Assorted topics in Lattice QCD

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Hadrons and Hadron Interactions in QCD 2024 (HHIQCD 2024)

Yukawa Institute of Theoretical Physics, Kyoto University

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Topics of the Talk

- Dibaryons with heavy quarks
  - Spin one - deuteron-like dibaryons
  - Spin zero dibaryons
- Continuous temperature simulation in Lattice QCD
  - General method
  - Simulate high temperature dependence of $\chi_{\rm top}$
- Precision scale setting at high temperatures
  - New method of relative scale setting
  - Subpercent precision

Heavy quark dibaryons

- Phys. Rev. Lett. 123, 162003 (2019)
- arXiv:2410.08519

The NN interaction

On the Neutron-Proton Interaction*

WILLIAM RARITA† AND JULIAN SCHWINGER
Department of Physics, University of California, Berkeley, California
(Received January 7, 1941)

$$S = 1,\ I = 0,\ \frac{1}{\sqrt{2}}(pn - np)$$

- Deuteron
- Only bound dibaryon system in Nature

$$S = 0, I = 1, (pp, \sqrt{2}pn, nn)$$

- Dineutron
- Still debated ...

- Wonder about heavy flavor extension ... ?

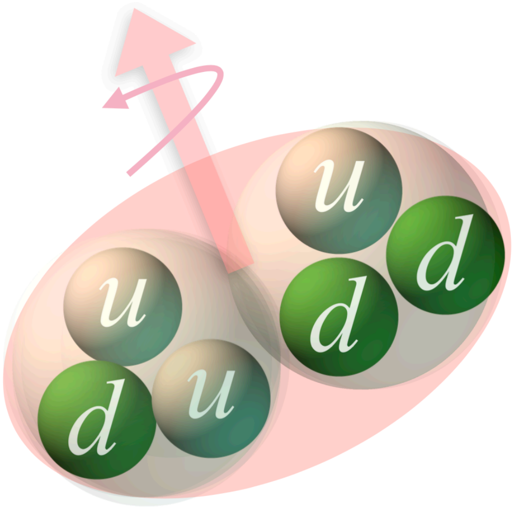
Spin-1 operators

✤ Construct deuteron operator :

$$D_{ud} = \frac{1}{\sqrt{2}}\big(N\ (C\gamma^j)\ P - P\ (C\gamma^j)\ N\big)$$

$$N_\alpha(x) = \epsilon_{abc}\ d^a_\alpha(x)\ \big(u^b_\mu(x)\ (C\gamma_5)_{\mu\nu}\ d^c_\nu(x)\big)$$

$$P_\alpha(x) = \epsilon_{abc}\ u^a_\alpha(x)\ \big(u^b_\mu(x)\ (C\gamma_5)_{\mu\nu}\ d^c_\nu(x)\big)$$



$$S = 1, \quad I = 0$$

Spin-1 operators

✤ **Construct deuteron operator :**

$$D_{ud} = \frac{1}{\sqrt{2}}\left(N\ (C\gamma^j)\ P - P\ (C\gamma^j)\ N\right)$$

$$N_\alpha(x) = \epsilon_{abc}\ d^a_\alpha(x)\ \left(u^b_\mu(x)\ (C\gamma_5)_{\mu\nu}\ d^c_\nu(x)\right)$$

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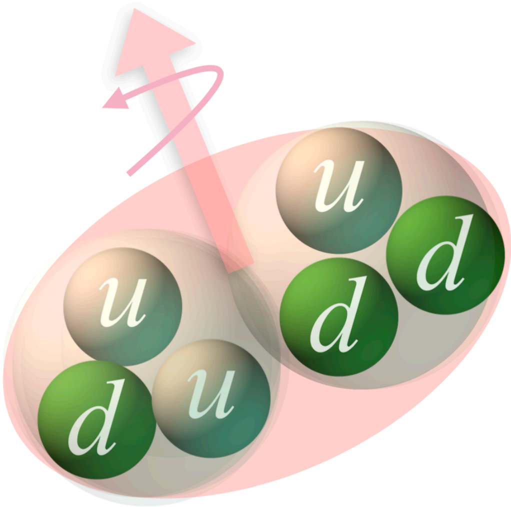


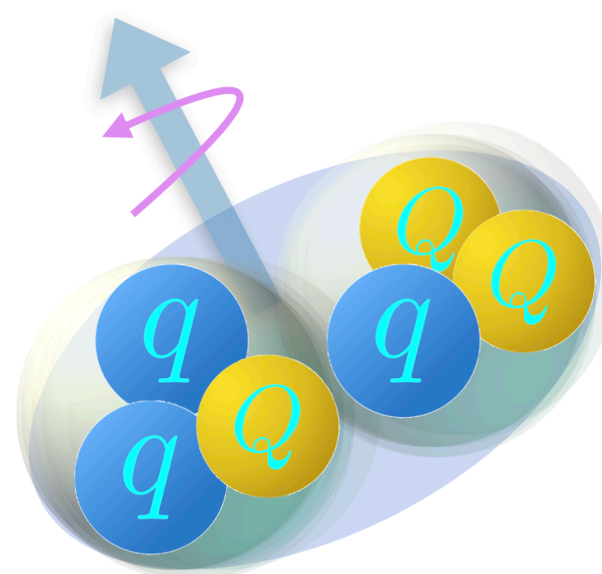
$S = 1, \quad I = 0$

✤ **Deuteron-like heavy operator :**

$$\mathcal{D}_{qQ} = \frac{1}{\sqrt{2}}\left(\Omega_{qqQ}(C\gamma^j)\Omega_{QQq} - \Omega_{QQq}(C\gamma^j)\Omega_{qqQ}\right)$$

$$(\Omega_{qqQ})_\alpha = \epsilon^{abc} q^a_\alpha(x) q^b_\mu(x) (C\gamma_5)_{\mu\nu} Q^c_\nu(x)$$

$$(\Omega_{QqQ})_\alpha = \epsilon^{abc} Q^a_\alpha(x) q^b_\mu(x) (C\gamma_5)_{\mu\nu} Q^c_\nu(x)$$



$S = 1,\ \mathrm{FA}$

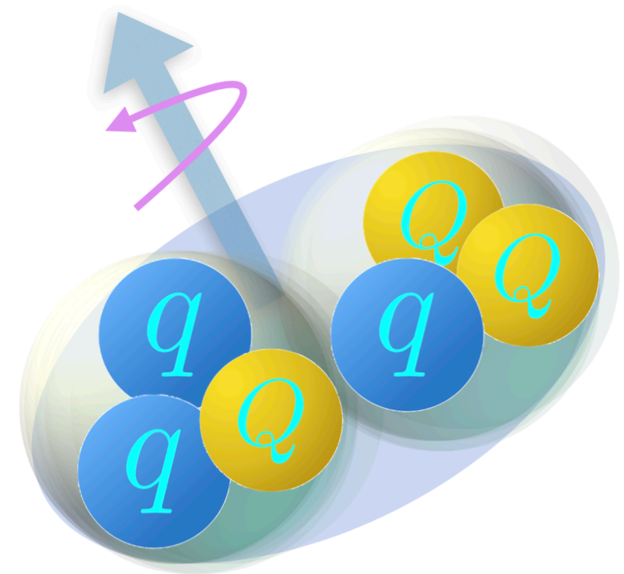
Spin-1 operators

- Deuteron-like heavy operator :

$$\mathcal{D}_{qQ} = \frac{1}{\sqrt{2}}\Big(\Omega_{qqQ}(C\gamma^j)\Omega_{QQq} - \Omega_{QQq}(C\gamma^j)\Omega_{qqQ}\Big)$$

$$(\Omega_{qqQ})_\alpha = \epsilon^{abc} q^a_\alpha(x) q^b_\mu(x)(C\gamma_5)_{\mu\nu} Q^c_\nu(x)$$

$$(\Omega_{QqQ})_\alpha = \epsilon^{abc} Q^a_\alpha(x) q^b_\mu(x)(C\gamma_5)_{\mu\nu} Q^c_\nu(x)$$



$S = 1,\ \mathrm{FA}$

- Explore different flavor combinations

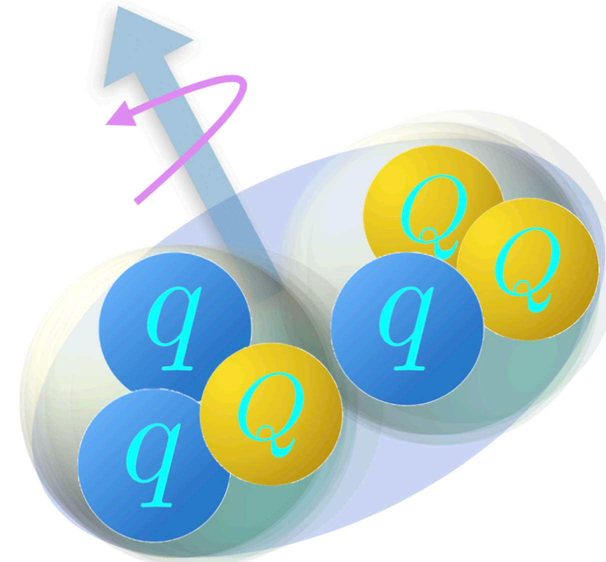
- Compute ground state and compare with non-interacting levels

$\mathcal{D}_{Qq}$	Interpolating fields
$\mathcal{D}_{bc}$	$\frac{1}{\sqrt{2}}\big(\Omega_{ccb}\Omega_{bbc} - \Omega_{bbc}\Omega_{ccb}\big)$
$\mathcal{D}_{bs}$	$\frac{1}{\sqrt{2}}\big(\Omega_{b}\Omega_{bb} - \Omega_{bb}\Omega_{b}\big)$
$\mathcal{D}_{cs}$	$\frac{1}{\sqrt{2}}\big(\Omega_{c}\Omega_{cc} - \Omega_{cc}\Omega_{c}\big)$
$\mathcal{D}_{bu}$	$\frac{1}{\sqrt{2}}\big(\Sigma_{b}\Xi_{bb} - \Xi_{bb}\Sigma_{b}\big)$
$\mathcal{D}_{cu}$	$\frac{1}{\sqrt{2}}\big(\Sigma_{c}\Xi_{cc} - \Xi_{cc}\Sigma_{c}\big)$

Spin-0 operators

- Operators

$$(FS, S=0) = \begin{cases} \Omega(llQ)_\alpha \ (C\gamma_5)_{\alpha\beta} \ \Omega(llQ)_\beta, \\ \sqrt{2}\, \Omega(llQ)_\alpha \ (C\gamma_5)_{\alpha\beta} \ \Omega(QlQ)_\beta, \\ \Omega(QlQ)_\alpha \ (C\gamma_5)_{\alpha\beta} \ \Omega(QlQ)_\beta. \end{cases}$$



$S = 0, FS$

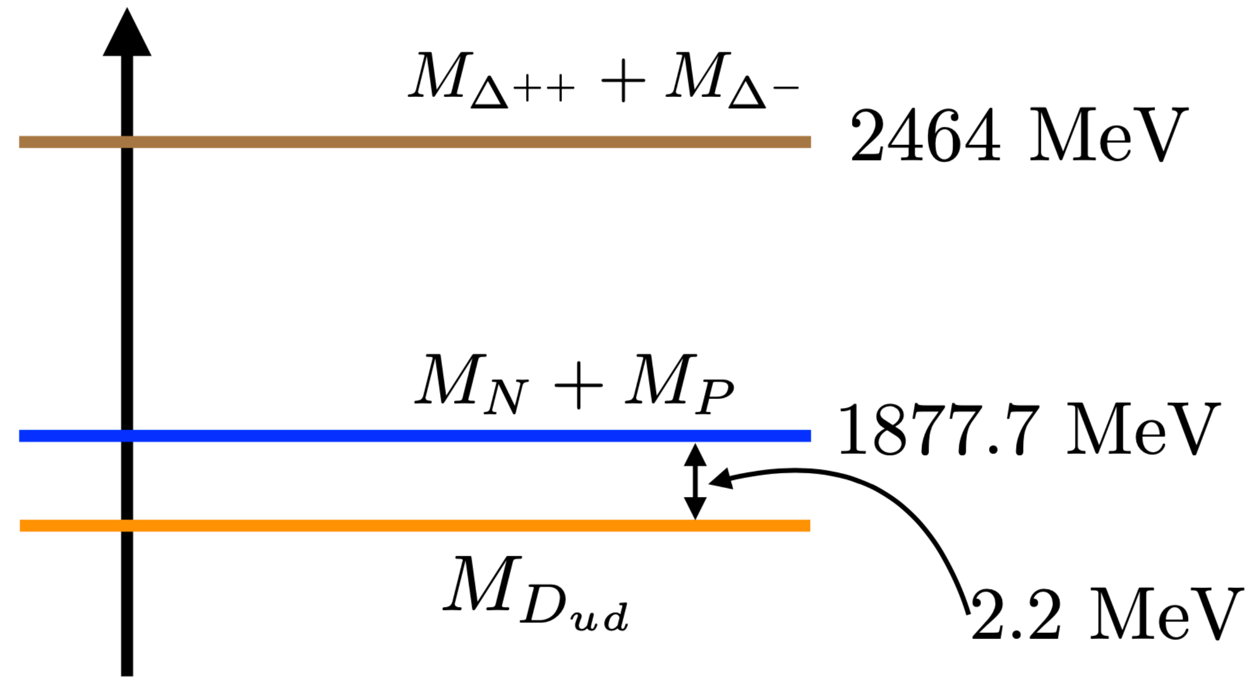
- Explore different flavor combinations

- Compute ground state and compare with non-interacting levels

Dibaryon	Lowest NI state
$\Omega_{bcb}\Omega_{bcb}$	$\Omega^{\frac{3}{2}}_{ccb}, \Omega^{\frac{3}{2}}_{bbb}$
$\Omega_{ccb}\Omega_{bcb}$	$\Omega^{\frac{3}{2}}_{ccc}, \Omega^{\frac{3}{2}}_{bbb}$
$\Omega_{ccb}\Omega_{ccb}$	$\Omega^{\frac{3}{2}}_{ccc}, \Omega^{\frac{3}{2}}_{cbb}$
$\Omega_{bb}\Omega_{bb}$	$\Omega^{\frac{3}{2}}_{b}, \Omega^{\frac{3}{2}}_{bbb}$
$\Omega_{b}\Omega_{bb}$	$\Omega^{\frac{3}{2}}_{sss}, \Omega^{\frac{3}{2}}_{bbb}$
$\Omega_{cc}\Omega_{cc}$	$\Omega^{\frac{1}{2}}_{cc}, \Omega^{\frac{1}{2}}_{cc}$
$\Omega_{c}\Omega_{cc}$	$\Omega^{\frac{1}{2}}_{c}, \Omega^{\frac{1}{2}}_{cc}$

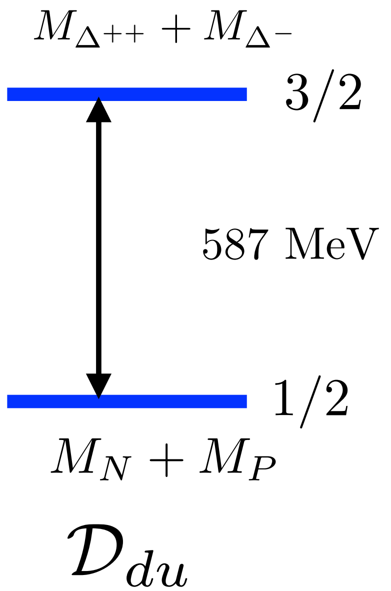
Non-interacting levels

- Consider deuteron energy spectrum



- Two levels spin 1/2 and 3/2

- Quark mass dependence of levels ?



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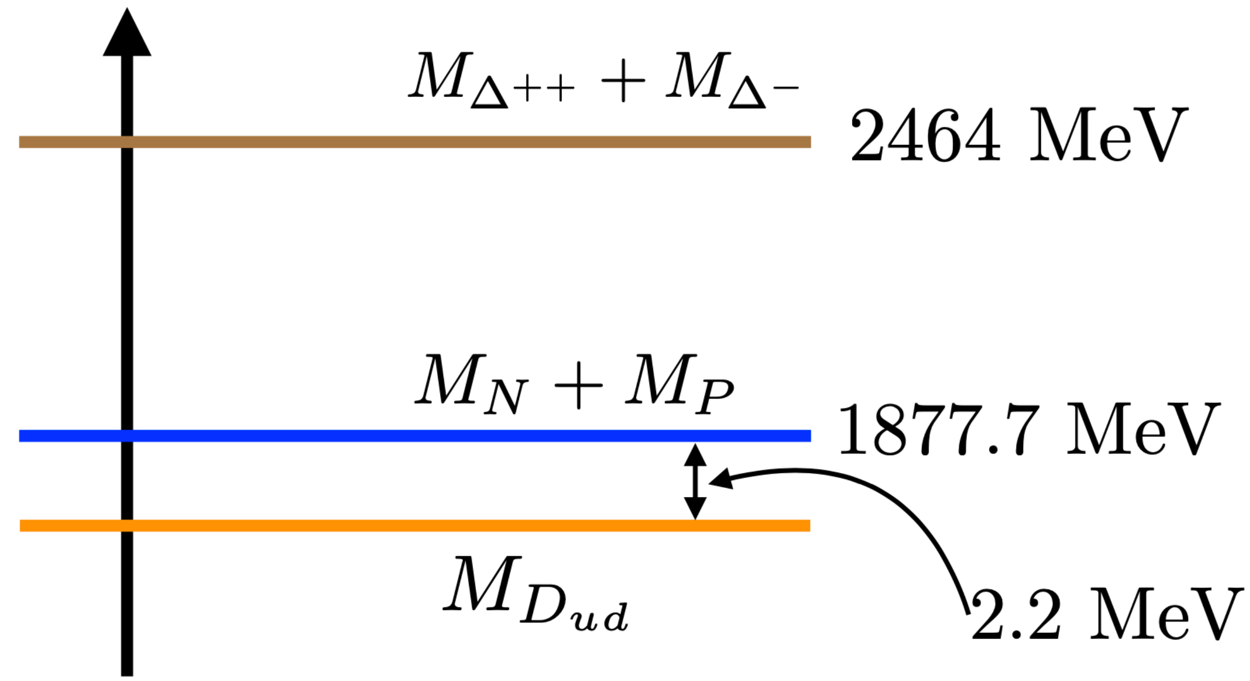

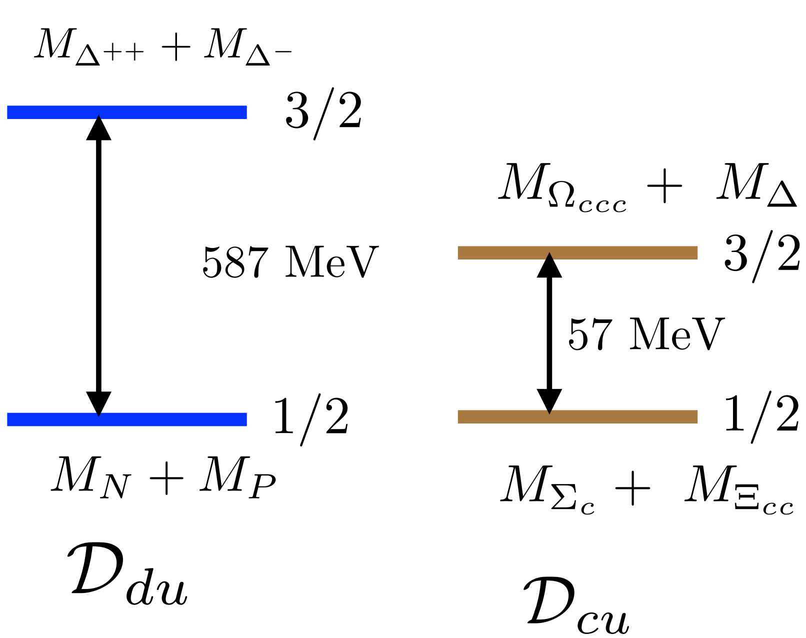

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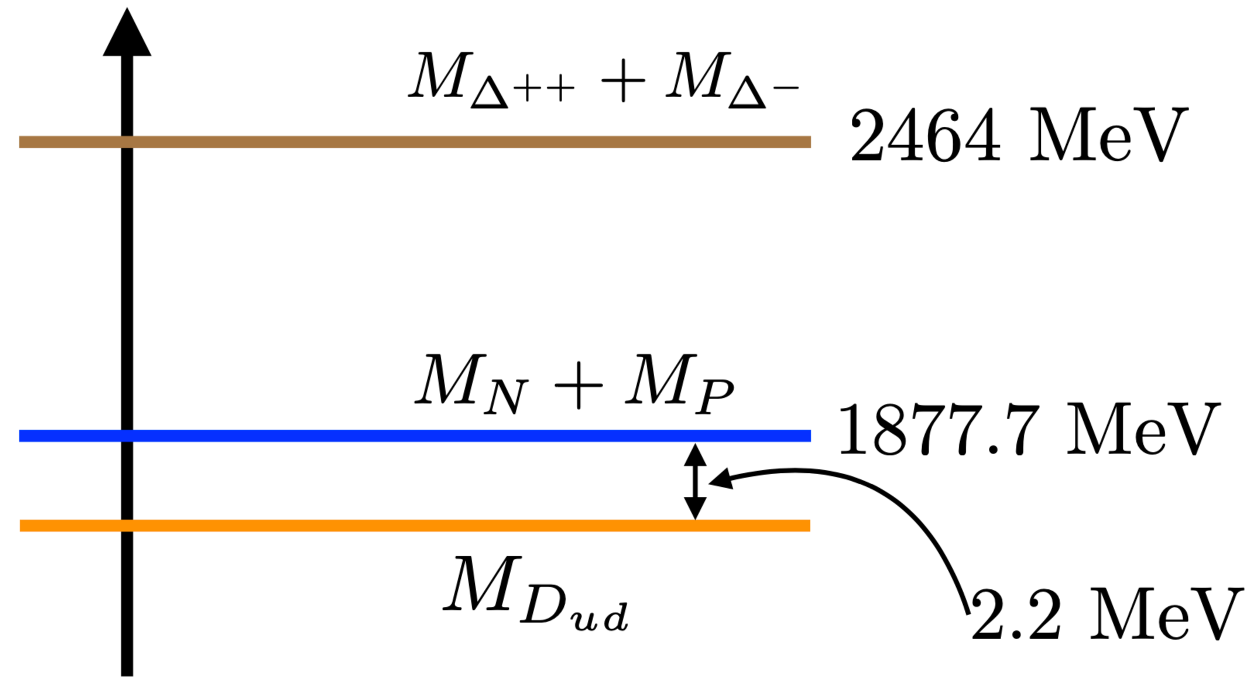

