Internal structure of T_{cc} and $X(3872)$ using compositeness

T. Kinugawa and T. Hyodo
Phys. Rev. C 109 , 045205 (2024).

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

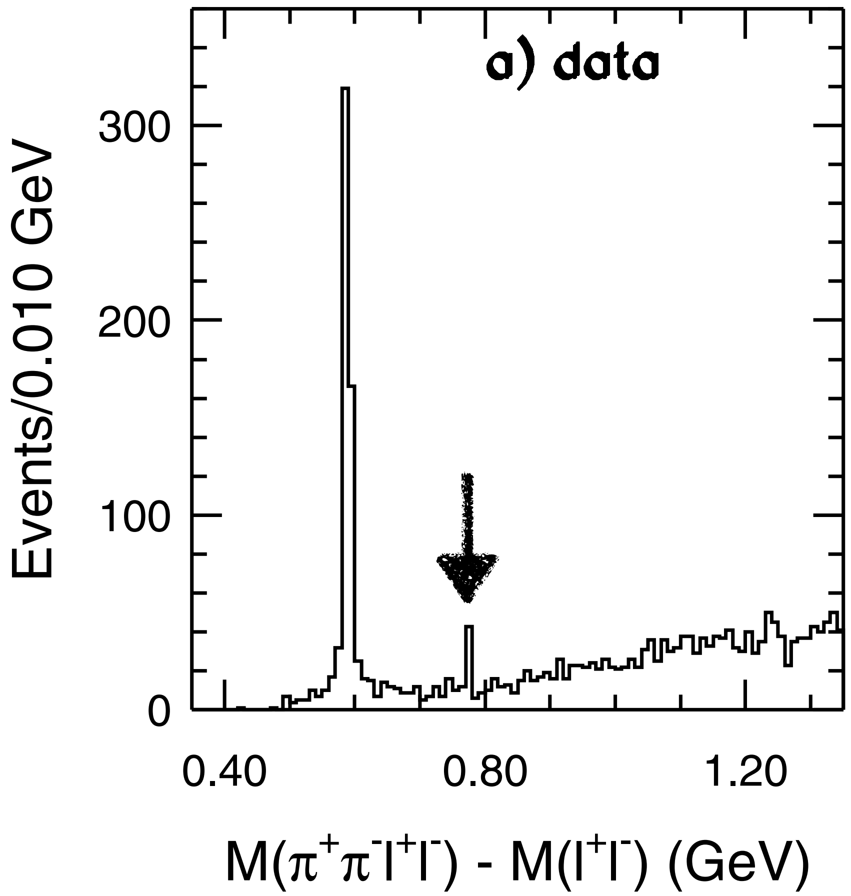
<u>Tomona Kinugawa</u> **Tetsuo Hyodo**

Department of Physics, Tokyo Metropolitan University

October 28th, HHIQCD2024

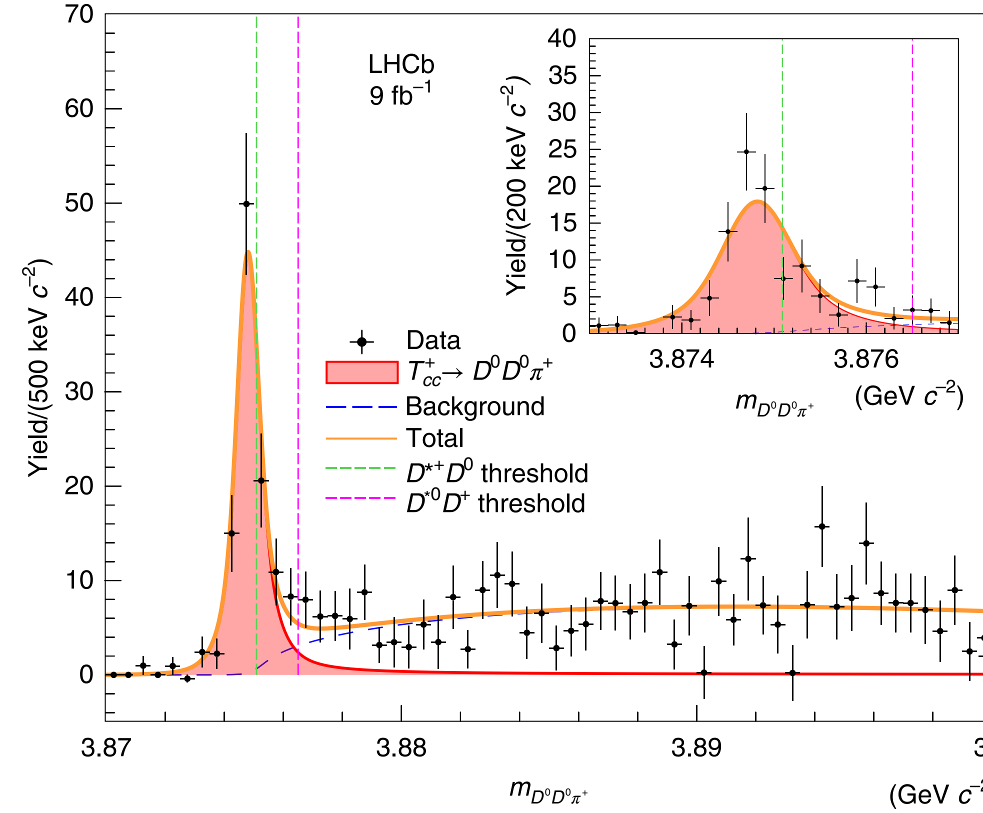
Near-threshold exotic hadrons

$X(3872) \to \pi^+\pi^- J/\psi$



S. K. Choi *et al.* (Belle), Phys. Rev. Lett. **91**, 262001 (2003).

$T_{cc} \to D^0 D^0 \pi^+ \ (cc\bar{u}\bar{d})$



LHCb Collaboration, Nature Phys. **18** (2022) no.7, 751-754;

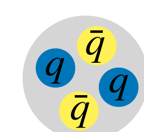
LHCb Collaboration, Nat. Commun. **13** 3351 (2022).

◉ internal structure?

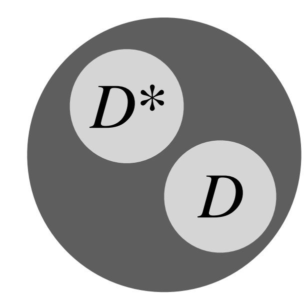
exotic hadron

$\neq$ qqq or $q\bar{q}$ ⟶ multiquarks

hadronic molecules



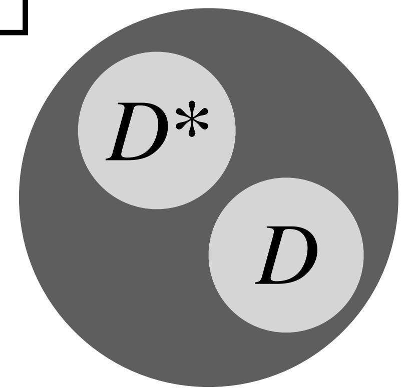
multiquarks

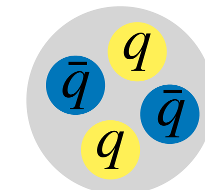


hadronic molecules

Compositeness

T. Hyodo, Int. J. Mod. Phys. A 28, 1330045 (2013).





◉ definition

hadron wavefunction

$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

compositeness **elementarity**

※ $0 \leq X \leq 1$ ⟶ $X > 0.5 \Leftrightarrow$ **composite dominant**

$X < 0.5 \Leftrightarrow$ **elementary dominant**

- **quantitative** analysis of internal structure

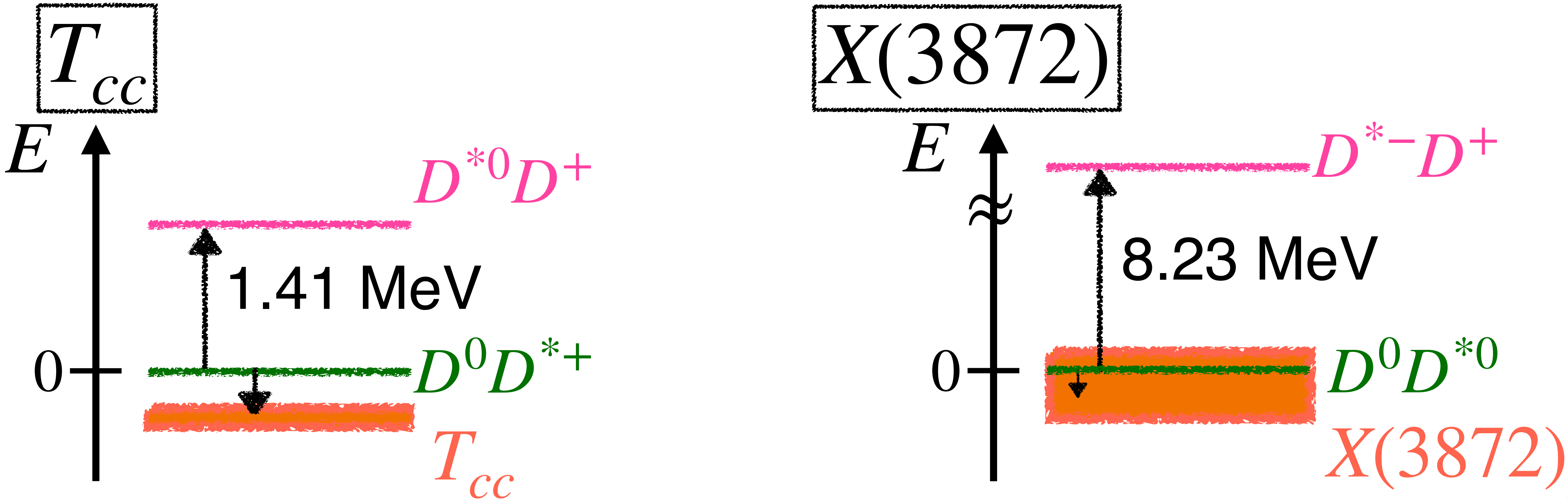
deuteron is not an elementary particle Weinberg, S. Phys. Rev. 137, 672–678 (1965).

$f_0(980)$, $a_0(980)$ Y. Kamiya and T. Hyodo, PTEP 2017, Phys. Rev. C 93, 035203 (2016); T. Sekihara, S. Kumano, Phys. Rev. D 92, 034010 (2015) etc.

$\Lambda(1405)$ T. Sekihara, T. Hyodo, Phys. Rev. C 87, 045202 (2013) ; Z.H. Guo, J.A. Oller, Phys. Rev. D 93, 096001 (2016) etc.

nuclei & atomic systems T. Kinugawa, T. Hyodo, Phys. Rev. C 106, 015205 (2022) etc.

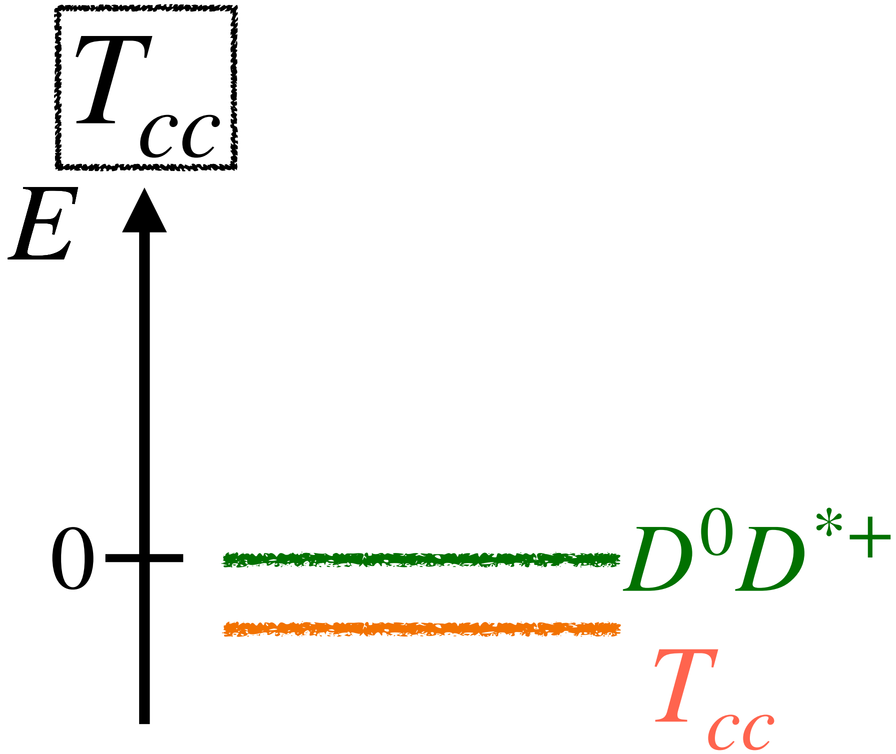
Near-threshold states



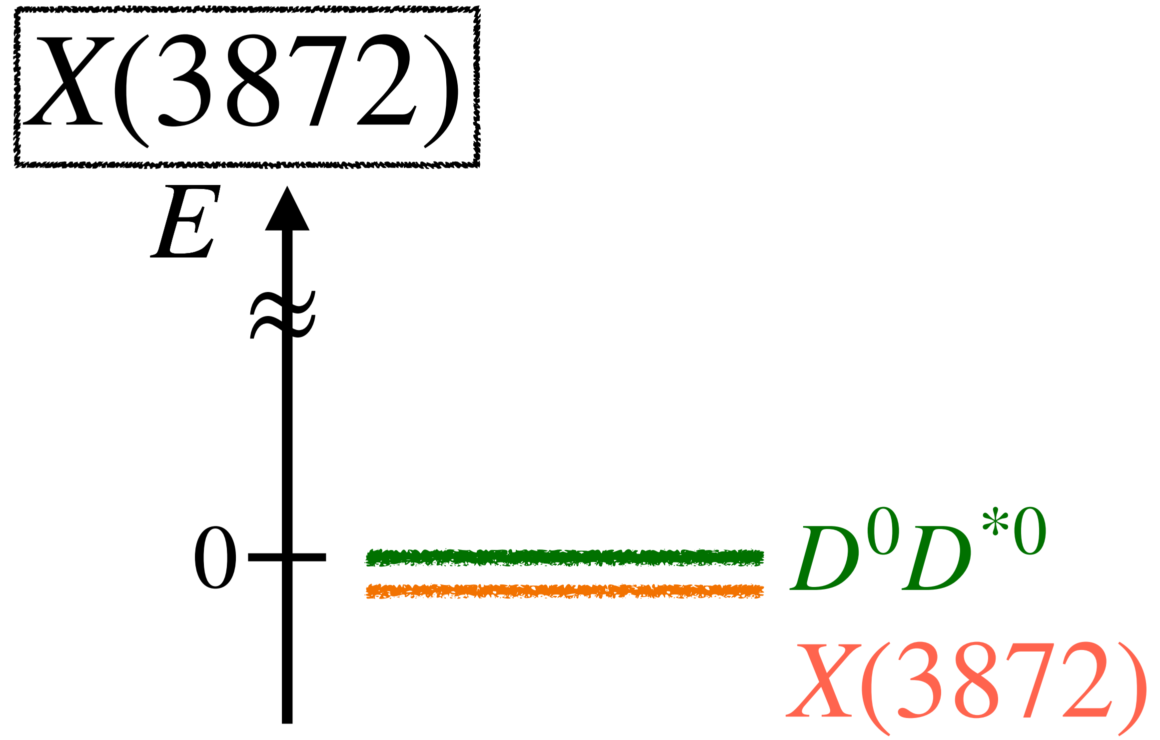
PDG

LHCb Collaboration, Nat. Commun **13** 3351 (2022).

Near-threshold states



LHCb Collaboration, Nat. Commun **13** 3351 (2022).



PDG

- compositeness $X = 1$ in $B \to 0$ limit (universality)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) .

shallow b.s. ($B \neq 0$) seems to be composite dominant

- however, elementary dominant states is realized with fine tuning

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
C. Hanhart, J. R. Pelaez, and G. Rios, Phys. Lett. B **739**, 375 (2014).

⟶ How finely tuning parameter?

Model

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008).

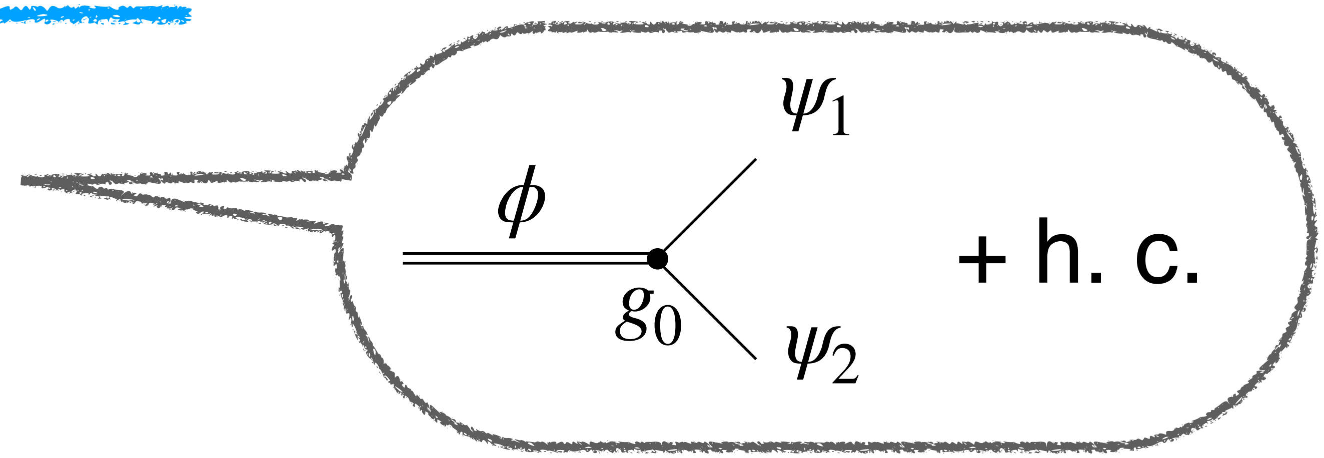
● single-channel resonance model

$$\mathscr{H}_{\text{free}} = \frac{1}{2m_1}\nabla\psi_1{}^\dagger\cdot\nabla\psi_1 + \frac{1}{2m_2}\nabla\psi_2{}^\dagger\cdot\nabla\psi_2 + \frac{1}{2m_\phi}\nabla\phi^\dagger\cdot\nabla\phi + \nu_0\phi^\dagger\phi,$$

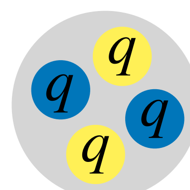
1.

$$\mathscr{H}_{\text{int}} = g_0(\phi^\dagger\psi_1\psi_2 + \psi_1{}^\dagger\psi_2^\dagger\phi)\,.$$

2.



