TECHNISCHE
UNIVERSITÄT
MÜNCHEN

Nonrelativistic (NR) bound systems
and
NR QCD effective field theories
with
applications to the XYZ exotics

erc Advanced Grant
EFT-XYZ

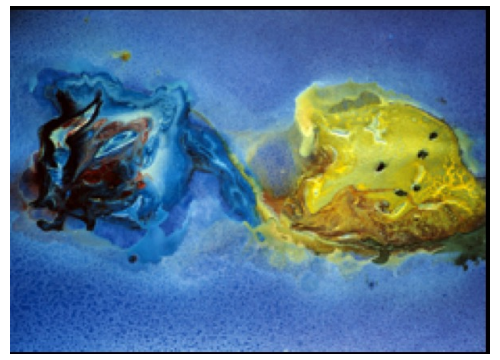
Nora Brambilla

T30f

TUMQCD
LATTICE
Collaboration

STRONG 2020

TUM-IAS Focus Group
Physics with Effective
Field Theories



Quark Confinement and
the Hadron Spectrum since 1994

MCQST

MDSI

Munich Data Science Institute

ORIGINS
Excellence Cluster

QwG

Nonrelativistic (NR) bound states lie at the core of quantum physics spanning particle to nuclear physics, and condensed matter to astrophysics

They are at the origin of several past and contemporary revolutions.

They are multiscale systems, which is an opportunity for the physics but a challenge for a QFT description

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Focus of the lecture

We introduce a nonrelativistic effective field theory (pNREFT) description that reinventes QM and allows precise calculations of observables

This framework is particularly suited to address strongly interacting systems

pNRQCD allows to use NR bound states to address contemporary challenges like:

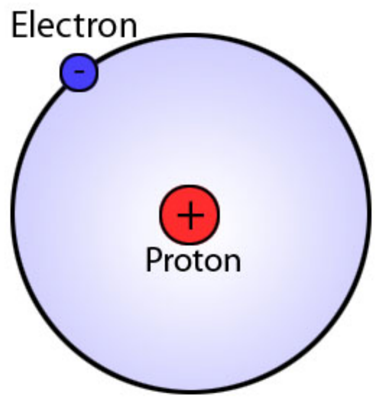
- the exotics XYZ states and the nature of the strong force
- the in medium heavy pairs evolution with impact e.g. on the nuclear phase diagram (and dark matter properties..)

Novel tools to bridge perturbative methods with lattice QCD are key to this program, as well as the combination between different EFTs

Past
Revolutions

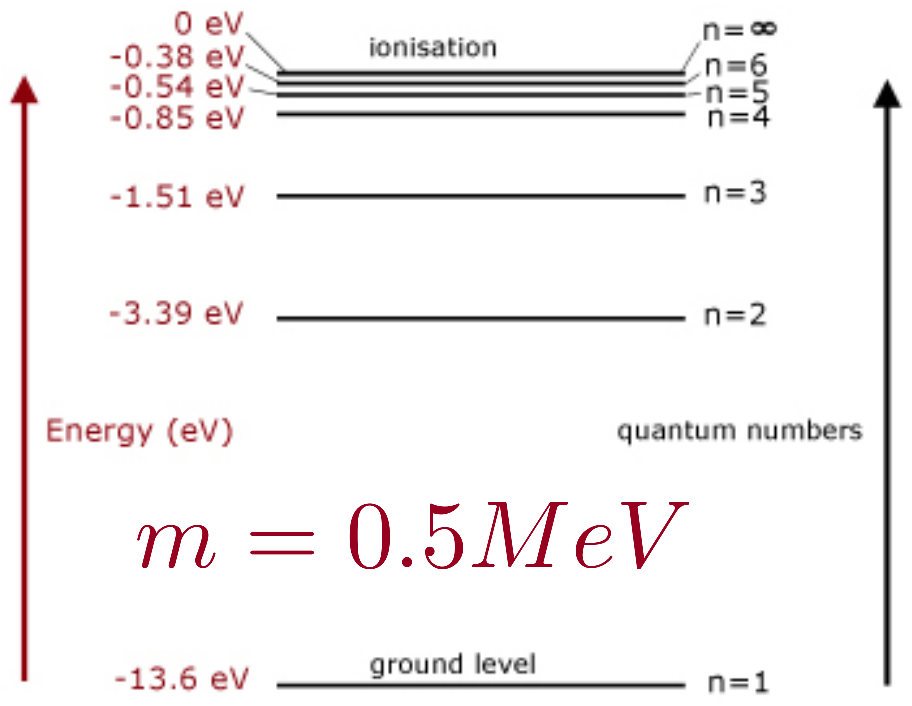
NR bound states accompanied the history of quantum theory from its beginning to the establishment of QFT

The prototype of NR system is the hydrogen atom and it is at the origin of the quantum revolution



$$v = \alpha = \frac{e^2}{4\pi}$$

fine structure constant



$$m = 0.5MeV$$

The relevant scales of the non-relativistic bound state dynamics are

- $E \sim \frac{\mathbf{p}^2}{2m} \sim V \sim mv^2,$
- $p \sim 1/r \sim mv;$

a crucial observation: if v(elocity)$\ll 1$, then $m \gg mv \gg mv^2.$

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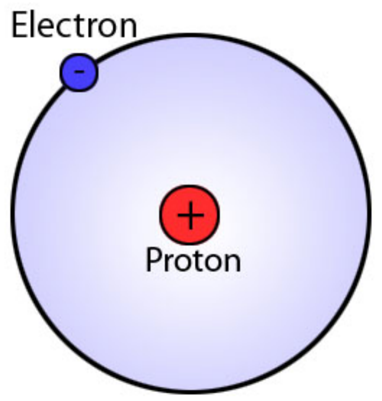

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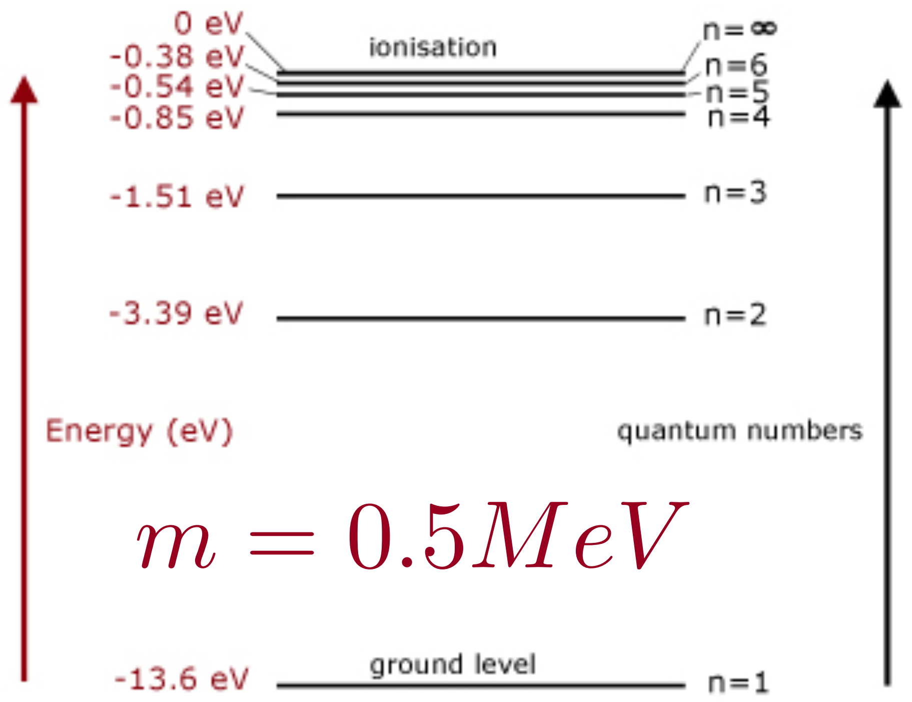

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To explain the energy levels of hydrogen, from QM…

- 1926 Schrödinger equation: $\left(\frac{\mathbf{p}^2}{2m} + V\right)\phi = E\phi$

- 1928 Dirac equation: $(i\not{D} - m)\,\psi = 0$

$$\begin{cases} g^D = g_0^D + g_0^D(-ie\not{A})g^D \\ g_0^D = \frac{i}{\not{p}-m} \end{cases}$$

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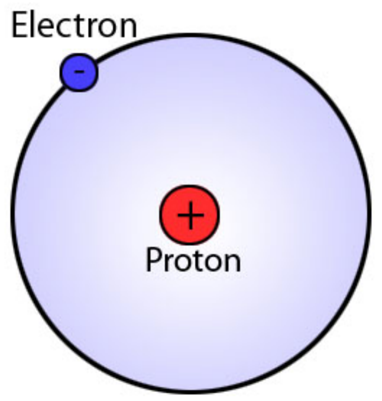

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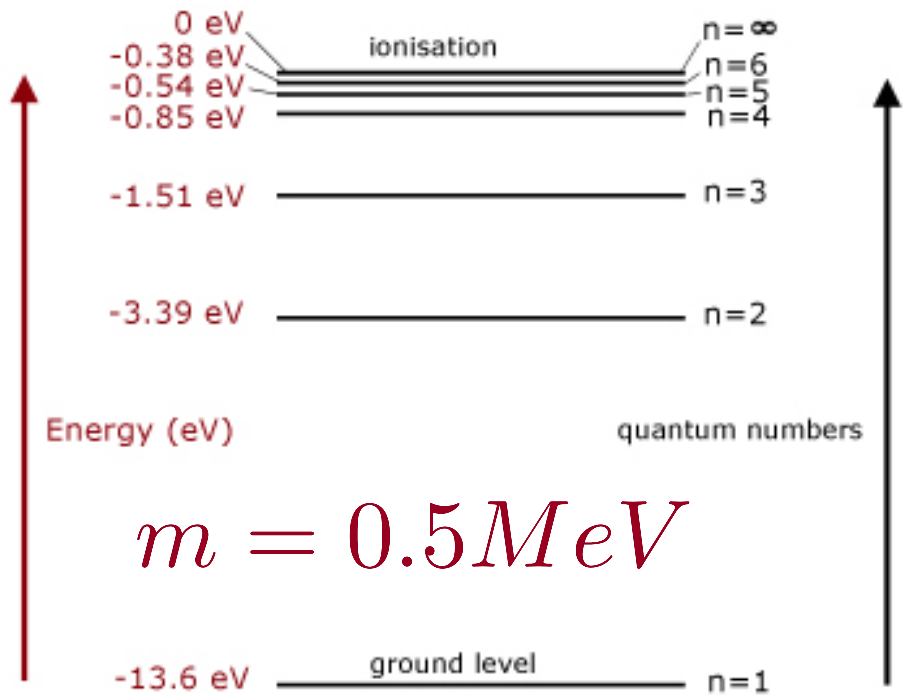

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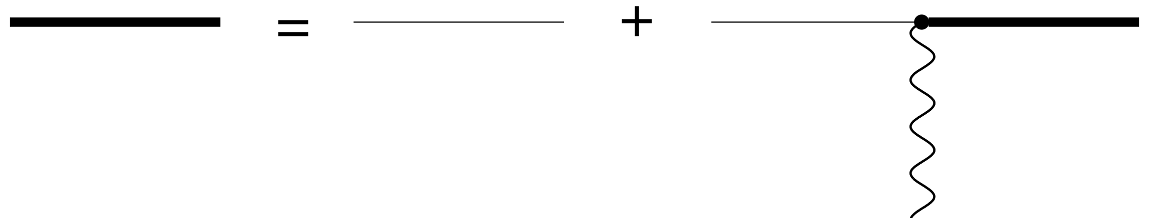
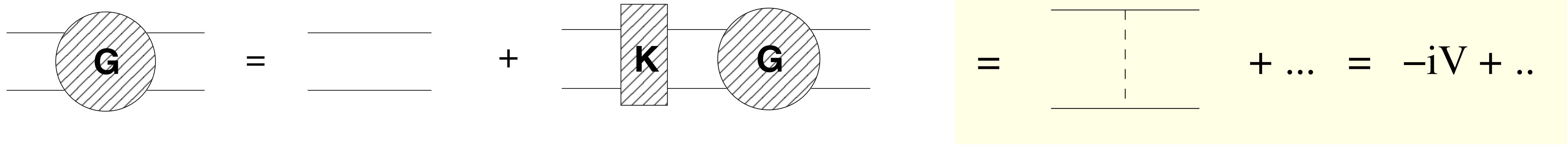


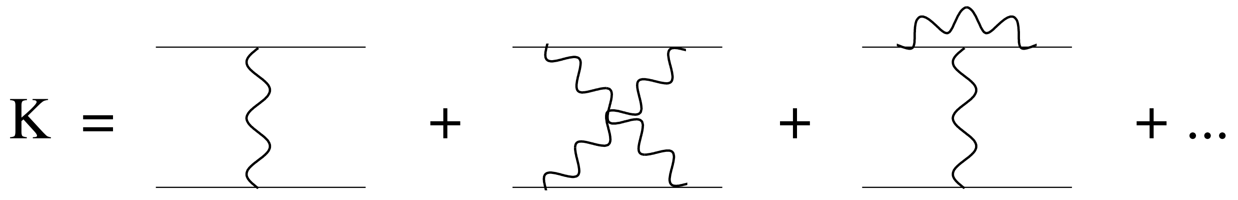
….to the relativistic quantum theory of bound states

- 1951 Bethe–Salpeter equation:

$$\begin{cases} G = G_0 + G_0KG \\ G_0 = g_0^D \otimes g_0^D \end{cases}$$



All the complexity of the field theory is in the kernel



which only in the non-relativistic limit reduces to the Coulomb potential, but, in general, keeps entangled all bound-state scales.

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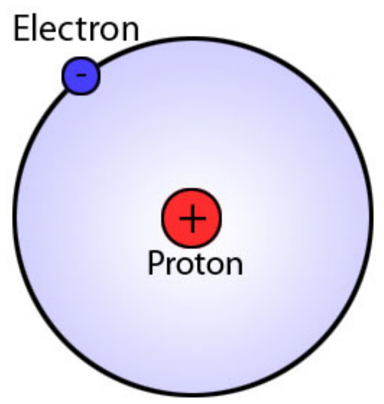

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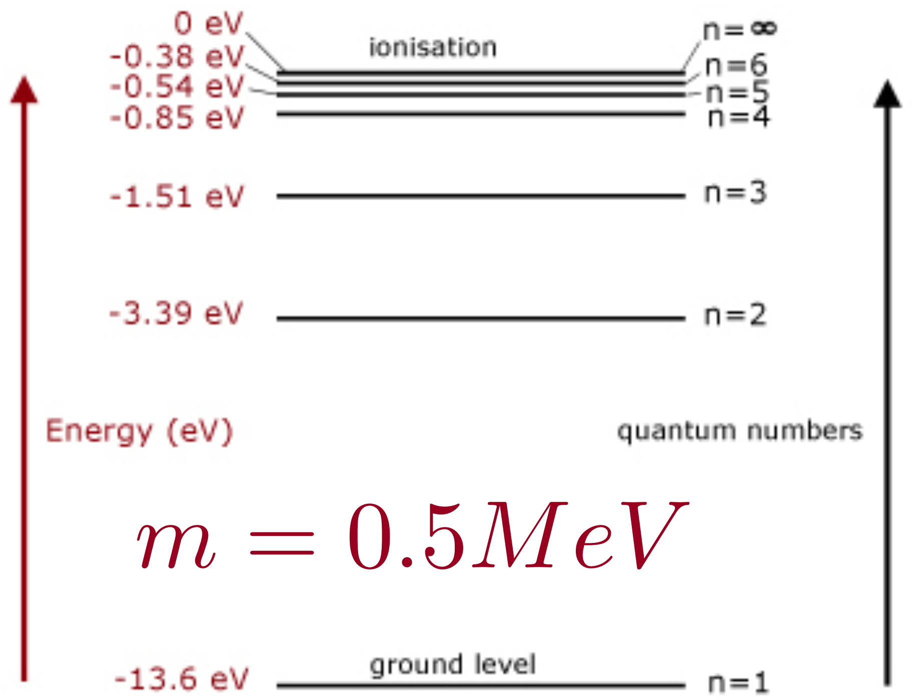

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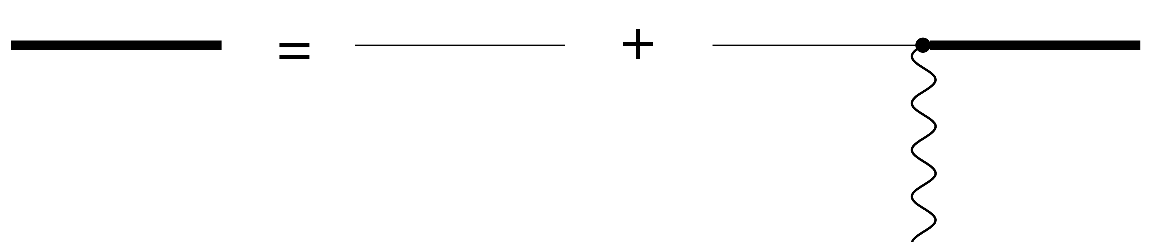


….to the relativistic quantum theory of bound states and its problems

- 1951 Bethe–Salpeter equation:

$$\begin{cases} G = G_0 + G_0 K G \\ G_0 = g_0^D \otimes g_0^D \end{cases}$$

- **cumbersome in perturbation theory :**

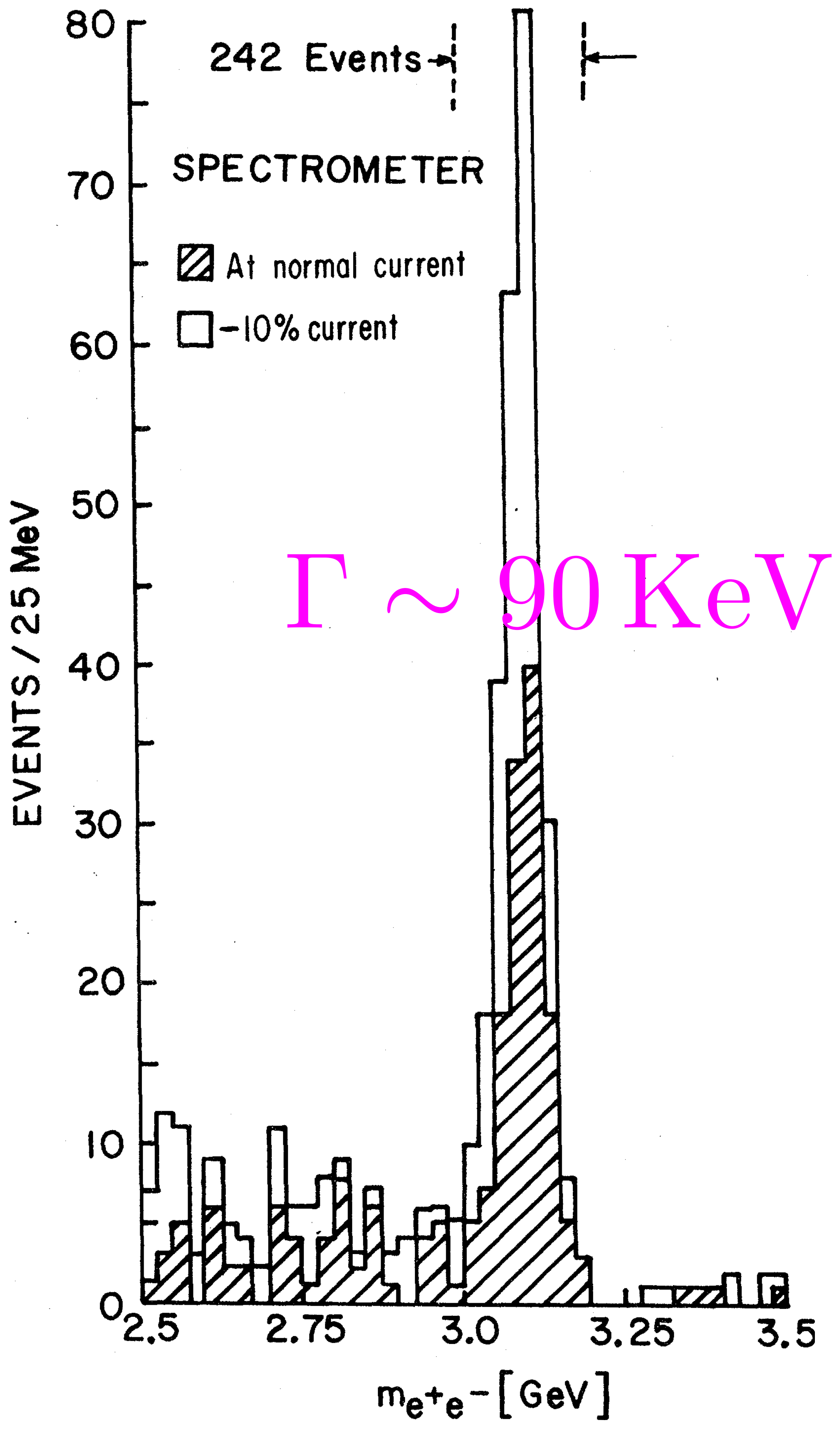
It shows the difficulty of the approach the fact that going from the calculation of the $m\alpha^5$ correction in the hyperfine splitting of the positronium ground state to the $m\alpha^6 \ln\alpha$ term took twenty-five years!

○ Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)36
Bodwin Yennie PR 43(78)267

- **poorly suited to achieve factorization (important in QCD)**

Past Revolutions

The discovery of the J/psi (ccbar lowest state) is at the origin of the November revolution in 1974



$\Gamma \sim 90\,\mathrm{KeV}$

Aubert et al. BNL 74

Samuel Ting: It was like to stumble in a village where people were living 70000 years

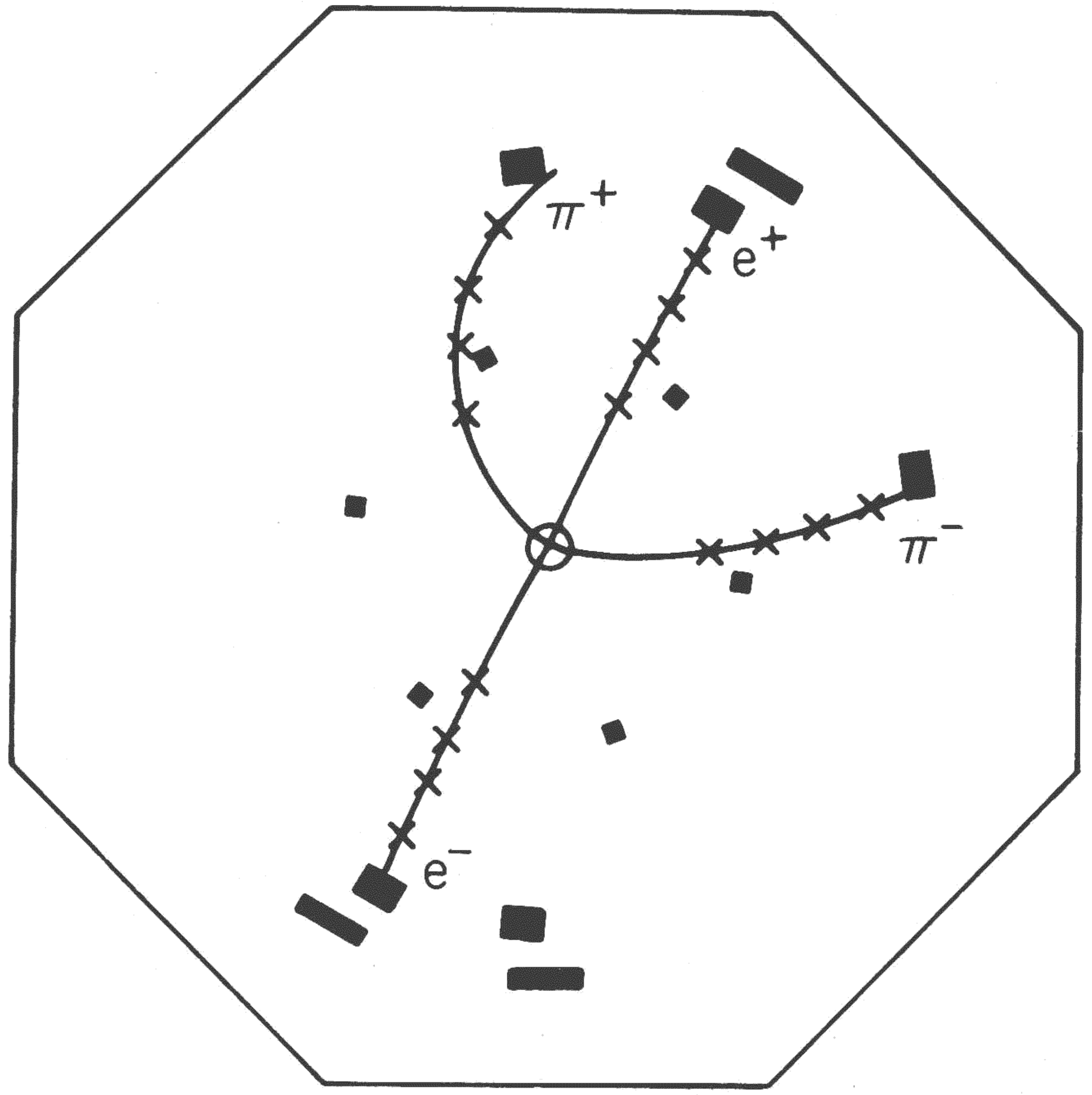
- Discovery of the first quark of heavy type Q (m_c > Lambda_QCD)
- Confirmation of the quark model and QCD

—>**narrow width and asymptotic freedom:** annihilation at large scale controlled by a small coupling constant $\alpha_s(2m_c) \ll 1$

—>**energy levels and confinement:** structure of levels controlled by a Cornell potential in a Schroedinger eq.

The November revolution in the '70s: more quarkonia

$e^+e^- \to \psi' \to \pi^+\pi^- J/\psi$

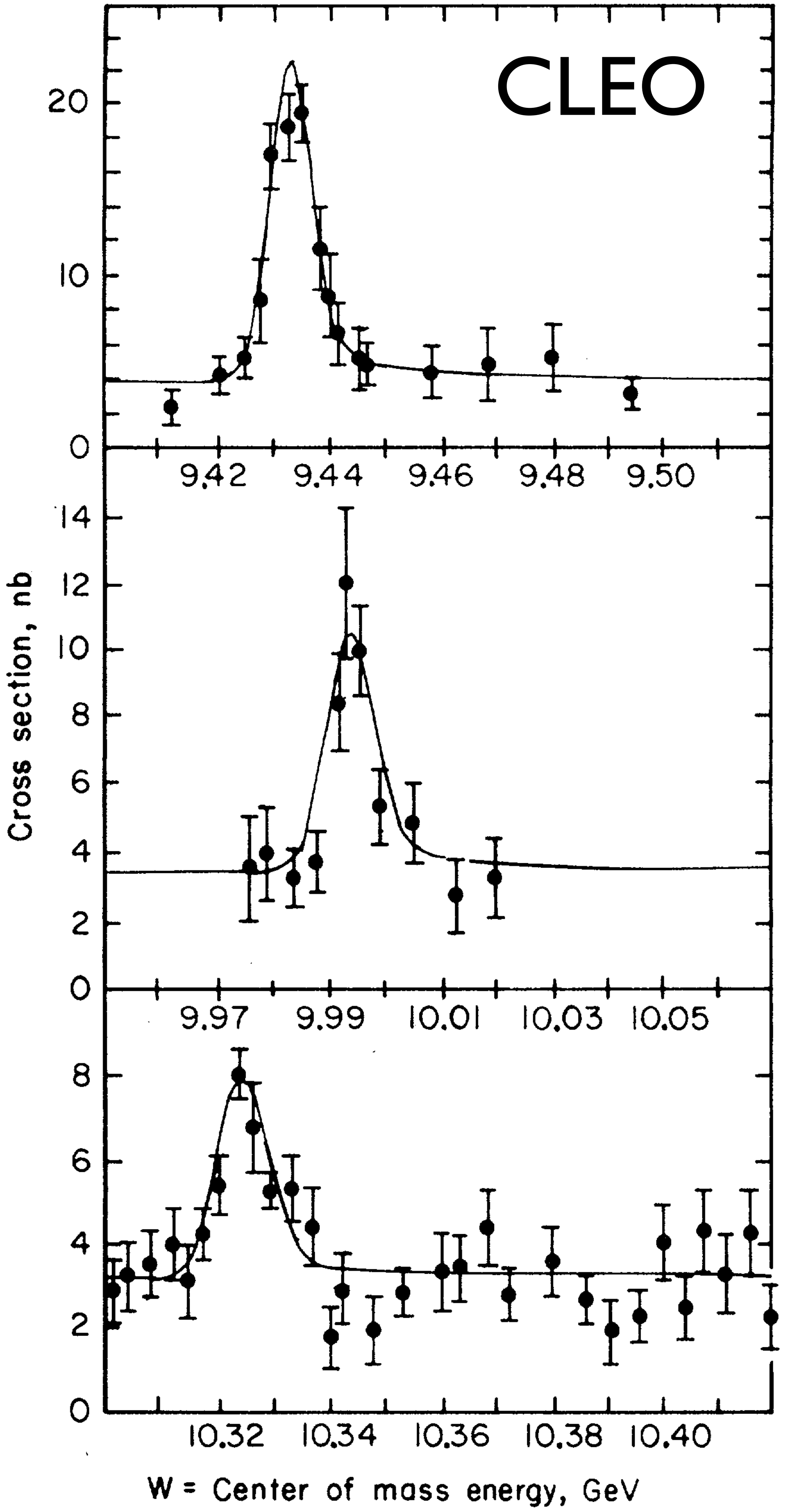


$\psi(2S)$

$\Upsilon(1S)$

$\Upsilon(2S)$

$\Upsilon(3S)$



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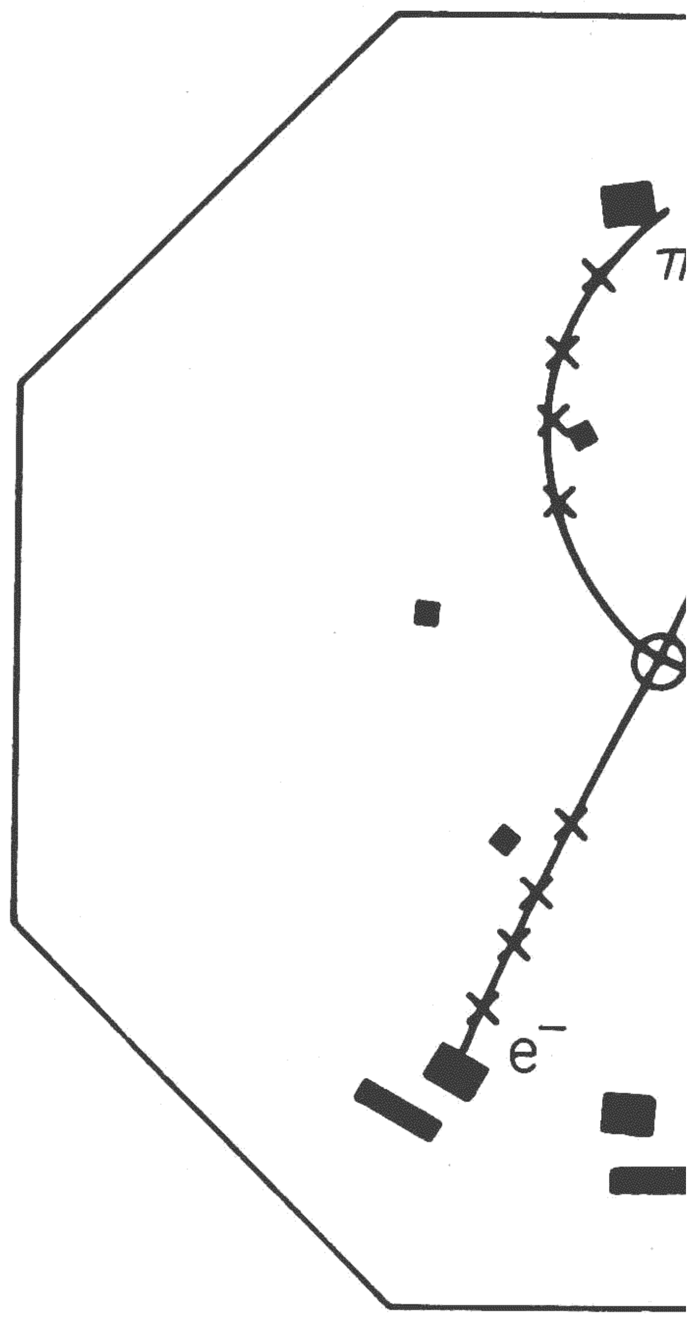

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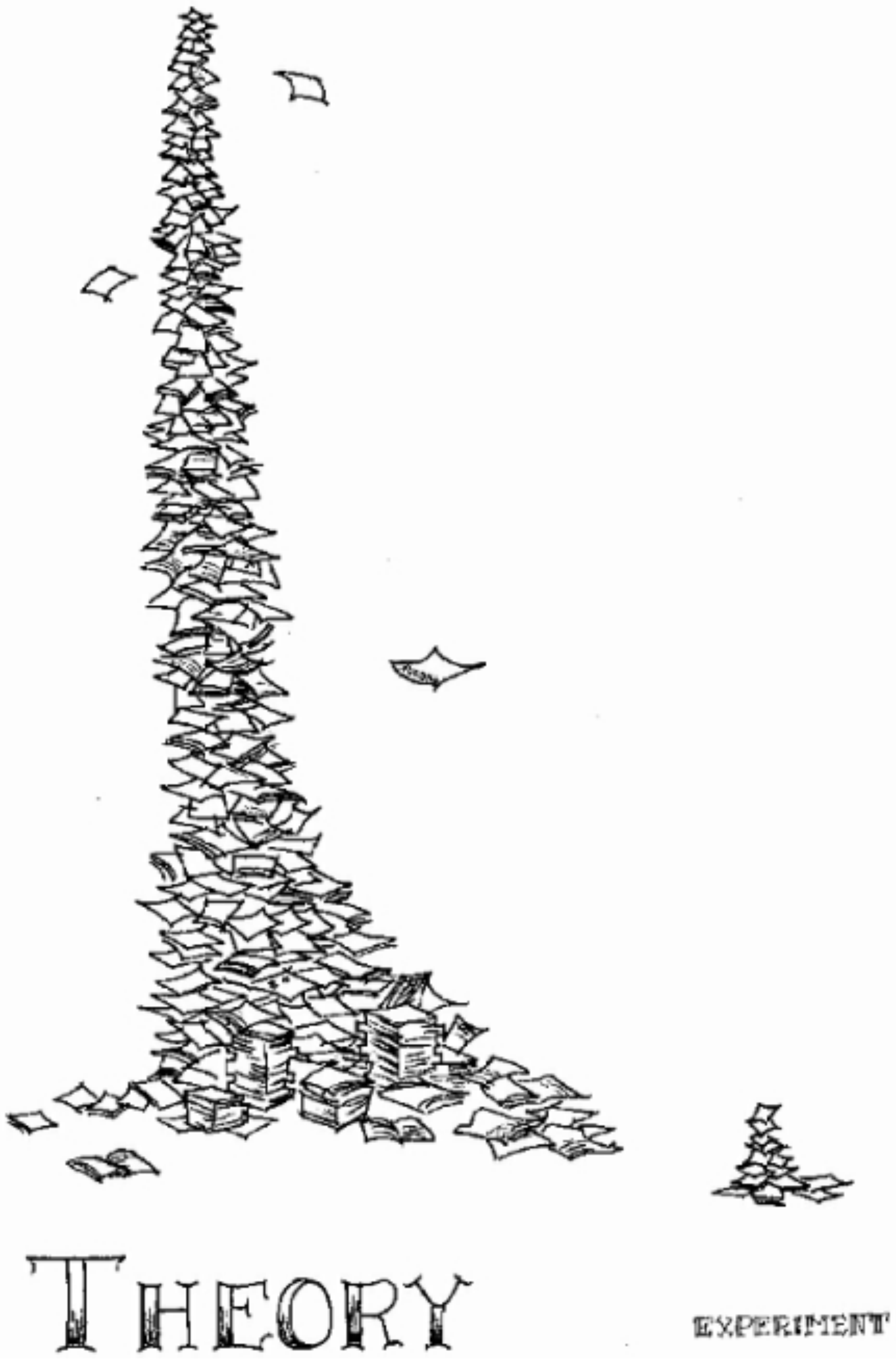
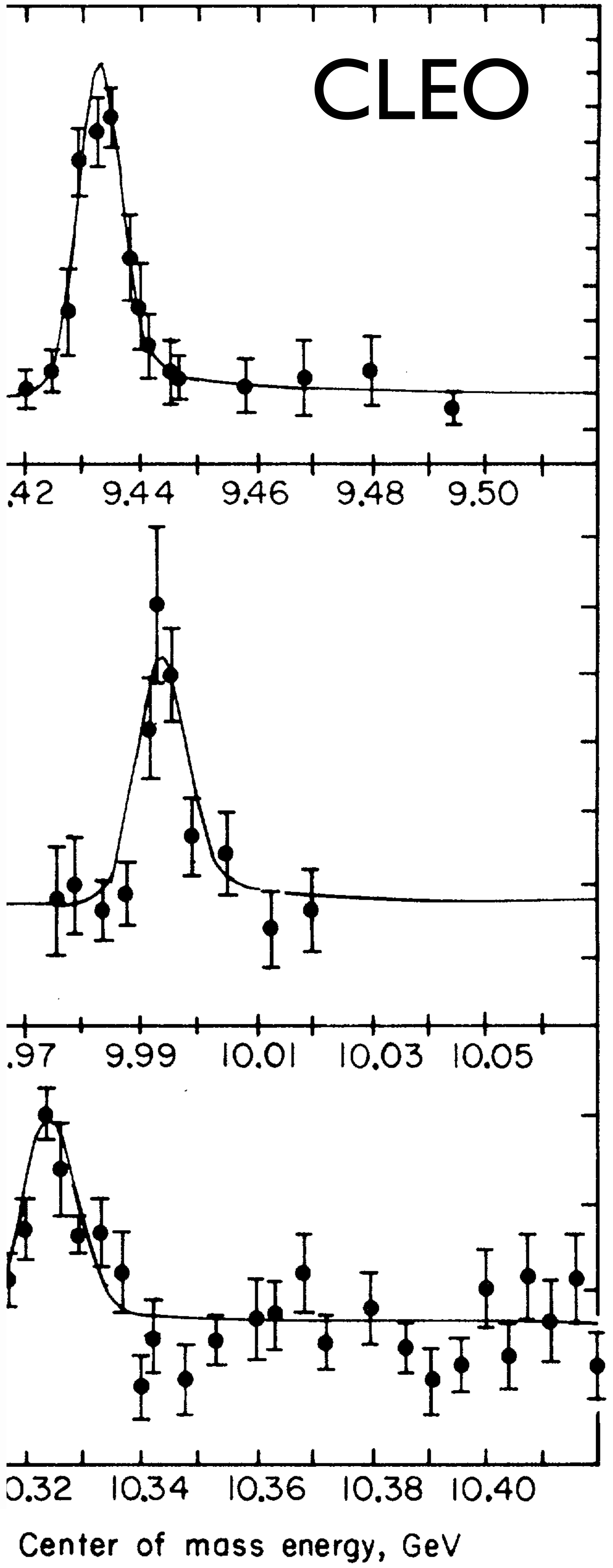
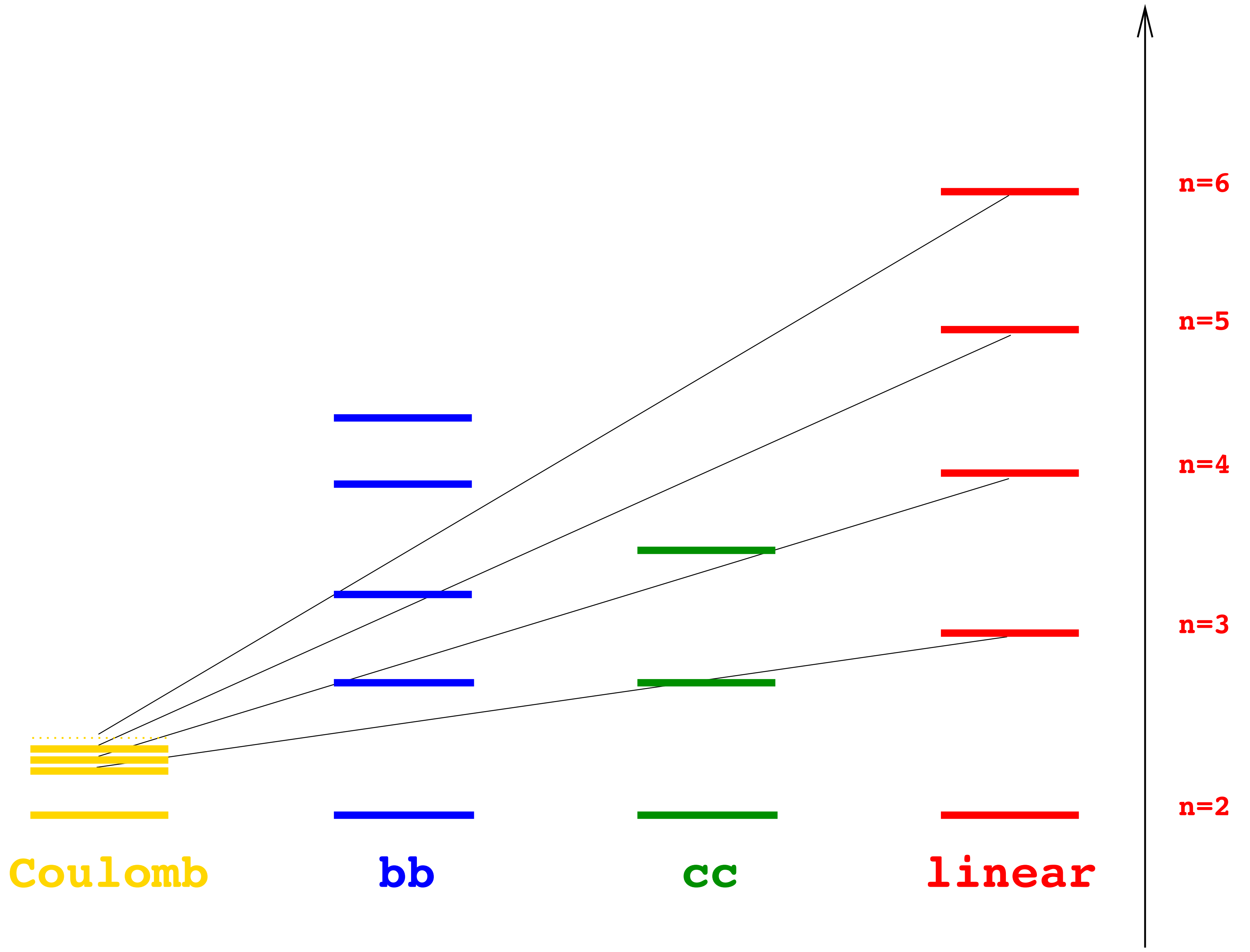


J D Jackson



The November revolution in the ’70s: more quarkonia

Cornell potential



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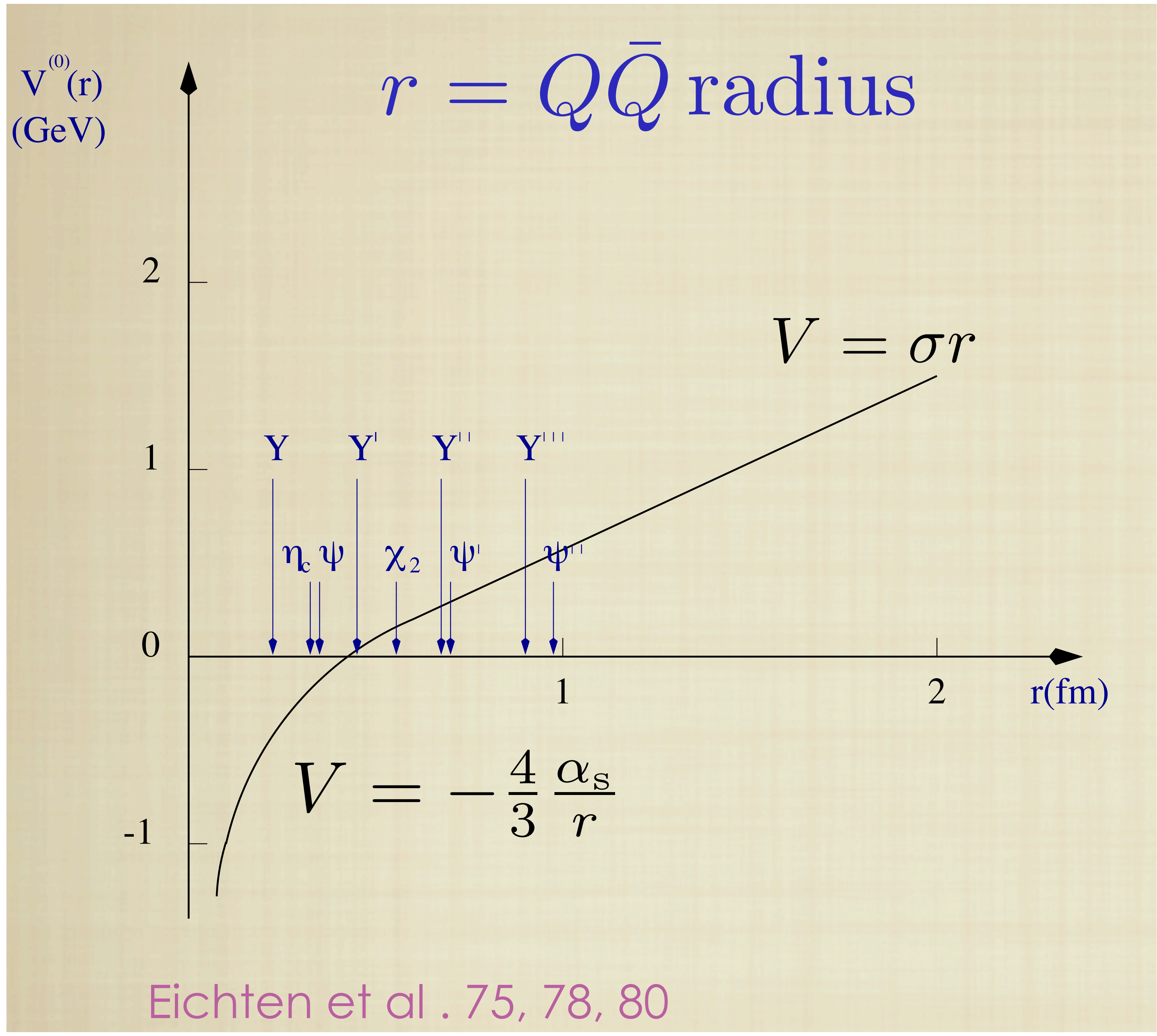
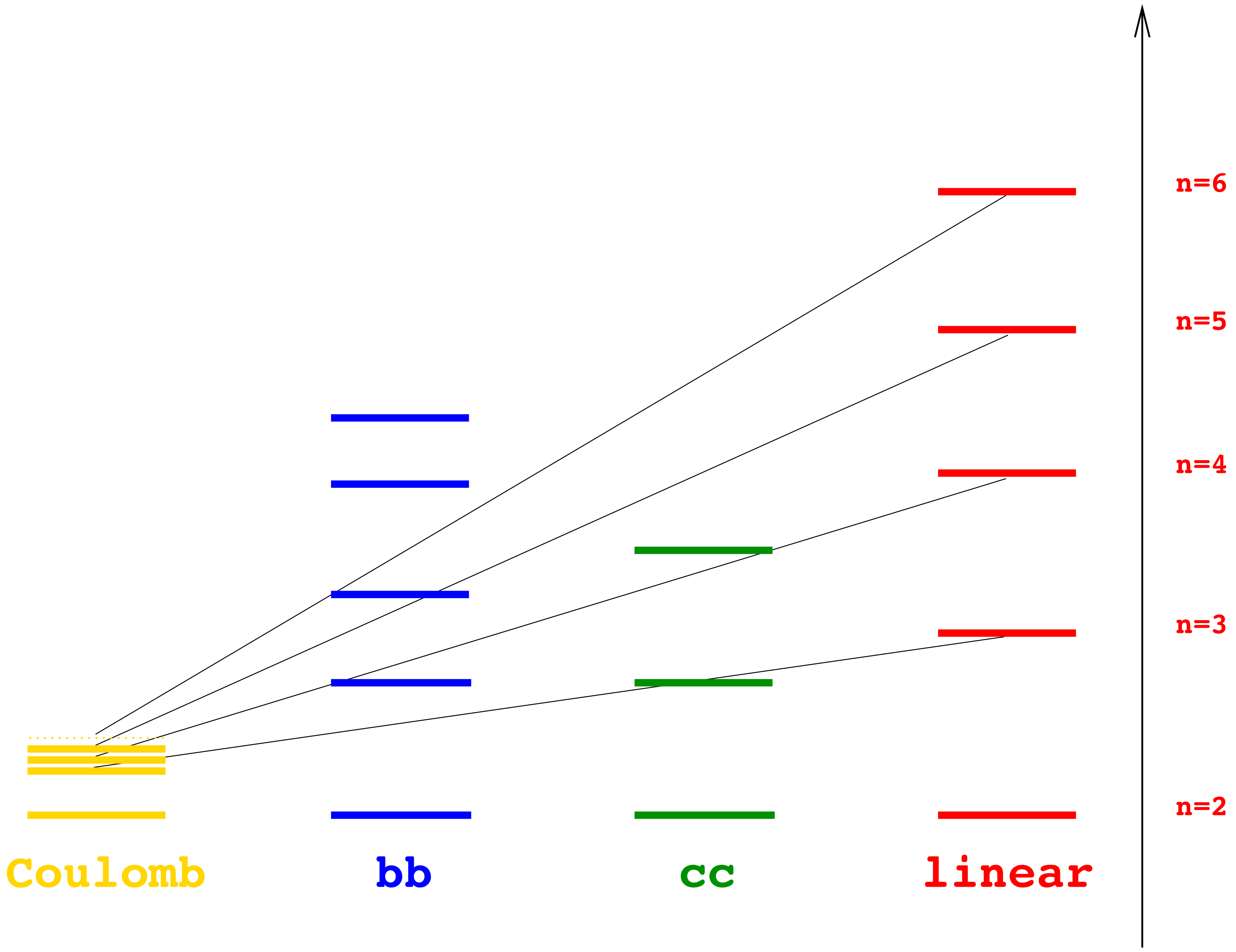


Eichten et al . 75, 78, 80



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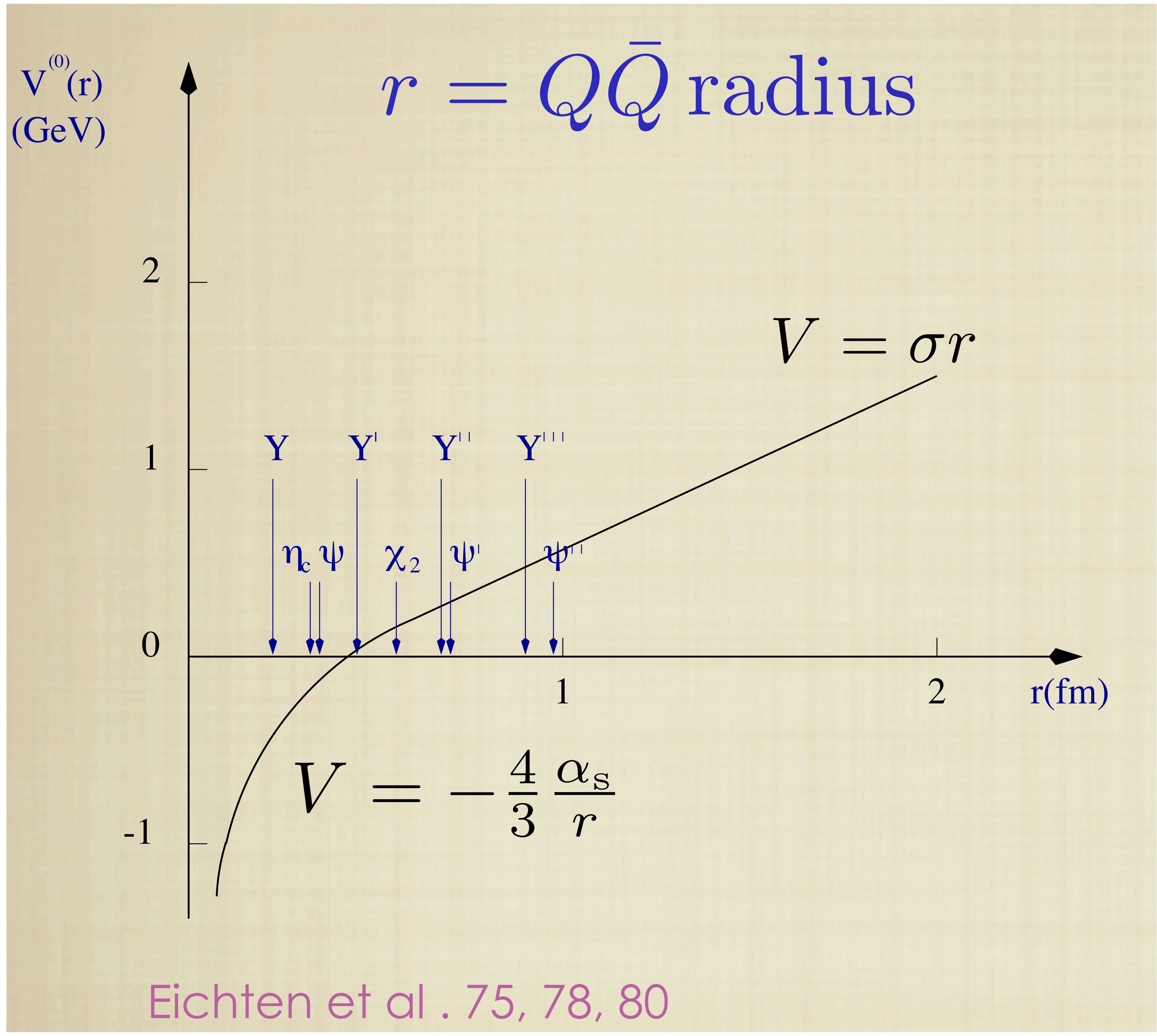

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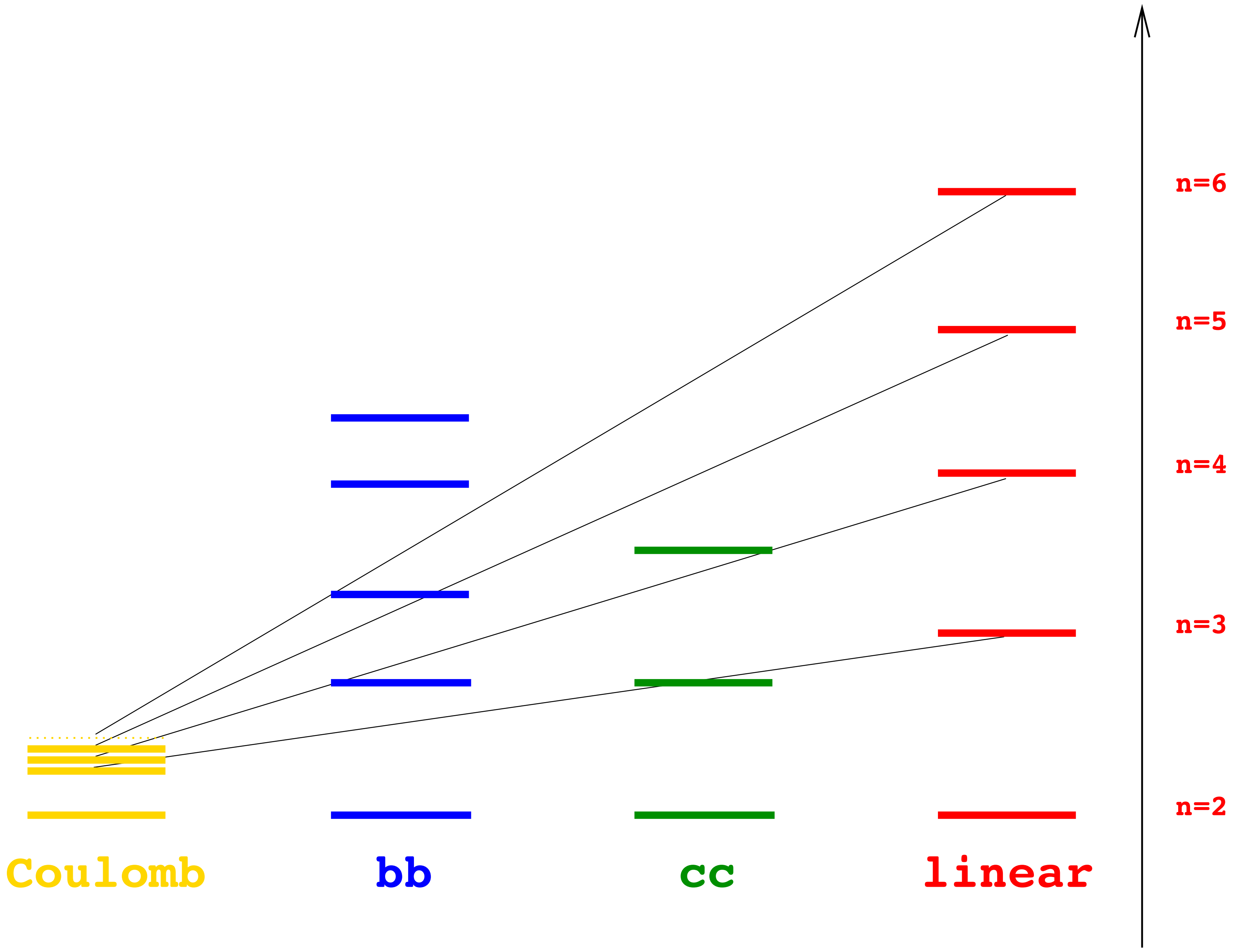
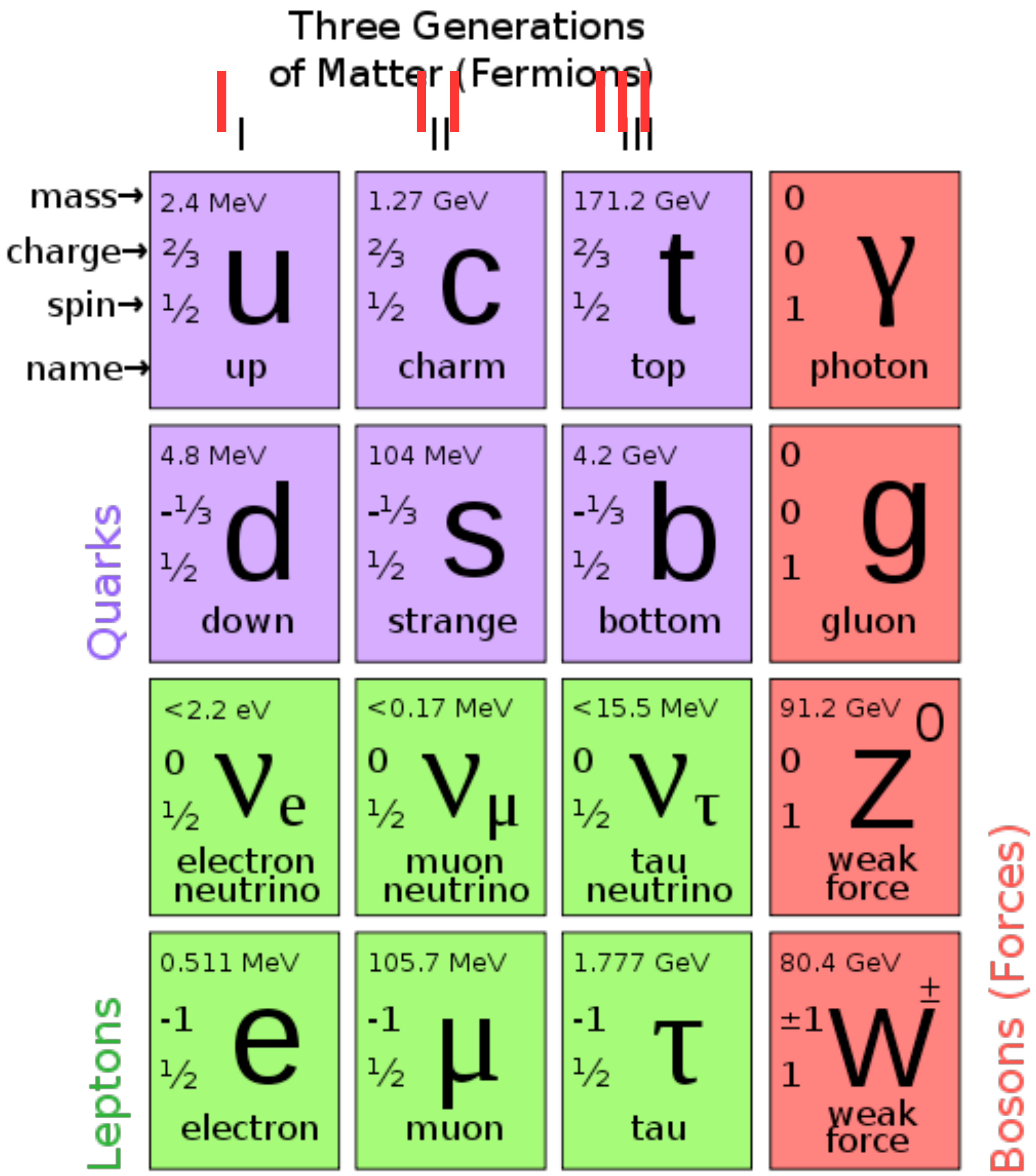


Variety of potential models used, including relativistic corrections confinement and asymptotic freedom--> **main properties of QCD**

our present knowledge of particle physics is in

Standard model of Particle Physics



Three Generations of Matter (Fermions)

		I	II	III	Bosons (Forces)
Quarks	mass→ charge→ spin→ name→	2.4 MeV ⅔ ½ u up	1.27 GeV ⅔ ½ c charm	171.2 GeV ⅔ ½ t top	0 0 1 γ photon
		4.8 MeV -⅓ ½ d down	104 MeV -⅓ ½ s strange	4.2 GeV -⅓ ½ b bottom	0 0 1 g gluon
Leptons		<2.2 eV 0 ½ ν_e electron neutrino	<0.17 MeV 0 ½ ν_μ muon neutrino	<15.5 MeV 0 ½ ν_τ tau neutrino	91.2 GeV 0 1 Z^0 weak force
		0.511 MeV -1 ½ e electron	105.7 MeV -1 ½ μ muon	1.777 GeV -1 ½ τ tau	80.4 GeV ±1 1 $W^\pm$ weak force

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Standard model of Particle Physics

A quantum field theory based
on the gauge principle
tested up to the Tev= $10^{12}\mathrm{eV}$
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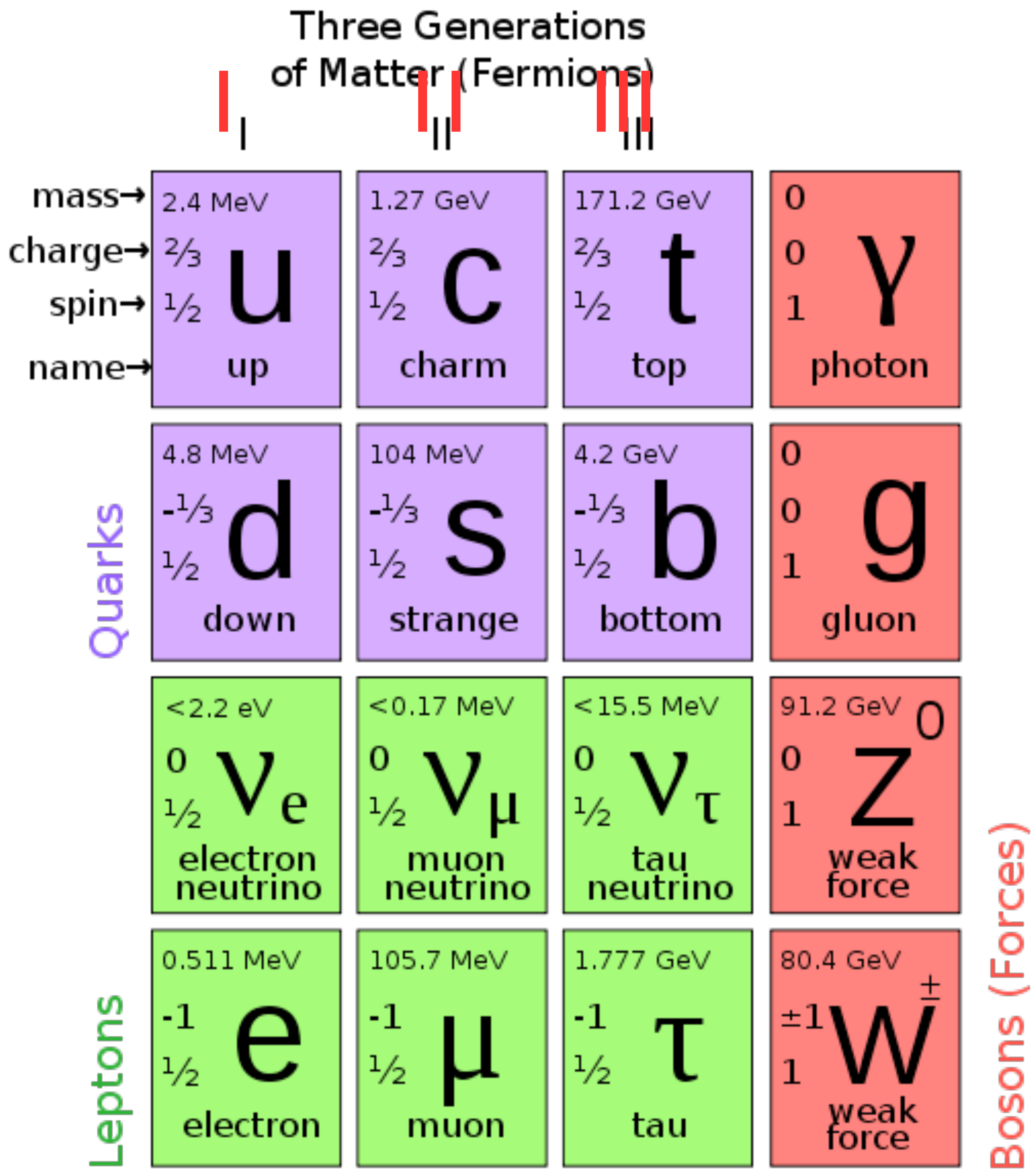

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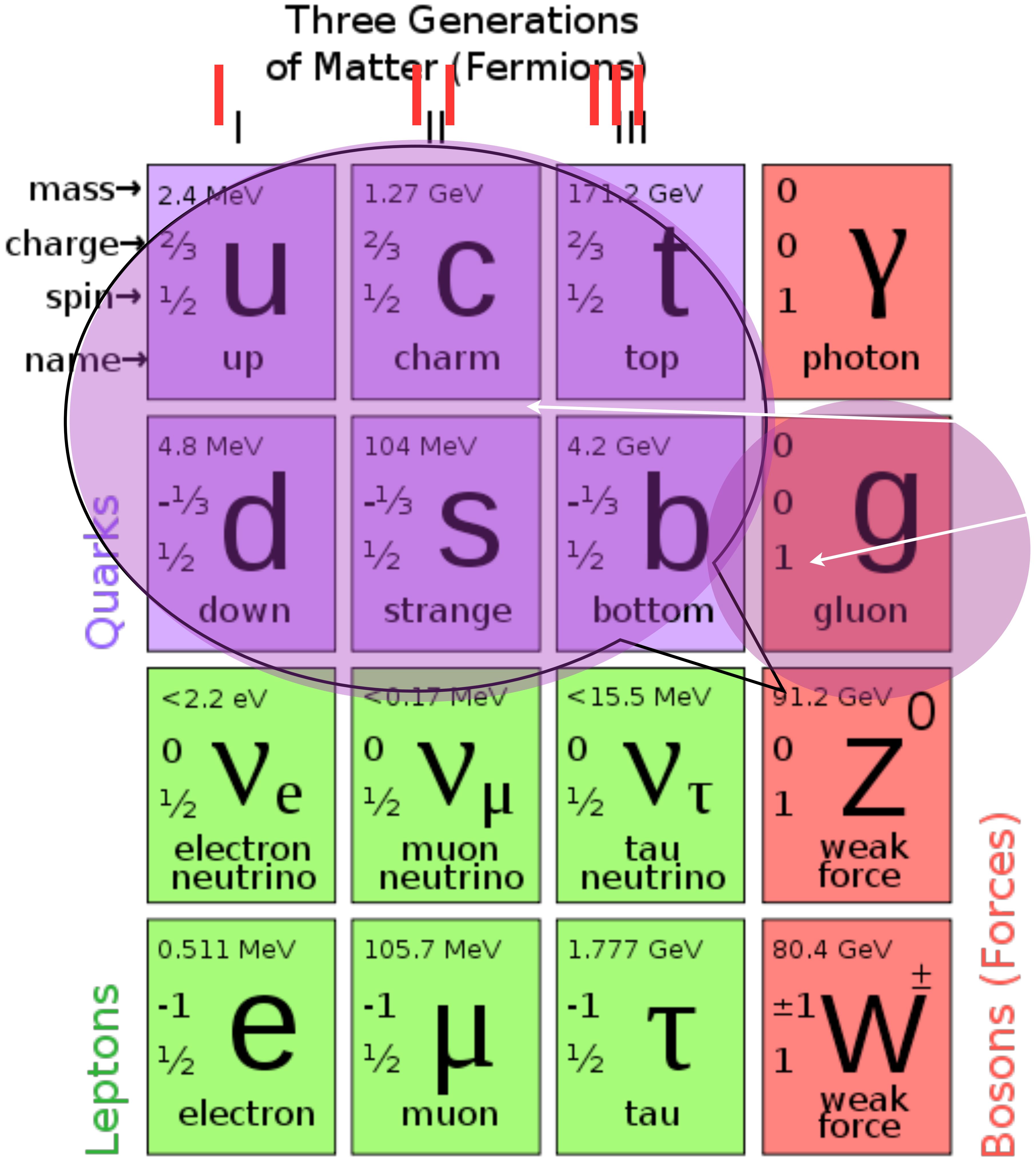

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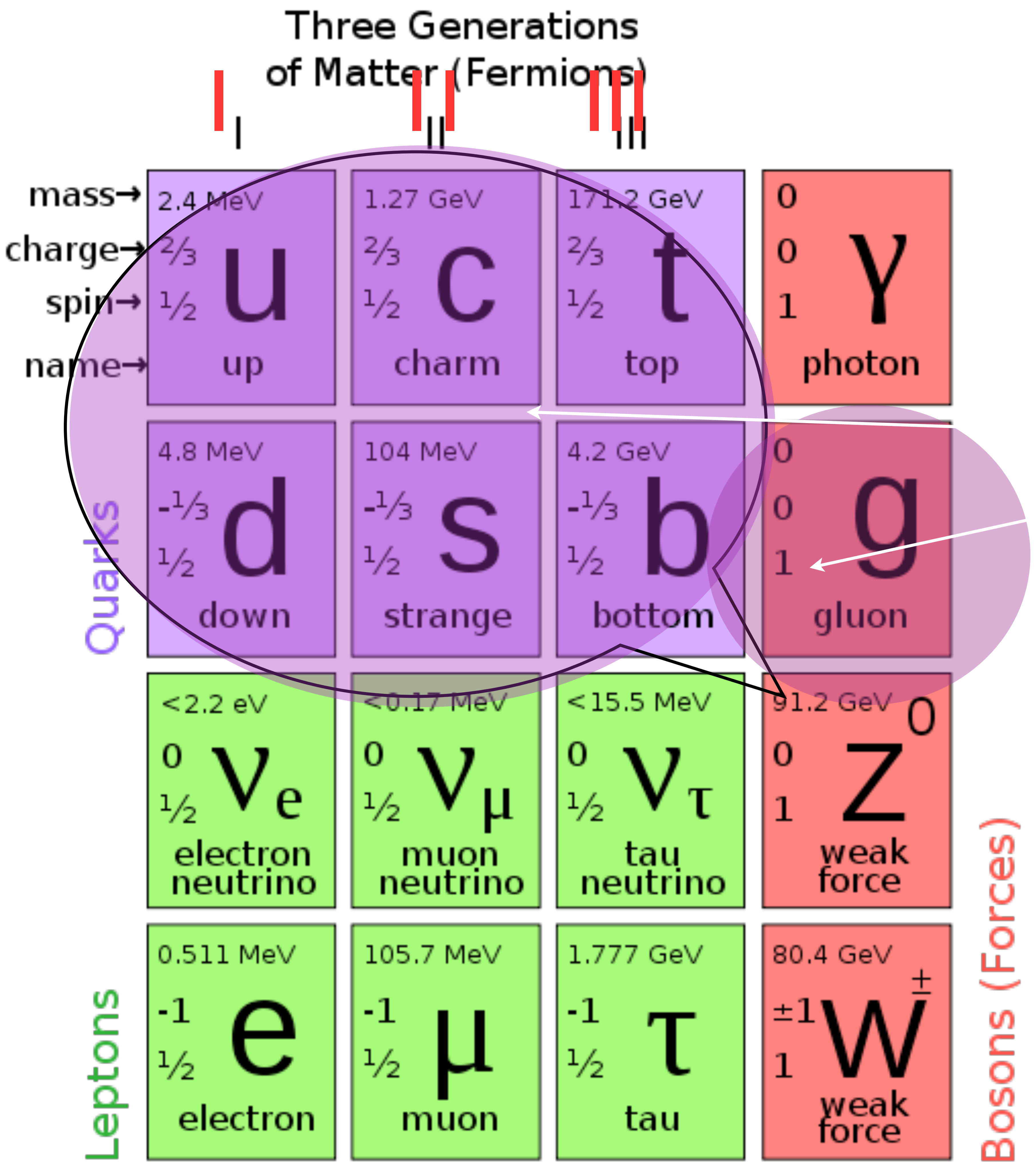

Differently from the other parts of the theory the low energy region of QCD cannot be studied by expanding in a small coupling constant, i.e. in perturbation theory. The non-perturbative nature of the QCD vacuum is a major difficulty that affects the determination of several observables in Particle Physics and some of the parameters of the Standard Model.