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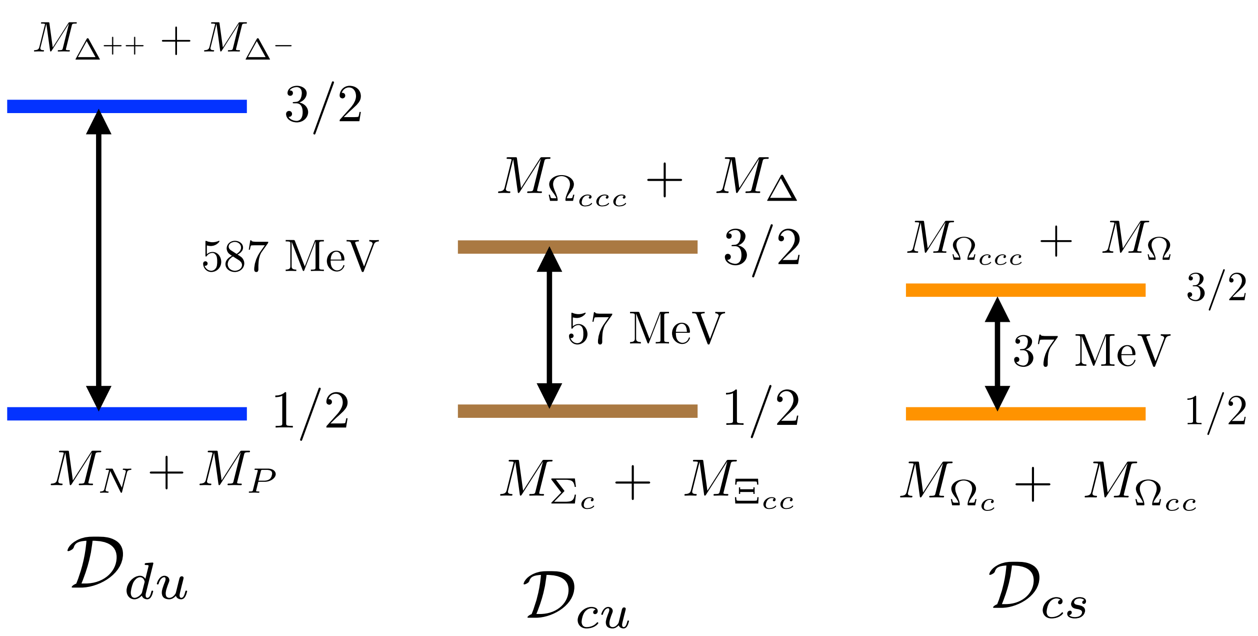

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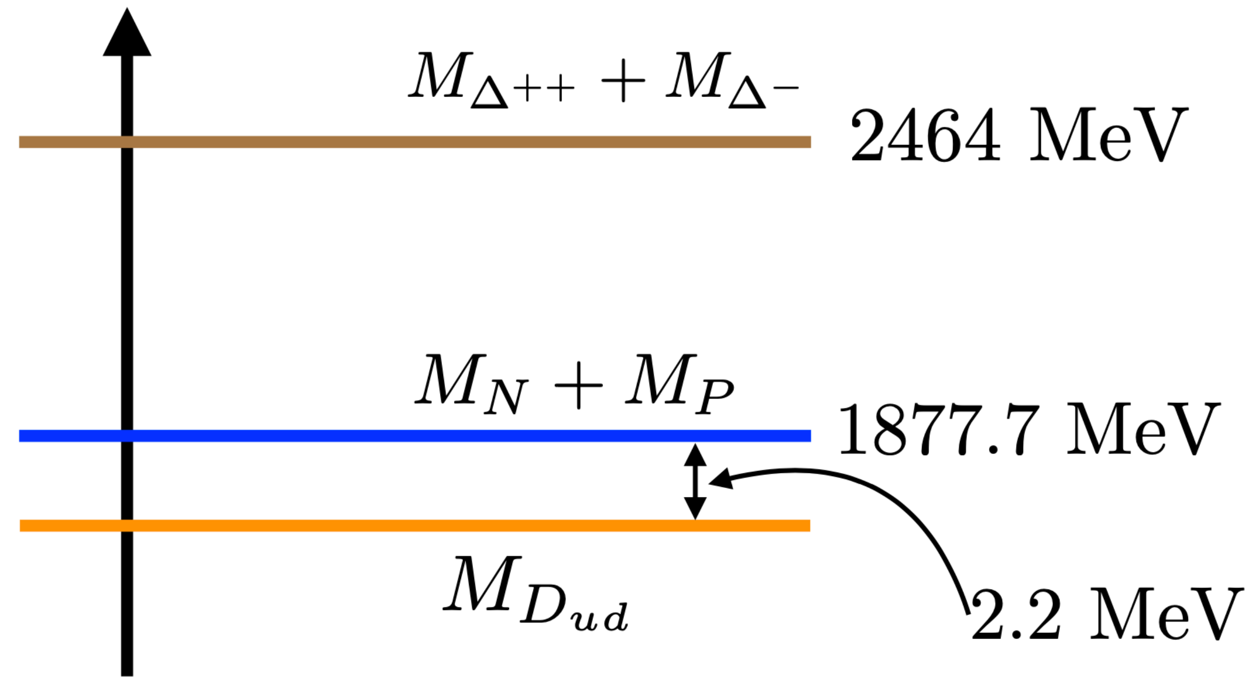

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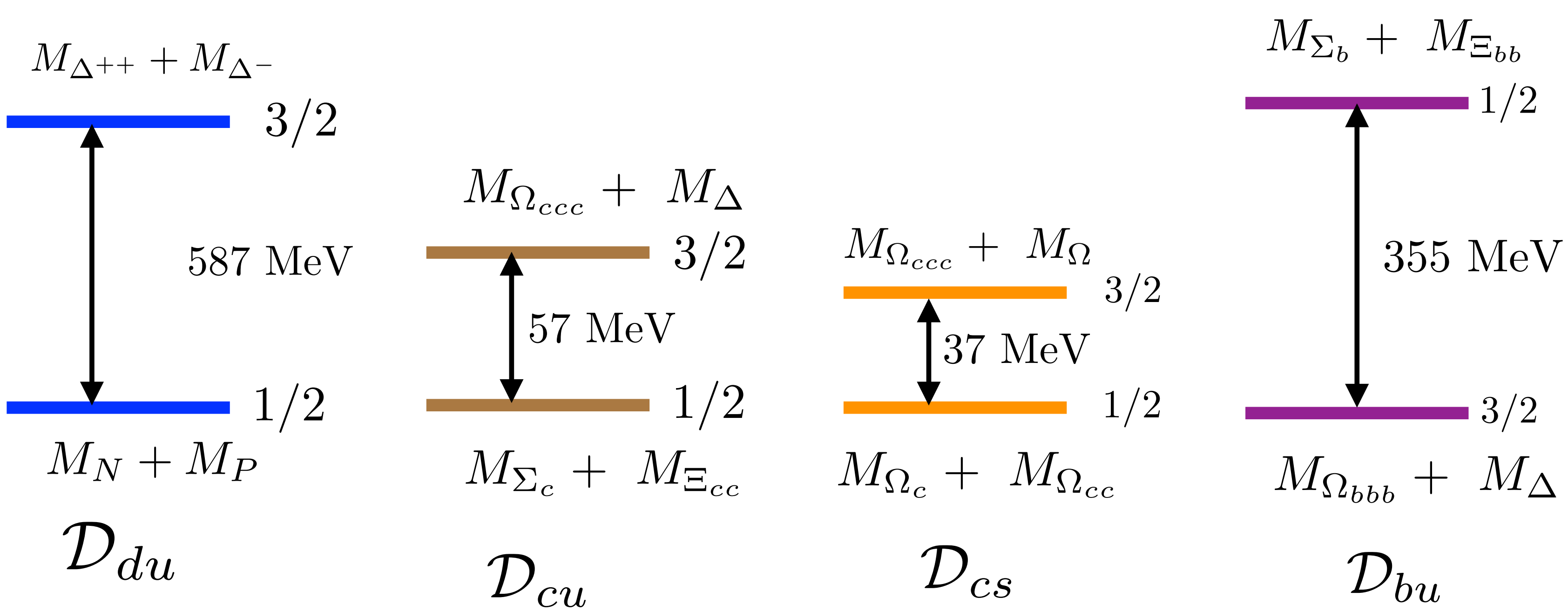

Non-interacting levels

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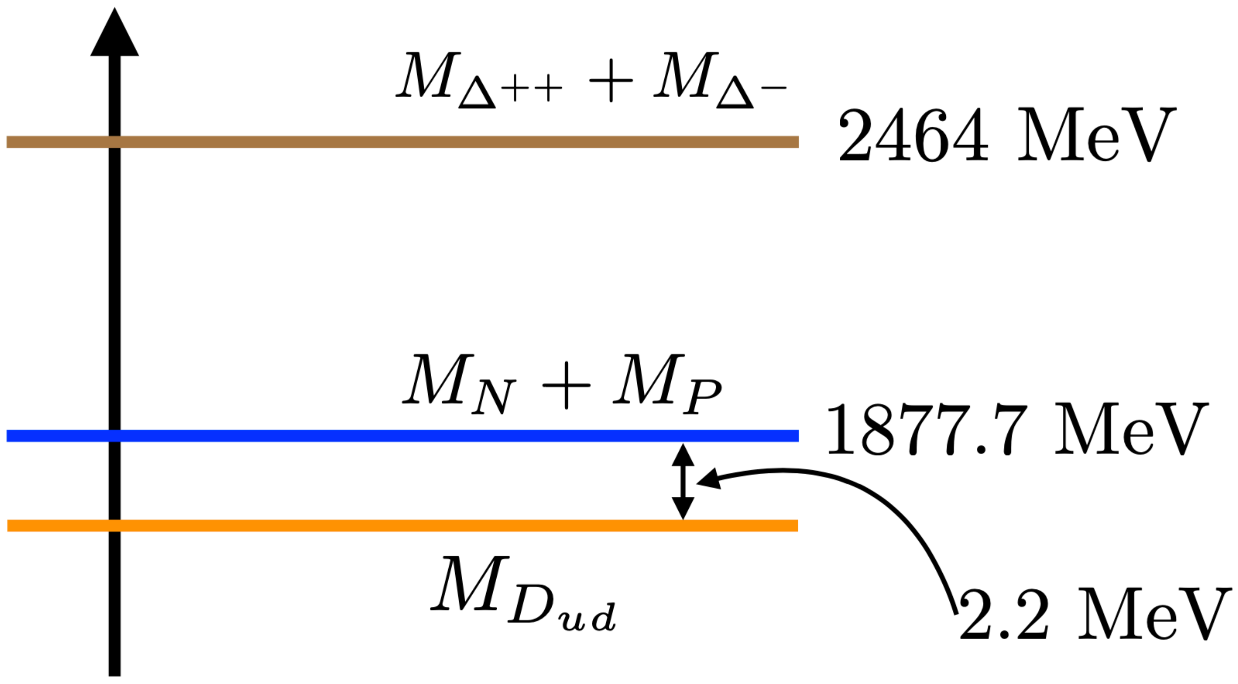

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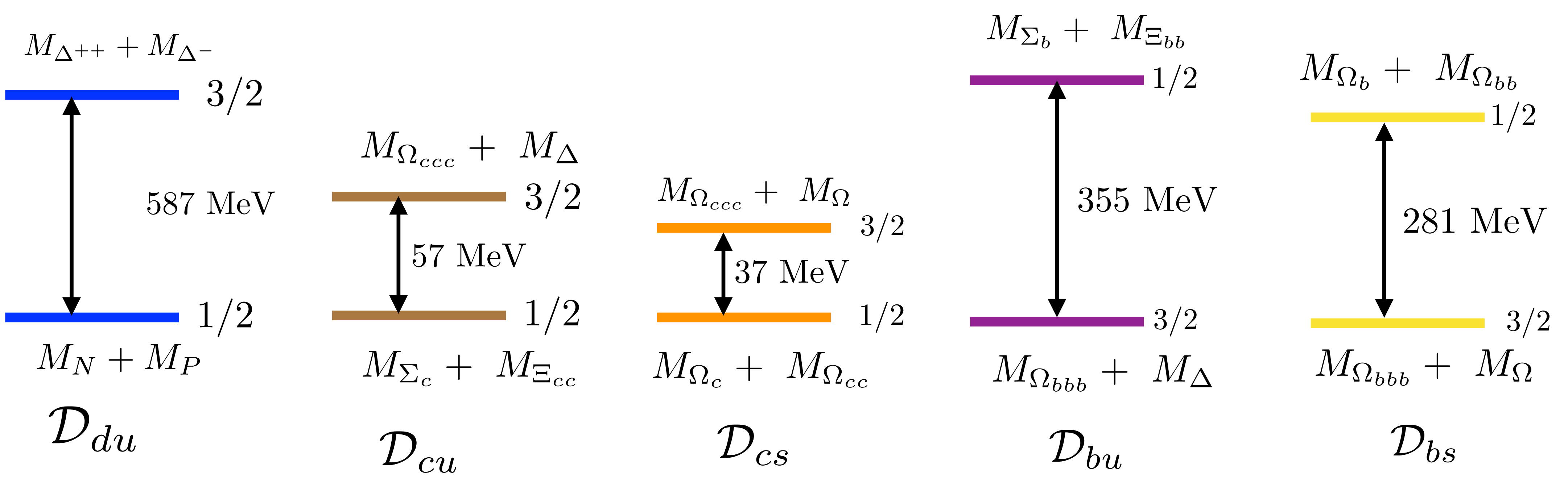

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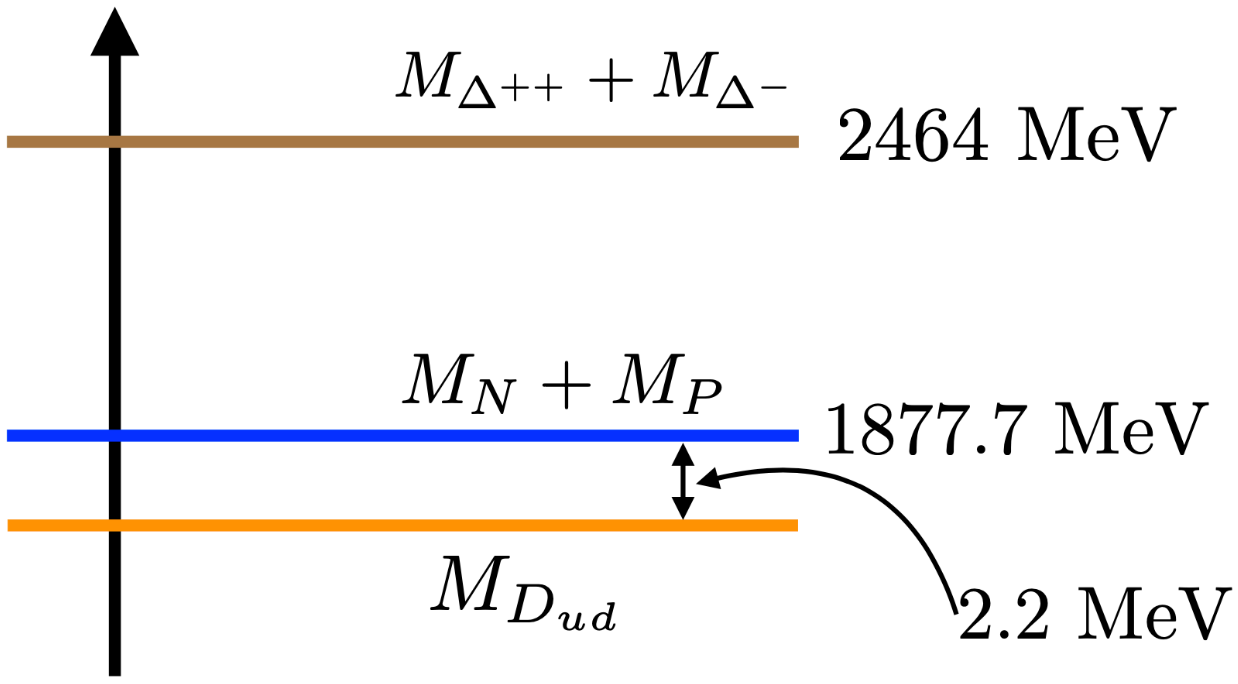

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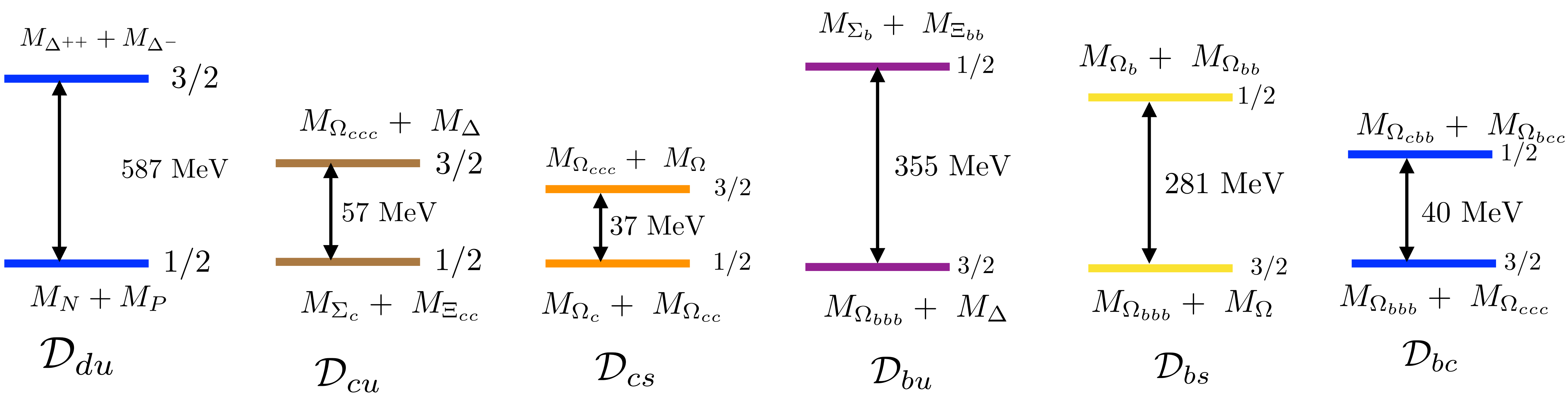


Lattice calculation

- We perform calculations at 3 lattice spacings

$L^3 \times T$	$m_\pi^{\rm sea}$ (MeV)	m_πL	a (fm)
$24^3 \times 64$	305.3	4.54	0.1207(11)
$32^3 \times 96$	312.7	4.50	0.0888(8)
$48^3 \times 144$	319.3	4.51	0.0582(5)

- Valence action DWF on MILC ensembles, NRQCD action for bottom

- Finest lattice spacing pion masses range from 0.3 - 1 GeV

- Compute dibaryon energy levels and compare them with NI levels

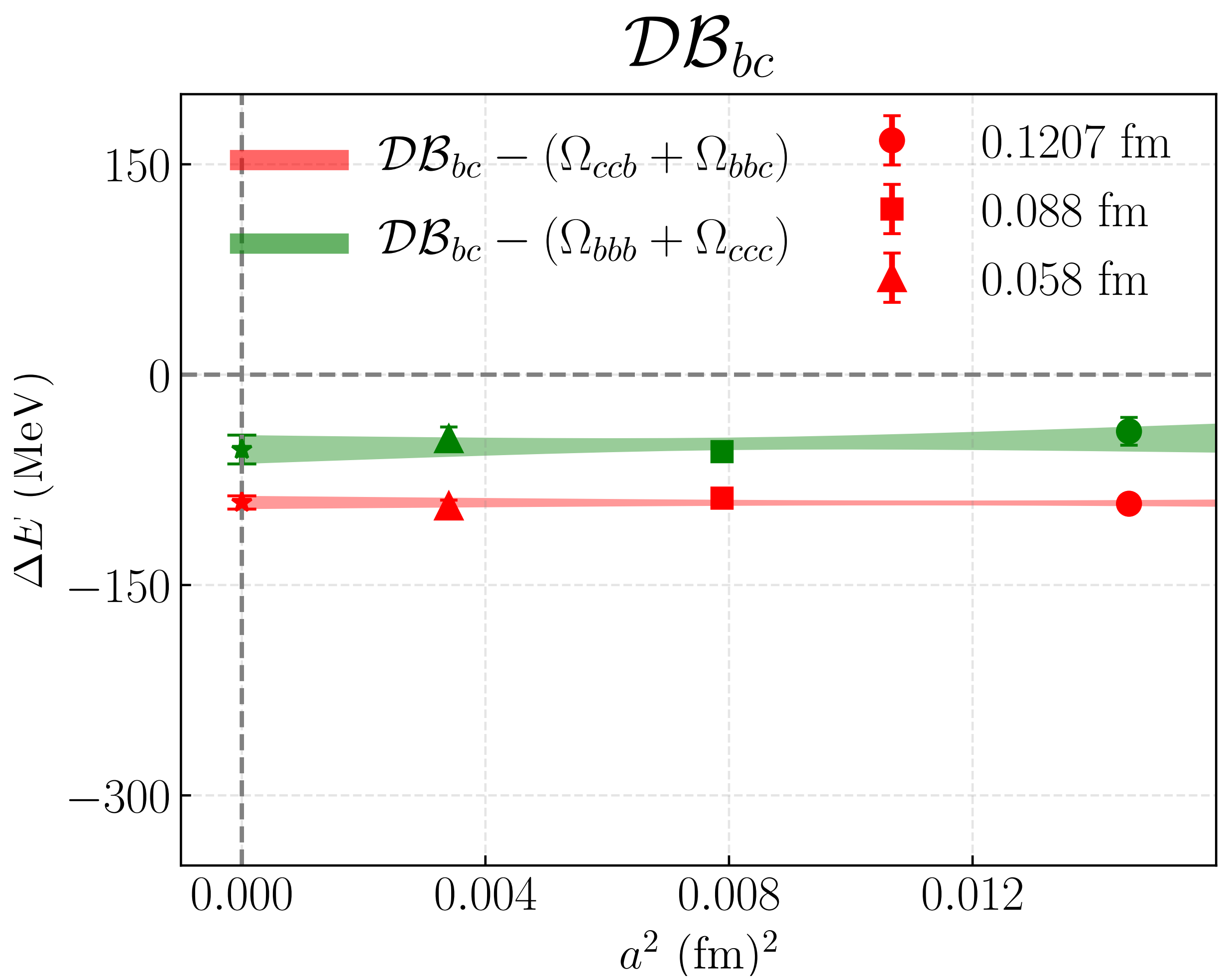
Results in spin-1 sector

- Fitted effective masses subtracted from NI level
- Two relevant NI levels

  Spin - 1/2 - red

  Spin - 3/2 - green

- Clear Indication of level below NI- Level
- No discretization effects



Phys. Rev. Lett. 123, 162003 (2019)

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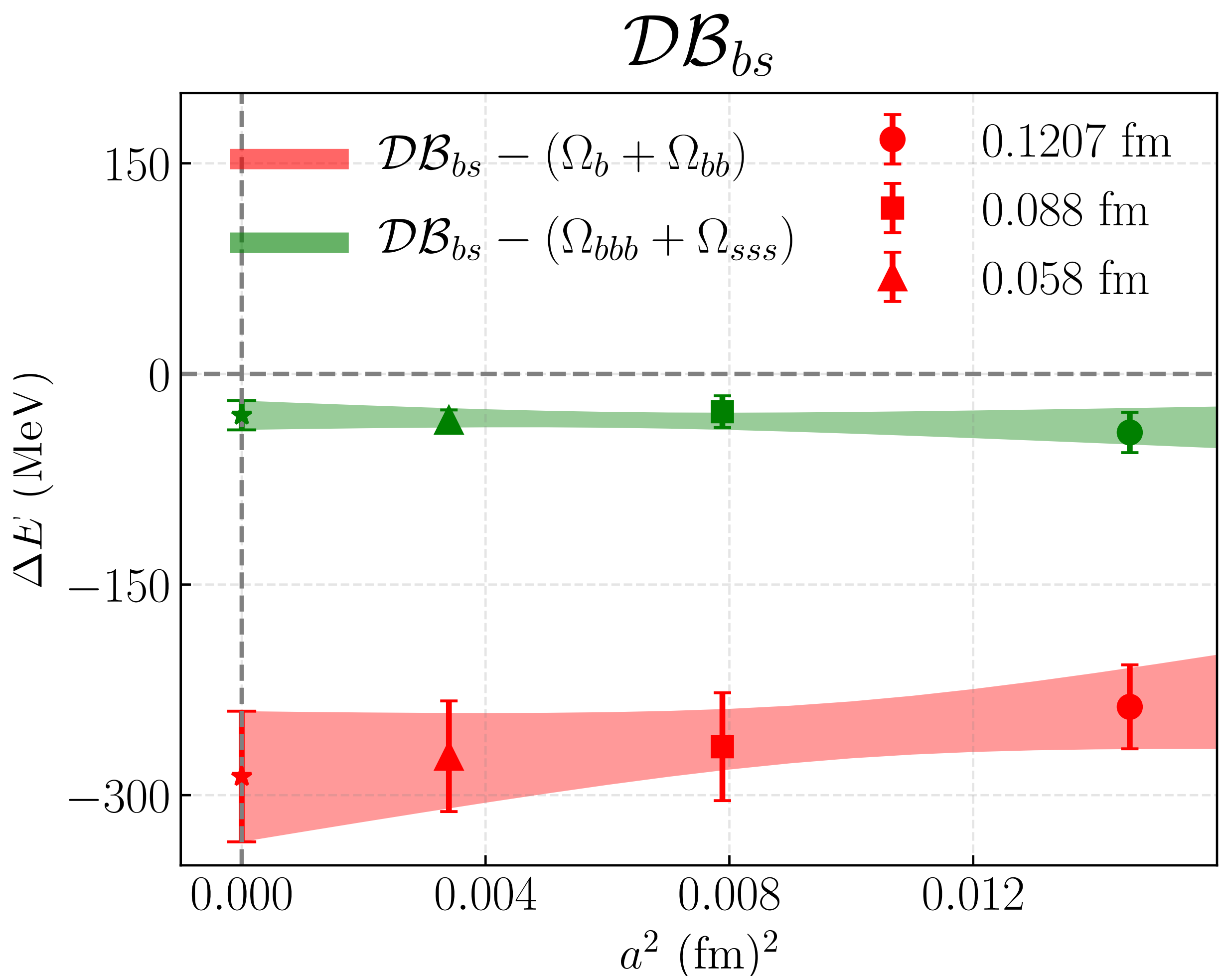