1. single-channel scattering
2. coupling to bare state ϕ



- coupling const. : g_0
- bare state energy : ν_0

● scattering amplitude

$$V = \frac{g_0^2}{E-\nu_0}, \quad G = -\frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right].$$

Λ : cutoff (finite)

$$T = \frac{1}{V^{-1}-G} \longrightarrow f(k) = -\frac{\mu}{2\pi}\left[\frac{\frac{k^2}{2\mu}-\nu_0}{g_0^2} + \frac{\mu}{\pi^2}\left[\Lambda + ik\arctan\left(\frac{\Lambda}{-ik}\right)\right]\right]^{-1}.$$

Model scales and parameters

◉ calculation of compositeness with given B

- typical energy scale : $E_{\rm typ} = \Lambda^2/(2\mu)$

1. reduce d.o.f. of model parameters g_0, ν_0, Λ

coupling const. g_0 : $$g_0^2(B, \nu_0, \Lambda) = \frac{\pi^2}{\mu}(B + \nu_0)\left[\Lambda - \kappa \arctan(\Lambda/\kappa)\right]^{-1}$$

$$\kappa = \sqrt{2\mu B}\,.$$

2. use dimensionless quantities with Λ

⟶ absorb Λ dependence

3. energy of bare quark state ν_0

varied in the region : $-B/E_{\rm typ} \leq \nu_0/E_{\rm typ} \leq 1$

∵ to have $g_0^2 \geq 0$ & applicable limit of model

Compositeness in model

◉ compositeness X

scattering amplitude : $T = \dfrac{1}{V^{-1} - G}$

Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

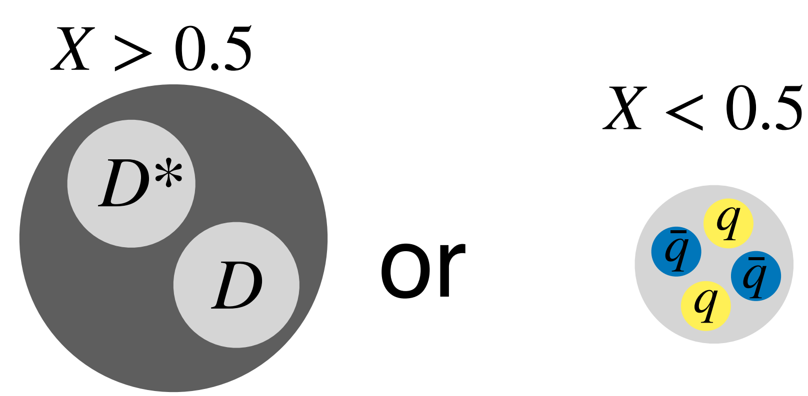
$$\longrightarrow X = \frac{G'(-B)}{G'(-B) - [V^{-1}(-B)]'}, \quad \alpha'(E) = d\alpha/dE$$

$$= \left[1 + \frac{\pi^2\kappa}{g_0^2\mu^2}\left(\arctan(\Lambda/\kappa) - \frac{\Lambda/\kappa}{1 + \left(\Lambda/\kappa\right)^2}\right)^{-1}\right]^{-1}.$$

- fix B to consider typical and shallow bound states
- ν_0 region : $-B/E_{\rm typ} \le \nu_0/E_{\rm typ} \le 1$

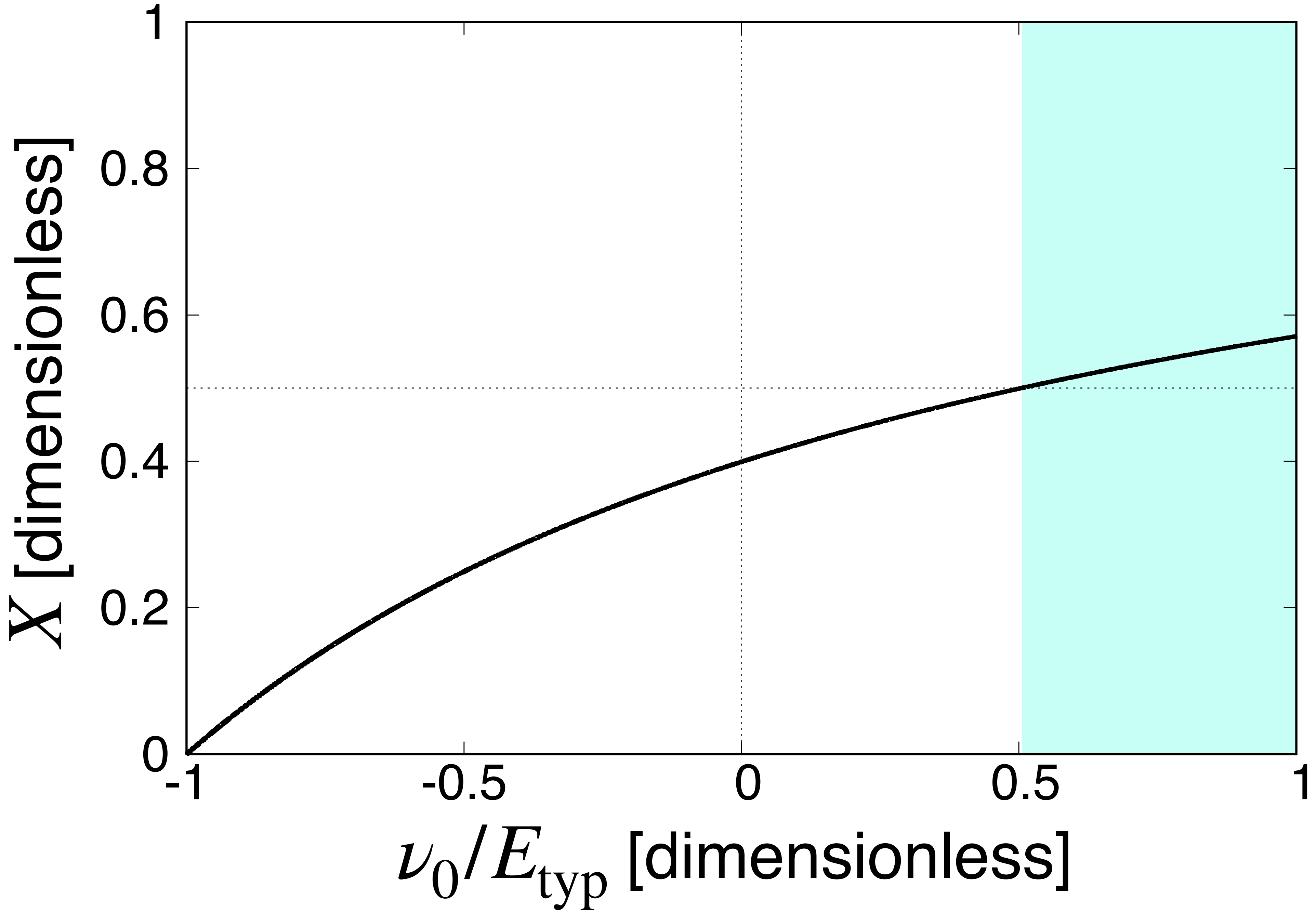
⟷ model dependence of X

compositeness X as a function of ν_0

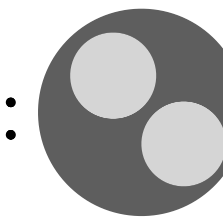


⟶ model dependence of structure of bound state?

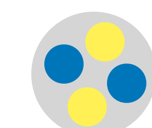
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = E_{\rm typ}$



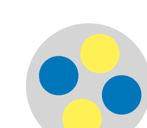
$X > 0.5$:



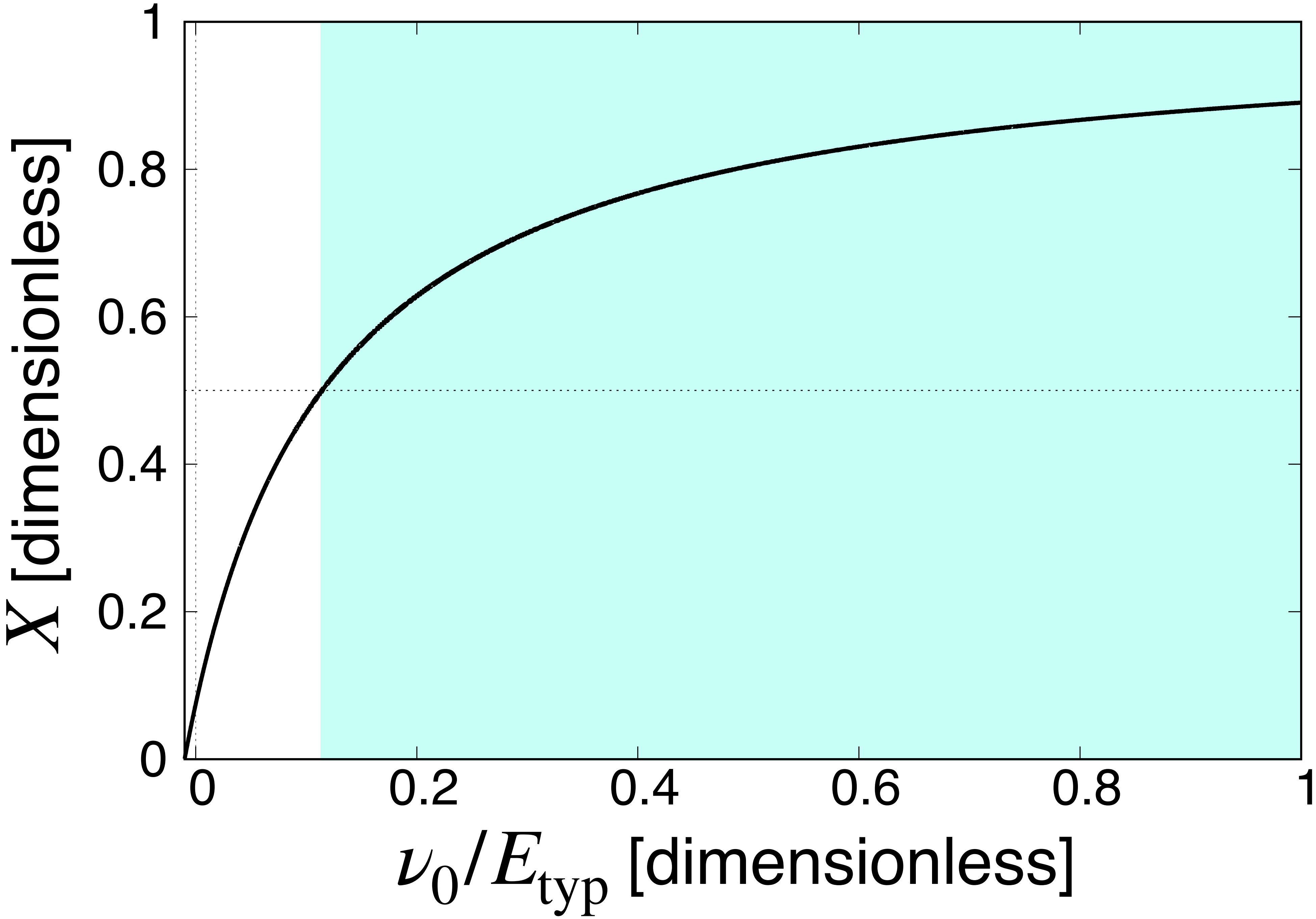
$X < 0.5$:



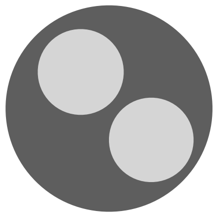
- typical energy scale : $B = E_{\rm typ} = \Lambda^2/(2\mu)$
- $X > 0.5$ only for $25\ \%$ of ν_0 $\because$ **bare state origin**



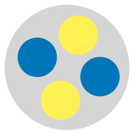
◉ X as a function of $\nu_0/E_{\rm typ}$ of bound state $B = 0.01E_{\rm typ}$



$X > 0.5$:



$X < 0.5$:



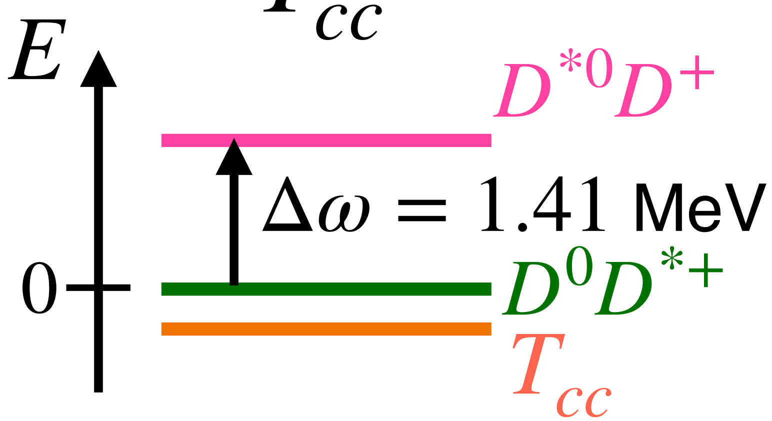
- weakly-bound state : $B = 0.01E_{\rm typ}$
- $X > 0.5$ for 88 % of ν_0 ⟶ **realization of universality**
- elementary dominant state can be realized with fine tuning

Application to T_{cc} and $X(3872)$

◉ <u>exotic hadron</u> ⟵ decay and coupled channel

small
Γ and $\Delta\omega$

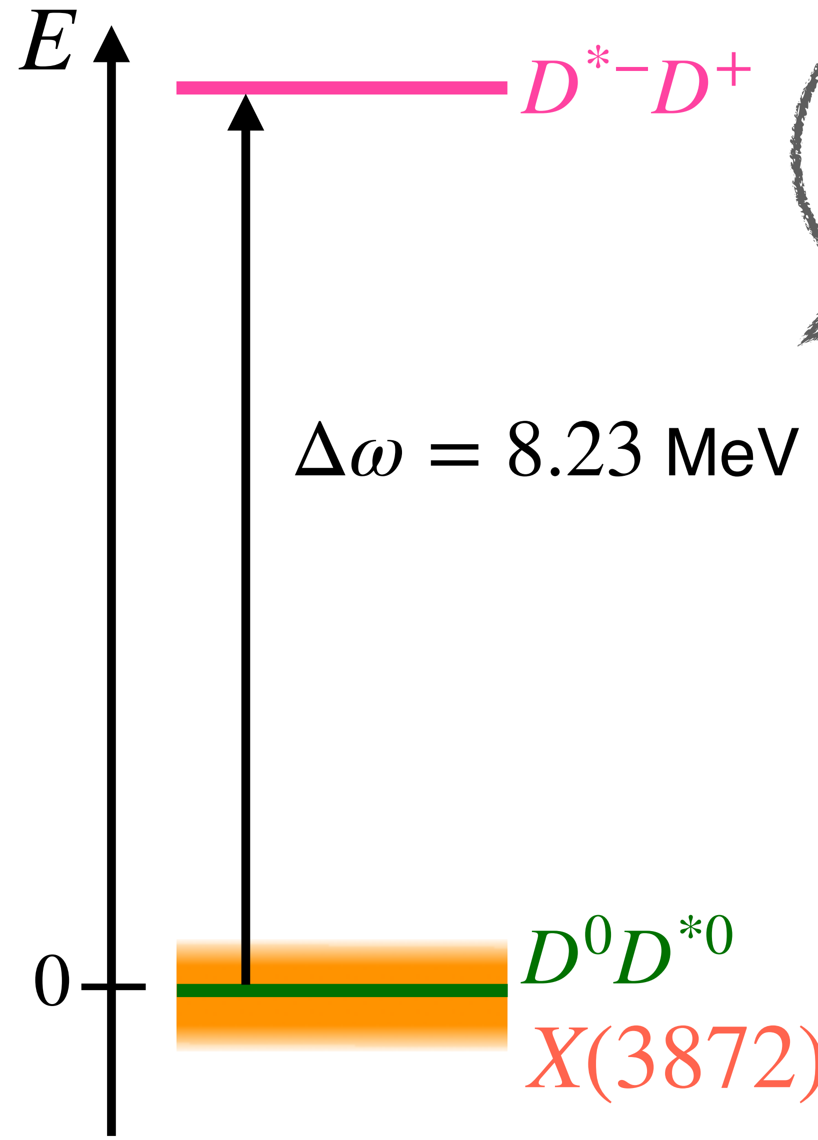
T_{cc}



$B = 0.36$ MeV
$\Gamma = 0.048$ MeV

LHCb Collaboration, Nat. Commun **13** 3351 (2022).

