark

	Lattice [MeV]	PDG [MeV]
π	137.1	138
N	939.7	938
$\bar{D}N$ (threshold)	2819.8	2807
$\bar{D}^*N$ (threshold)	2957.5	2949



$L \simeq 8$ fm
$m_\pi \simeq 137$ MeV

- statistics: 360 configurations $\times$ 96 source $\times$ 4

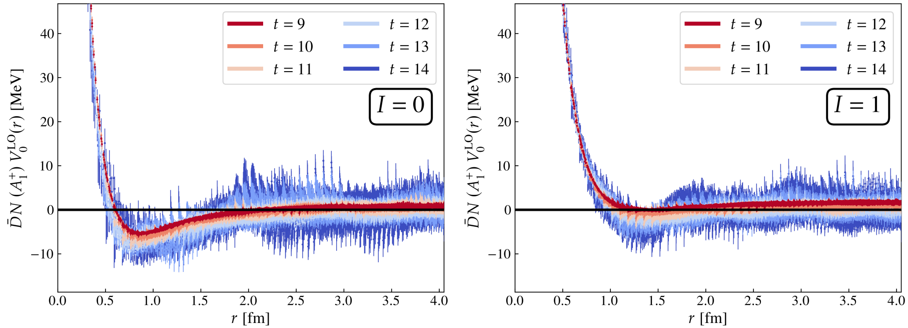
- HALQCD Analysis: Single-channel $\bar{D}N$ system ($I = 0, I = 1$)

$\bar{D}N$ potential: A_1^+, Derivative expansion LO

LO term of the derivative expansion computed from the A_1^+ projected NBS function

$$V(\vec{r},\vec{r}\,') = V_{\mathrm{LO}}(r)\delta(\vec{r}-\vec{r}\,') + \mathcal{O}(\nabla)\delta(\vec{r}-\vec{r}\,')$$

$$V_{\mathrm{LO}}(r) = R^{-1}(r,t)\left[\frac{1+3\delta^2}{8\mu}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right]R(r,t)$$

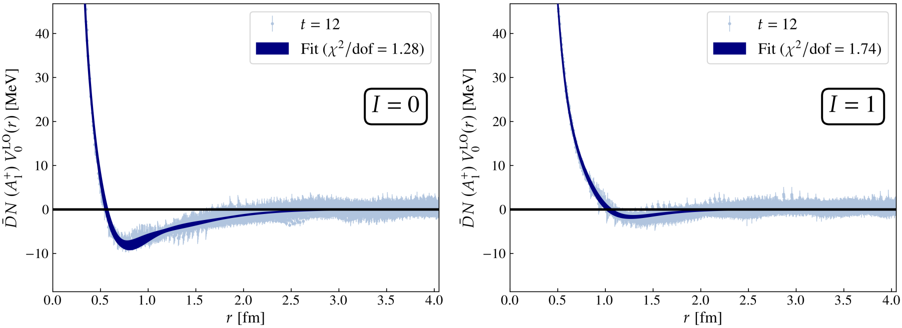


- $I = 0$: Short-range repulsive core + Attractive pocket (~10 MeV)
- $I = 1$: Short-range repulsive core

$\bar{D}N$ potential: A_1^+, Derivative expansion LO

Phenomenological fit of V_{LO} (uncorrelated)

$$V_{\mathrm{LO}}(r) \approx a_0 e^{-a_1 r^2} + a_2 e^{-a_3 r^2} + a_4 (1 - e^{-a_5 r^2})^2 \frac{e^{-a_6 r}}{r^2} \equiv V_{\mathrm{fit}}(r)$$



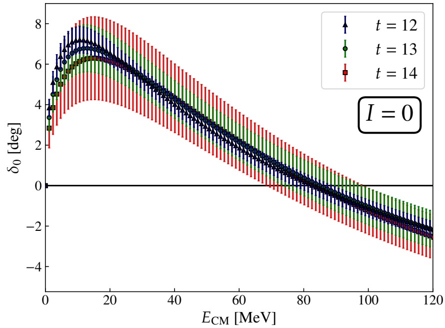
- χ^2/dof

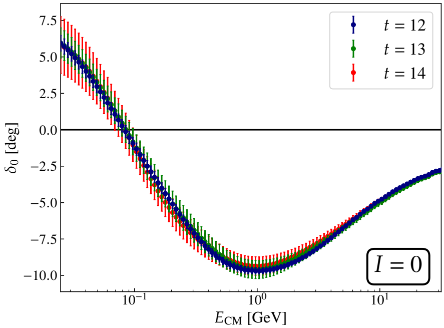
	$t = 12$	$t = 13$	$t = 14$
$I = 0$	1.28	1.25	1.16
$I = 1$	1.74	1.36	1.17

Phase shift δ_0 $(I = 0)$

Computed phase shift by solving the schrödinger equation with potential $V_{\text{fit}}^{(I=0)}$

$$\psi(R) = \frac{\sin(kR + \delta)}{kR} e^{i\delta} \quad (R = 48a)$$



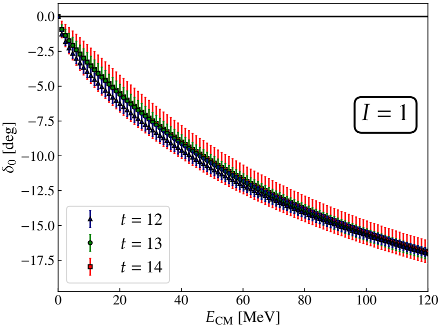


- Attractive behavior (small attraction) in the low-energy region
  Repulsive behavior in the higher energy region
- No bound-state: $\delta_0(0) - \delta_0(\infty) = 0$