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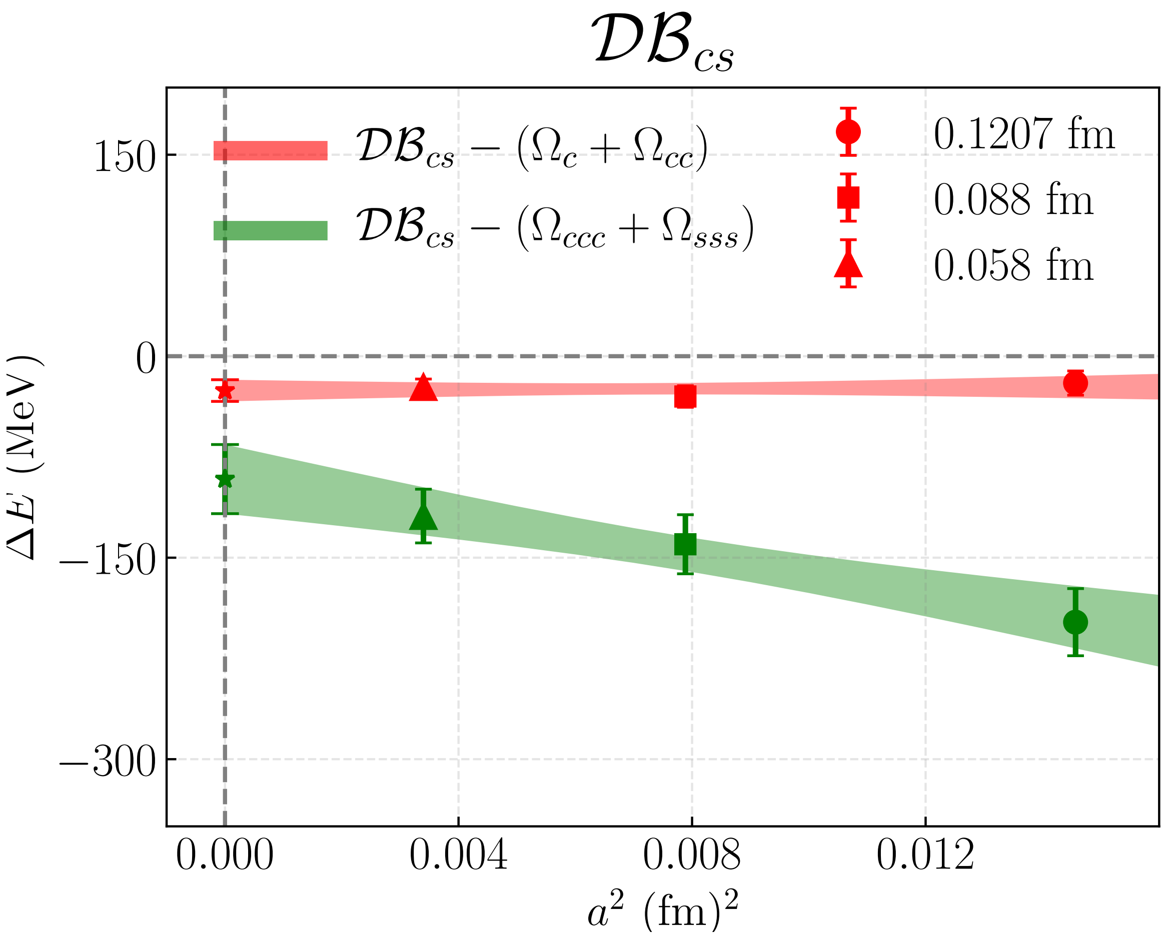
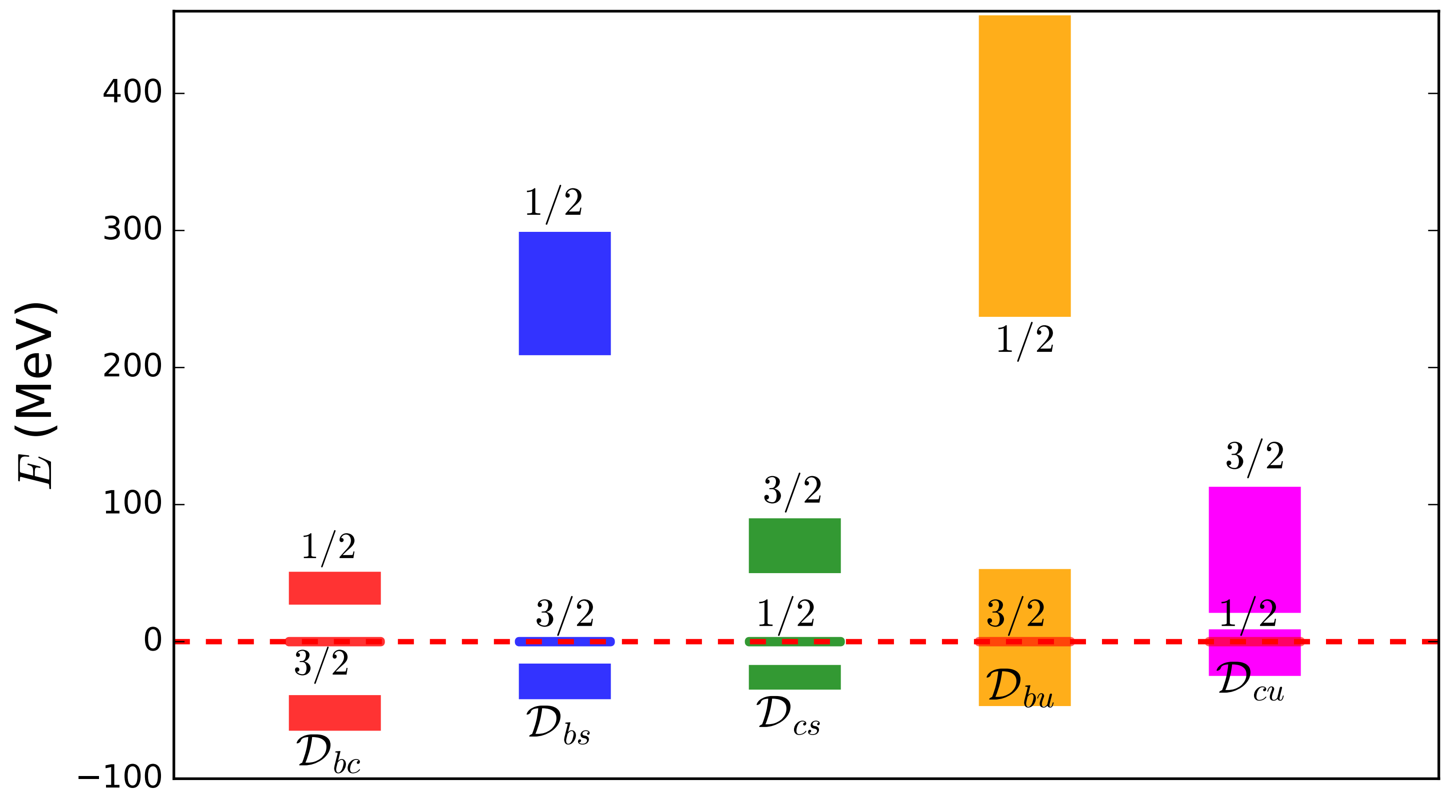
Results in spin-1 sector

- Mild discretization for spin - 1/2 charm



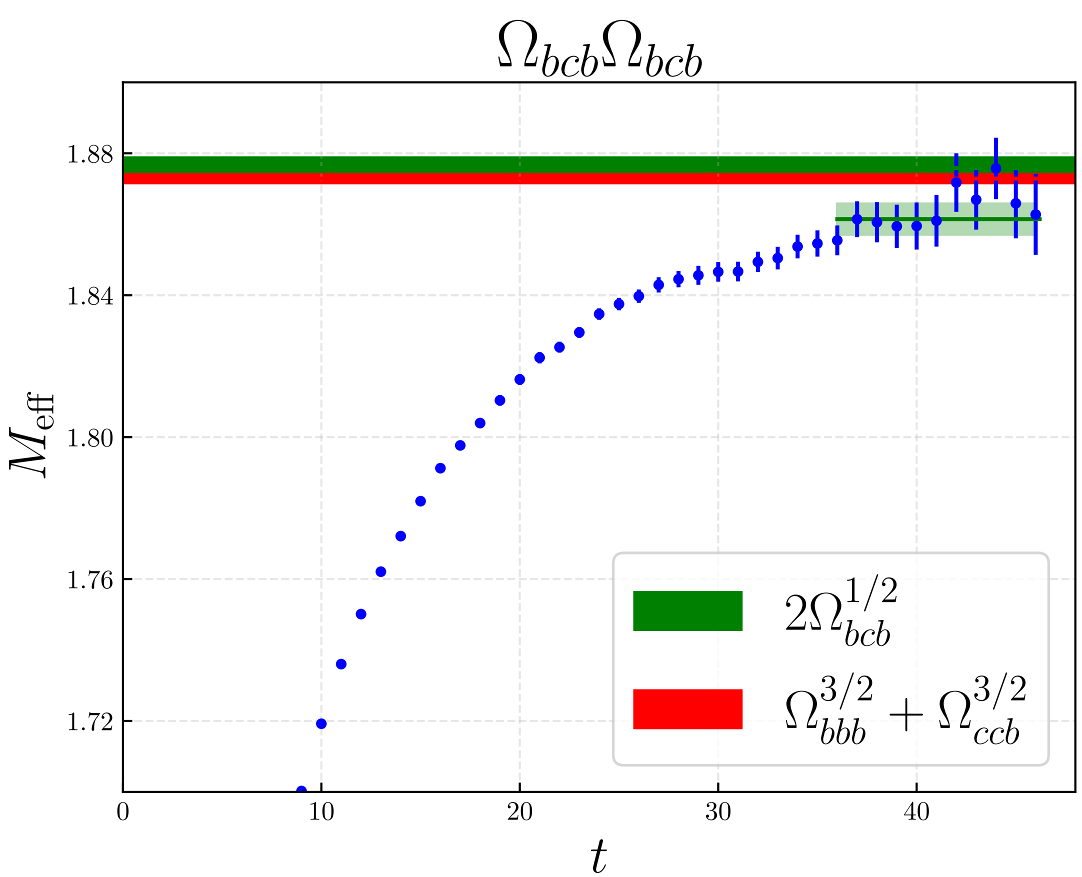


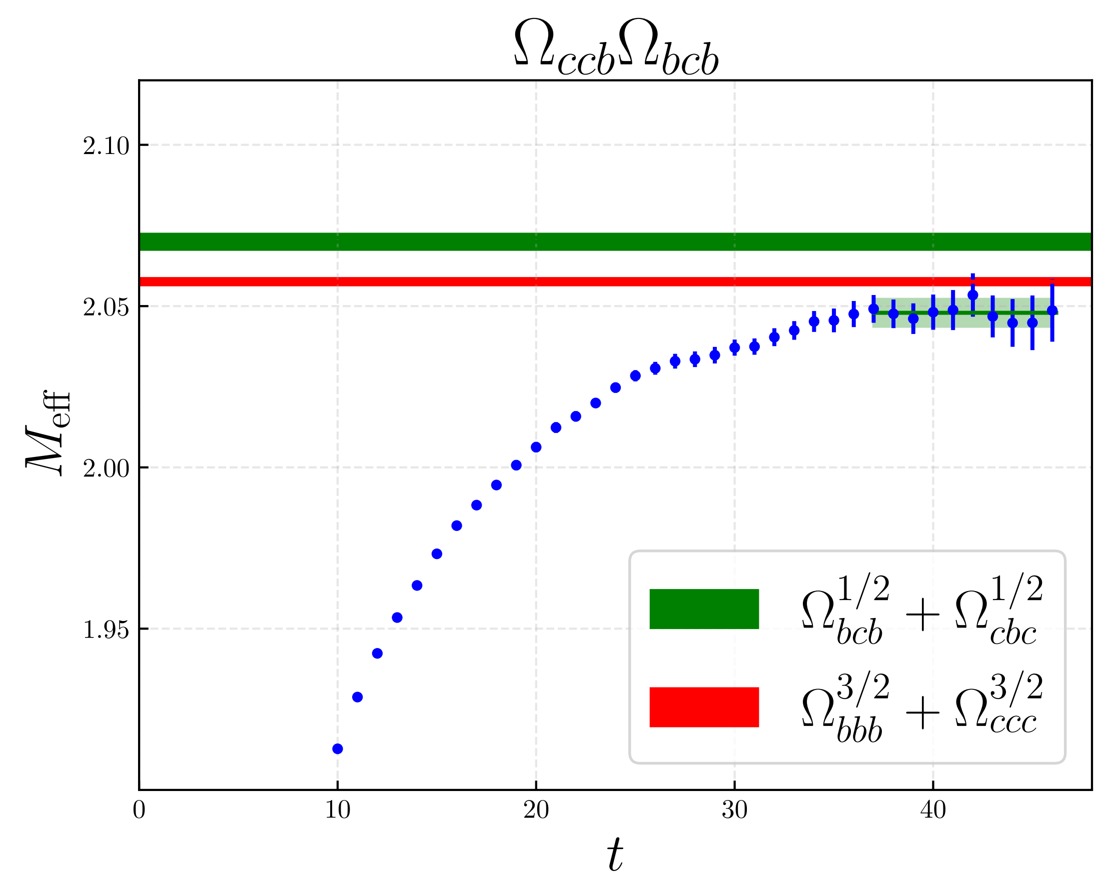
- No conclusive results for $\mathcal{D}_{bu},\ \mathcal{D}_{cu}$

Results in Singlet sector

- Flavor symmetric states in bottom-charm sector

Spin-1 cousin state



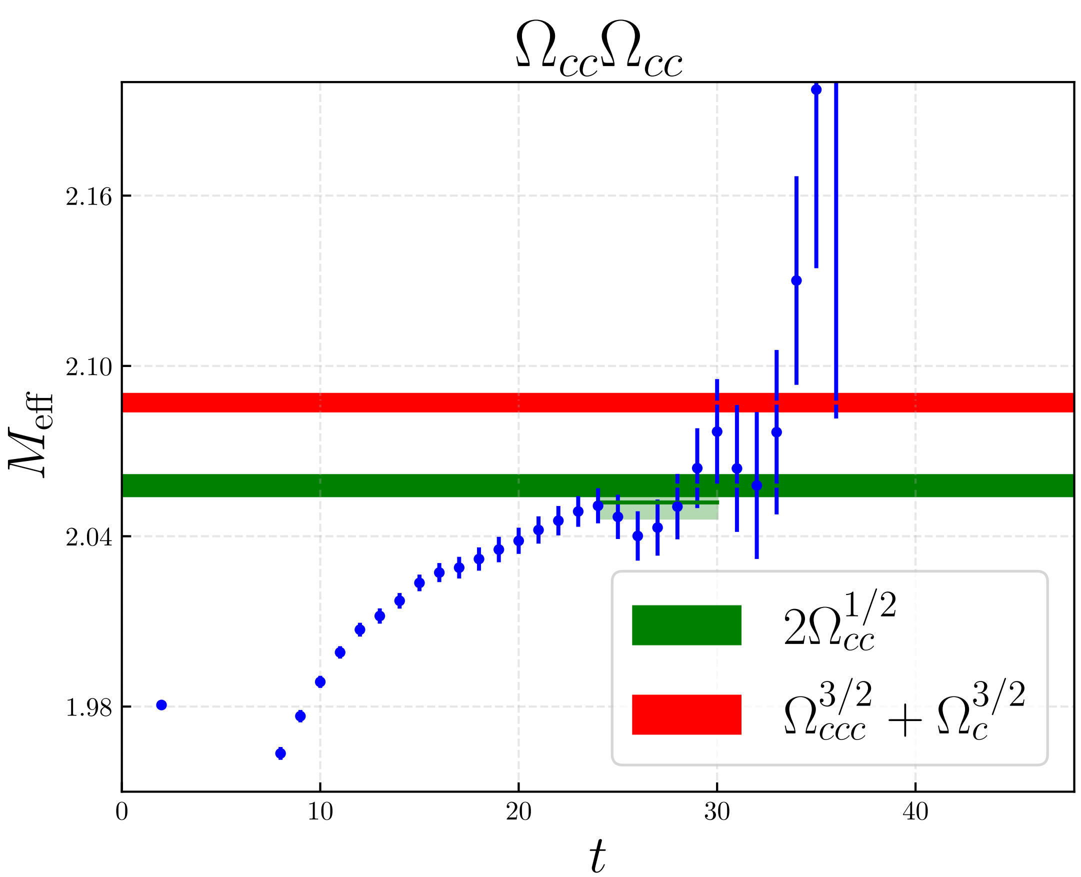


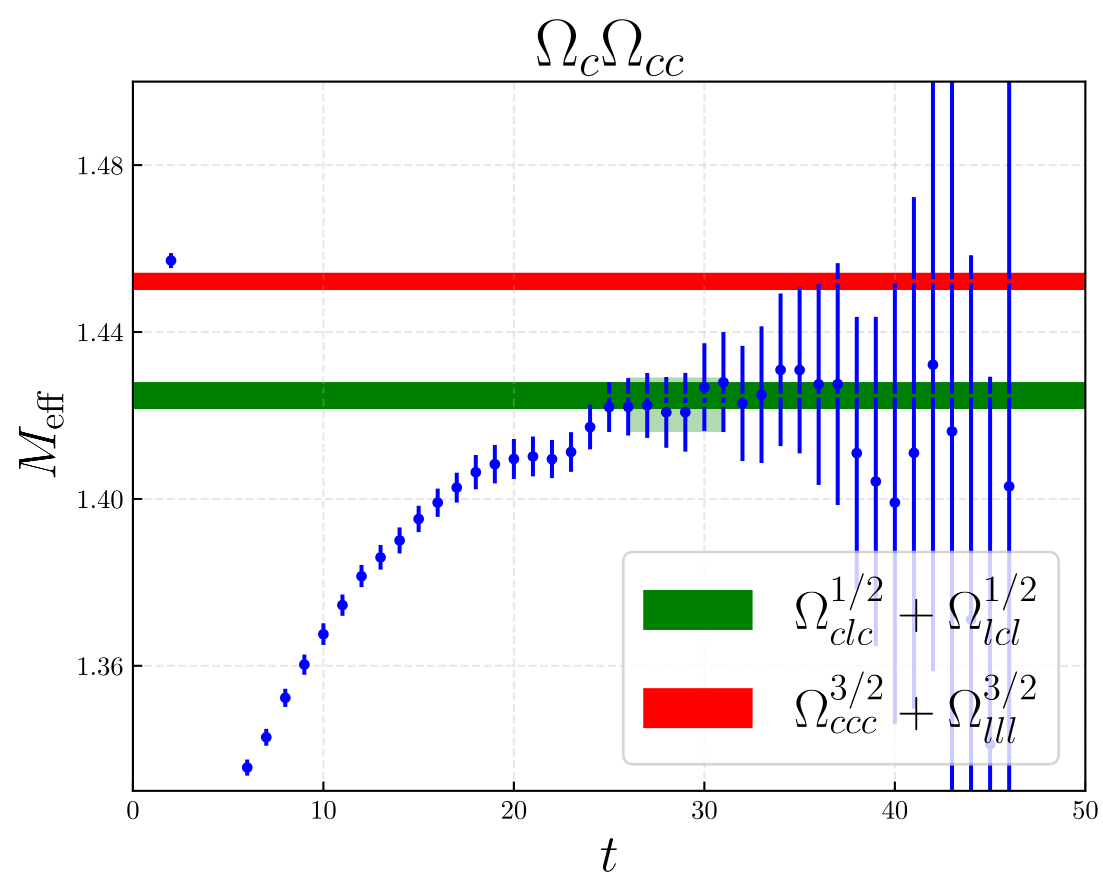
- Clear indication of level below NI-level