$X(3872)$



large
Γ and $\Delta\omega$

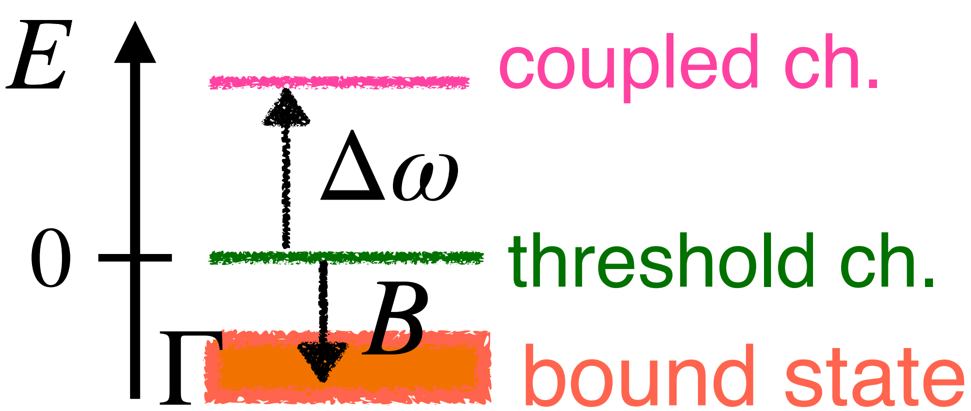
$B = 0.04$ MeV
$\Gamma = 1.19$ MeV

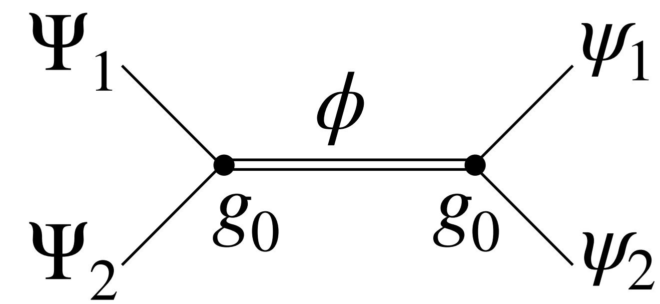
PDG

Effect of decay & coupled channel

$|\Psi\rangle = \sqrt{X_1}\,|\text{threshold ch}\rangle + \sqrt{X_2}\,|\text{coupled ch}\rangle + \sqrt{1-(X_1+X_2)}\,|\text{others}\rangle$

- threshold energy difference $\Delta\omega$
- ch. 1 couples to ch. 2 through ϕ with same coupling const.





- decay width $E = -B - i\Gamma/2$
- effectively introduced : coupling const. $g_0 \in \mathbb{C}$

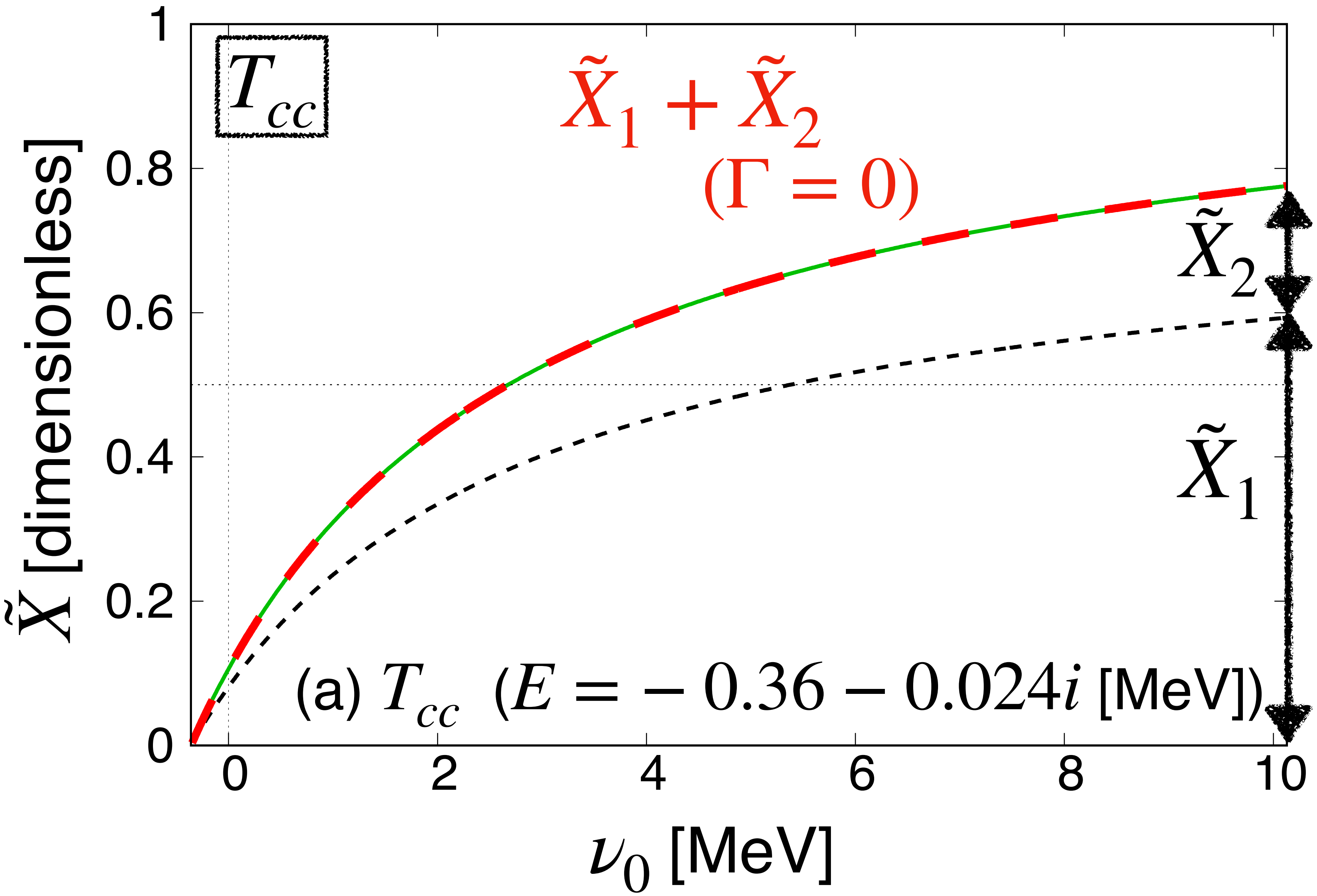
◉ compositeness T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

$$\tilde{X}_j = \frac{|X_j|}{\sum_j |X_j| + |Z|}, \quad (j = 1,2)$$

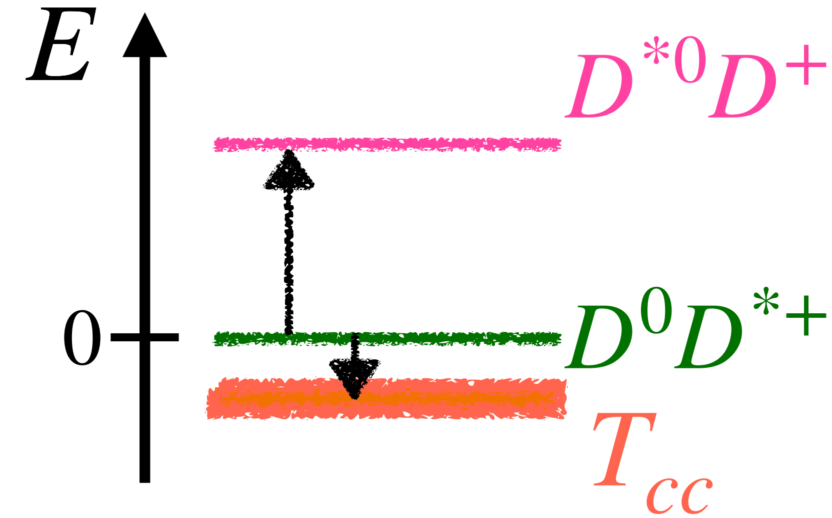
$\tilde{X}_1$: threshold ch. compositeness

$\tilde{X}_2$: coupled ch. compositeness

Application to T_{cc}



$\Lambda = 140$ MeV
(π meson)



$\tilde{X}_1 \sim D^0D^{*+}$

$\tilde{X}_2 \sim D^{*0}D^+$

- $\tilde{X}_2$ is not negligible

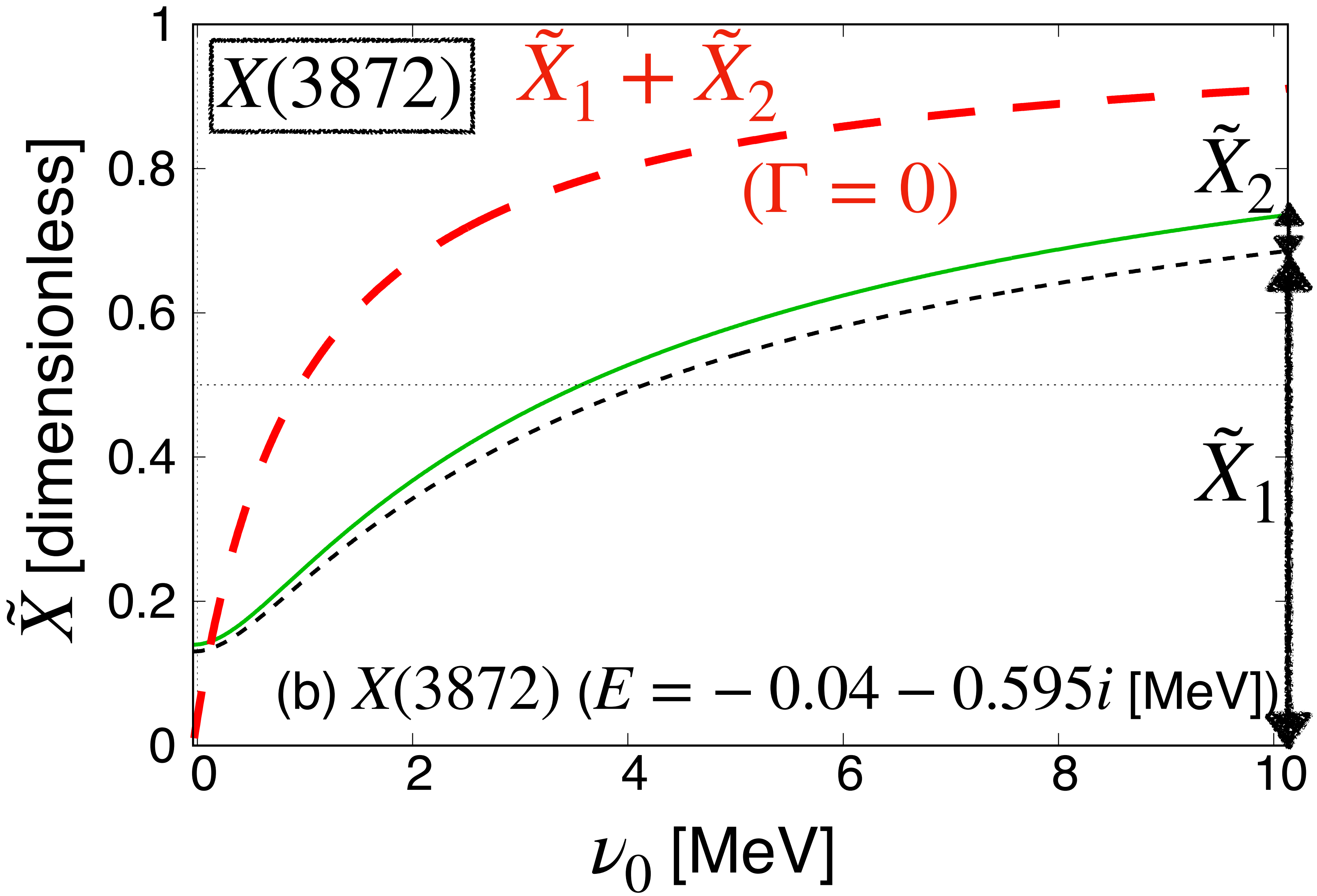
  $\because$ coupled ch. contribution (small $\Delta\omega$)

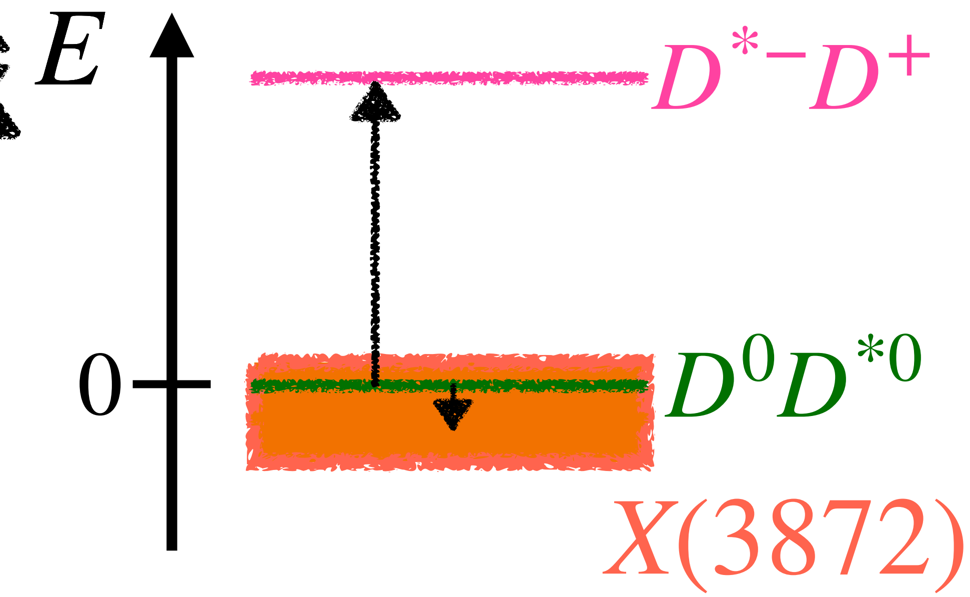
- difference of $\tilde{X}_1 + \tilde{X}_2$ $(\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is too small

  ⟶ We can neglect decay contribution $\because \Gamma \ll B$

Application to $X(3872)$

$\Lambda = 140$ MeV
(π meson)





$\tilde{X}_1 \sim D^0 D^{*0}$

$\tilde{X}_2 \sim D^{*-}D^{+}$

- difference of $\tilde{X}_1 + \tilde{X}_2 (\Gamma = 0)$ and $\tilde{X}_1 + \tilde{X}_2$ is large
  - ∵ large decay width contribution
- $\tilde{X}_2$ is much smaller than $\tilde{X}_1$
  - → coupled ch. effect is small

New interpretation scheme

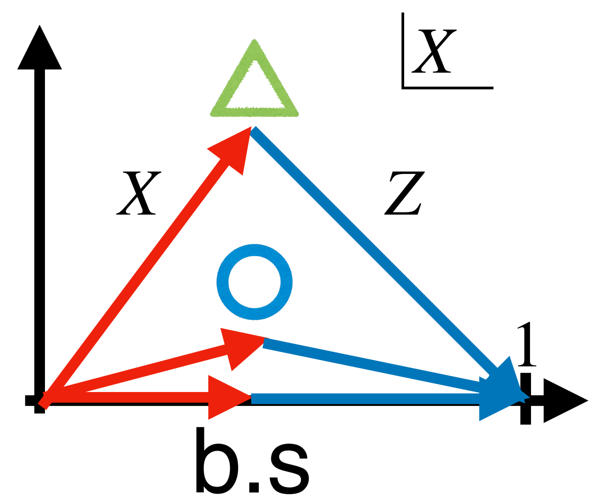
T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

- interpretation for complex X of resonances?