Quantum chromodynamics (QCD)

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]\right)^2 + \bar{\psi}_f \left(i\not{\partial} - g\not{A} - m_f\right)\psi_f$$

$$A_\mu^a, a = 1, 8$$

Gluon field

$$\psi^j{}_f, j = 1, 3, f = 1, 6$$

Quark field

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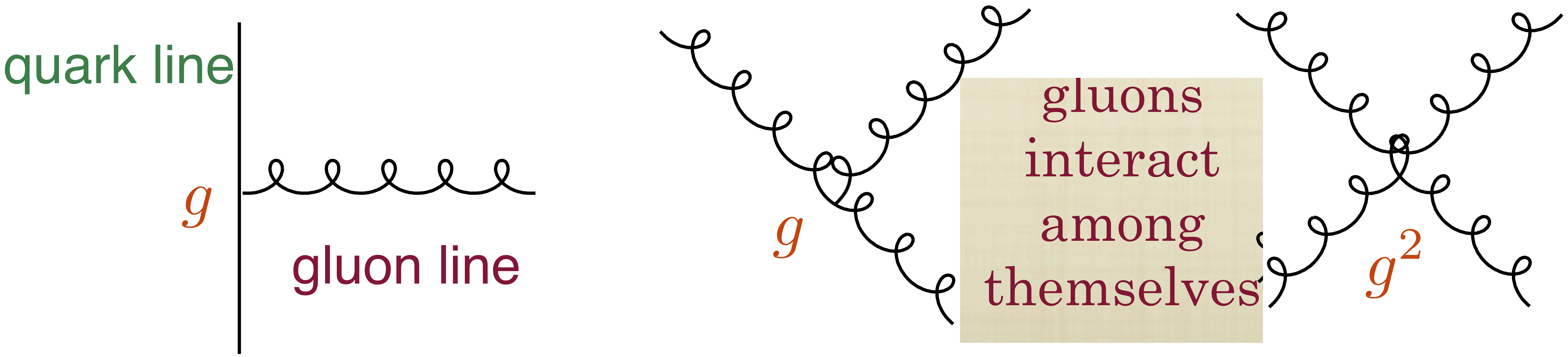
Interaction vertices



gluons interact among themselves

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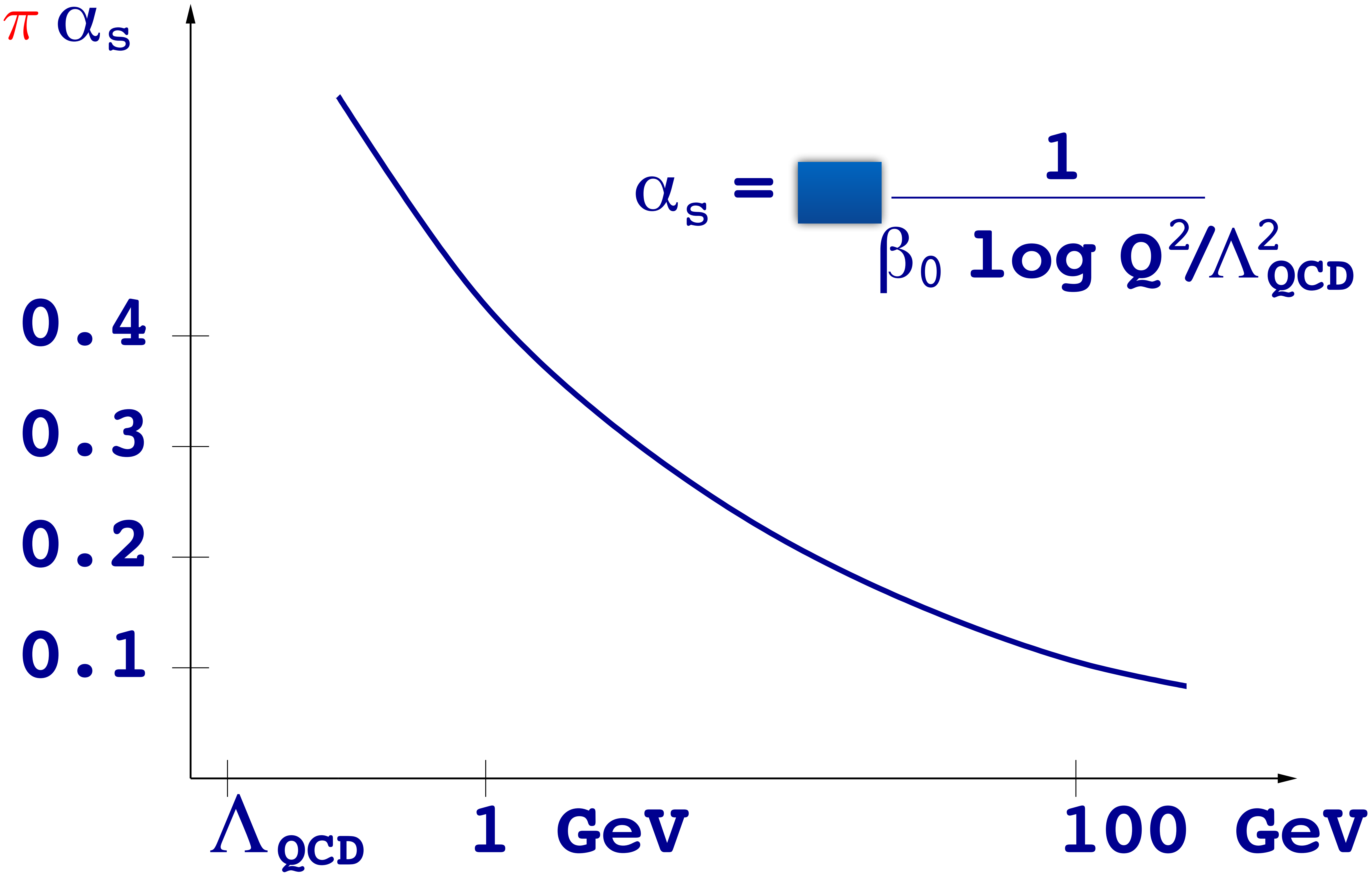

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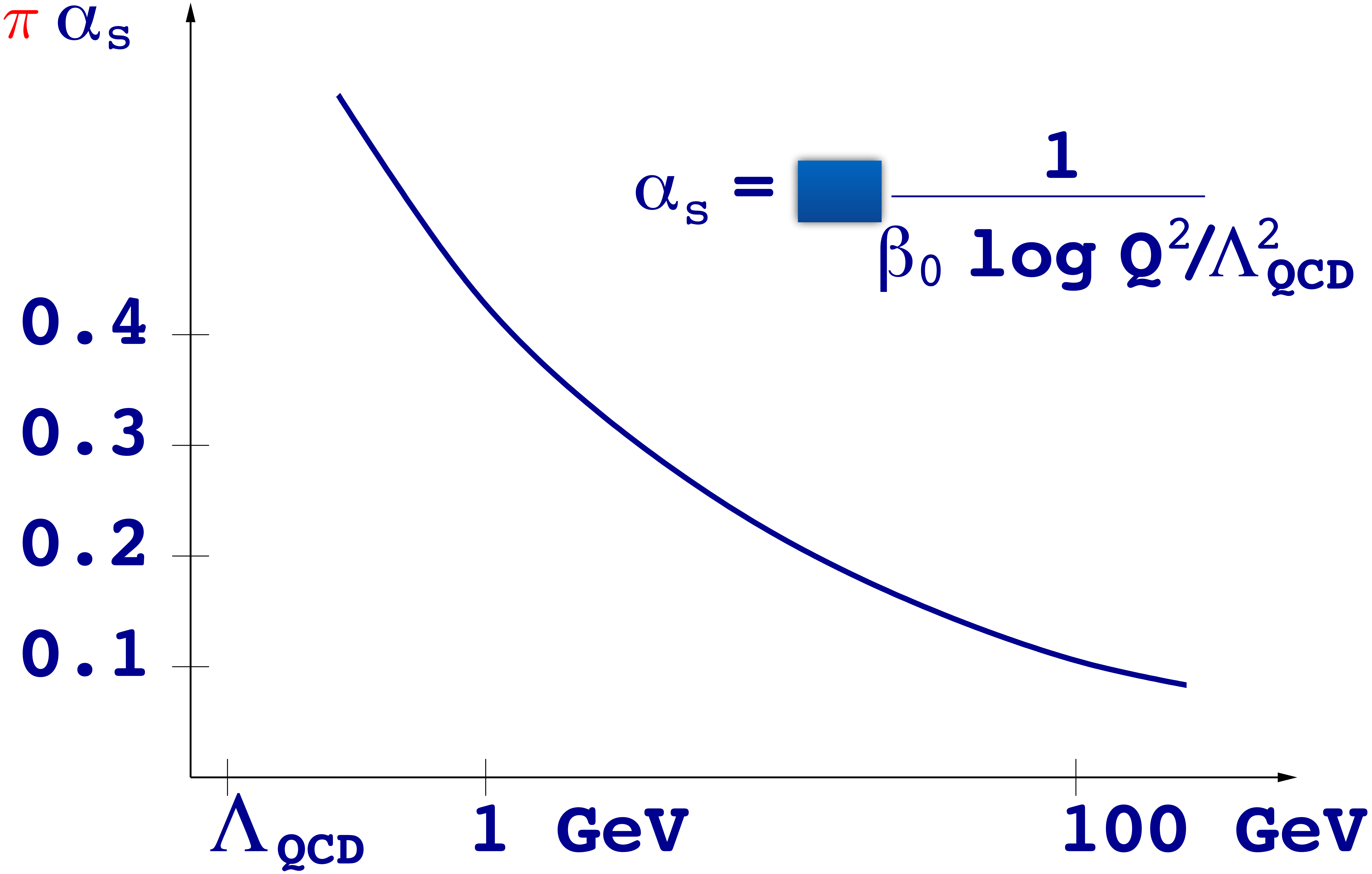

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Λ_{QCD} is the scale where nonperturbative effects dominate

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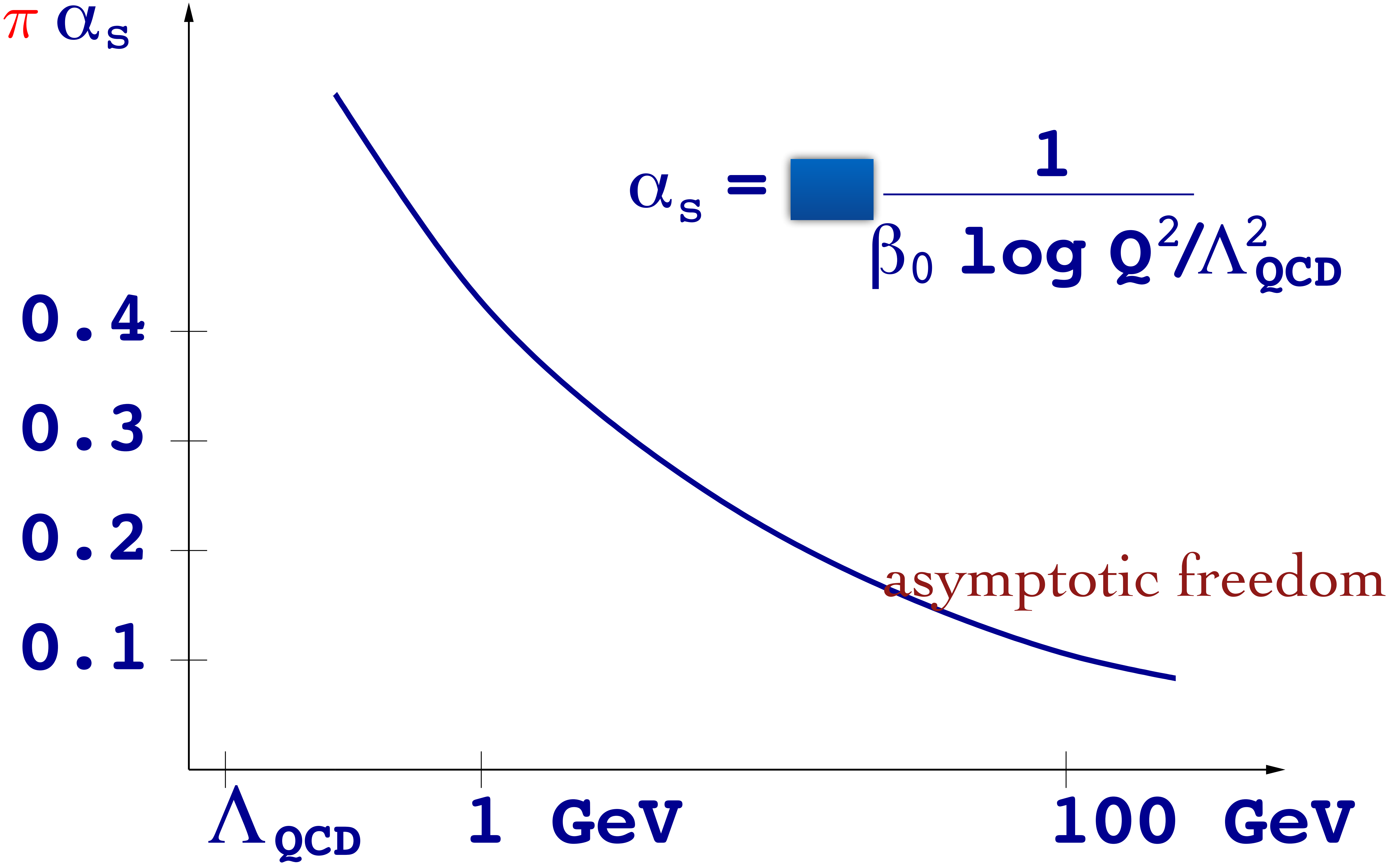

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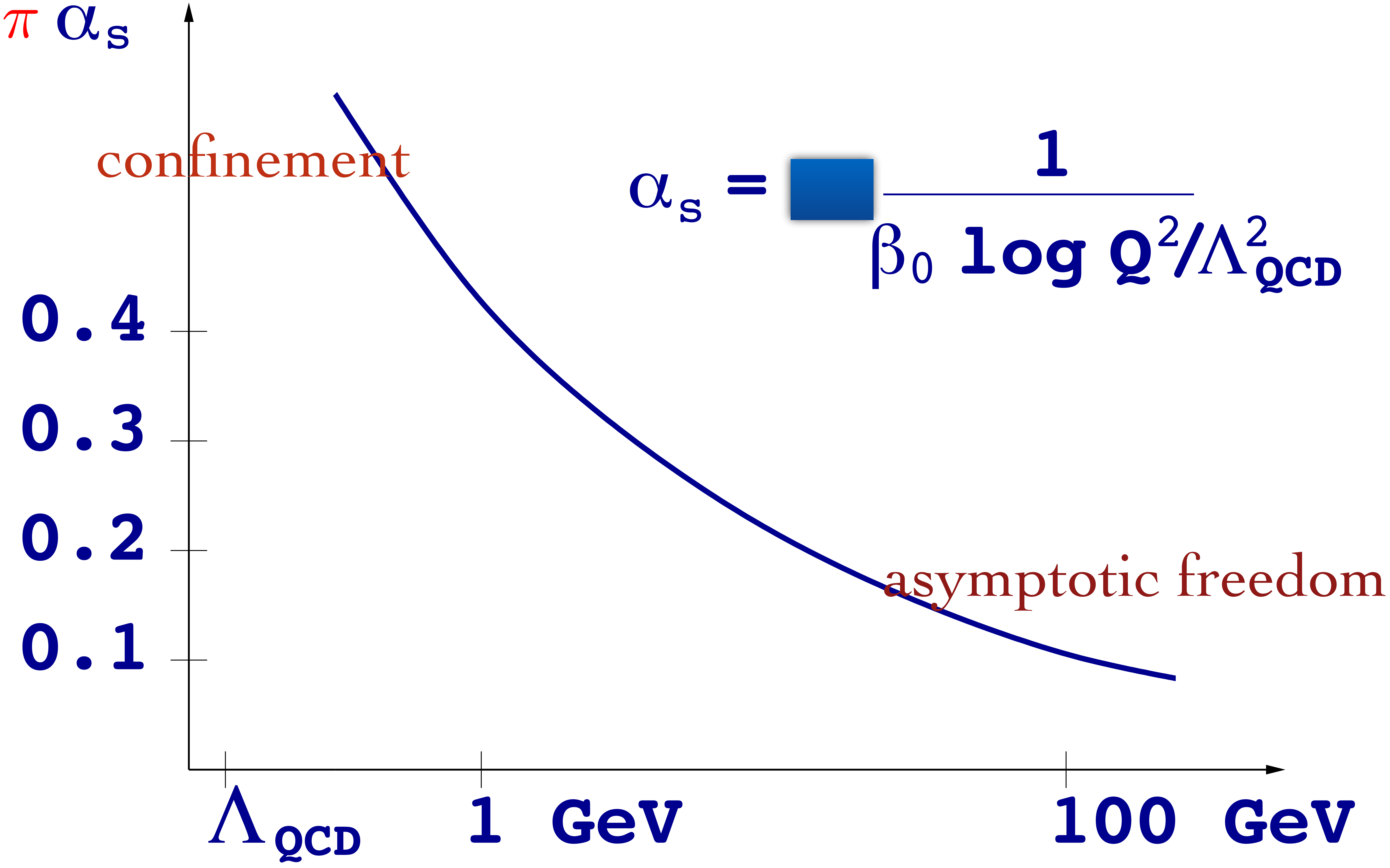

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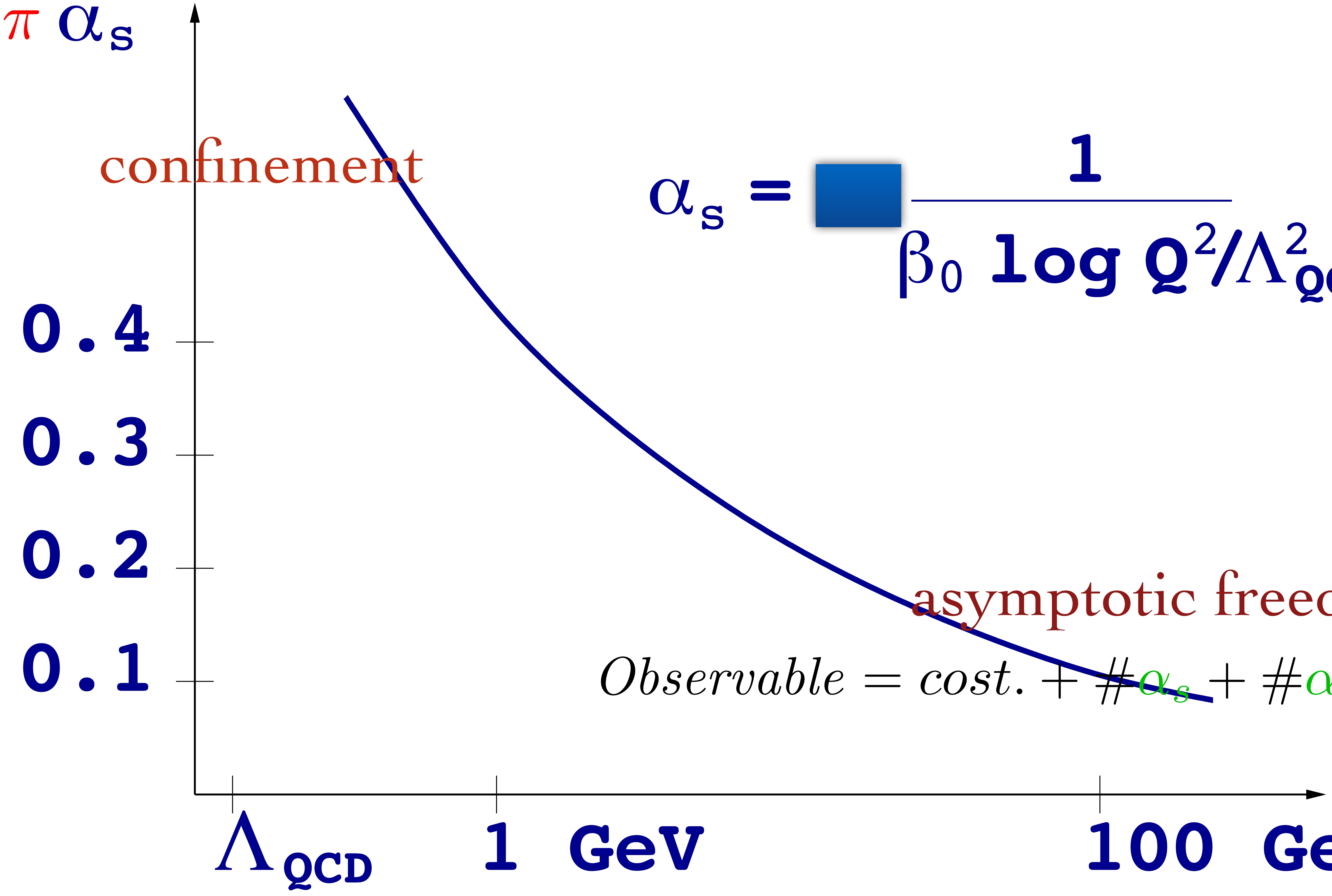

confinement

$$\alpha_s = \blacksquare\,\frac{1}{\beta_0 \log Q^2/\Lambda^2_{QCD}}$$

asymptotic freedom

$$Observable = cost. + \#\alpha_s + \#\alpha_s^2 + \ldots +$$

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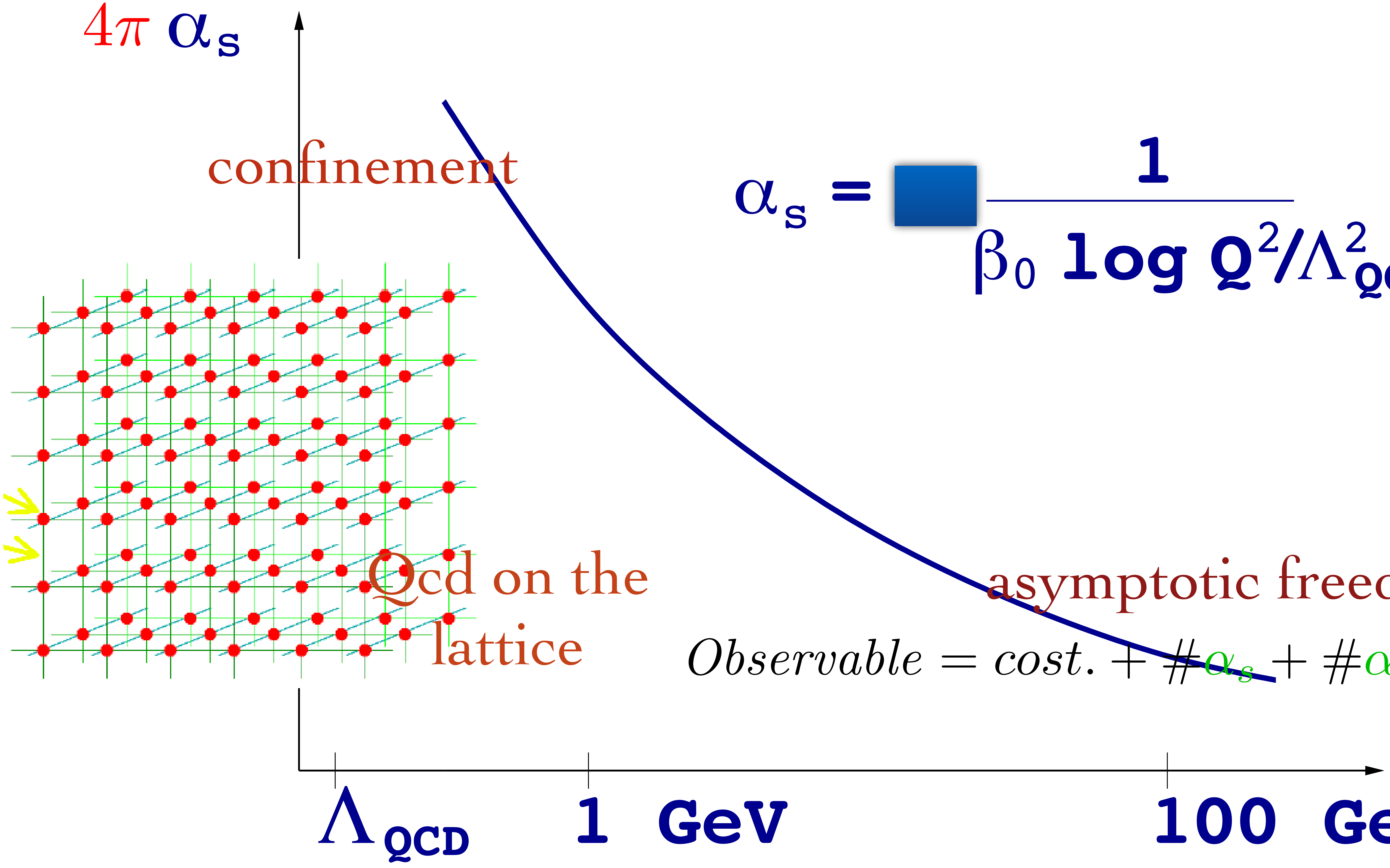

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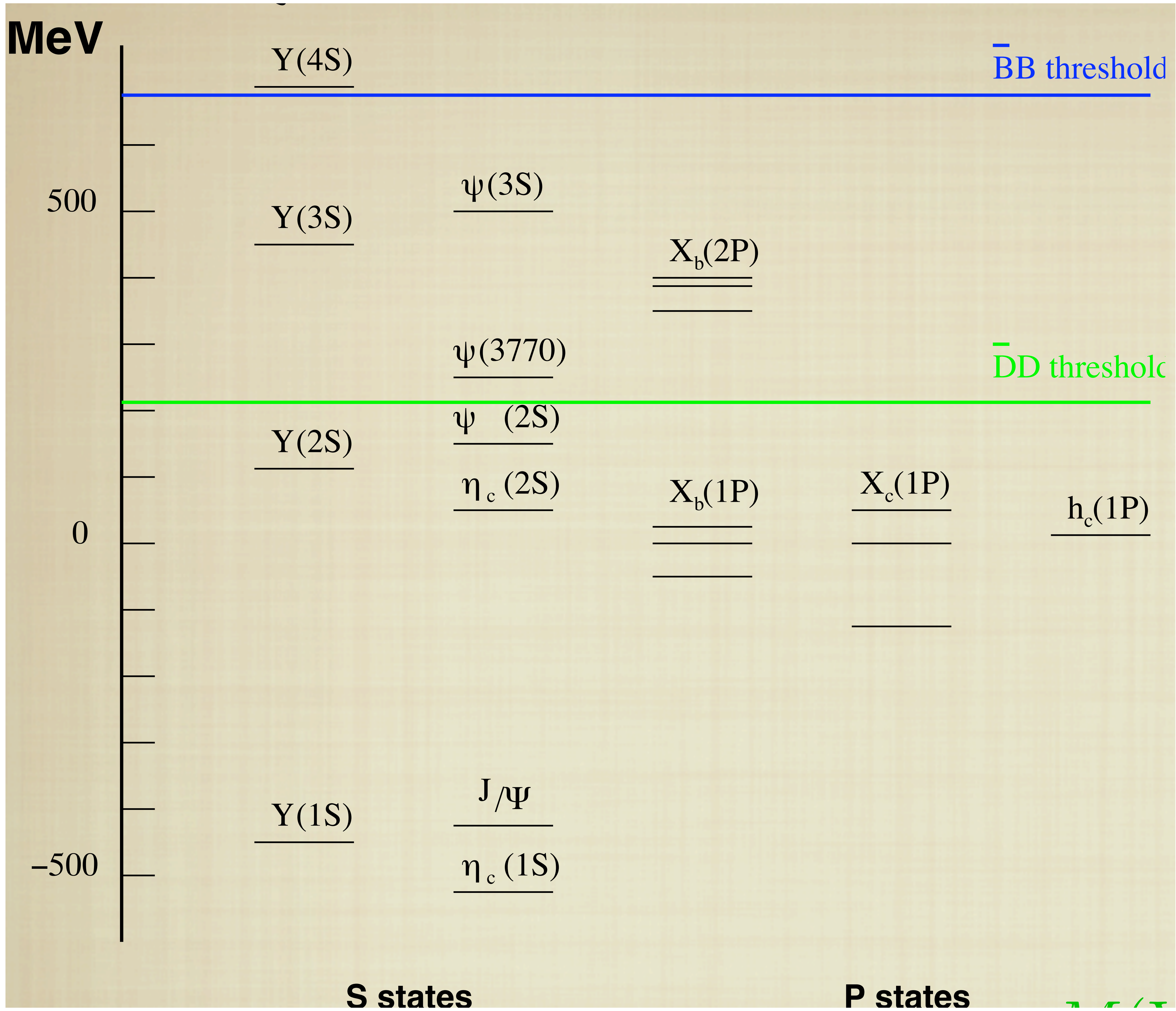
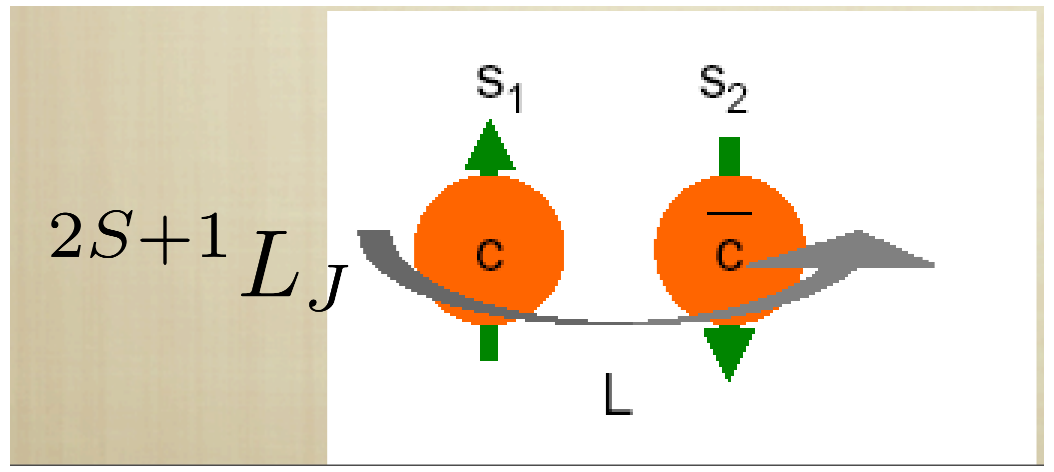
Quarkonium scales



Levels normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$



$$M(\Upsilon(1S)) = 9.460\,\text{GeV}$$

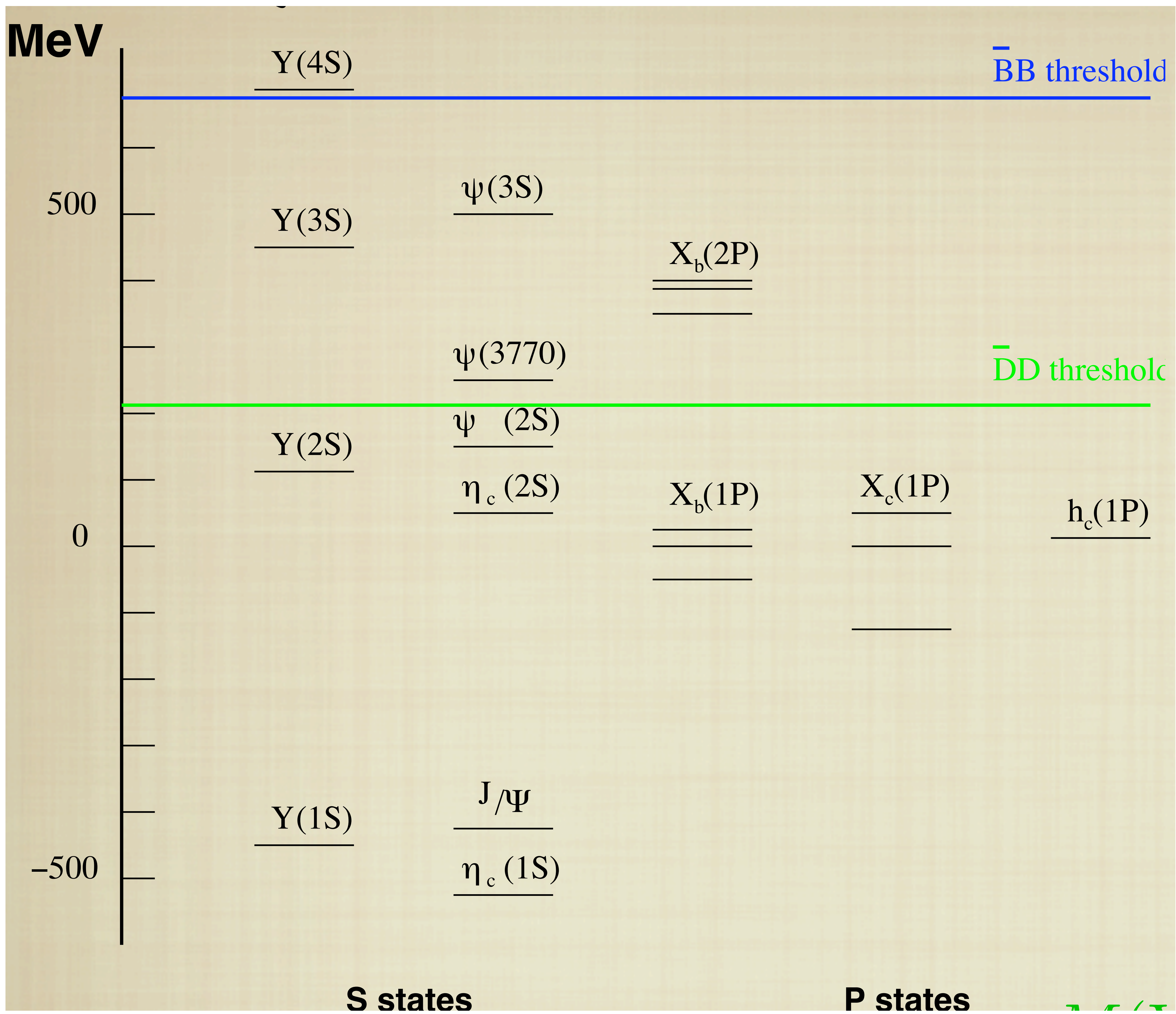
$$M(J/\psi) = 3.097\,\text{GeV}$$

THE MASS SCALE IS PERTURBATIVE

$$m_Q \gg \Lambda_{\text{QCD}}$$

$$m_b \simeq 5\,\text{GeV};\, m_c \simeq 1.5\,\text{GeV}$$

Quarkonium scales



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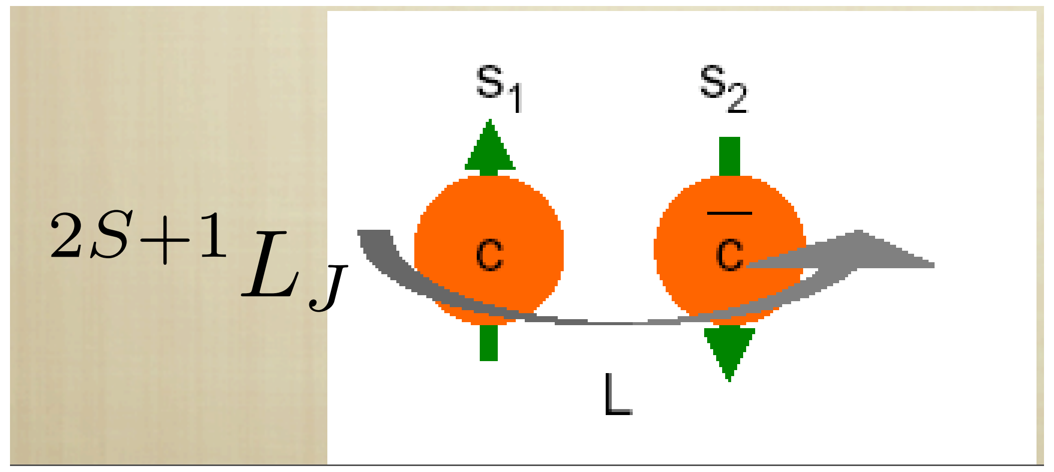

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$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

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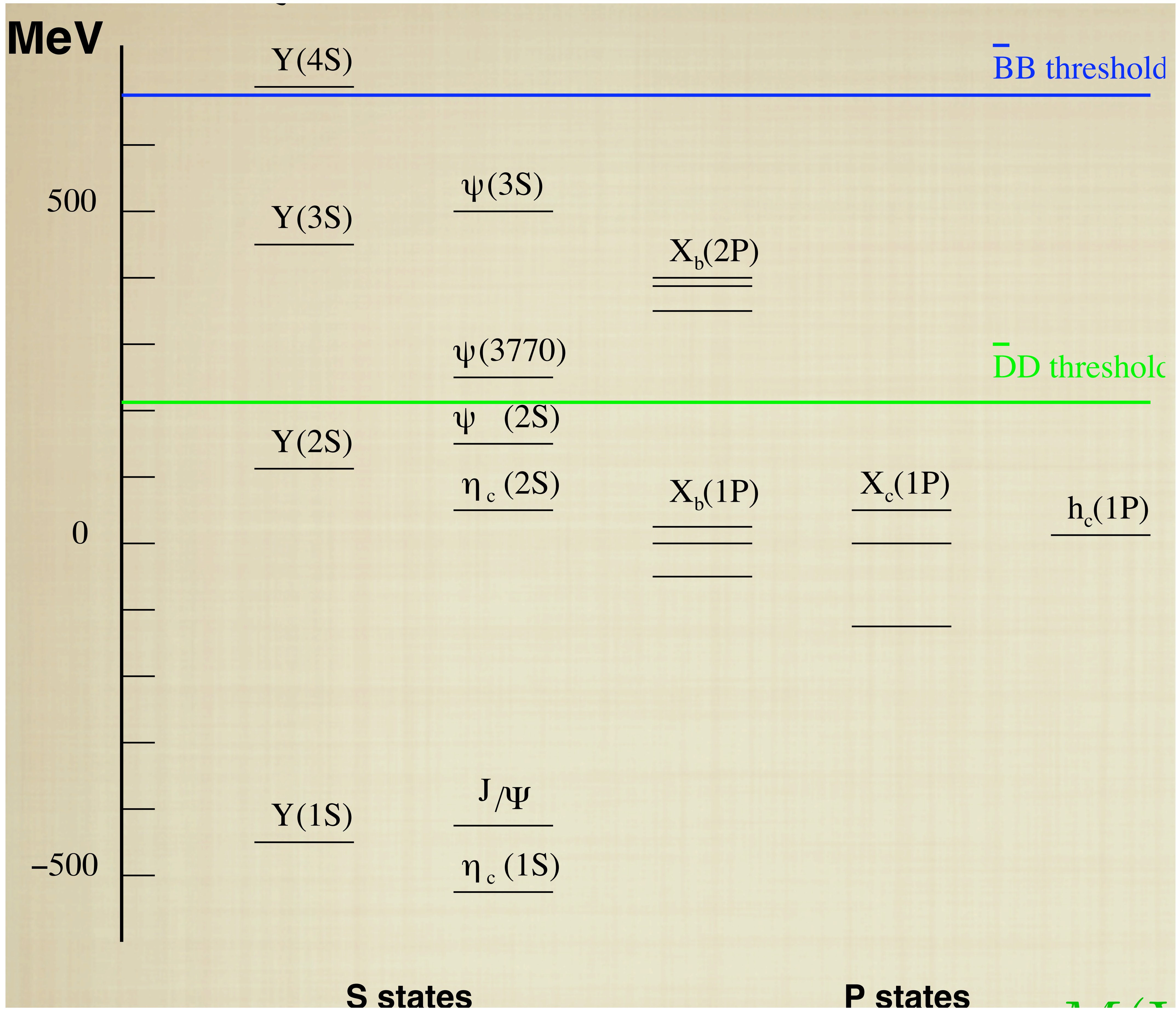

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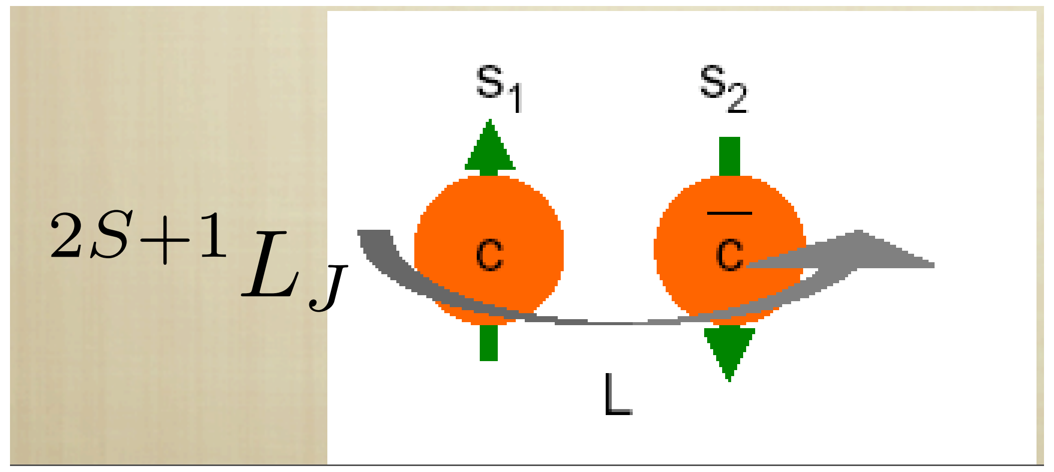

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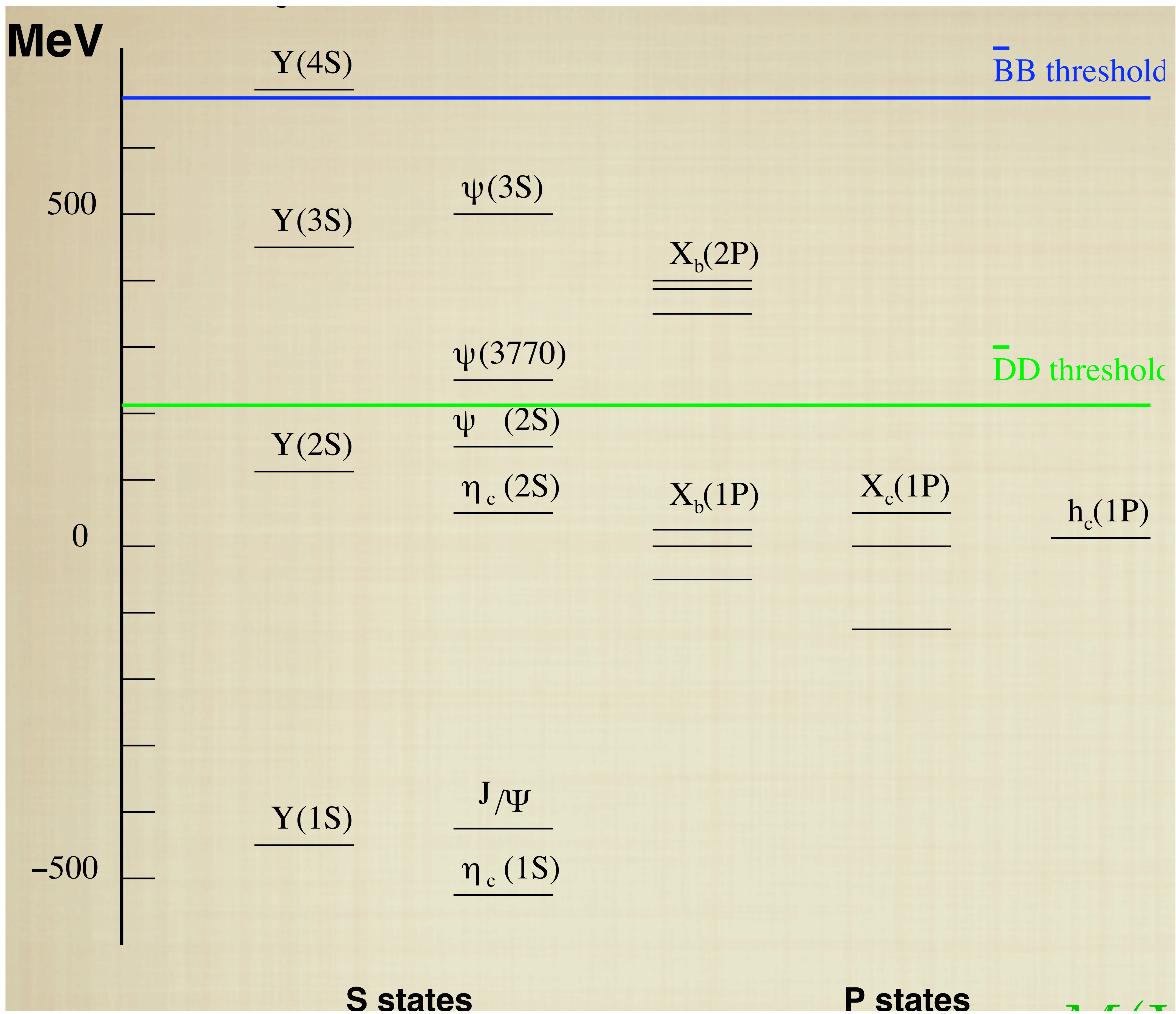

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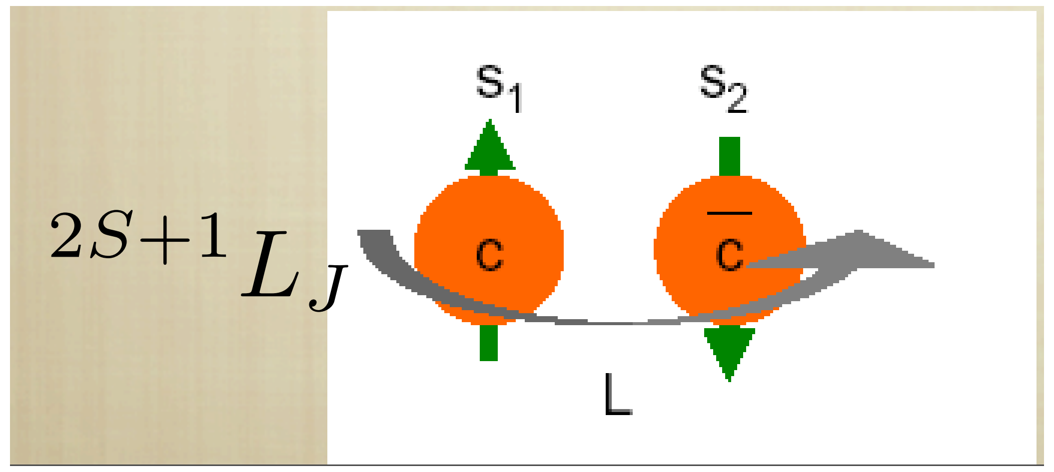

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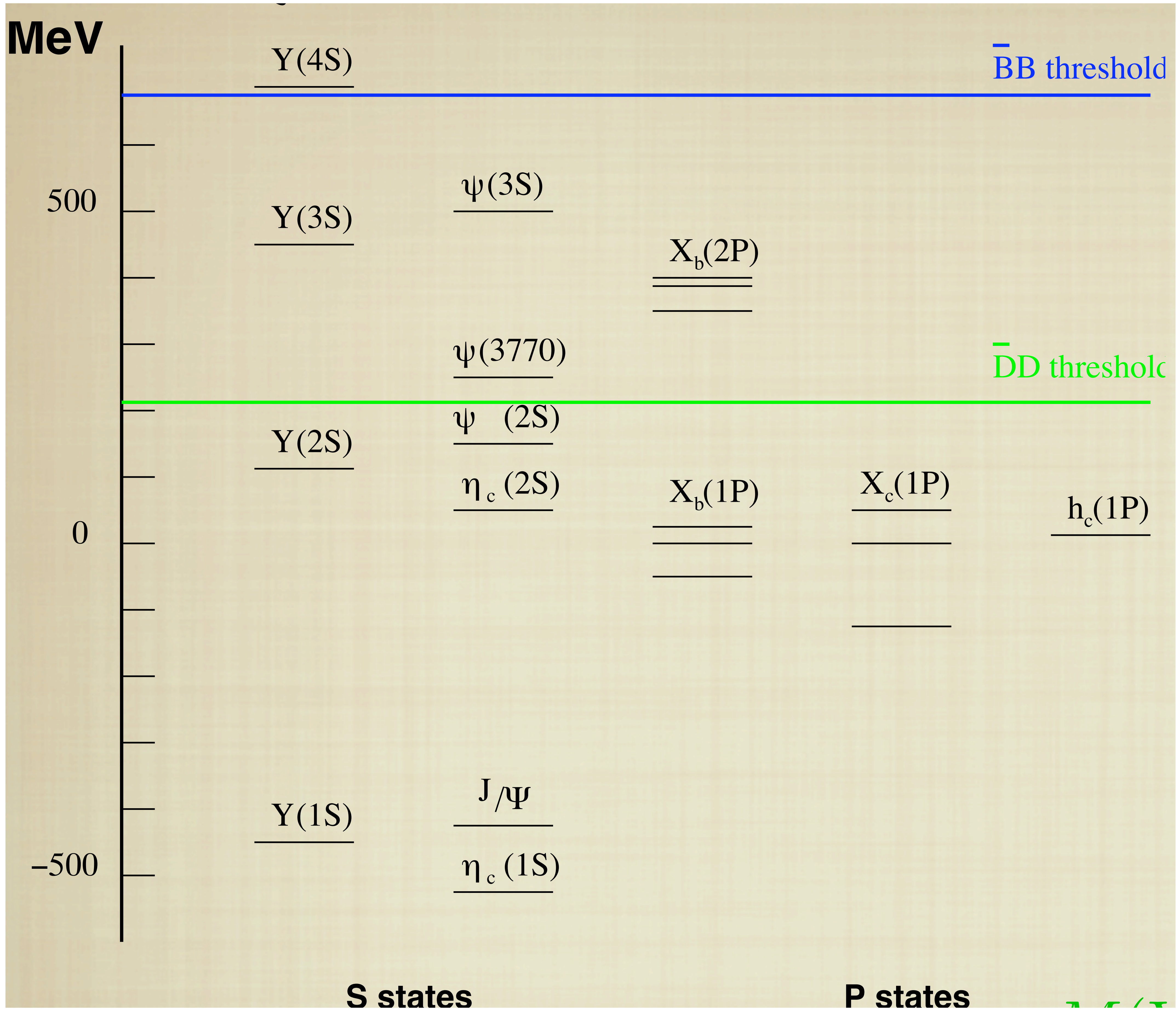

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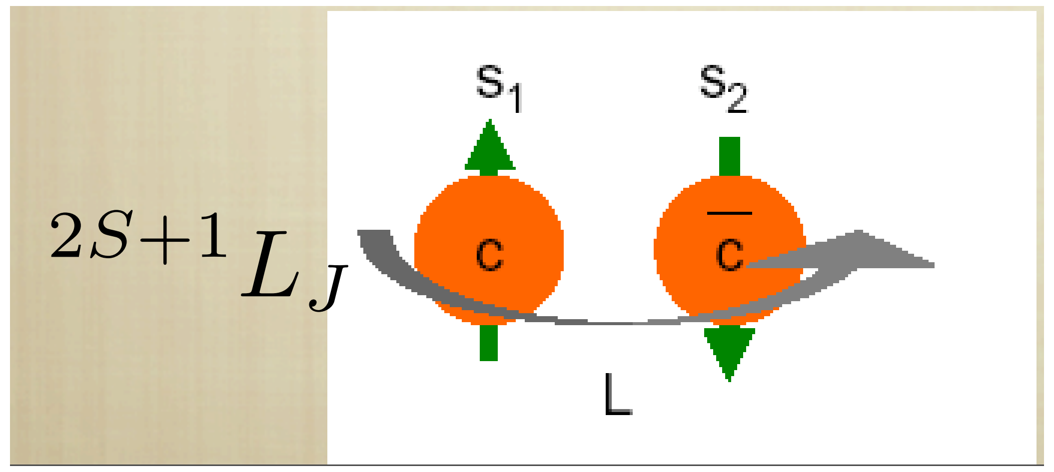

$^{2S+1}L_J$

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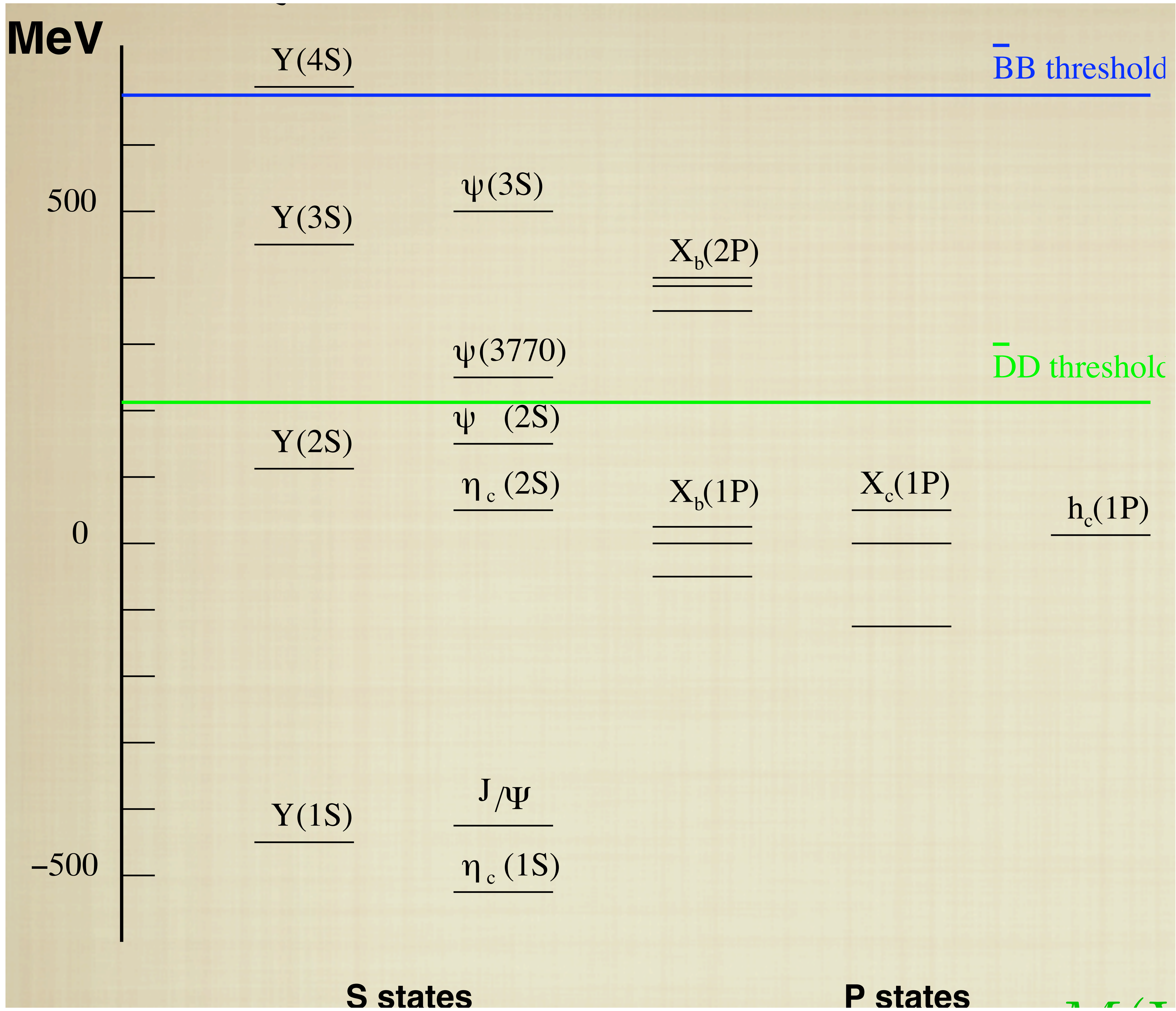

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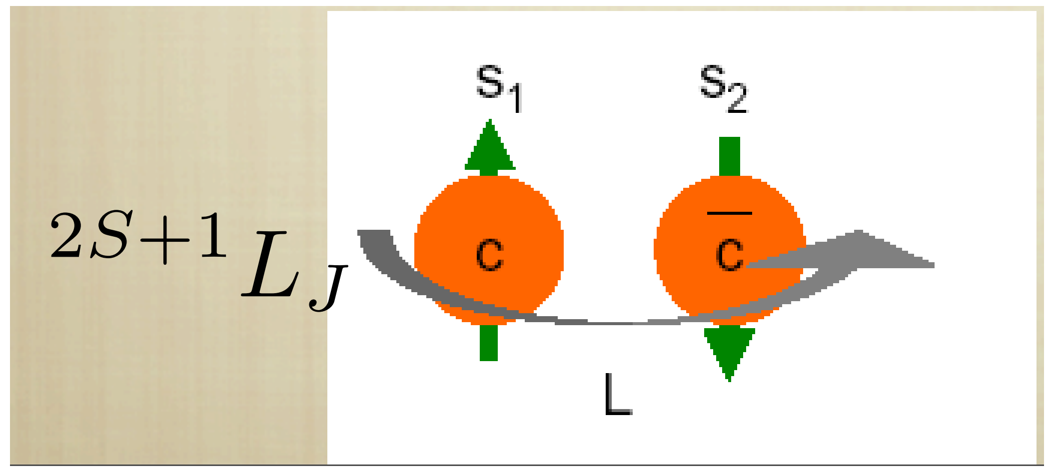

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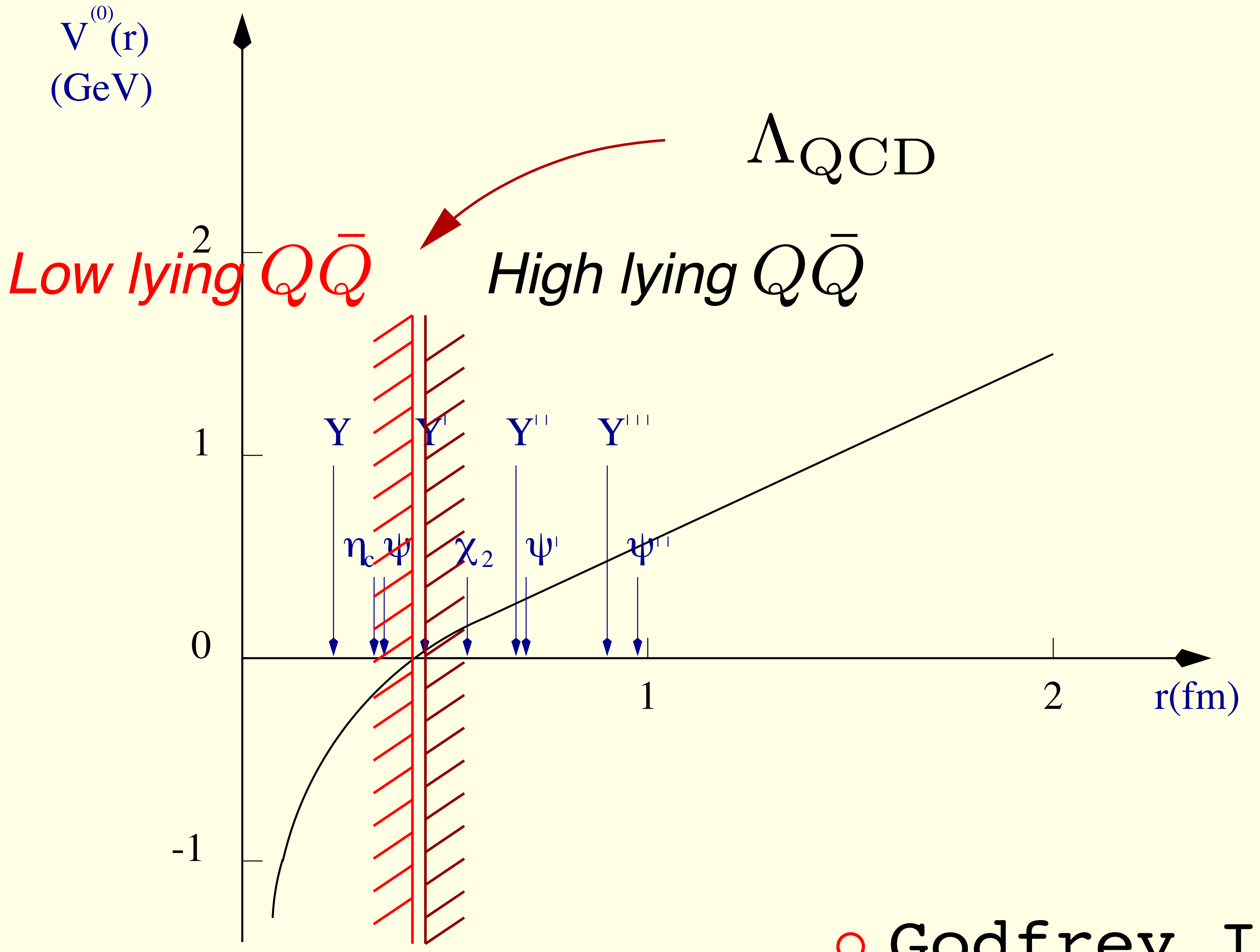
Heavy quarkonia are nonrelativistic systems
multiscale systems to be treated in QCD

The rich structure of separated energy scales makes QQbar systems an ideal probe of strong interactions