arXiv:2410.08519

Results in Singlet sector

- Flavor symmetric states in charm-strange sector





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Results in Singlet sector

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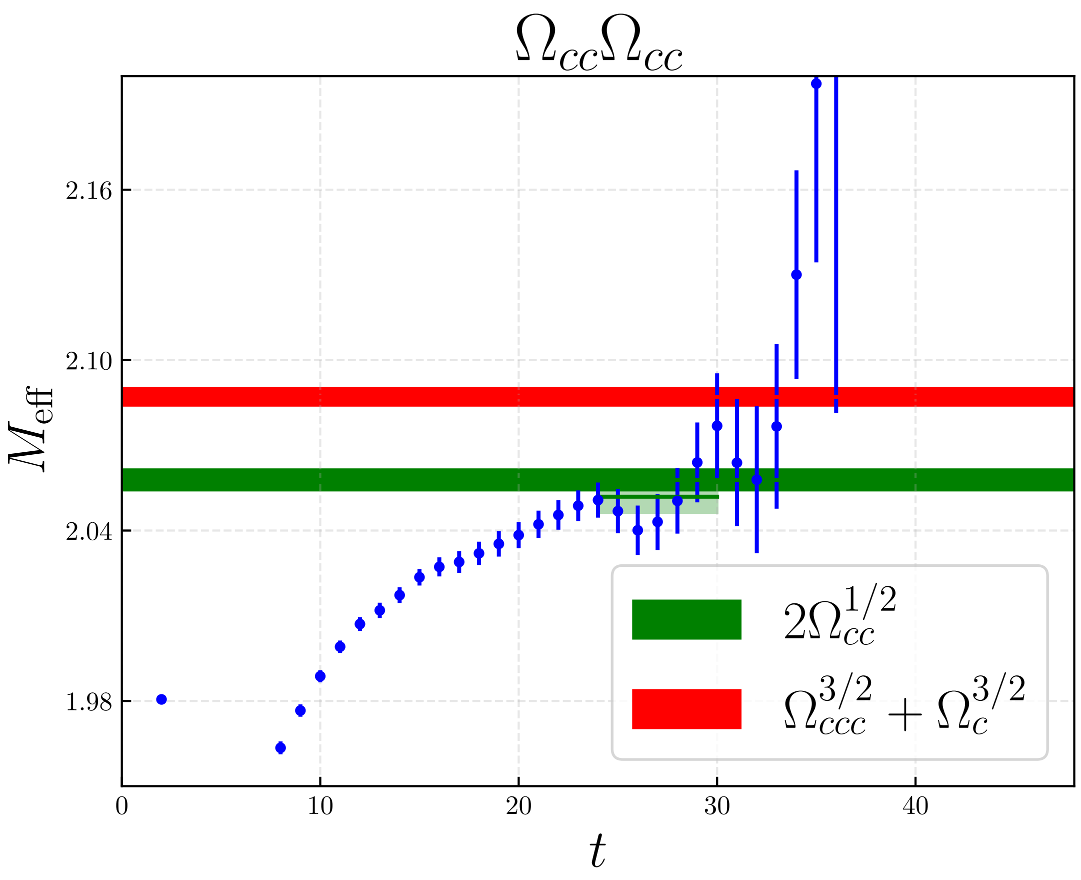
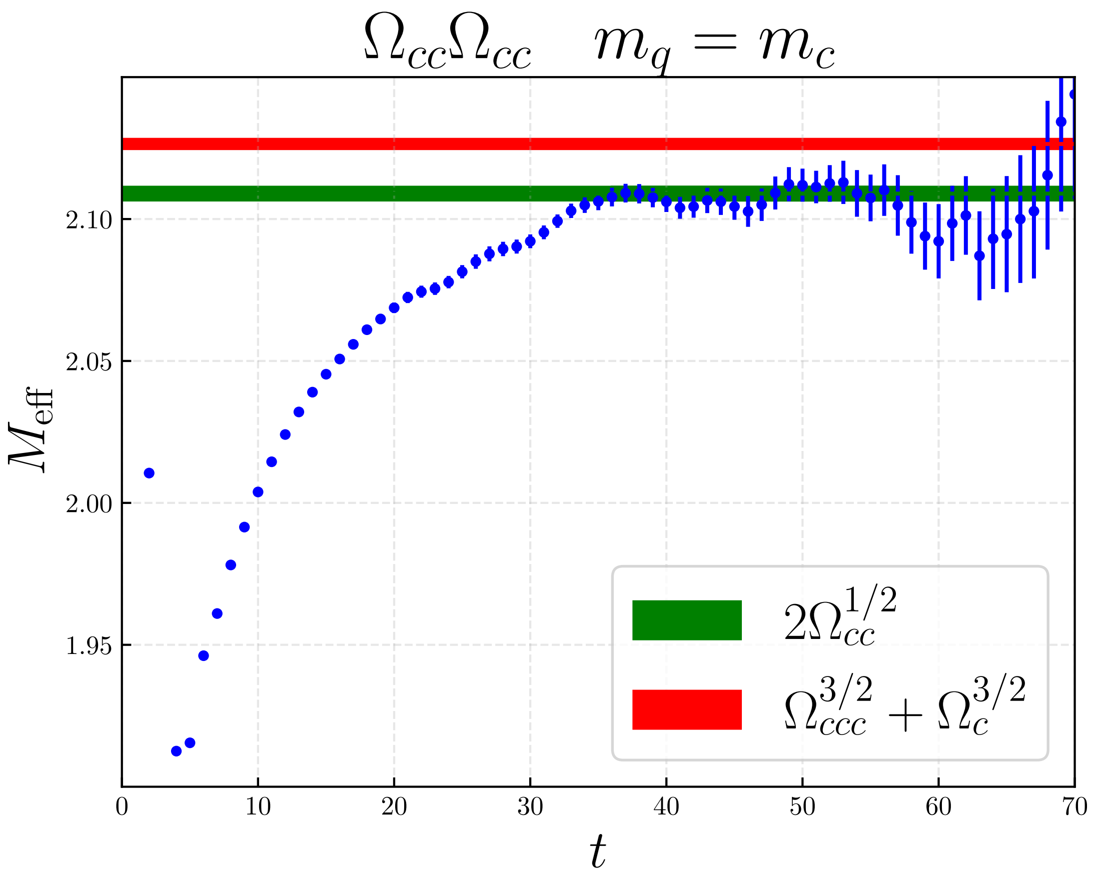
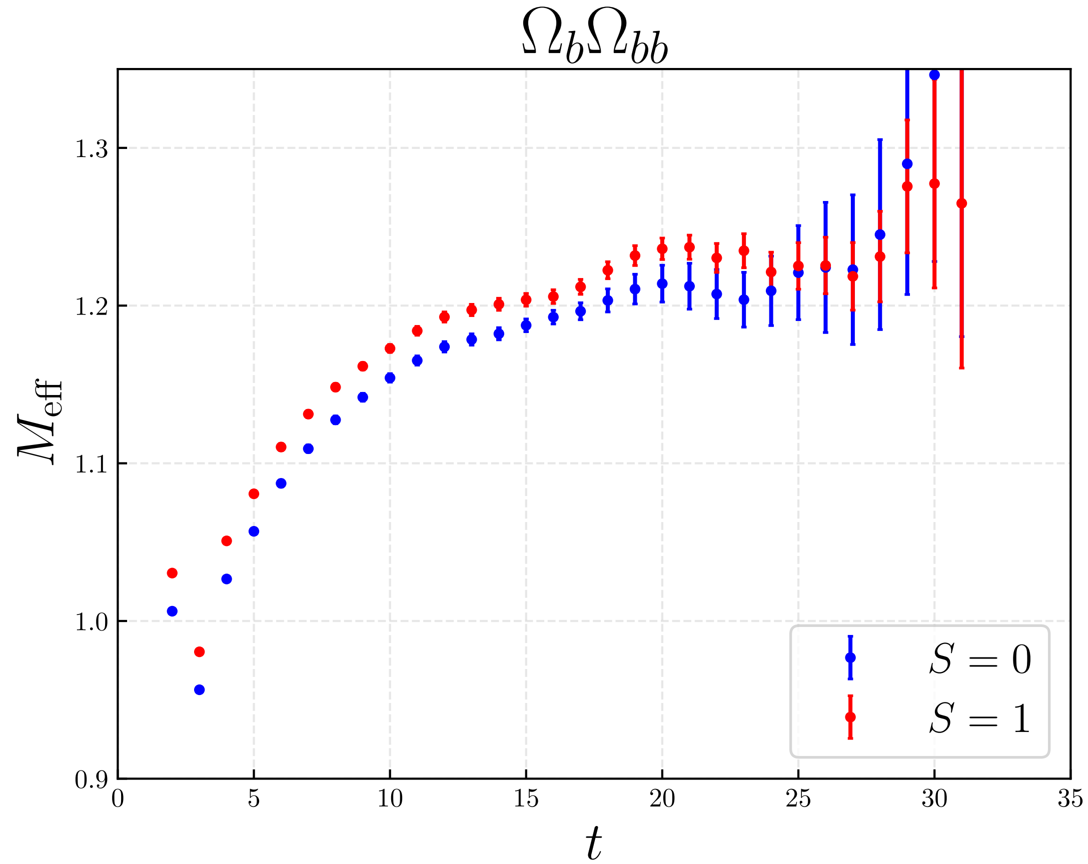




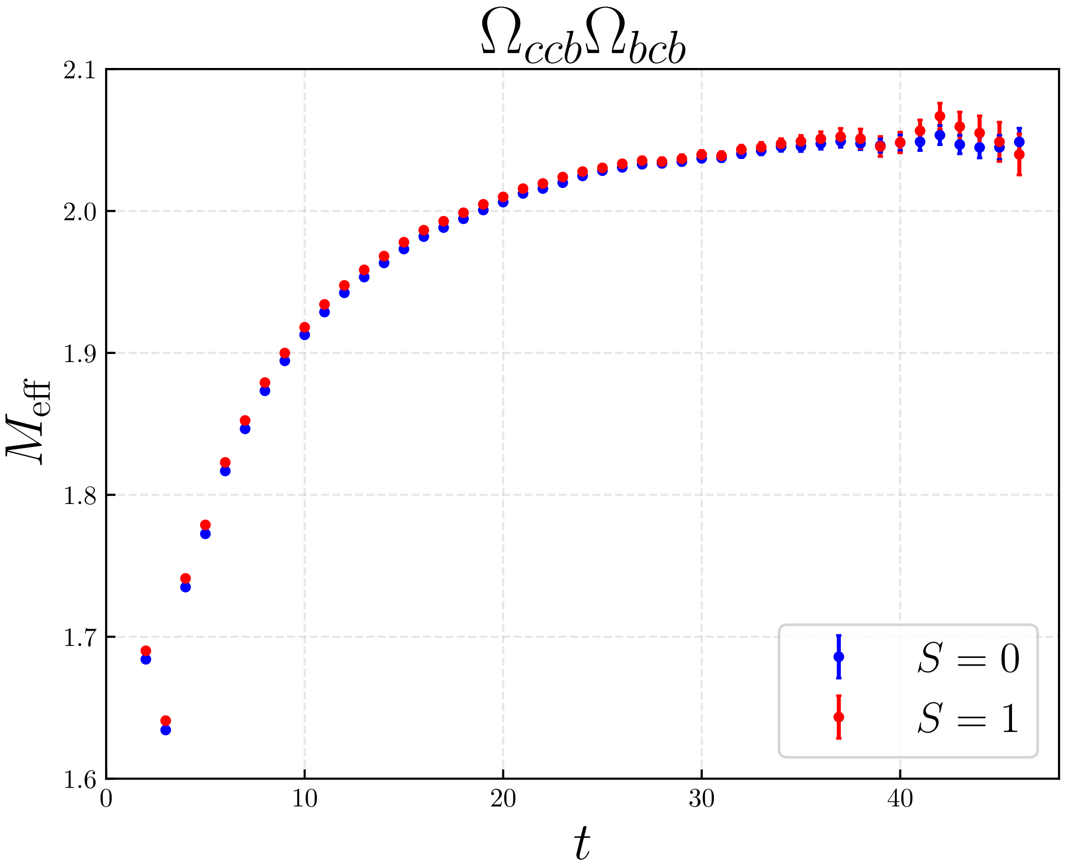
- No Clear indication of level below NI-level

Heavy quark spin symmetry

- Compare of correlation functions of : $S = 0, \quad S = 1$
- Consistent pattern $S = 1 > S = 0$



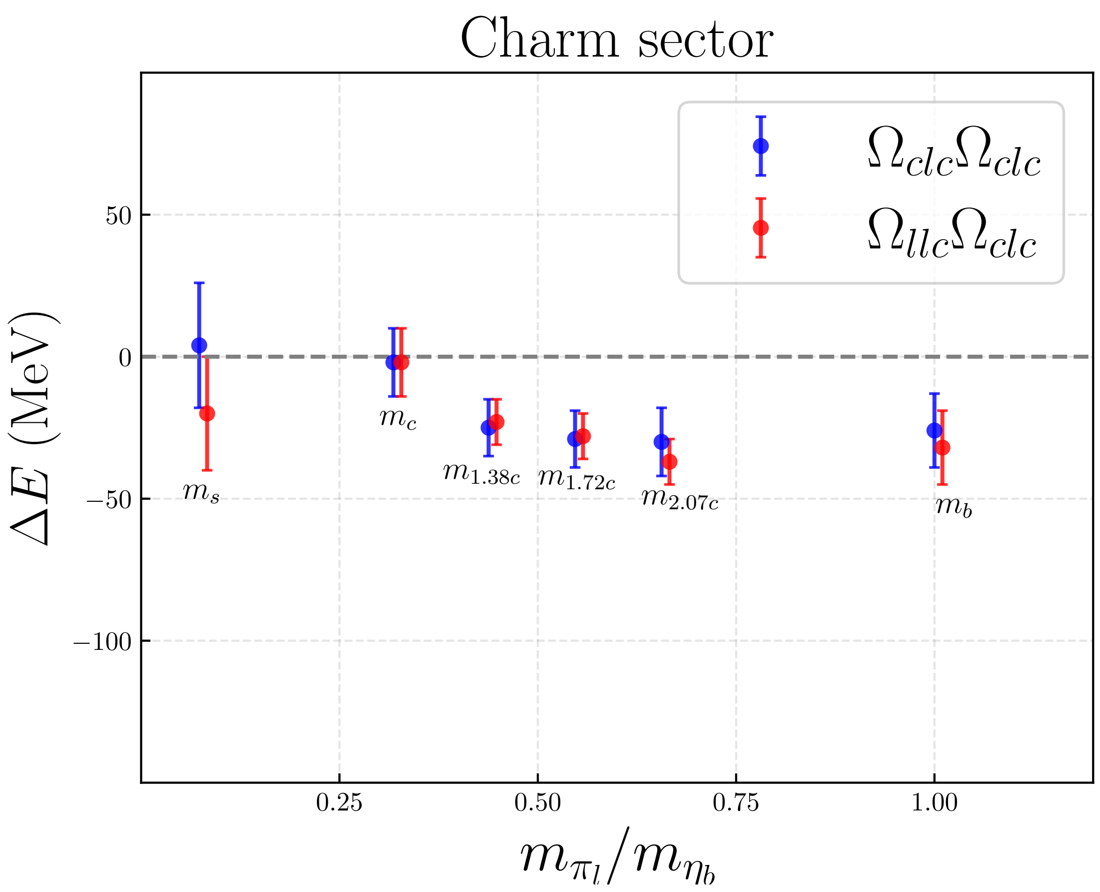
$$M^{S=1}_{\Omega_{bb}\Omega_b} - M^{S=0}_{\Omega_{bb}\Omega_b} = 70(30)\ \text{MeV}.$$

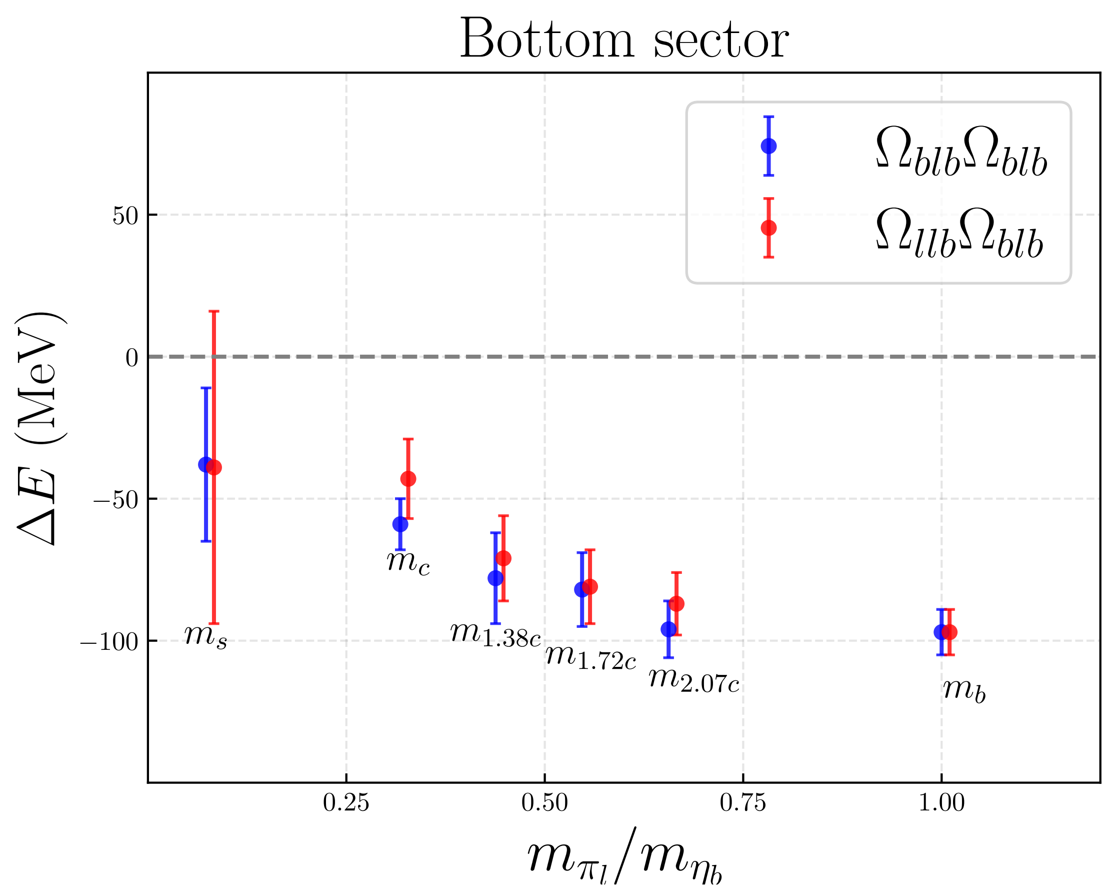


$$M^{S=1}_{\Omega_{ccb}\Omega_{bcb}} - M^{S=0}_{\Omega_{ccb}\Omega_{bcb}} = 2(6)\ \text{MeV}.$$

New phenomenology ?

- Binding energy at unphysically heavy charm quarks





- Deepening of binding energy as bottom quark becomes heavier

- Indicative of new mechanism of binding for heavy quarks ?

Outlook

- Bound states in spin-1 and spin-0 states of charm-bottom

$$\Delta\mathcal{D}_{bc} = -52(13)\ \mathrm{MeV} \qquad \Delta\mathcal{D}_{bs} = -29(13)\ \mathrm{MeV}$$

$$\Delta E_{\Omega_{bcb}\Omega_{bcb}} = -66(11)\ \mathrm{MeV} \qquad \Delta E_{\Omega_{ccb}\Omega_{bcb}} = -48(13)\ \mathrm{MeV}$$

- No clear indication in bottom-strange and charm-strange

- Trend of deepening of binding energy with heavy quark mass

- Perhaps evidence of new phenomenology ... ?

Continuous temperature sampling in Lattice QCD

Phys. Rev. D 104, 014502 (2021)

Motivation

- Consider a lattice calculation of $\langle \mathcal{O}_\beta \rangle$

$$\langle \mathcal{O}_\beta \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U e^{-\beta S[U]} \mathcal{O}[U]$$

Single coupling corresponds to a single temperature

- We are interested in the temperature and cutoff dependence

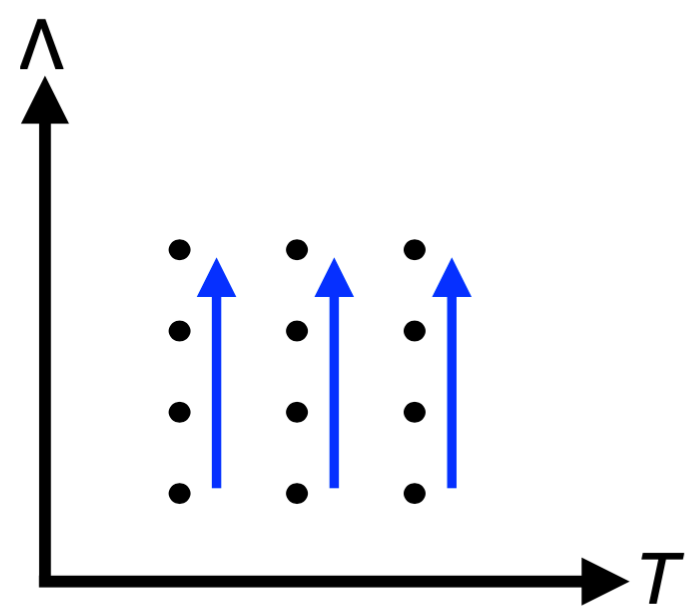
Motivation

- Consider a lattice calculation of $\langle \mathcal{O}_\beta \rangle$

$$\langle \mathcal{O}_\beta \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U e^{-\beta S[U]} \mathcal{O}[U]$$

Single coupling corresponds to a single temperature

Simulate at every temperature and cutoff



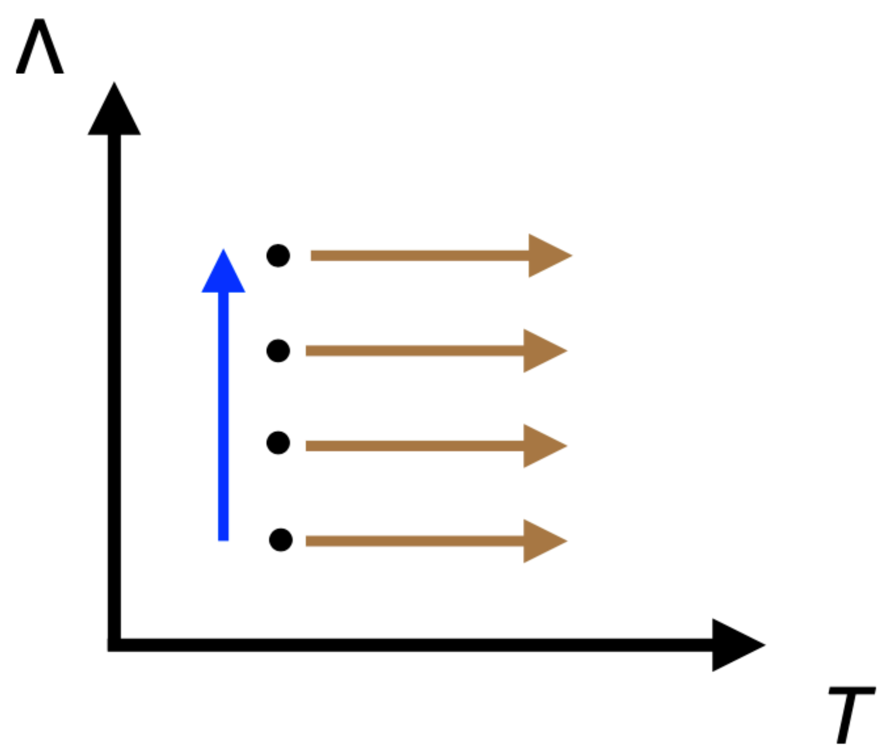
Could be expensive if the temperature range is really large

Perhaps there is a better way ?