$X \in \mathbb{C}$ and $X + Z = 1$

- If Im X is large, it seems that reasonable interpretation is impossible △

complex X plane



◉ **our proposal** T. Berggren, Phys. Lett. B 33, 547 (1970).

i) $\mathcal{X}$: probability of certainly finding $|\mathrm{composite}\rangle$
ii) $\mathcal{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$
iii) $\mathcal{Y}$: probability of uncertain identification

◉ **conditions for sensible interpretation**

- normalization : $\mathcal{X} + \mathcal{Y} + \mathcal{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathcal{X} \to X$, $\mathcal{Z} \to Z$ and $\mathcal{Y} \to 0$

$\mathcal{Y}$ characterizes uncertainty of resonance

structure of T_{cc} & $X(3872)$

◉ compositeness X in previous model with our scheme

- fix ν_0 from quark models
- $X_2 + Z \to Z$

T_{cc} $\nu_0 = 7$ MeV M. Karliner and J. L. Rosner, Phys. Rev. Lett. 119, 202001 (2017).

$$X_{D^0 D^{*+}} = 0.541 - 0.007i$$

$$\longrightarrow \mathscr{X} = 0.537,\ \mathscr{Y} = 0.008,\ \mathscr{Z} = 0.456$$

higher channel contribution

$X(3872)$ $\nu_0 = 78.36$ MeV S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).

$$X_{D^0 D^{0*}} = 0.919 - 0.079i$$

$$\longrightarrow \mathscr{X} = 0.890,\ \mathscr{Y} = 0.028,\ \mathscr{Z} = 0.081$$

large bare state energy $\nu_0 \gg E_{\text{typ}}$

structure of T_{cc} & $X(3872)$

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- fix ν_0 from quark models
- $X_2 + Z \to Z$

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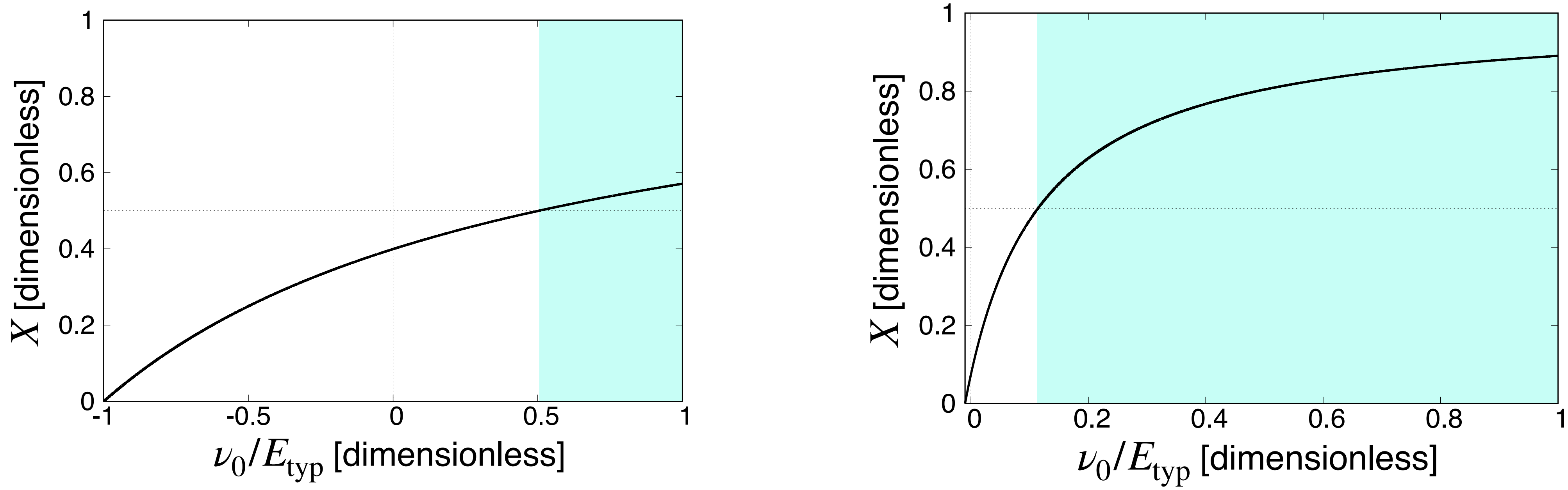
Summary

T. Kinugawa and T. Hyodo Phys. Rev. C 109 , 045205 (2024).
T. Kinugawa and T. Hyodo, arXiv:2403.12635 [hep-ph].

- internal structure of exotic hadrons ⟵ compositeness

- shallow bound states

⟶ fine tuning is necessary to realize elementary dominant state



typical b.s. shallow b.s.

T_{cc} : important coupled ch. effect with negligible decay effect

$X(3872)$: important decay effect with negligible coupled ch. effect

Back up

Effect of decay

◉ introducing decay effect

- formally : introducing decay channel in lower energy region than binding energy

  ⟶ eigenenergy becomes complex

- effectively : coupling const. $g_0 \in \mathbb{C}$! ⟵ this work

$$\mathscr{H}_{\rm int} = g_0(\phi^\dagger \psi_1 \phi_2 + \phi_1{}^\dagger \psi_2^\dagger \phi)\,.$$

$$E = -B \longrightarrow E = -B - i\Gamma/2$$

compositeness

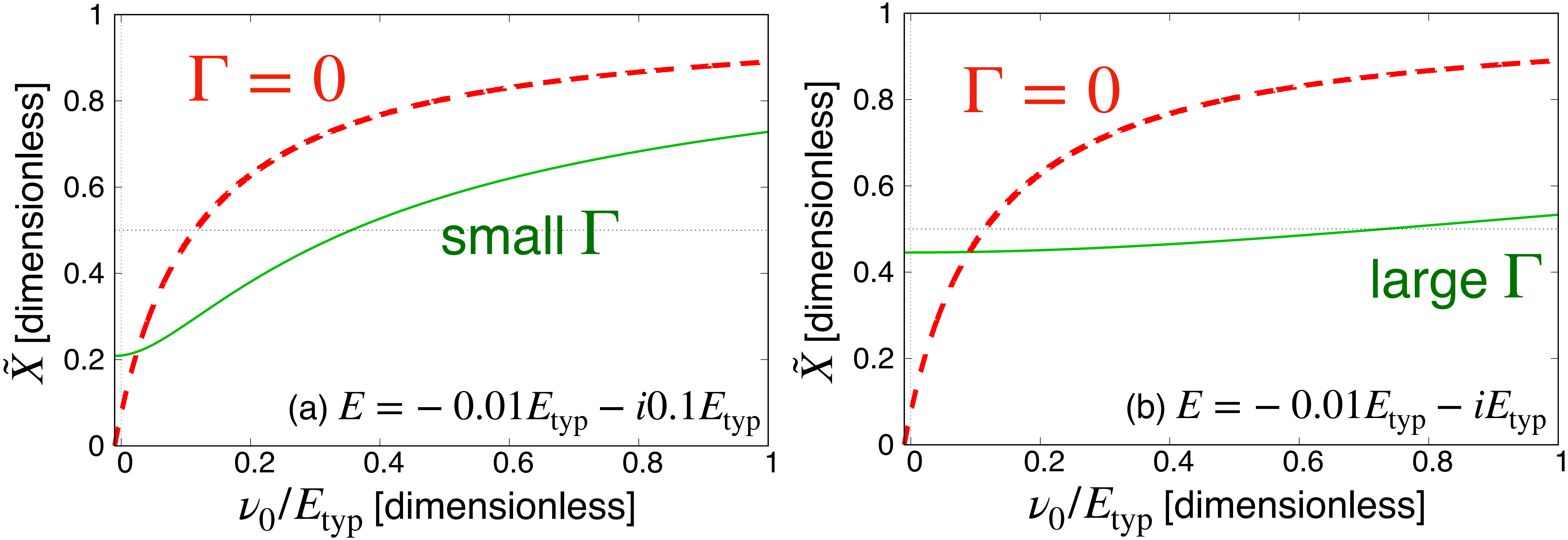
$$X \in \mathbb{R} \longrightarrow X \in \mathbb{C}$$

$$\tilde{X} = \frac{|X|}{|X| + |1 - X|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC 93, 035204 (2016).

Effect of decay

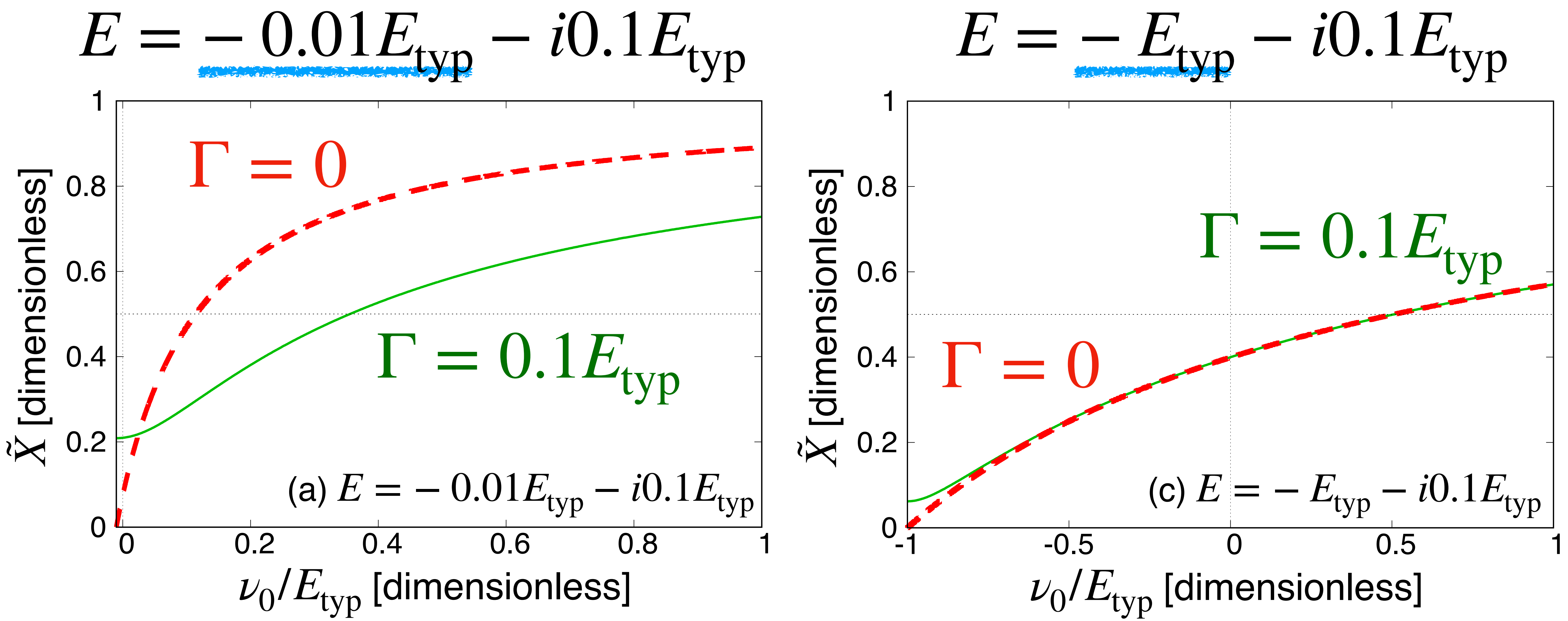
$E = -0.01 E_{\rm typ} - i0.1 E_{\rm typ}$ $\qquad$ $E = -0.01 E_{\rm typ} - i E_{\rm typ}$



- $\tilde{X}$ is suppressed by decay effect

∵ threshold ch. component ($\tilde{X}$) decreases with inclusion of decay ch. component ($1 - \tilde{X}$)

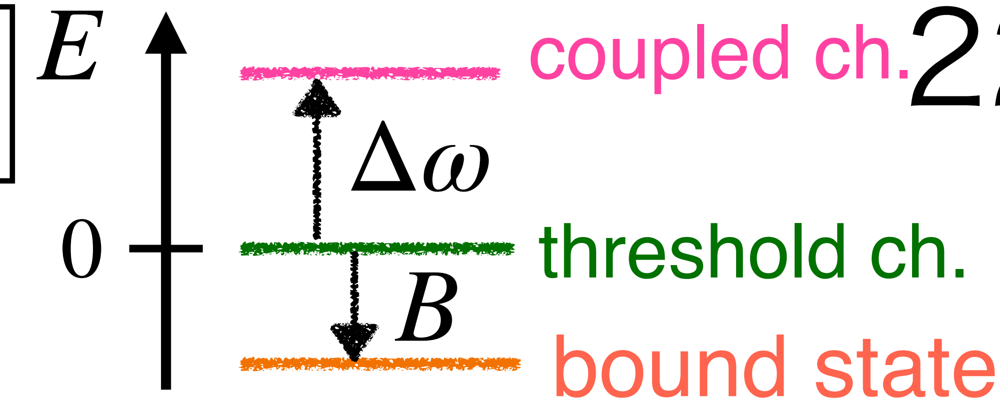
Effect of decay



compositeness is more suppressed when B is small

- suppression of $\tilde{X}$ is determined by ratio of B to Γ

Effect of coupled channel



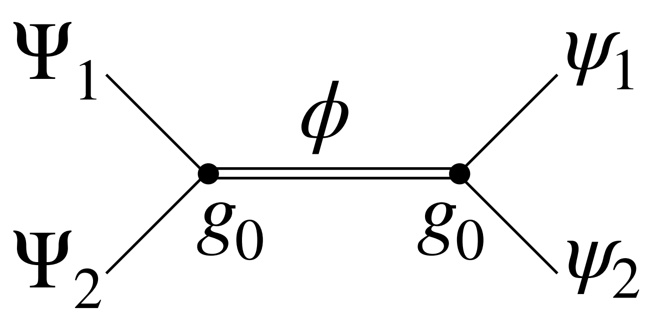
◉ introducing coupled channel $\Psi_1\Psi_2$

$$|\Psi\rangle = \sqrt{X_1}\,|\text{threshold ch}\rangle + \sqrt{X_2}\,|\text{coupled ch}\rangle + \sqrt{1-(X_1+X_2)}\,|\text{others}\rangle$$

$$\mathscr{H}_{\text{free}} = (\text{kinetic terms of } \psi_{1,2}, \Psi_{1,2}, \phi) + \omega_1\Psi_1\Psi_1^\dagger + \omega_2\Psi_2\Psi_2^\dagger + \nu_0\phi^\dagger\phi,$$

$$\mathscr{H}_{\text{int}} = g_0(\phi^\dagger\psi_1\psi_2 + \psi_1^\dagger\psi_2^\dagger\phi + \phi^\dagger\Psi_1\Psi_2 + \Psi_1^\dagger\Psi_2^\dagger\phi)\,.$$

- threshold energy difference $\Delta\omega = \omega_1 + \omega_2$
- ch. 1 couples to ch. 2 through ϕ with same coupling const.



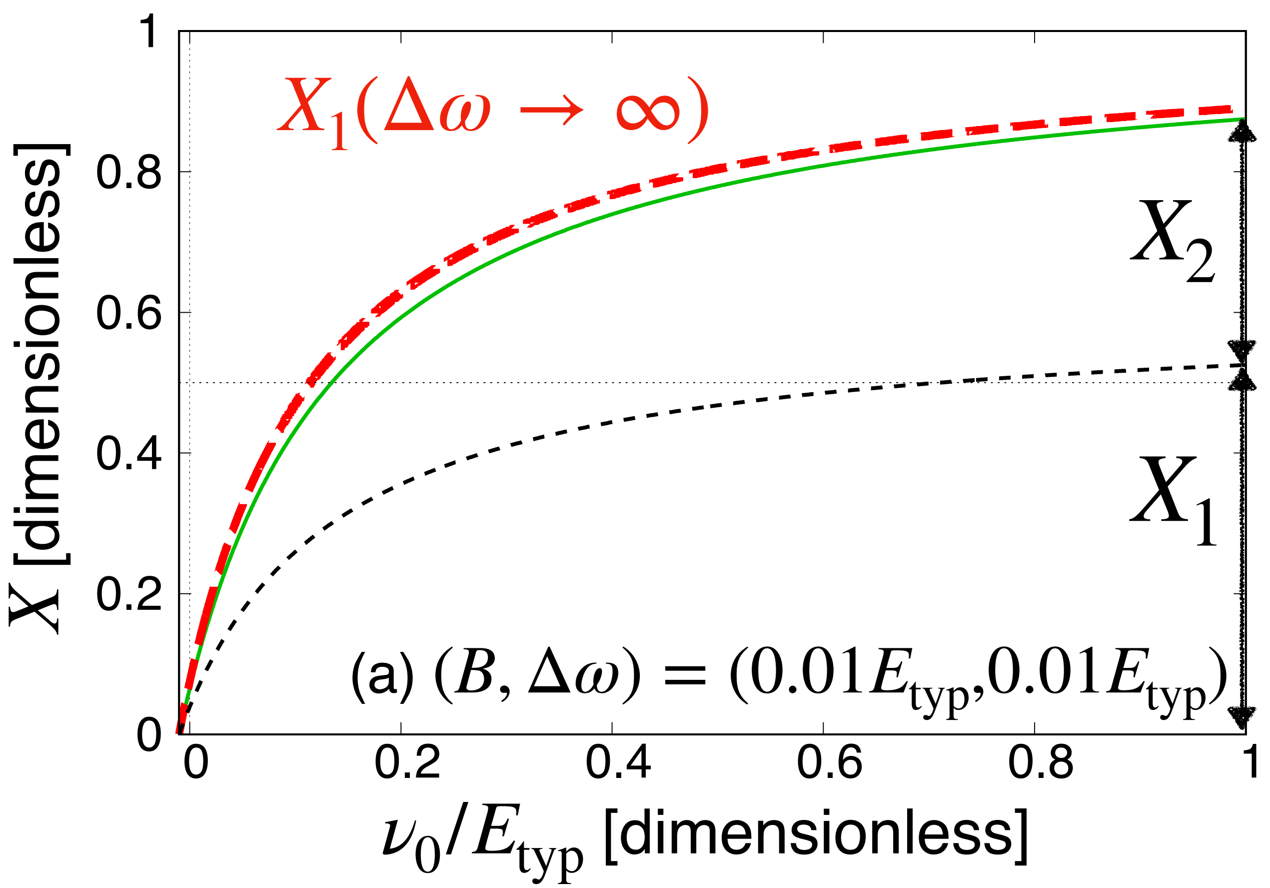
- low-energy universality with coupled-channel effect

$X_1 \sim 1$ (threshold channel)