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



○ Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

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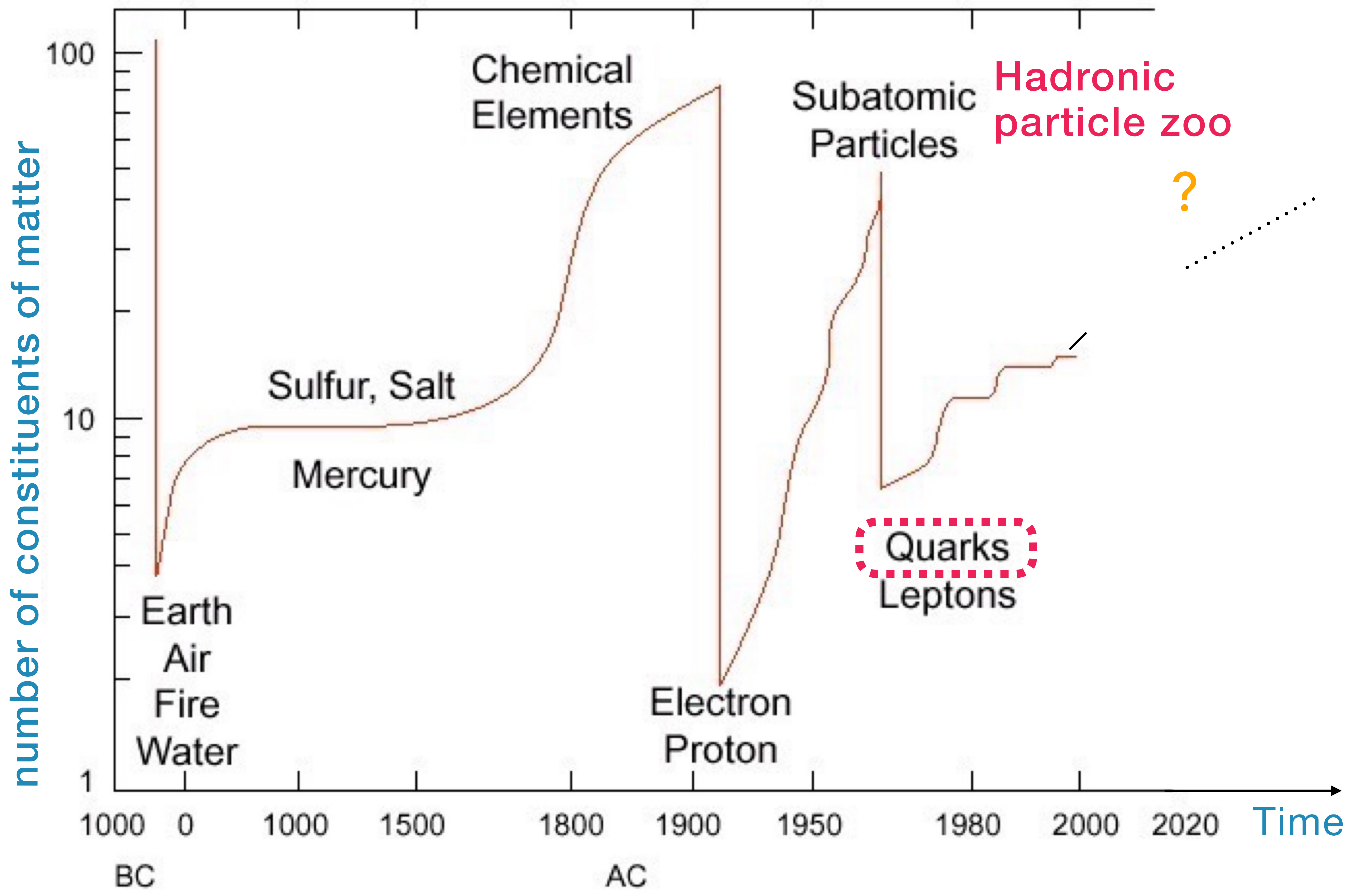
Today NR bound systems are at the center of new Revolutions

Constituents of matter and fundamental forces

history of constituents of matter



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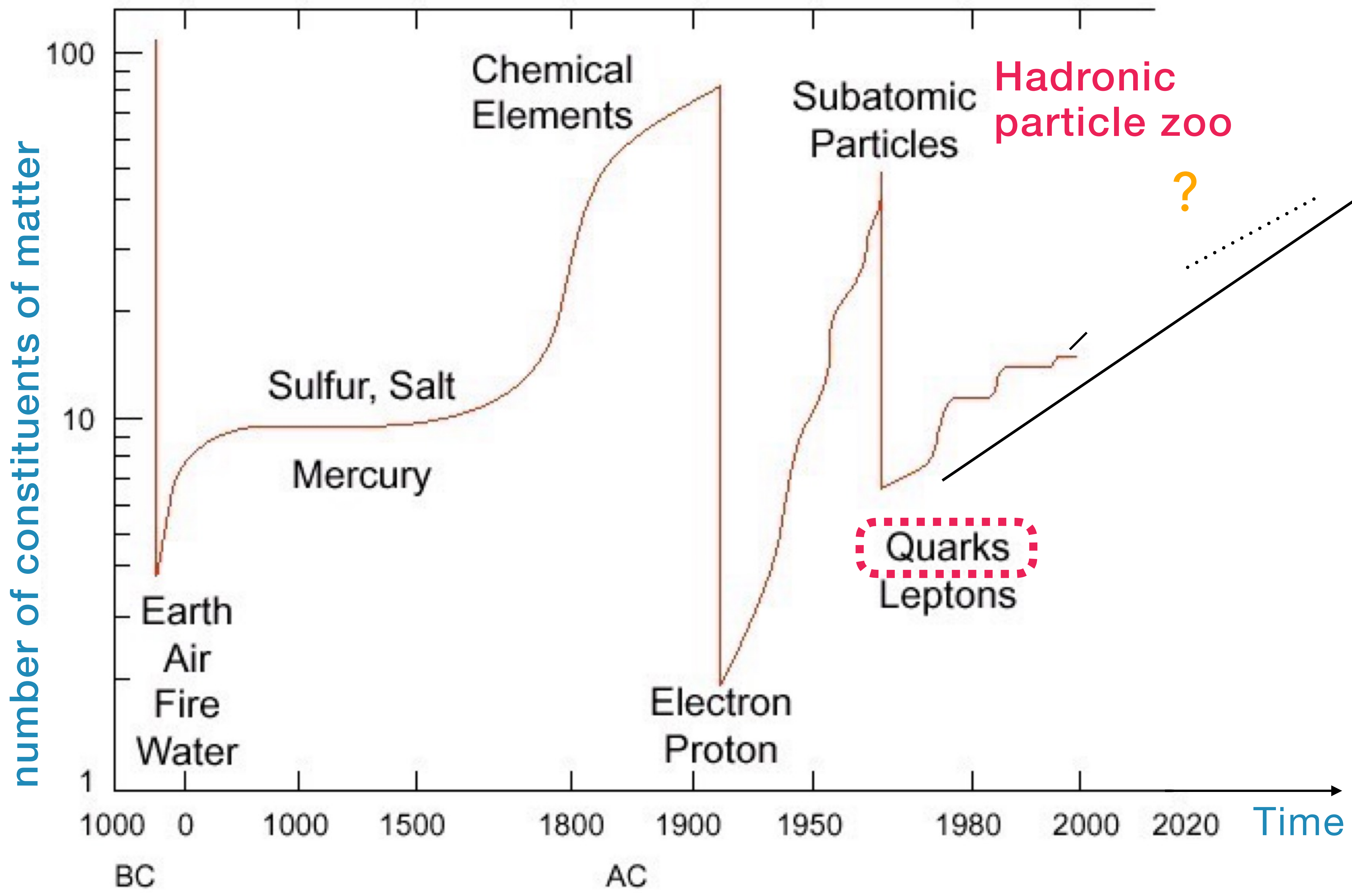
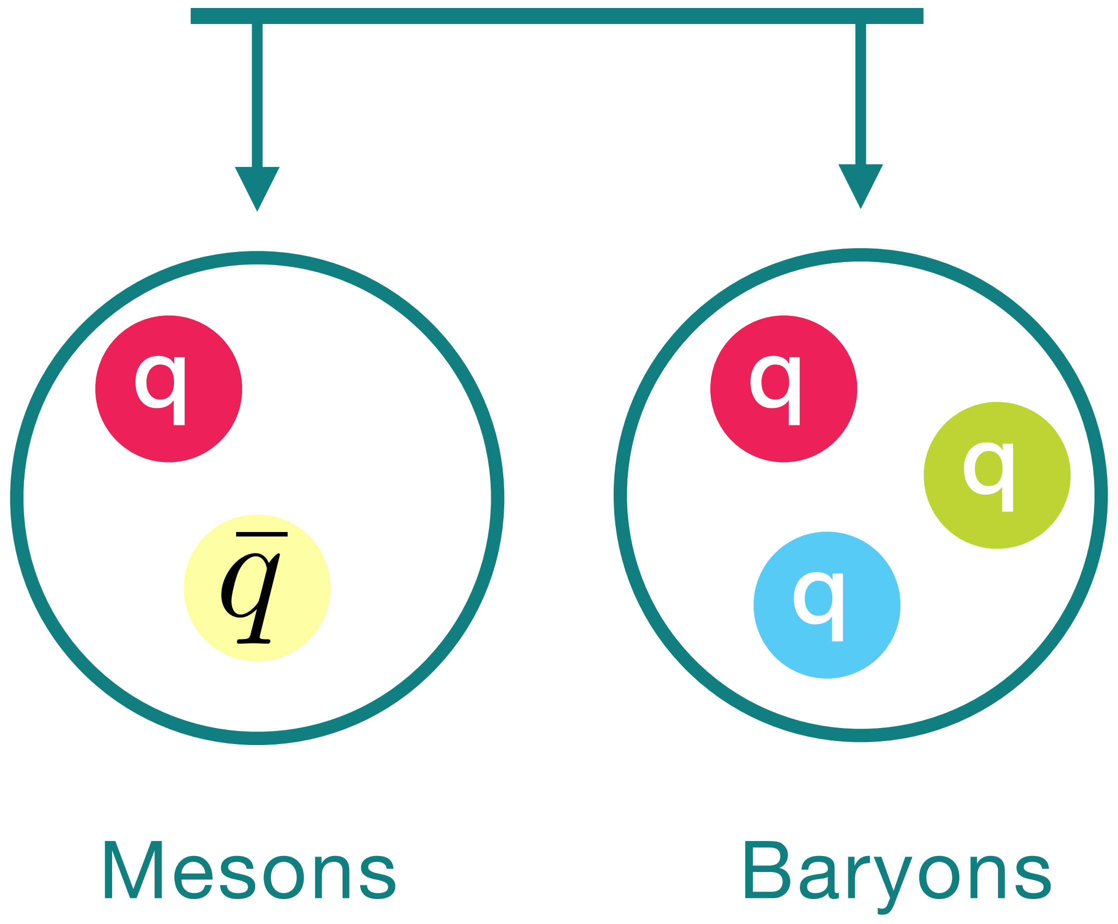


Quark Model 1964
Gell-Mann Zweig

observed only



$qq\bar{q}\bar{q}$ $qqqq\bar{q}\ldots$

possible but not observed

Constituents of matter and fundamental forces

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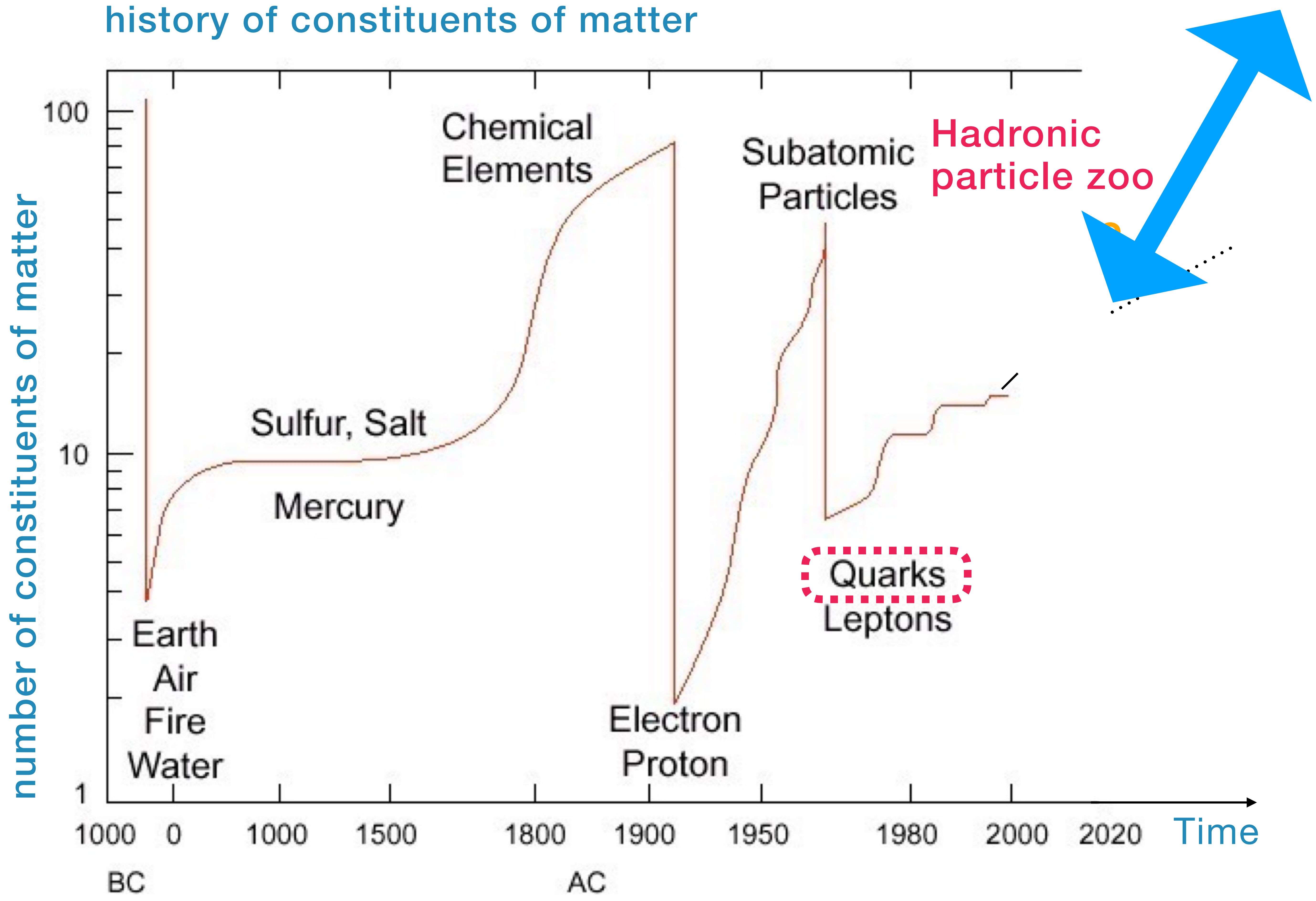
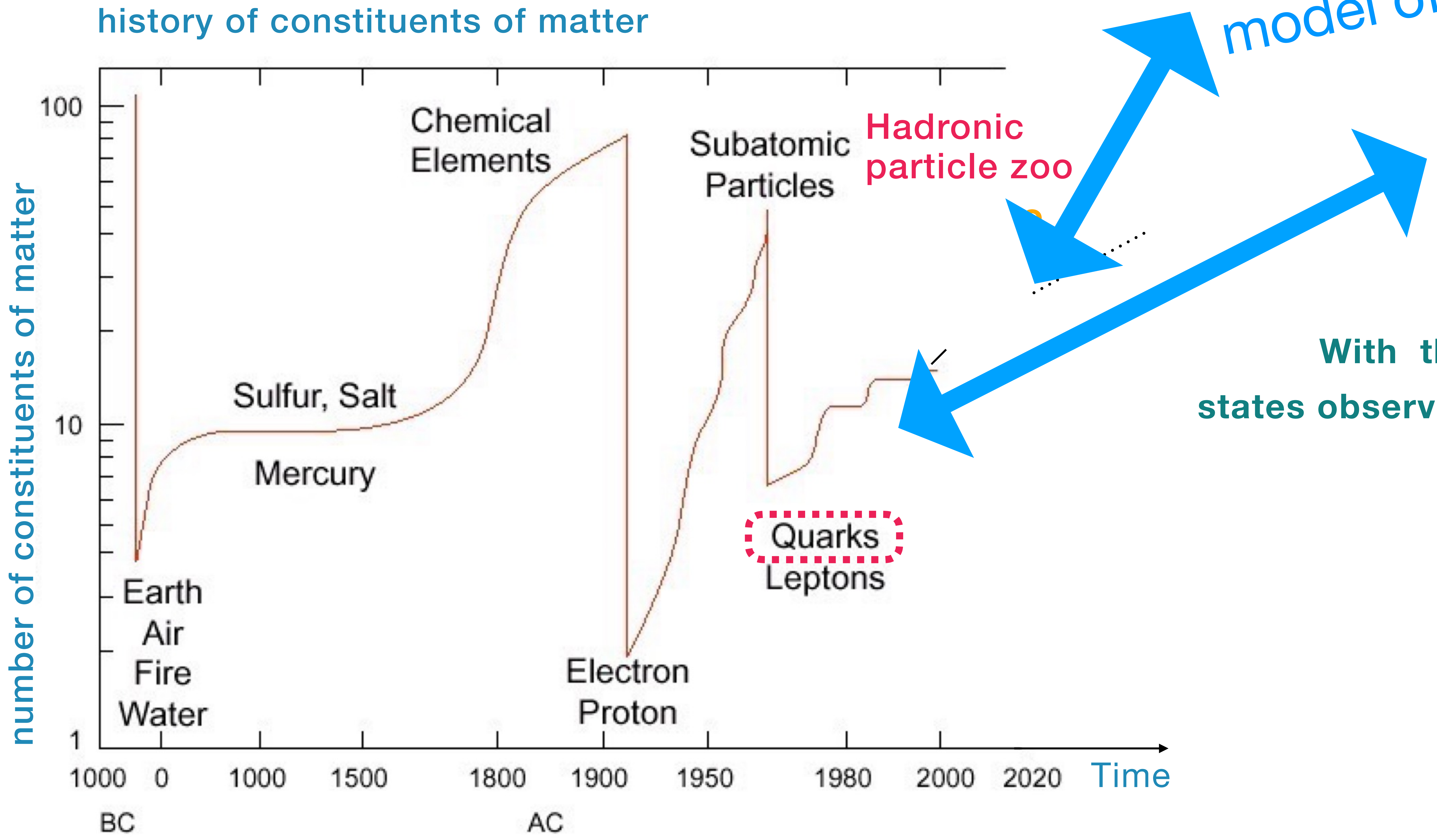


Beyond the standard model of particle physics

Constituents of matter and fundamental forces



Beyond the standard model of particle physics

Beyond the standard quark model

With the XYZ exotic states discovery, states observed in the sector with two heavy quarks

THE ECONOMIC TIMES

Scientists at CERN observe three "exotic" particles for first time

HARD SCIENCE — JULY 16, 2022

Tetraquarks and pentaquarks: "Unnatural" forms of exotic matter have been found

Scientists have found three new examples of a very exotic form of matter made of quarks. They can yield insights into the early Universe.

JARY 26, 2016 | 3 MIN READ

ysicists May Have Discovered a New "Tetraquark" Particle

Data from the DZero experiment shows evidence of a particle containing four different types of qu

WIRED

'Impossible' Particle Adds a Piece to the Strong Force Puzzle

The unexpected discovery of the double-charm tetraquark gives physicists fresh insight into the strongest of nature's fundamental forces.

INDIA TODAY

Mysterious 'X' particles that formed moments after the big bang found in Large Hadron Collider

CORRIERE DELLA SERA

Nuova straordinaria particella scoperta al Cern: il pentaquark

Consentirà di saperne di più sulla «forza forte» che tiene unite le particelle nel nucleo

i e sui componenti della materia

Le Monde

Les surprises du tétraquark, « collage » de particules élémentaires

La découverte d'une nouvelle particule à la structure particulièrement stable pourrait permettre aux chercheurs de vérifier leurs théories sur l'interaction forte.

of unknown nature discover

sector with tw

BBC

Pentaquarks: scientists find new "exotic" configurations of quarks

Scientists have found new ways in which quarks, the tiniest particles known to humankind, group together.

ZEIT ONLINE

Cern-Forscher entdecken neues Teilchen

Die Physiker am Kernforschungszentrum in Genf haben die Existenz des Pentaquark-Teilchens nachgewiesen. Bislang war es nur in theoretischen beschrieben worden.

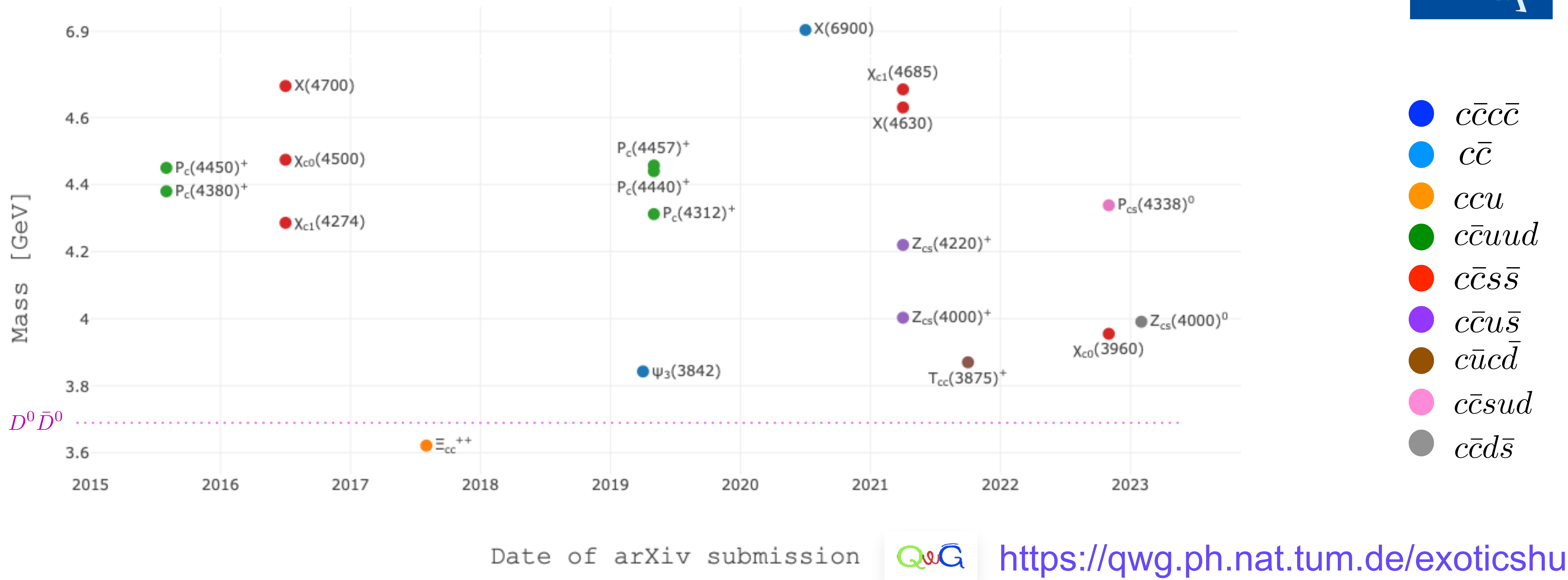
LHCb discovers longest-lived exotic matter yet

08/04/21 | By Sarah Charley

The newly discovered tetraquark provides a unique window into the interactions of the particles that make up atoms.

symmetry

XYZ Revolution: Discoveries of Exotic States in the Sector With Two Heavy Quarks: Beyond the Standard Quark Model!



https://qwg.ph.nat.tum.de/exoticshub/

INTERPLAY AMONG MANY EXPERIMENTS:

UPCOMING EXPERIMENTS:

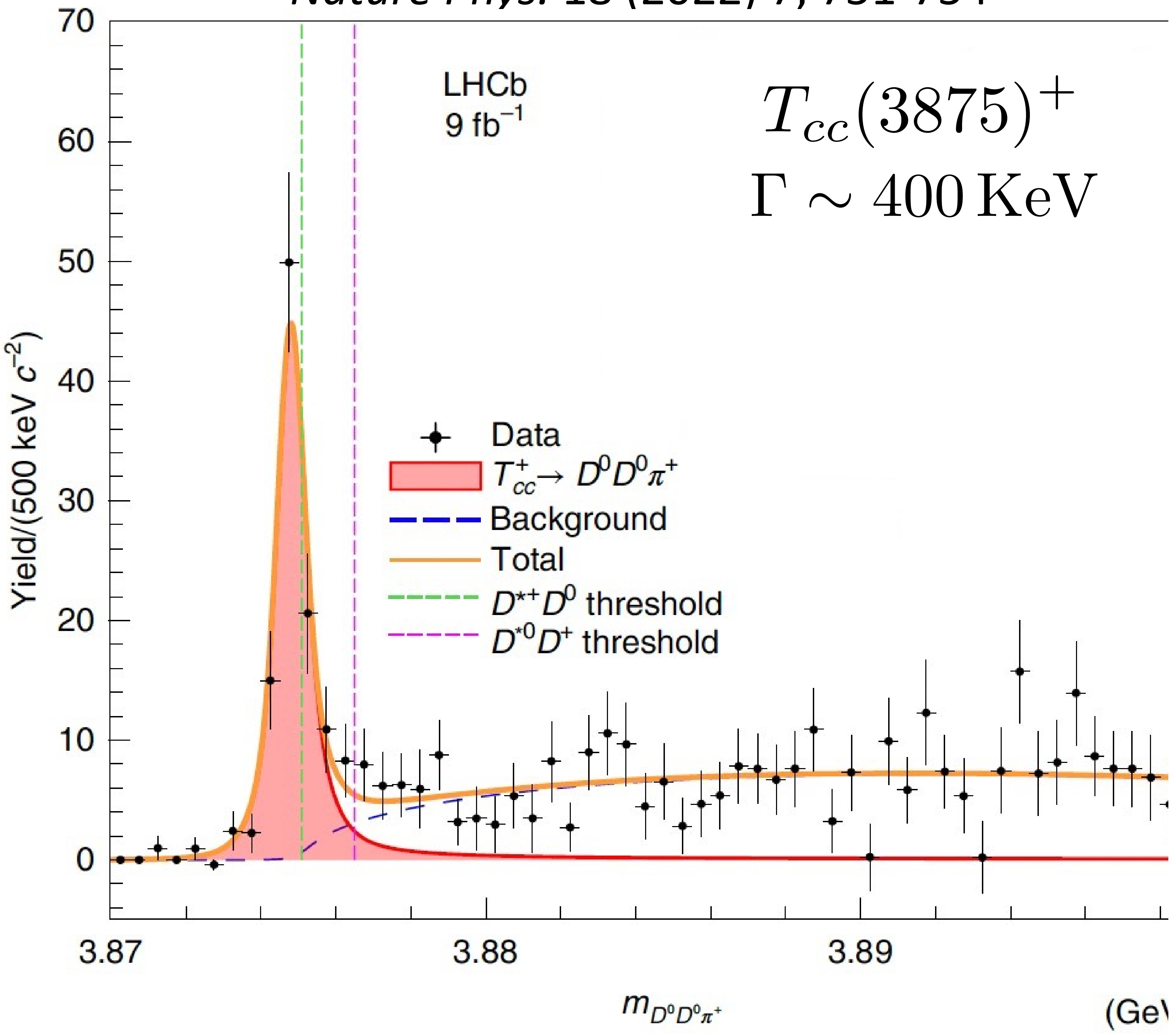
STCF

Electron ion

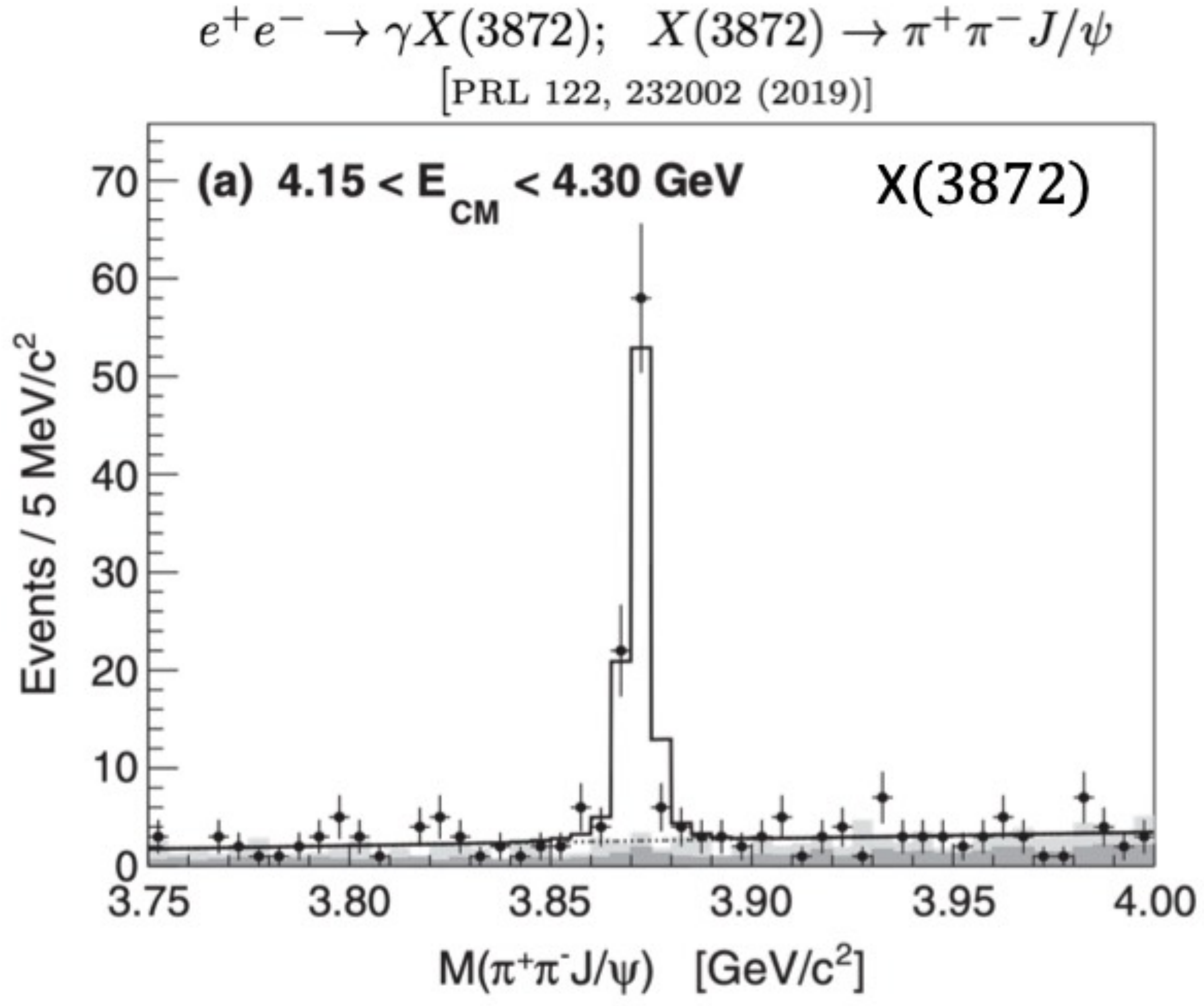
XYZ REVOLUTION: **A New Spectroscopy Is Born!**

Some surprisingly narrow states even if above/at strong decay thresholds

Nature Phys. 18 (2022) 7, 751-754

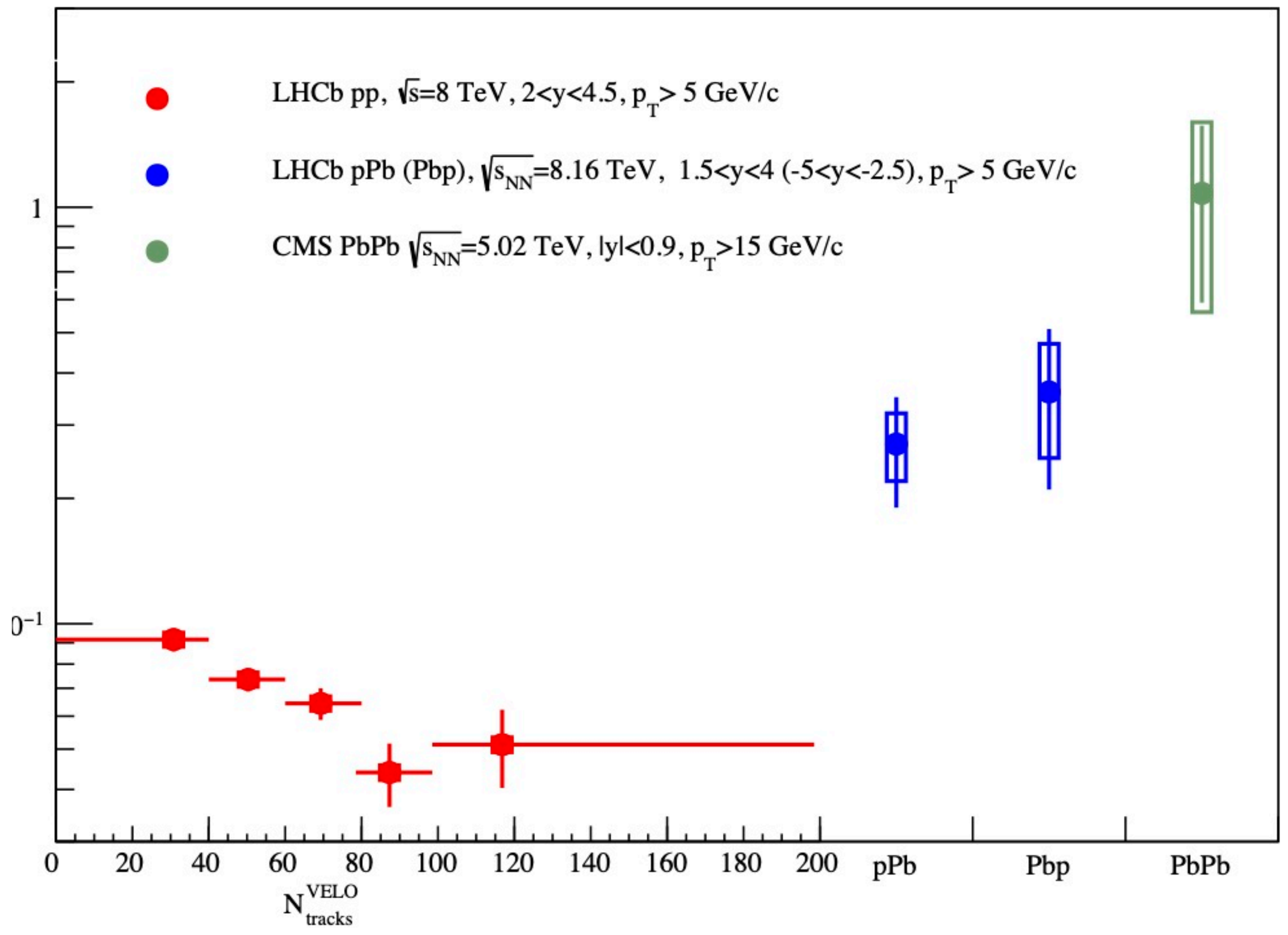


$$M_{T_{cc}(3875)^+} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06 MeV$$



$$M_{X(3872)} - M_{D^0 D^{*0}} = 0.01 \pm 0.14 MeV$$

Produced in heavy ions where the deconfined strongly coupled QCD medium (Quark Gluon Plasma-QGP) is formed



New perspectives for XYZ studies!

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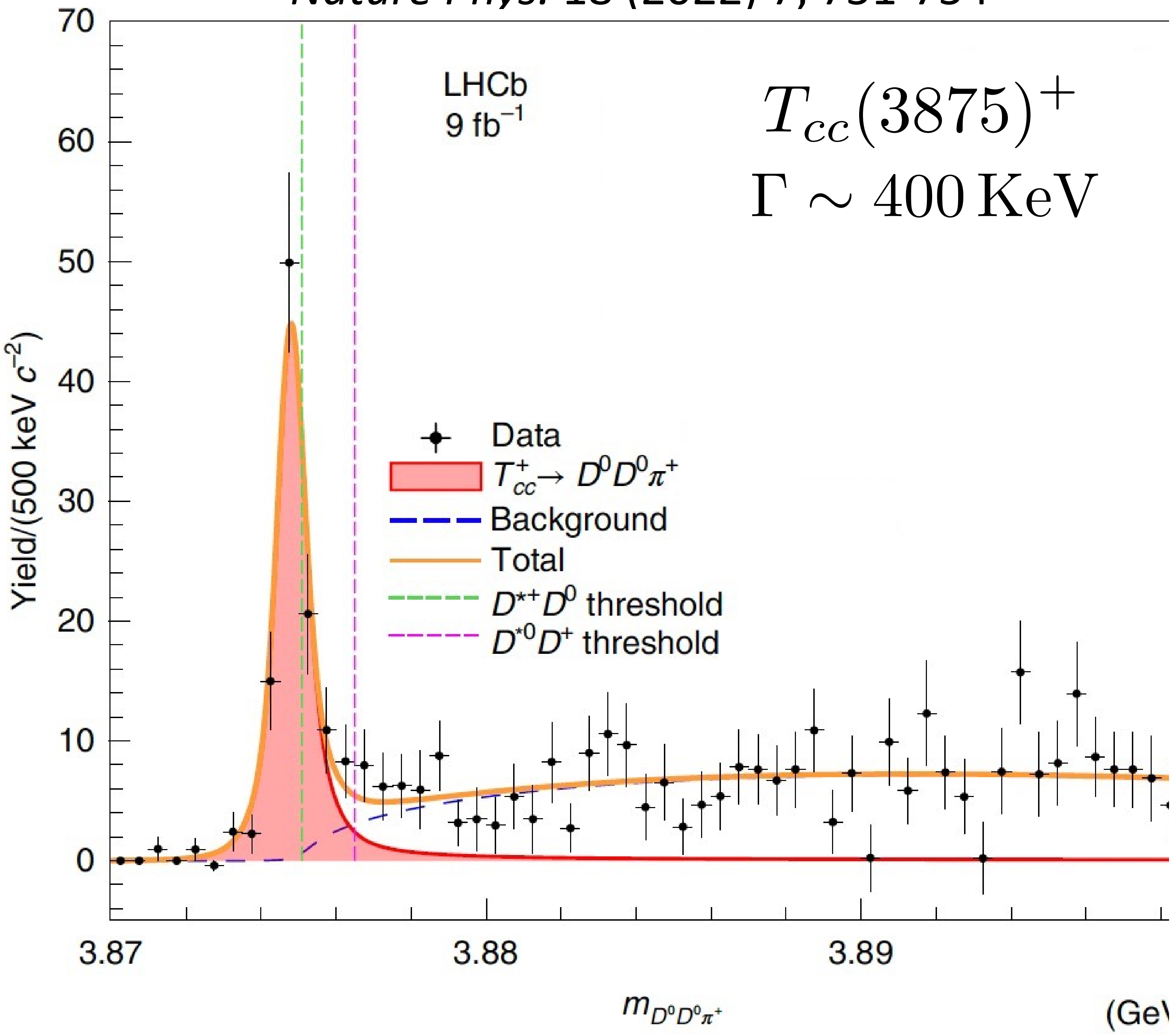

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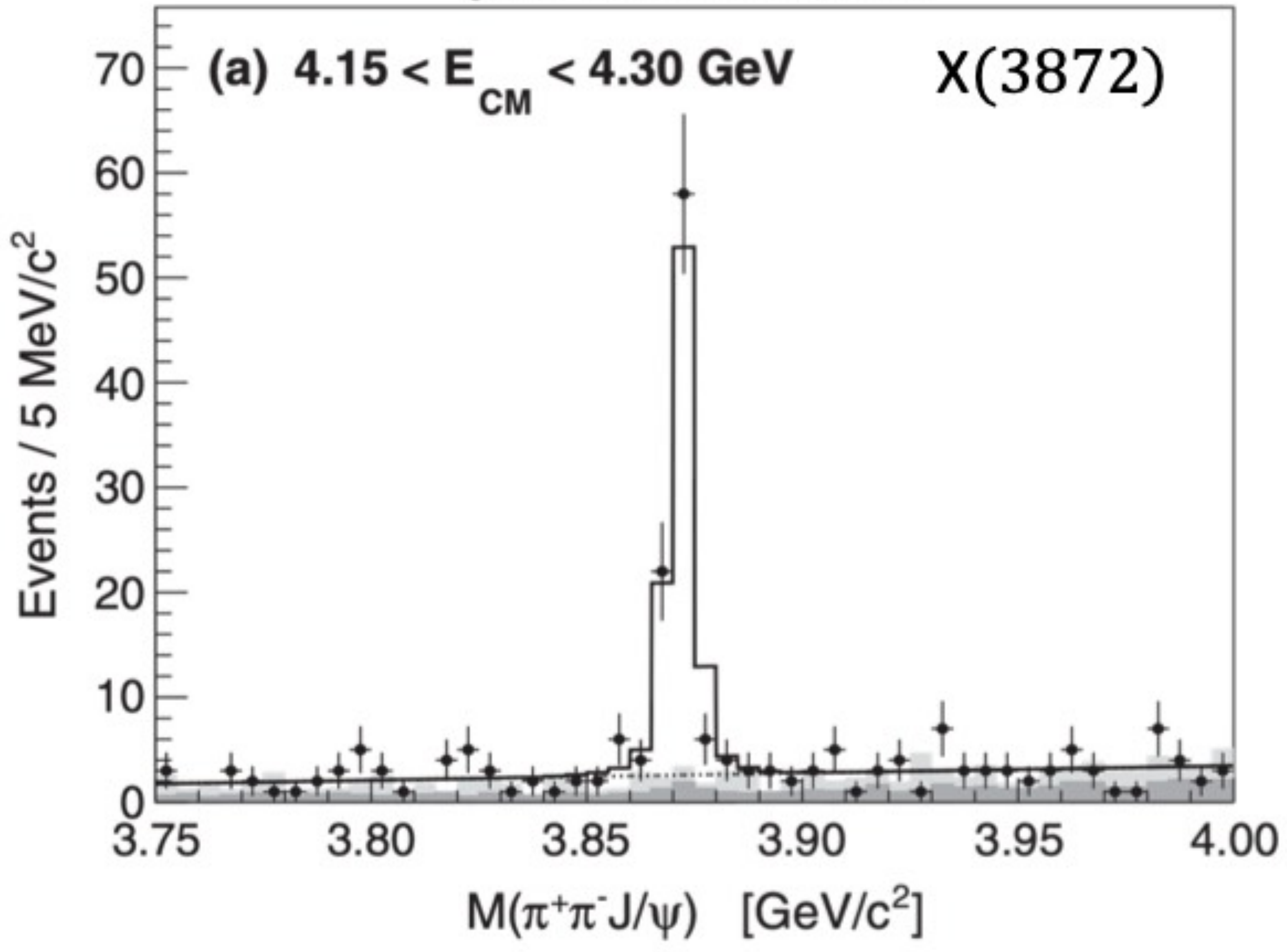
$e^+e^- \to \gamma X(3872)$; $X(3872) \to \pi^+\pi^- J/\psi$

[PRL 122, 232002 (2019)]



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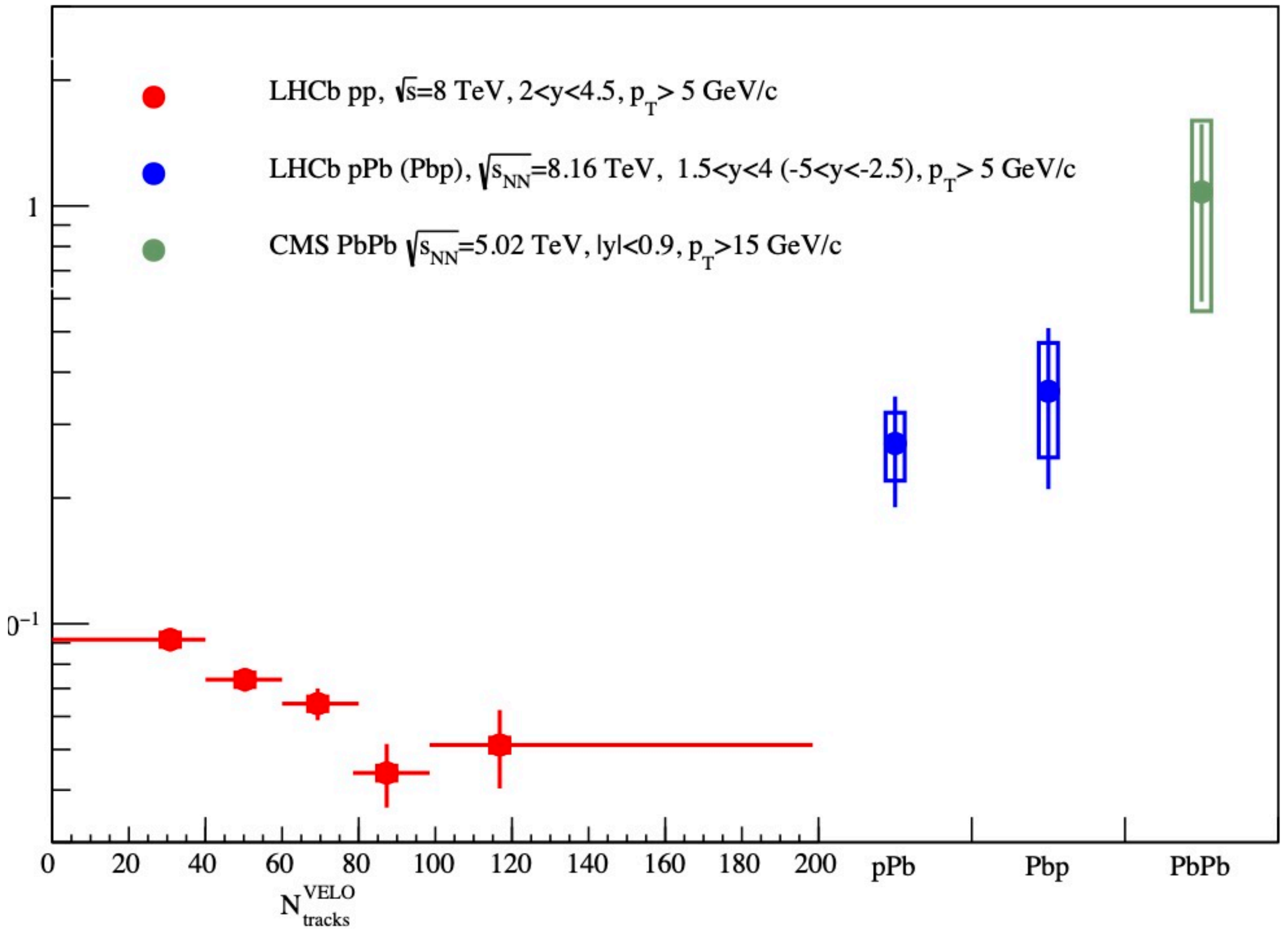


New perspectives for XYZ studies!

XYZs not merely composite particles, have unique properties

Novel strongly correlated exotics systems

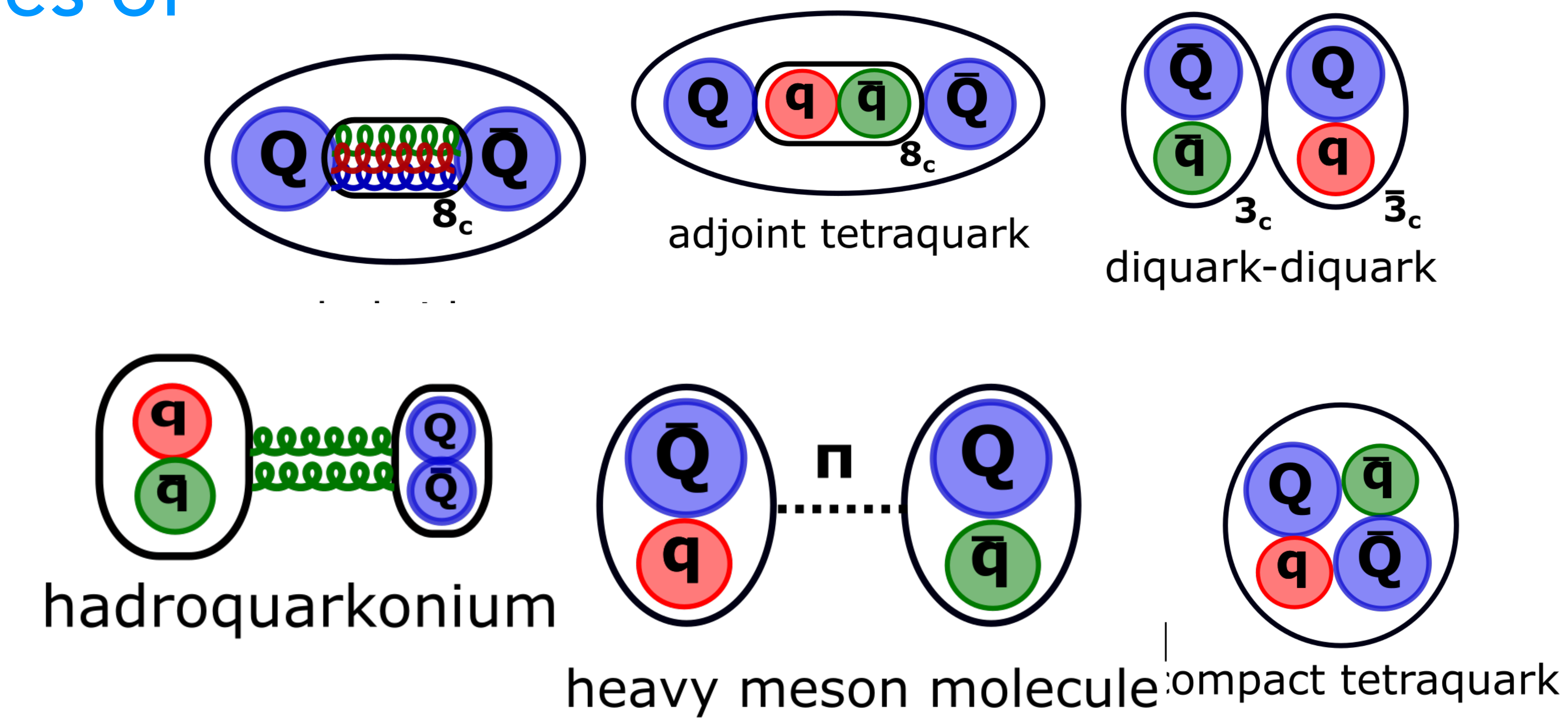
The present revolution: XYZ a great theoretical challenge

Close/above threshold new degrees of freedom like glue and light quarks are nonperturbative part in the binding.

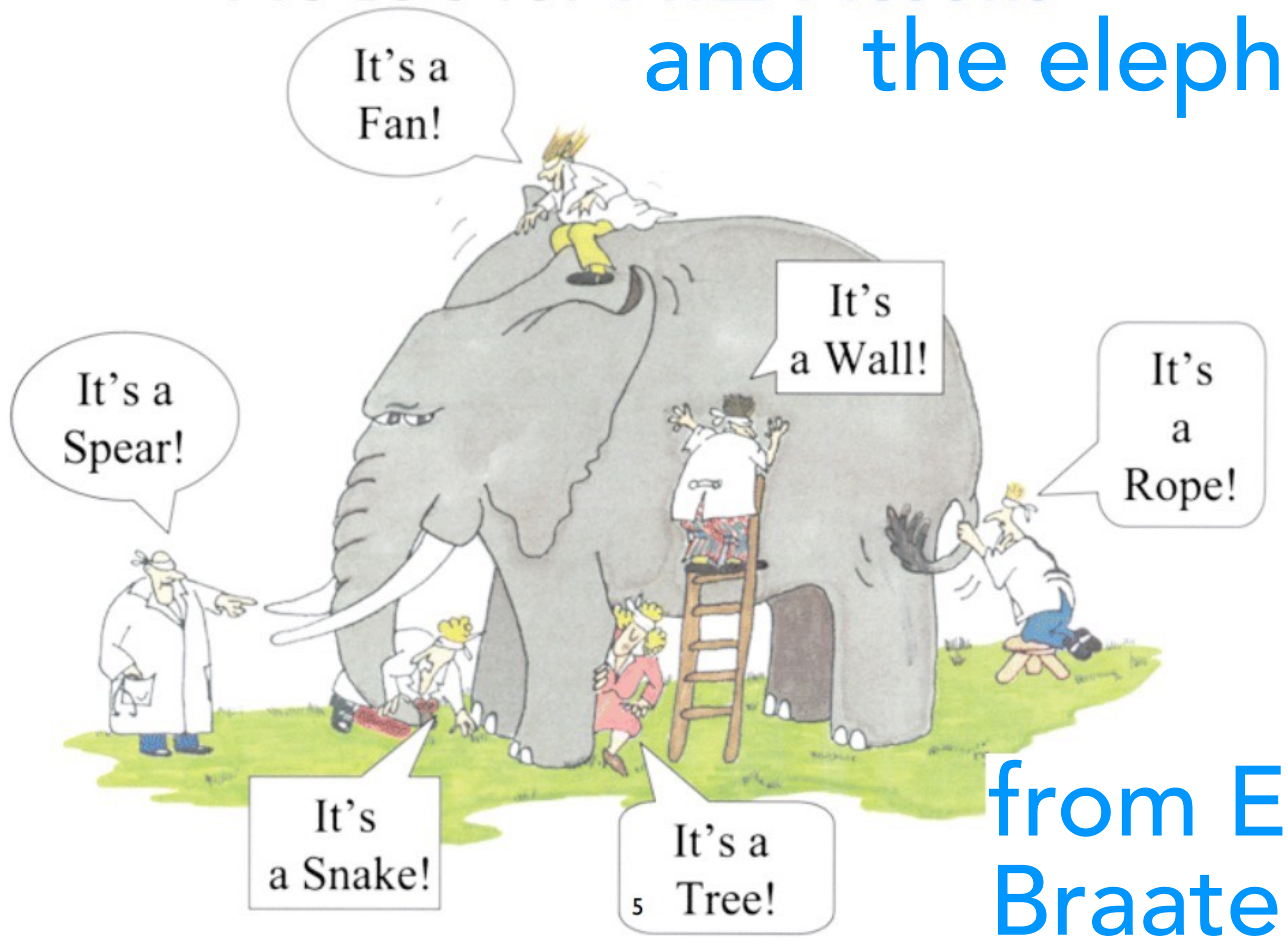
The present revolution: XYZ a great theoretical challenge

Close/above threshold new degrees of freedom like glue and light quarks are nonperturbative part in the binding.

- **Models** assume some special degrees of freedom and a model interaction



the blind men
and the elephant



from E. Braaten

- **Lattice** calculation of exotics masses are limited by the large number of open decay modes and they are not Immediately suited for production and in medium studies

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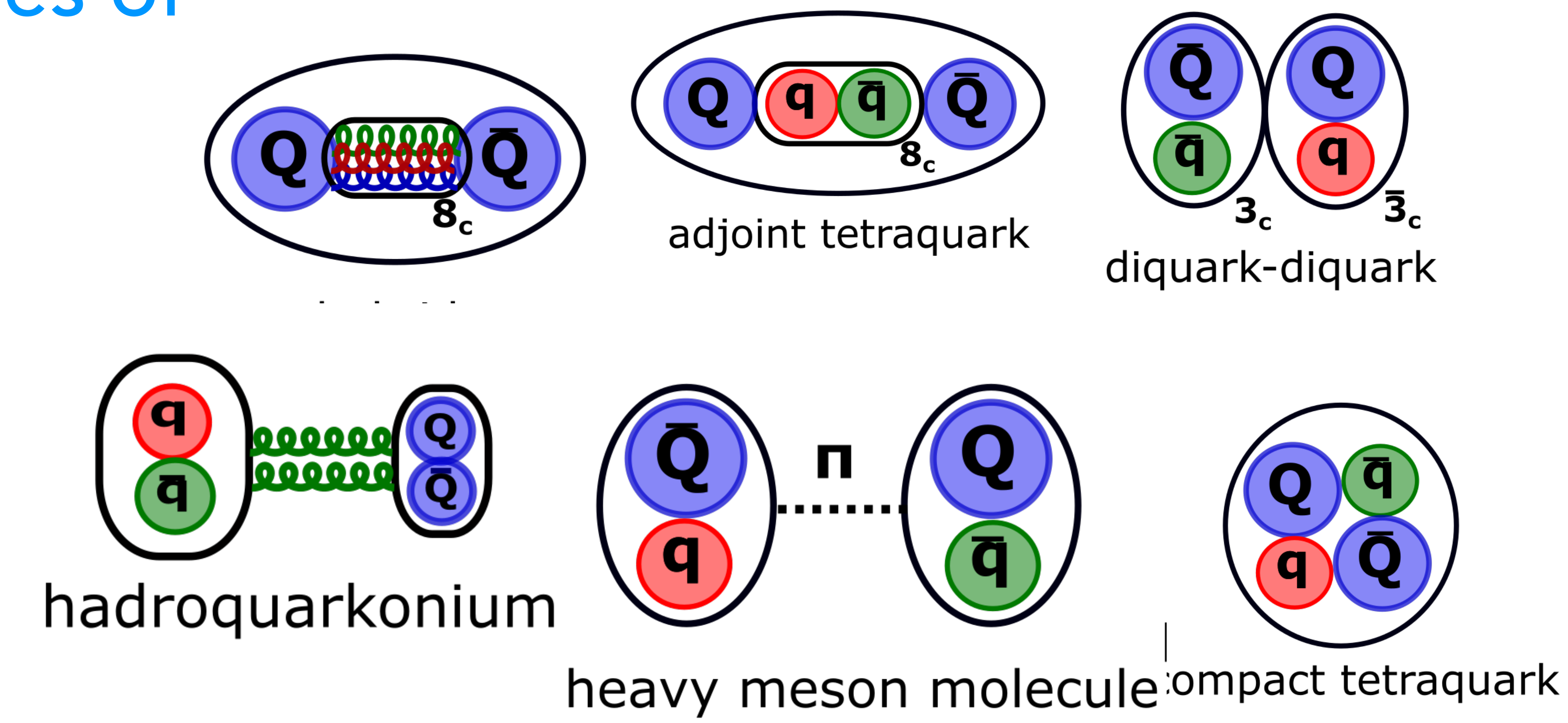

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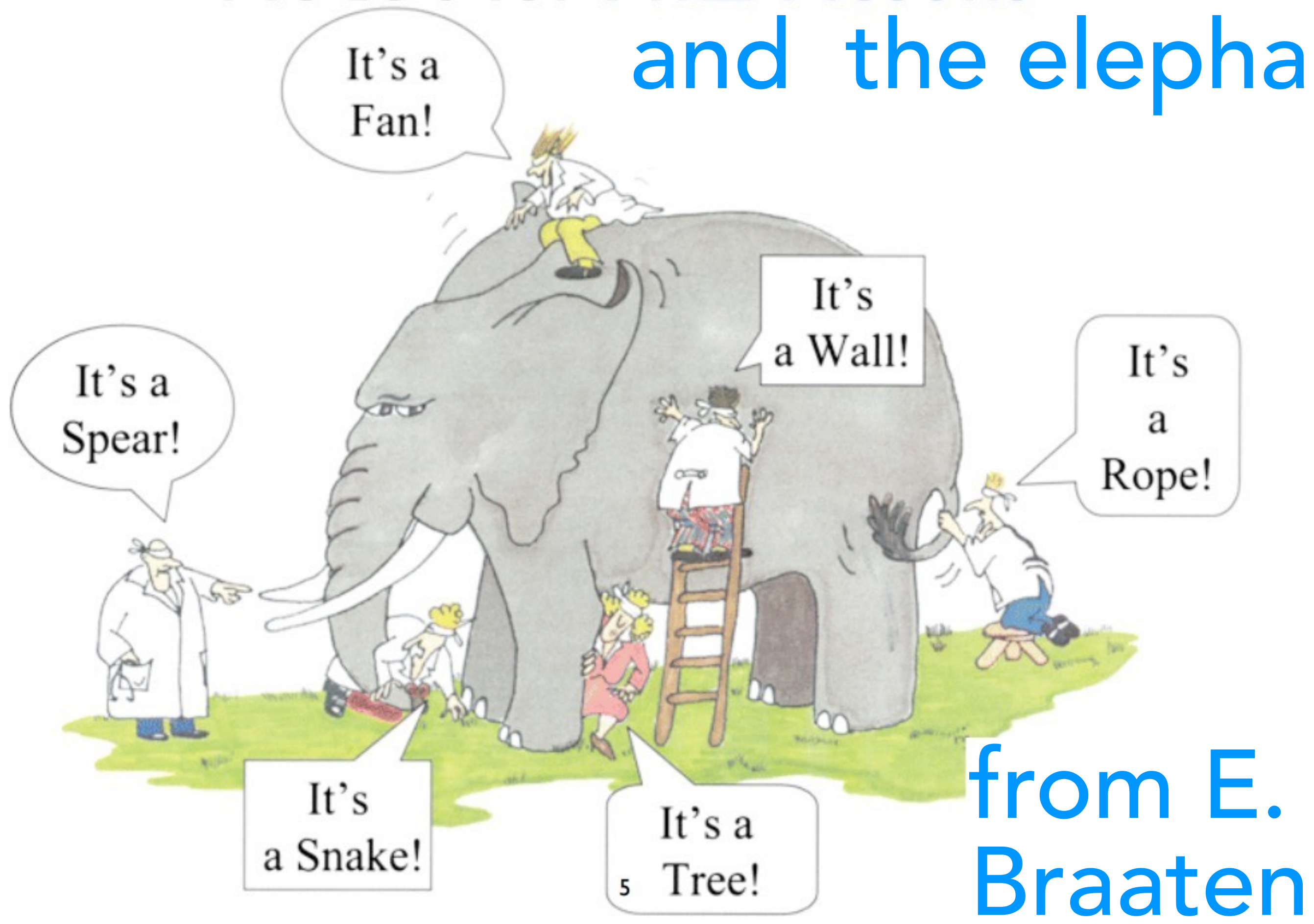

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A flexible approach rooted in QCD that can address all properties of XYZ, spectra, transitions, production, propagation in medium is needed allowing also to study the nature of the QCD force

The present revolutions: nuclear matter phase diagram

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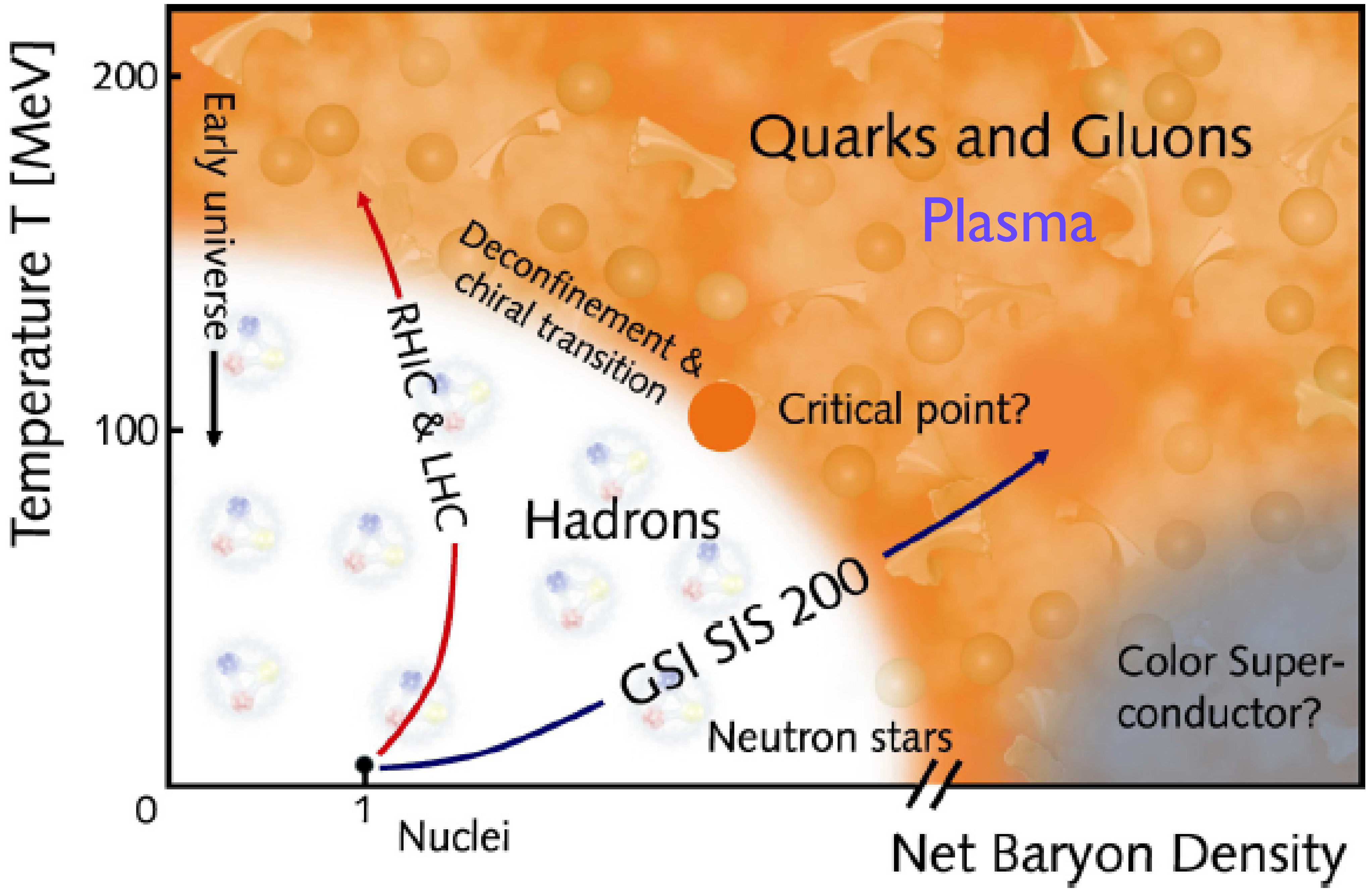

Nuclear Matter

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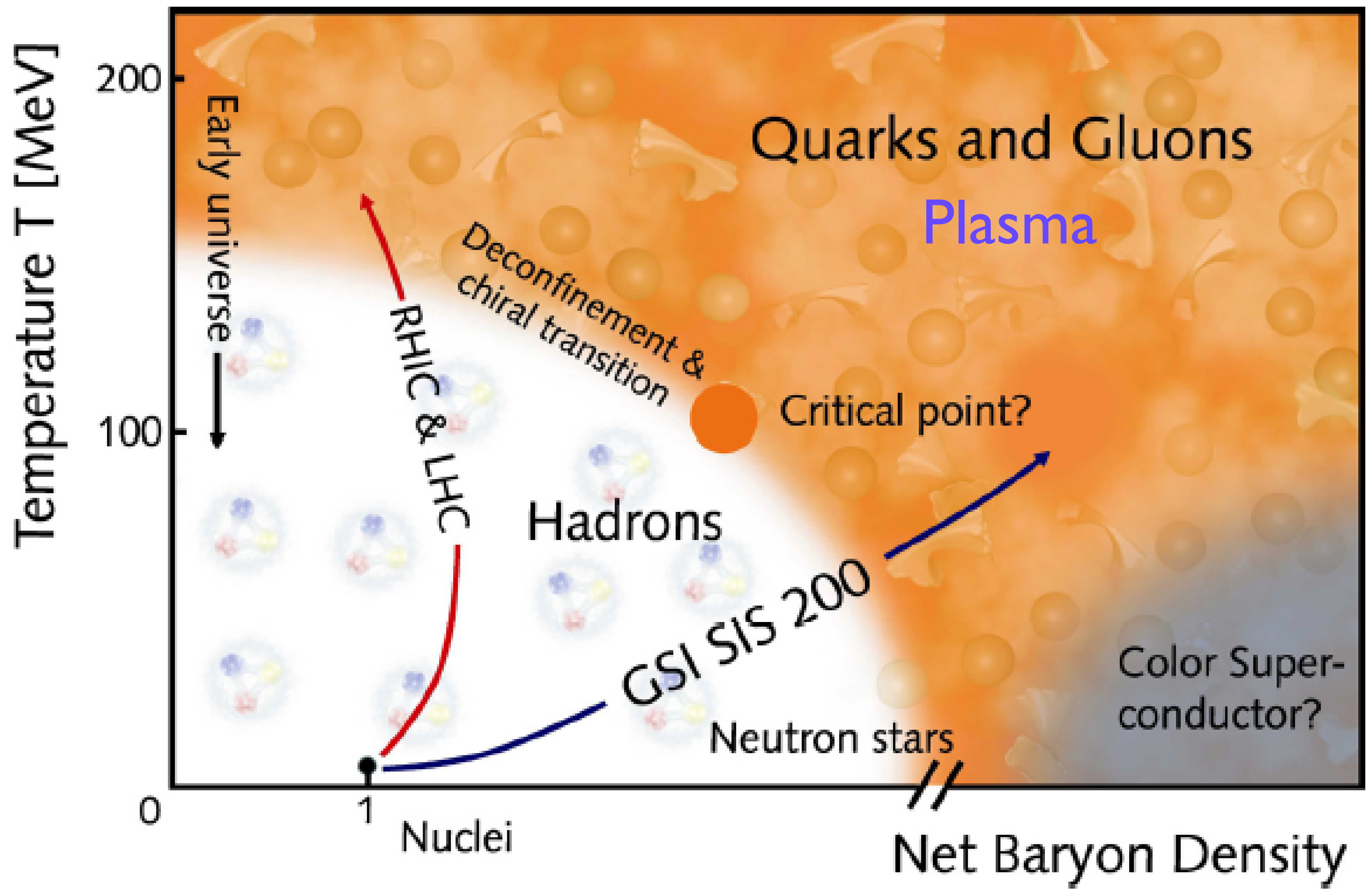