Temperature reweighting

- A novel way …. Sample temperatures continuously

- Employ a reweighting in temperature at a fixed cutoff



Could be very cost effective

Temperature reweighting

- Consider a lattice calculation of $\langle \mathcal{O}_\beta \rangle$

$$\langle \mathcal{O}_\beta \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U e^{-\beta S[U]} \mathcal{O}[U] \qquad \mathcal{Z} = \int \mathcal{D}U e^{-\beta S[U]}$$

Temperature reweighting

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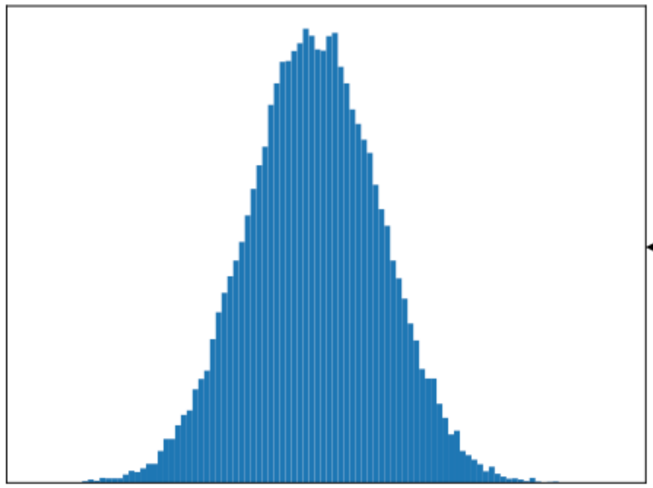
Single β simulates single temperature



Action histogram approx gaussian

Temperature reweighting

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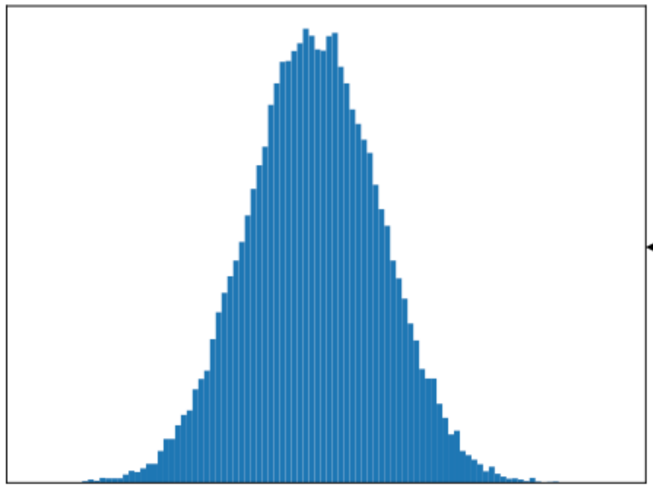
Single β simulates single temperature

To simulate continuous temperatures, simulate continuous β



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Temperature reweighting

- Consider a lattice calculation of $\langle \mathcal{O}_\beta \rangle$

$$\langle \mathcal{O}_\beta \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}U e^{-\beta S[U]} \mathcal{O}[U] \qquad \mathcal{Z} = \int \mathcal{D}U e^{-\beta S[U]}$$

- Replace the weight βS

$$\mathcal{Z} = \int \mathcal{D}U e^{-\beta S[U]}$$

$$\downarrow$$

$$\int \mathcal{D}U e^{-W(S)}$$

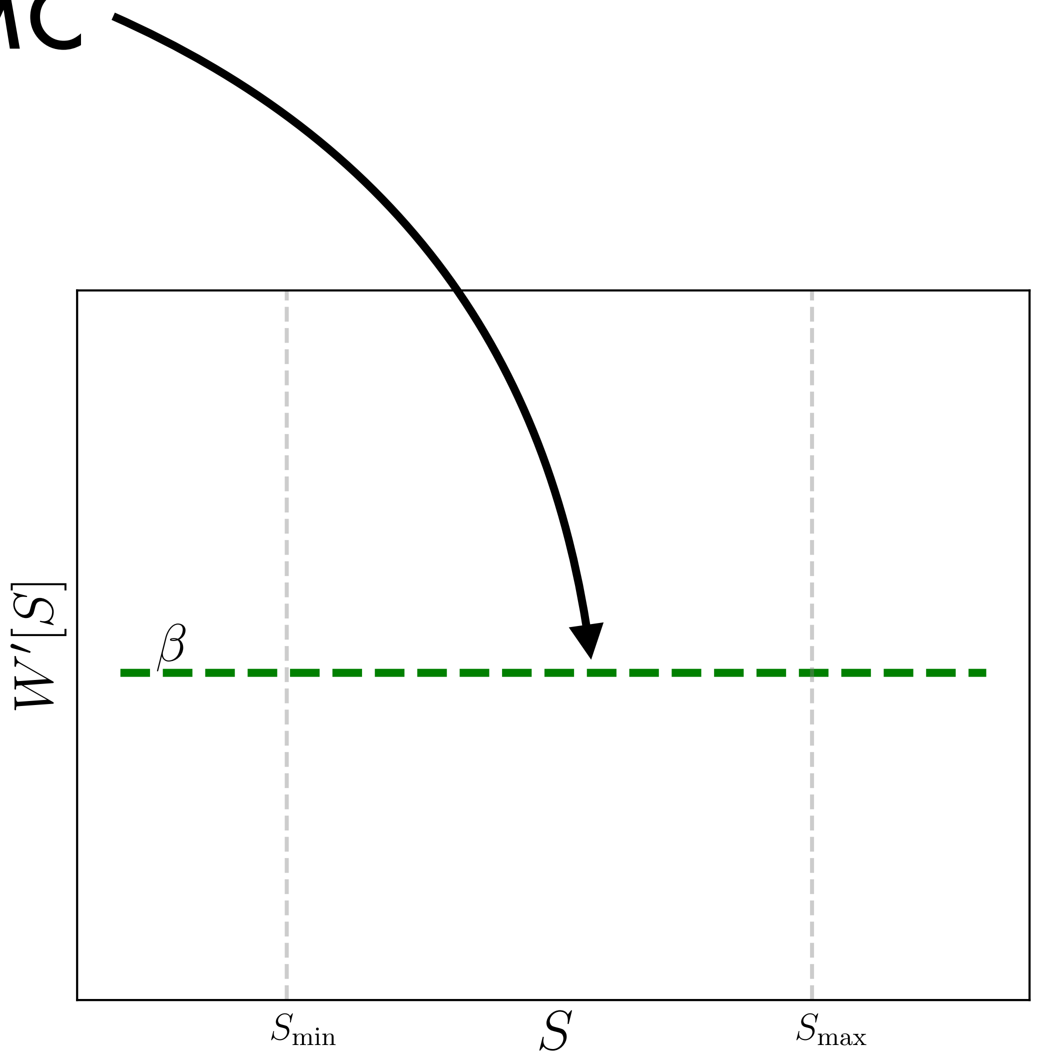
$$W(S) = \beta S$$

$$W'(S) = \frac{\mathrm{d}W(S)}{\mathrm{d}S} = \beta$$

Continuous temperatures → Continuous gauge couplings → Continuous $W'(S)$

Temperature reweighting

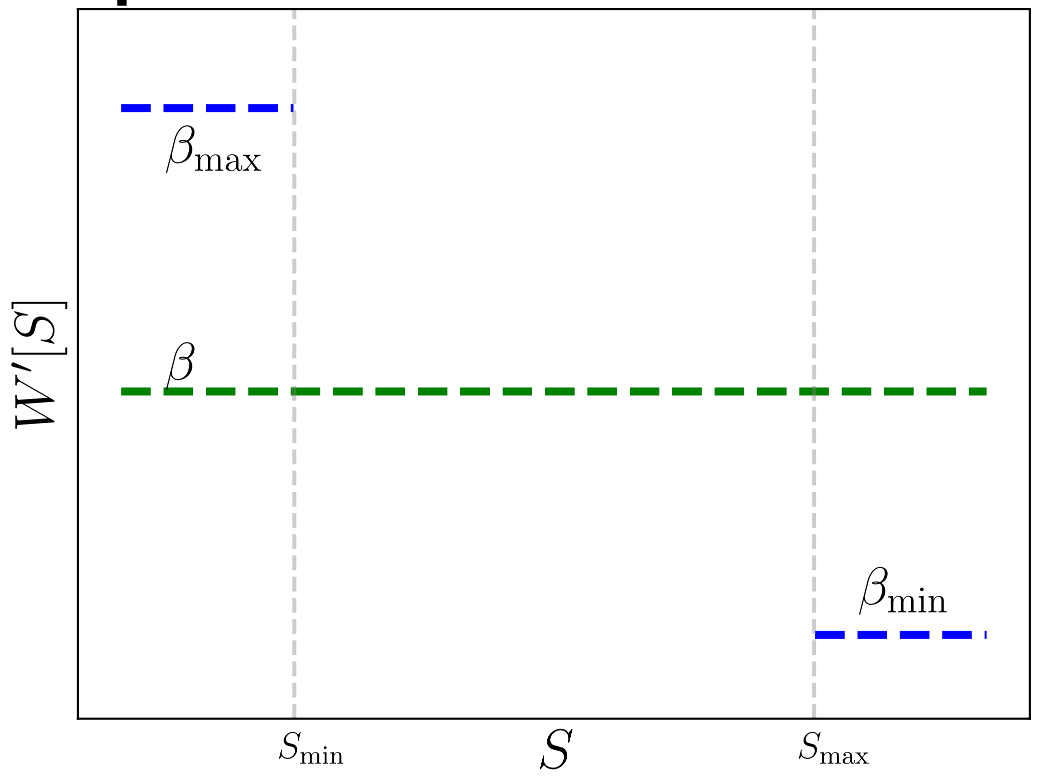
- Single temperature/coupling MC



Temperature reweighting

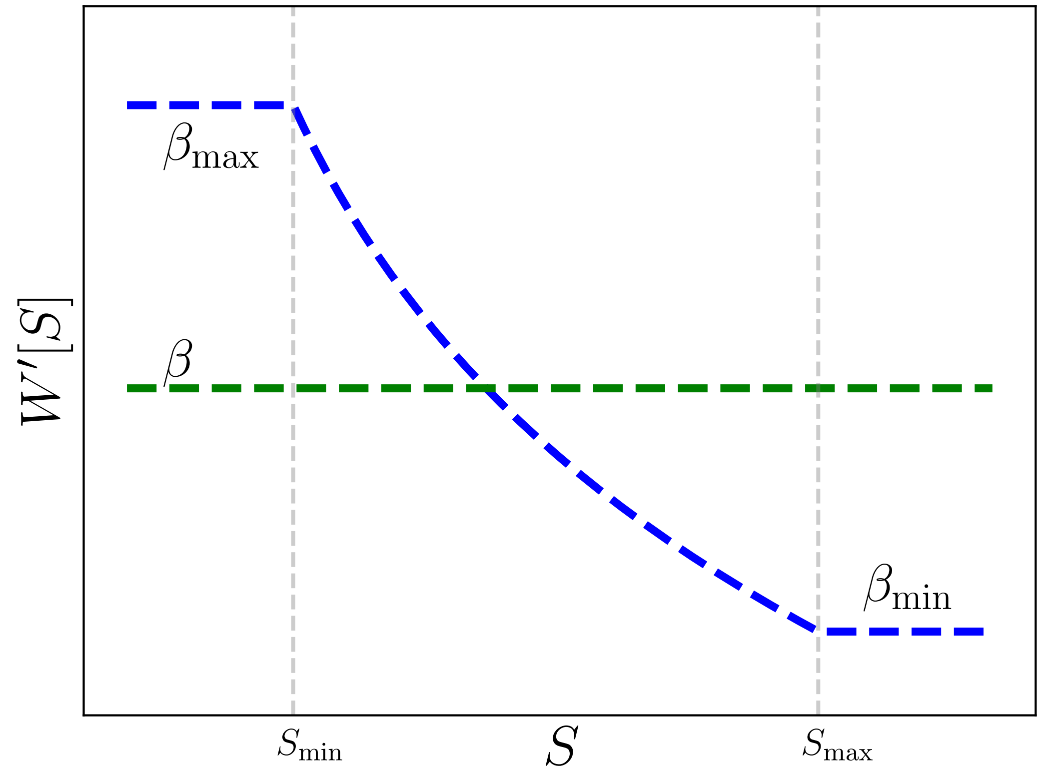
- Single temperature/coupling MC

- Define a range of continuous temperatures



Temperature reweighting

- Single temperature/coupling MC
- Continuous sampling of S, then samples continuous β



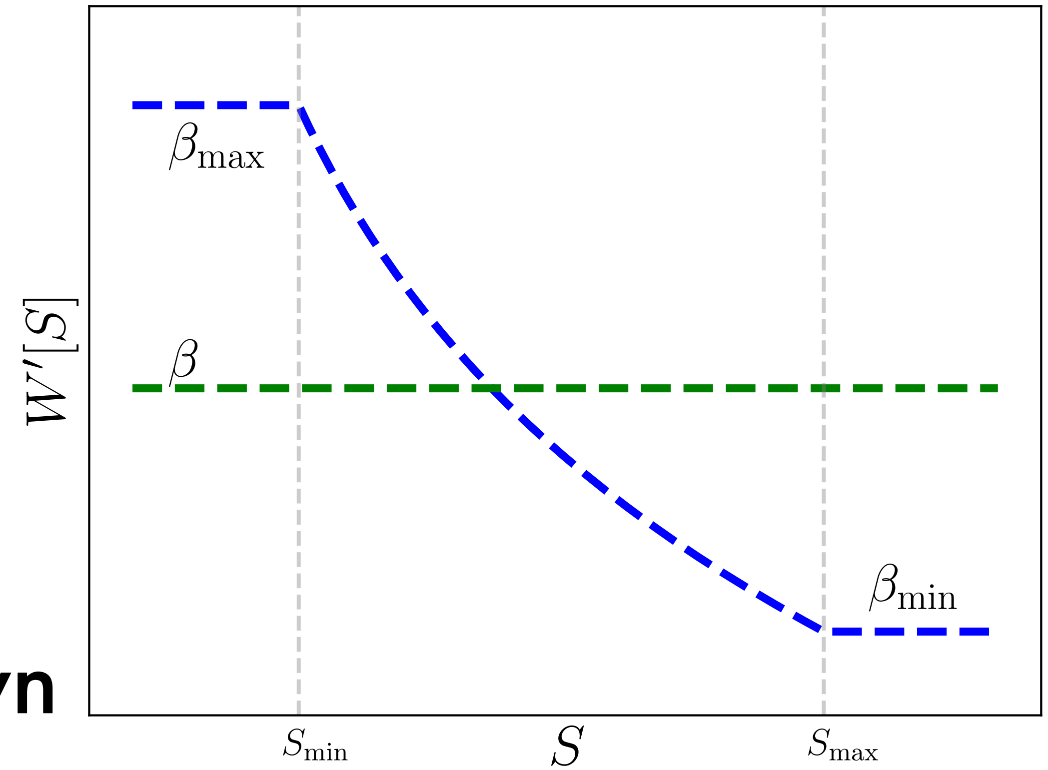
Temperature reweighting

- Single temperature/coupling MC
- Computing $\mathcal{Z}(\beta_0)$ from $\int \mathcal{D}U e^{-W(S)}$

Assuming W(s) is known

$$\mathcal{Z}(\beta_0) = \int \mathcal{D}U e^{-W(S)} \; e^{W(S)-\beta_0 S}$$

$$\propto \sum_i \; e^{W(S_i)-\beta_0 S_i}$$



- Overall normalization unknown

- Regularisation issues with $\mathcal{Z}(\beta_0)$

Topological Susceptibility

- Compute QCD $\chi_{\rm top}(\beta)$ at high temperatures :

$$\chi_{\rm top}(\beta) a^4(\beta) = \frac{1}{V_L} \frac{Z_1(\beta)}{Z_0(\beta)}$$

Topological Susceptibility

- Compute QCD $\chi_{\text{top}}(\beta)$ at high temperatures :

$$\chi_{\text{top}}(\beta)a^4(\beta) = \frac{1}{V_L}\frac{Z_1(\beta)}{Z_0(\beta)}$$

- Computing ratios of $\chi_{\text{top}}(\beta)$

$$\frac{\chi_{\text{top}}(\beta_1)a^4(\beta_1)}{\chi_{\text{top}}(\beta_2)a^4(\beta_2)} = \frac{Z_1(\beta_1)}{Z_1(\beta_2)}\;\frac{Z_0(\beta_2)}{Z_0(\beta_1)}$$

Allows for two single topology simulations

Topological Susceptibility

- Compute QCD $\chi_{\rm top}(\beta)$ at high temperatures :

$$\chi_{\rm top}(\beta)a^4(\beta) = \frac{1}{V_L}\frac{Z_1(\beta)}{Z_0(\beta)}$$

- Computing ratios of $\chi_{\rm top}(\beta)$

$$\frac{\chi_{\rm top}(\beta_1)a^4(\beta_1)}{\chi_{\rm top}(\beta_2)a^4(\beta_2)} = \frac{Z_1(\beta_1)}{Z_1(\beta_2)}\ \frac{Z_0(\beta_2)}{Z_0(\beta_1)}$$

- Temperature reweighting :

$$\frac{\chi_{\rm top}(\beta_1)a^4(\beta_1)}{\chi_{\rm top}(\beta_2)a^4(\beta_2)} = \frac{\sum_{iQ}\ e^{W(S_{iQ})-\beta_1 S_{iQ}}}{\sum_{iQ}\ e^{W(S_{iQ})-\beta_2 S_{iQ}}}\frac{\sum_{i}\ e^{W(S_{i})-\beta_1 S_{i}}}{\sum_{i}\ e^{W(S_{i})-\beta_2 S_{i}}}$$

How to simulate continuous temperatures

- We would like to generate : $\mathrm{d}P[U] = \frac{\mathcal{D}U\ e^{-W(S[U])}}{\int \mathcal{D}U\ e^{-W(S[U])}}$

- Construct an MD Hamiltonian :

$$\mathcal{H}(\pi, U) = \sum_{x,\mu} \frac{1}{2} \left((\pi_\mu(x))^2 + W(S[U]) \right)$$

How to simulate continuous temperatures

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- Construct an MD Hamiltonian :

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- Solve Hamilton's EOM :

$$\frac{\mathrm{d}U}{\mathrm{d}t} = i\pi U \quad \frac{\mathrm{d}\pi}{\mathrm{d}t} = iU^\dagger \frac{\partial W(S[U])}{\partial U} = iU^\dagger \frac{\partial W(S[U])}{\partial S[U]} \frac{\partial S[U]}{\partial U}$$

- Metropolis accept/reject :

$$\Delta H = \mathcal{H}(\pi_f, U_f) - \mathcal{H}(\pi_i, U_i)$$

How to compute W(S)

- W(S) needs to be computed in a separate simulation :

- Initial W(S) obtained from interpolation :

$$(\beta_{min}, \beta_{mid}, \beta_{max}) \longrightarrow (S_{max}, S_{mid}, S_{min})$$

- Action S is split in N intervals $(S_1, S_2, \cdots S_N)$

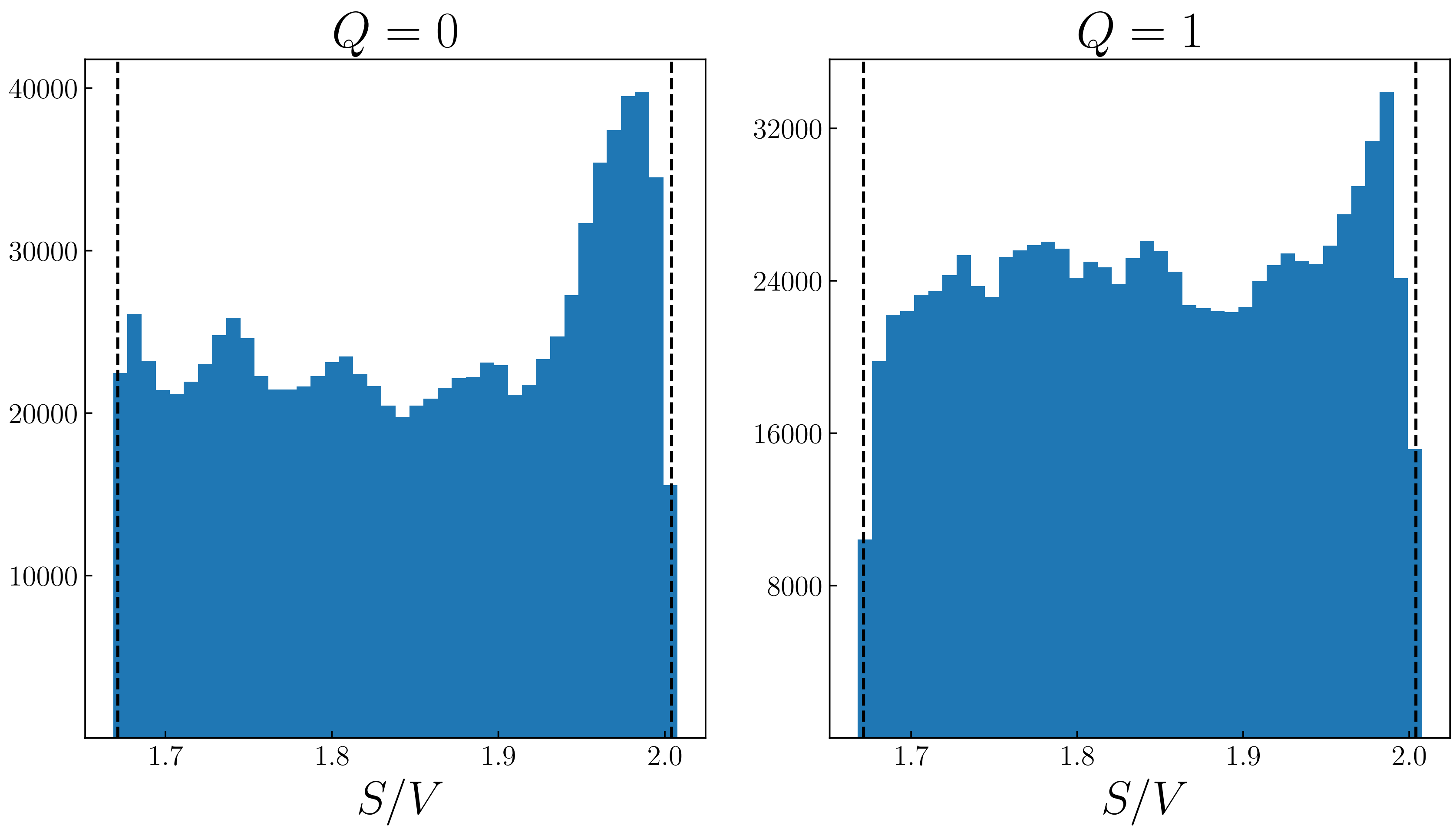
$$W(S) = \begin{cases} W_i + s_r x (W_{i+1} - W_i), & S_{max} < S_i < S_{min}, \\ \beta_{max} S, & S < S_{min} \\ \beta_{min} S, & S > S_{max} \end{cases} \qquad x = \frac{S - S_i}{S_{i+1} - S_i}$$