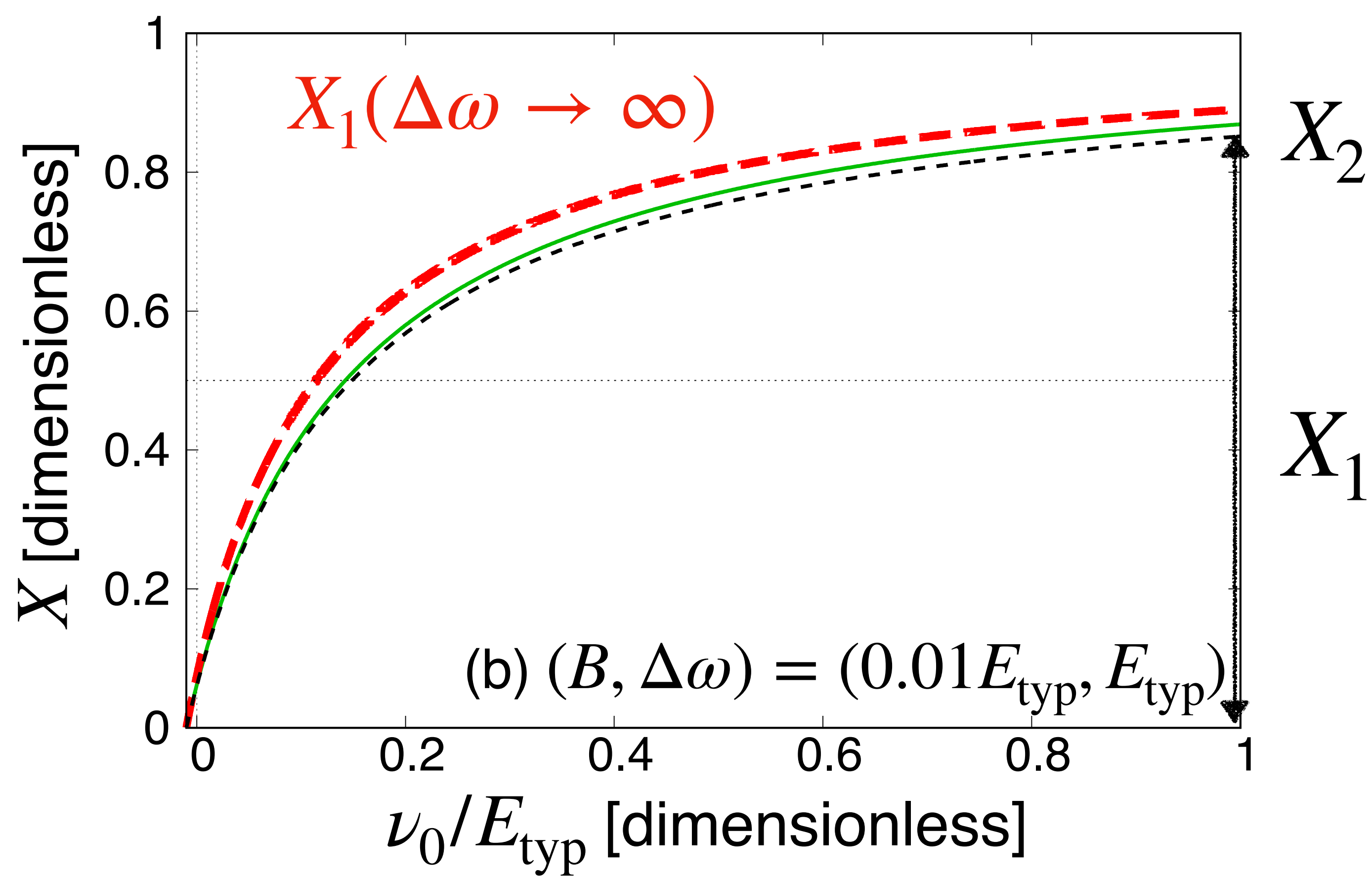
$X_2 \sim 0$ and $Z \sim 0$ (other channel)

Effect of coupled channel

$(B, \Delta\omega) = (0.01E_{\mathrm{typ}}, 0.01E_{\mathrm{typ}})$



$(B, \Delta\omega) = (0.01E_{\mathrm{typ}}, E_{\mathrm{typ}})$



- X_1 is suppressed by channel coupling

  ∵ threshold ch. component (X_1) decreases with inclusion of coupled ch. component (X_2)

- $Z = 1 - (X_1 + X_2)$ is stable

Compositeness for two-channel case

$$V(k) = \begin{pmatrix} v(k) & v(k) \\ v(k) & v(k) \end{pmatrix}, \quad v(k) = \frac{g_0^2}{\frac{k^2}{2\mu_1} - \nu_0}.$$

$$G(k) = \begin{pmatrix} G_1(k) & 0 \\ 0 & G_2(k) \end{pmatrix}, \quad G_1(k) = -\frac{\mu_1}{\pi^2}\left[\Lambda + ik\arctan\left(-\frac{\Lambda}{ik}\right)\right], \quad G_2(k') = -\frac{\mu_2}{\pi^2}\left[\Lambda + ik'\arctan\left(-\frac{\Lambda}{ik'}\right)\right].$$

$$k = \sqrt{2\mu_1 E}, \quad k'(k) = \sqrt{2\mu_2(E - \Delta\omega)} = \sqrt{\frac{\mu_2}{\mu_1}k^2 - 2\mu_2\Delta\omega}.$$

$$\longrightarrow \quad X_1 = \frac{G_1'}{(G_1' + G_2') - [v^{-1}]'}, \quad X_2 = \frac{G_2'}{(G_1' + G_2') - [v^{-1}]'}.$$

Universality for near-th. resonances

- near-threshold **bound** (and virtual) states

$a_0 \to \infty$ and universality holds in $B \to 0$ limit

→ $X \to 1$ (completely composite)

T. Hyodo, Phys. Rev. C **90**, 055208 (2014) ;
T. Kinugawa, T. Hyodo, 2303.07038 [hep-ph].

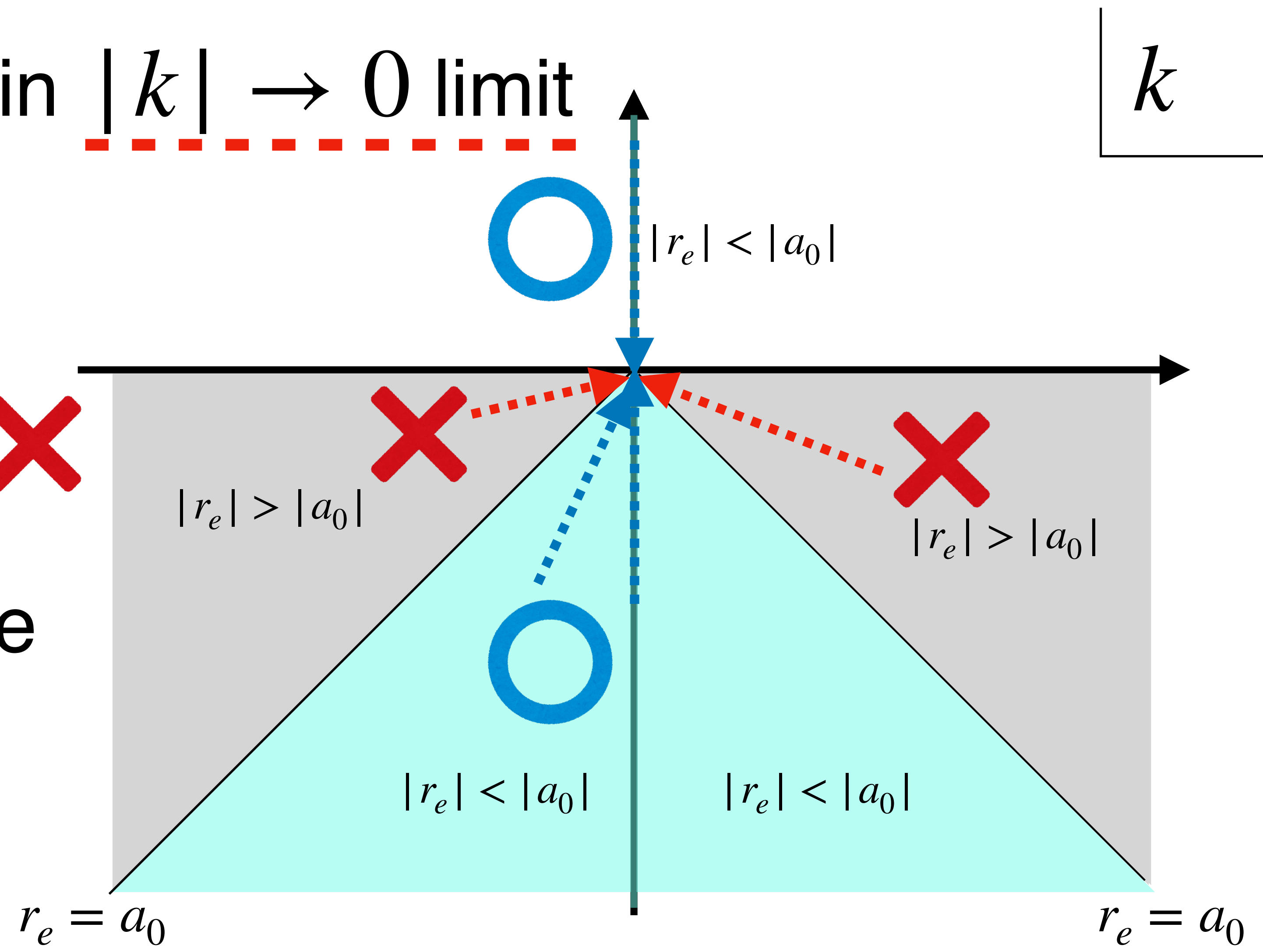
- near-threshold **resonances**

$a_0 \to \infty$ but also $|r_e| \to \infty$ in $|k| \to 0$ limit

$\because |a_0| \leq |r_e|$

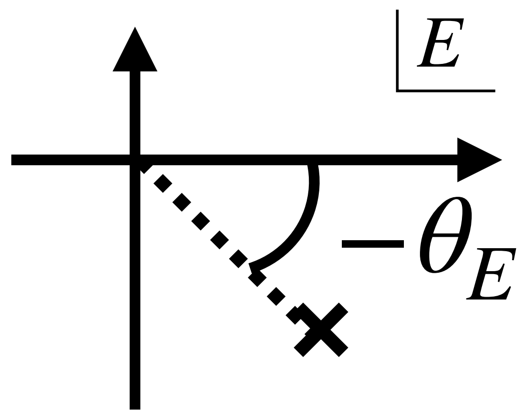
→ universality does not hold

Near-threshold resonances are **not** necessarily composite dominant



Compositeness in ERE

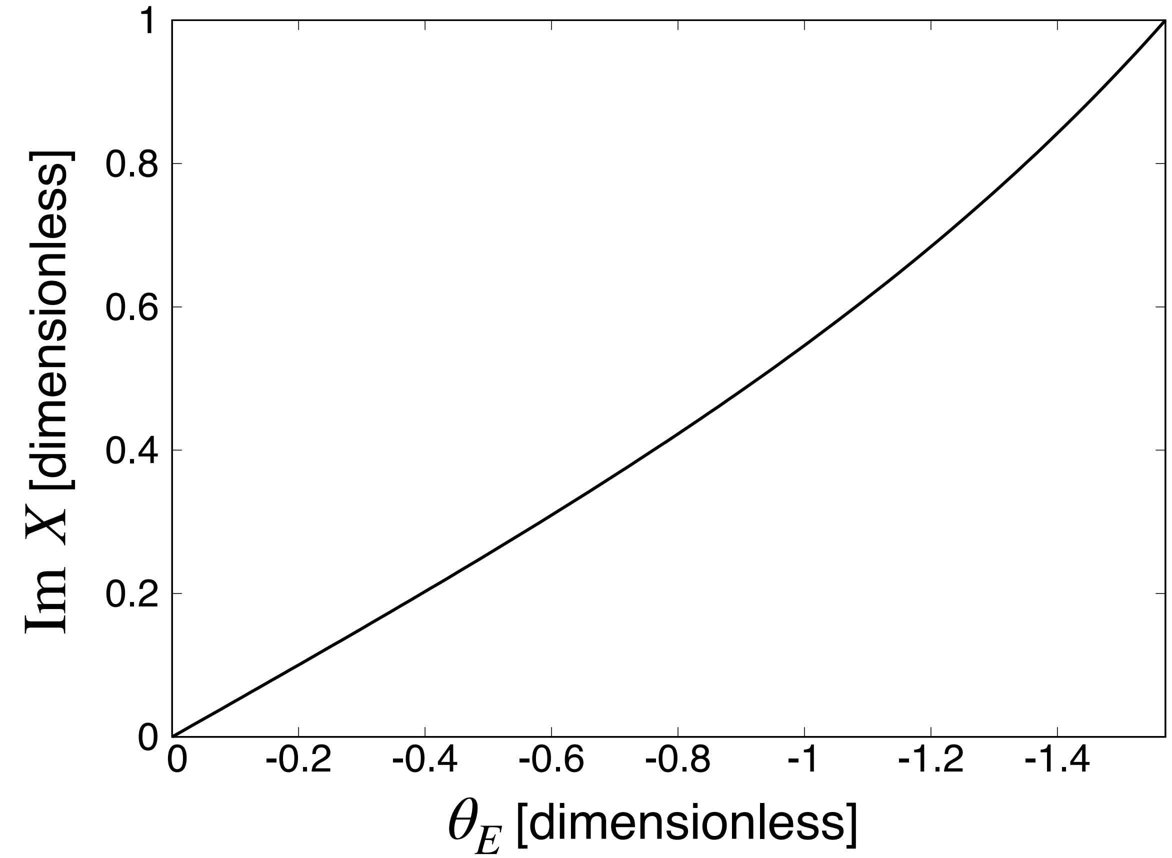
$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i\tan\theta_k = -i\tan(\theta_E/2)$$



$$(k = |k|e^{i\theta_k}, E = |E|e^{i\theta_E})$$

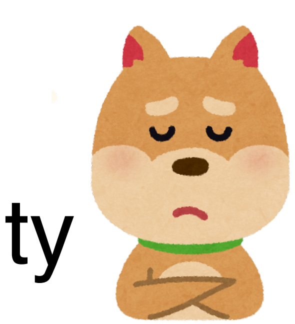
→ X in ERE is pure imaginary

Im X as a function of θ_E



- in general, compositeness X of unstable resonances becomes **complex** by definition

- complex X **cannot** be directly interpreted as a probability



Complex compositeness

- probabilistic interpretation?

$X \in \mathbb{C}$ and $X + Z = 1$

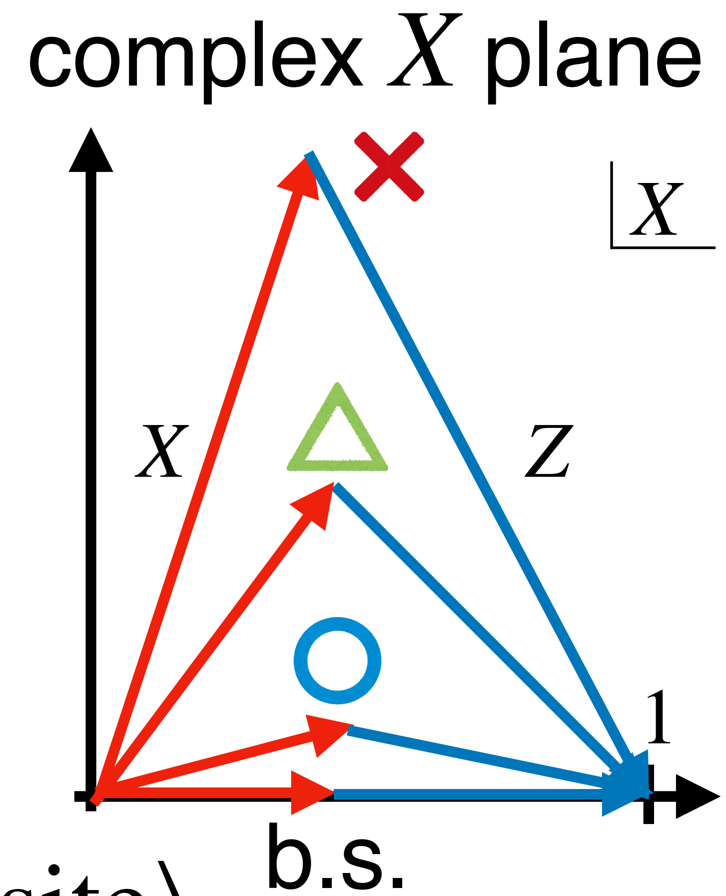
- If Im X is large, it seems that reasonable interpretation is impossible ✕ △

- our proposal

i) $\mathscr{X}$: probability of certainly finding $|\mathrm{composite}\rangle$

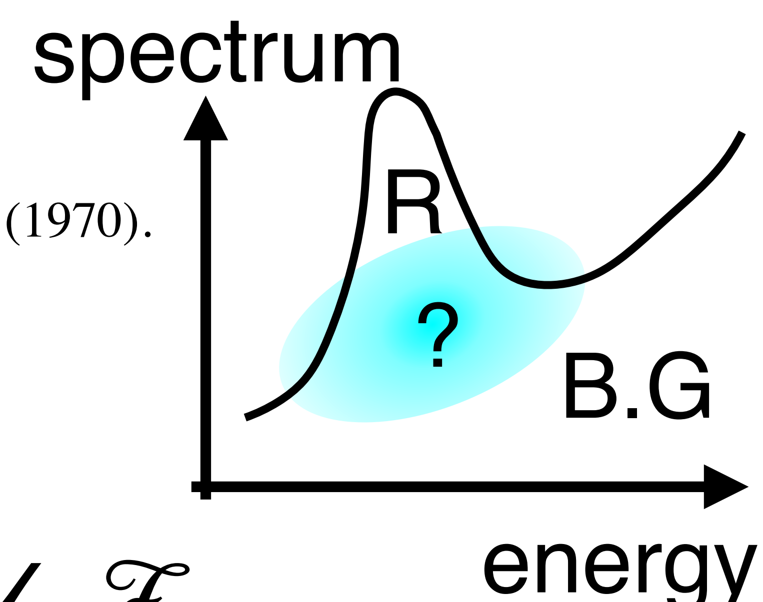
ii) $\mathscr{Z}$: probability of certainly finding $|\mathrm{elementary}\rangle$

iii) $\mathscr{Y}$: probability of uncertain identification



uncertain appears from T. Berggren, Phys. Lett. B 33, 547 (1970).