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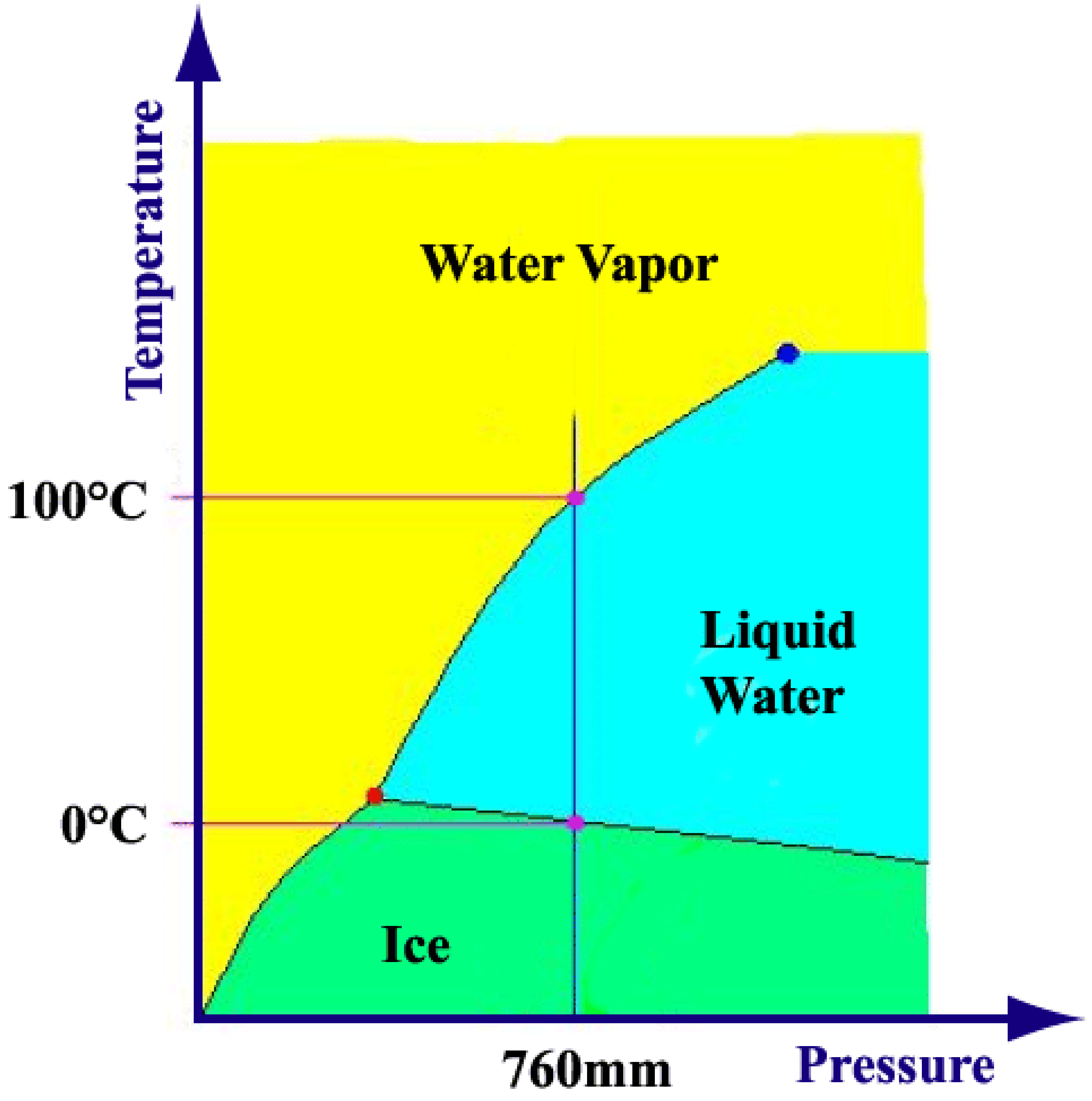

Water

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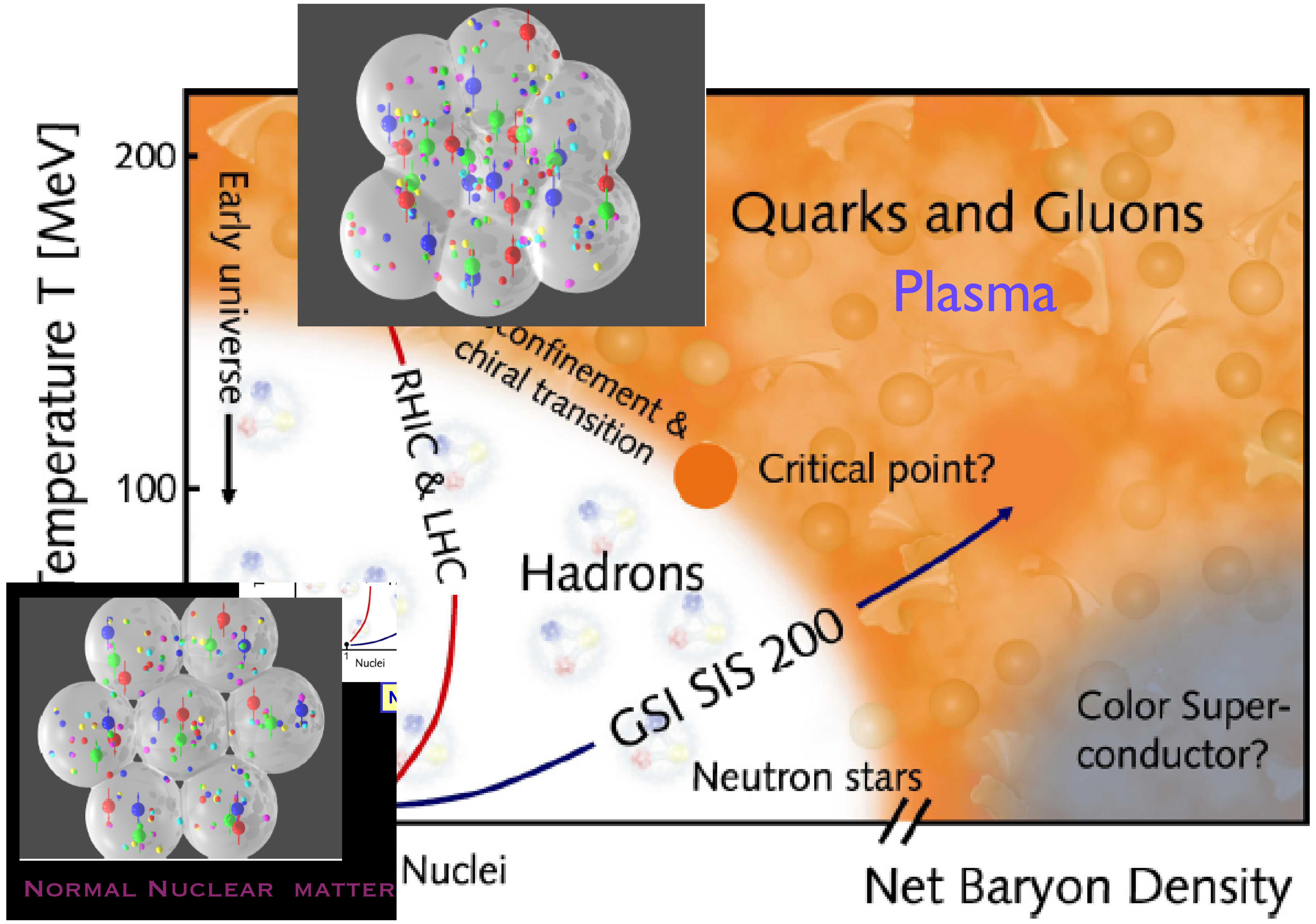

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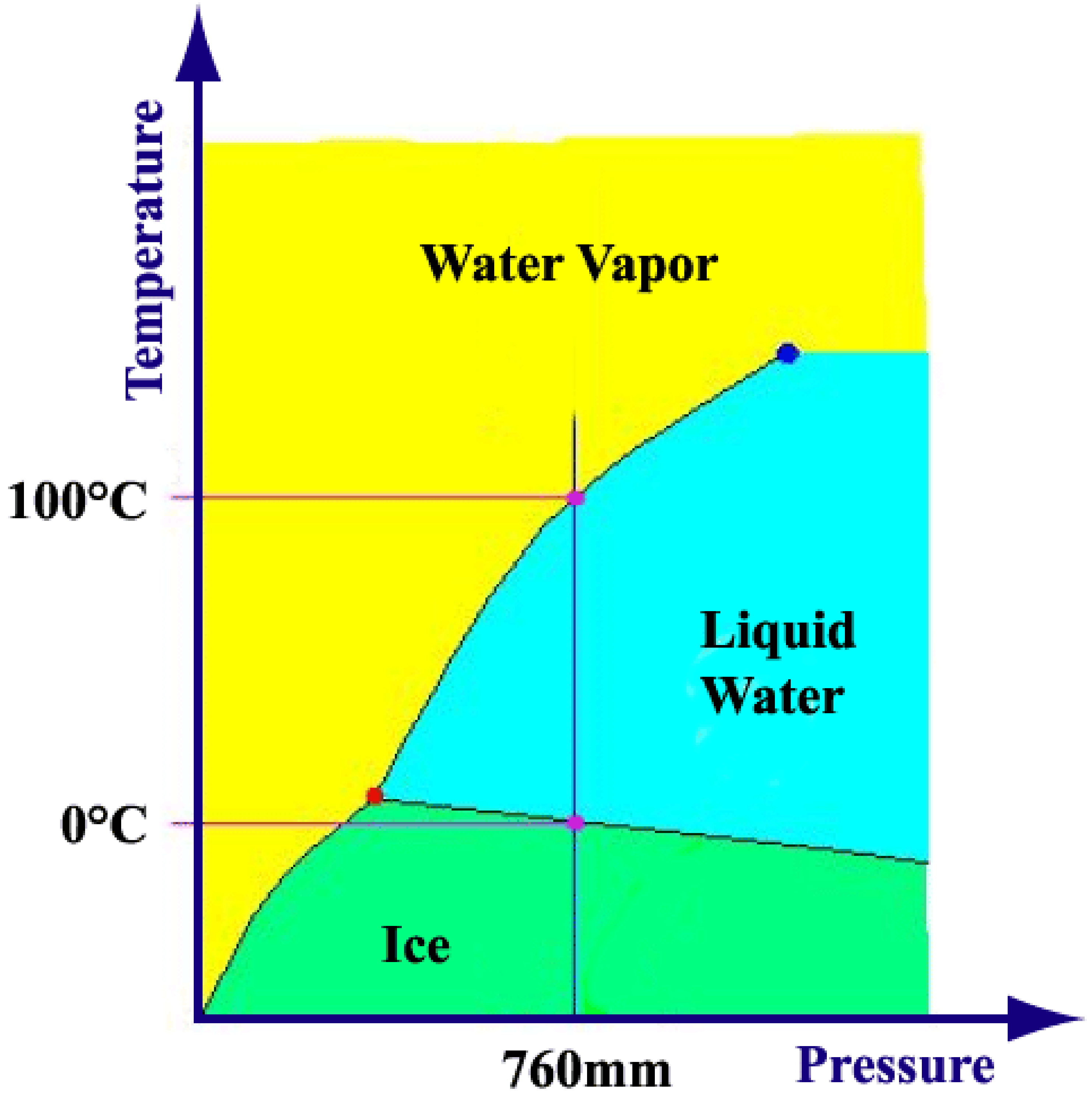
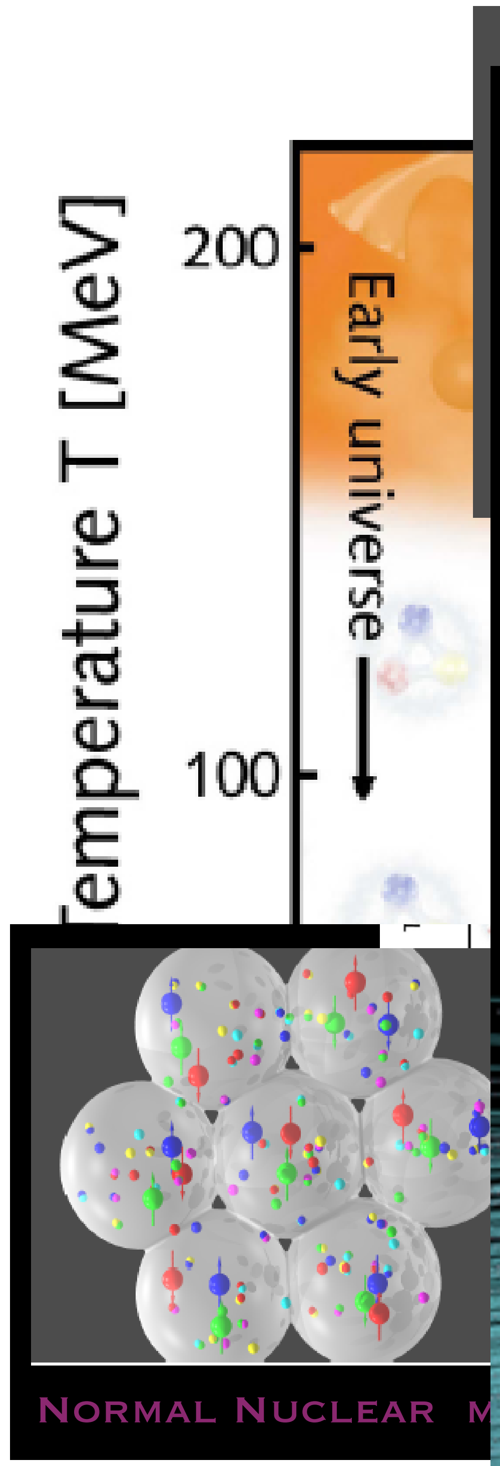
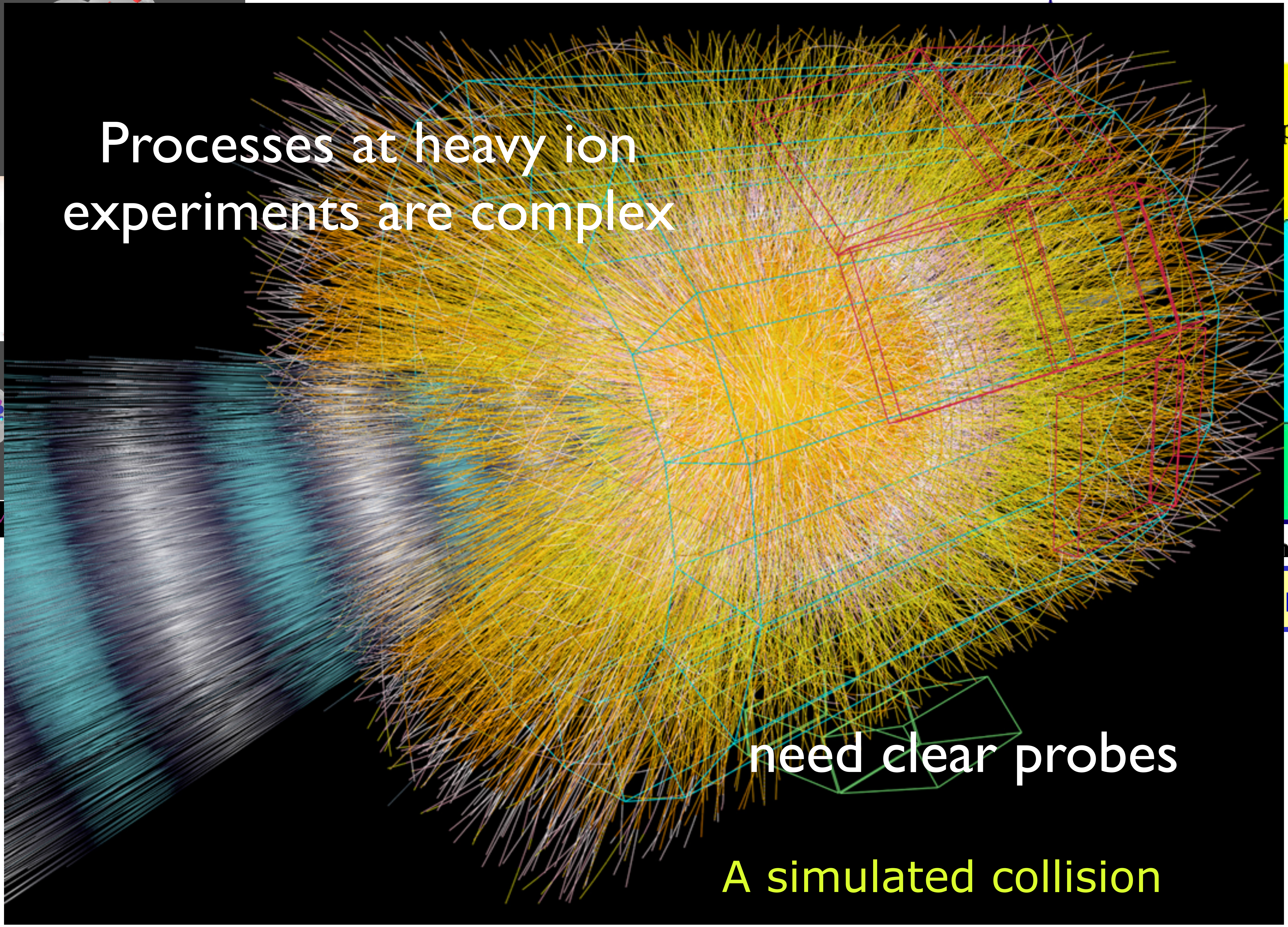


Water

The present revolutions: nuclear matter phase diagram

investigated in heavy ions collision at the LHC at CERN and RHIC USA (5.36 TeV per nucleon pair)

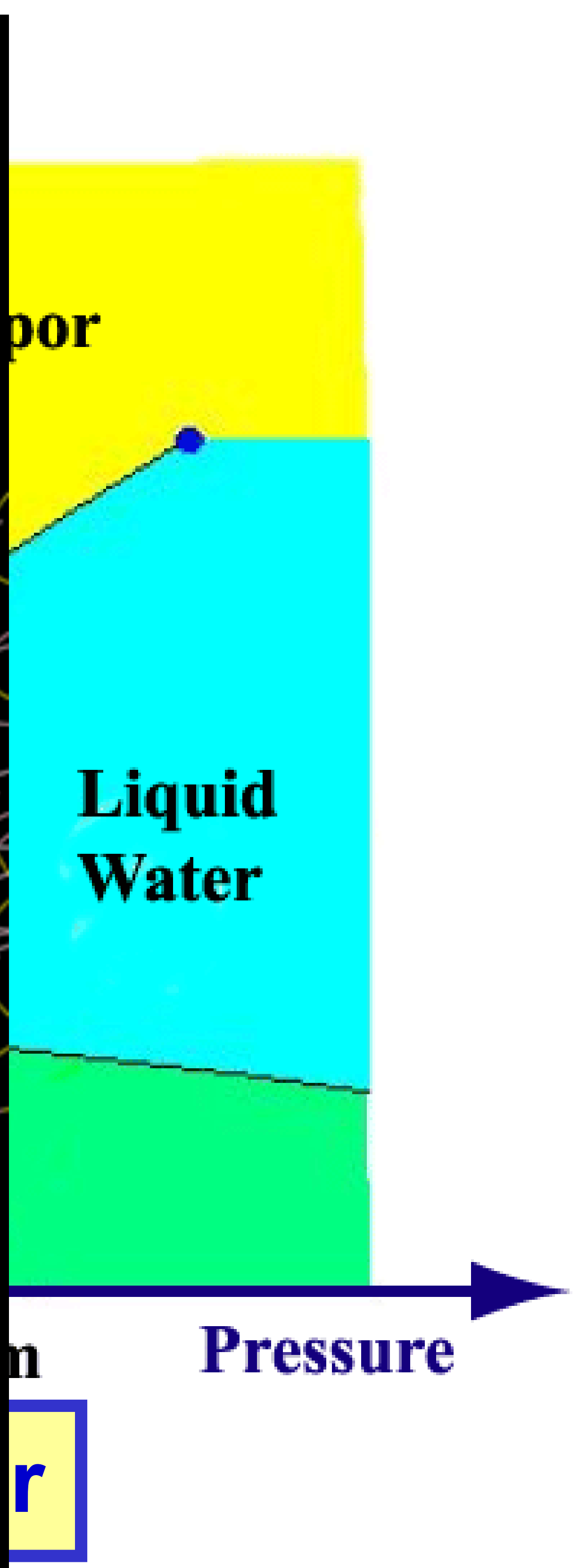




Processes at heavy ion experiments are complex

need clear probes

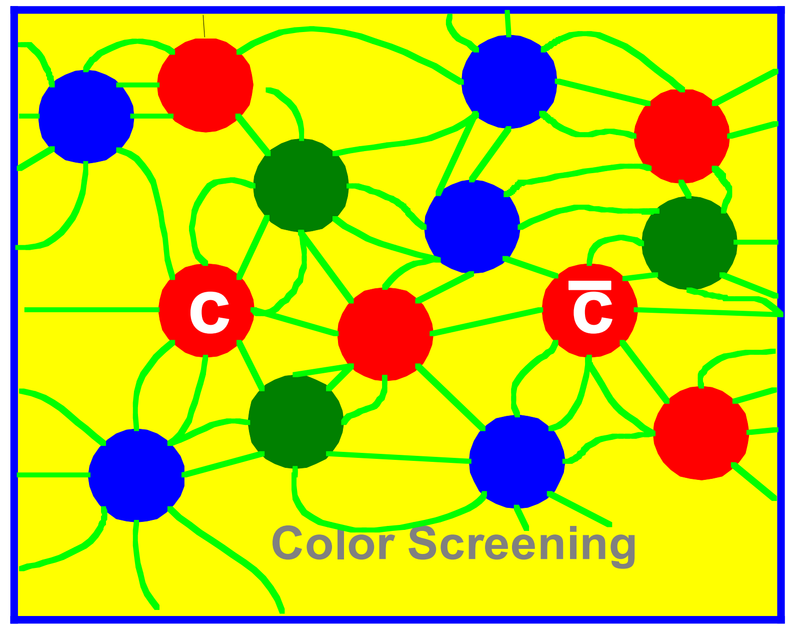
A simulated collision



The present revolutions: nuclear matter phase diagram

Quarkonia are probe of QGP formation

Matsui Satz 1986 idea **of color screening in medium**

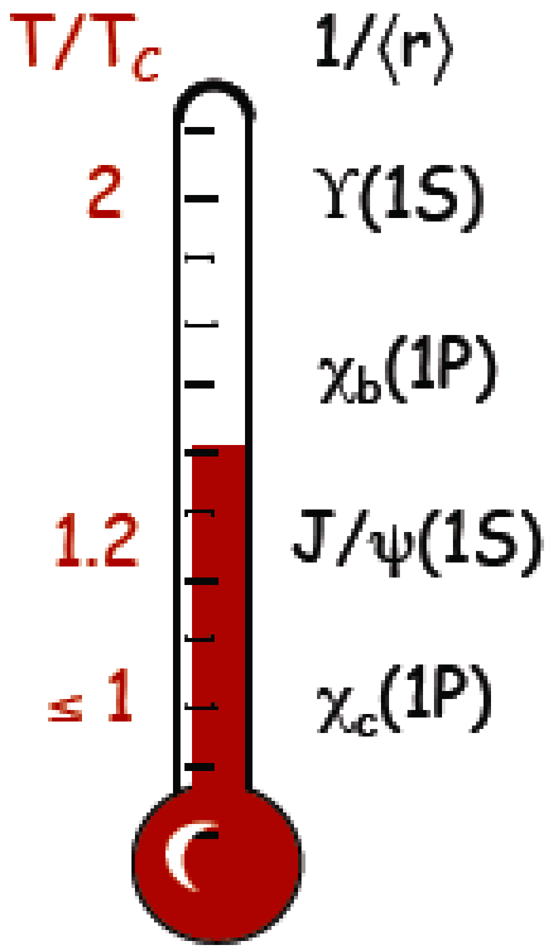


Debye charge screening

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$m_D \sim gT$$

$$r \sim \frac{1}{m_D} \xrightarrow{\text{Bound state dissolve}}$$



Sequential Melting at different Temperature

The present revolutions: nuclear matter phase diagram

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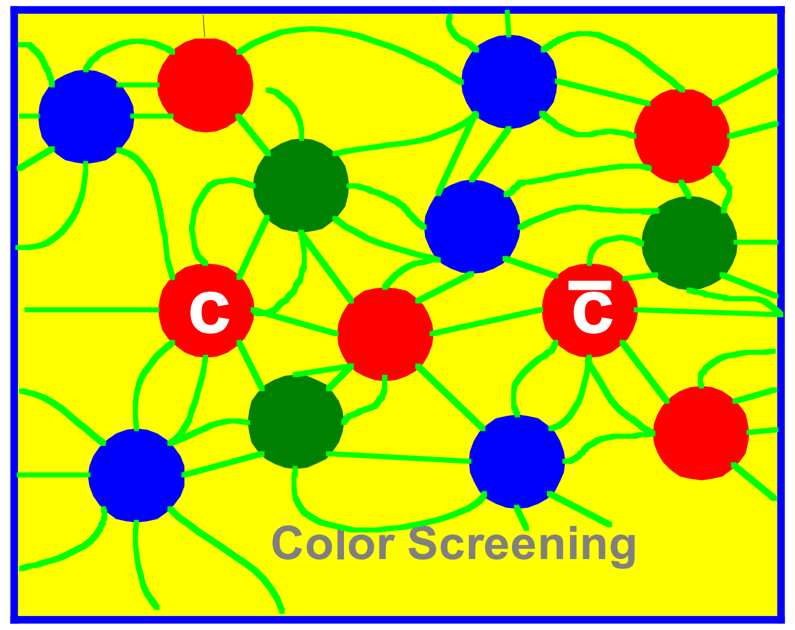

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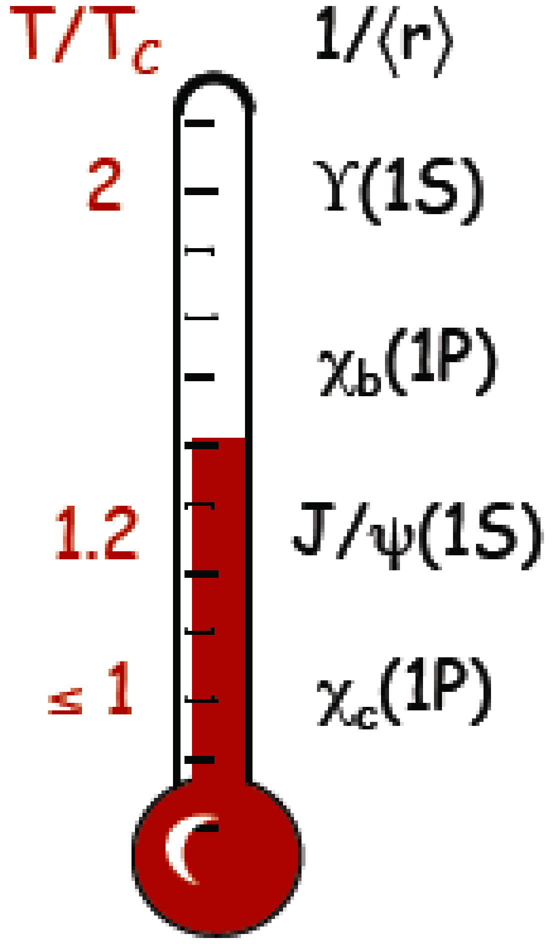

Sequential
Melting at
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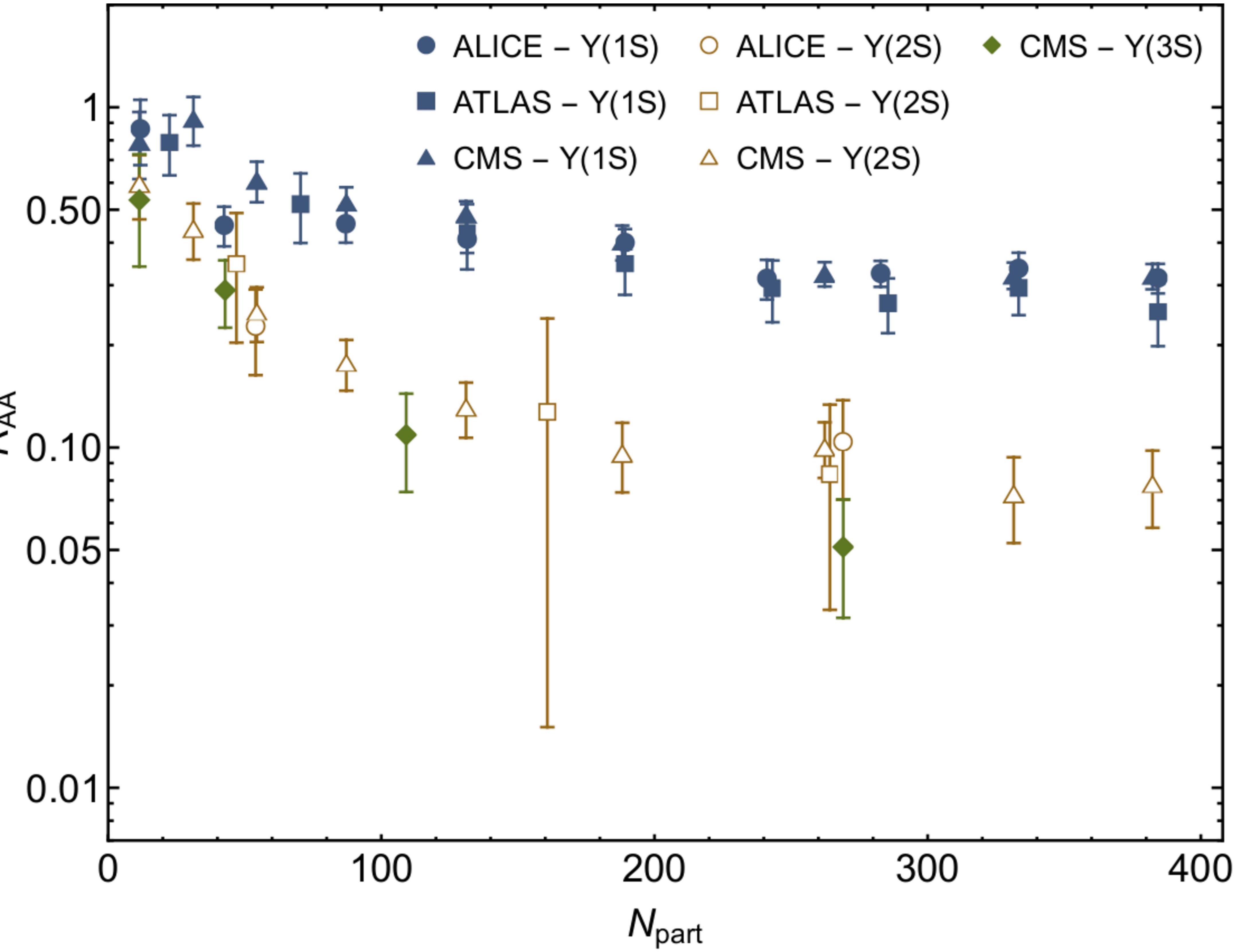
Temperature

Experimental measurements:

R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.



○ CMS PLB 790 (2019) 270
ALICE PLB 822 (2021) 136579
ATLAS PRC 107 (2023) 054912

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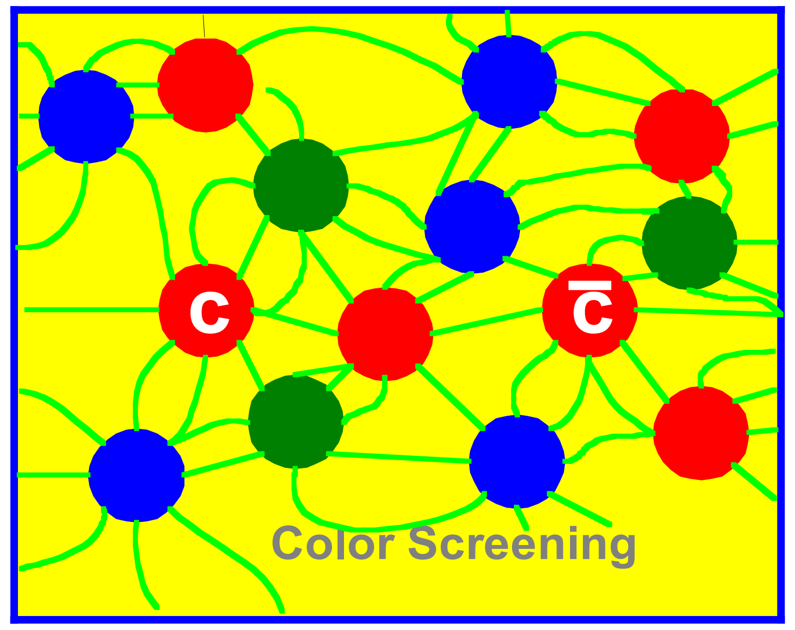

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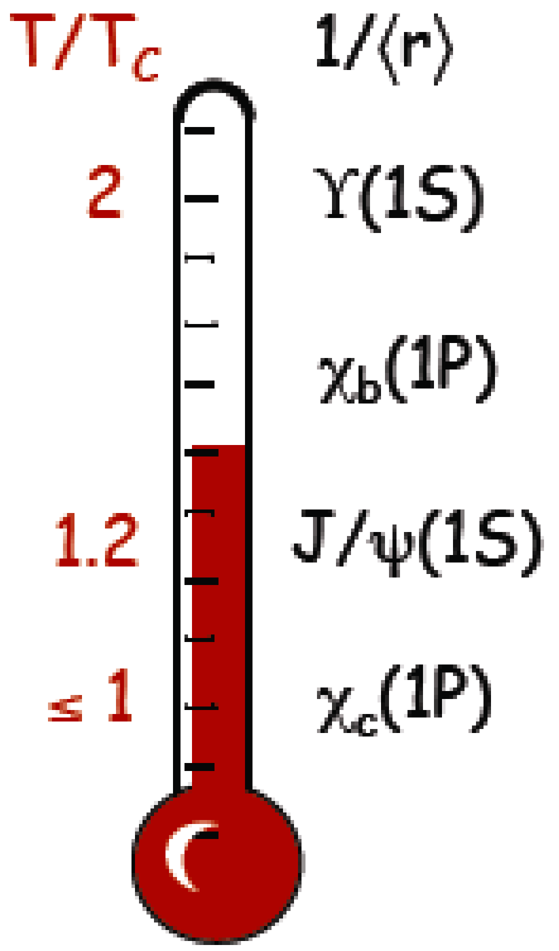

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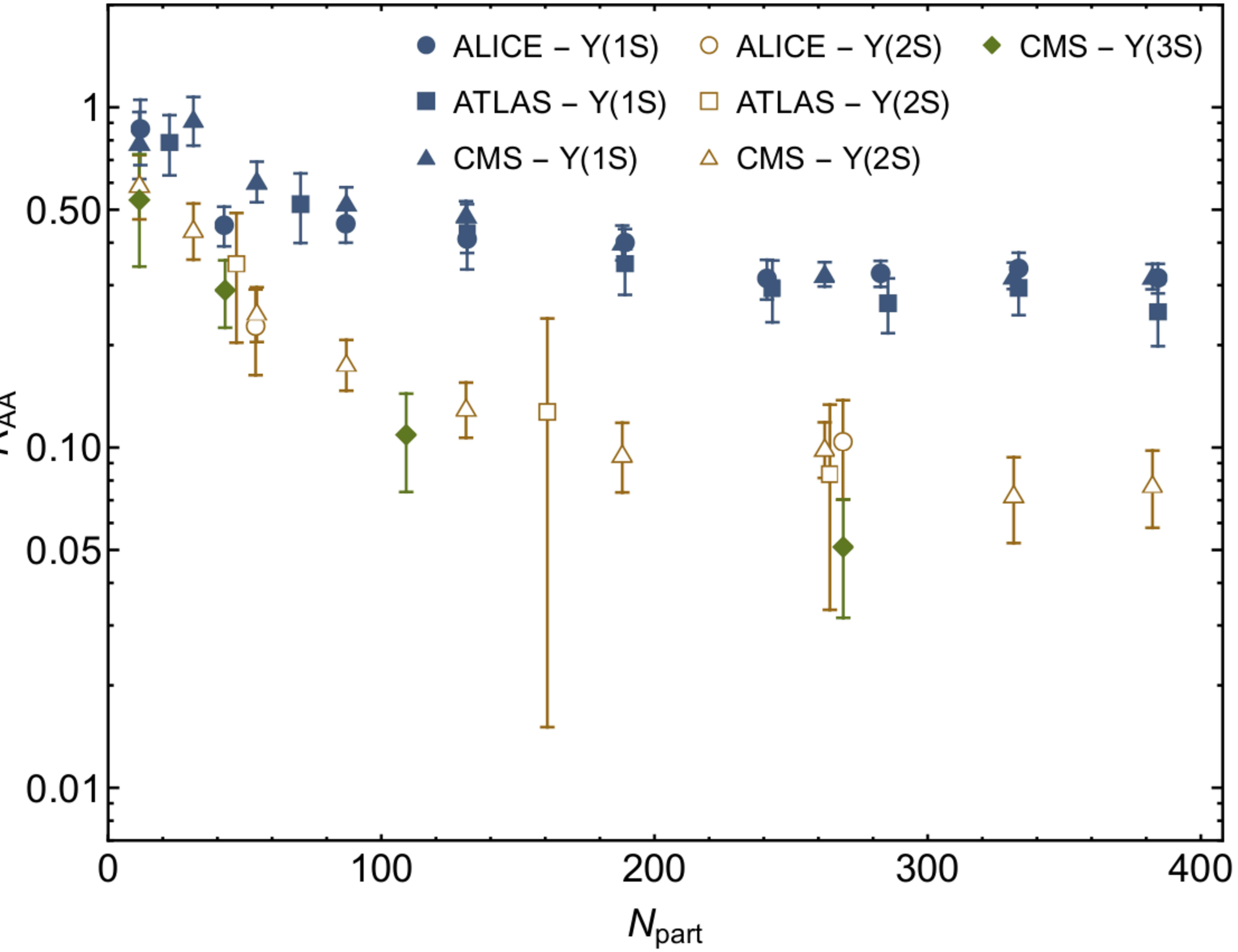

○ CMS PLB 790 (2019) 270
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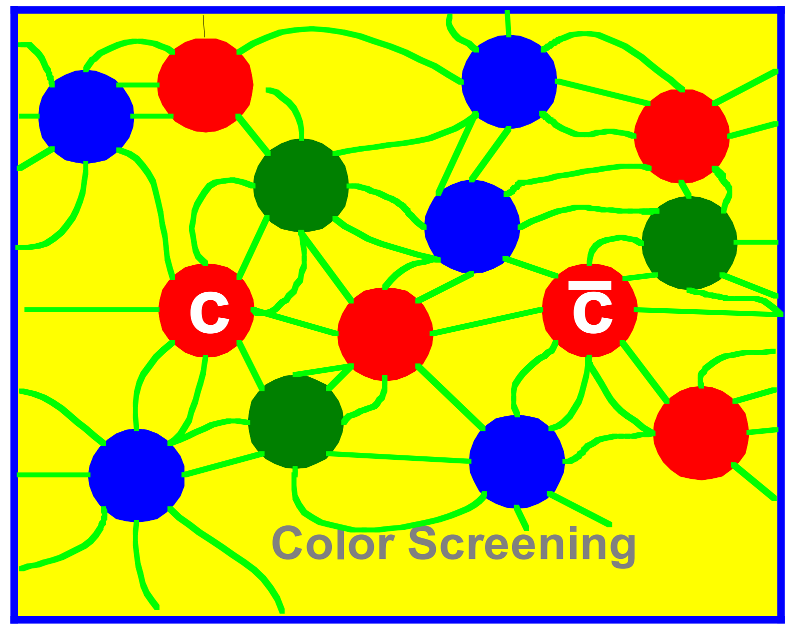
ATLAS PRC 107 (2023) 054912

Today a new paradigm emerged **beyond screening** relating the R_AA to the **nonequilibrium evolution of the heavy pair in medium:** medium induced dissociation and color singlet/octet recombination. **Quantum phenomenon to be addressed with quantum master equations**

The present revolutions: nuclear matter phase diagram

Quarkonia are probe of QGP formation

Matsui Satz 1986 idea **of color screening in medium**

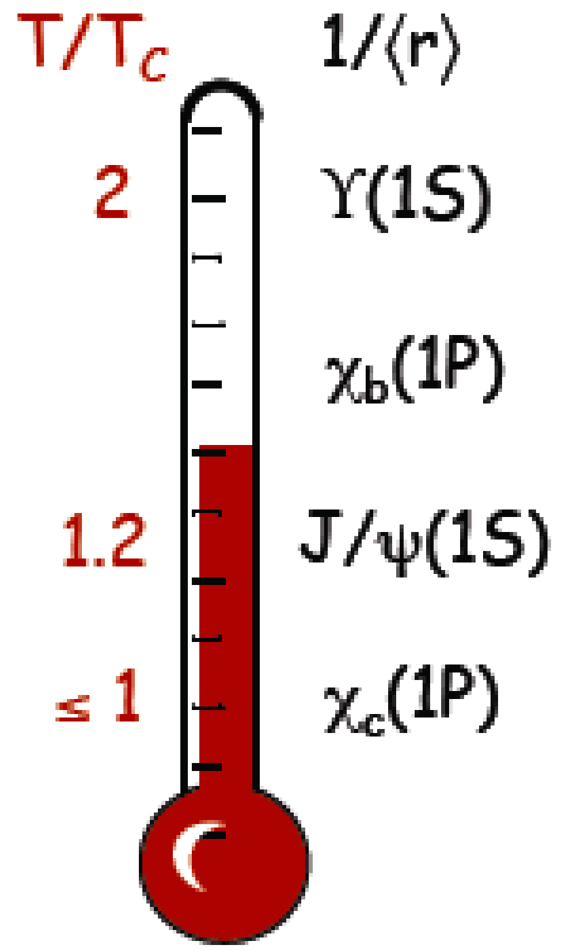


Debye charge screening

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$m_D \sim gT$$

$$r \sim \frac{1}{m_D} \xrightarrow[\text{dissolve}]{\text{Bound state}}$$



Sequential Melting at different Temperature

Experimental measurements:

R_{AA} is the nuclear modification factor = yield of quarkonium in PbPb / yield in pp.

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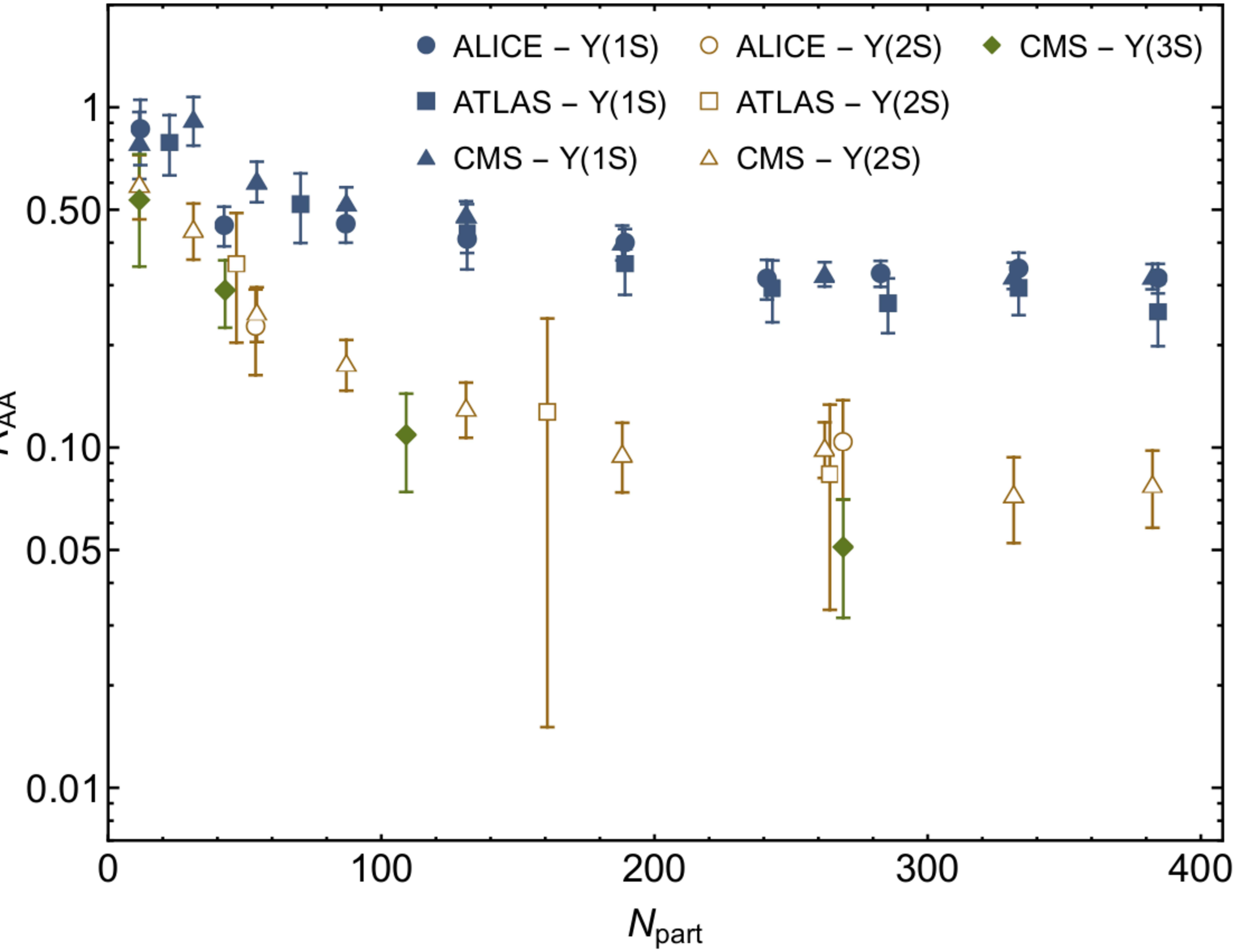


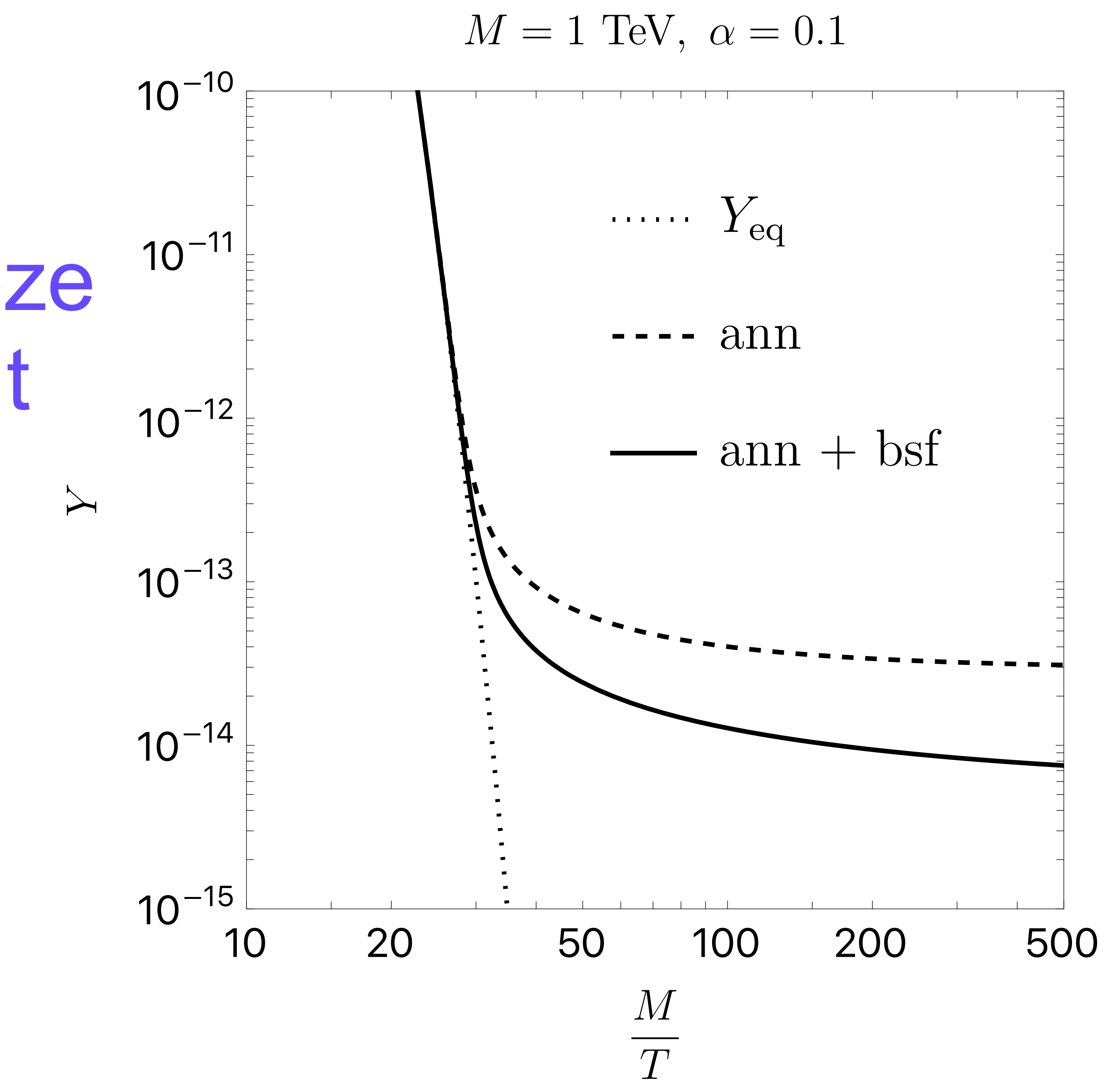
Today a new paradigm emerged **beyond screening** relating the R_AA to the **nonequilibrium evolution of the heavy pair in medium:** medium induced dissociation and color singlet/octet recombination. **Quantum phenomenon to be addressed with quantum master equations**

XYZ states are also produced and evolve in heavy ion collisions

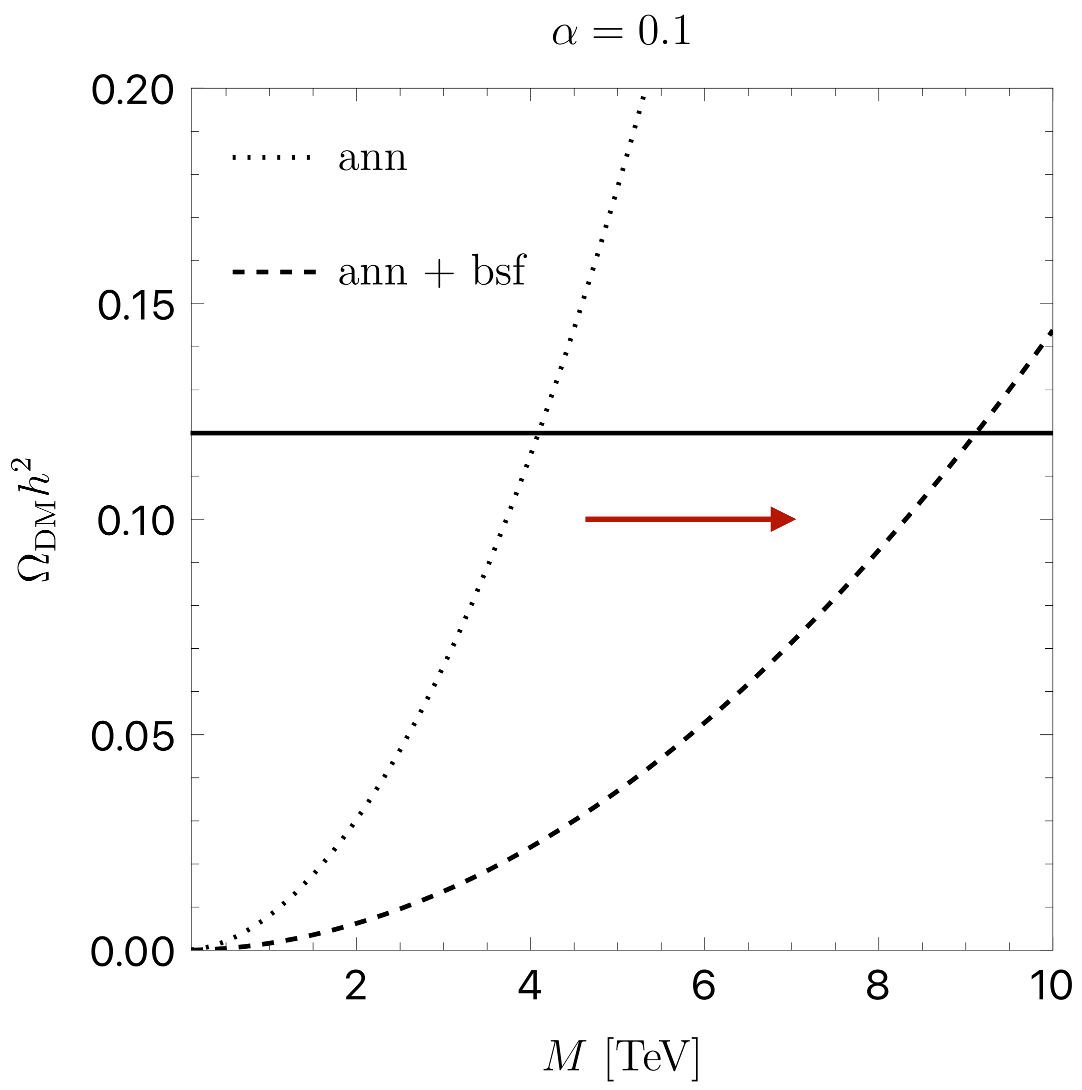
The present revolutions: dark matter evolution in early universe or neutrino evolution (fast flavour conversion)in supernova

A similar description can be applied to other NR system evolving in medium: e.g. **heavy dark matter pairs evolving in the early universe**: in order to predict the cosmological abundance of dark matter an estimation of particle rates in an expanding thermal environment is needed. Bound state effects at finite T may have large impact on the result

Freeze Out



- Early universe ($T \gtrsim M$): Heavy DM in thermal equilibrium with dark medium
- Expanding universe ($T \lesssim M$): T cools down $\rightarrow$ detailed balance lost
- Evolution equation: $(\partial_t + 3H)n = -\frac{1}{2}\langle\sigma_{\rm eff}v_{\rm rel}\rangle(n^2 - n_{\rm eq}^2)$
- Accurate prediction of DM relic density requires precise determination of the relevant interaction rates in expanding thermal environment
- Observed DM relic abundance implies heavy DM: $\Omega_{\rm DM}h^2 \sim 3\times 10^{11}\frac{M}{\rm TeV}Y_0 \quad \rightarrow \quad M \sim \rm TeV$
- During and after chemical freeze-out, DM is non-relativistic: $H \sim \langle\sigma_{\rm eff}v_{\rm rel}\rangle n_{\rm eq} \quad \rightarrow \quad T \sim M/25$



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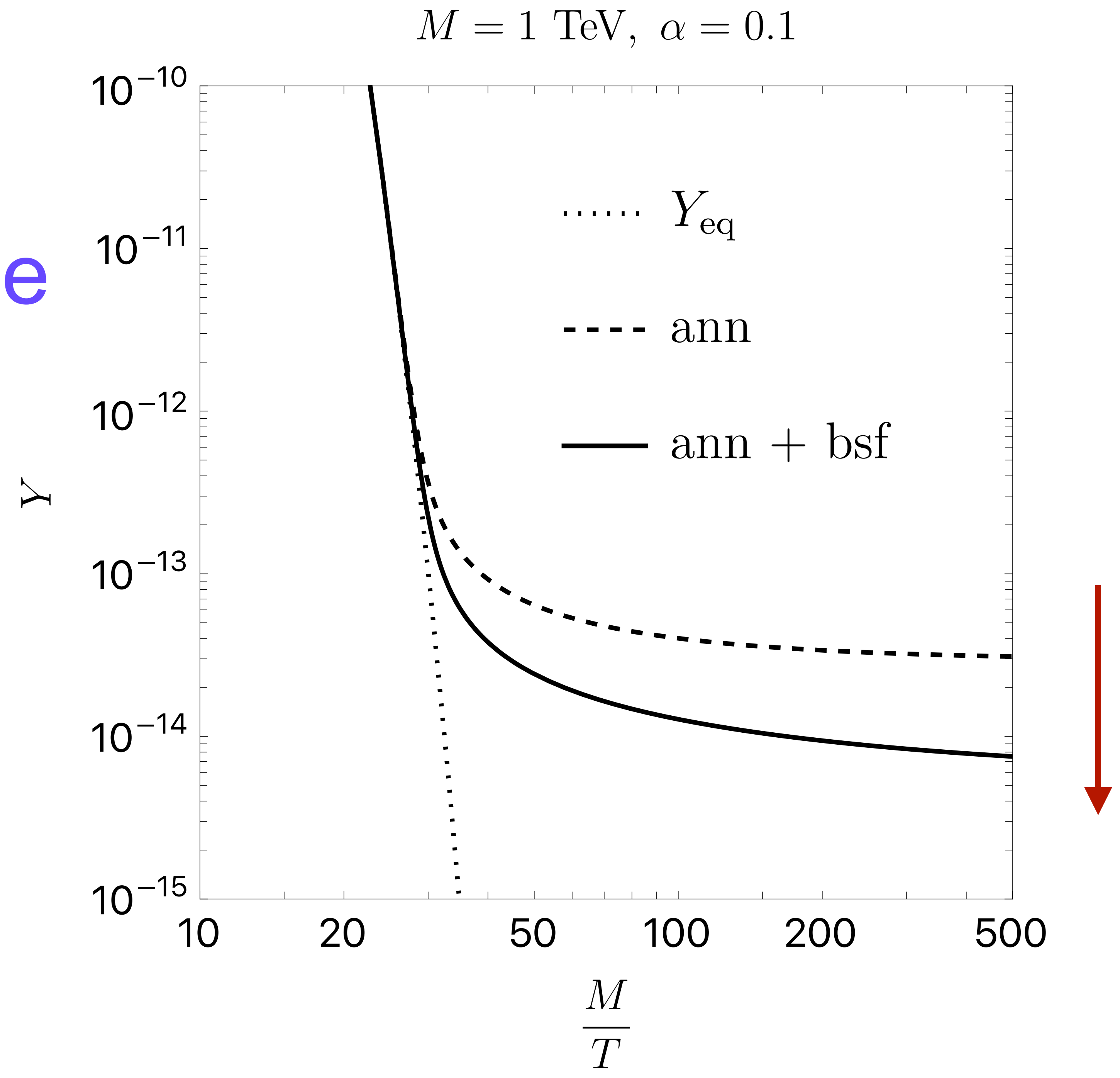

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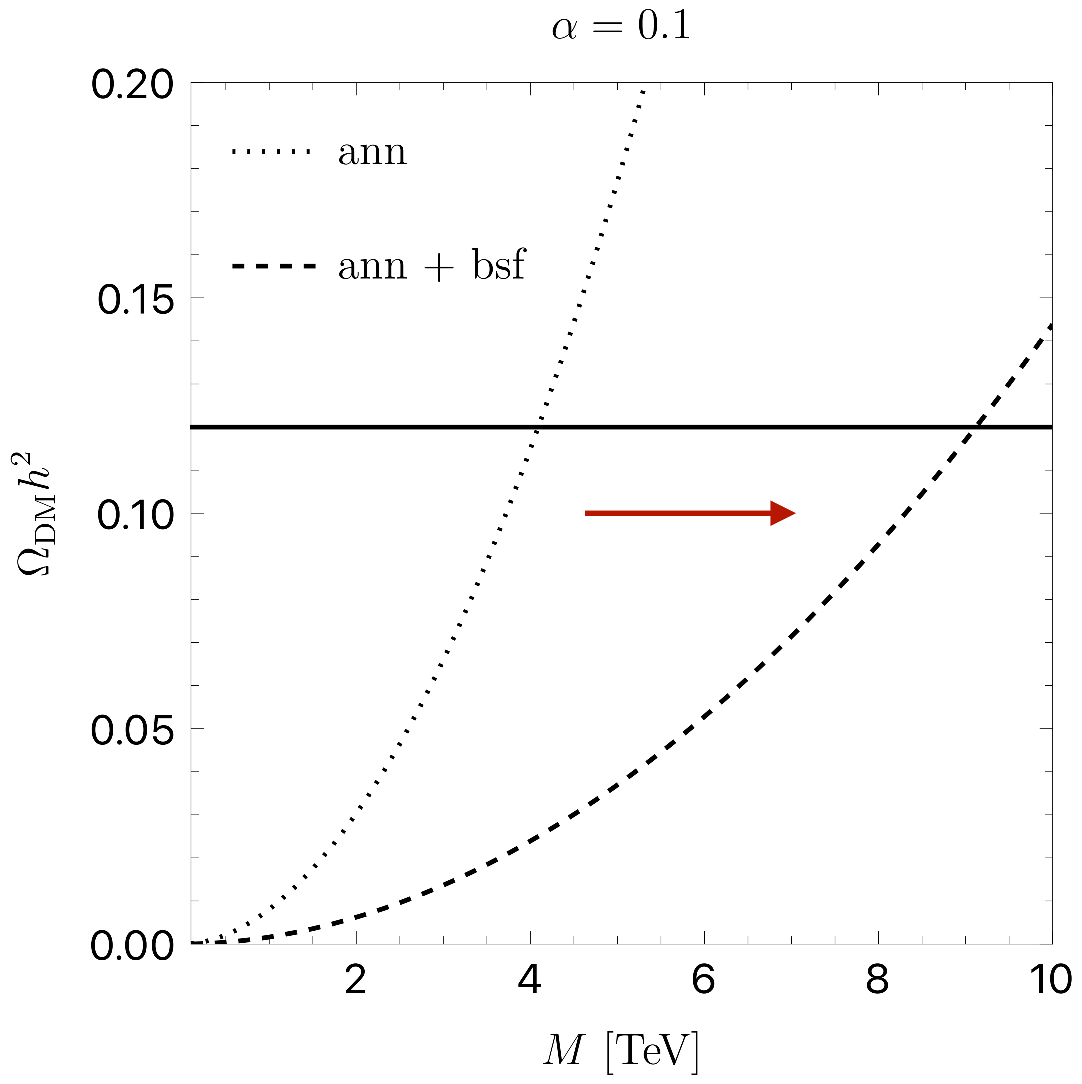


->obtain the quantum nonequilibrium evolution of DM pairs in the early universe Including dissipation, decoherence and recombination

QQbar systems (and NR bound systems) are important tools to address significant problems at the frontier of particle physics

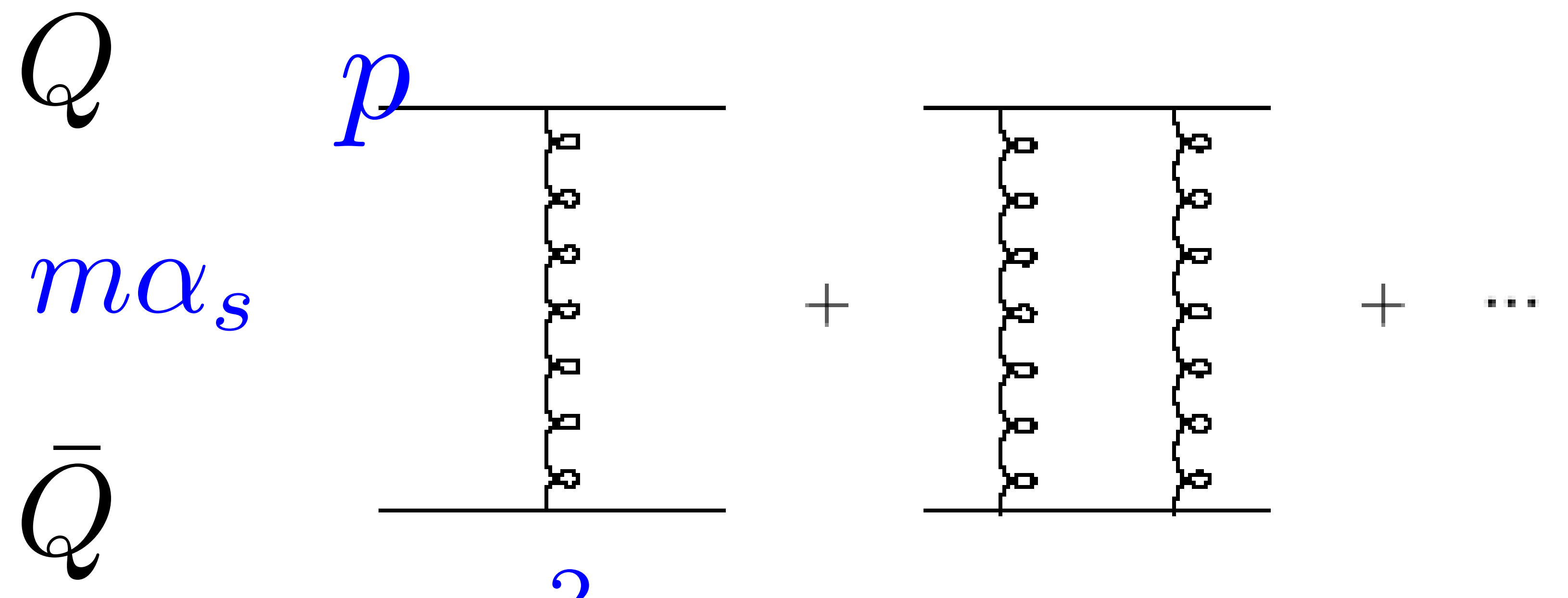
To this aim they should be addresses in QFT, QFT at finite T , finite mu

However...