- After S traverses all intervals and back, s_r is reduced

- Separate simulation needed for Q=0,1

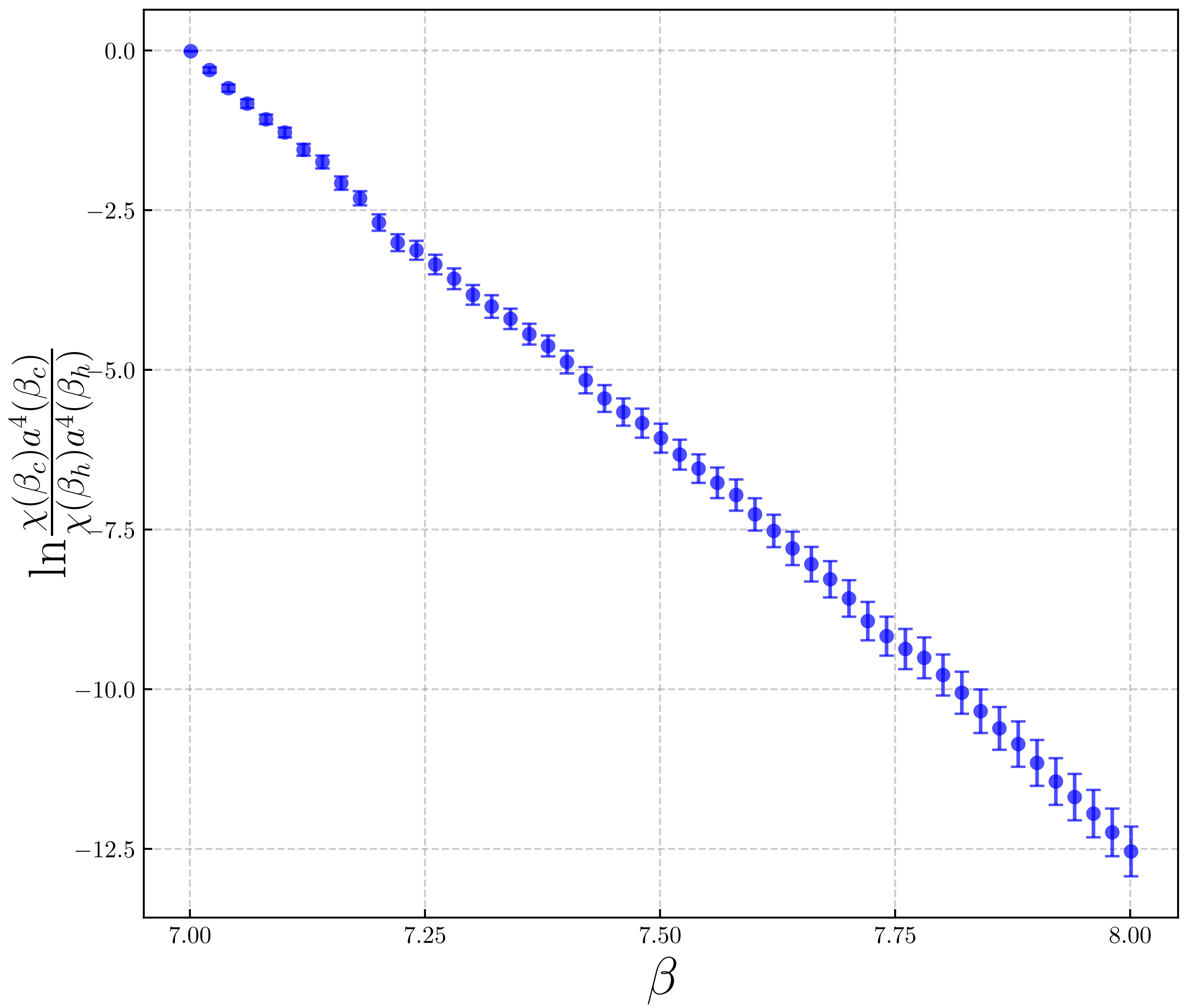
Results for action distribution

- Simulation with Wilson gauge action
- Temperature 2.5 Tc - 9.4 Tc

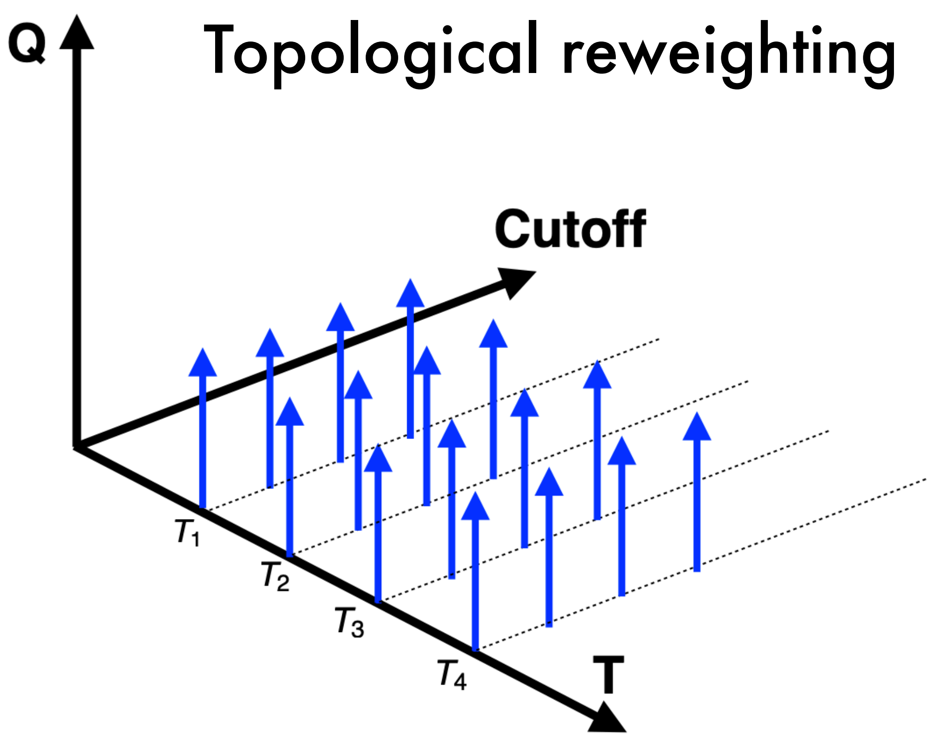


Results for Susceptiblity

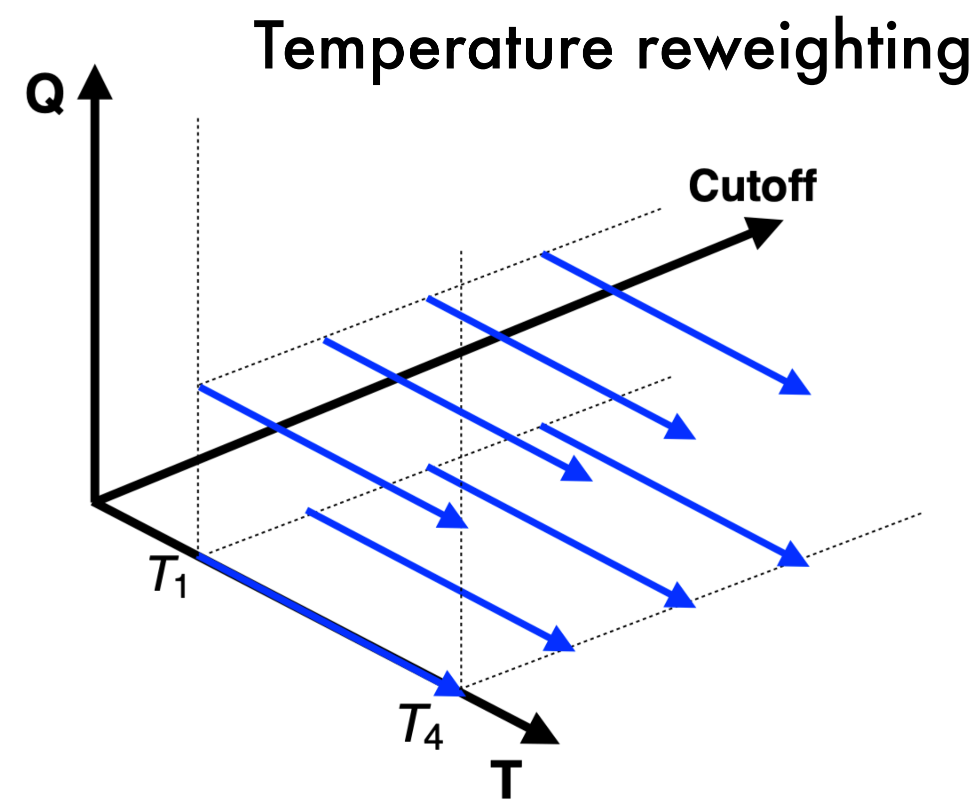
- Method accomplishes continuous sampling of temperatures.
- Results in agreement with topological reweighting.
- Scale agnostic determination.
- Method comparable to current methods for high statistics determination.
- Inefficient for less precise determination



Outlook



- Needs to be simulated at every cutoff and temperature.
- No of simulations = x
- Simulation for reweighting function cheap
- Cheaper for lower statistics



- Needs to simulated at two topologies and every cutoff.
- No of simulations = x/2
- Simulation for reweighting function expensive
- Expensive for lower statistics

Scale setting at high temperature

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Motivation

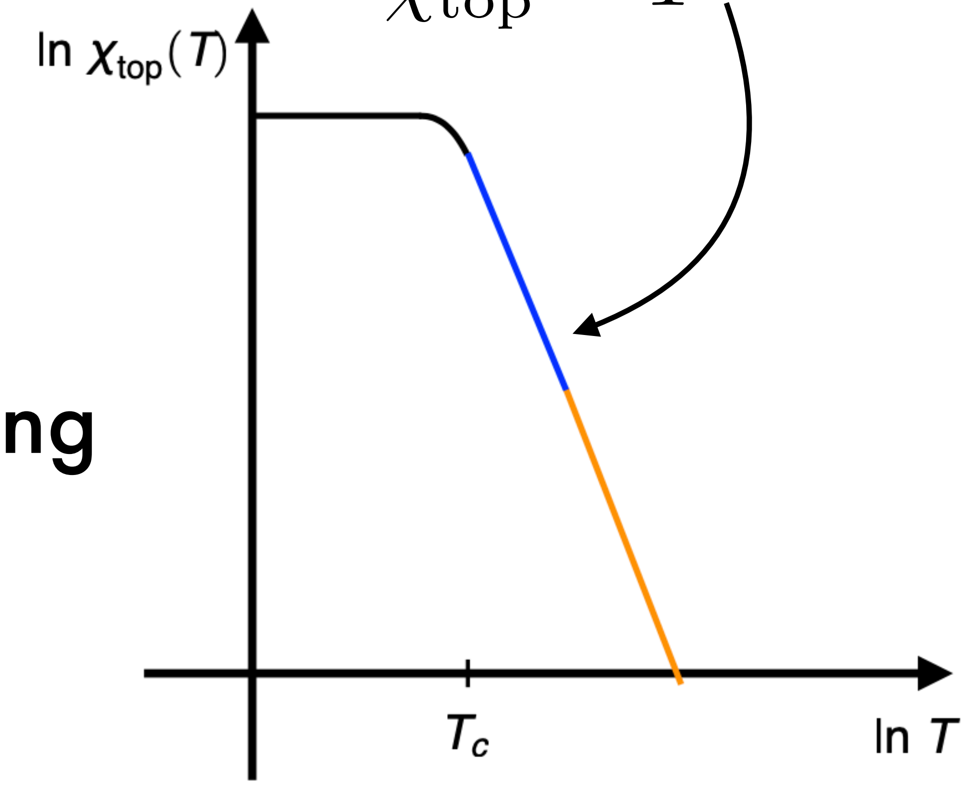
- Determine lattice cutoff – subpercent precision
  – very fine lattices $\sim 0.015\ \mathrm{fm}$

- Target – Full QCD determination of $\chi_{\rm top}$

Motivation

- Temperature dependence of $\chi_{\rm top}$
- Large power dependence on T
- Worse dependence on lattice spacing

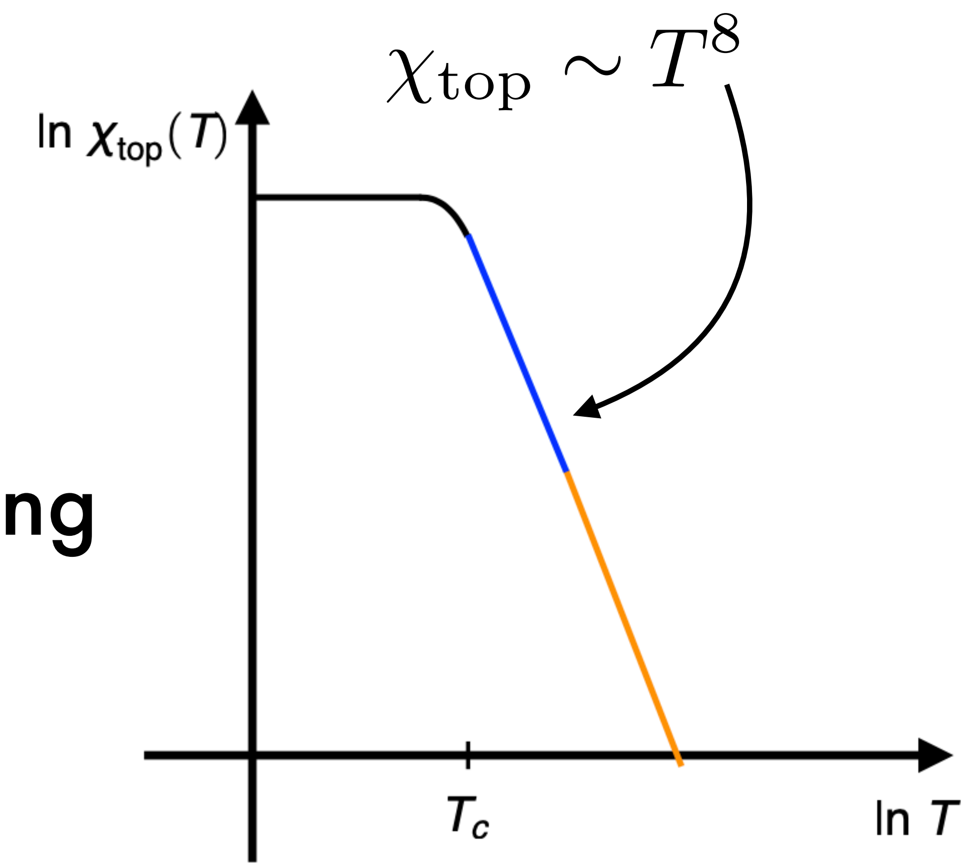
$\chi_{\rm top} \sim T^8$



$\chi_{\rm top} a^4 \sim a^{12}$

Motivation

- Temperature dependence of χ_{top}
- Large power dependence on T
- Worse dependence on lattice spacing
- 2% σ in T ⟶ 24% σ χ_{top}
- Need % level determination of a.
- Temp 1 GeV – Box size $N_t = 14$ ⟶ $a \sim 0.01$ fm



$\chi_{\text{top}} a^4 \sim a^{12}$

Scale setting in Lattice QCD

- Lattice calculations use bare parameters (g_0, am_l, am_s, am_c)

Parameters need to describe physics.

Scale setting in Lattice QCD

- Lattice calculations use bare parameters (g_0, am_l, am_s, am_c)
- Determine the renormalized trajectory
- Determining the cutoff - "Set" the scale

Scale setting in Lattice QCD

- Lattice calculations use bare parameters (g_0, am_l, am_s, am_c)
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  - Tune bare parameters to physical observables
  - One observable for each parameter
  - Determine the Line of Constant Physics - Renormalised Trajectory
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Scale setting in Lattice QCD

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  - Lattice quantities computed as a ratio with the cutoff
  - Determines the overall precision

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- Determining the cutoff - "Set" the scale
  - Lattice quantities computed as a ratio with the cutoff
  - Determines the overall precision
- Closer look at scale setting

Scale setting observables

- Desirable properties : *R.Sommer, arXiv:1401.3270v1*
  - Low numerical effort
  - Good statistical precision
  - Small systematic error
  - Weak quark mass dependence

Scale setting observables

- **Desirable properties :** *R.Sommer, arXiv:1401.3270v1*
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  - Small systematic error
  - Weak quark mass dependence

- **Classification of observables**

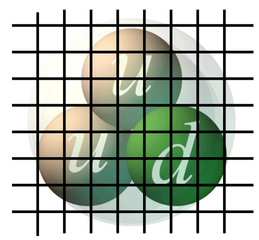
Phenomenological scales → Direct experimental observables

Theory scales → Observables defined from phenomenology. Indirectly related to experiment.

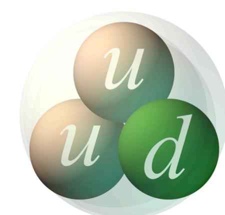
Phenomenological Scales

- Most Naive observable – Proton

Compute proton mass on lattice



aM_N



M_N

Compare it to experiment

$$\mathcal{R} = \frac{aM_N}{M_N}$$

Phenomenological Scales

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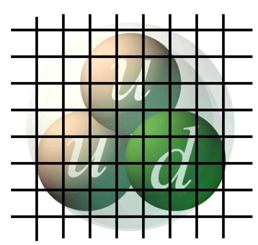
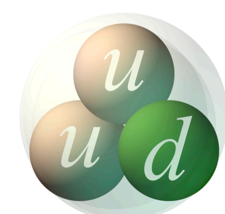


aM_N



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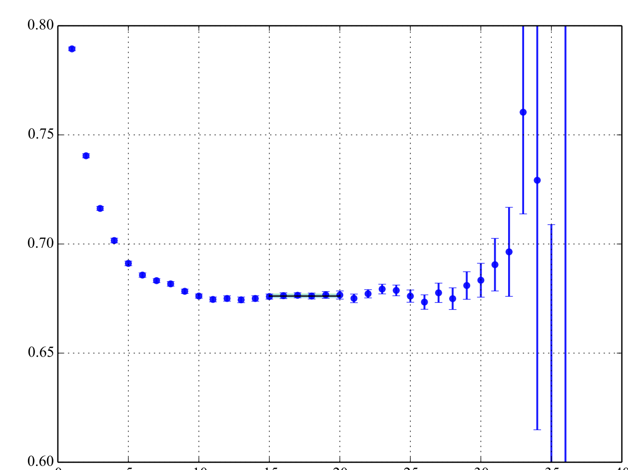
Compare it to experiment

$$\mathcal{R} = \frac{aM_N}{M_N}$$

- Highly sensitive to quark mass

- Issue with precision – Low signal/noise



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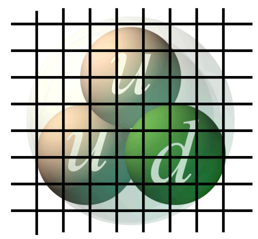
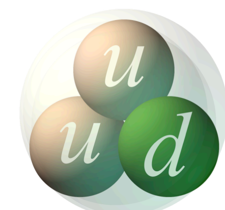


aM_N

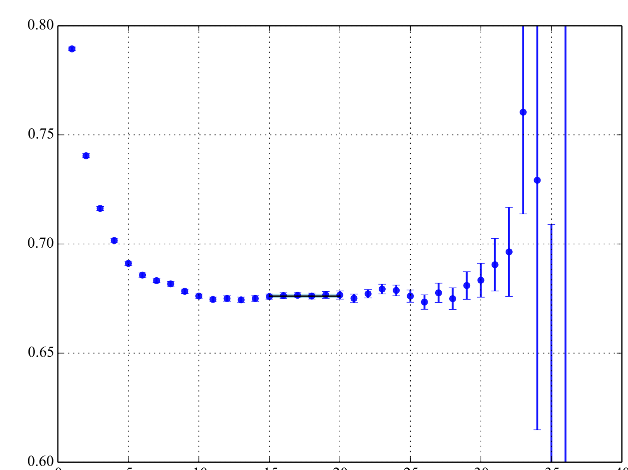


M_N

Compare it to experiment

$$\mathcal{R} = \frac{aM_N}{M_N}$$

- Highly sensitive to quark mass
- Issue with precision – Low signal/noise
- Use Omega baryon – scale setting



Phenomenological Scales

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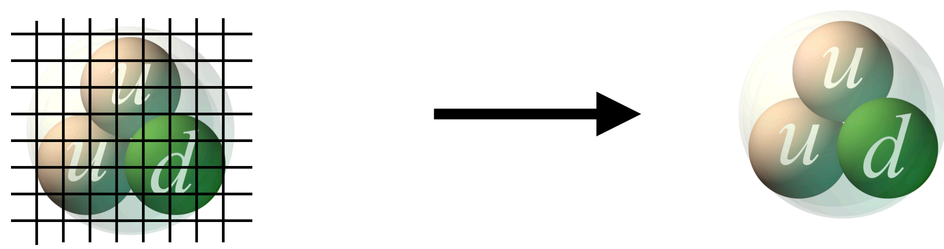
Compute proton mass on lattice



Compare it to experiment

aM_N $\qquad$ M_N $\qquad$ $\mathcal{R} = \frac{aM_N}{M_N}$

- Pseudoscalar decay constants –
  - Match lattice computations of f_π, f_K to experiment.
  - Reliable lattice calculation — Good chiral behavior.
  - Experimental results depend on precision of $V_{ud},\ V_{us}$

Theory scales

- Observables defined from phenomenology
  → Indirectly related to experiment

- Sommer parameters : r_0, r_1
  - Use static quark potential to define a hadronic scale.
  - Defined as $r^2 F(r)|_{r=R(c)} = c$, choice $c = 1.65 \to r_0 = 0.5 \text{ fm}$
  - Related to the spectra of $\bar{b}b, \bar{c}c$ to Cornell potential
  - Cheap computation, high statistical precision

- Gradient flow observables :

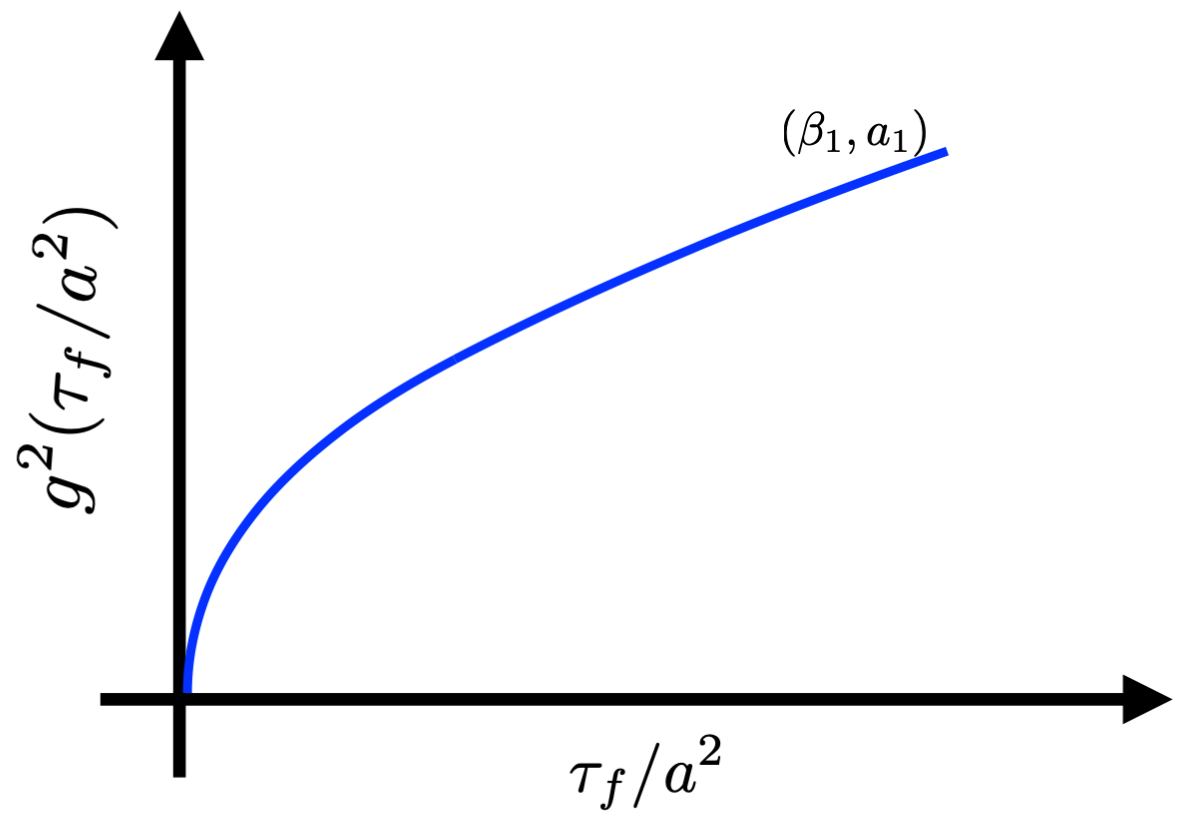
Gradient flow

✤ Gradient flow - smoothing procedure for gauge fields

$$\partial_t B_\nu(x,t) = D_\mu G_{\mu\nu}, \quad B_\mu(x,0)|_{t=0} = A_\mu.$$ Covariant diffusion equation

LO in coupling - diffusion equation

$$B_\mu(x,t) = \int d^4y \frac{e^{-(x-y)^2/4t}}{(4\pi t)^2} A_\mu(y)$$



Compute YM energy at LO

$$\langle E_t \rangle = \frac{24}{128\pi^2}\frac{g^2}{\tau_f^2}$$

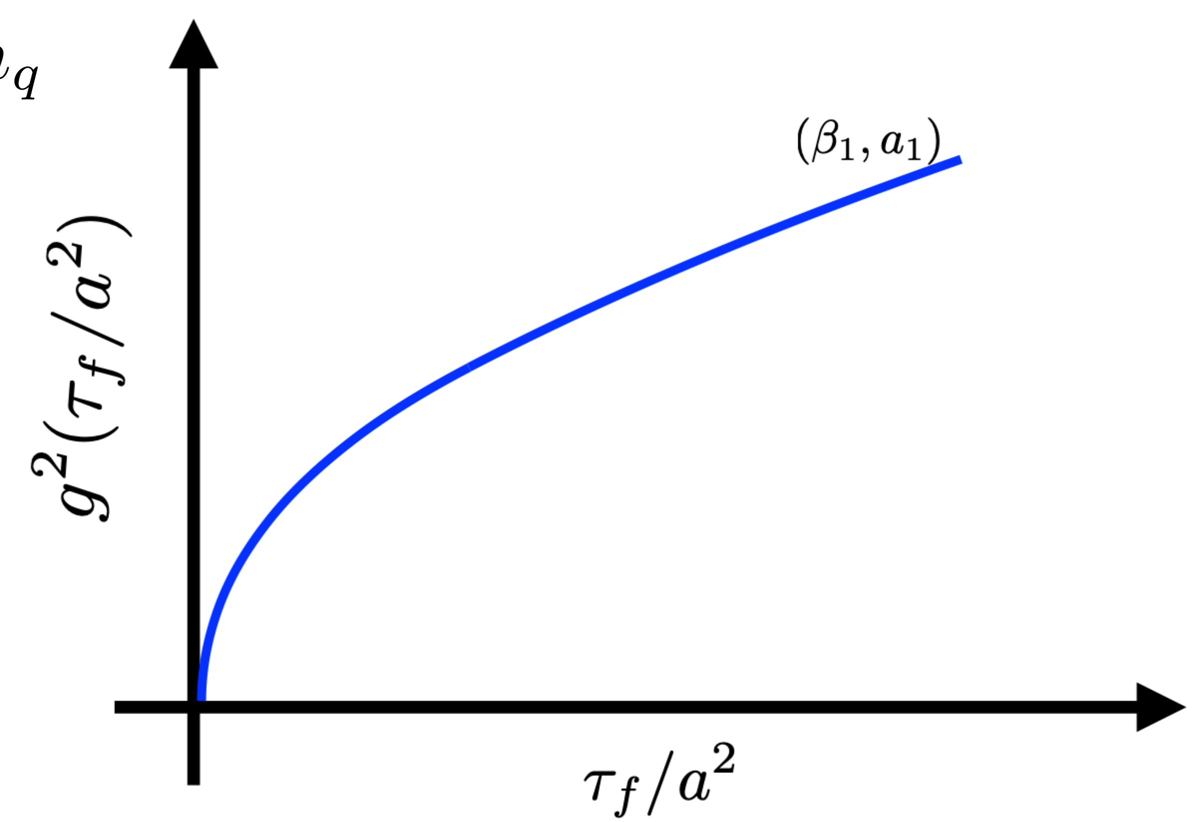
Use this to define the coupling $$g^2_{\text{flow}}(\tau_f) \equiv \frac{128\pi^2\tau_f^2\langle E_t(\tau_f)\rangle}{24}$$

Gradient flow observable

- Use simple prescriptions $\tau^2\langle E_t\rangle|_{\tau=\tau_0} = c_1, \quad \left(\tau\frac{\mathrm{d}(\tau^2\langle E_t\rangle)}{\mathrm{d}\tau}\right)_{\tau=w_0^2} = c_2$

  c_1, c_2 are chosen to reduce FV, discretization effects.