- finite lifetime (uncertainty in energy)
- separation from B.G.



complex compositeness $X \in \mathbb{C} \longrightarrow \mathscr{X}, \mathscr{Y}, \mathscr{Z}$

Definition

T. Kinugawa and T. Hyodo
arXiv:2403.12635 [hep-ph].

◉ conditions for sensible interpretation

- normalization : $\mathscr{X} + \mathscr{Y} + \mathscr{Z} = 1$ for probabilistic interpretation
- in bound state limit : $\mathscr{X} \to X$, $\mathscr{Z} \to Z$ and $\mathscr{Y} \to 0$

$\mathscr{Y}$ characterizes uncertainty of resonance

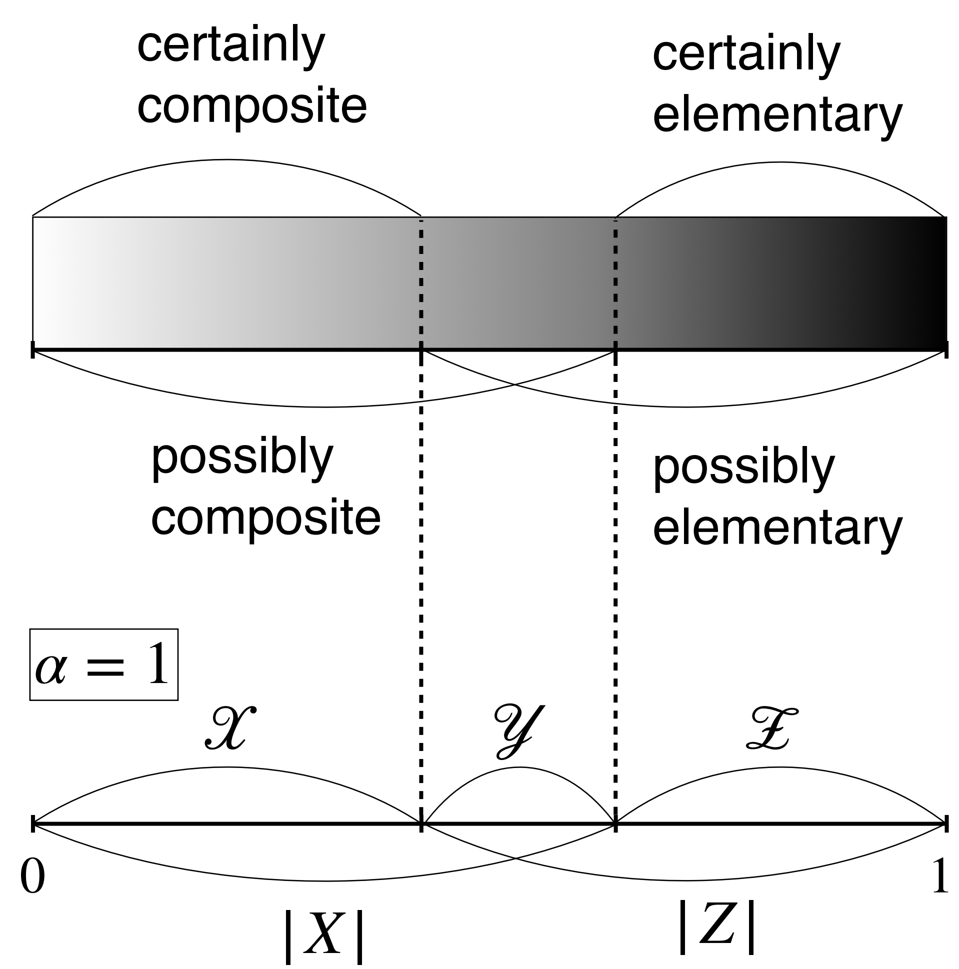
◉ new interpretation

$$\mathscr{X} + \alpha\mathscr{Y} = |X|, \quad \mathscr{Z} + \alpha\mathscr{Y} = |Z|$$

$$\mathscr{X} = \frac{(\alpha - 1)|X| - \alpha|Z| + \alpha}{2\alpha - 1}$$

$$\mathscr{Z} = \frac{(\alpha - 1)|Z| - \alpha|X| + \alpha}{2\alpha - 1}$$

$$\mathscr{Y} = \frac{|X| + |Z| - 1}{2\alpha - 1}$$

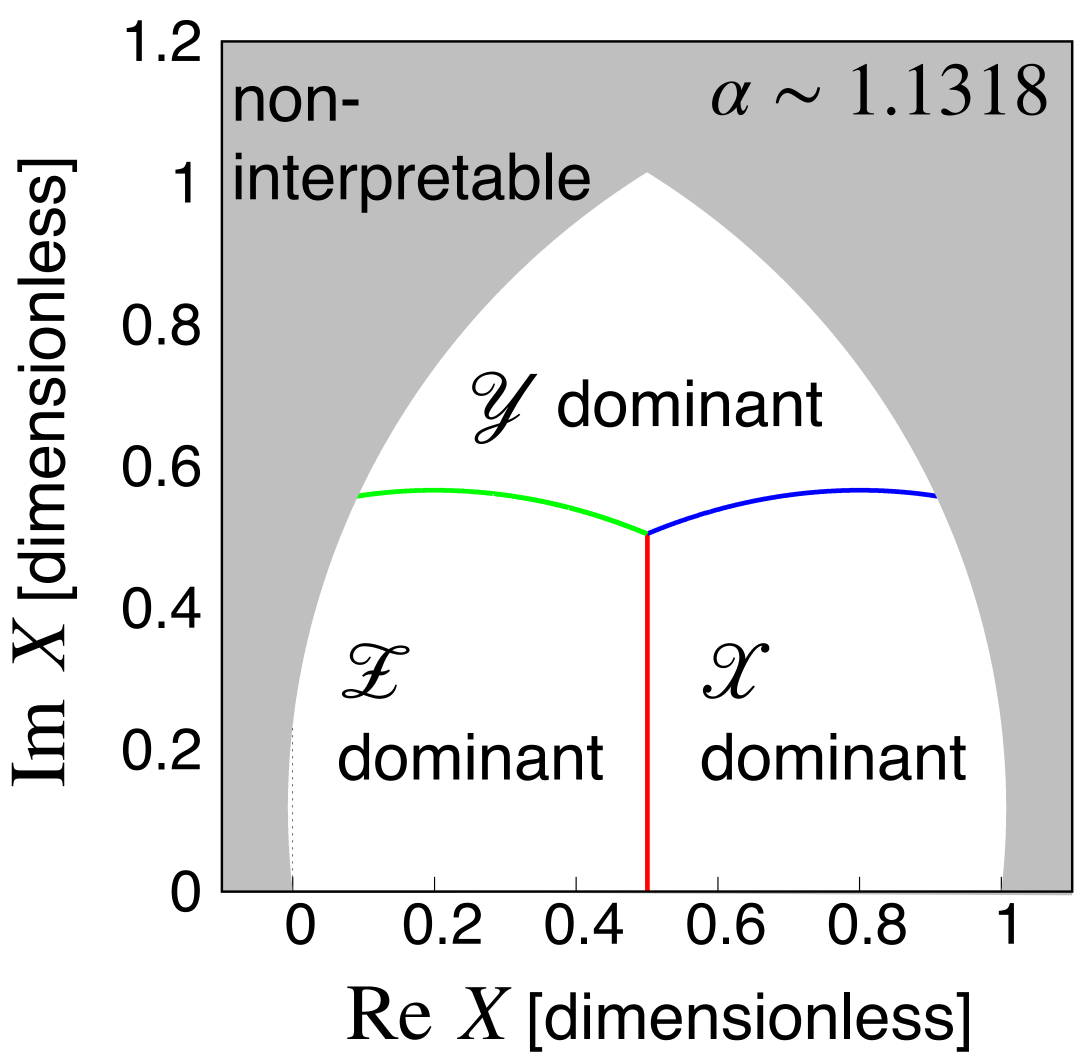


α reflects uncertain nature of resonances

Definition

- if $\alpha > 1/2$, $\mathscr{Y}$ is always positive but $\mathscr{X}$, $\mathscr{Z}$ can be negative

	$\mathcal{X} > \mathcal{Y}, \mathcal{Z}$	composite dominant
$\mathcal{X} \geq 0$ and $\mathcal{Z} \geq 0$	$\mathcal{Z} > \mathcal{Y}, \mathcal{X}$	elementary dominant
	$\mathcal{Y} > \mathcal{X}, \mathcal{Z}$	uncertain
$\mathcal{X} < 0$ or $\mathcal{Z} < 0$		non-interpretable



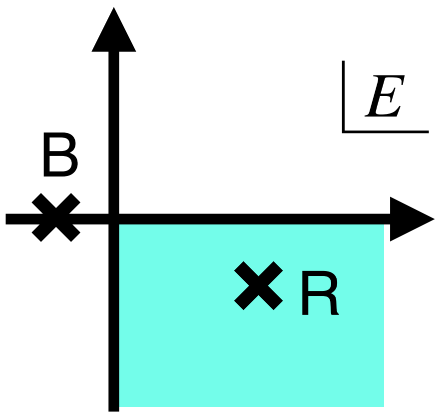
<u>our criterion for physical "state"</u>

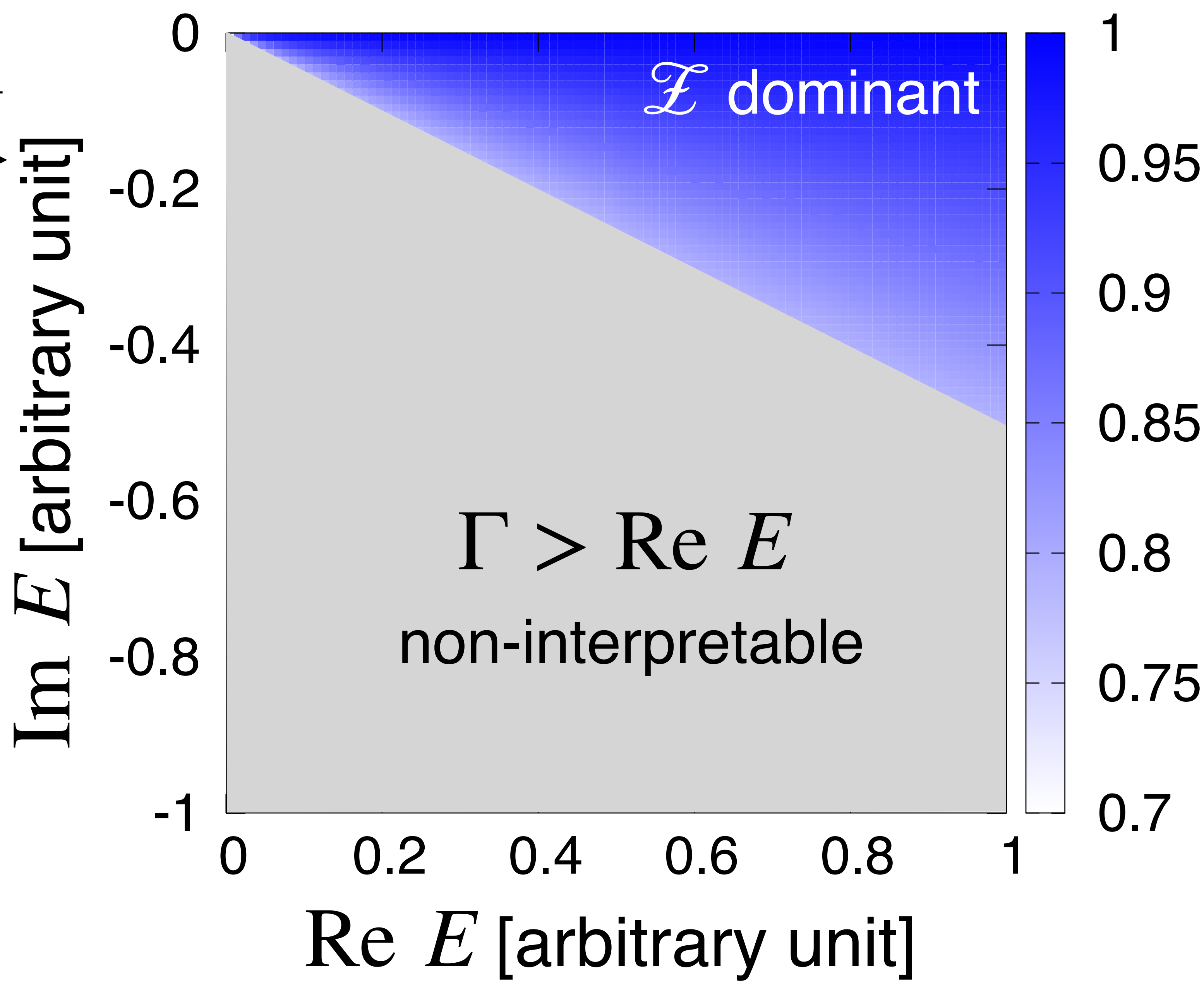
$$\Gamma \leq \mathrm{Re}\ E$$

- exclude poles which we cannot regard as physical "state" from probabilistic interpretation

$\mathscr{X}$, $\mathscr{Y}$, $\mathscr{Z}$ dominant regions
and
non-interpretable region

Structure of near-th. resonances





- resonances are **not composite dominant state** ($\mathcal{Z} \gtrsim 0.8$)
- different from near-threshold bound states (composite dominant $X \sim 1$ and $Z \sim 0$)

resonances with previous works

$$\bar{Z} = 1 - \sqrt{\left|\frac{1}{1-2r_e/a_0}\right|}$$

T. Hyodo, Phys. Rev. Lett. **111**, 132002 (2013).

$$\tilde{Z}_{\rm KH} = \frac{1+|Z|-|X|}{2}$$

Y. Kamiya and T. Hyodo, Phys. Rev. C **93**, 035203 (2016).

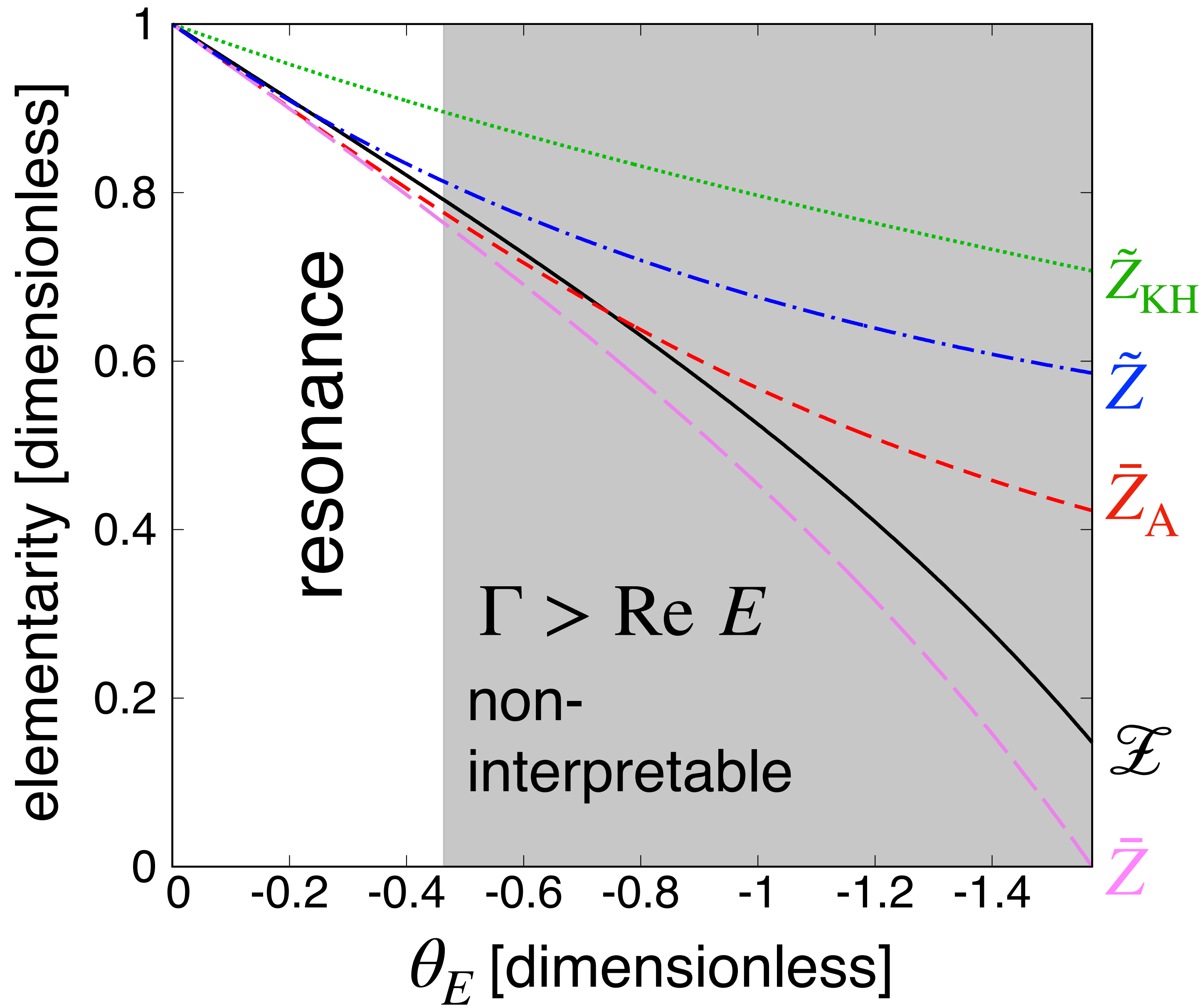
$$\tilde{Z} = \frac{|Z|}{|X|+|Z|}$$

T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, PRC **93**, 035204 (2016).