For quarkonium to become a probe of strong interactions, it should be treated in QCD :a very hard problem

Close to the bound state $\alpha_s \sim v$

$p \sim m\alpha_s$



$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $(\frac{p^2}{m} + V)\phi = E\phi \to p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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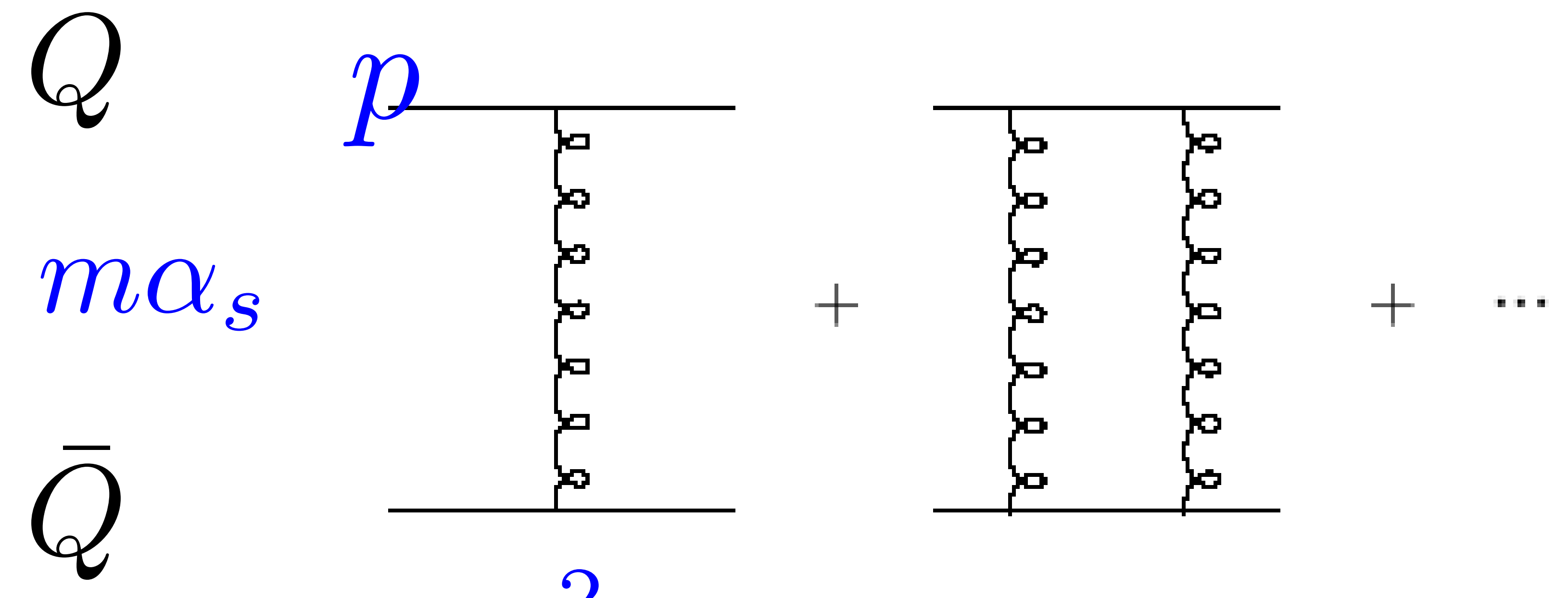

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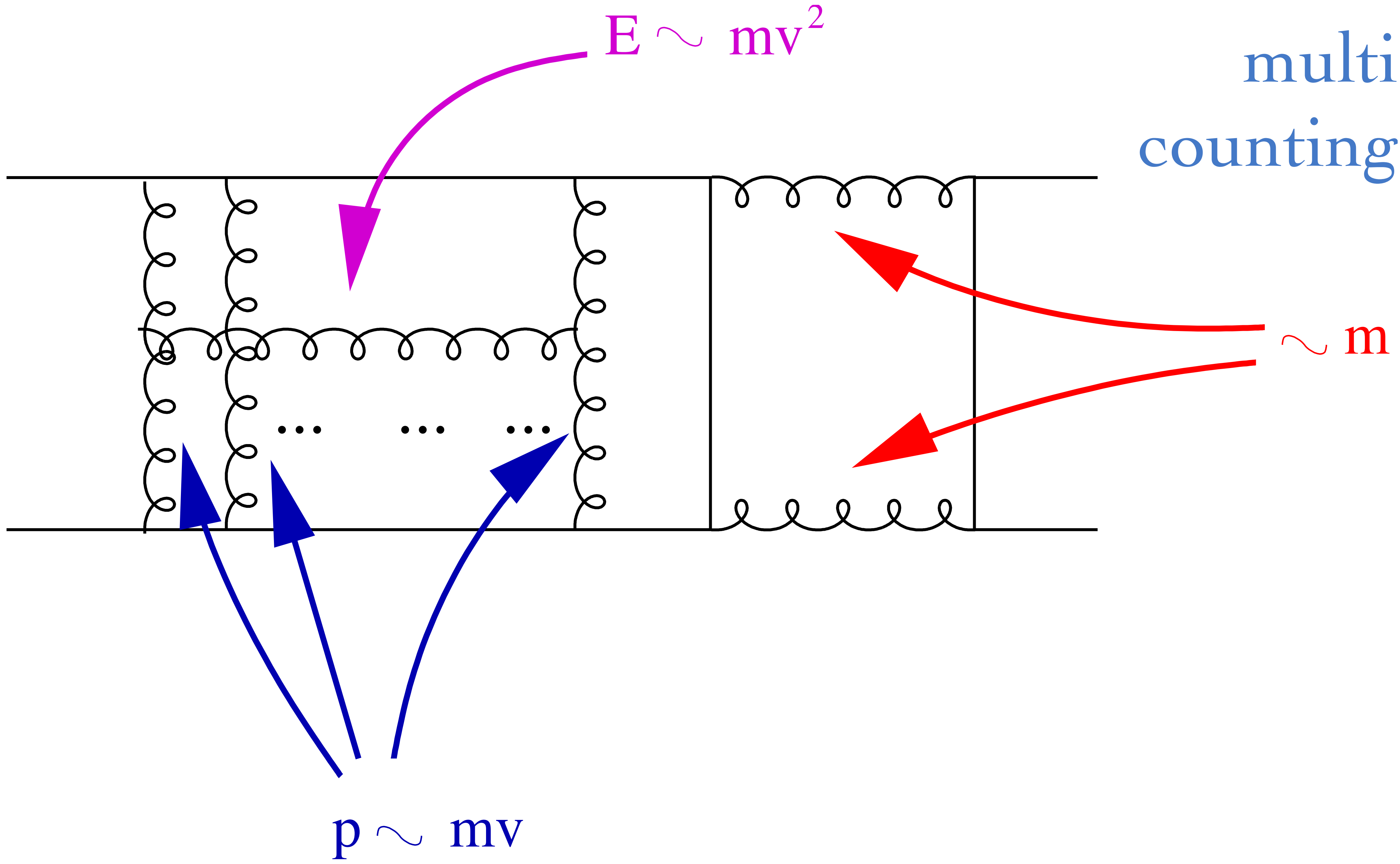

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

Difficult also
for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

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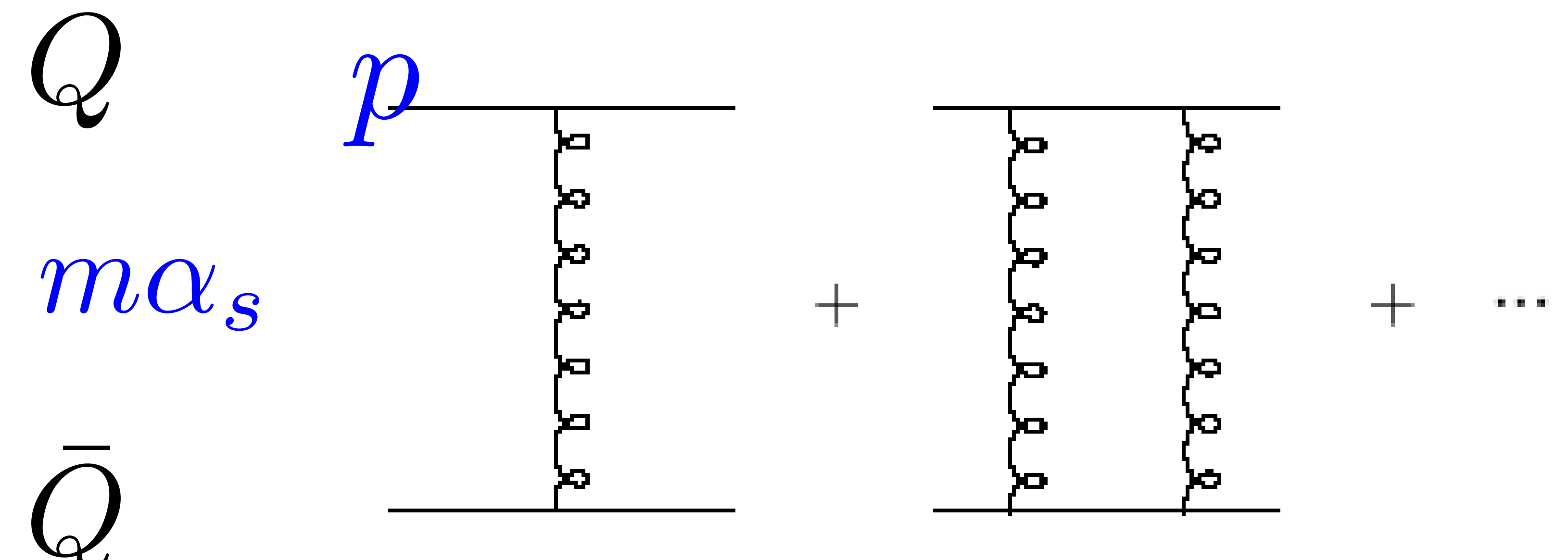

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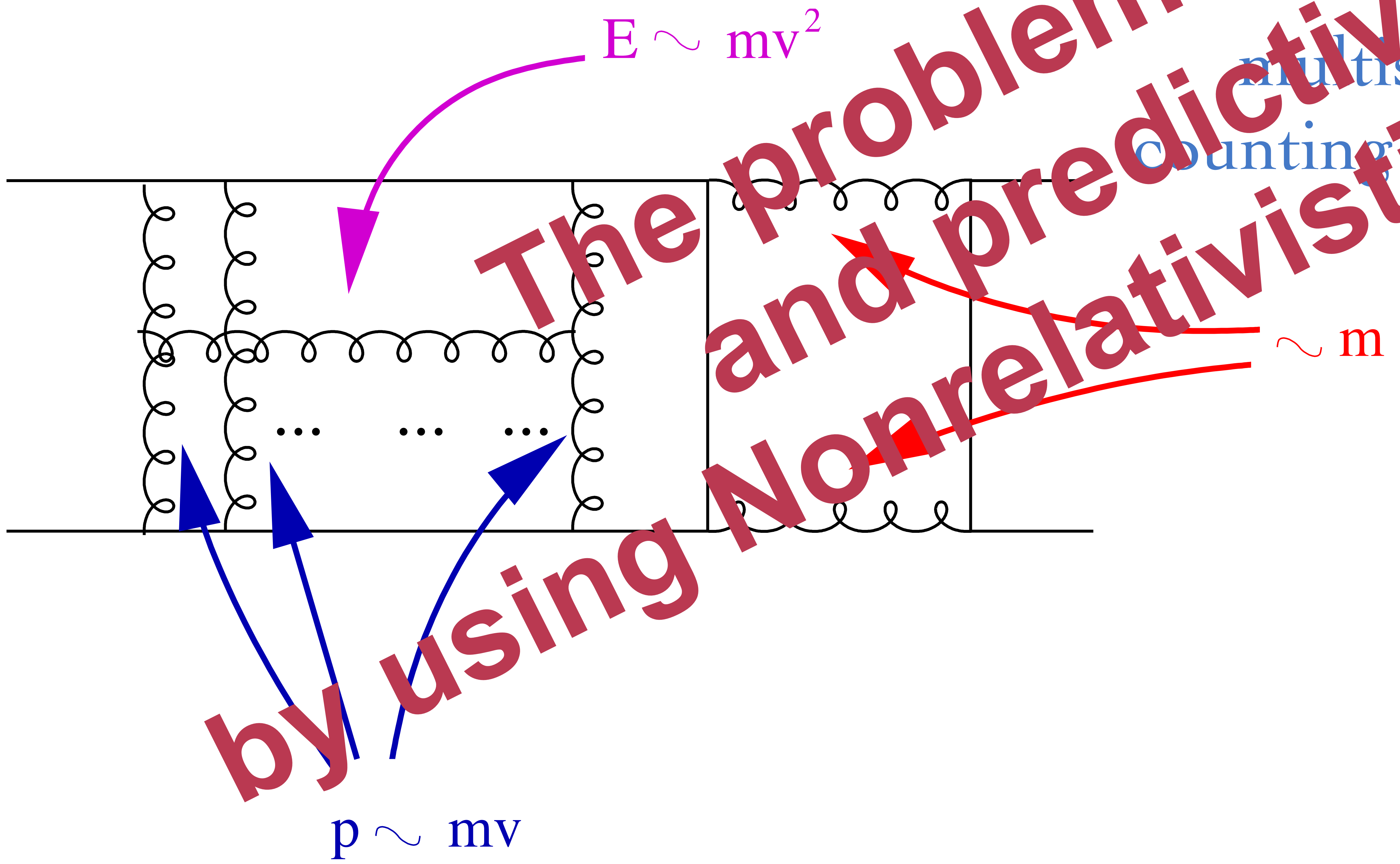
The problem is greatly simplified and predictivity is achieved by using Nonrelativistic Effective Field Theories



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What is the bound state wave function of hydrogen atom? Does it depend on the coupling constant? Does this helps you to understand th origin of the difference of a bound stte calculation with respect to a on shell scattering calculations? is the bound state problem in QED nonperturbative? in which sense? what is the difference with the bound state problem in QCD?

Disentangling the bound state scales at the Lagrangian level has advantages

I. It facilitates higher order perturbative calculations

Relevant for: physics: Hydrogen atom (e.g. proton radius), positronium (e.g. width, hfs), muonium ttbar threshold production; Dark matter annihilation and production close to threshold; SUSY particles annihilation and production; QQbar, QQq and QQQ with small radius; extraction of SM parameters

II. In QCD (or in a strongly coupled theory) it factorizes automatically high energy contributions (perturbative) from low-energy (nonperturbative, thermal) ones

Relevant for:pionium and precision chiral dynamics; nucleon-nucleon systems; Quarkonium, Exotic X, Y, Z states, Quarkonium in hot QCD medium in heavy ion collisions; confinement and nonperturbative effects

III. It allows to integrate out hierarchically other scales using other EFTs (for example the temperature T using Hard Thermal Loop (HTL) EFT) and to apply lattice directly on the low energy factorized part

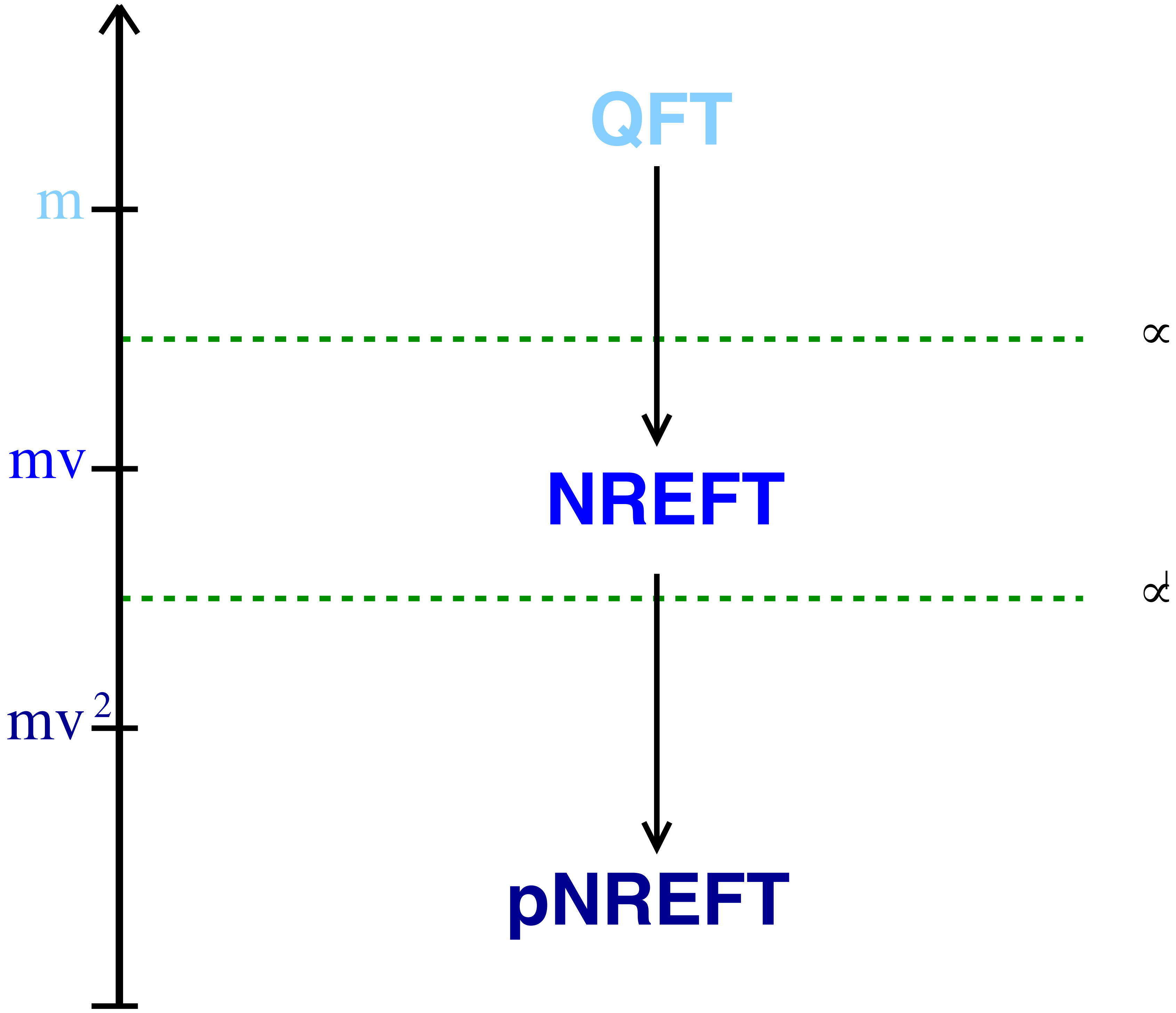
IV. It allows to define in QFT objects of great importance like potentials both in the perturbative and in the nonperturbative regime

V. More conceptually It provides a field theoretical foundation of the Schroedinger equation

Disentangling the bound state scales at the Lagrangian level has advantages : pNREFT

$$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

separates the Schroedinger dynamics of the two particle field ϕ from the low energy dynamics encoded in $\Delta\mathcal{L}$



QFT

NREFT

pNREFT

pNREFT is the lowest energy EFT that can be constructed for the NR bound system.

Notice: if QFT = QED, pNRQED gives a proper version of Quantum Mechanics

N B., A. Pineda, J. Soto, A.Vairo. Rev. Mod. Phys 77 (2005) 1423

The lowest dynamical energy and the corresponding pNREFT depend on the system in consideration, e.g. to describe Van der Waals interaction between bound states the lowest scale is lower than mv^2

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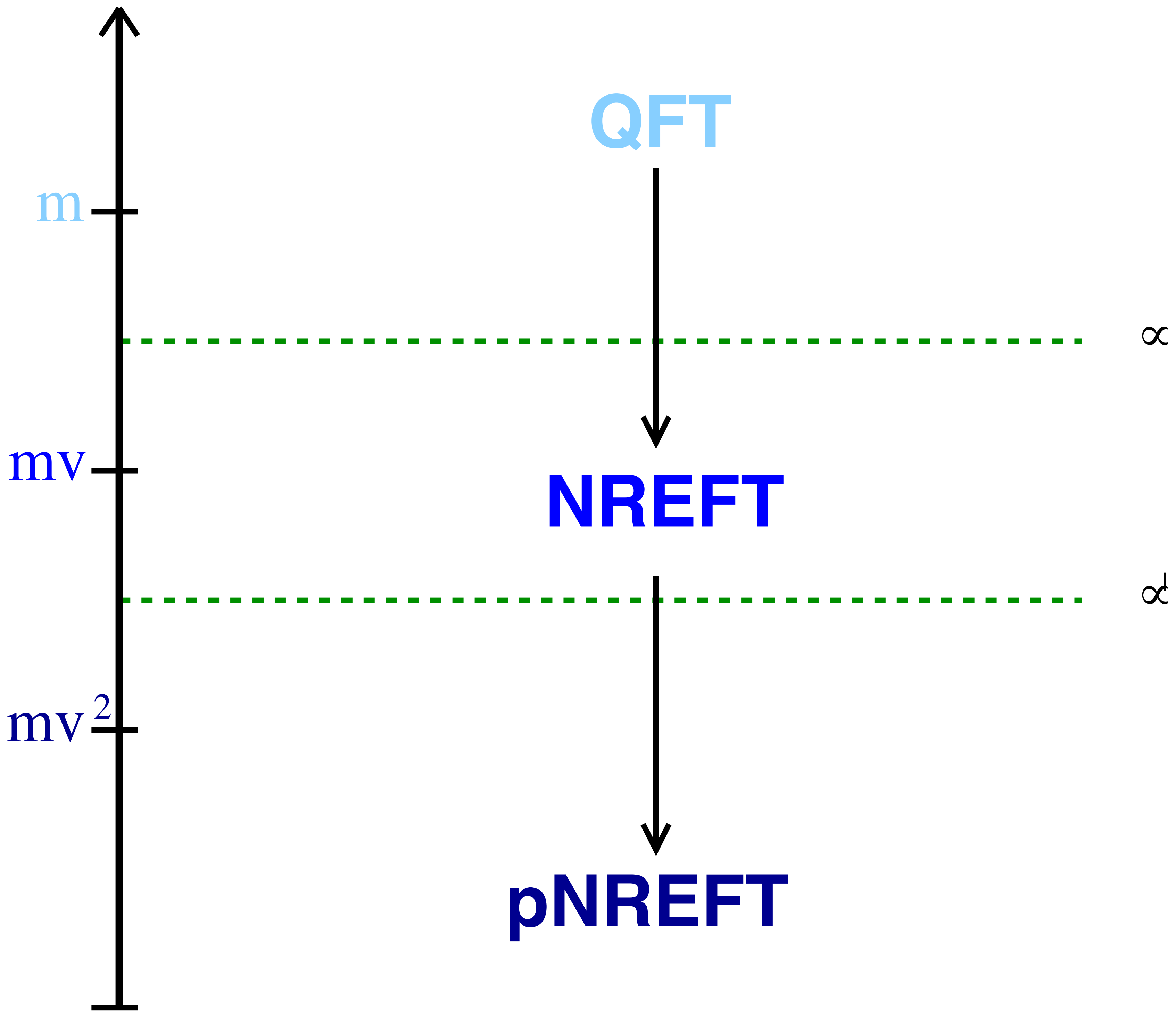

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It implements the Schroedinger eq. as zero order problem, define the potentials at the level of the QFT, implements systematically retardation corrections (Lamb shift), it encodes Poincare' invariance, and it is equivalent at any given order of the expansion to the underlying QFT

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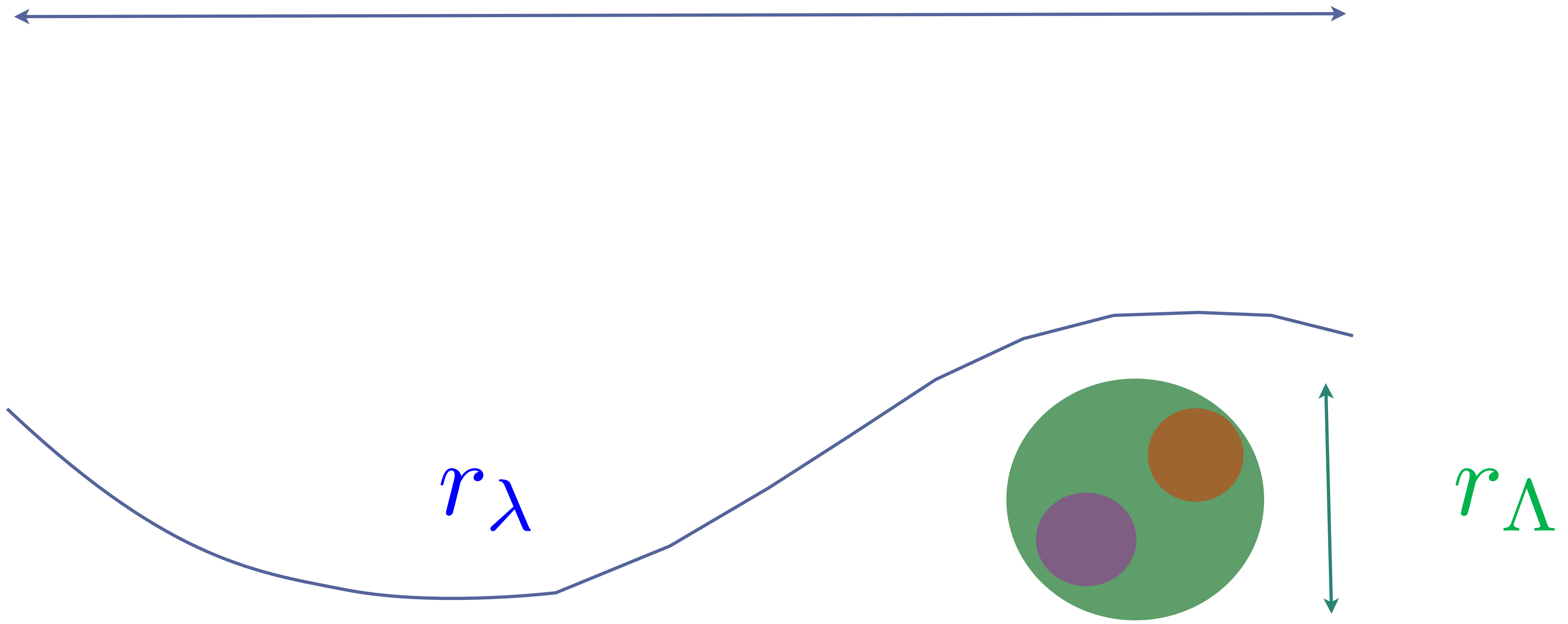
a hierarchy of EFTS can be formulated in correspondence to the hierarchy of scales



An effective field theory makes the expansion in λ/Λ explicit at the Lagrangian level.

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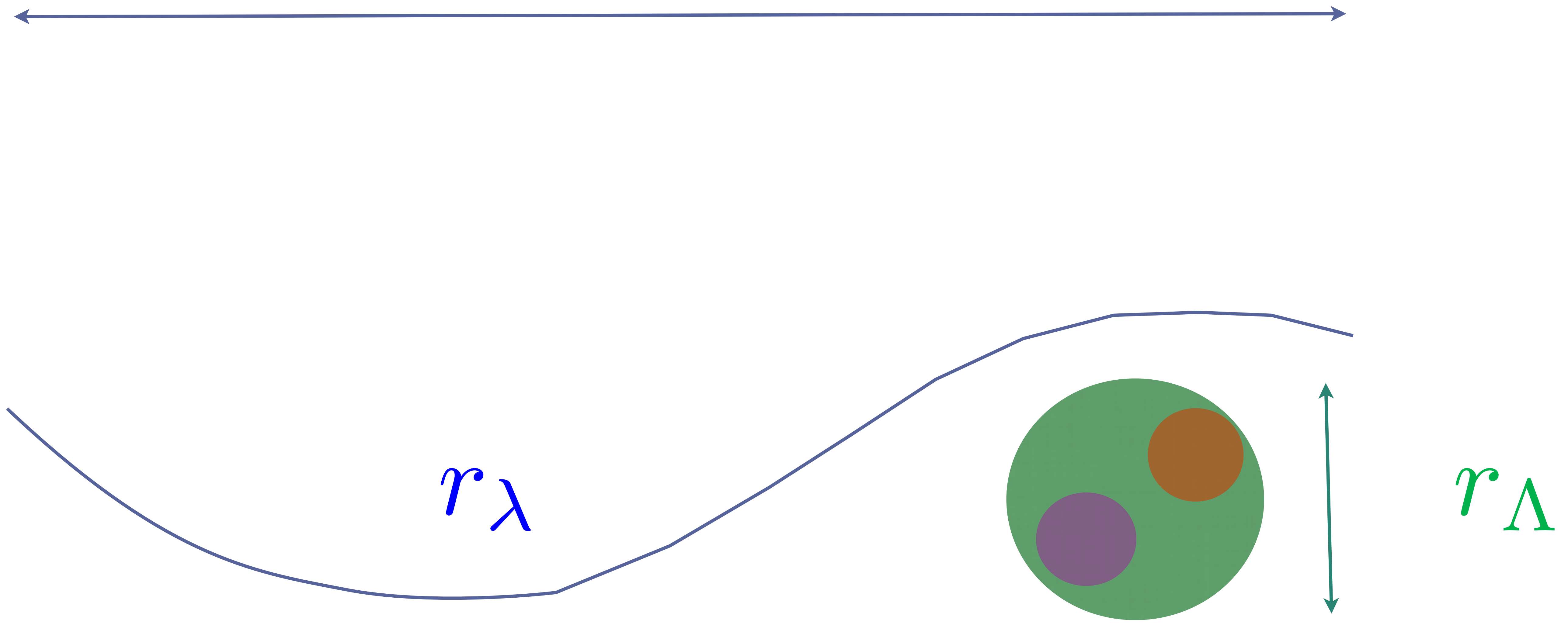

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The EFT Lagrangian, $\mathcal{L}_{\mathrm{EFT}}$, suitable to describe H at scales lower than Λ is defined by

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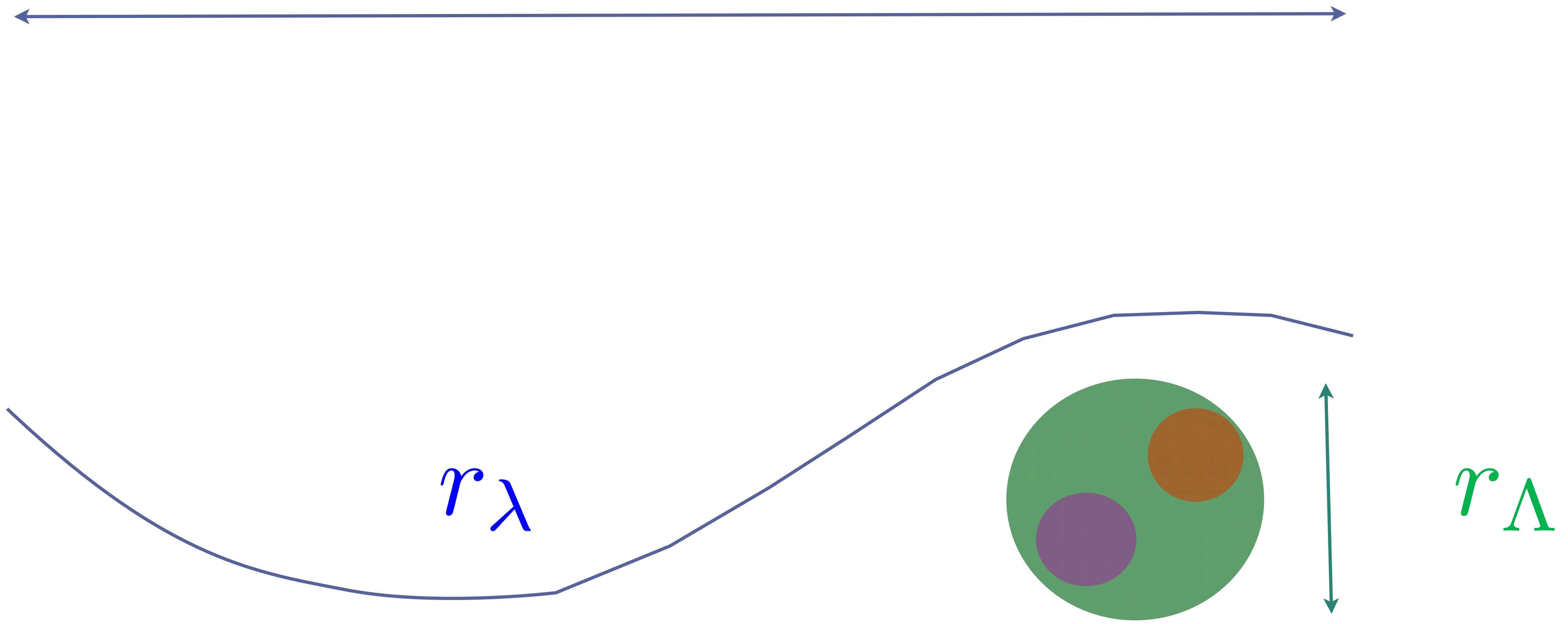

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RANGE OF VALIDITY OF THE EFT: ENERGY $< \mu$

$\Rightarrow$ $\mathcal{L}_{\rm EFT}$ is made of all operators O_n that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}$.

How to build EFTS

$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(\Lambda, \mu) \frac{O_n(\mu, \lambda)}{\Lambda^n}$$

Tower of interactions beginning with conventional renormalizable interactions but going on to include nonrenormalizable interactions of arbitrarily high dimensions

How to build EFTS

low energy operator

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Wilson coefficient

large scale

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- The matching coefficients $c_n(\Lambda/\mu)$ encode the non-analytic behaviour in Λ. They are calculated by imposing that $\mathcal{L}_{\mathrm{EFT}}$ and $\mathcal{L}$ describe the same physics at any finite order in the expansion: matching procedure.

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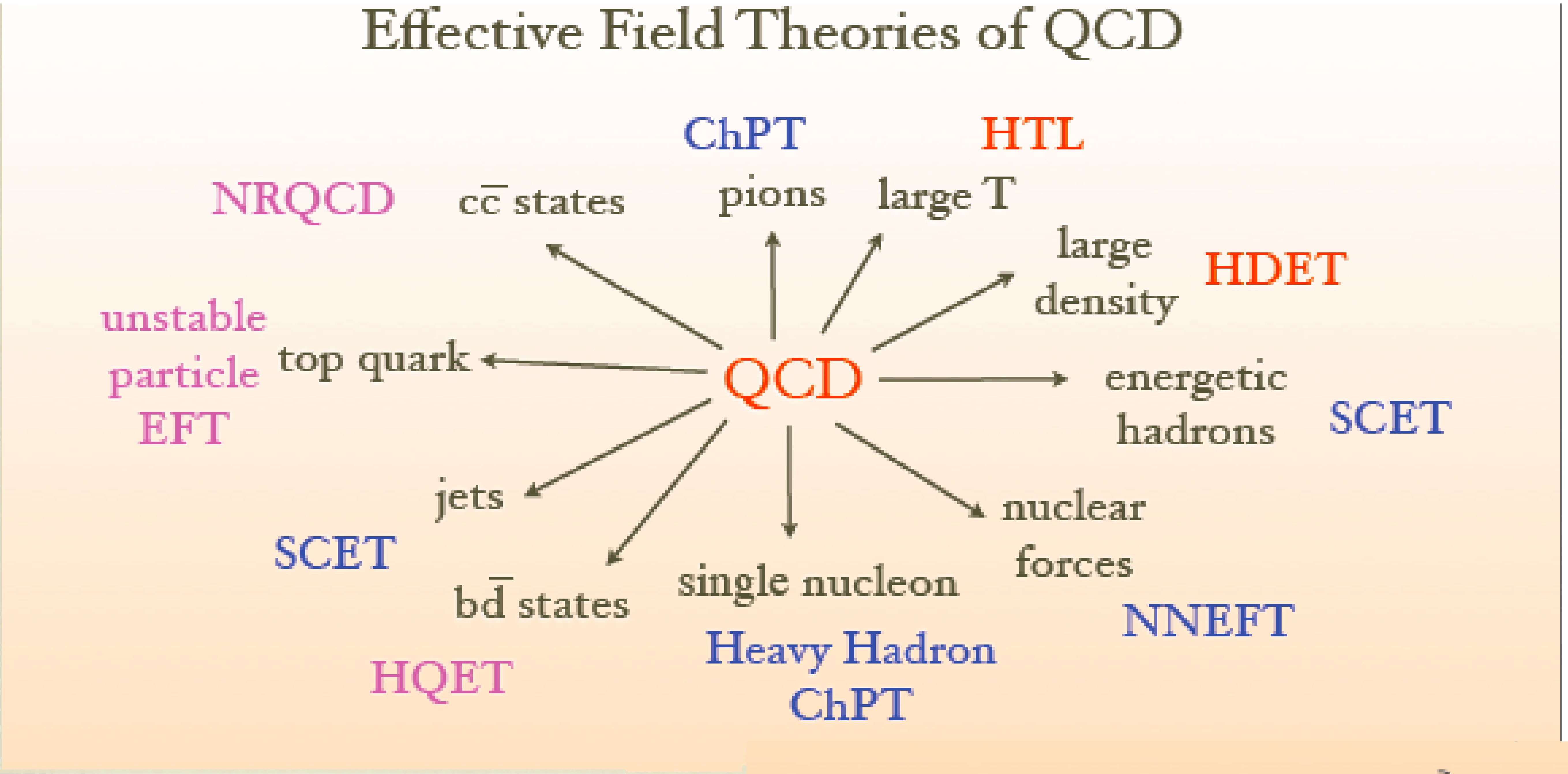
Matching

- If $\Lambda \gg \Lambda_{\rm QCD}$ then $c_n(\Lambda/\mu)$ may be calculated in perturbation theory.

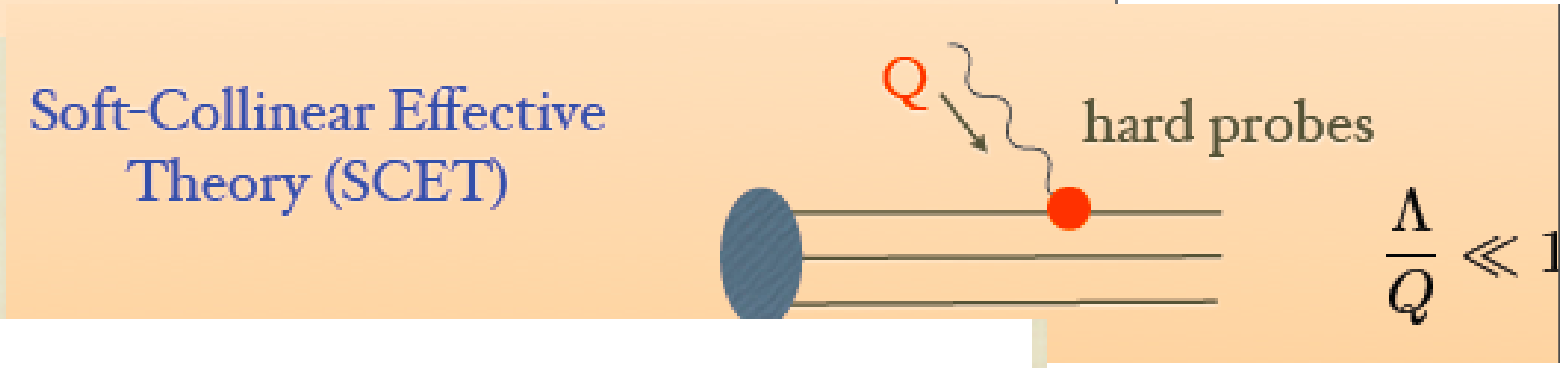
- Symmetries of the system become manifest;
- Large log(Λ/λ) can be resummed via RG. (Renormalization group)

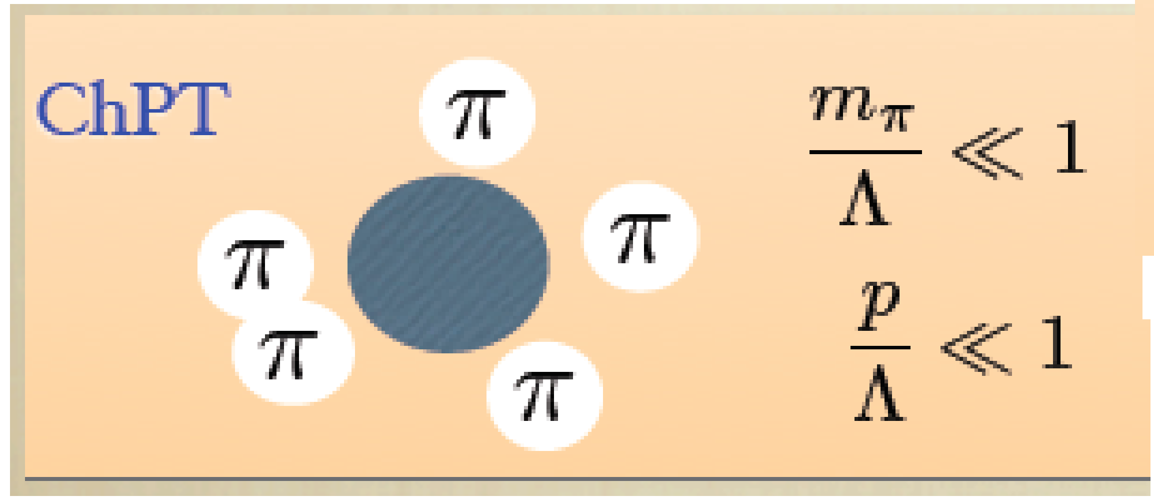
To address the research frontier of strong interactions we need to construct effective field theories and complement them with lattice

Effective Field Theories of QCD



- Heavy quark effective theory (HQET): $\frac{\lambda}{\Lambda} = \frac{\Lambda_{QCD}}{m}$


Soft-Collinear Effective Theory (SCET)



Lattice QCD $\equiv$ Effective Field Theory ($\Lambda = \pi/a$).

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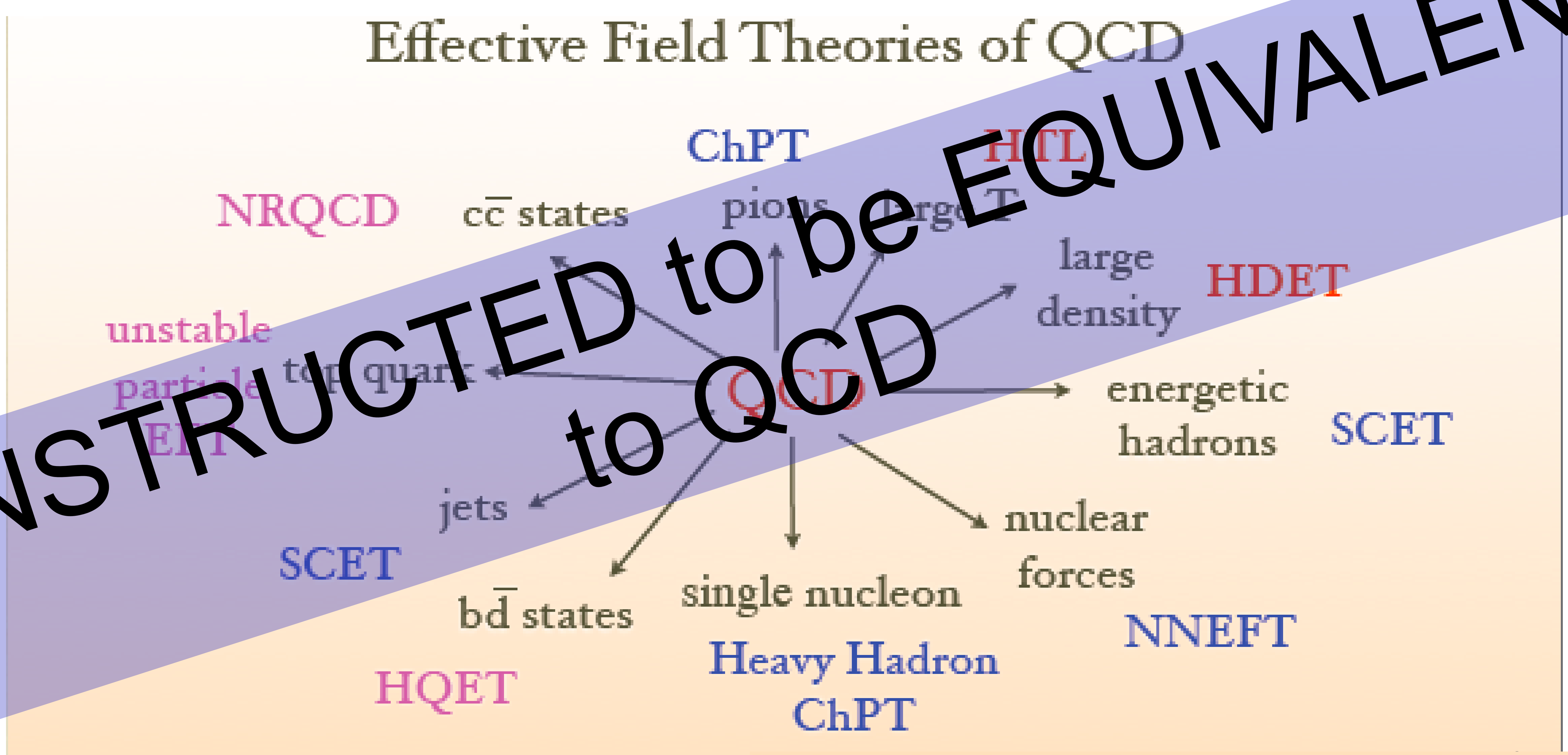

CONSTRUCTED to be EQUIVALENT to QCD

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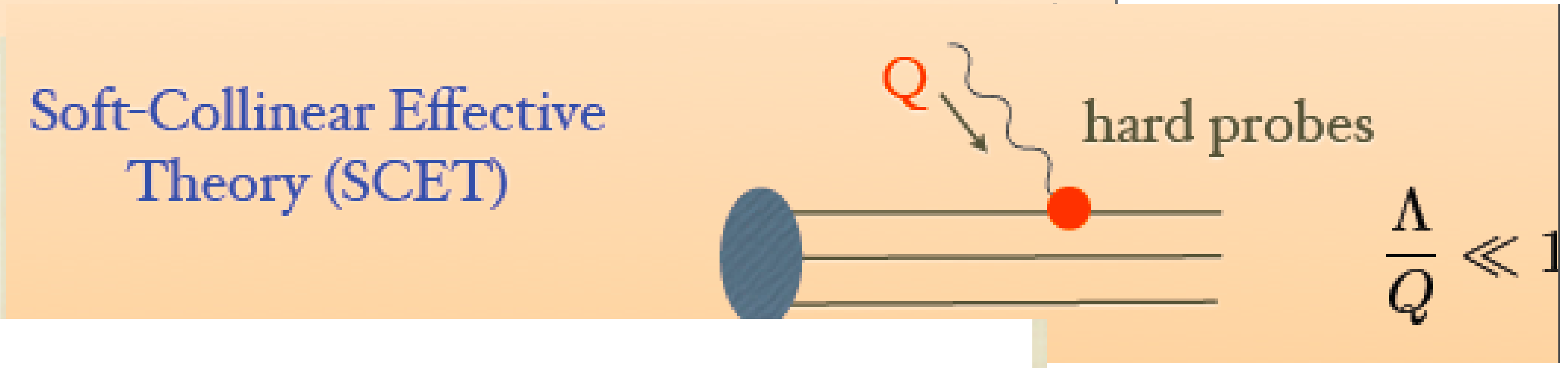

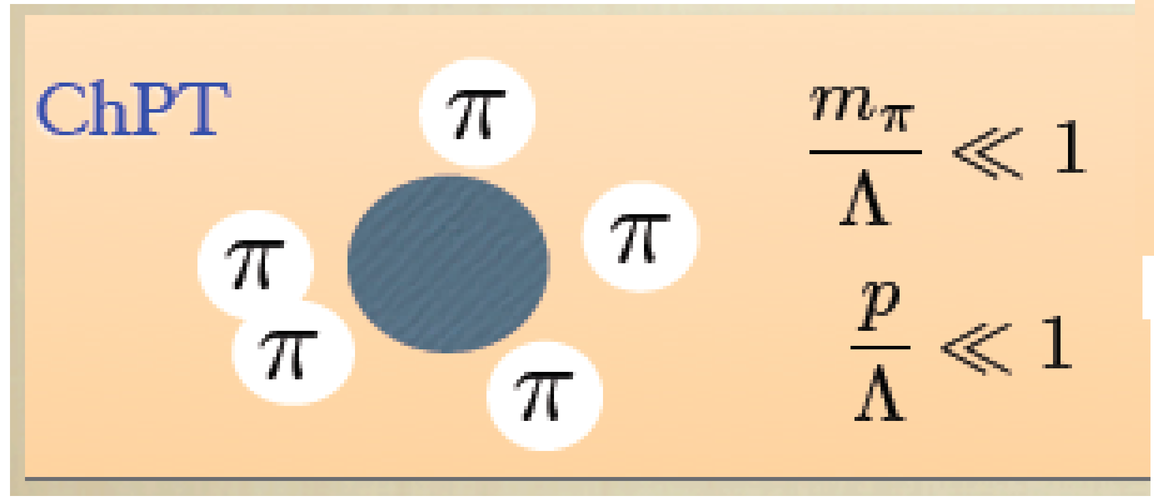

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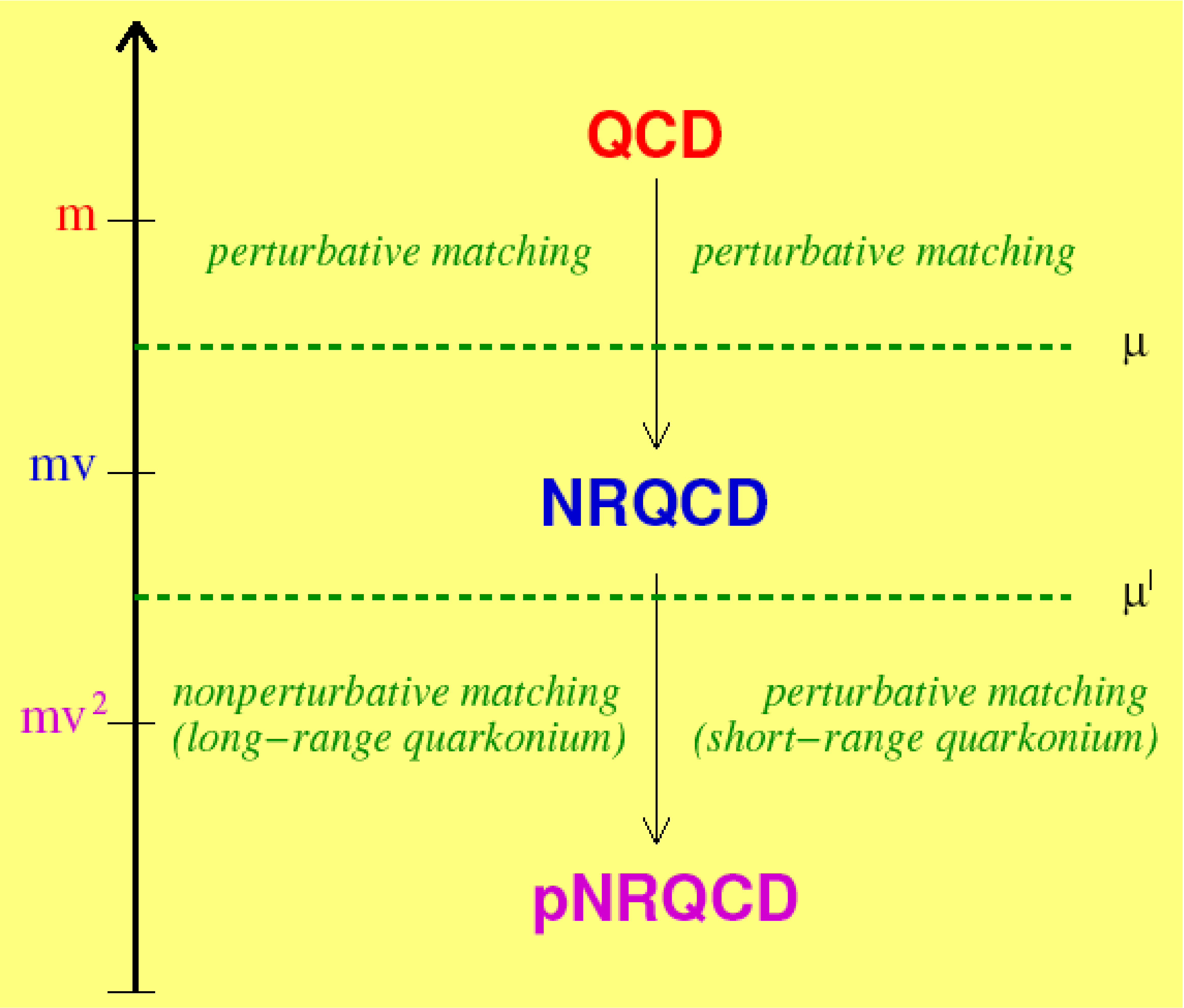
Quarkonium with NR EFT



Color degrees of freedom
3X3=1+8
singlet and octet QQbar

Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

Quarkonium with NR EFT

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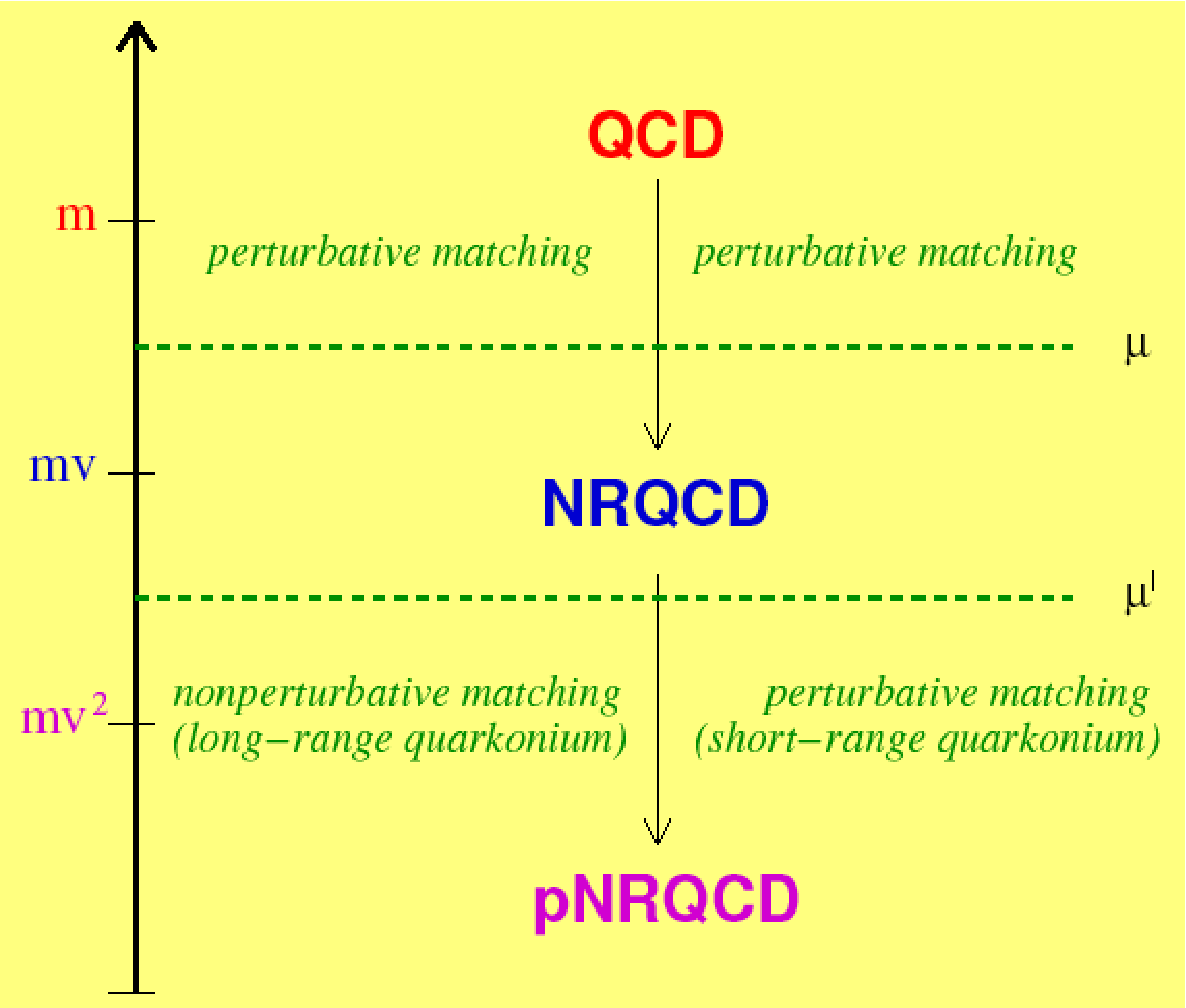

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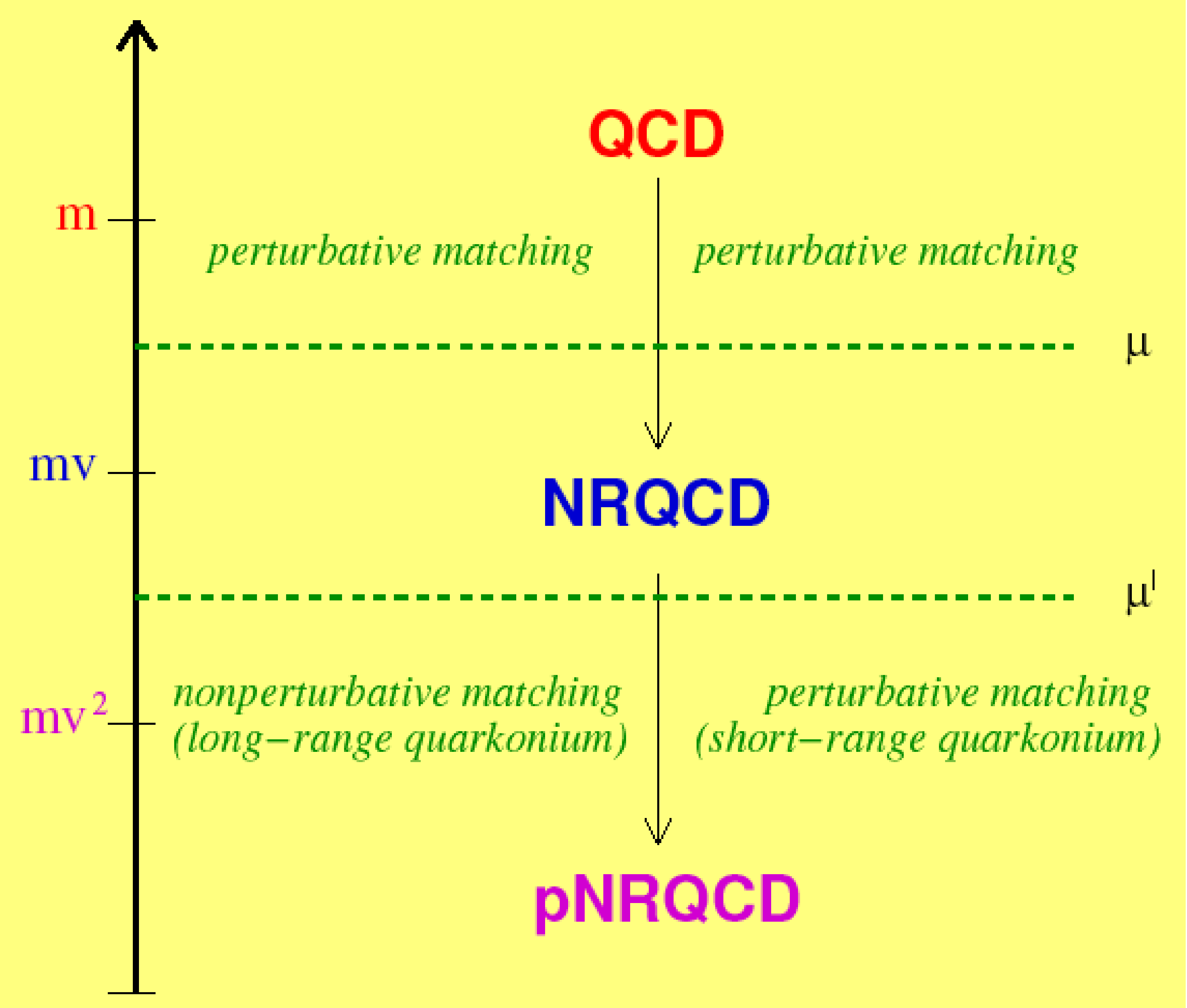

QCD

m

perturbative matching *perturbative matching*

μ

mv

NRQCD

μ'

mv^2

nonperturbative matching
(long−range quarkonium)

perturbative matching
(short−range quarkonium)

pNRQCD

Hard

Soft
(relative
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Ultrasoft
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$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda} \qquad \langle O_n\rangle \sim E_\lambda^n$$

Quarkonium with NR EFT

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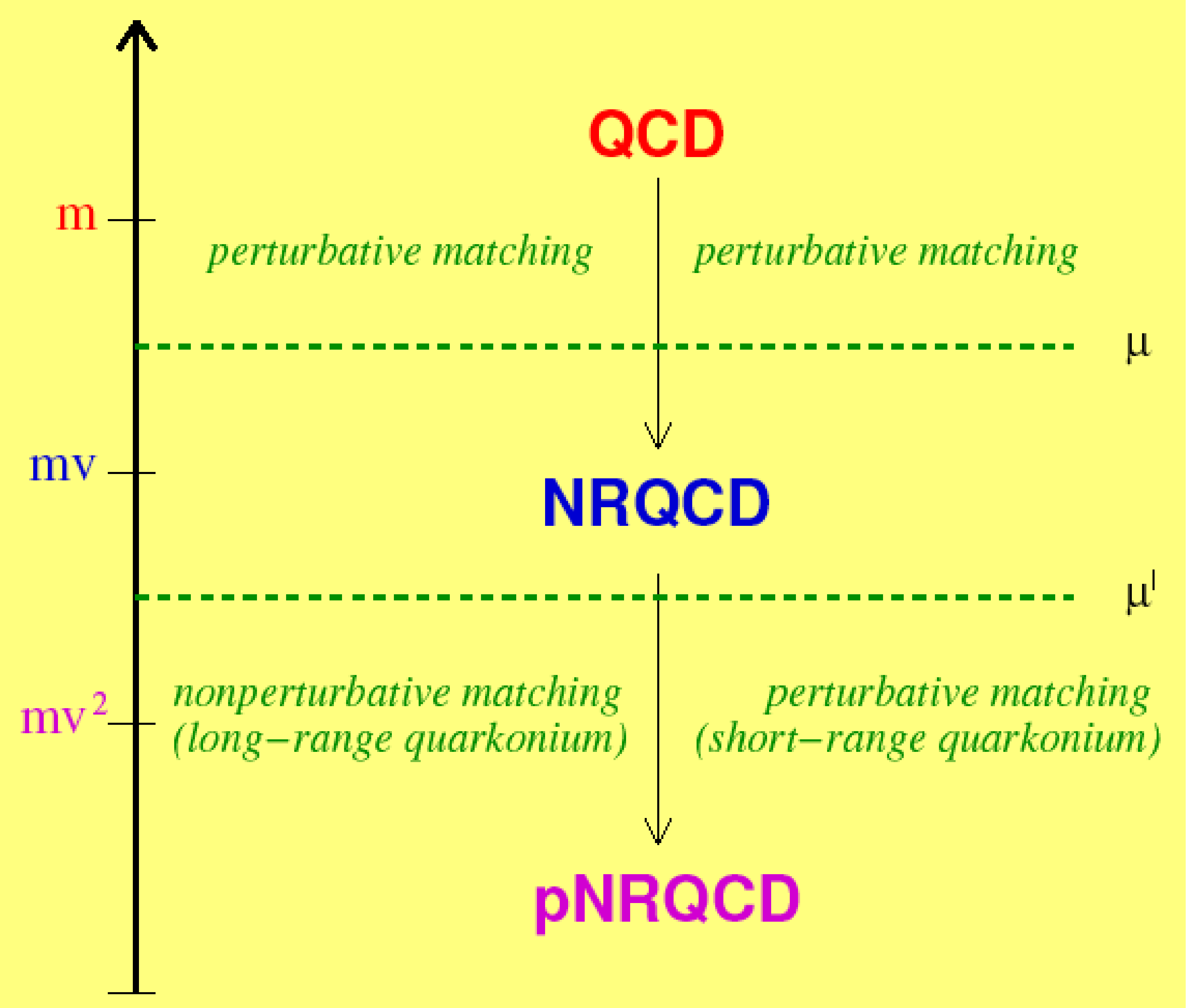

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mv^2

QCD

perturbative matching *perturbative matching*

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nonperturbative matching
(long–range quarkonium)

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(relative
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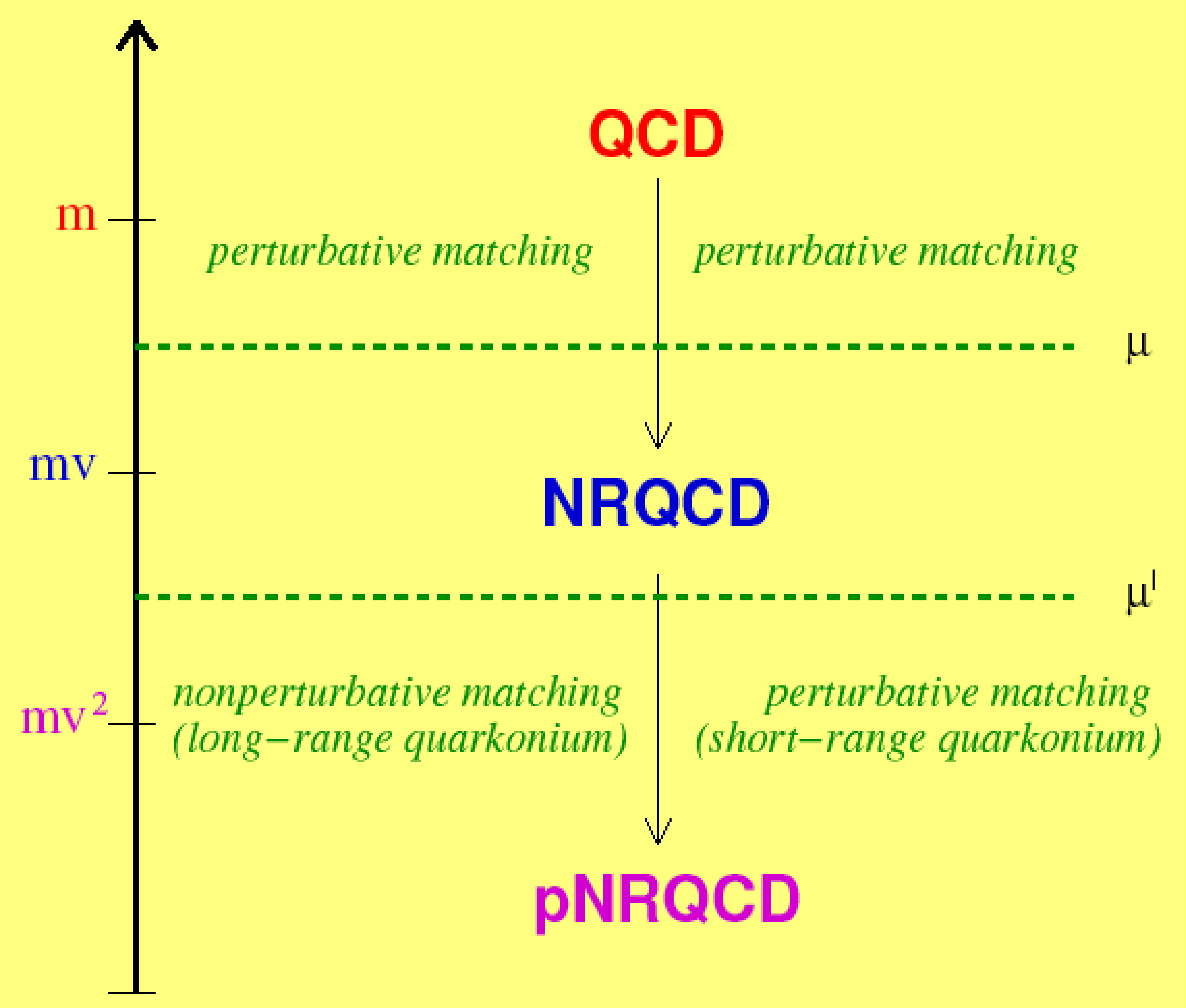

m

QCD

perturbative matching *perturbative matching*

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mv

NRQCD

μ'

mv²

nonperturbative matching
(long–range quarkonium)

perturbative matching
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pNRQCD

Hard

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

Soft
(relative momentum)