- At a given lattice spacing, measure $\sqrt{\tau_0}/a, w_0/a$ at different m_q



- Perform a $\chi\mathrm{PT}$ extrapolation

  $\sqrt{\tau_0}_{\mathrm{phys}}, (w_0)_{\mathrm{phys}}$ Obtained from decay constants

- Compute scale by matching.

Issues with scale observables

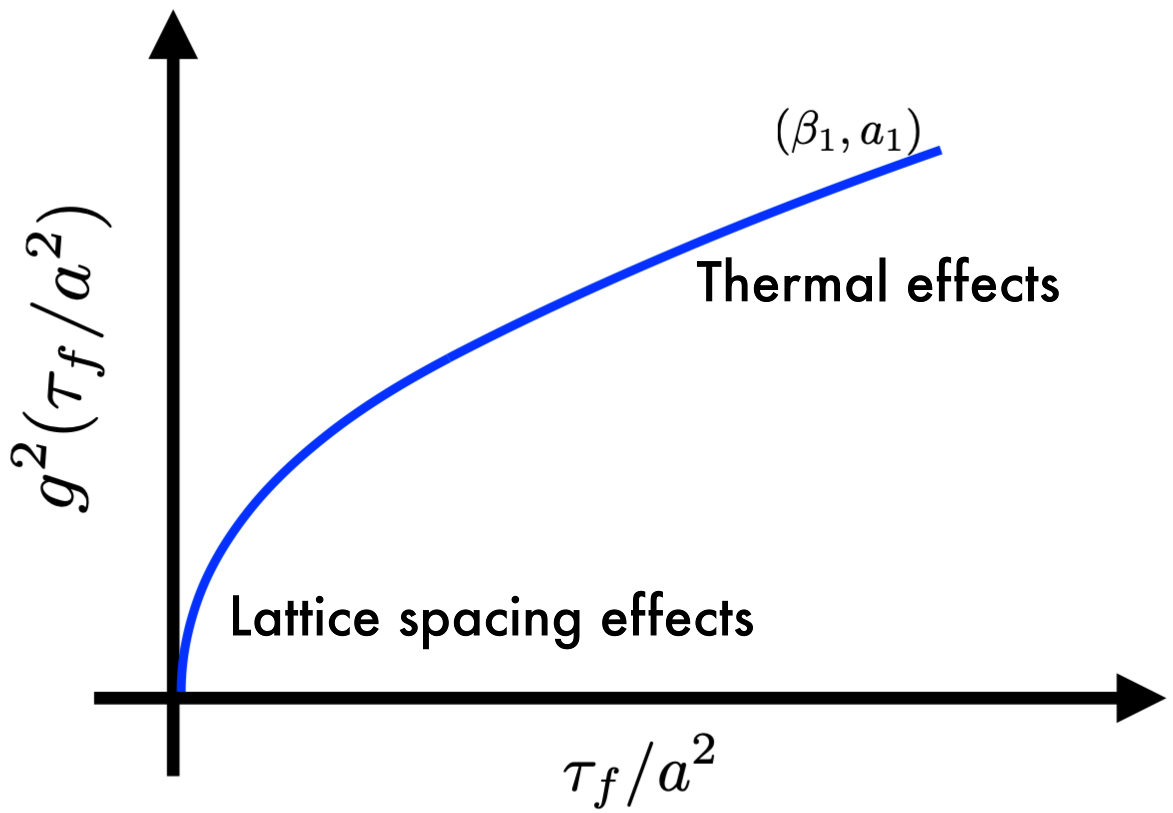
- Phenomenological scales
  - Decay constants : determined from weak processes $\pi \to l\nu, \quad V_{ud} f_\pi$

    Precision is depended on precision of V_{ud}
  - Baryon masses : Signal/noise problem prohibits precision
- Theory scales :
  - Sommer parameter : poor behavior at fine lattice spacings - Small S/N
  - Topological freezing problem at fine lattices with gradient flow
- New method : Relative scale setting with gradient flow

Systematics of coupling

- Lattice spacing effects :

- Thermal effects :



Systematics of coupling

- Lattice spacing effects :

$$\langle E_t \rangle_{\rm latt} = \langle E_t \rangle_{\rm cont} \left(1 + \frac{a^2}{\tau_f} C_1 + \frac{a^4}{\tau_f^2} C_2 + \mathcal{O}\left(\frac{a^6}{\tau_f^3} \right) \right) \qquad R_{\rm latt} \equiv \frac{\langle E_t \rangle_{\rm latt}}{\langle E_t \rangle_{\rm cont}}$$

C1 and C2 depend on lattice QCD action, flow type and observable

- Thermal effects :

Systematics of coupling

- **Lattice spacing effects :**

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C1 and C2 depend on lattice QCD action, flow type and observable

- **Thermal effects :**

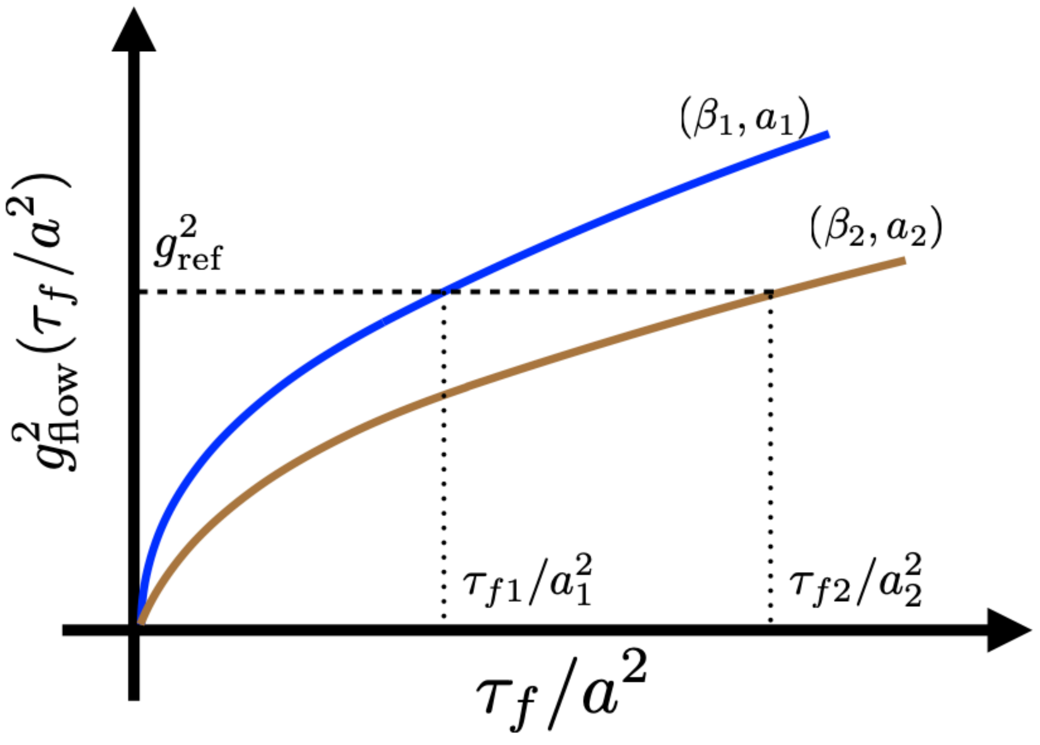
$$\langle E_t \rangle (T) = \langle E_t \rangle (T=0) \sum_{n \in Z} e^{-n^2/8\tau_f T^2} \qquad R_T \equiv \frac{\langle E_t \rangle (T)}{\langle E_t \rangle (T=0)}$$

Computed from LO perturbative free theory

- **Full coupling** $$g^2_{\rm flow}(\tau_f) = \frac{128\pi^2}{24\tau_f^2} \frac{\langle E_t \rangle_{T,{\rm latt}}}{R_{\rm latt} R_T} + ({\rm higher\ order}).$$

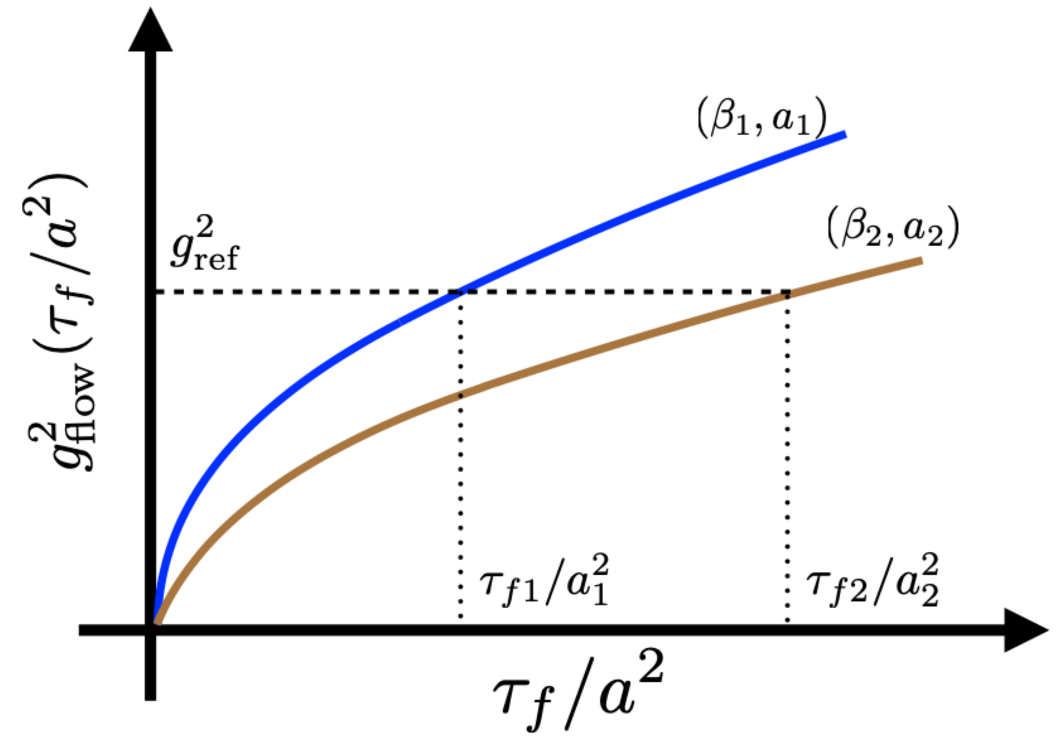
Relative Scale setting

- Two lattice ensembles at same temperature $T \begin{cases} (\beta_1, N_{\tau_1}, a_1) \\ (\beta_2, N_{\tau_2}, a_2) \end{cases}$



Relative Scale setting

- Two lattice ensembles at same temperature $T \begin{cases} (\beta_1, N_{\tau_1}, a_1) \\ (\beta_2, N_{\tau_2}, a_2) \end{cases}$
- Relation between τ_{f1}/a_1^2 and τ_{f2}/a_2^2 for given



Relative Scale setting

- Two lattice ensembles at same temperature T → $(\beta_1, N_{\tau_1}, a_1)$, $(\beta_2, N_{\tau_2}, a_2)$

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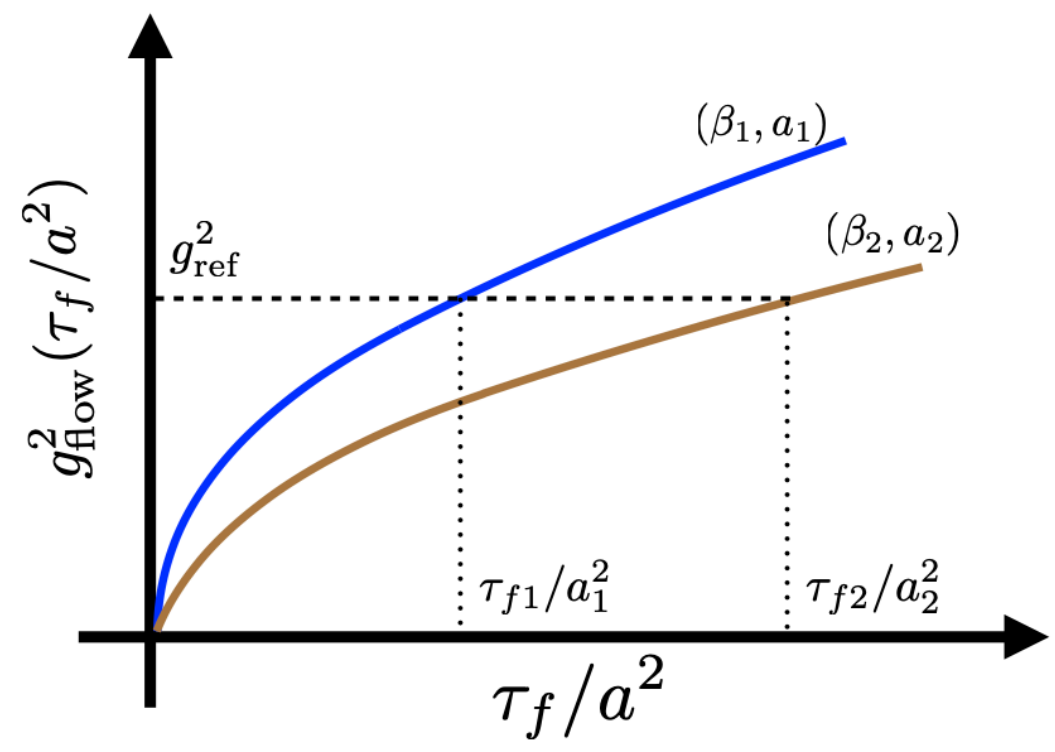
$$\frac{\tau_{f1}}{N_{\tau_1}^2 a_1^2} = \frac{\tau_{f2}}{N_{\tau_2}^2 a_2^2}$$



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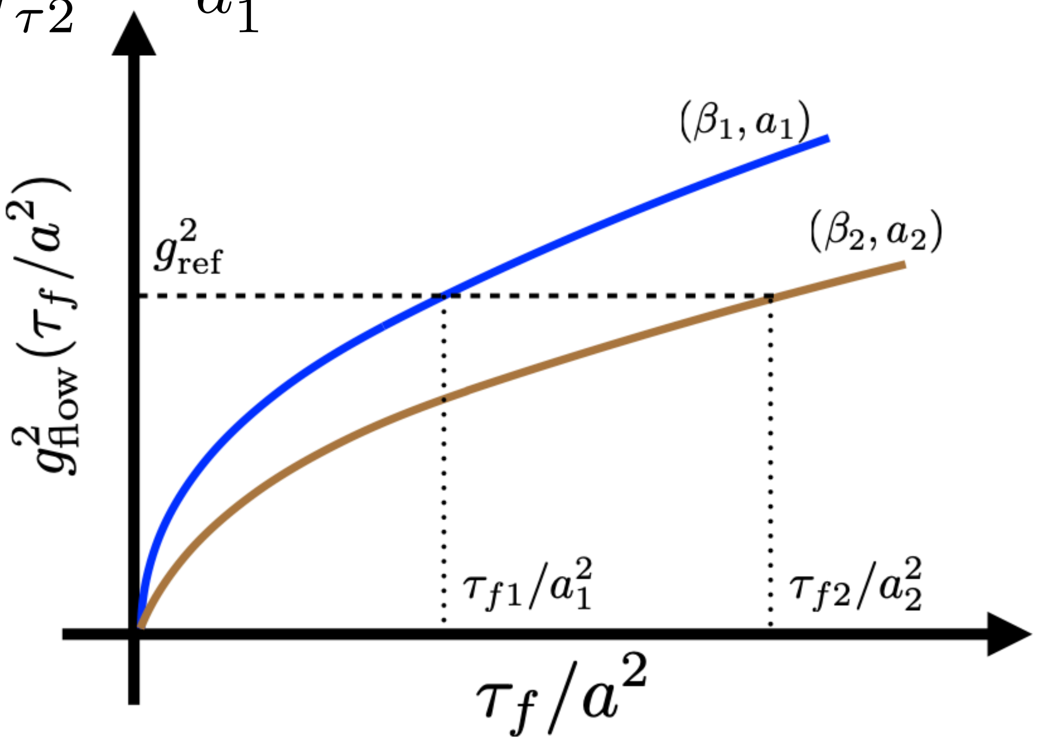

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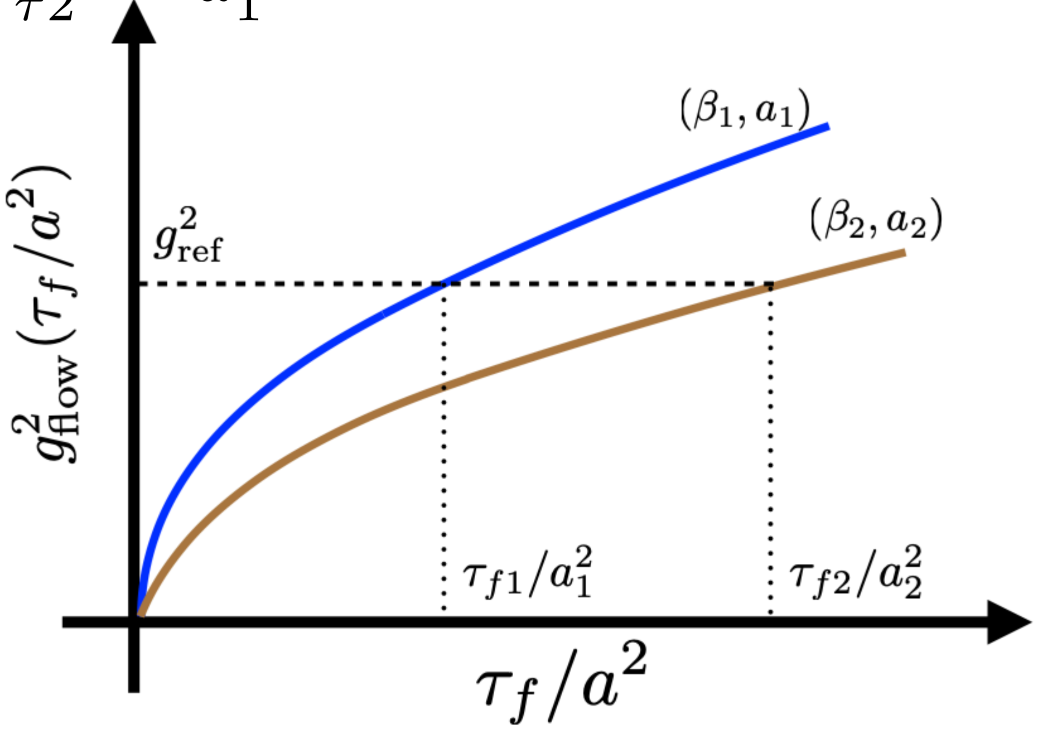
- Can use this to set scale



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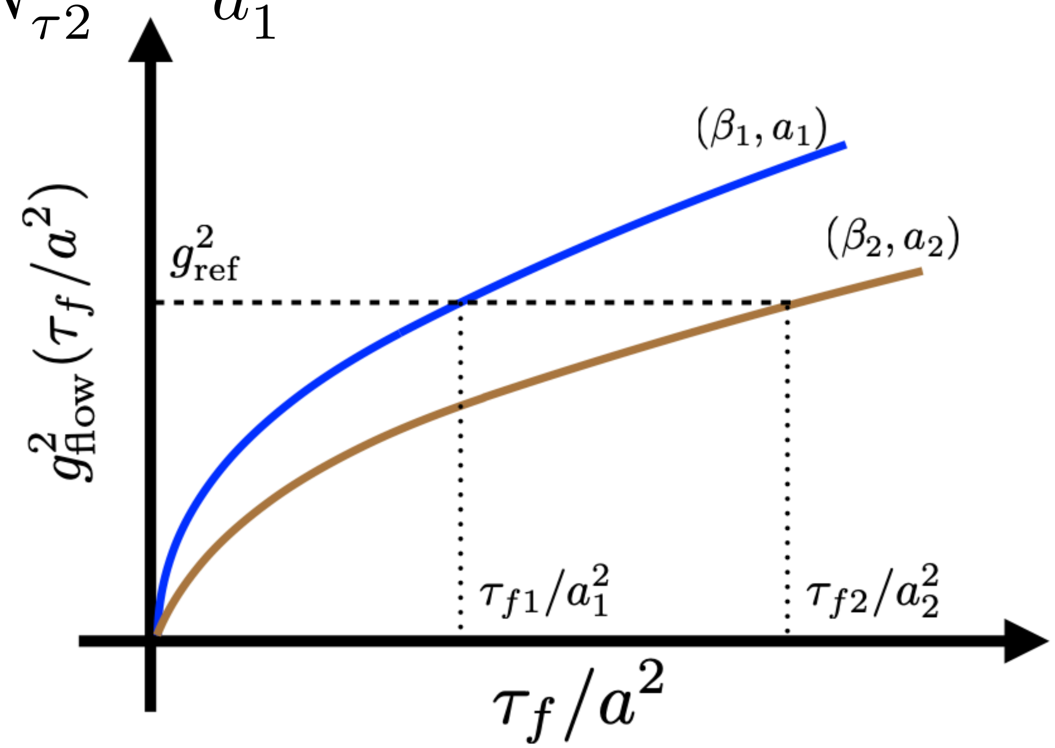
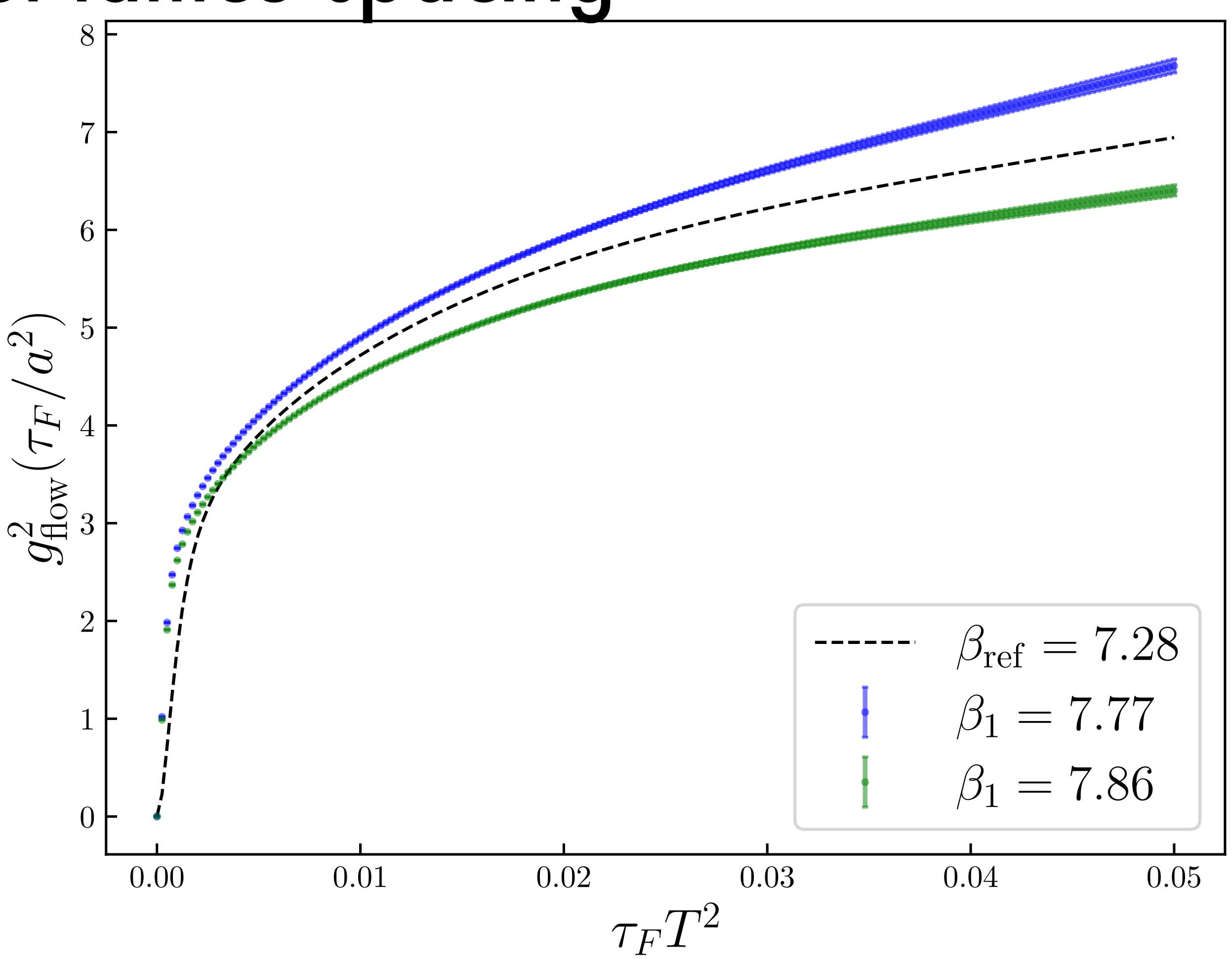