$$\bar{Z}_A = 1 - \sqrt{\frac{1}{1+|2r_e/a_0|}}$$

I. Matuschek, V. Baru, F.-K. Guo, and C. Hanhart, Eur. Phys. J. A **57**, 101 (2021).

interpretations as a function of θ_E



- all interpretations show resonances are elementary dominant

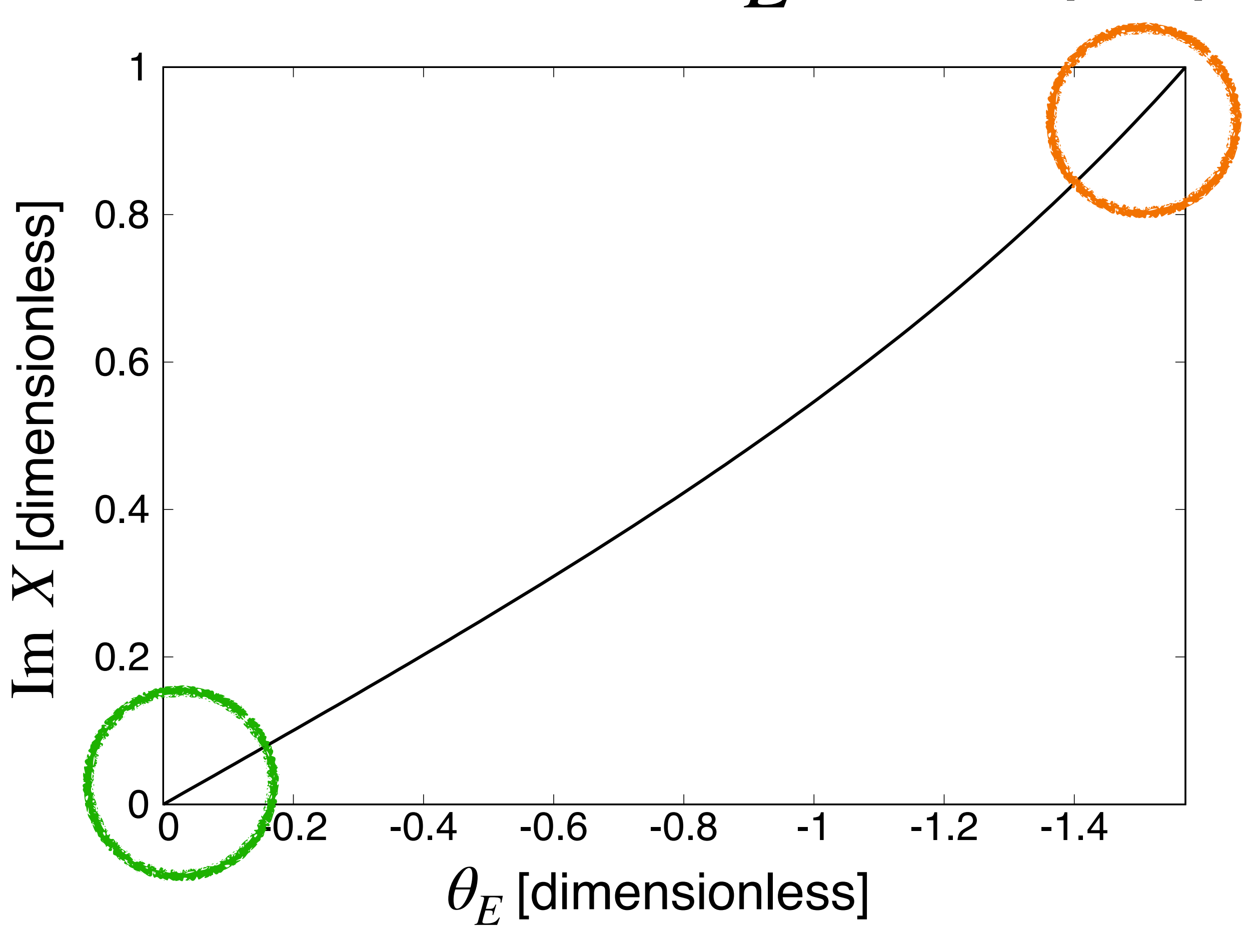
Compositeness in ERE

$$X = \sqrt{\frac{1}{1 - \frac{2r_e}{a_0}}} = -i \tan \theta_k$$

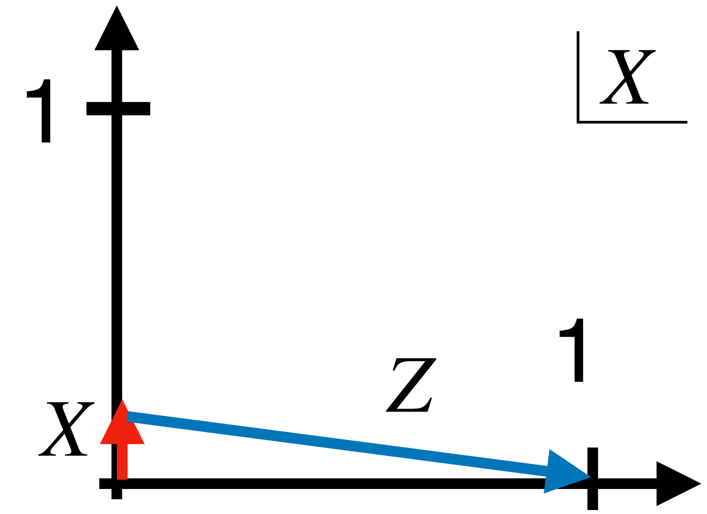
$$(k = |k| e^{i\theta_k})$$

→ X in ERE is pure imaginary

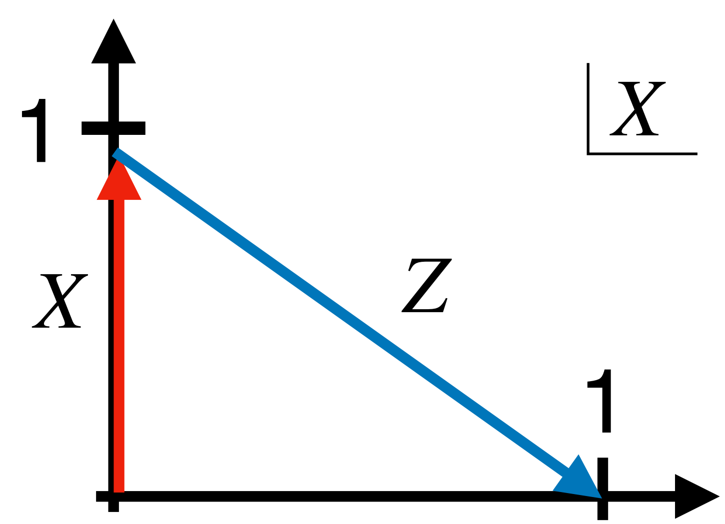
X as a function of θ_E $(E = |E| e^{i\theta_E})$



small width $(\theta_E \sim 0)$

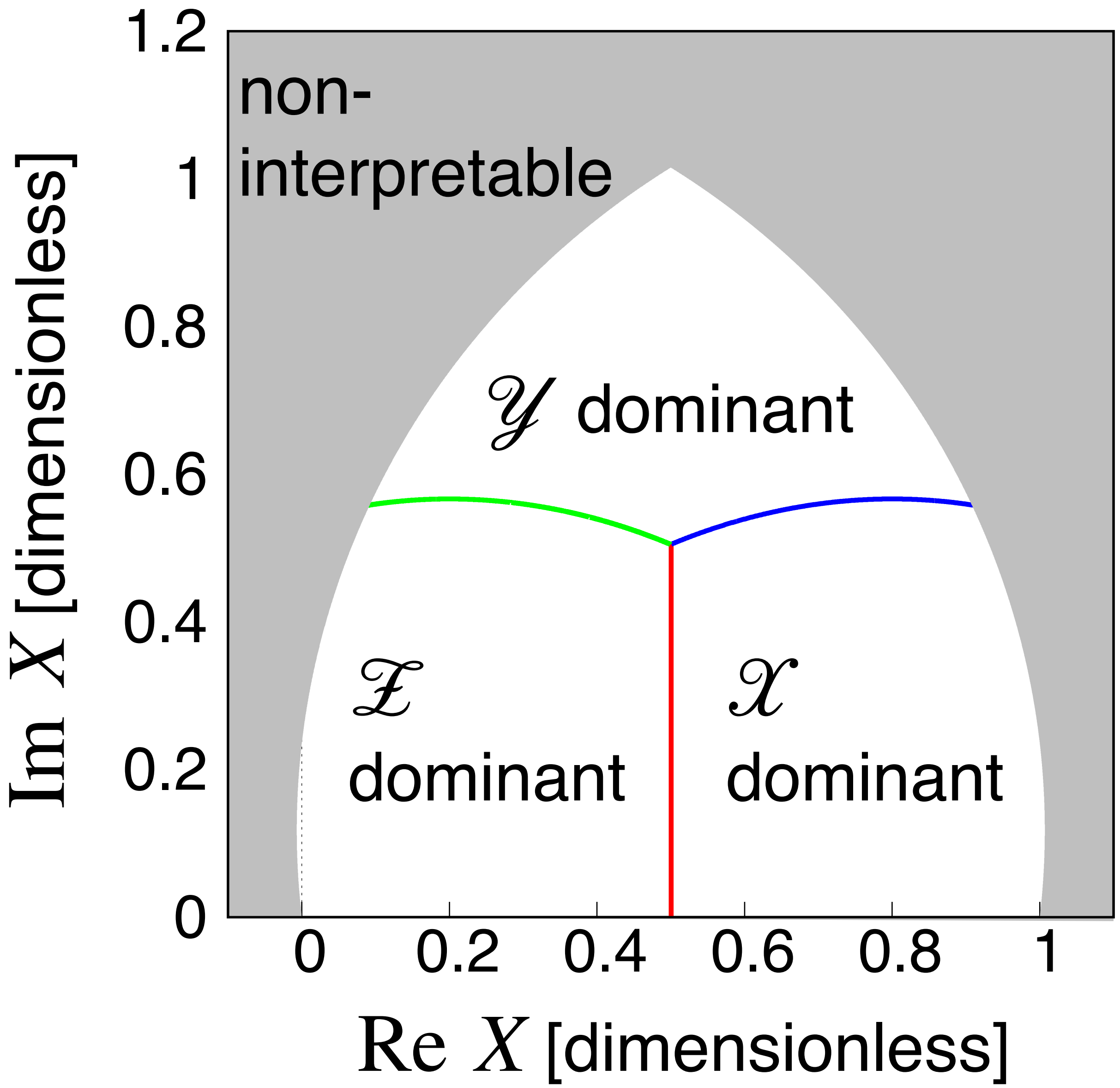


large width $(\theta_E \sim -\pi/2)$



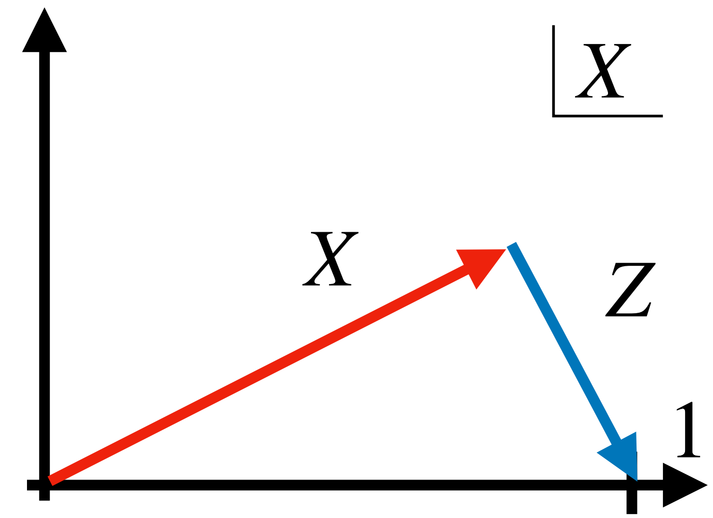
$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region

- $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region in complex X plane

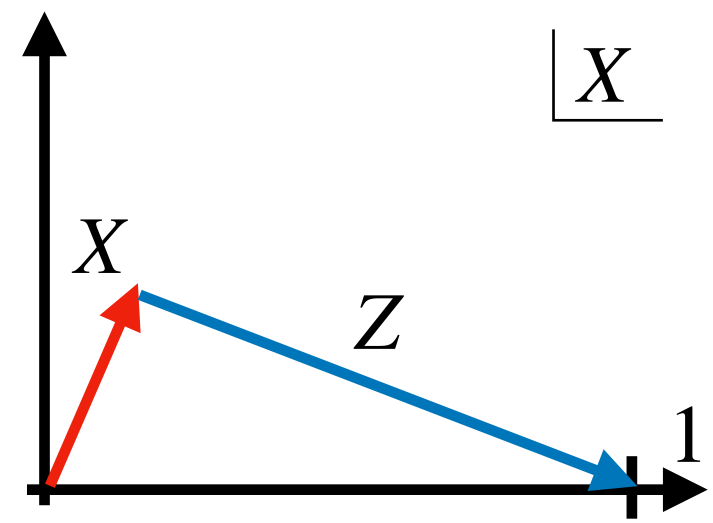


large Im X is assigned to non-interpretable cases

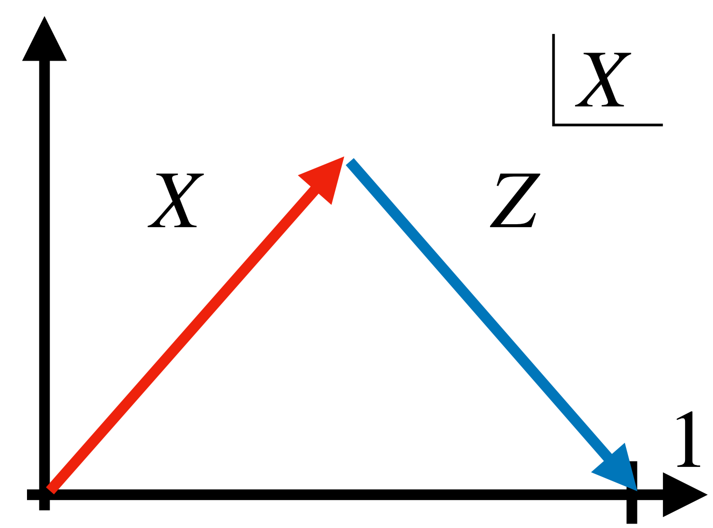
composite dominant



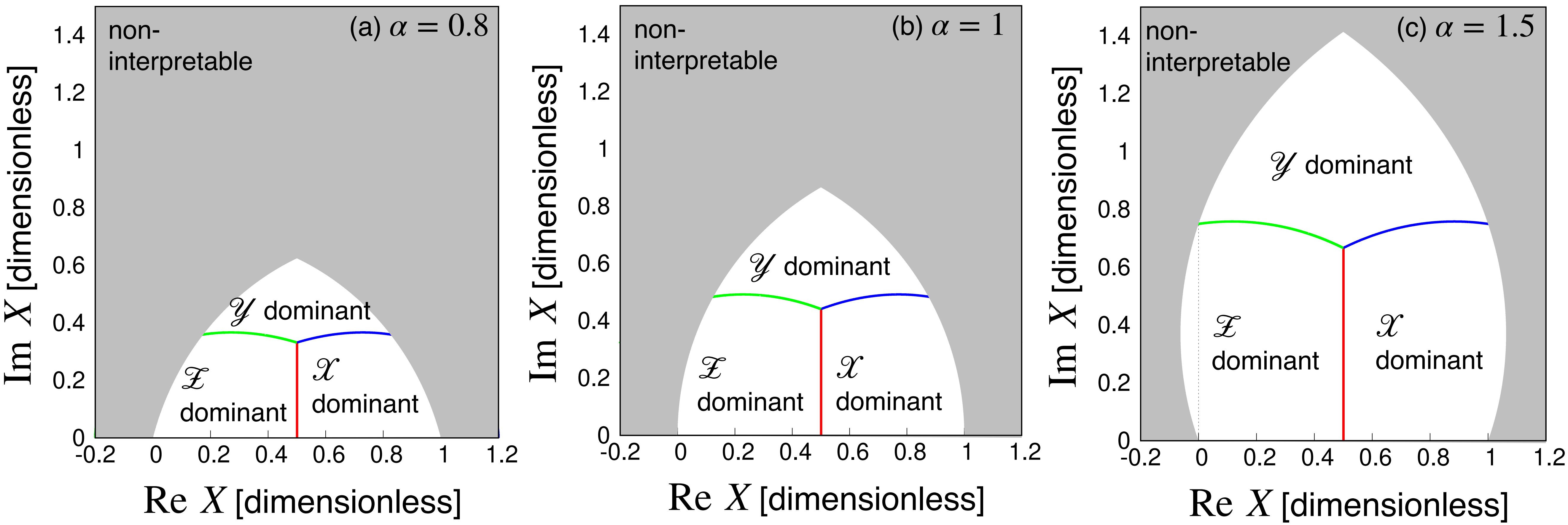
elementary dominant



uncertain



$\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ dominant region



- Interpretable regions become large with increase of α

$\alpha \to \infty$ ⟶ $\mathscr{X}, \mathscr{Y}, \mathscr{Z}$ reduce to interpretation in previous work