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv}$$

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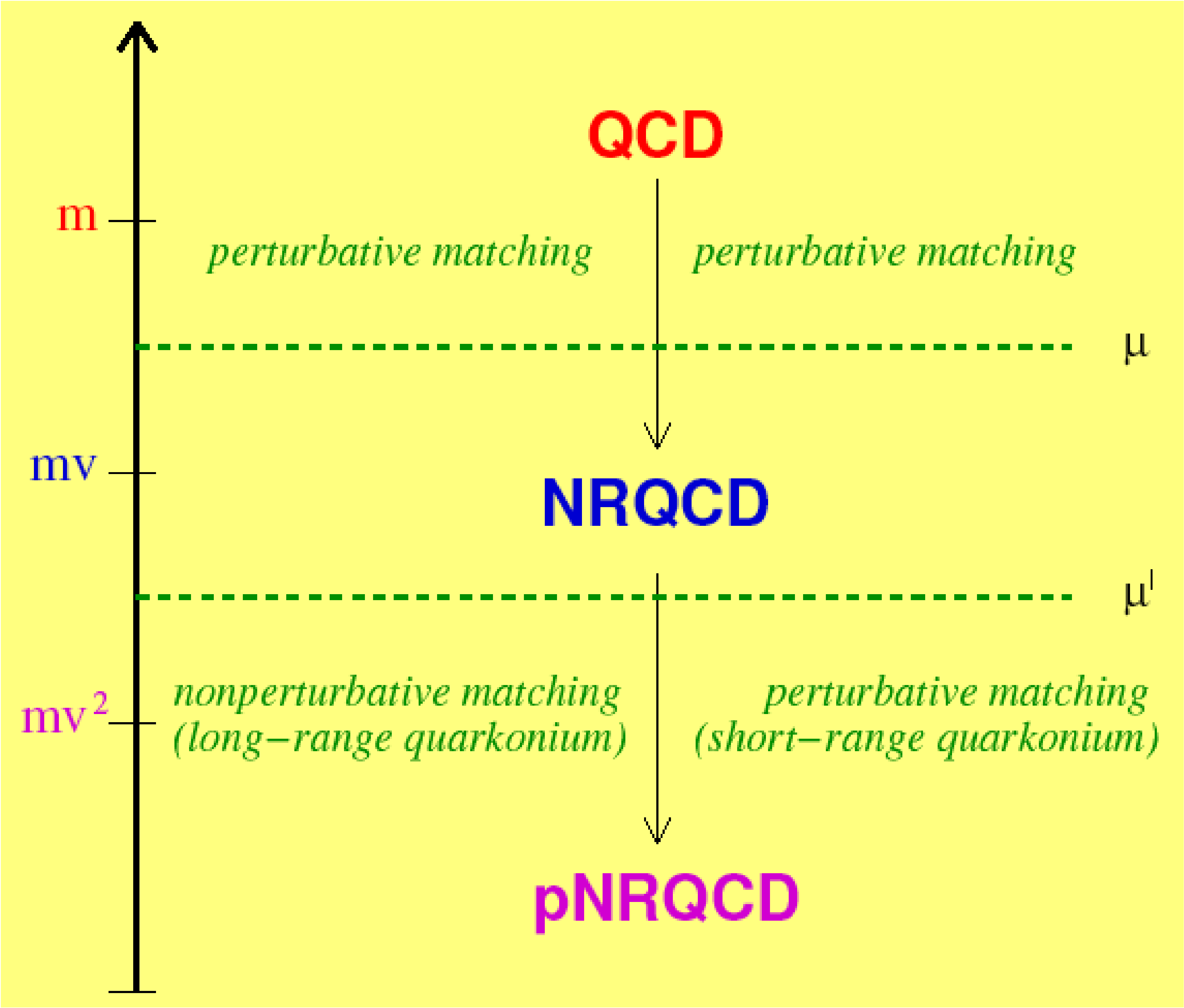
$$\mathcal{L}_{\mathrm{EFT}} = \sum_n c_n(E_\Lambda/\mu)\frac{O_n(\mu,\lambda)}{E_\Lambda}$$

$$\langle O_n \rangle \sim E_\lambda^n$$

QQbar systems with NR EFT: Non Relativistic QCD (NRQCD)

Caswell, Lepage 86, Lepage Thacker 88,
Bodwin, Braaten, Lepage 95



QCD

m

perturbative matching *perturbative matching*

μ

mv

NRQCD

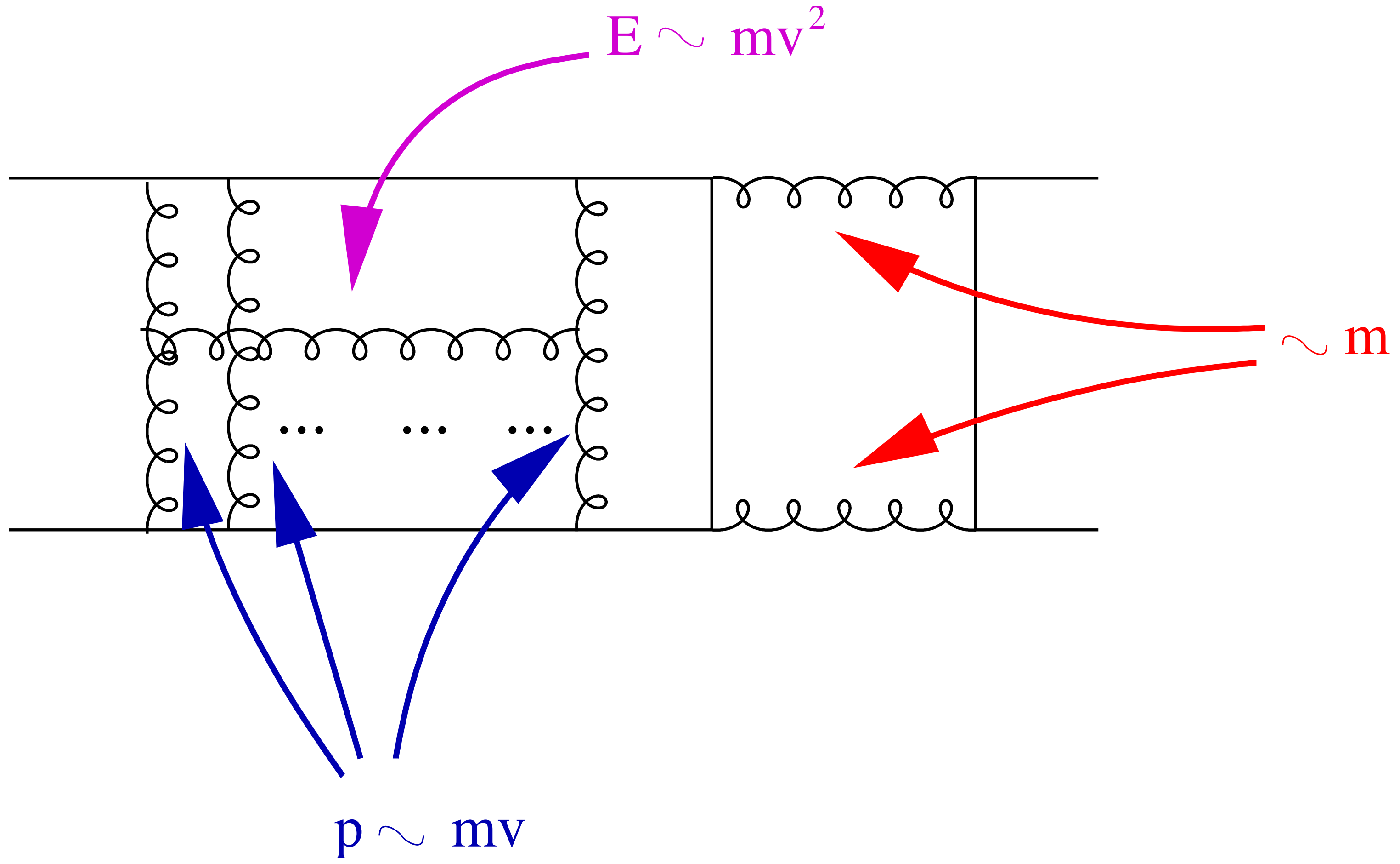
μ'

mv^2

nonperturbative matching
(long–range quarkonium)

perturbative matching
(short–range quarkonium)

pNRQCD



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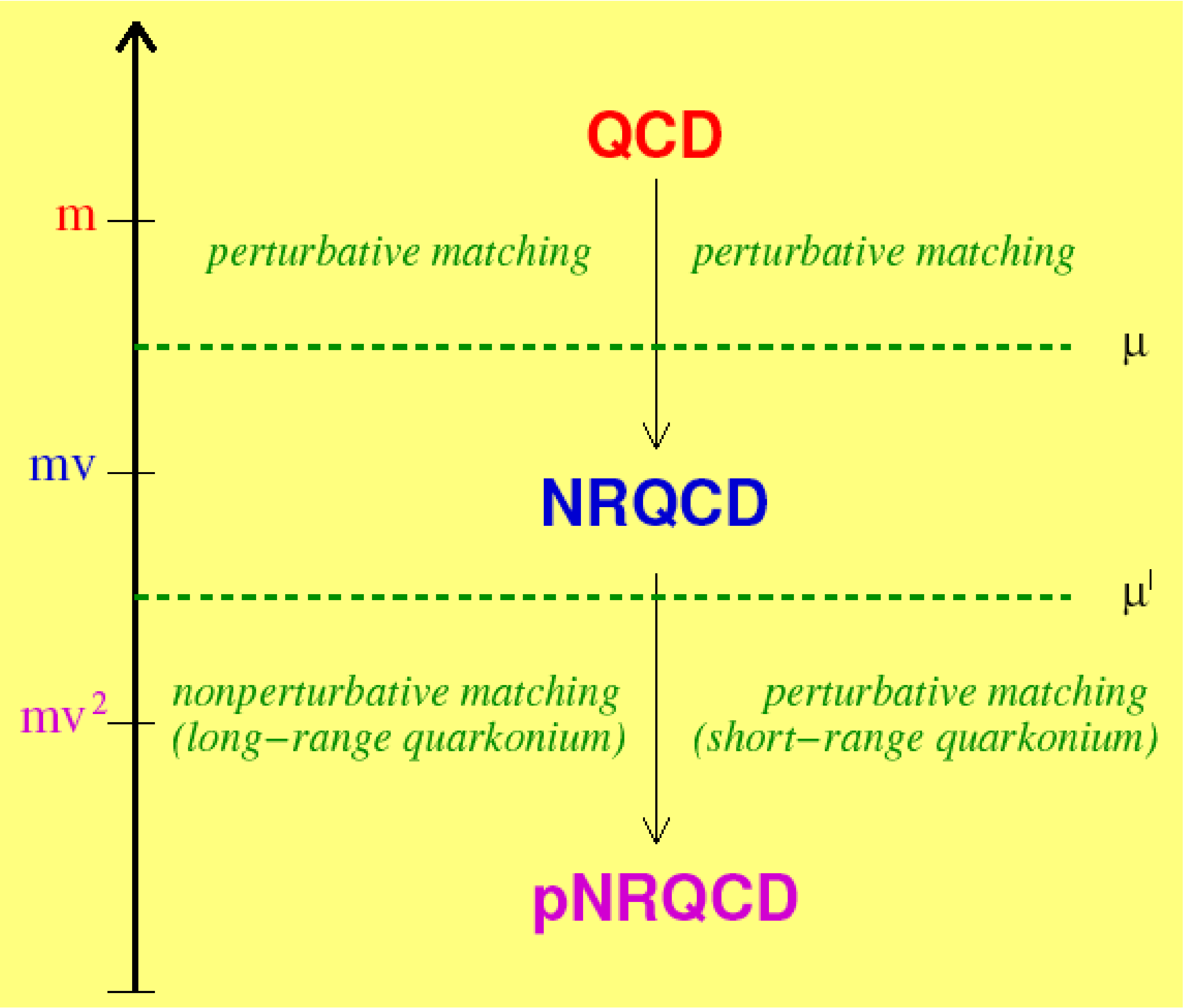

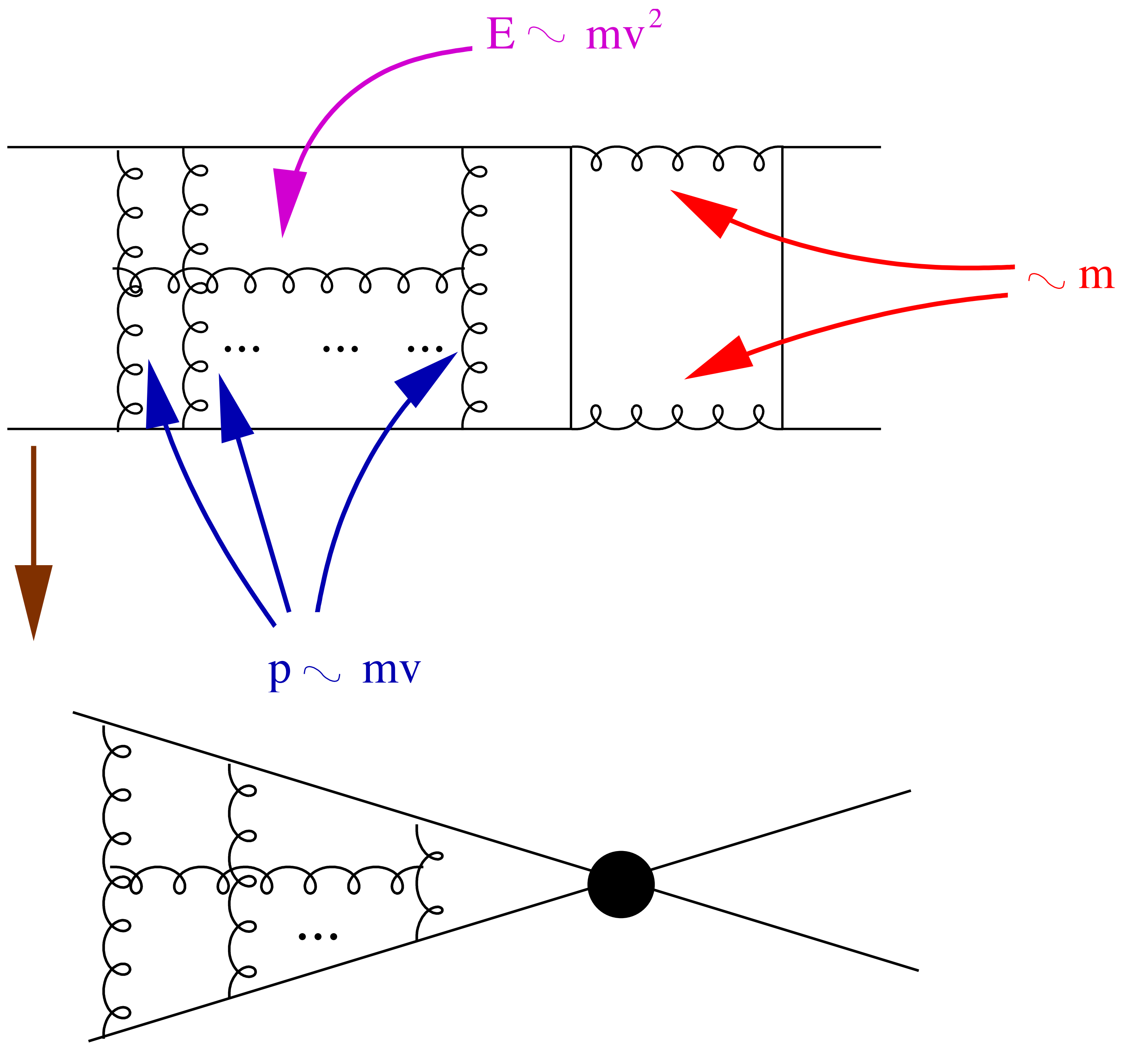

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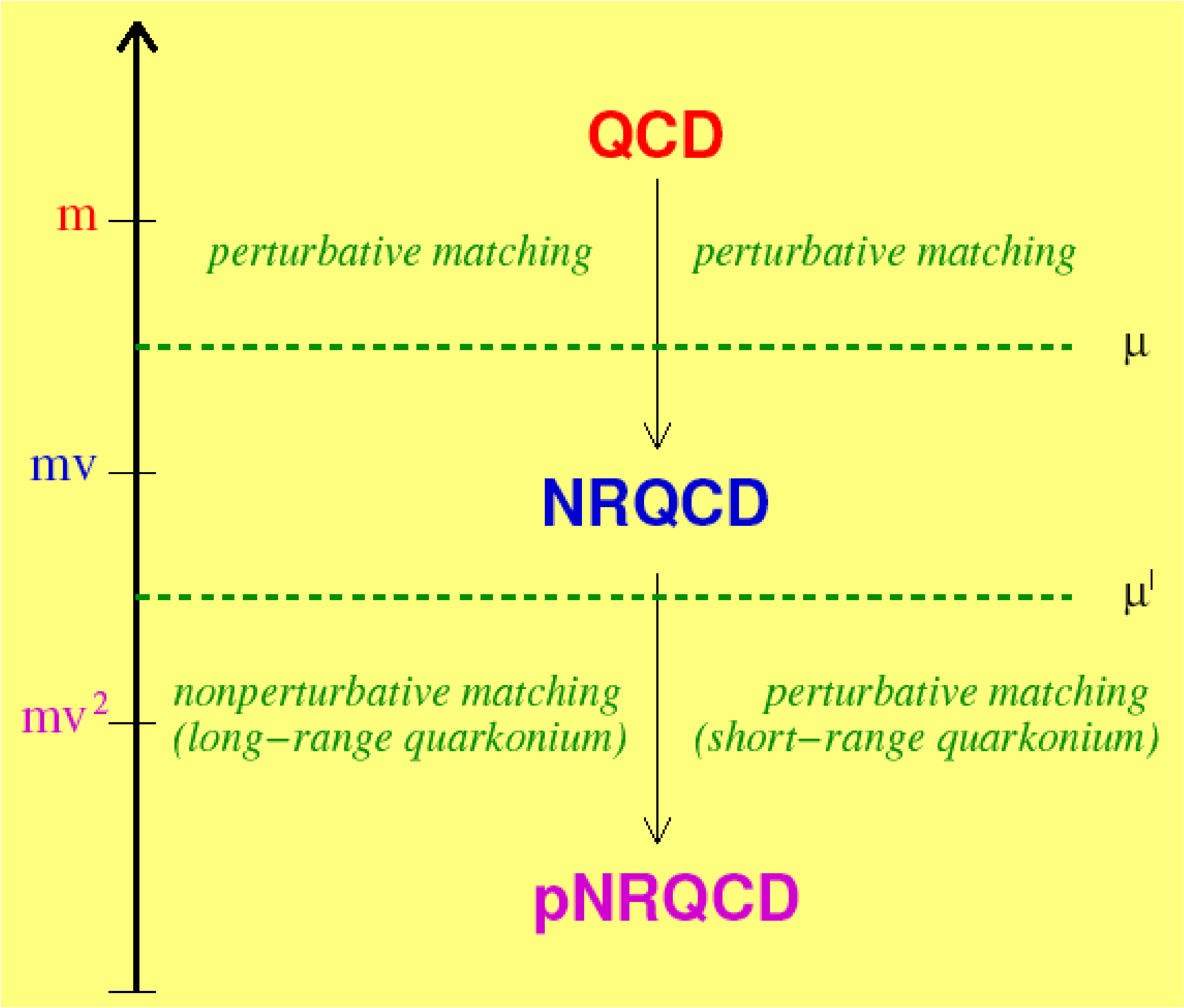

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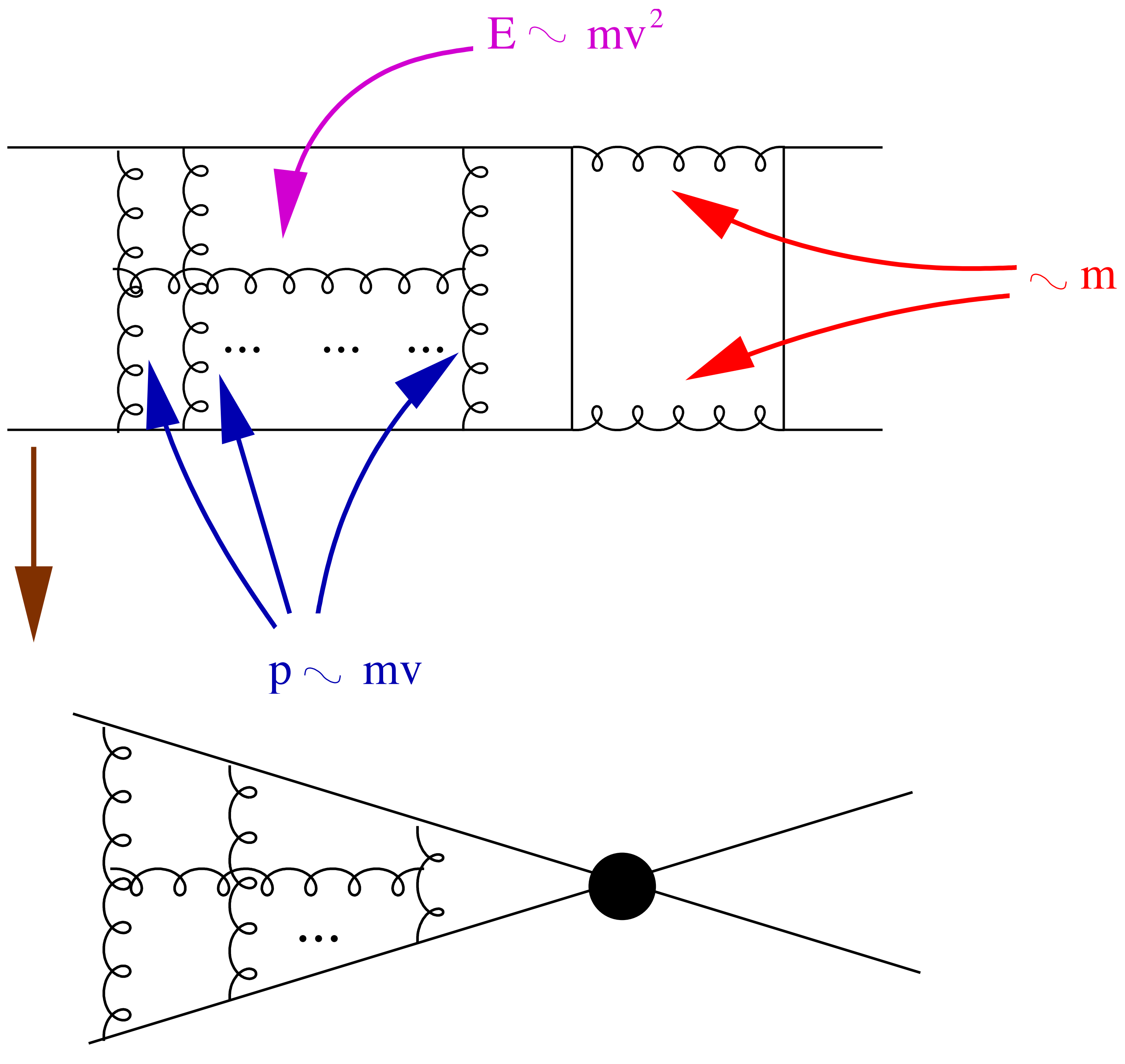
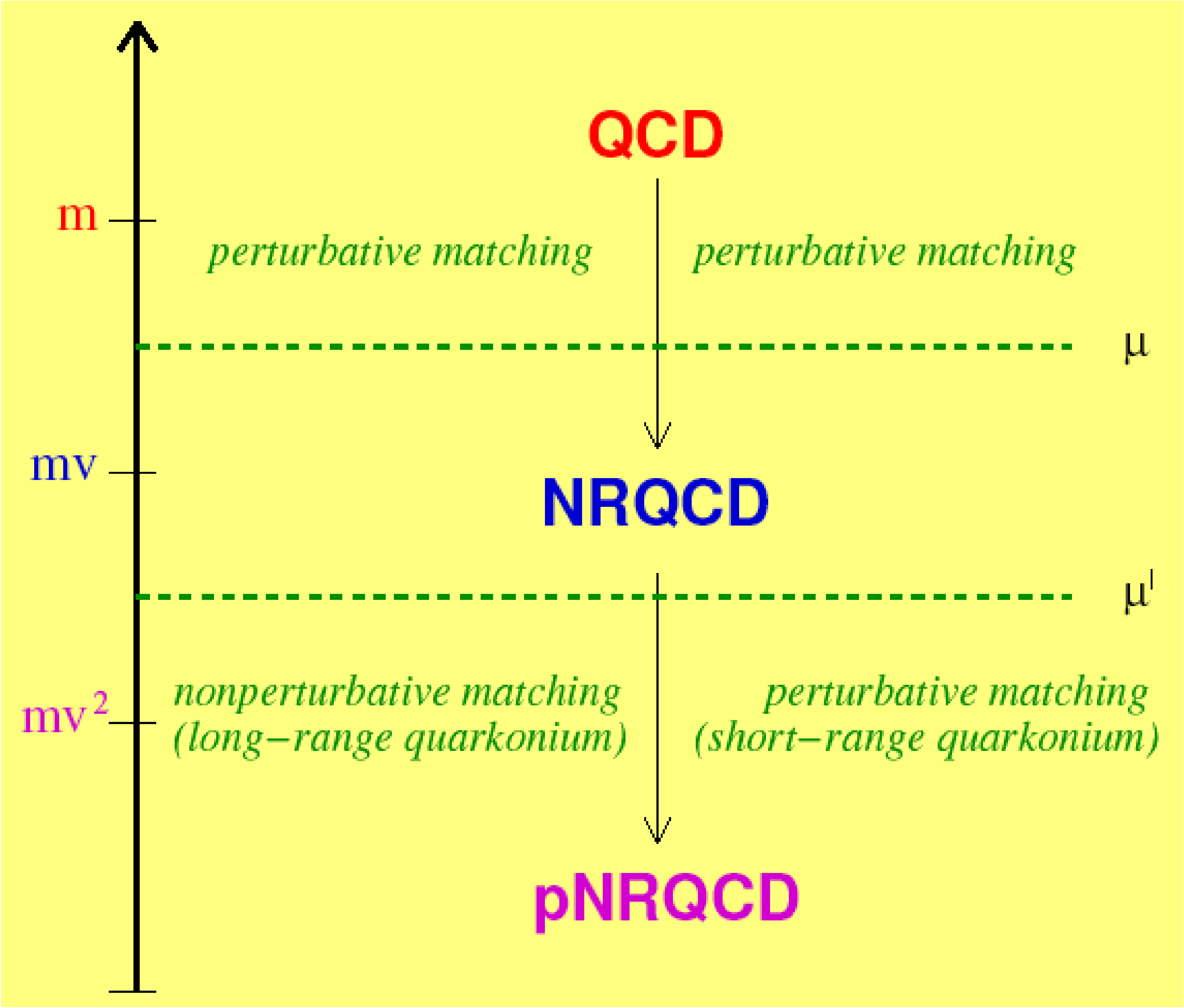


$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n}$$

Applications to productions,
Lattice NRQCD…
Still two scales entangled

Quarkonium with NR EFT: potential Non Relativistic QCD (pNRQCD)

Pineda Soto 97, N. B., Pineda, Soto, Vairo 98



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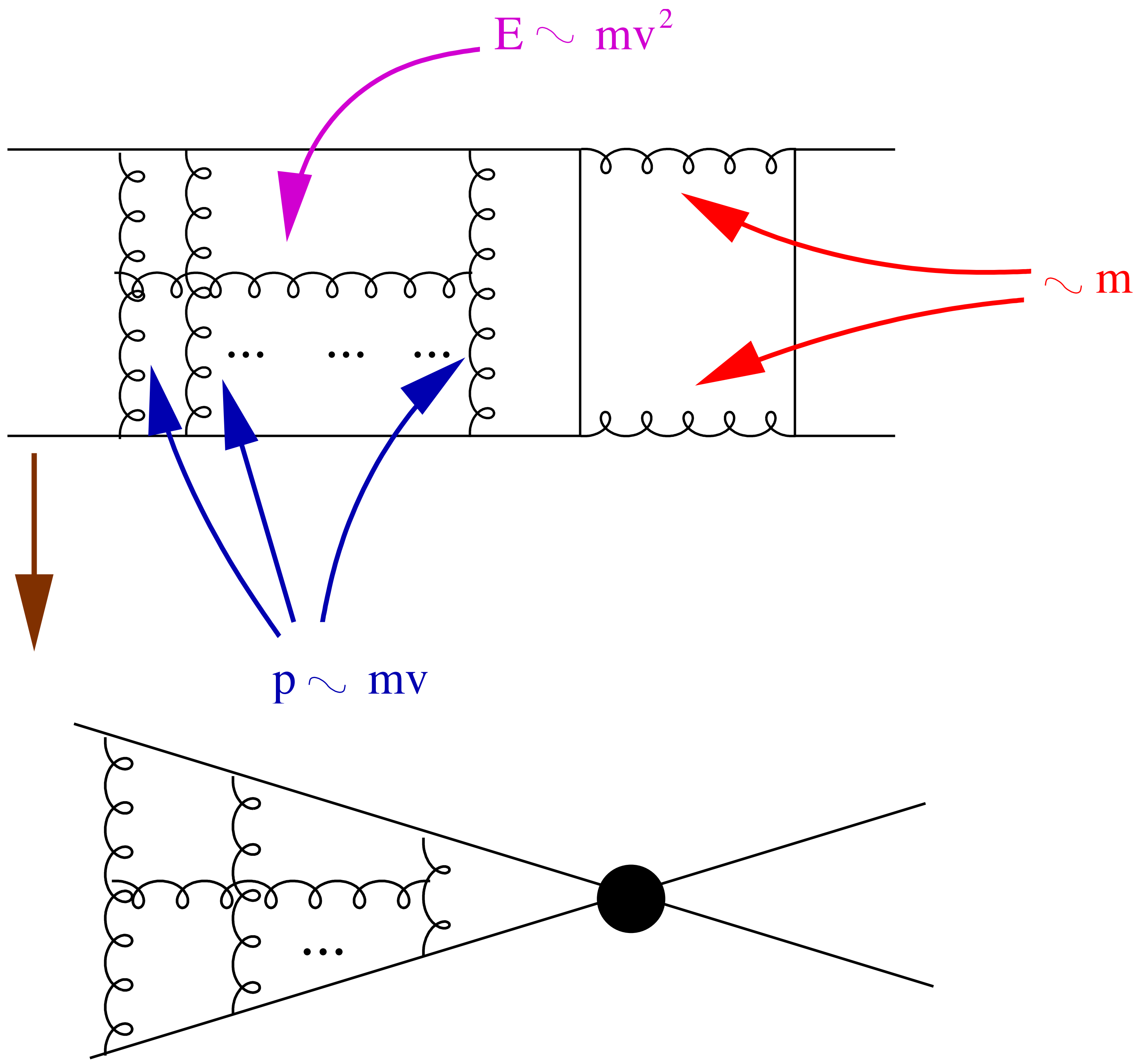

For mv >>Lambda_QCD

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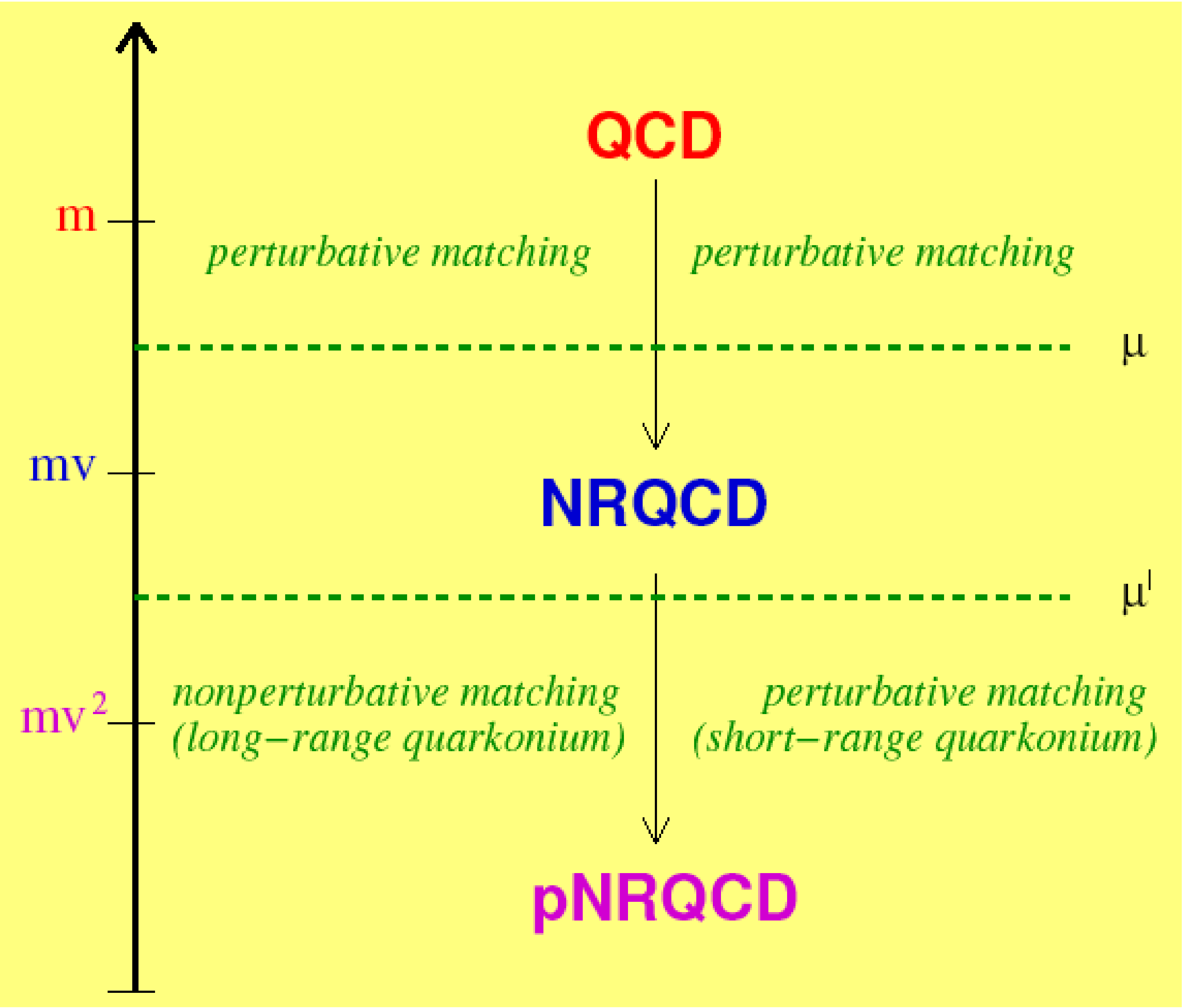

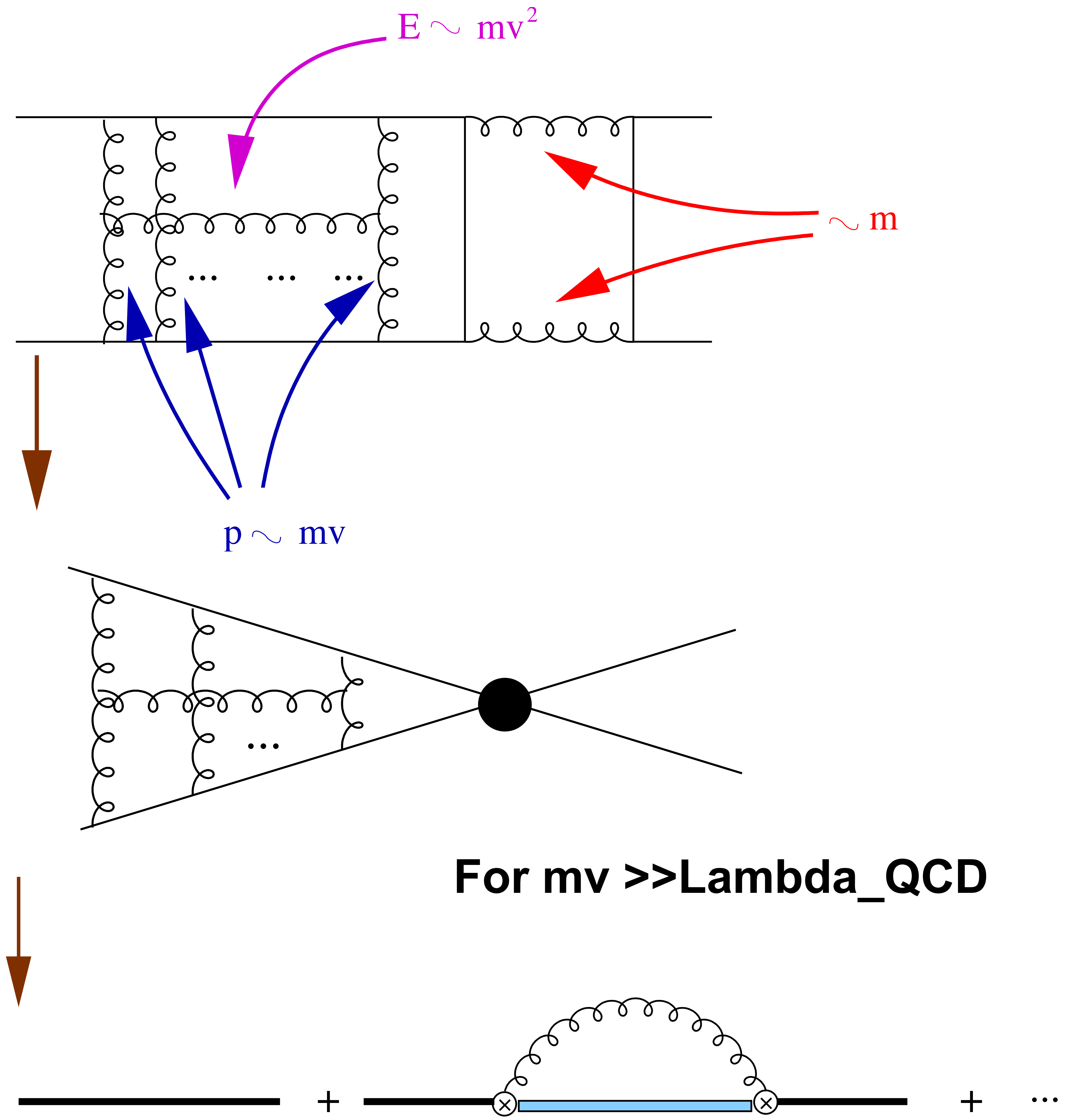

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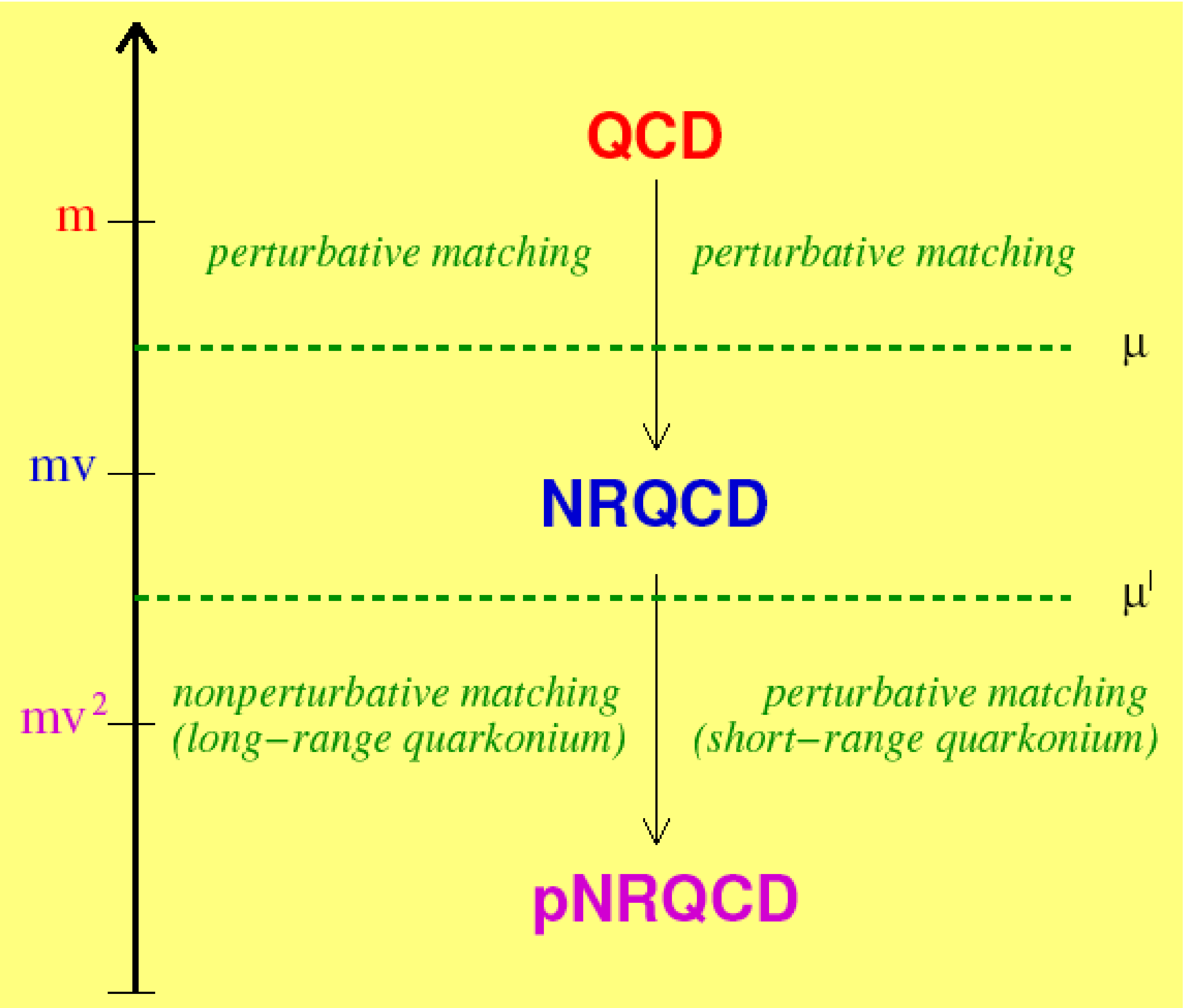

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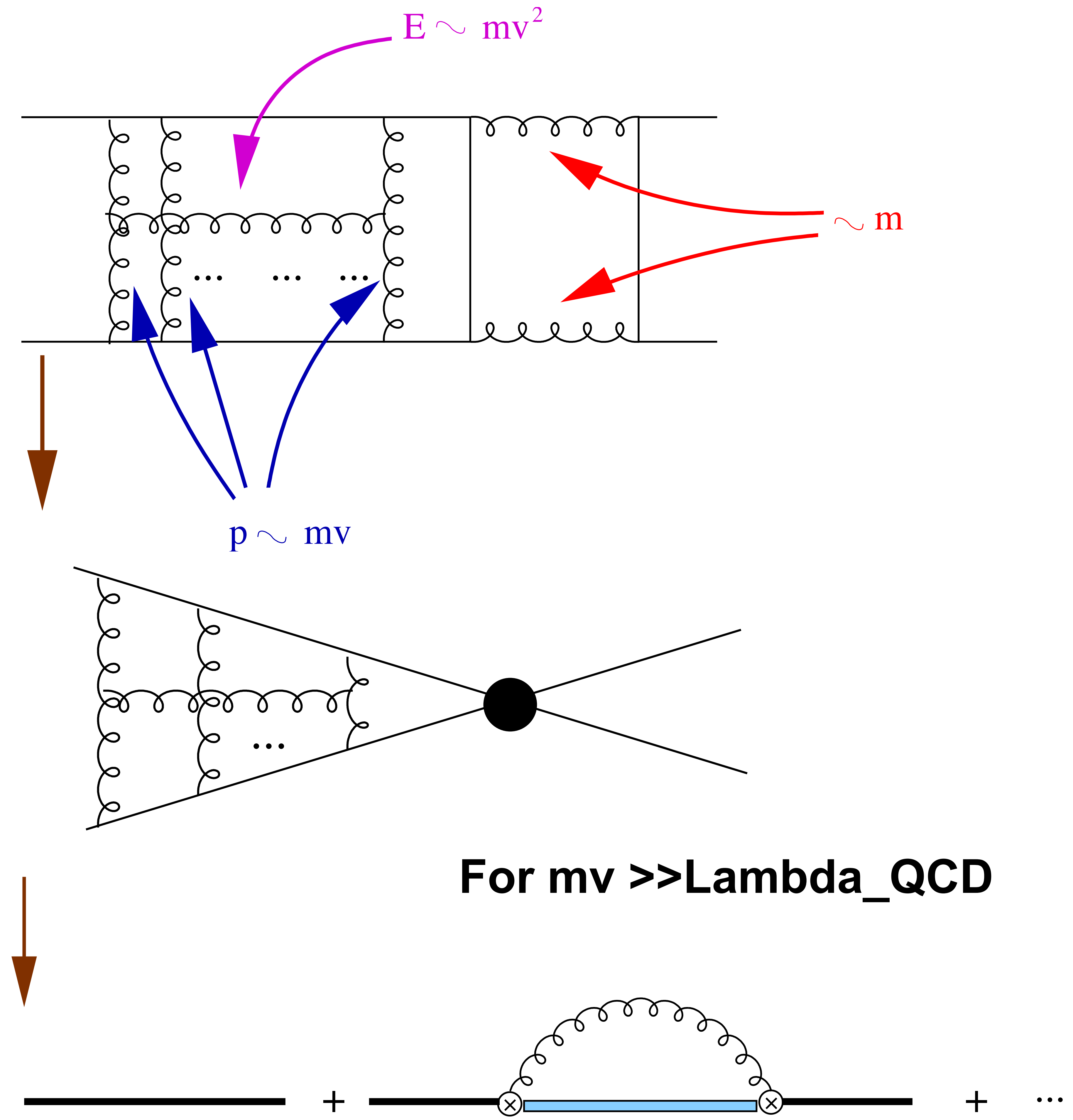

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$$\mathcal{L}_{\mathrm{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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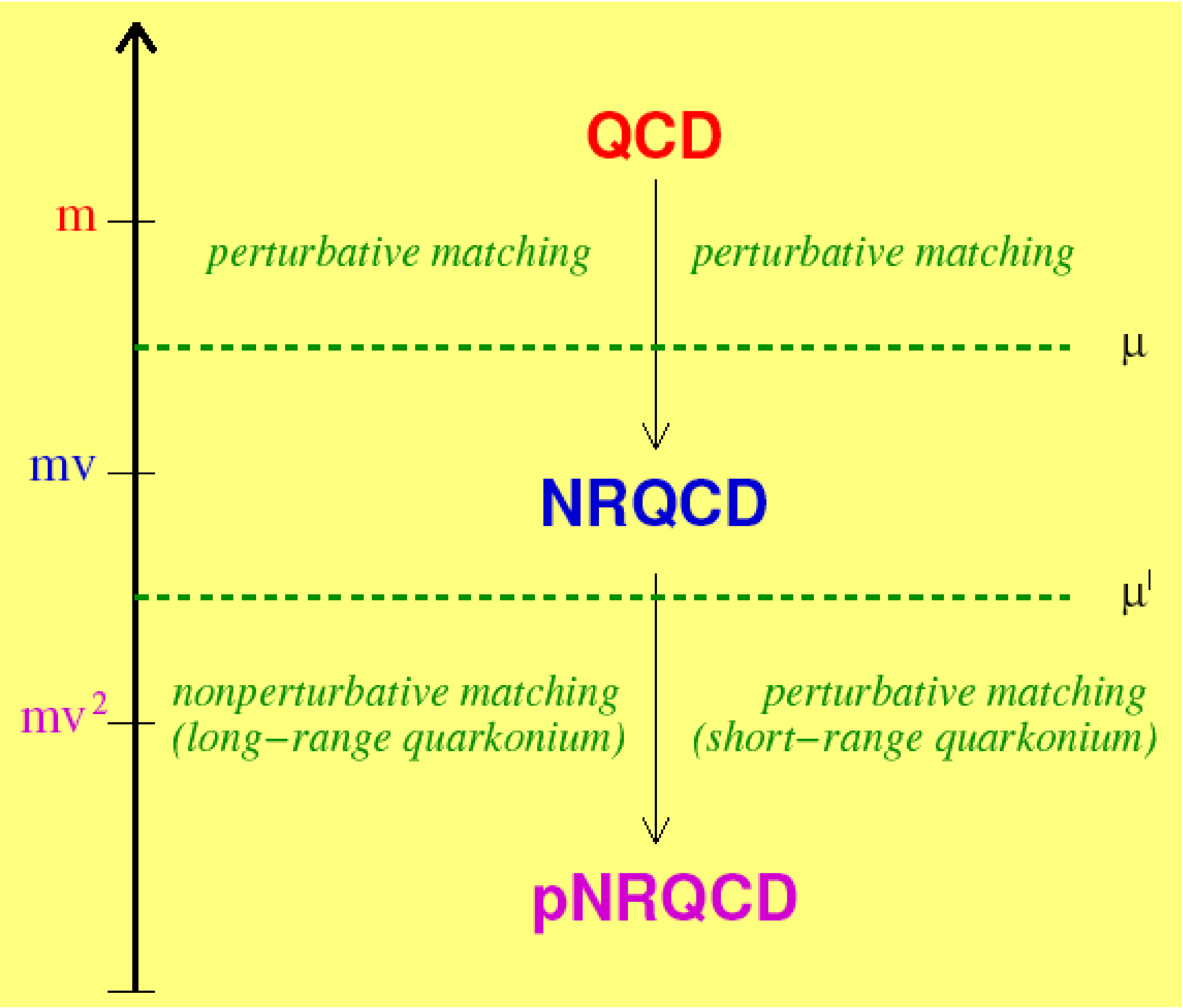

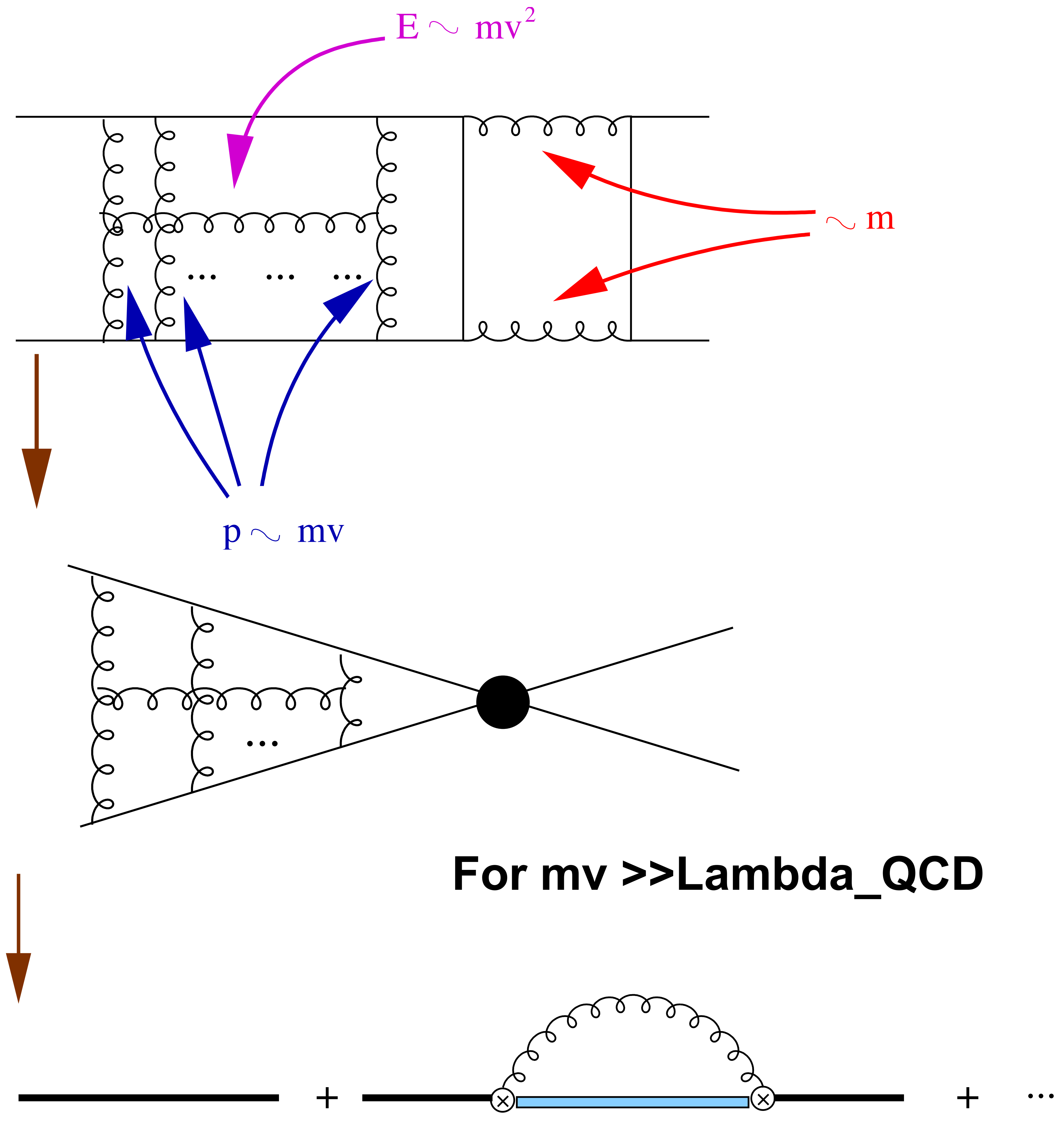

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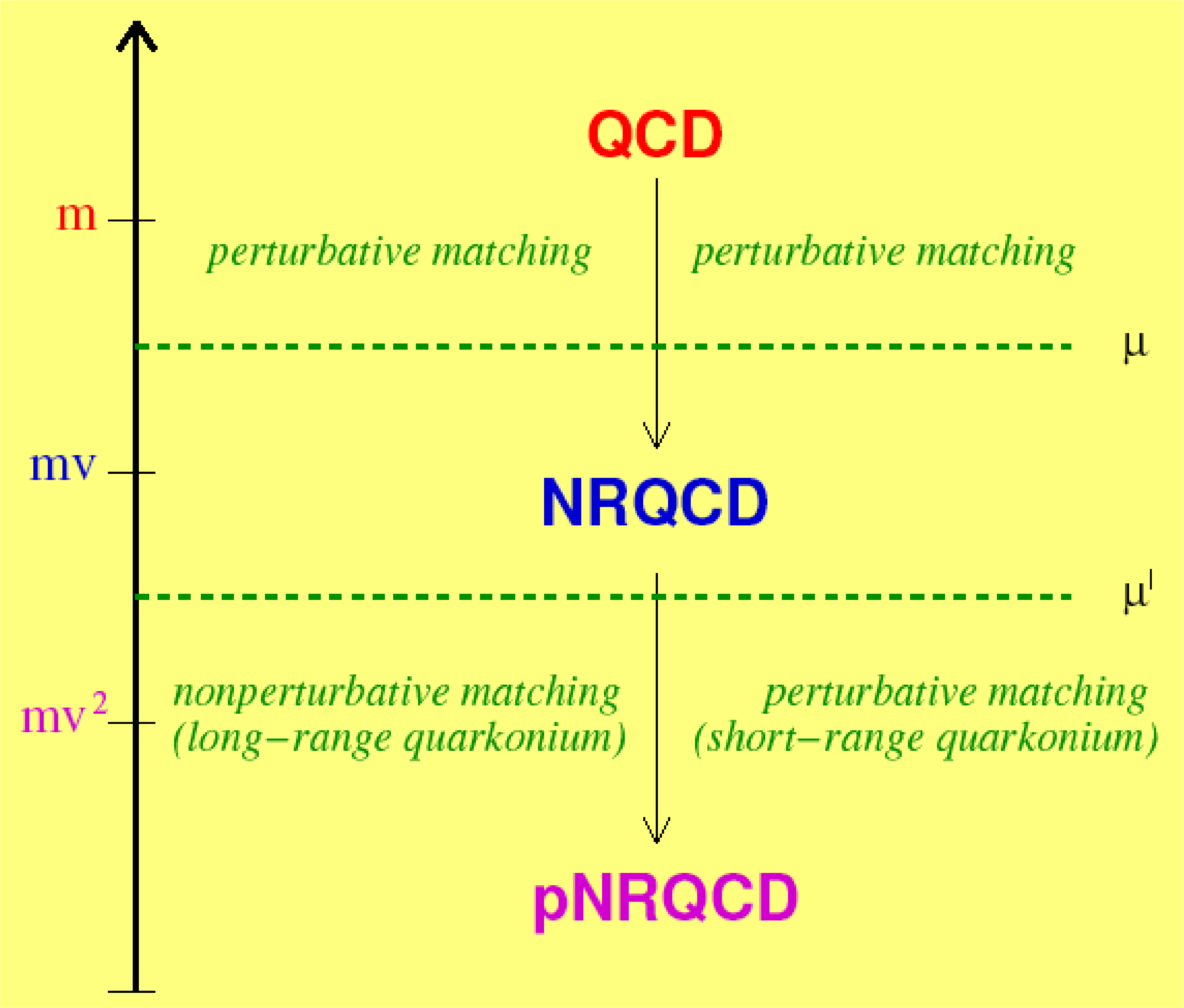
Quarkonium with NREFT: pNRQCD



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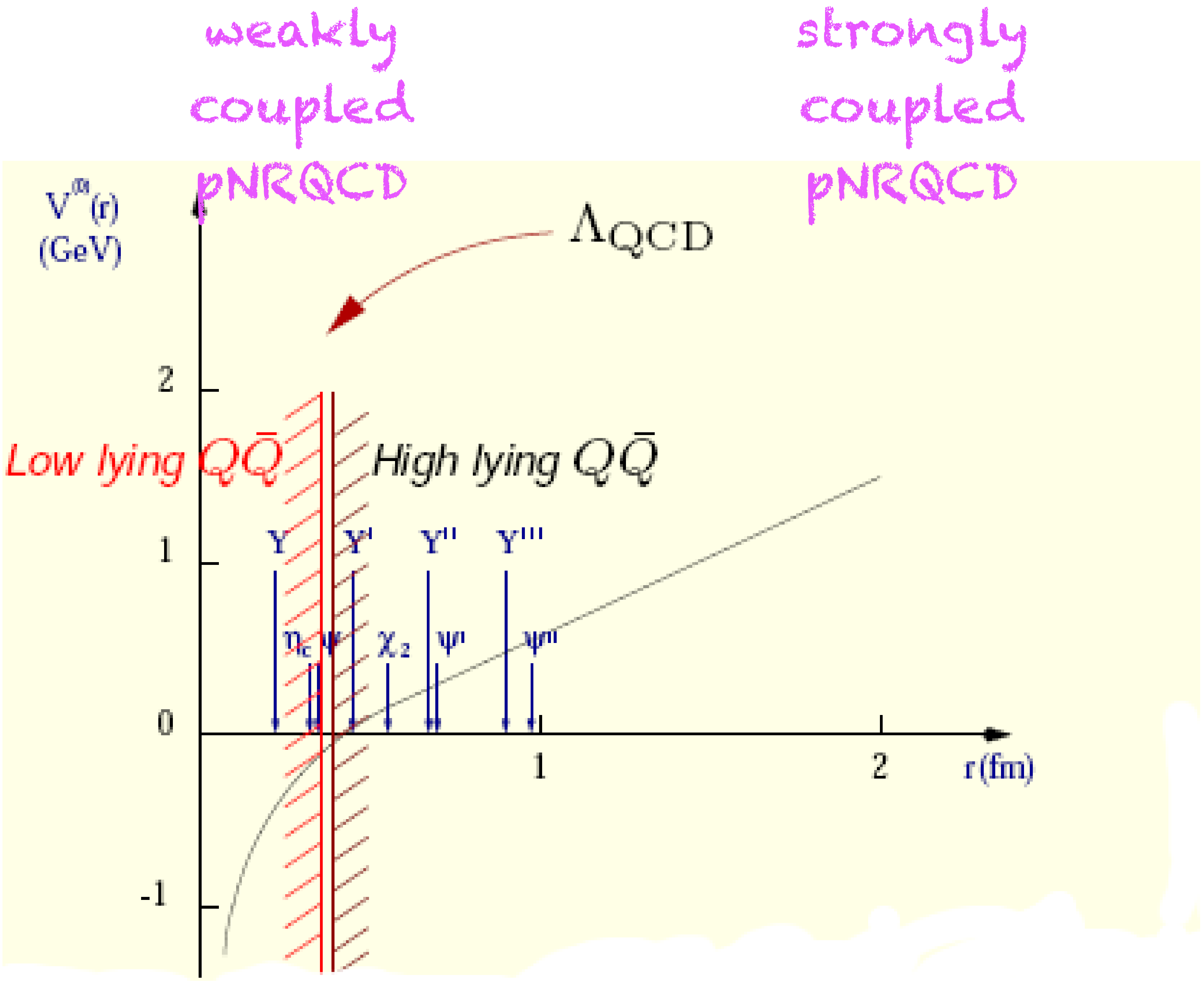
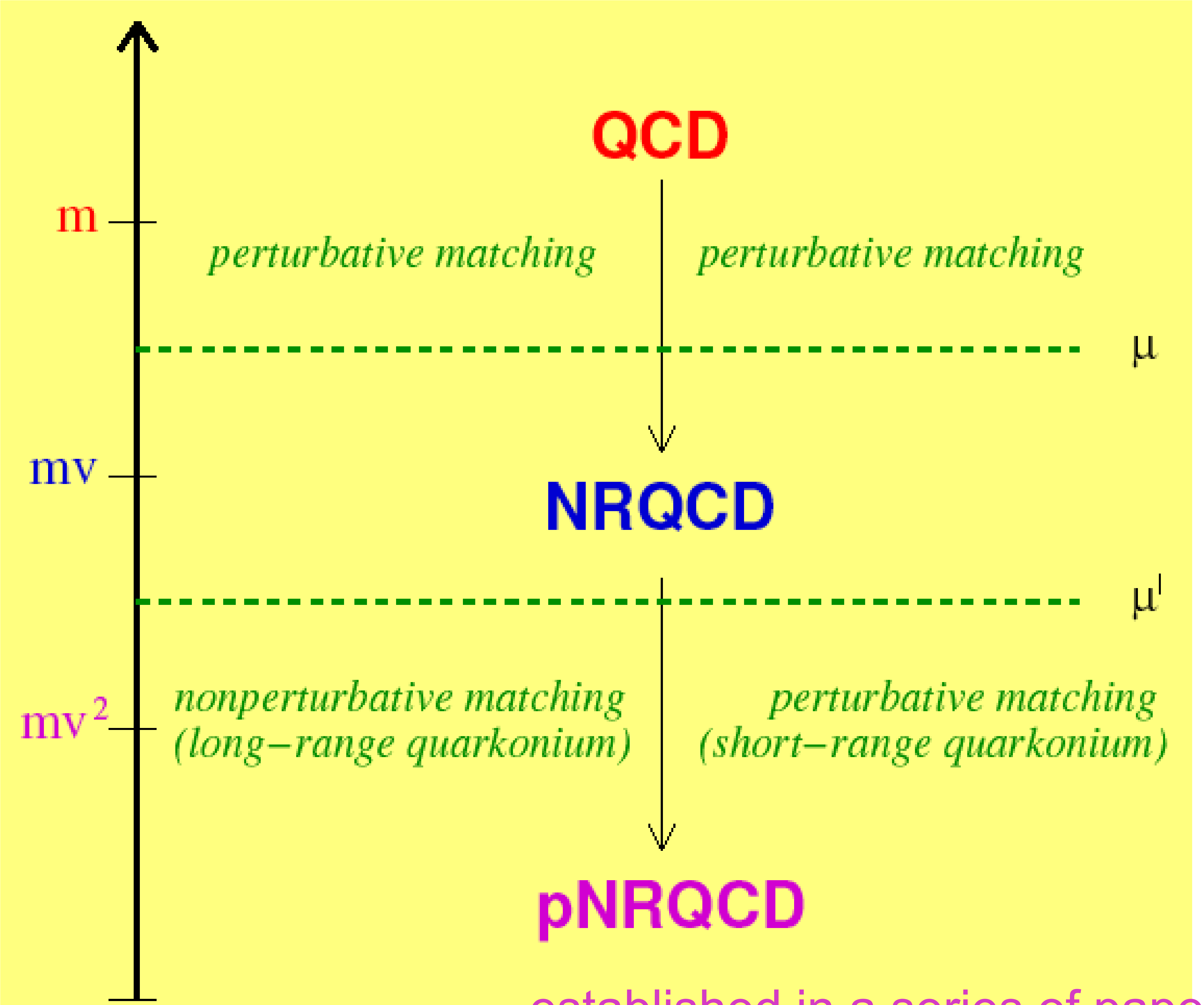


A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
- the potential is non-perturbative if $mv \sim \Lambda_{QCD}$

In QCD another scale is relevant Λ_{QCD}

Quarkonium with NREFT



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Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, et al. 00–024
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

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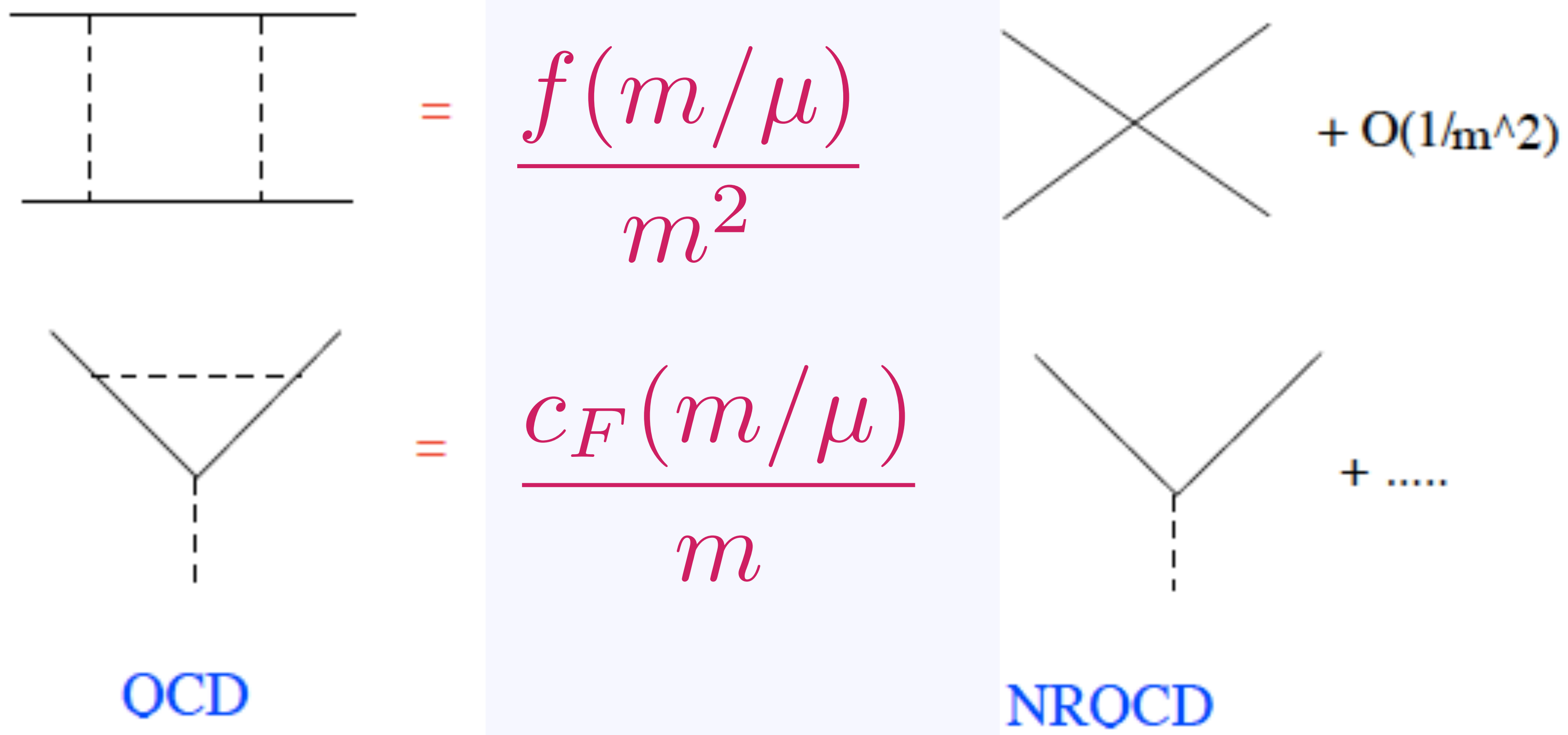
example QCD—> NRQCD, integrate out m

Manohar 97

$$QCD \qquad \int d^4q f(q,m,|\mathbf{p}|,E) = \int d^4q f(q,m,0,0) + \mathcal{O}\left(\frac{E}{m},\frac{|\mathbf{p}|}{m}\right) \sim C(\frac{\mu}{m})(\text{tree level})|_{NRQCD}$$

$$NRQCD \qquad \int d^4q f(q,|\mathbf{p}|,E) = \int d^4q f(q,0,0) = 0!$$

here p is an external momentum smaller than the cutoff



$$= \frac{f(m/\mu)}{m^2}$$

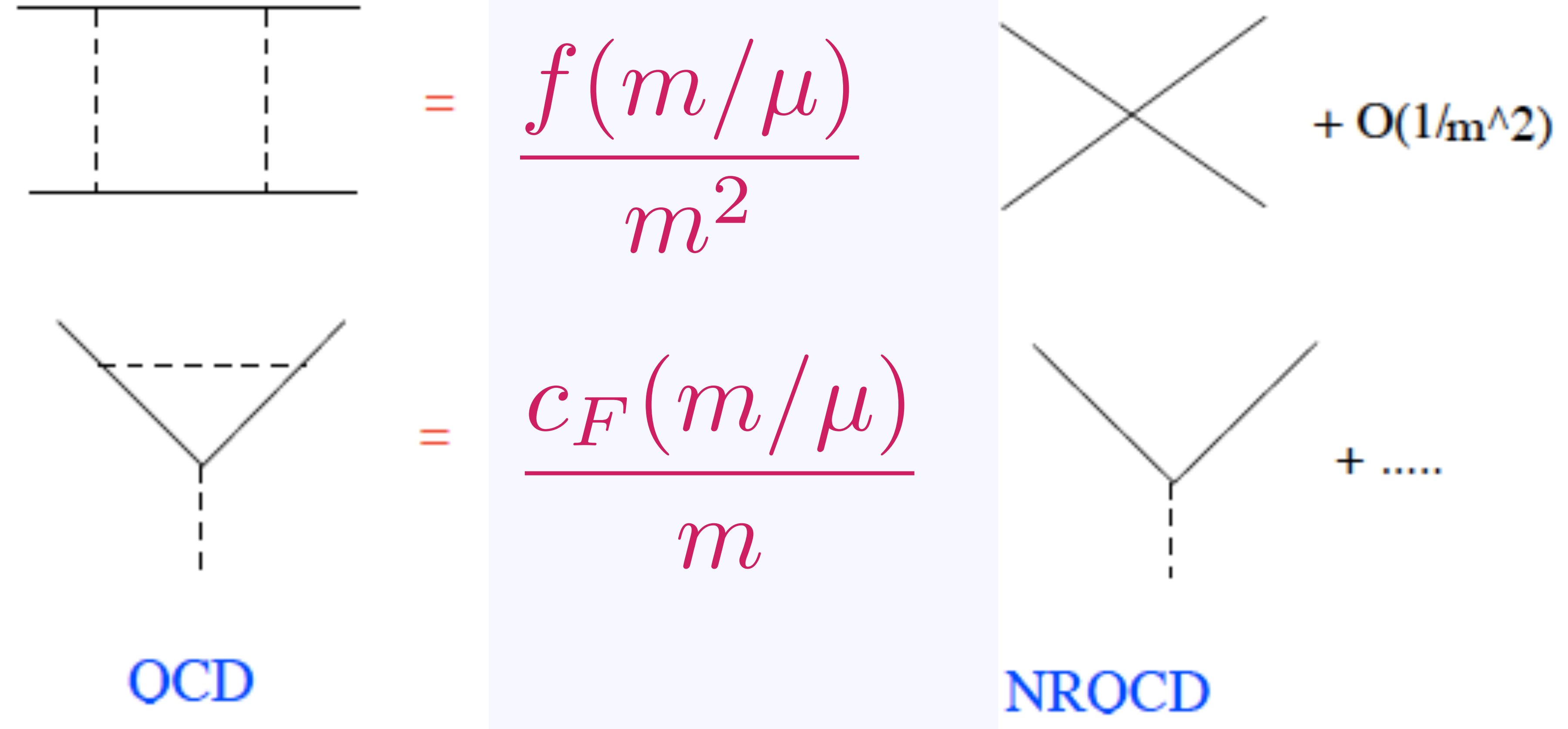
+ O(1/m^2)

$$= \frac{c_F(m/\mu)}{m}$$

+ .....