Phase shift δ_0 $(I = 1)$

Computed phase shift by solving the schrödinger equation with potential $V_{\text{fit}}^{(I=1)}$

$$\psi(R) = \frac{\sin(kR + \delta)}{kR} e^{i\delta} \quad (R = 48a)$$

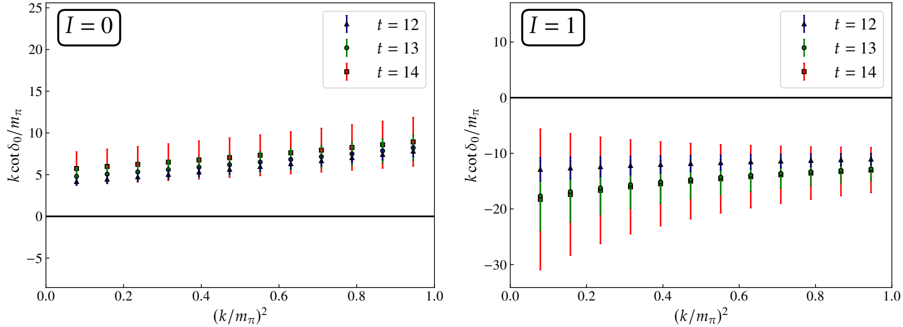


- Repulsive behavior in all energy regions
- No bound-state: $\delta_0(0) - \delta_0(\infty) = 0$

Scattering Length, Effective Range

Effective-Range expansion:

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \cdots$$



- Scattering length a_0 [fm]

	$I = 0$	$I = 1$
$t = 12$	0.370 (42)	−0.110 (18)
$t = 13$	0.320 (65)	−0.080 (30)
$t = 14$	0.267 (93)	−0.081 (54)

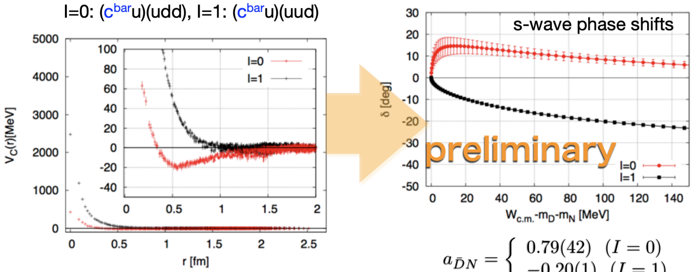
- Effective Range r_0 [fm]

	$I = 0$	$I = 1$
$t = 12$	10.26 ± 0.71	8.24 ± 5.33
$t = 13$	9.84 ± 1.41	24.09 ± 23.74
$t = 14$	9.20 ± 2.60	26.88 ± 50.19

m_π dependence

Comparison to results with heavier pion mass m_π = 410 MeV (PACS-CS config.)

Y. Ikeda slides 10th APCTP-BLTP/JINR-RCNP-RIKEN Joint Workshop Aug.2016



$$a_{\bar{D}N} = \begin{cases} 0.79(42) & (I=0) \\ -0.20(1) & (I=1) \end{cases}$$

- $I = 0$: smaller pion mass, shallower attractive pocket
  ($m_\pi \simeq 137$ MeV case) has smaller scattering length but is still positive
- $I = 1$: ($m_\pi \simeq 137$ MeV case) has slightly smaller repulsion
  scattering length closer to zero

Comparison: EFT Models, Femtoscopy

Preliminary Results from HALQCD $m_\pi \simeq 137$ MeV

- No bound state in $I = 0$, $I = 1$
- Scattering length a_0 [fm]

	$I = 0$	$I = 1$
$t = 12$	0.370 (42)	−0.110 (18)
$t = 13$	0.320 (65)	−0.080 (30)
$t = 14$	0.267 (93)	−0.081 (54)

Scattering length of various models

Table from Hosaka, Hyodo, Sudoh, Yamaguchi, Yasui, PPNP 96, 88 (2017)

Model	$a^{I=0}_{\bar{D}N}$	$a^{I=1}_{\bar{D}N}$	$a_{\bar{D}}$
SU(4) contact [185]	−0.16	−0.26	−0.24
Meson exchange [194]	0.07	−0.45	−0.32
Pion exchange [192]	−4.38	−0.07	−1.15
Chiral quark model [219]	0.03–0.16	0.20–0.25	0.16–0.23

$D^- p$ correlation function (femtoscopy): ALICE PRD 106, 052010 (2022)

- $a^{-1}_{I=0} \in [-0.4, 0.9]$ fm^{-1}

Summary

Summary

- $\bar{D}N$ system

  Possibility of Pentaquark, Charmed nuclei…
- Limited experimental data, No lower open channels, no $q\bar{q}$ annihilation

  $\rightarrow$ good system for Lattice simulations
- Preliminary results HALQCD ($m_\pi \simeq 137$ MeV)
  - (Small) Attractive behavior in the low energy region of $I = 0$ channel
  - Repulsive behavior in the $I = 1$ channel
  - No bound states

Future work

- Coupled-channel analysis of $\bar{D}N$-$\bar{D}^*N$

  Coupling of $\bar{D}N$-$\bar{D}^*N$ is important to explain the attraction in $I = 0$ channel
- Femtoscopy analysis using the $\bar{D}N$ HAL potential