$\mathscr{X} \to \tilde{X}, \mathscr{Z} \to \tilde{Z}, \mathscr{Y} \to 0$

Y. Kamiya and T. Hyodo,
Phys. Rev. C **93**, 035203 (2016).

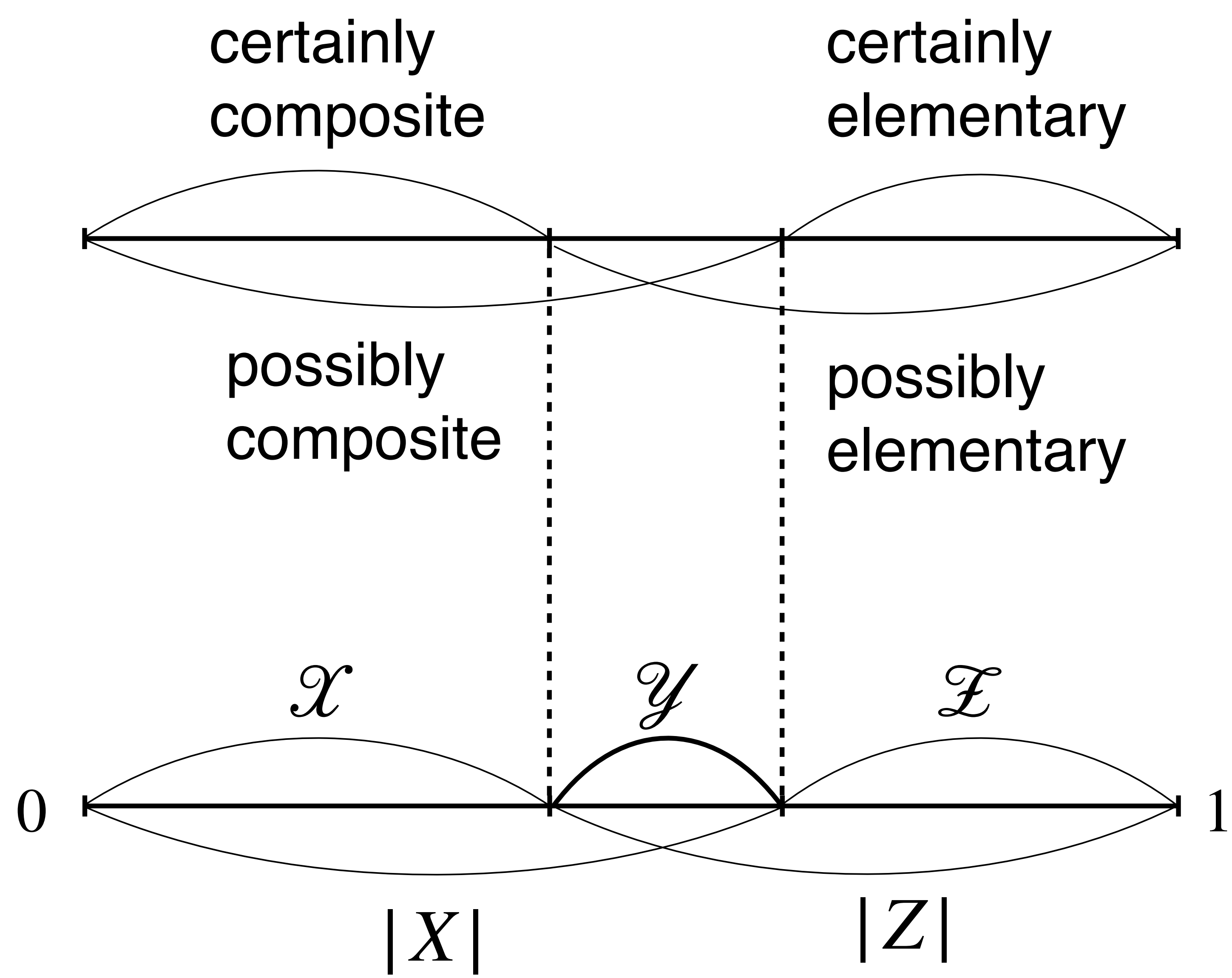
Definition

◉ new interpretation of complex compositeness & elementarity

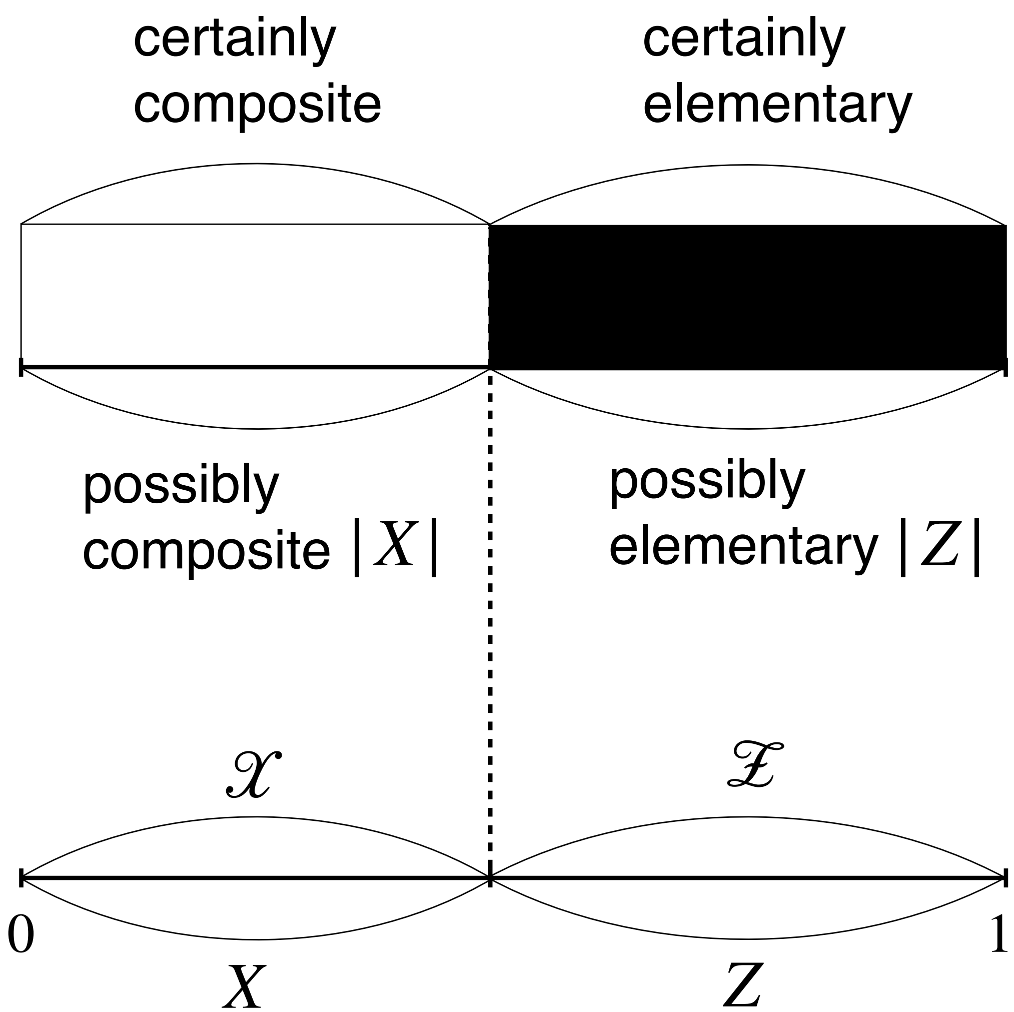
→ from Berggren's idea T. Berggren, Phys. Lett. B 33, 547 (1970).

$$\mathscr{X} + \mathscr{Y} = |X| \text{ \& } \mathscr{Z} + \mathscr{Y} = |Z|$$

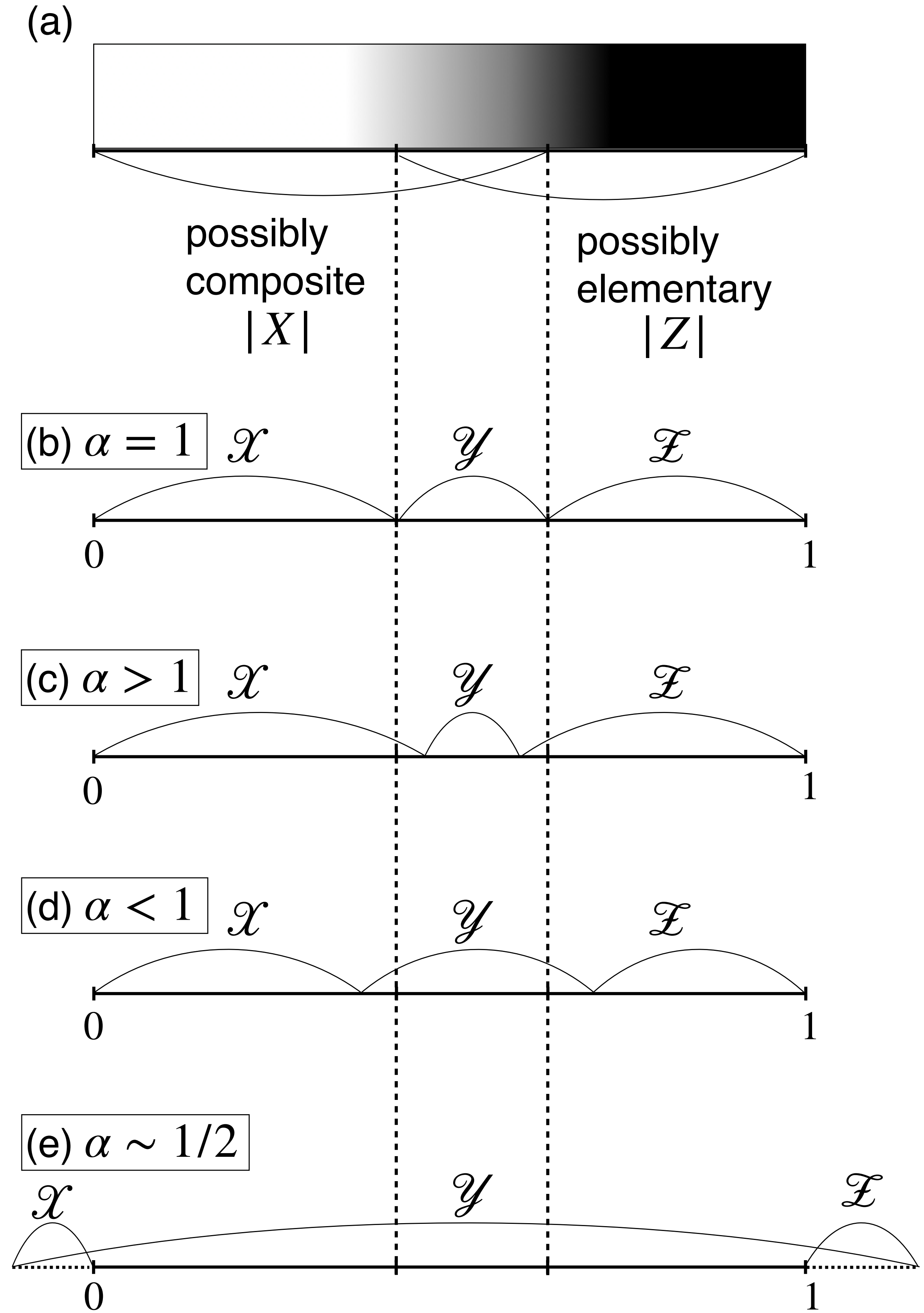
$$\mathscr{X} = 1 - |Z|$$
$$\mathscr{Z} = 1 - |X|$$
$$\mathscr{Y} = |X| + |Z| - 1$$



interpretation

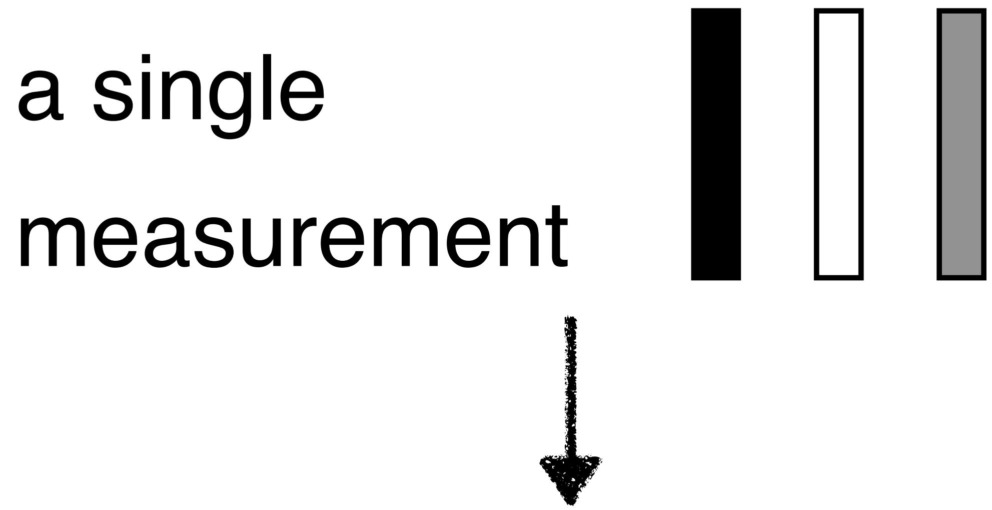


bound state

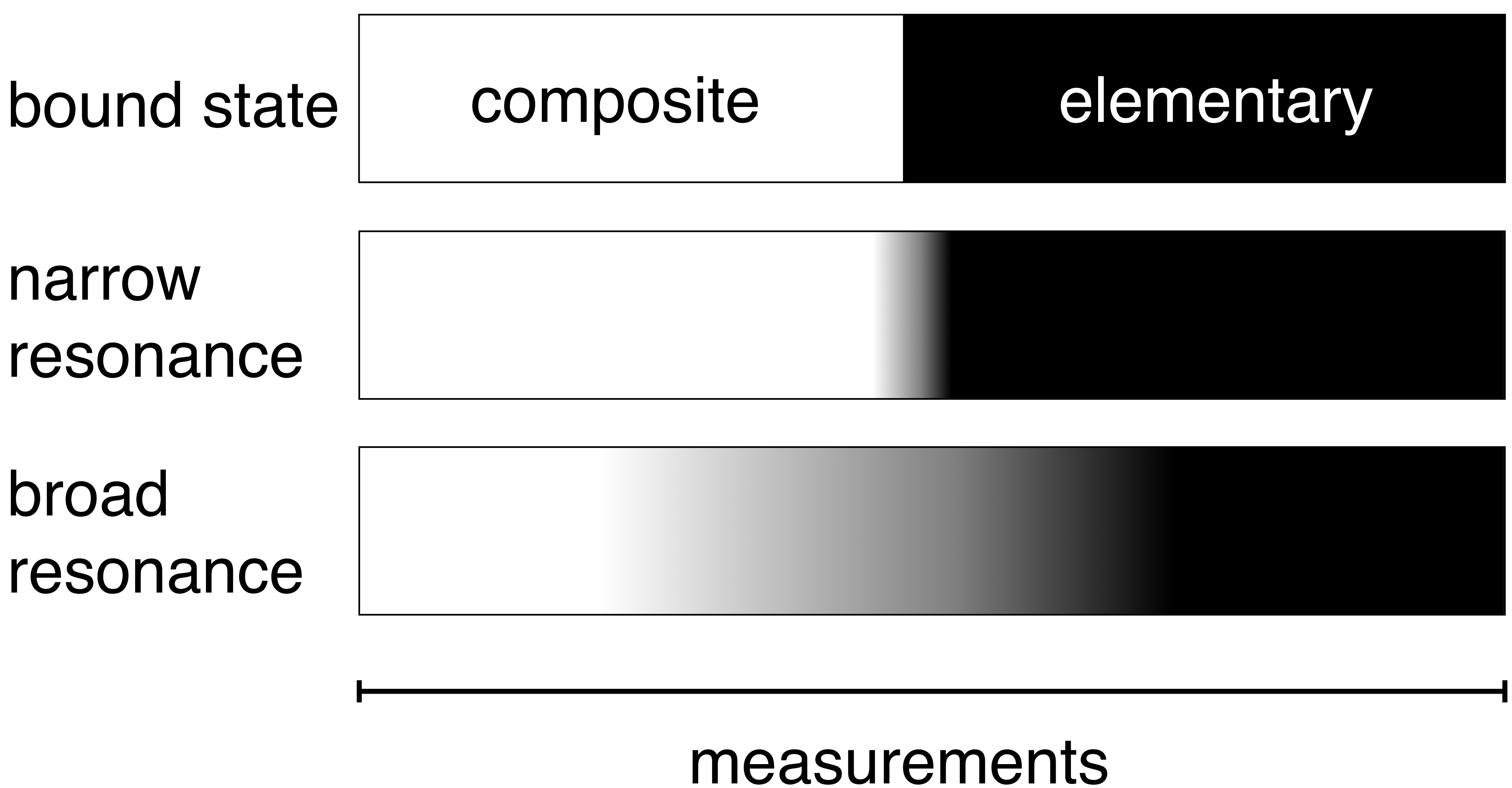


resonance

uncertainty in resonances



sum of measurements of a bound states / resonances



Structure of near-th. resonances