QCD

NRQCD



$$= \frac{f(m/\mu)}{m^2} \qquad + O(1/m^2)$$

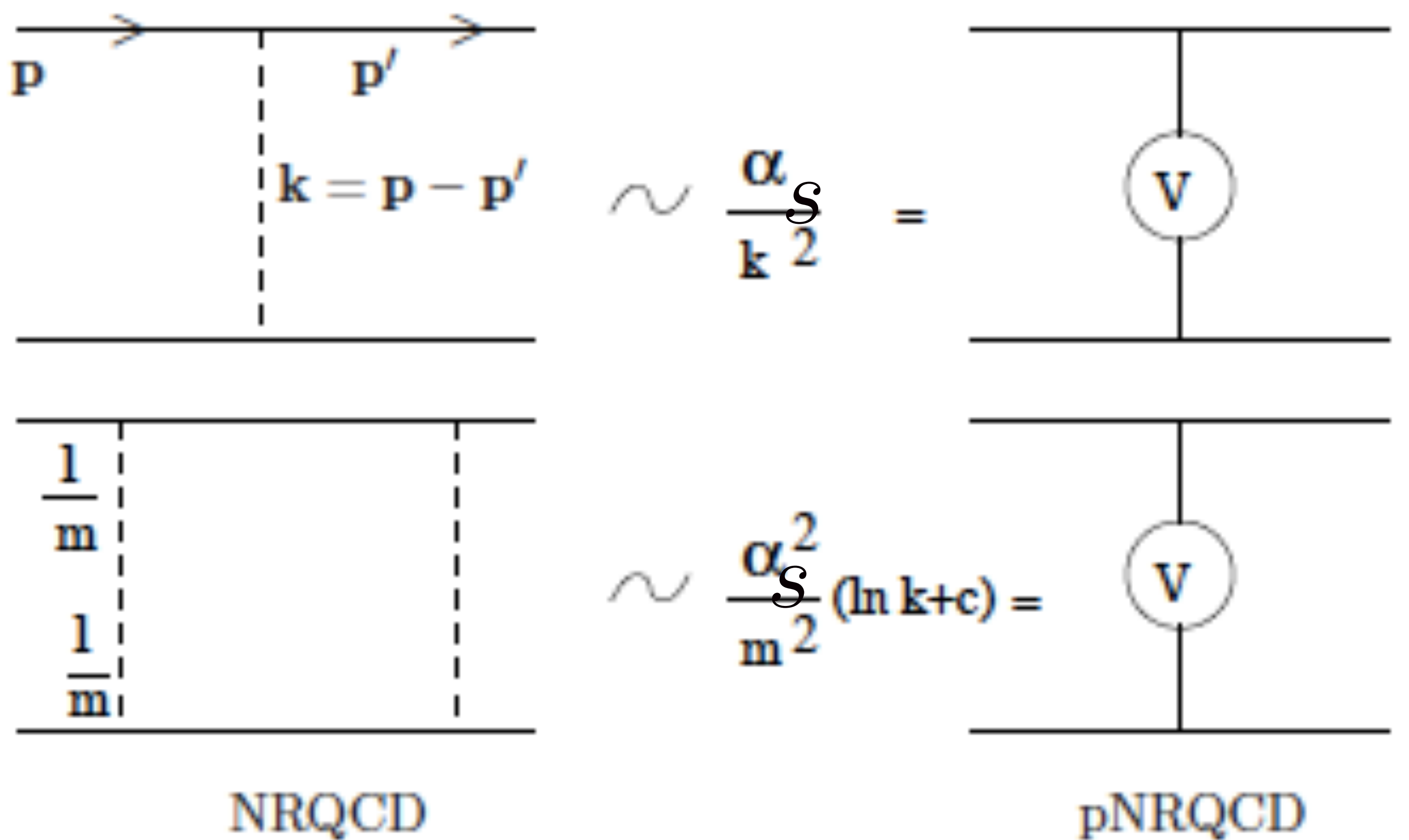
$$= \frac{c_F(m/\mu)}{m} \qquad + ...$$

Example: NRQCD—> pNRQCD, integrate out k (transfer momentum between quark and antiquark)

$$NRQCD \qquad \int d^4q f(q,k,|\mathbf{p}|,E) = \int d^4q f(q,k,0,0) + \mathcal{O}\left(\frac{E}{k},\frac{|\mathbf{p}|}{k}\right) \sim V(k)$$

(tree level pNRQCD)

$$pNRQCD \qquad \int d^4q f(q,|\mathbf{p}|,E) = \int d^4q f(q,0,0) = 0$$



$$\sim \frac{\alpha_s}{k^2} =$$

$$\sim \frac{\alpha_s^2}{m^2}(\ln k+c) =$$

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they are gauge invariant and scheme dependent—->
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-if $\Lambda_{QCD} \sim mv = k$

the matching is nonperturbative and the potentials are given in terms of gauge invariant generalised Wilson loops to be evaluated on the lattice

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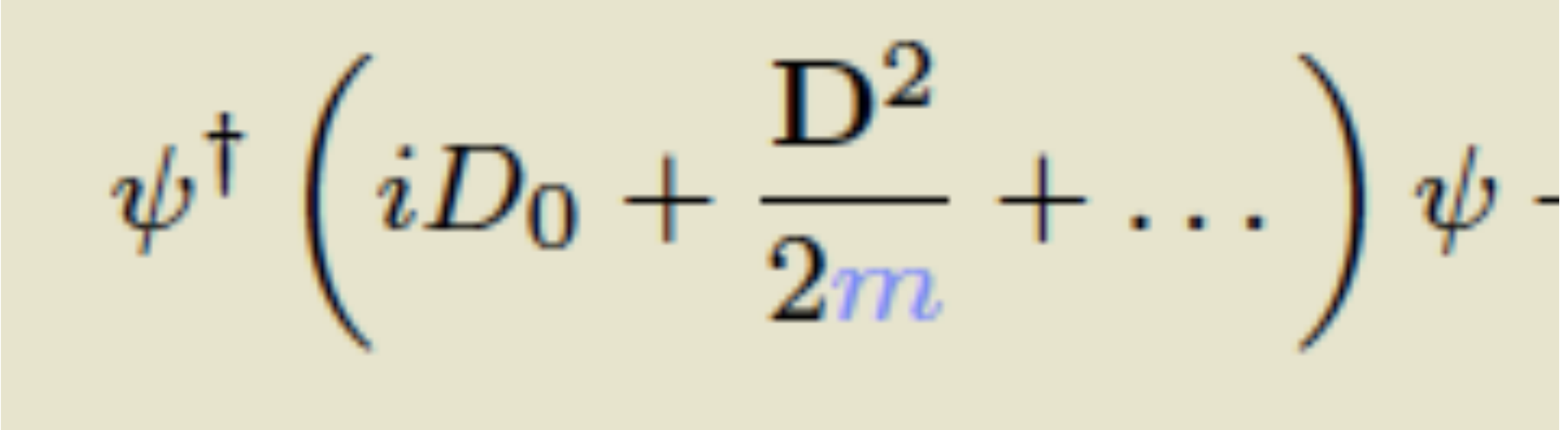
The propagators of the NREFT
have a noncovariant structure—>
we have created a dedicated automatic
symbolic calculation program to address
these calculations (tree, 1 loop) see
'FeynOnium' 2006.15451
N.B., H. S. Chung, V. Shtabovenko, A. Vairo

NRQCD/HQET

HQET and NRQCD is obtained by integrating out modes associated with the scale m

First two terms:



$$\psi^\dagger\left(iD_0+\frac{\mathbf{D}^2}{2m}+\ldots\right)\psi$$

$\psi = \mathrm{Pauli\,spinor}$

The power counting, the zero order problem and the propagator depends on the physics

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Heavy-light mesons, next scale after m: Lambda_QCD——> HQET

$D_0 \sim \Lambda_{QCD}$ $\frac{\mathbf{D}^2}{2m} \sim \frac{\Lambda^2_{QCD}}{m}$

propagator $i/(k^0 + i\epsilon)$,

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Quarkonium, next scale after m: mv——> NRQCD

$D_0 \sim mv^2$ $\frac{\mathbf{D}^2}{2m} \sim mv^2$ propagator $\frac{i}{(k^0-\mathbf{k}^2/2m+i\epsilon)}$.

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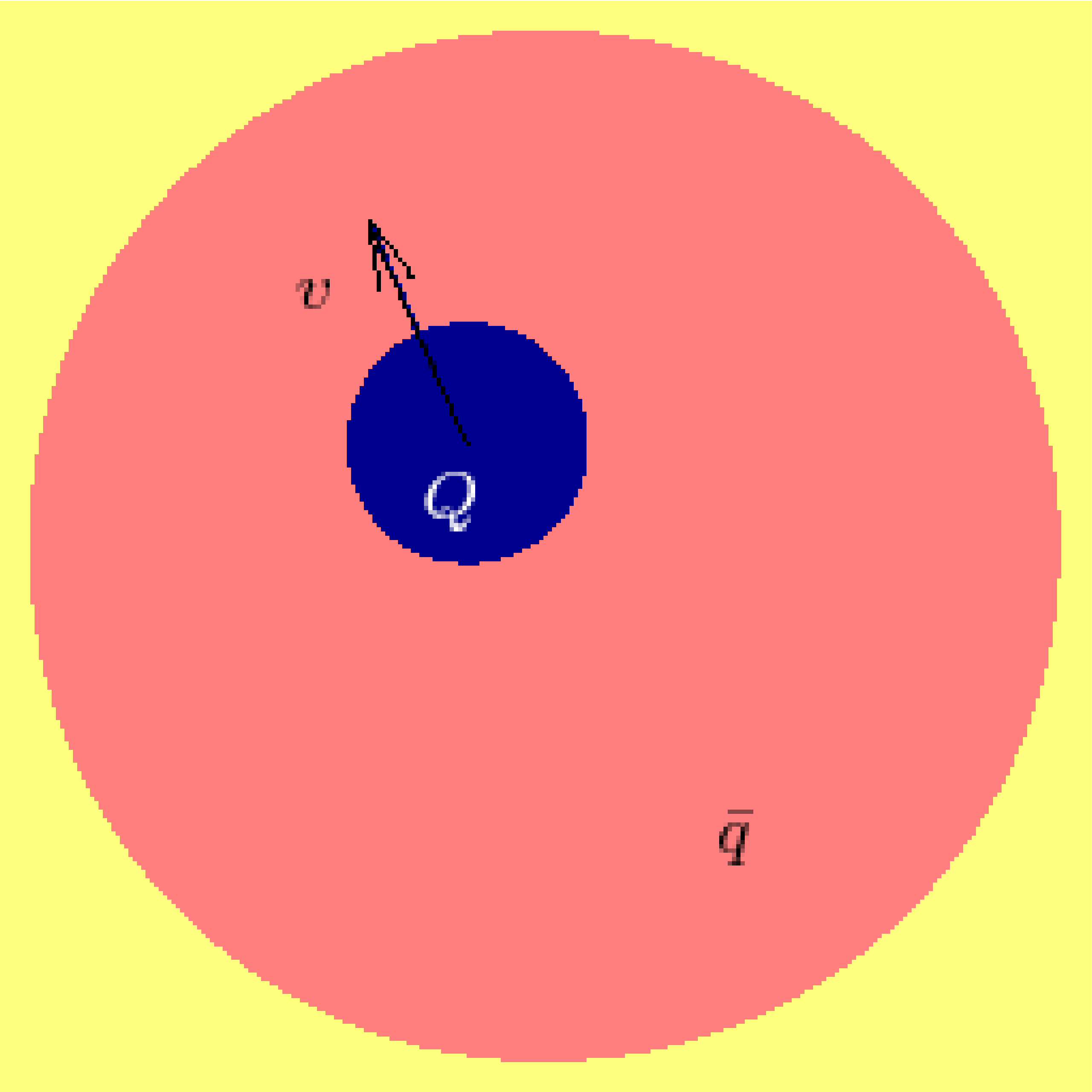
propagator $\frac{i}{(k^0-\mathbf{k}^2/2m+i\epsilon)}.$

the fermion bilinear part is the same in NRQCD and HQET but the physics and the power counting are different

Heavy-light system: Heavy Quark EFT

$Q\bar{q}$ scales and HQET



The mass scale is perturbative:

$m \gg \Lambda_{\rm QCD}$

The system is characterized by the scales:

$m \gg 1/r \sim \Lambda_{\rm QCD}$

At scales lower than m the symmetries hidden in QCD become manifest:

$SU(2)_s \times SU(2)_f$

$$\mathcal{L} = Q^\dagger \left(iD_0\right) Q + Q^\dagger \left(\frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + i c_S \frac{\mathbf{S}\cdot[\mathbf{D}\times, \mathbf{gE}]}{4m^2} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \dots \right) Q$$

--Can you understand why the heavy-light case
and the leading HQET display a
SU(2)_spinX SU(2)_flavor symmetry?
Is this symmetry realized in the heavy-light
mesons? have a look at PDG and find it out!
Is this symmetry already there in QCD or is it an
additional symmetry? Apart from the antiquark
term and the four fermion terms this Lagrangian
is equal to the NRQCD Lagrangian.
However the physics of QQbar and of Qq system
is very different....so, how can it be?
(the power counting is different)

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \dots \right) \psi$$
$$+ \chi^\dagger \left(\qquad \dots \qquad \right) \chi$$
$$+ \sum_K \frac{f}{m^2} \psi^\dagger K \chi \chi^\dagger K \psi + \cdots$$
$$- \frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i \not{D} q + \dots$$

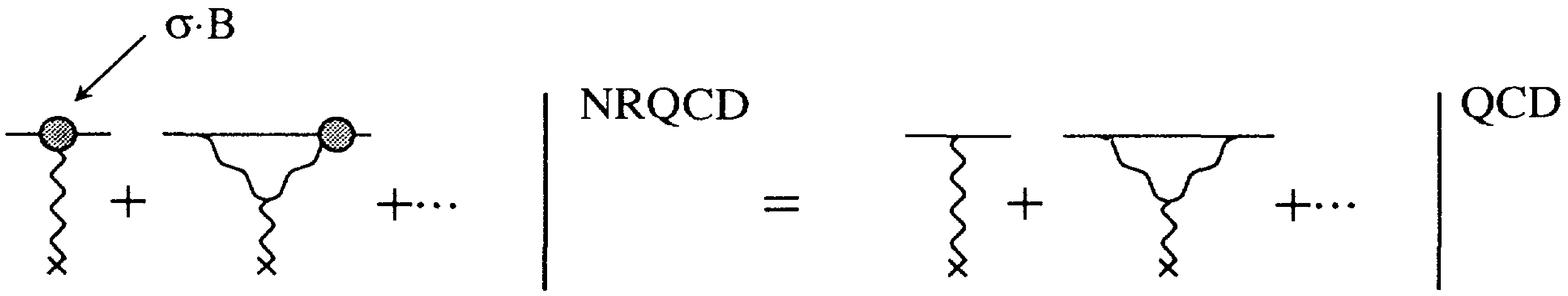
Caswell Lepage 86, Bodwin Braaten Lepage 95, Manohar 97

ψ (χ) is the field that annihilates (creates) the (anti)fermion.

- The relevant dynamical scales of NRQCD are: mv, mv^2, ...
- Low-energy scales may be set to zero while matching.

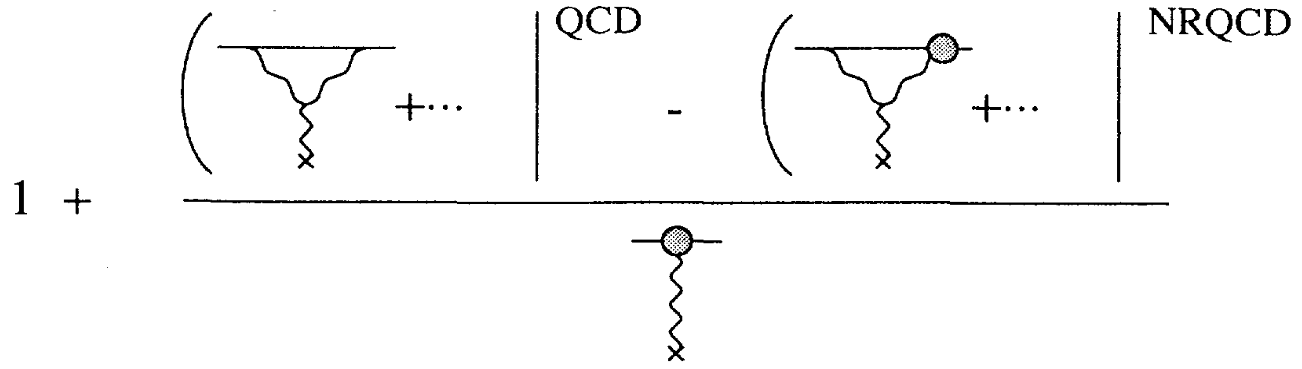
-- is this Lagrangian the same
that you would get with a nonrelativistic
expansion of the QCD Lagrangian? what is the
difference? and what is the origin of such
difference?

Matching calculation : c_F



amplitude for spin flip scattering calculated in QCD and in NRQCD

c_F=



extraction of c_F

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \dots \right) \psi$$

$$1 + (\cdots)\,\alpha_{\rm s} + \dots$$

$$f = \mathrm{Re}f + i\mathrm{Im}f$$

$$+ \sum_K \frac{f}{m^2} \psi^\dagger\, K\, \chi \chi^\dagger\, K\, \psi + \cdots$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i\not{D}\, q + \dots$$

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* Counting in $\alpha_s(m)$:

$$c_F = 1 - \frac{C_A}{2} \frac{\alpha_s}{\pi} \log\frac{m}{\mu} + \dots \qquad c_S = 2c_F - 1$$

$$c_D = 1 + \left(\frac{2}{3} C_A + \frac{8}{3} C_F \right) \frac{\alpha_s}{\pi} \log\frac{m}{\mu} + \dots$$

$$f = \mathcal{O}(\alpha_s) \qquad \mathrm{Im} f = \mathcal{O}(\alpha_s^2)$$

* Counting in v: *The power counting is not unique.*

1) $\int d^3\mathbf{x}\, \psi^\dagger \psi \simeq 1 \Rightarrow |\psi|^2 \sim \frac{1}{(\Delta x)^3} \sim m^3 v^3$

2) $K^{(d)} \sim (mv)^d$ (e.g. $g\mathbf{E}, g\mathbf{B} \sim m^2 v^2$, $\mathbf{D} \sim mv$)

3) $D_0 \sim mv^2$ (virial theorem)

NRQCD

$$\mathcal{L} = \psi^\dagger \left(iD_0 + \frac{\mathbf{D}^2}{2m} + c_F \frac{\mathbf{S}\cdot g\mathbf{B}}{m} + c_D \frac{[\mathbf{D}\cdot, g\mathbf{E}]}{8m^2} + \dots \right) \psi$$

$$1 + (\cdots)\,\alpha_s + \dots$$

$$f = \mathrm{Re} f + i \mathrm{Im} f$$

$$+ \sum_K \frac{f}{m^2} \psi^\dagger\, K\, \chi\chi^\dagger\, K\, \psi + \cdots$$

$$-\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum^{n_f} \bar{q}\, i\not{D}\, q + \dots$$

* Counting in $\alpha_s(m)$:

$$c_F = 1 - \frac{C_A}{2}\frac{\alpha_s}{\pi} \log\frac{m}{\mu} + \dots \qquad c_S = 2c_F - 1$$

$$c_D = 1 + \left(\frac{2}{3}C_A + \frac{8}{3}C_F\right)\frac{\alpha_s}{\pi}\log\frac{m}{\mu} + \dots$$

$$f = \mathcal{O}(\alpha_s) \qquad \mathrm{Im} f = \mathcal{O}(\alpha_s^2)$$

The power counting is not unique.

E.g. in Lepage et al. 92 ("standard NRQCD power counting"):

$g A_0 \sim m v^2$, $g\mathbf{A} \sim m v^3$, $g\mathbf{E} \sim m^2 v^3$, $g\mathbf{B} \sim m^2 v^4$.

* Counting in v: *The power counting is not unique.*

1) $\int d^3\mathbf{x}\, \psi^\dagger\psi \simeq 1 \Rightarrow |\psi|^2 \sim \frac{1}{(\Delta x)^3} \sim m^3 v^3$

2) $K^{(d)} \sim (mv)^d$ (e.g. $g\mathbf{E}, g\mathbf{B} \sim m^2 v^2$, $\mathbf{D} \sim mv$)

3) $D_0 \sim m v^2$ (virial theorem)

Octets

$$|H\rangle = \left(|(Q\bar{Q})_1\rangle + |(Q\bar{Q})_8 g\rangle + \cdots\right) \otimes |nljs\rangle$$

$$\mathcal{O}(1) \qquad \mathcal{O}(v)$$

$$\psi^\dagger K^{(n)} \chi \chi^\dagger K'^{(n)} \psi = \begin{cases} O_1({}^{2S+1}L_J) \\ O_8({}^{2S+1}L_J) \end{cases}$$

indicate the
angular momentum state
of the QQ pair that is
created or anihilated by the
operator

$$\psi^\dagger T^a \chi \chi^\dagger T^a \psi = O_8({}^1S_0) \qquad \psi^\dagger \mathbf{D} \chi \chi^\dagger \mathbf{D} \psi = O_1({}^1P_1) \qquad \ldots$$

$$\mathcal{O}(1) \qquad \mathcal{O}(v^2)$$

once you have constructed the EFT, established the power counting and calculated the matching coefficients you can start applications to calculations of spectra, decays, production..

NRQCD is used in lattice calculations: spectroscopy, decays

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NRQCD is used to study quarkonium production at LHC and B factories

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pNRQCD

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with respect to $$\mathcal{L}_{\rm pNREFT} = \int d^3r \phi^\dagger (i\partial_0 - \frac{\mathbf{p}^2}{m} - V)\phi + \Delta\mathcal{L}$$

- QFT = QCD
- It is obtained by integrating out hard and soft gluons with p or E scaling like m, mv.
- The d.o.f. are $Q\bar{Q}$ pairs (sometimes cast in color singlet S and color octet O) and ultrasoft modes (e.g. light quarks, low-energy gluons):
  $\phi = S$
- The Lagrangian is organized as an expansion in $1/m$ and r.
- The form of $\Delta\mathcal{L}$ and of the ultrasoft modes depends on the low energy dynamics.
- The power counting is
  $\rightarrow p \sim 1/r \sim mv$ (soft scale),
  $\rightarrow E \sim \mathbf{p}^2/2m \sim V^{(0)} \sim \mathbf{P}_{\rm cm} \sim 1/\mathbf{R}_{\rm cm} \sim mv^2$ (ultrasoft scale),
  $\rightarrow$ operators in $\Delta\mathcal{L}$ scale like $(mv^2)^{\rm dimension}$.

○ Brambilla Pineda Soto Vairo RMP 77 (2005) 1423

○ Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275

Weakly coupled pNRQCD

○ Pineda Soto NP PS 64 (1998) 428
Brambilla Pineda Soto Vairo NPB 566 (2000) 275

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

  Non-analytic behaviour in $r \to$ matching coefficients V

The gauge fields are multipole expanded:
$A(R,r,t) = A(R,t) + \mathbf{r}\cdot\boldsymbol{\nabla}A(R,t) + \dots$

$\mathbf{R}$ = center of mass
$\mathbf{r}$ = $Q\bar{Q}$ distance

$$\mathcal{L}^{\rm pNRQCD} = \int d^3r\, {\rm Tr}\,\{S^\dagger(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_S + \cdots)S + O^\dagger(iD_0 - \frac{\mathbf{p}^2}{m} - V_O + \cdots)O +$$

LO in r

$$+V_A(S^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger \mathbf{r}\cdot g\mathbf{E}S) + \frac{V_B}{2}(O^\dagger \mathbf{r}\cdot g\mathbf{E}O + O^\dagger O\mathbf{r}\cdot g\mathbf{E})\} + \dots$$

NLO in r

$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\, a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

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NLO in r

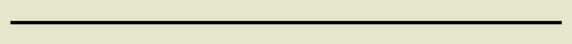
$$-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\, i\not{D} q_i$$

The matching coefficients are the Coulomb potential

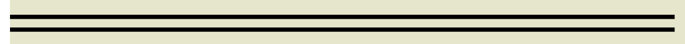
$$V_S(r) = -C_F\frac{\alpha_s}{r} + \dots, \qquad V_o(r) = \frac{1}{2N}\frac{\alpha_s}{r} + \dots,$$

$$V_A = 1 + \mathcal{O}(\alpha_s^2),\ V_B = 1 + \mathcal{O}(\alpha_s^2).$$

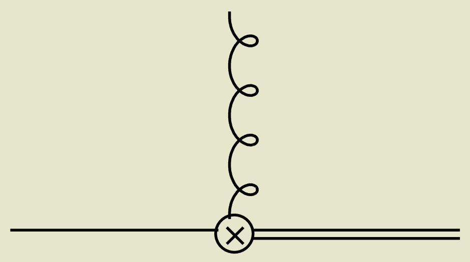
Feynman rules



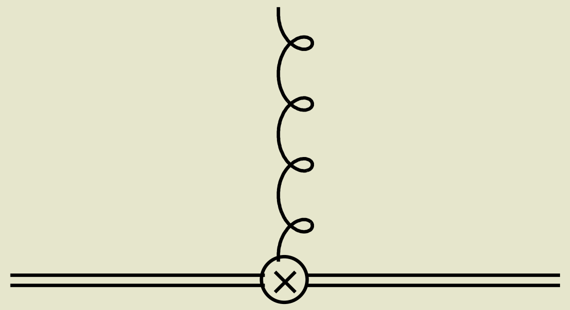
$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V)}$



$= \theta(t)\, e^{-it(\mathbf{p}^2/m+V_o)}\left(e^{-i\int dt\, A^{\rm adj}}\right)$



$= \mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\,\mathrm{S}$



$= \mathrm{O}^\dagger\{\mathbf{r}\cdot g\mathbf{E},\, \mathrm{O}\}$

Matching the potential

○ Brambilla Pineda Soto Vairo PRD 60 (1999) 091502

$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \dots$$

- The potential is a matching coefficient of the EFT that may be computed from first principle by matching Green's functions in QCD with Green's function in pNRQCD, it is scheme and scale dependent, and undergoes renormalization.
  It may be organized as an expansion in $1/m$:
- The interaction terms contained in $\Delta\mathcal{L}$ provide corrections to the quantum mechanical picture.

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- The static potential:

  in weakly coupled pNRQCD

NRQCD static energy E_0

$$V^{(0)}(r) \;=\; \lim_{T\to\infty}\frac{i}{T}\ln\langle \boxed{} \rangle - \Delta\mathcal{L}\,\text{effects}; \qquad \boxed{} = \exp\left\{ig\oint_{r\times T} dz^\mu A_\mu\right\}$$

Wilson loops (as matching Green's functions) guarantee gauge invariance.

Perturbation theory describes $E_0(r)$ in the short range ($r\Lambda \ll 1$, $\alpha_s(1/r) < 1$):

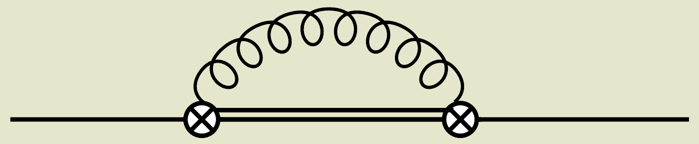
$$E_0(r) = \Lambda_s - \frac{C_F\alpha_s}{r}(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3\ln\alpha_s + \#\alpha_s^4\ln^2\alpha_s + \#\alpha_s^4\ln\alpha_s + \dots)$$

- $E_0(r)$ is known at three loops.
- $\ln\alpha_s$ signals the cancellation of contributions coming from different energy scales:

$$\ln\alpha_s = \ln\frac{\mu}{1/r} + \ln\frac{\alpha_s/r}{\mu}$$

○ Brambilla Pineda Soto Vairo PRD 60 (1999) 091502

$$V^{(0)}(r,\mu') = \lim_{T\to\infty}\frac{i}{T}\ln\left\langle \boxed{} \right\rangle -$$



$$+\cdots$$

$$= E_0(r) + \frac{i}{N}\int_0^\infty dt\, e^{-it(V_o-V)}\langle \mathrm{Tr}\,\mathbf{r}\cdot g\mathbf{E}(t)\,\mathbf{r}\cdot g\mathbf{E}(0)\rangle(\mu') + \cdots$$

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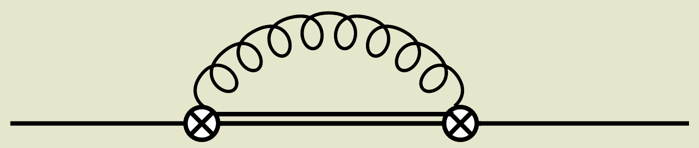

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The QCD static potential at N^4LO

$$\begin{aligned}
V^{(0)}(r,\mu') = & -C_F\frac{\alpha_s(1/r)}{r}\left\{1+\frac{\alpha_s(1/r)}{4\pi}a_1+\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 a_2\right. \\
& +\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3\left[\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu' + a_3\right] \\
& \left.+\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\left[a_4^{L2}\ln^2 r\mu' + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5+6\ln 2)\right)\ln r\mu' + \cdots\right]\right\}
\end{aligned}$$

a_1	Billoire 80
a_2	Schroeder 99, Peter 97
coeff $ln r\mu$	N.B. Pineda, Soto, Vairo 99
a_4^{L2}, a_4^L	N.B., Garcia, Soto, Vairo 06

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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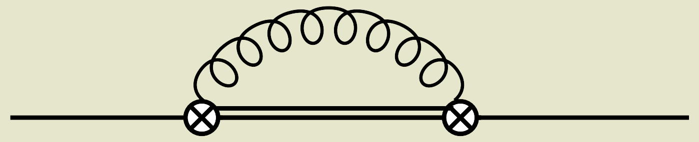

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at the logs level

3LOOPS REDUCES TO 1LOOP IN THE EFT

4LOOPS REDUCES TO 2LOOPS IN THE EFT

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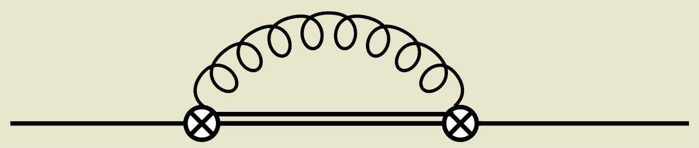

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Two problems:

1)Bad convergence of the series due to large beta_0 terms

2) Large logs

The eft cures both:

1) Renormalon subtracted scheme Beneke 98, Hoang, Lee 99, Pineda 01, N.B. Pineda Soto, Vairo 09

2) Renormalization group summation of the logs

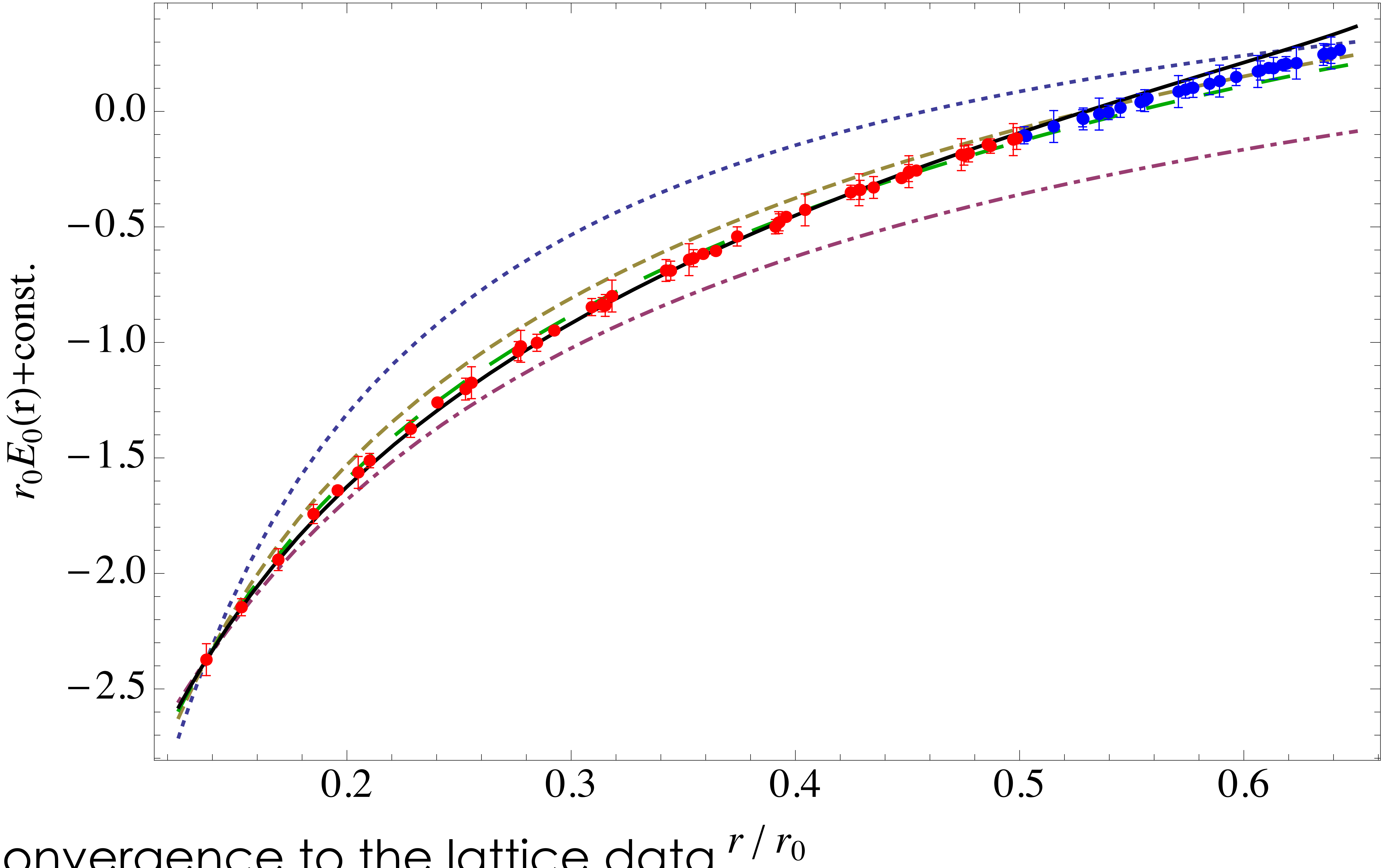
up to N^3LL $(\alpha_s^{4+n}\ln^n\alpha_s)$ N. B Garcia, Soto Vairo 2007, 2009, Pineda, Soto

α_s

from quarkonium

QQbar singlet static energy at NNNLL in pNRQCD in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014, with Weber 2019



Good convergence to the lattice data

Can exclude linear

nonperturbative corrections at short distance

α_s

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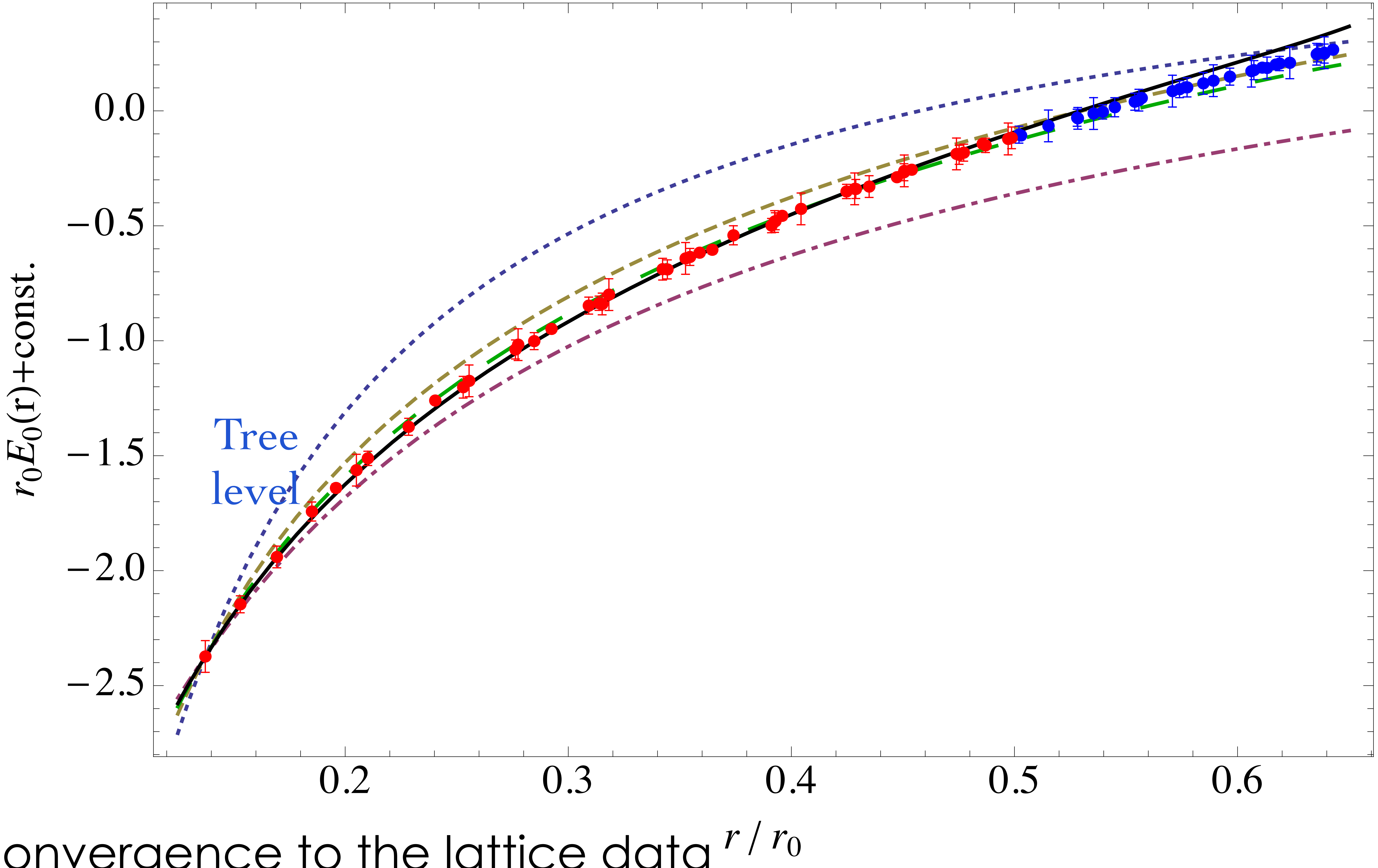

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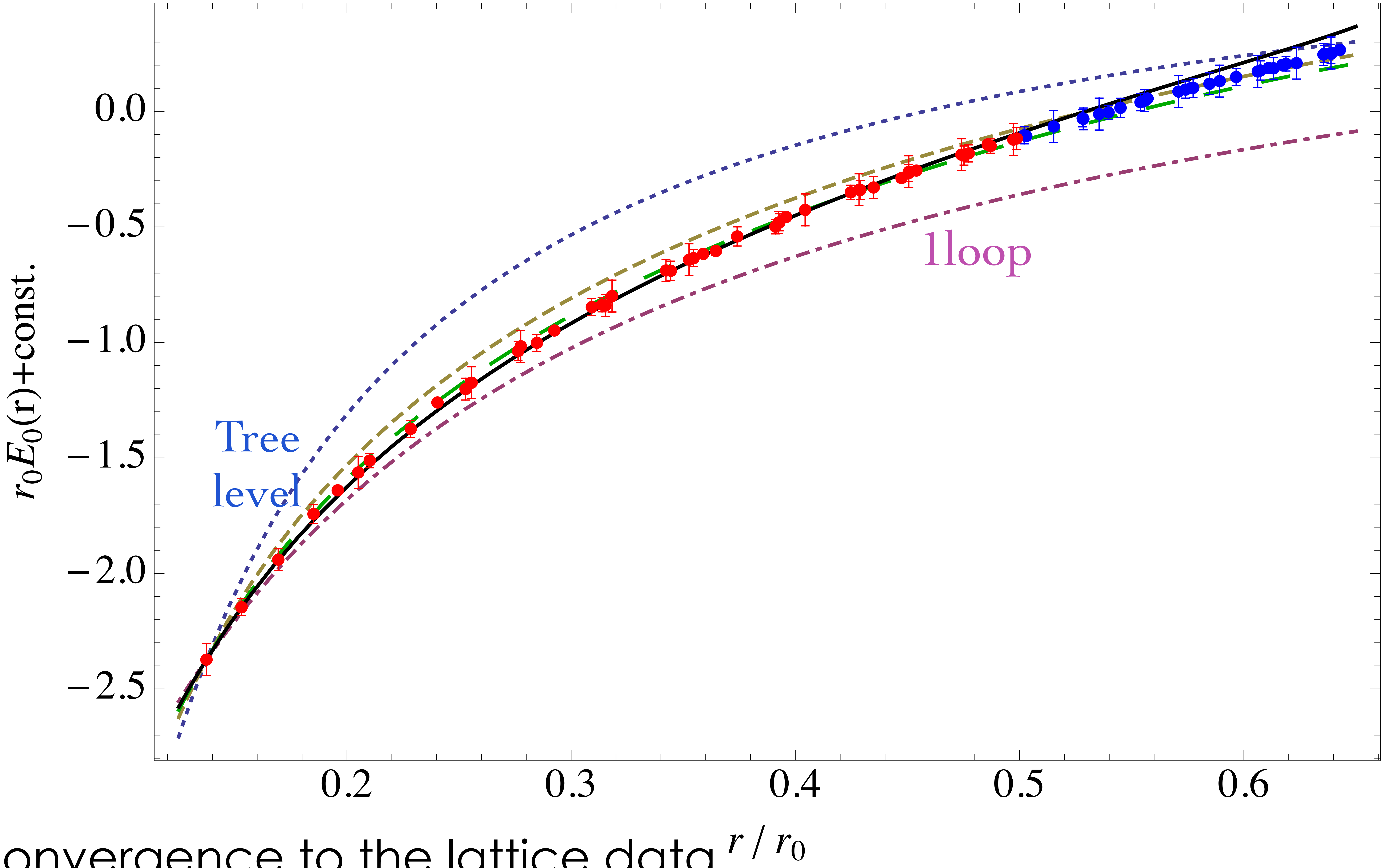

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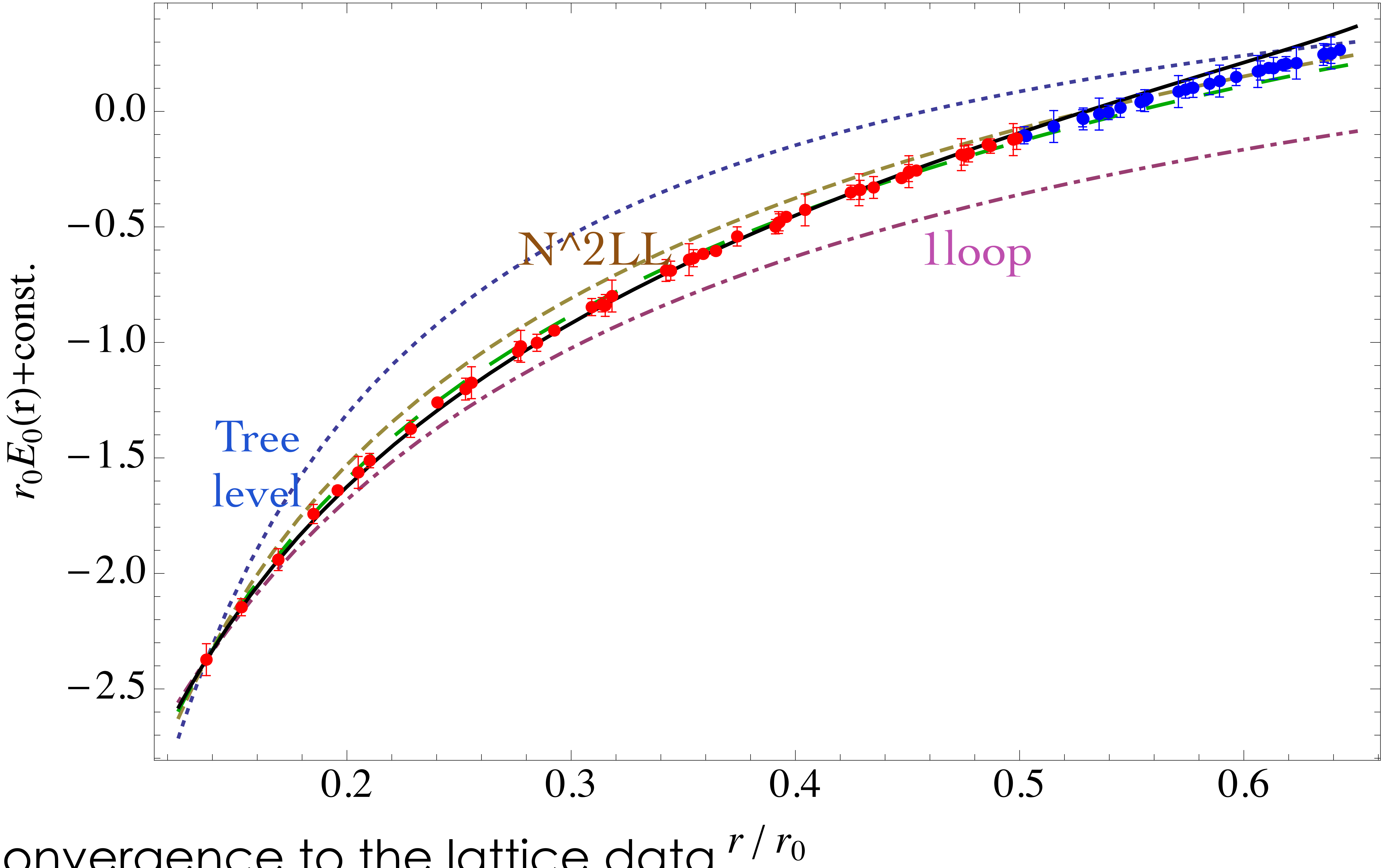

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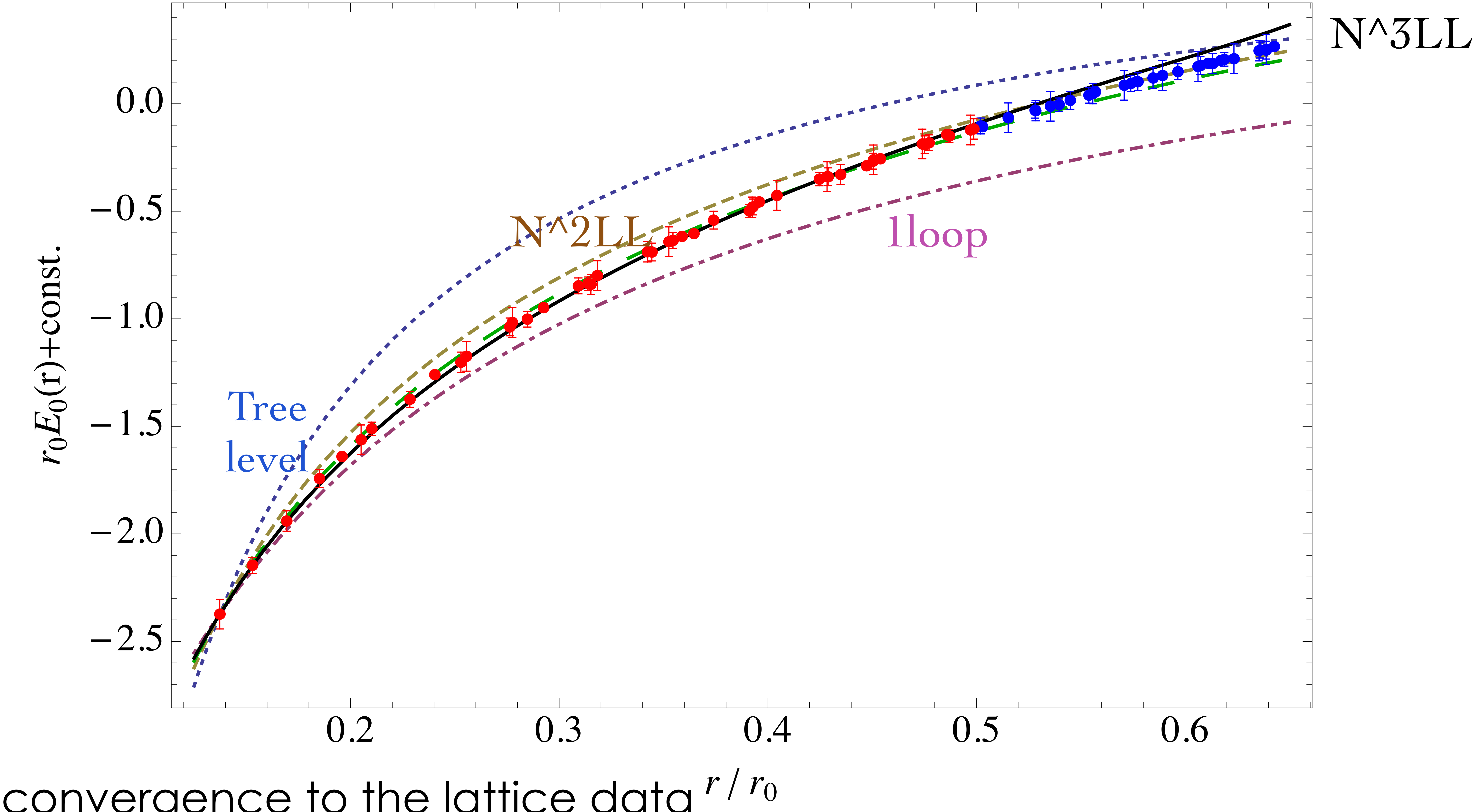

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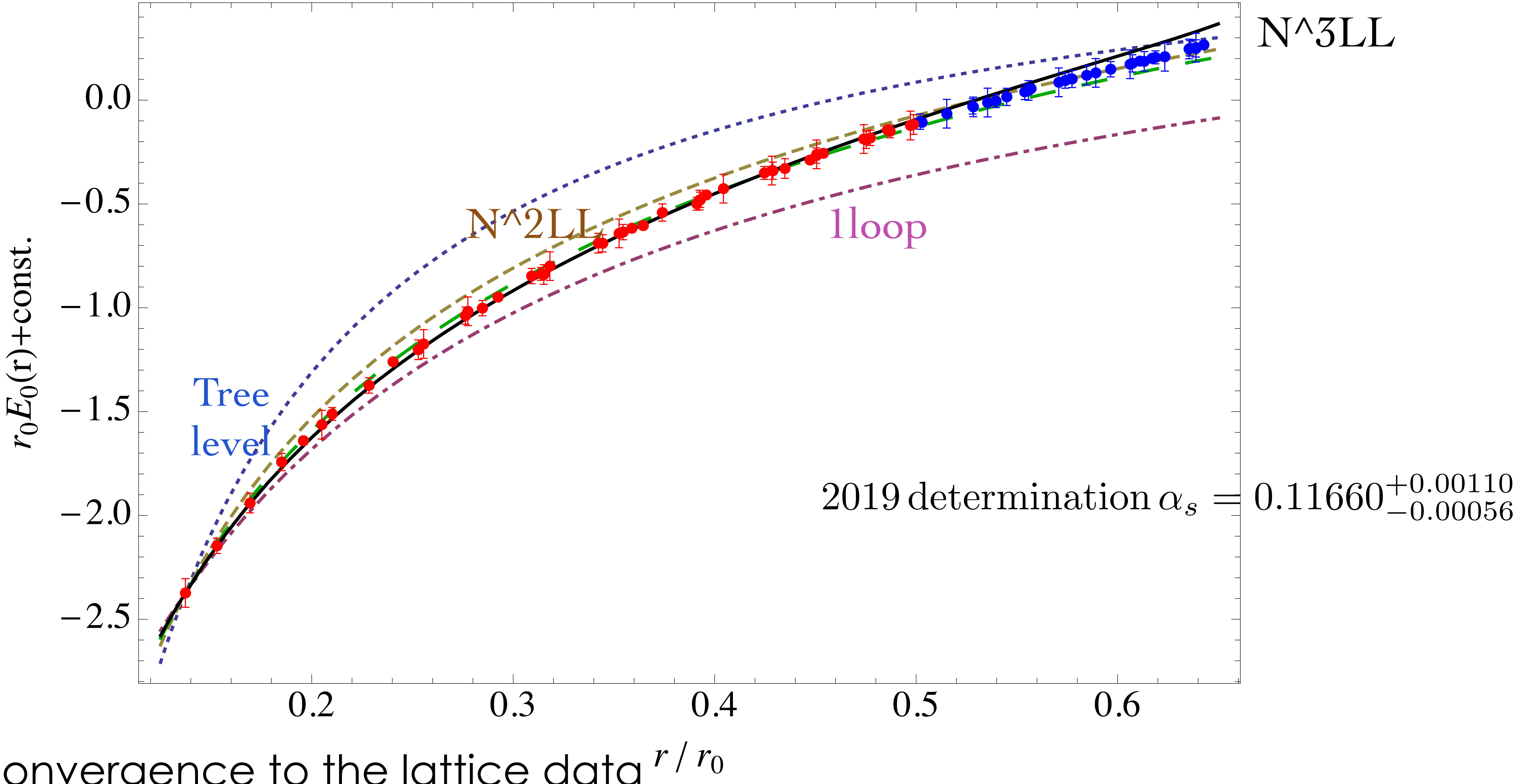

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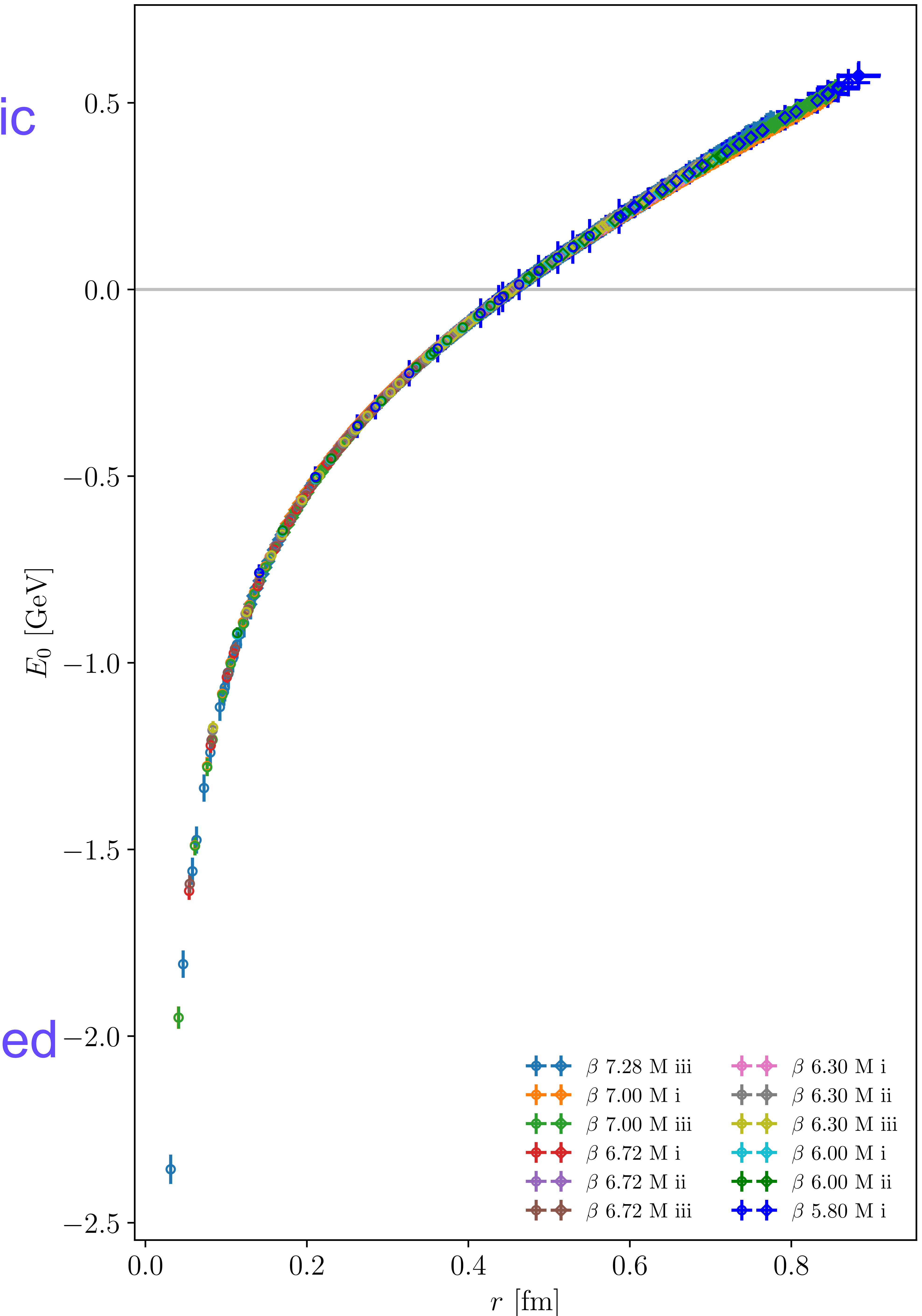
First lattice
calculation of the QCD static
energy with n_f=2+1+1,
I.e. with charm effects

Can be used to
extract alpha_s with
n_f=2+1+1

Finite charm mass effect
Should be implemented in
perturbation theory
Charm decoupling is observed



TUMQCD
N.B., Delgado, Kronfeld, Leino, Petreczky,
Steinbeisser, Vairo, Weber *Phys.Rev.D* 107
(2023) 7, 074503 •

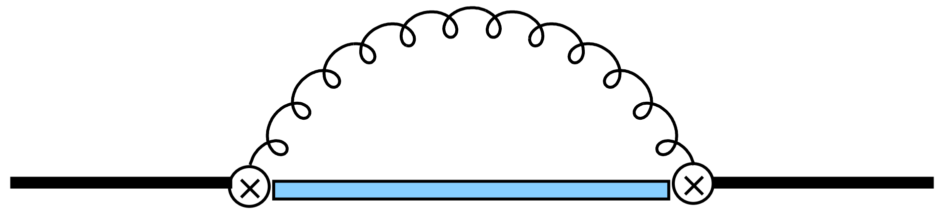
TUMQCD
Our lattice QCD collaboration with mission to
complement EFT methods and lattice QCD
to obtain strong interaction observables

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln\alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

NNNLL Peset Pineda et al 2018,2019, Kiyo Sumino 2014, Beneke, Kiyo Schuler 05,08

$$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$$



$$|n\rangle$$

$\sim e^{i\Lambda_{\mathrm{QCD}}t}$

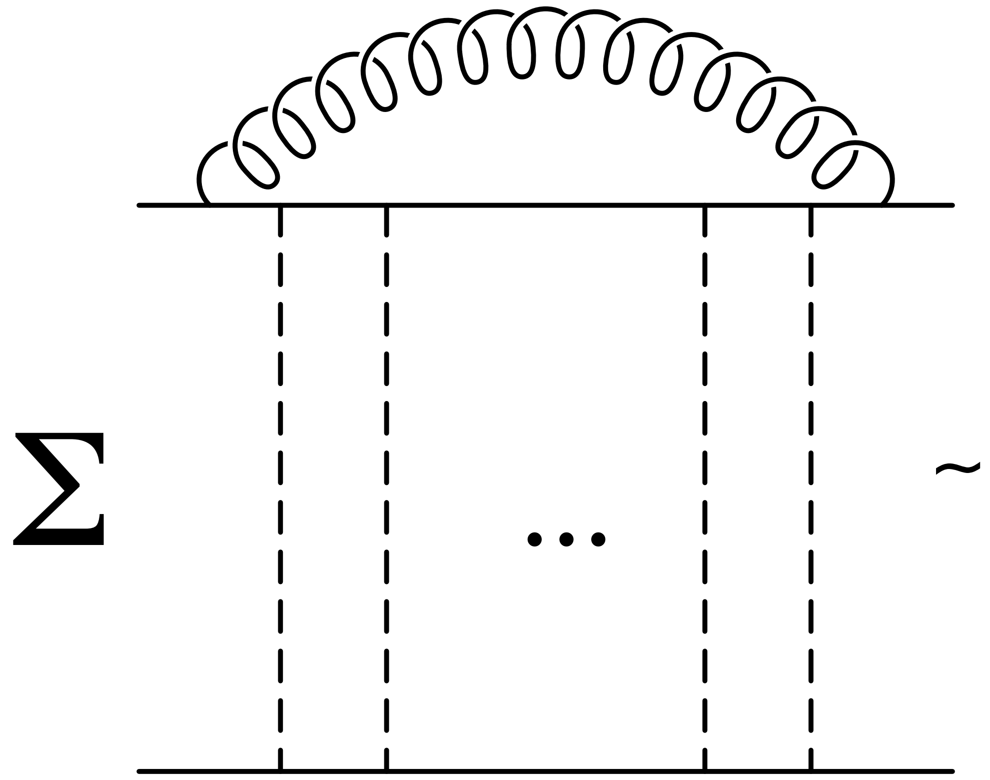
$$E_n = \langle n|H_s(\mu)|n\rangle - i\frac{g^2}{3N_c}\int_0^\infty dt\,\langle n|\mathbf{r}e^{it(E_n^{(0)}-H_o)}\mathbf{r}|n\rangle\,\langle\mathbf{E}(t)\,\mathbf{E}(0)\rangle(\mu)$$

$E_n^{(0)} - H_o \gg \Lambda_{\mathrm{QCD}} \Rightarrow \langle\mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle\mathbf{E}^2(0)\rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\mathrm{QCD}} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED



$\sim \alpha_s^5 \ln\alpha_s$ (Lamb-shift)

Energies at order m alpha^5 (NNNLO)

$m\alpha_s^5 \ln \alpha_s$ Brambilla Pineda Soto Vairo 99, Kniehl Penin 99
$m\alpha_s^5$ Kniehl Penin Smirnov Steinhauser 02 NNLL Pineda 02

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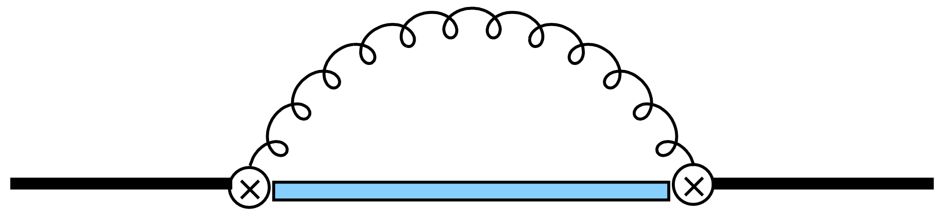
$E_n = 2m + \langle n|\frac{p^2}{m} + V_s|n\rangle + \langle n|$  $|n\rangle$

$\sim e^{i\Lambda_{\rm QCD}t}$

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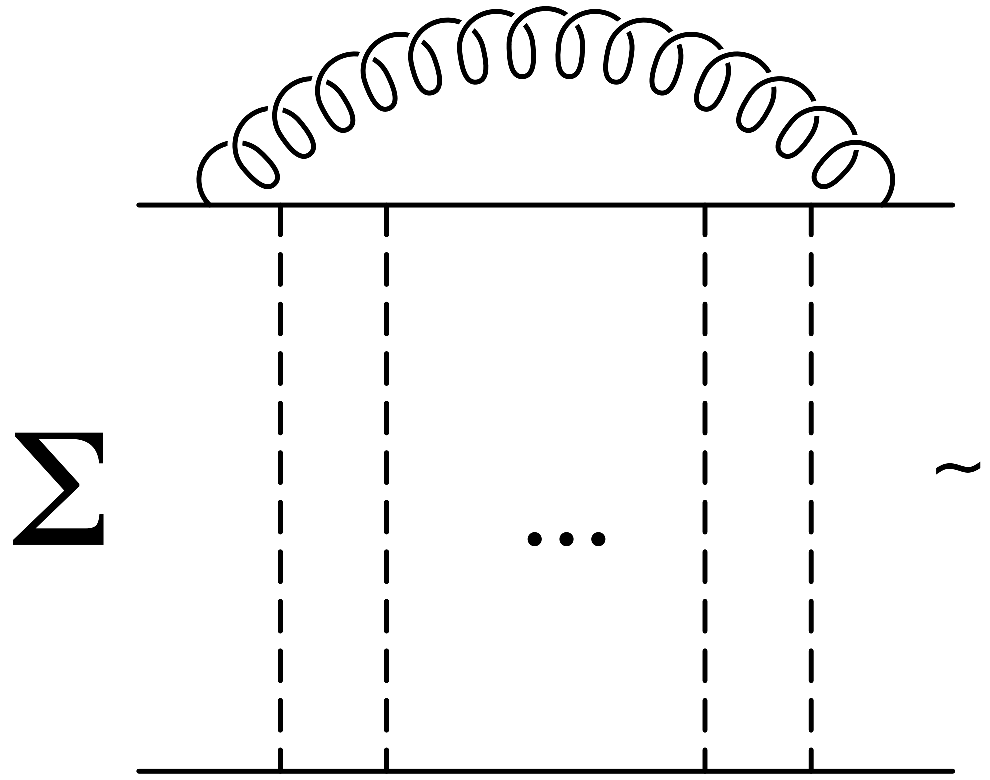
$E_n^{(0)} - H_o \gg \Lambda_{\rm QCD} \Rightarrow \langle \mathbf{E}(t)\mathbf{E}(0)\rangle(\mu) \to \langle \mathbf{E}^2(0)\rangle$

local condensates as predicted in a paper by Misha Voloshin in 1979

—>used to extract precise (NNNLO) determination of m_c and m_b

$E_n^{(0)} - H_o \sim \Lambda_{\rm QCD} \Rightarrow$ no expansion possible, non-local condensates, analogous to the Lamb shift in QED