- Two Issues :
  - Assumes β_2 is tuned
  - Exists a procedure that accomplishes this

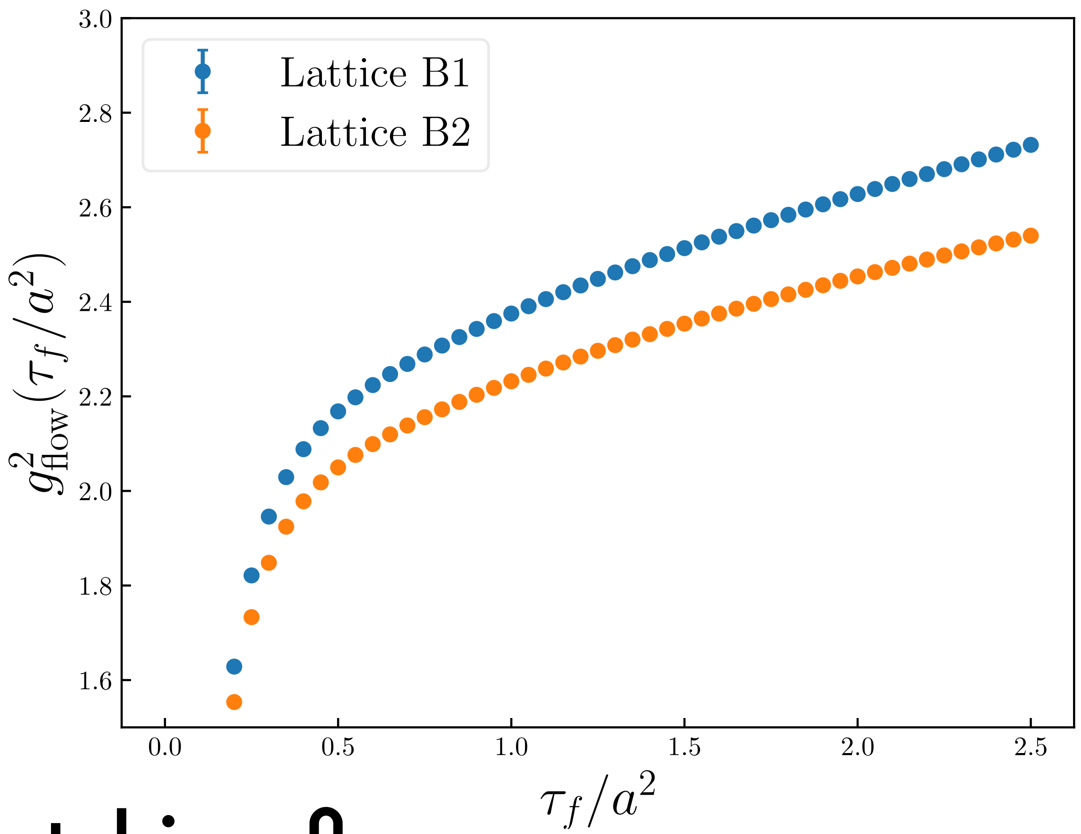
Gauge coupling tuning

- Relative scale setting implies prior knowledge of LCP
- MILC provides LCP data up to 0.04 fm
- Interpolate LCP data to target lattice spacing
- With 20% deviation, choose two couplings
- Interpolate in beta over several flow depths



Scale determination procedure

- Two gradient flows
  - B1 has well determined scale
  - Need to set the scale for tuned B2
- Would like to match two flows



- Chi-square like minimization for matching flows

$$\Delta G(g_1^2(x_i), x_i, s) = \frac{1}{\sqrt{x_i}}\left(g_1^2(x_i) - p(x_i/s^2)\delta(A, B, x_i)\right)$$

$$\chi^2(s, A, B, p(x)) = \Delta G(g_1^2(x_i), x_i, 1)\ C^{-1}_{1,ij}\ \Delta G(g_1^2(x_j), x_j, 1)$$
$$+\Delta G(g_2^2(s\ x_l), s\ x_l, s)\ C^{-1}_{2,lm}\ \Delta G(g_2^2(s\ x_m), s\ x_m, s)$$

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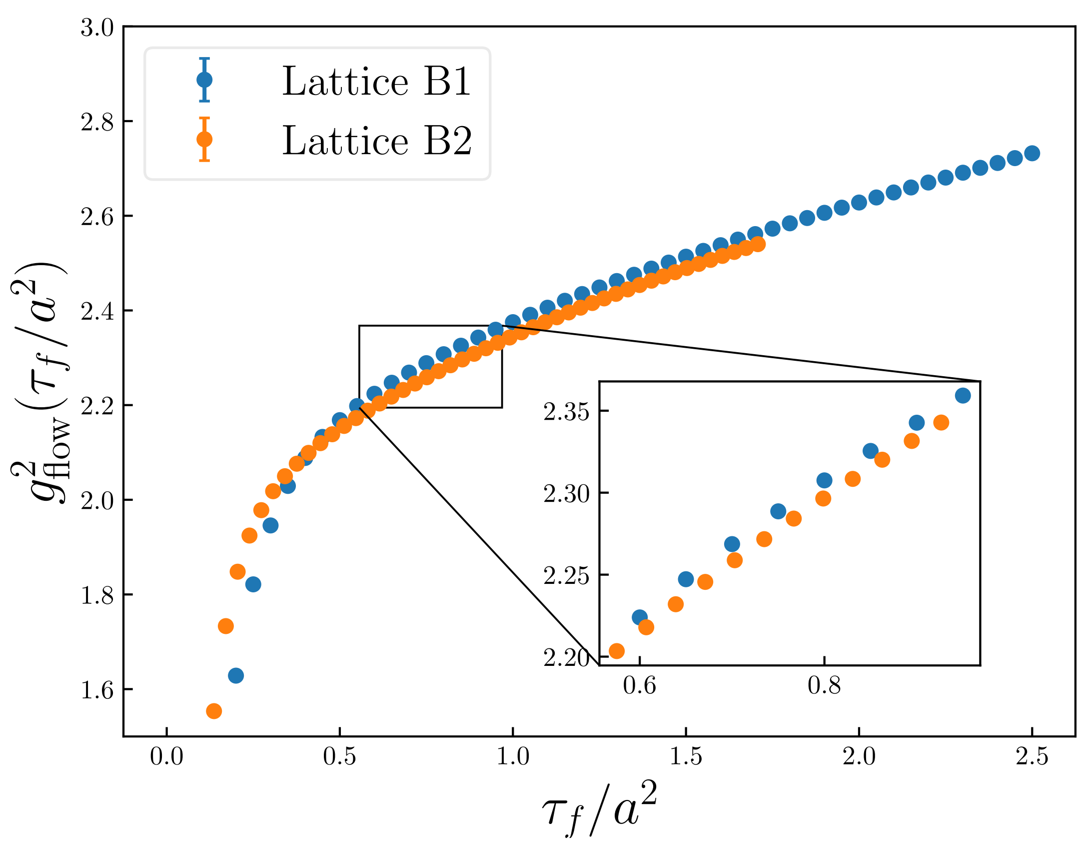

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Quark mass tuning

- Tune quark masses in $N_f = 2+1+1$ set up
- Tune only charm quarks and the rest from $m_l = \frac{m_s}{27.3}, \quad m_s = \frac{m_c}{11.783}$
- Compute screening correlation functions of :

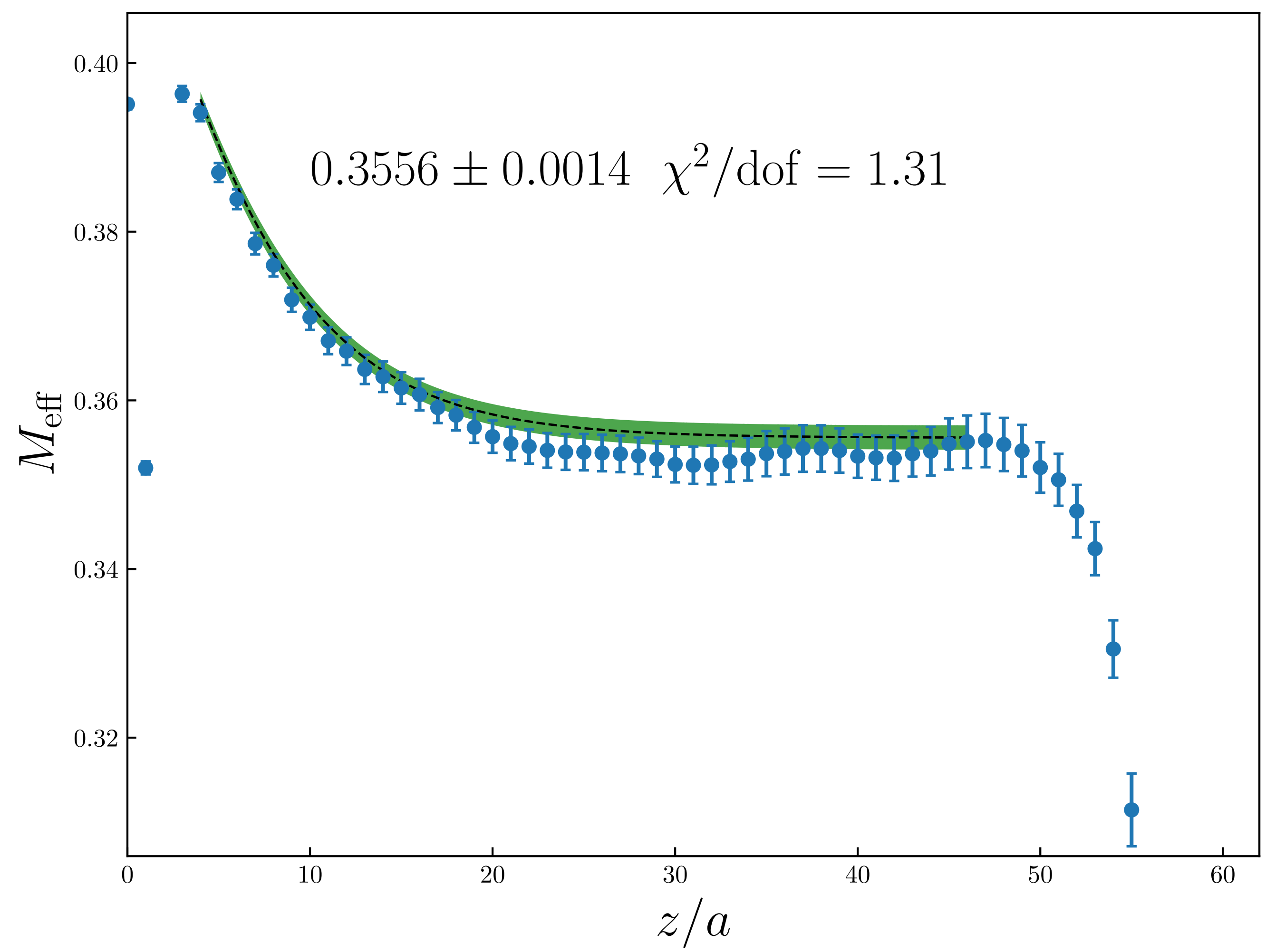
$$D_s(x) = \bar{c}^j_\alpha (\gamma_5)_{\alpha\beta} s^j_\beta(x) \qquad D_c(x) = \bar{c}^j_\alpha (\gamma_5)_{\alpha\beta} c^j_\beta(x)$$

Complementary check

- Maintain z direction 6X longer than other spatial
- Generate 5 guesses with 20% deviation from original

Quark mass tuning

- Momentum projected 12 equidistant sources
- Correlators averaged over sources
- Uncertainties account for autocorrelations
- Correlated two exponential fit.



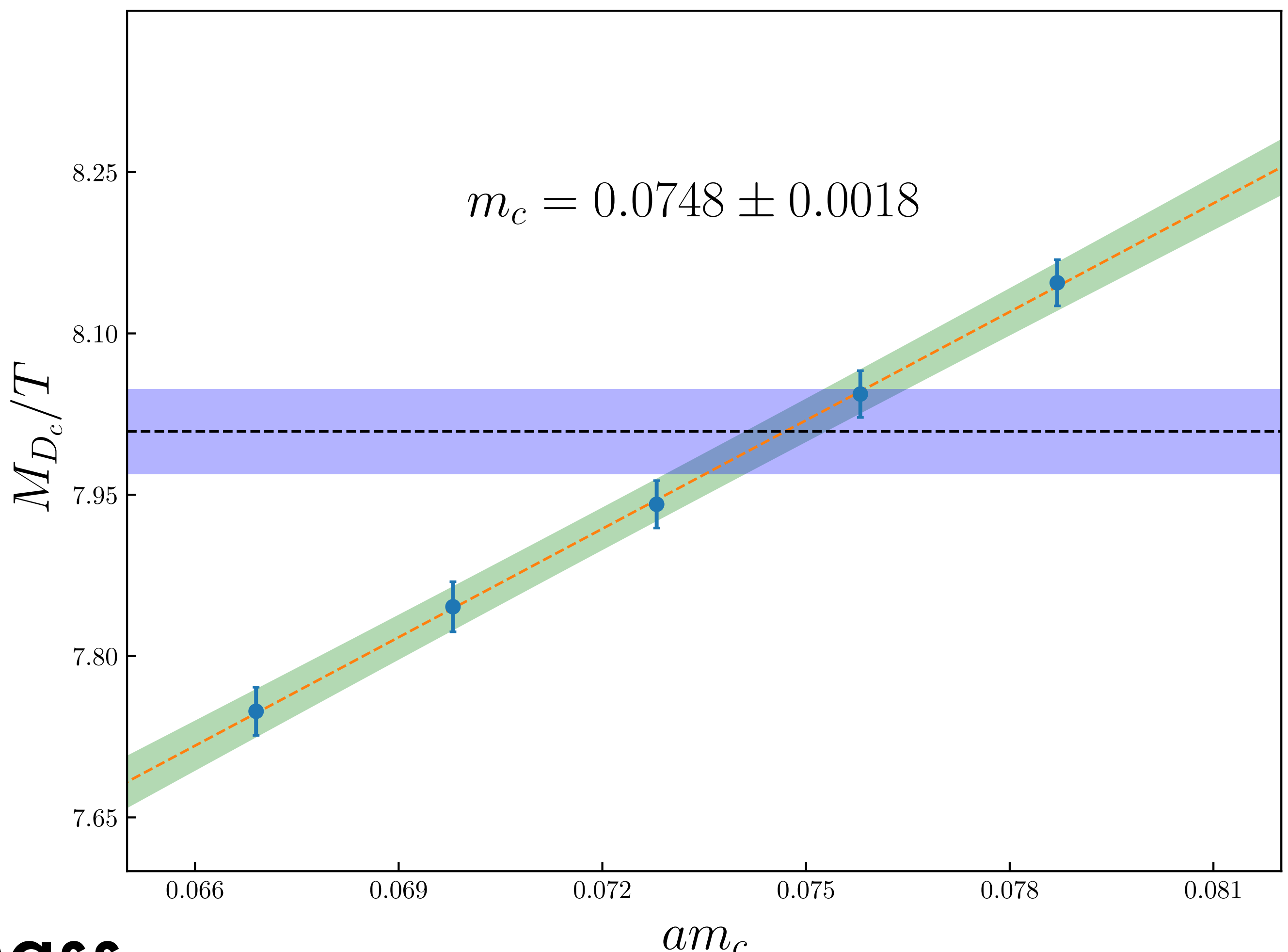
Quark mass tuning

- Blue band – Screening mass corresponds to tuned quark mass

- Data points – Screening mass from each guess

- Green band – Linear fit to the data

- Shaded overlapping region – tuned quark mass



Final results

Extension of MILC scale to 0.013 fm

435 MeV

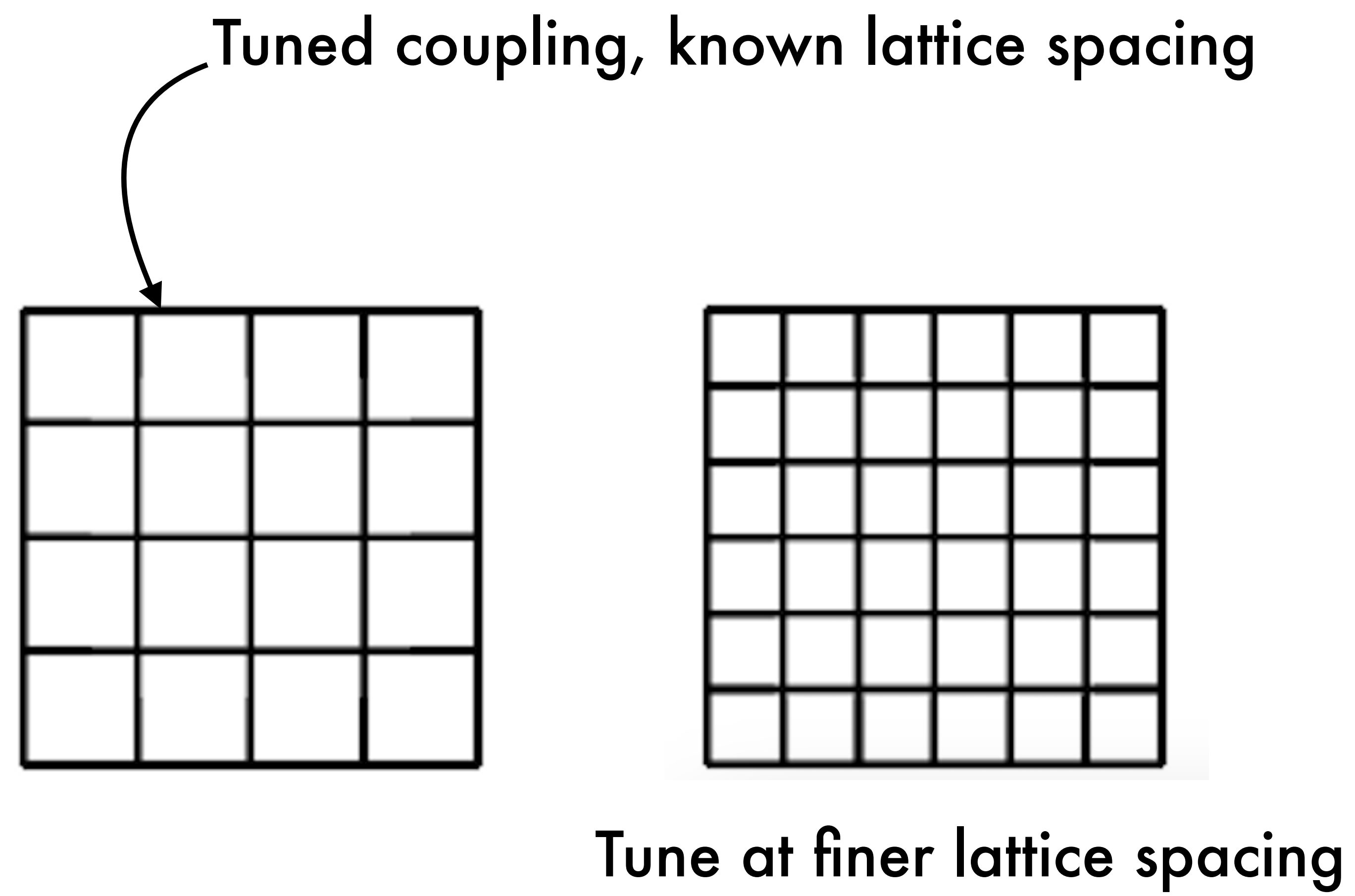
$\beta = 6.72,\ a = 0.05662(13)\ \mathrm{fm}$

341 MeV

$\beta = 7.28,\ a = 0.03215(13)\ \mathrm{fm}$

511 MeV

$\beta = 7.28,\ a = 0.03215(13)\ \mathrm{fm}$



Final results

Extension of MILC scale to 0.013 fm

435 MeV

$\beta = 6.72,\ a = 0.05662(13)\ \text{fm}$

$$\begin{cases} \beta = 6.95,\ a = 0.04531(26)\ \text{fm} \\ \beta = 7.13,\ a = 0.03786(12)\ \text{fm} \end{cases}$$

341 MeV

$\beta = 7.28,\ a = 0.03215(13)\ \text{fm}$

$$\begin{cases} \beta = 7.15,\ a = 0.03626(9)\ \text{fm} \\ \beta = 7.39,\ a = 0.02877(10)\ \text{fm} \\ \beta = 7.715,\ a = 0.02084(15)\ \text{fm} \end{cases}$$

511 MeV

$\beta = 7.28,\ a = 0.03215(13)\ \text{fm}$

$$\begin{cases} \beta = 7.60,\ a = 0.02400(4)\ \text{fm} \\ \beta = 7.82,\ a = 0.01936(19)\ \text{fm} \\ \beta = 8.045,\ a = 0.01599(18)\ \text{fm} \\ \beta = 8.22,\ a = 0.01372(11)\ \text{fm} \end{cases}$$

Thank you