Applications of weakly coupled pNRQCD include:
ttbar production, quarkonia spectra, decays, E1 and M1 transitions, QQq and QQQ energies, thermal masses and potentials

Applications to Quarkonium physics: systems with small radius

for references see the QWG doc

arXiv:1010.5827

- c and b masses at NNLO, N^{3}LO*, NNLL*;
- B_c mass at NNLO;
- B_c^*, η_c, η_b masses at NLL;
- Quarkonium $1P$ fine splittings at NLO;
- $\Upsilon(1S)$, η_b electromagnetic decays at NNLL;
- $\Upsilon(1S)$ and J/ψ radiative decays at NLO;
- $\Upsilon(1S) \to \gamma\eta_b$, $J/\psi \to \gamma\eta_c$ at NNLO;
- $t\bar{t}$ cross section at NNLL;
- QQq and QQQ baryons: potentials at NNLO, masses, hyperfine splitting, ... ;
- Thermal effects on quarkonium in medium: potential, masses (at $m\alpha_s^5$), widths, ...;

Penin et al 04

Kniehl et al 04

N. B. et al 010

The group of

Yu. Sumino, H.Takaura,

Yu. Kiyo calculates higher

order perturbative corrections

(N4LO)

and renormalon subtraction

$$\mathcal{B}(J/\psi \to \gamma\eta_c(1S)) = (1.6 \pm 1.1)\%$$

$$\mathcal{B}(\Upsilon(1S) \to \gamma\eta_b(1S)) = (2.85 \pm 0.30) \times 10^{-4}$$

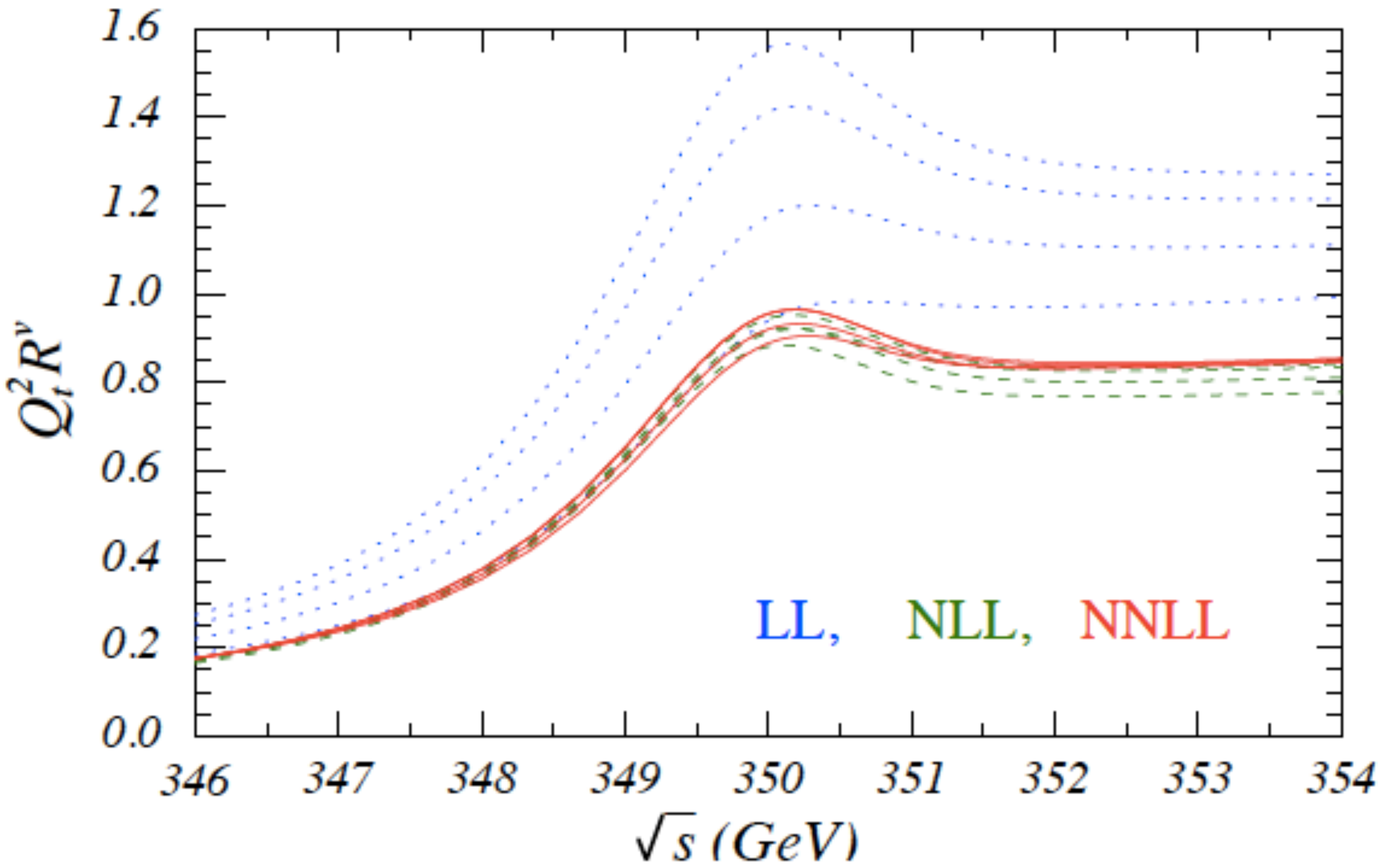
N. B. Yu Jia A. Vairo 2005

$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV} .$$

$$\Gamma(\eta_b(1S) \to \text{LH}) = 7\text{-}16 \text{ MeV}$$

Y. Kiyo, A. Pineda, A. Signer 2010

Threshold $t\bar{t}$ cross section



Hoang Teubner 01

Bound systems with a typical radius $\sim \Lambda_{\mathrm{QCD}}^{-1}$

Strongly coupled pNRQCD and Born Oppenheimer EFT

A nonperturbative problem: construct a pNREFT description on the basis of scale separations and symmetries

Strongly coupled pNRQCD and Born Oppenheimer EFT

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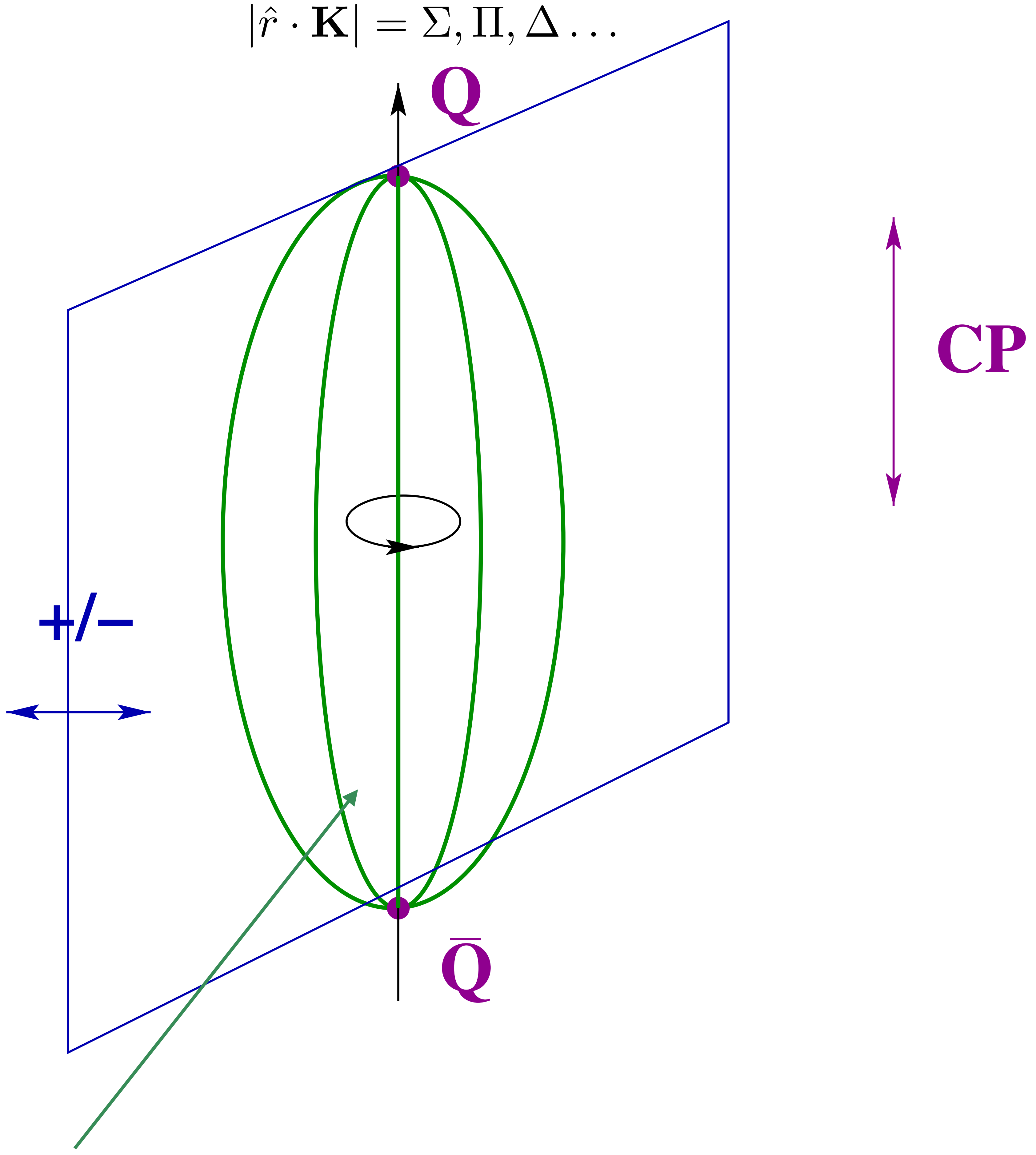
Two heavy quarks with large mass $m \gg \Lambda_{\rm QCD}$ and residual scale separation $\Lambda_{\rm QCD} \gg E$



Nonperturbative light degrees of freedom
glue and light quarks

produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$

Irreducible representations of $D_{\infty h}$

- $\boldsymbol{K}$: angular momentum of light d.o.f.
  $\lambda = \hat{\boldsymbol{r}} \cdot \boldsymbol{K} = 0, \pm 1, \pm 2, \pm 3, \ldots$
  $\Lambda = |\lambda| = 0, 1, 2, 3, \ldots \ (\Sigma,\ \Pi,\ \Delta,\ \Phi,\ \ldots)$
- Eigenvalue of CP: $\eta = +1\,(g),\ -1\,(u)$
- σ: eigenvalue of relfection about a plane containing $\hat{\boldsymbol{r}}$ (only for Σ states)

$$\Lambda_\eta^\sigma$$

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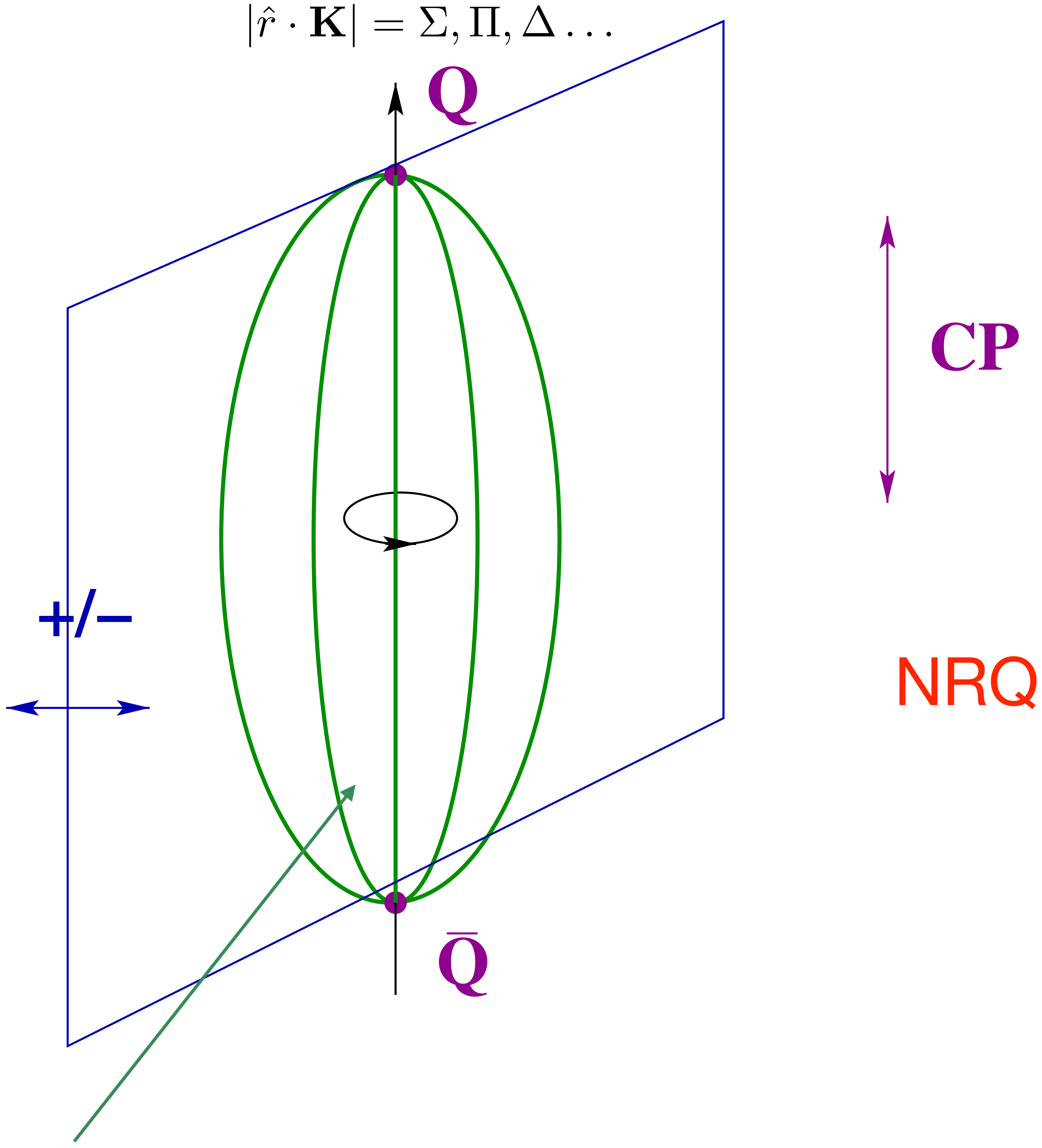
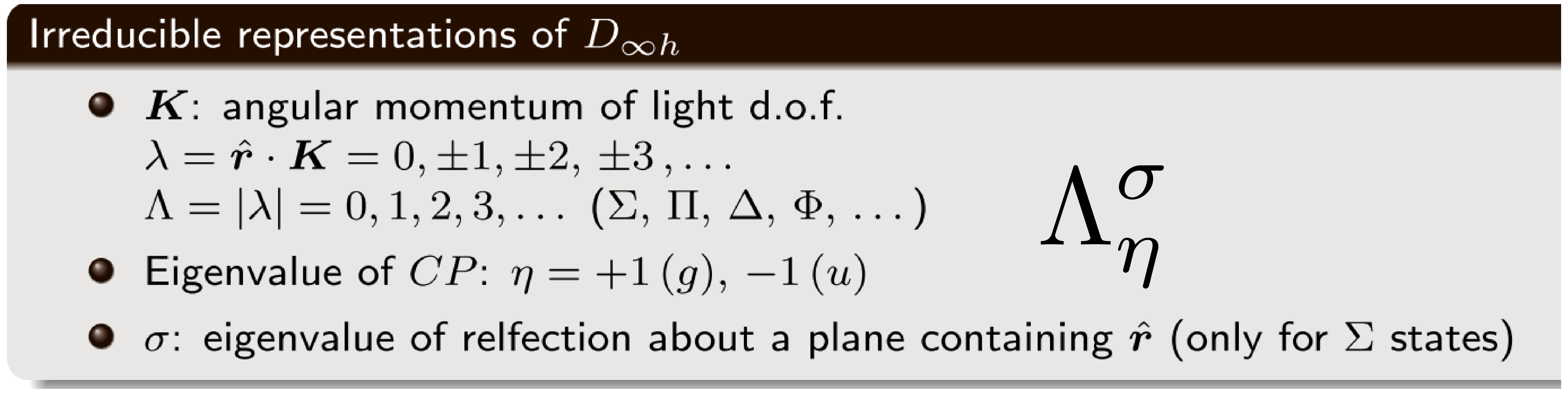


Nonperturbative light degrees of freedom
glue and light quarks

NRQCD

produce a hierarchy of NRQCD static energies identified by the quantum number of $D_{\infty h}$



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$$\mathcal{H}^{(0)}|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1,\mathbf{x}_2)|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2)|n;\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}$$

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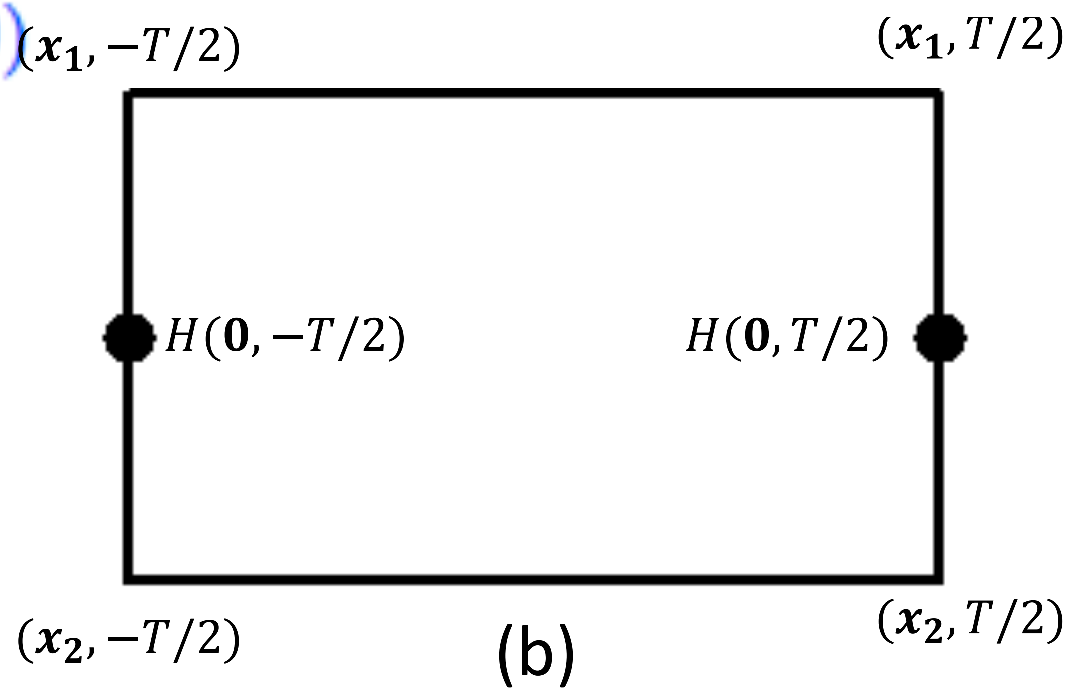

Phi =Wilson lines and H= gluonic and light quarks

Strongly coupled pNRQCD and Born Oppenheimer EFT

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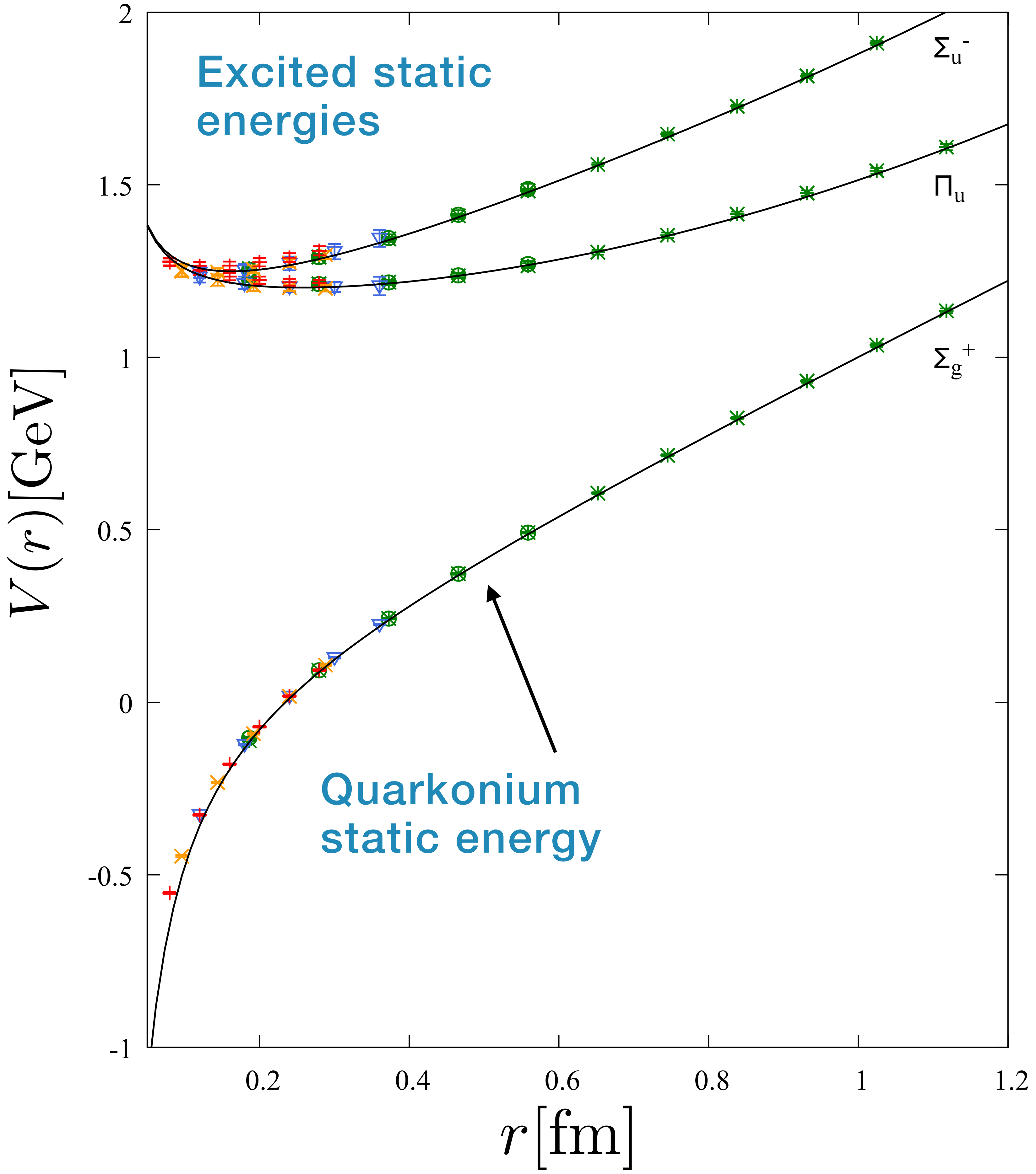

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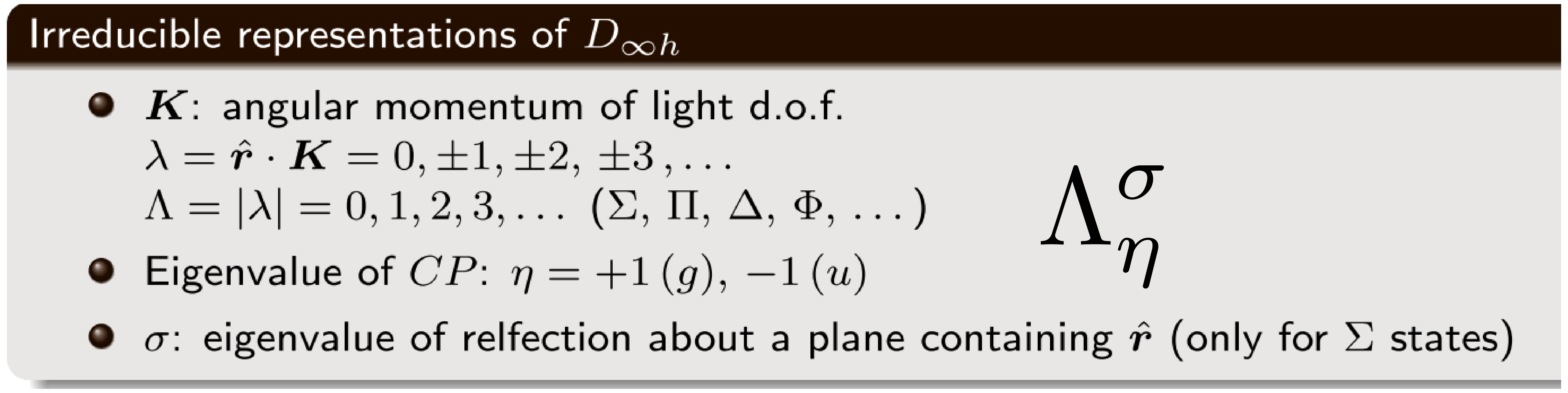

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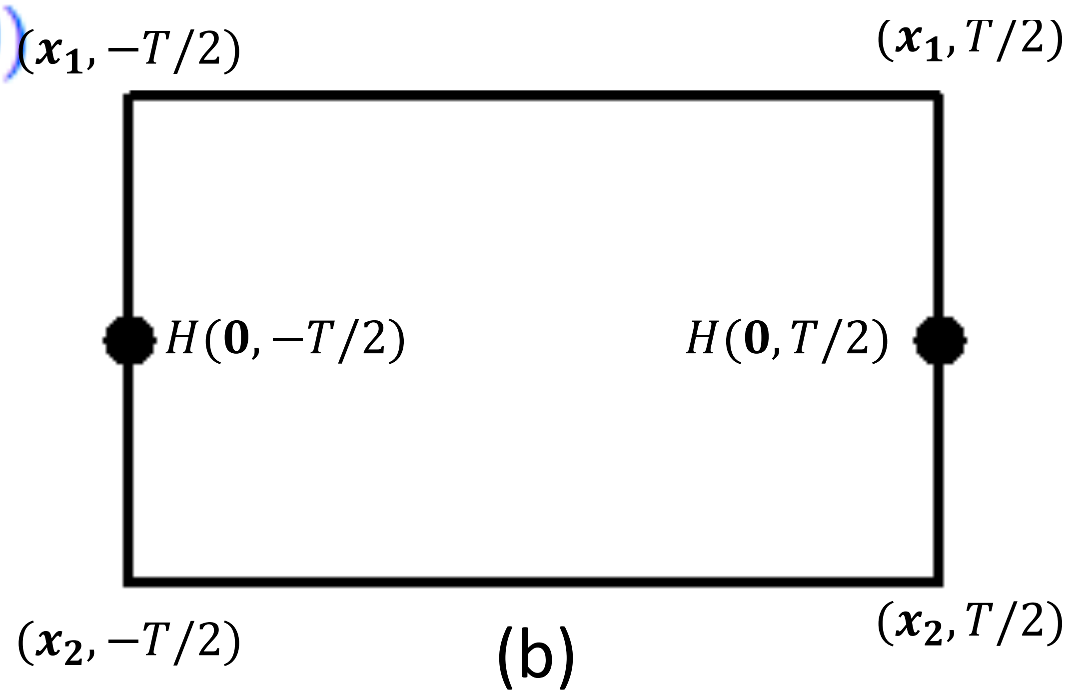

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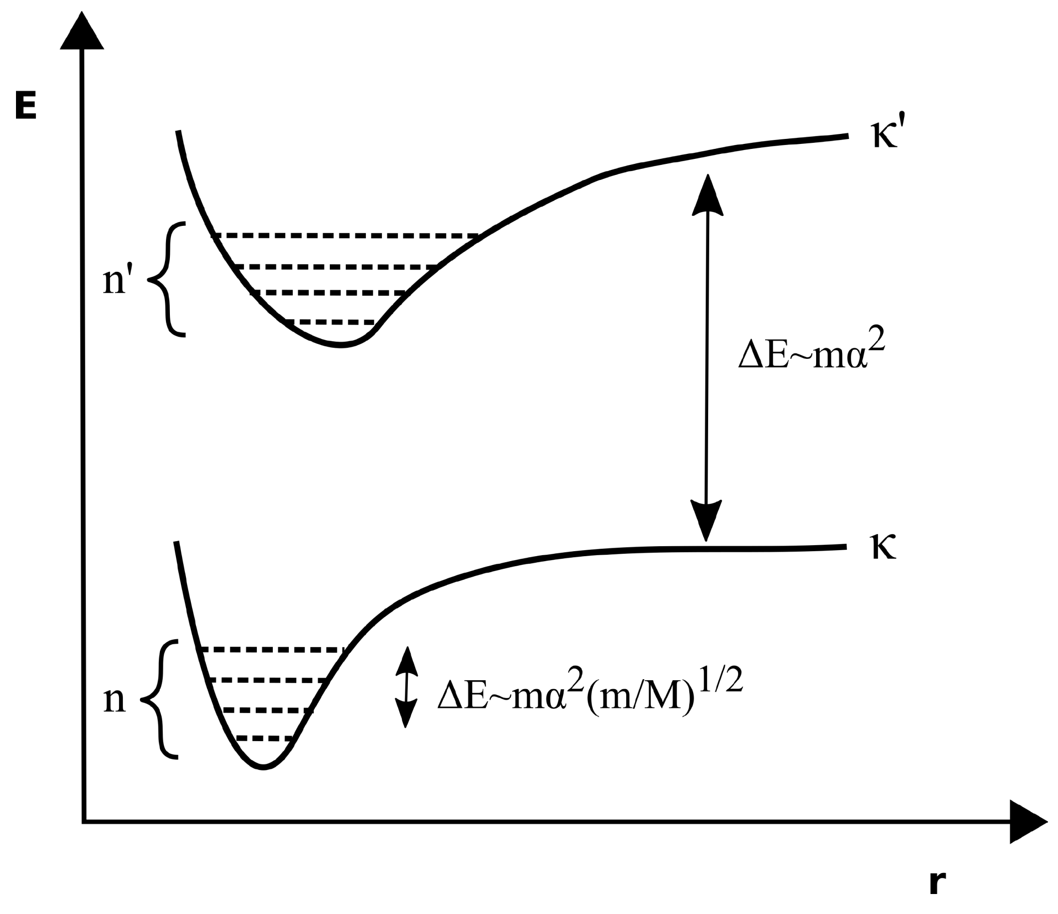


QED —

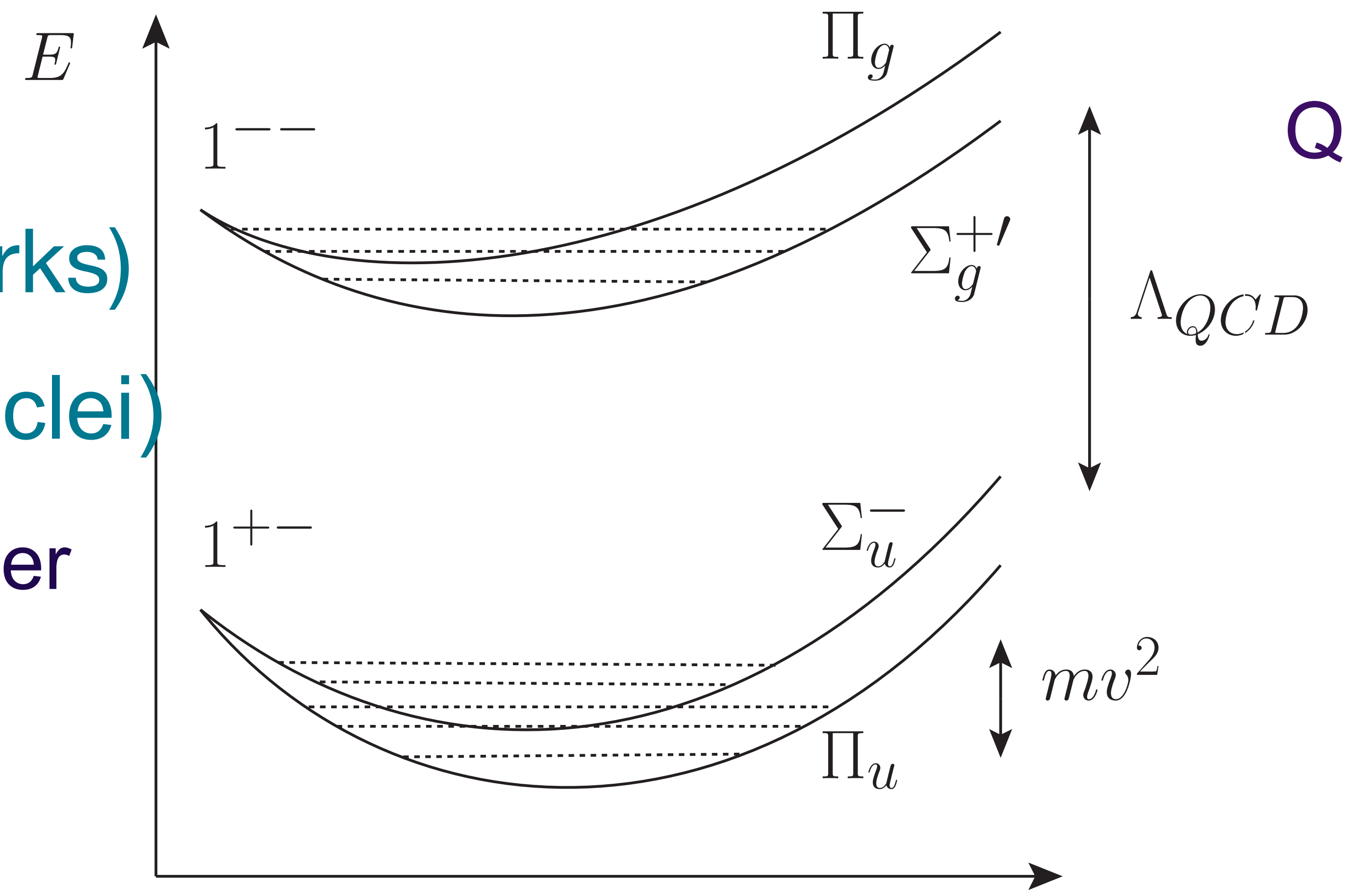
$$\Lambda_{QCD} > mv^2$$

fast (gluons, light quarks) and slow (heavy quarks)

like in molecular physics (fast-electrons, slow nuclei)

Born Oppenheimer
Description

`Braaten PRL 111 (2013) 162003`
`Braaten Langmack Smith PRD 90 (2014) 014044`

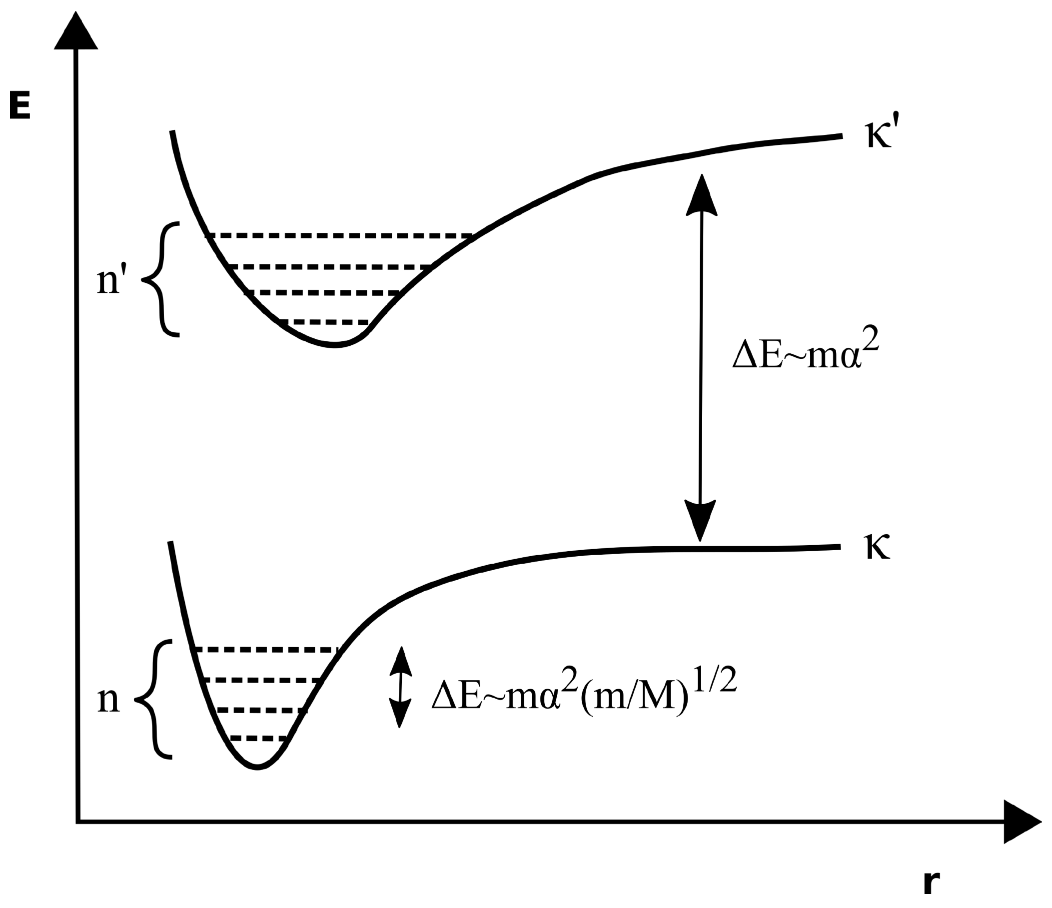


QCD

Higher excitations
develop a gap of order Lambda_QCD

Introducing a finite mass m:

- The spectrum of the mv^2 fluctuations around the lowest static energy is the quarkonium spectrum
- The spectrum of the mv^2 fluctuations around the higher excitations is the exotic spectrum (hybrids and tetraquarks)



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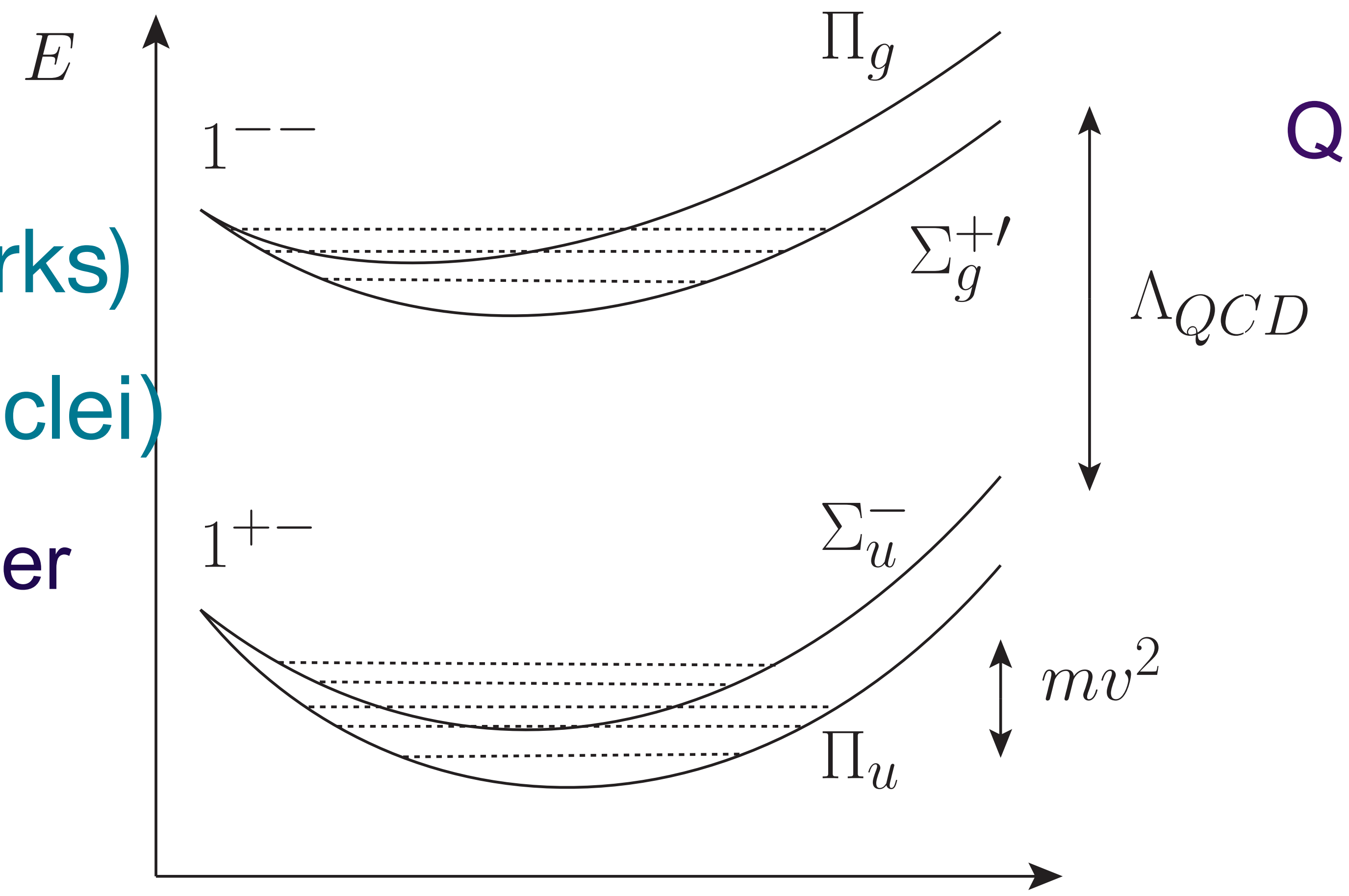

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Nonperturbative matching to the pNREFT

systematically

$$\langle H|\mathcal{H}|H\rangle = \langle nljs|\frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n}|nljs\rangle$$

expand quantomechanically NRQCD states and
energies in 1/m around the
zero order and identify the QCD potentials

$$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle -> \ |(Q\bar{Q})_1\rangle \rightarrow$$ Quarkonium Singlet

$$E_0(r) -> V_0(r)$$ pNRQCD

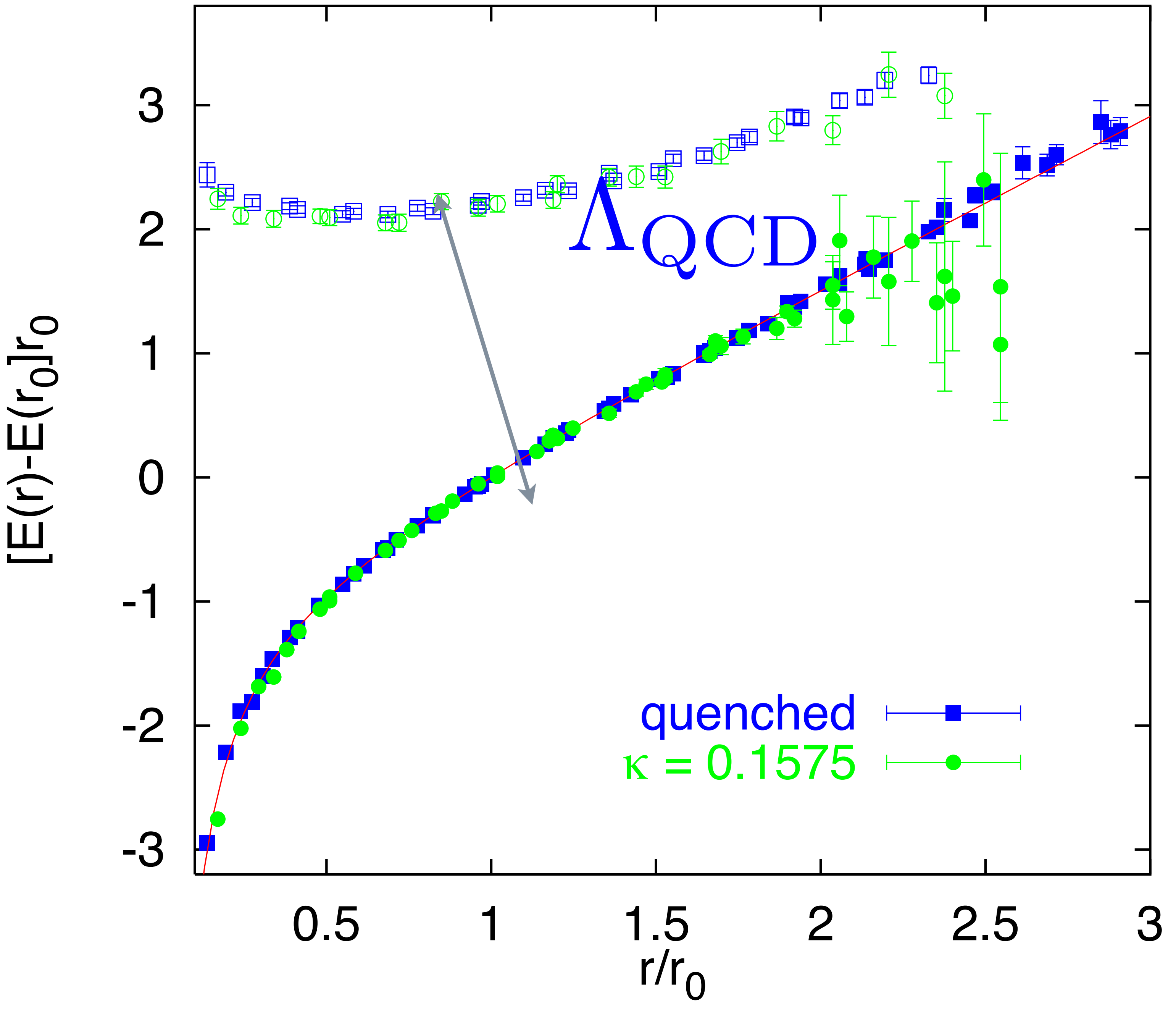
$$|\underline{n}>0;\mathbf{x}_1\mathbf{x}_2\rangle -> \ |(Q\bar{Q})g^{(n)}\rangle \rightarrow$$ Higher Gluonic Excitations

$$|Q\bar{Q}q\bar{q}\rangle$$ Tetraquarks

$$E_n^{(0)}(r) -> V_n^{(0)}(r)$$ BOEFT

Strongly coupled pNRQCD for quarkonium

$mv \sim \Lambda_{QCD}$



Bali et al. 98

pNRQCD and the potentials come from integrating out all scales up to mv^2

- gluonic excitations develop a gap $\Lambda_{\rm QCD}$ and are integrated out

⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

Brambilla Pineda Soto Vairo 00

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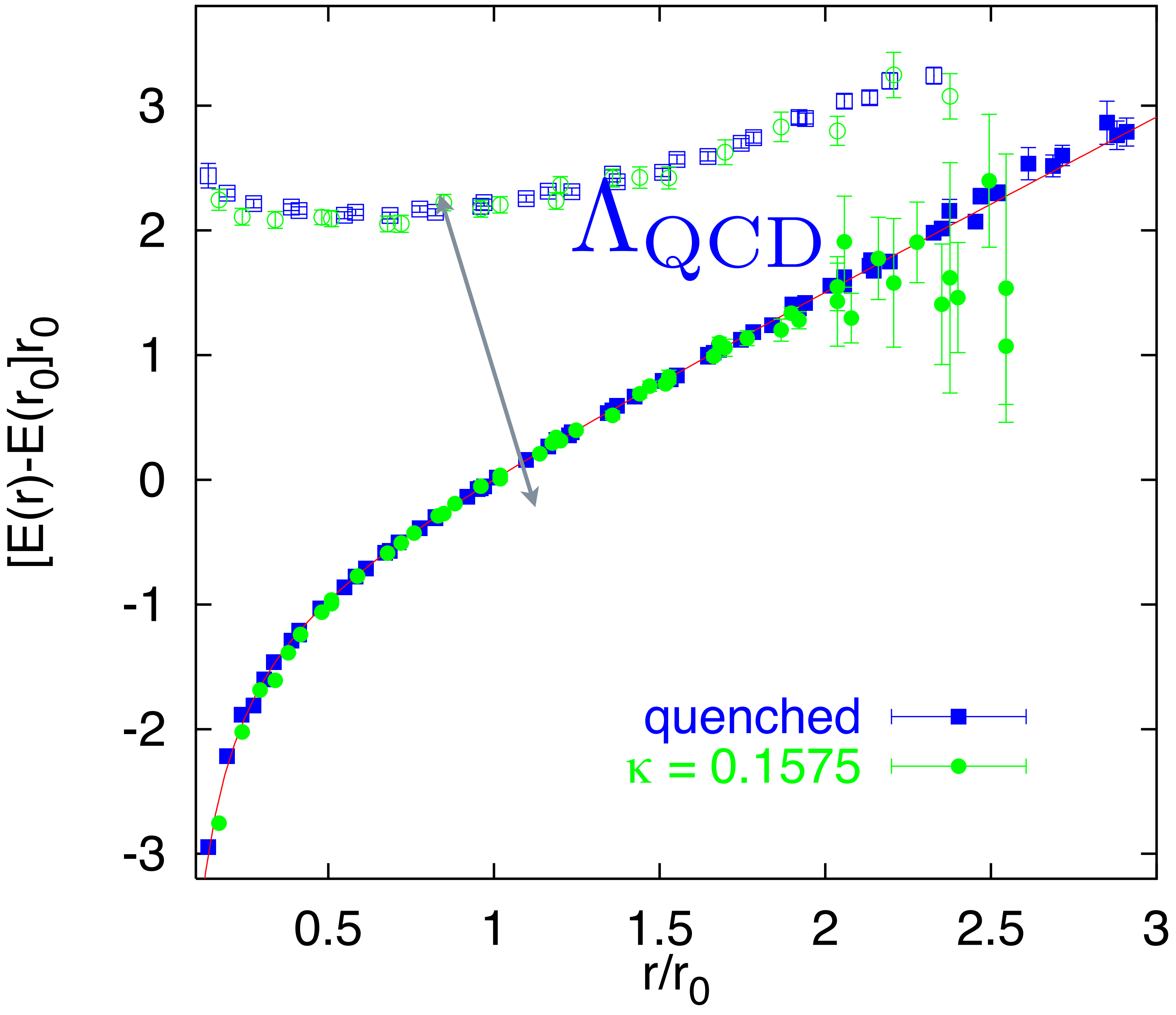

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$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\} + \Delta\mathcal{L}(\text{US light quarks})$$

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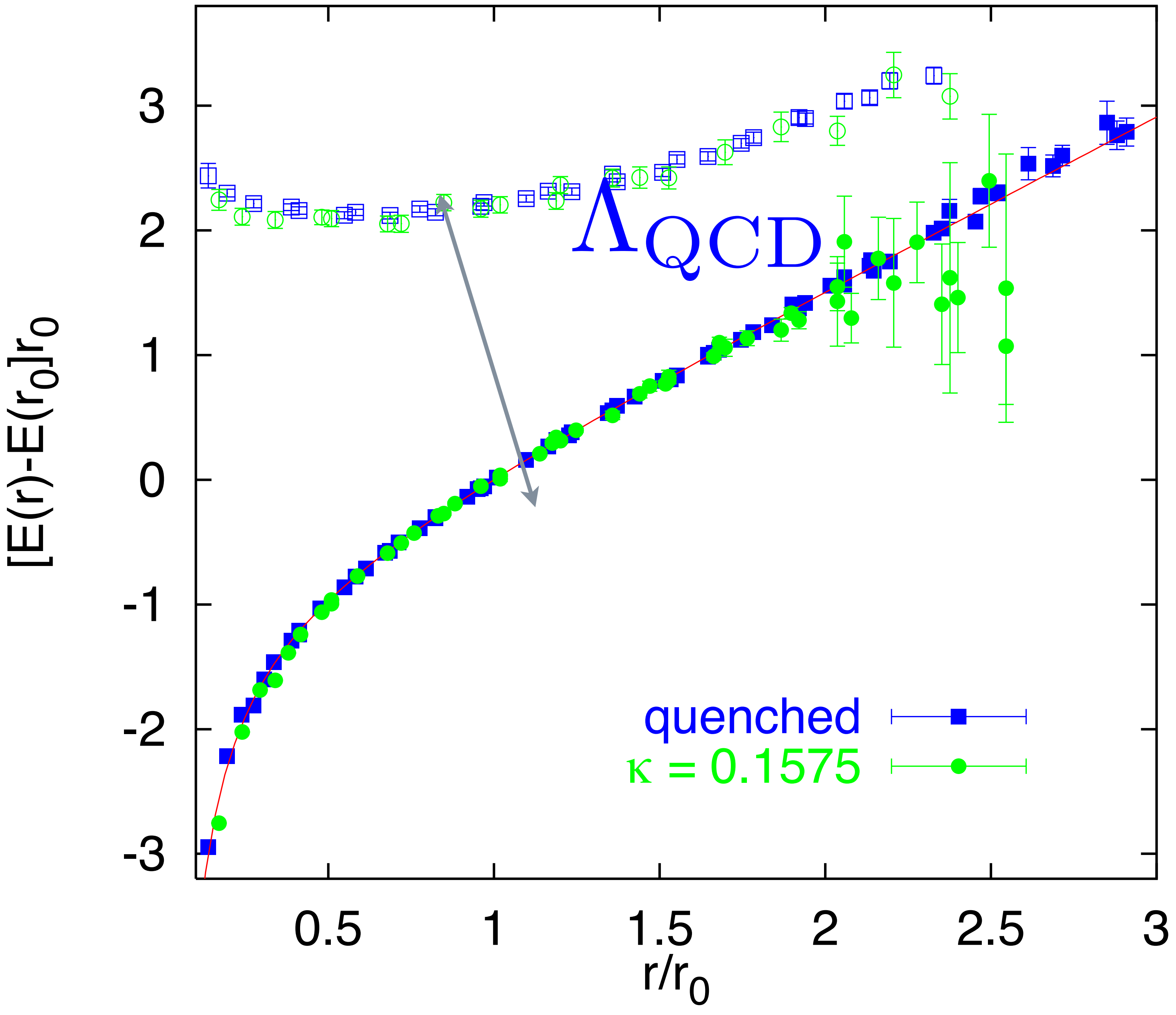

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- We obtain the form of the nonperturbative potentials V in terms of generalized Wilson loops (stat that are low energy pure gluonic correlators: all the flavour dependence is pulled out

Strongly coupled pNRQCD for quarkonium

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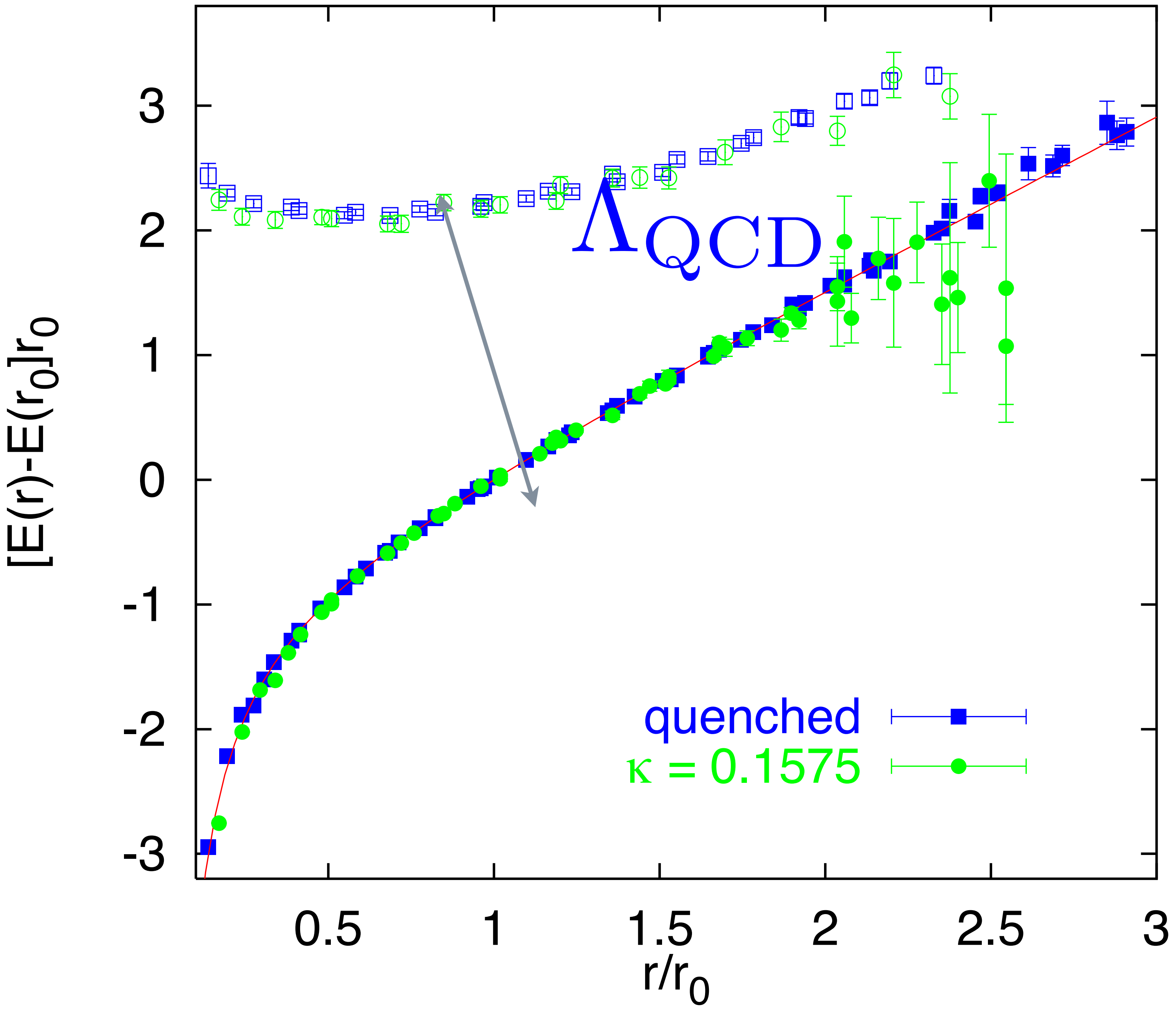

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Applications regard: Spectrum, decays, production at LHC, studies of confinement

Strongly coupled pNRQCD: quantum mechanical matching

the matching conditions are :

$$\langle H|\overset{\text{NRQCD}}{\mathcal{H}}|H\rangle = \langle nljs|\,\frac{\mathbf{p}^2}{m}+\sum_n \frac{V_s^{(n)}}{m^n}\,|nljs\rangle$$

$$|\underline{0};\mathbf{x}_1\mathbf{x}_2\rangle - > S^\dagger(\mathbf{x}_1\mathbf{x}_2)|\text{vac}\rangle$$

expand quantomechanically NRQCD states and energies in 1/m around the zero order and identify the QCD potentials

$$\overset{\text{NRQCD}}{\mathcal{H}} = \mathcal{H}^{(0)} + \frac{\delta\mathcal{H}^{(1)}}{m} + \frac{\delta\mathcal{H}^{(2)}}{m^2} + \frac{\delta\mathcal{H}^{(3)}}{m^3} + \frac{\delta\mathcal{H}^{(4)}}{m^4} + ..$$

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$$\delta\mathcal{H}^{(1)} = -\int d^3\mathbf{x}\,\psi^\dagger\left(\frac{\mathbf{D}^2}{2} + c_F\,g\,\mathbf{S}\cdot\mathbf{B}\right)\psi + \text{antip.}$$

$$|H\rangle \longrightarrow |\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle \otimes |nljs\rangle$$

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$$\times\frac{{}^{(0)}\langle\underline{n};\mathbf{z}_1,\mathbf{z}_2|\delta\mathcal{H}^{(1)}|\underline{0};\mathbf{x}_1,\mathbf{x}_2\rangle^{(0)}}{E_0^{(0)}(z) - E_n^{(0)}(x)} + \dots$$

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$$V = V^{(0)} + \frac{V^{(1)}}{m} + \frac{V^{(2)}}{m^2} + \dots$$

$$V^{(0)}(r) \;=\; \lim_{T\to\infty}\frac{i}{T}\ln\langle\,\boxed{}\,\rangle$$

given in terms of gauge invariant generalised Wilson loops

$$\boxed{} = \exp\left\{ig\oint_{r\times T} dz^\mu A_\mu\right\}$$

The singlet potential has the general structure

$$V = \underbrace{V_0}_{\text{static}} + \frac{1}{m}V_1 + \frac{1}{m^2}(\underbrace{V_{SD}}_{\text{spin dependent}} + \underbrace{V_{VD}}_{\text{velocity dependent}})$$

the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

The singlet potential has the general structure

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static

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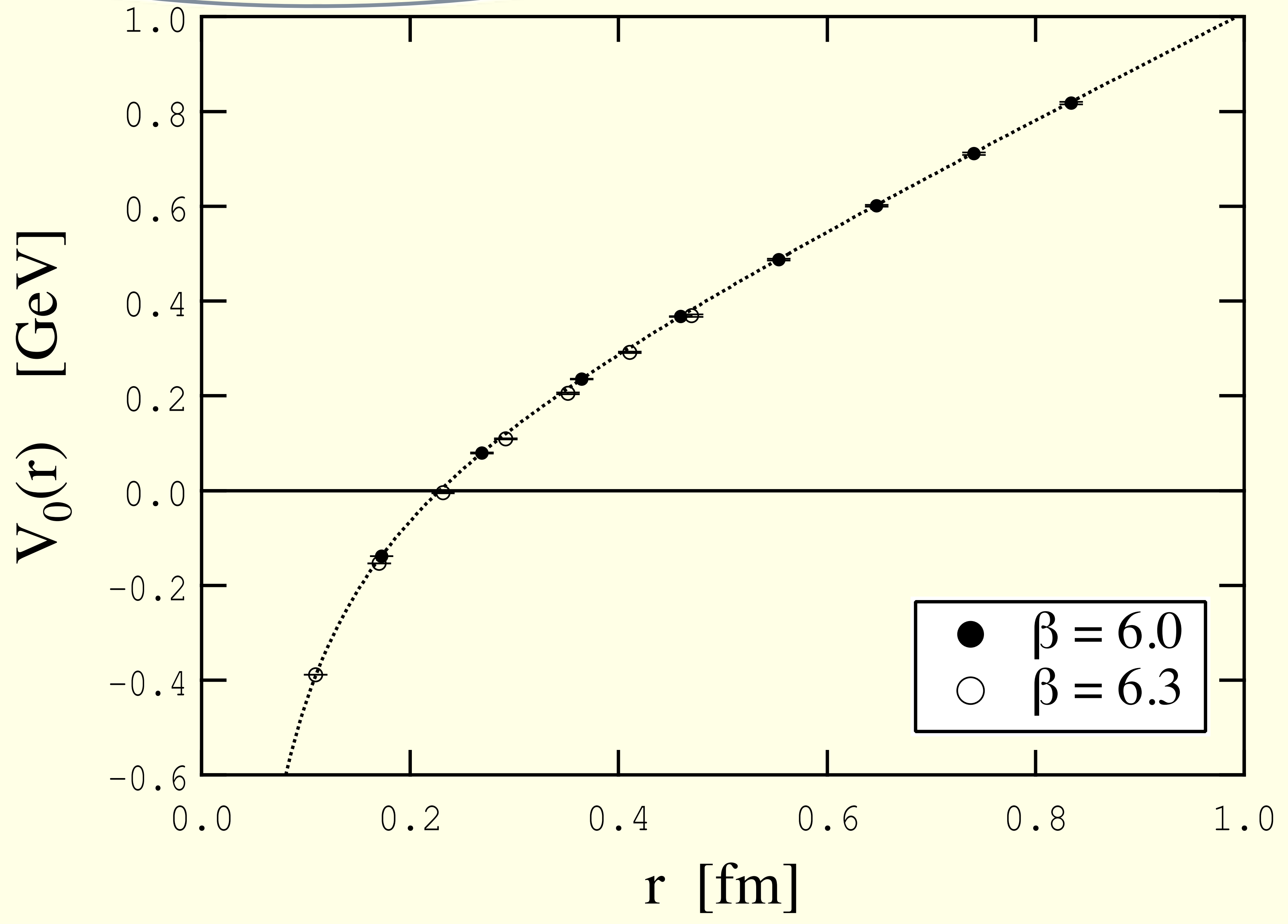
velocity dependent

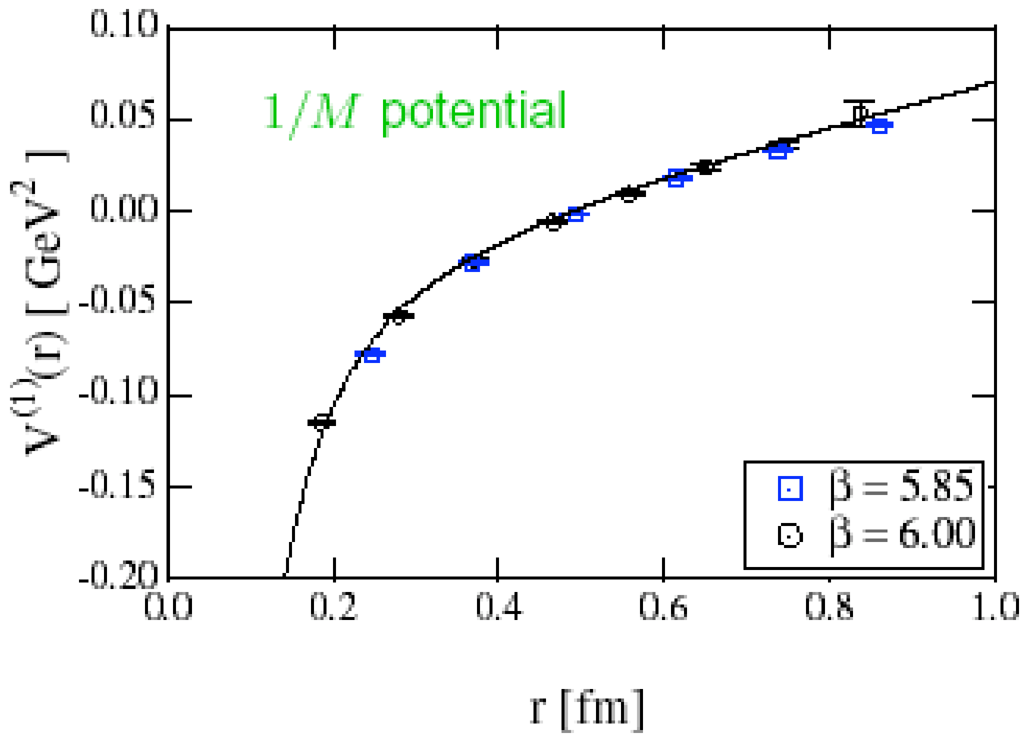
the fact that spin dependent corrections appear at order 1/m^2 is called Heavy Quark Spin Symmetry

$$V_s^{(0)} = \lim_{T\to\infty}\frac{i}{T}\ln\langle W(r\times T)\rangle = \lim_{T\to\infty}\frac{i}{T}\ln\langle \boxed{} \rangle$$

$$W = \langle \exp\{ig\oint A^\mu dx_\mu\}\rangle$$

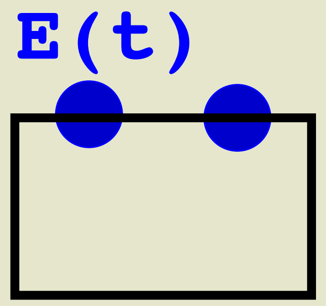


○ Koma Koma NPB 769(07)79



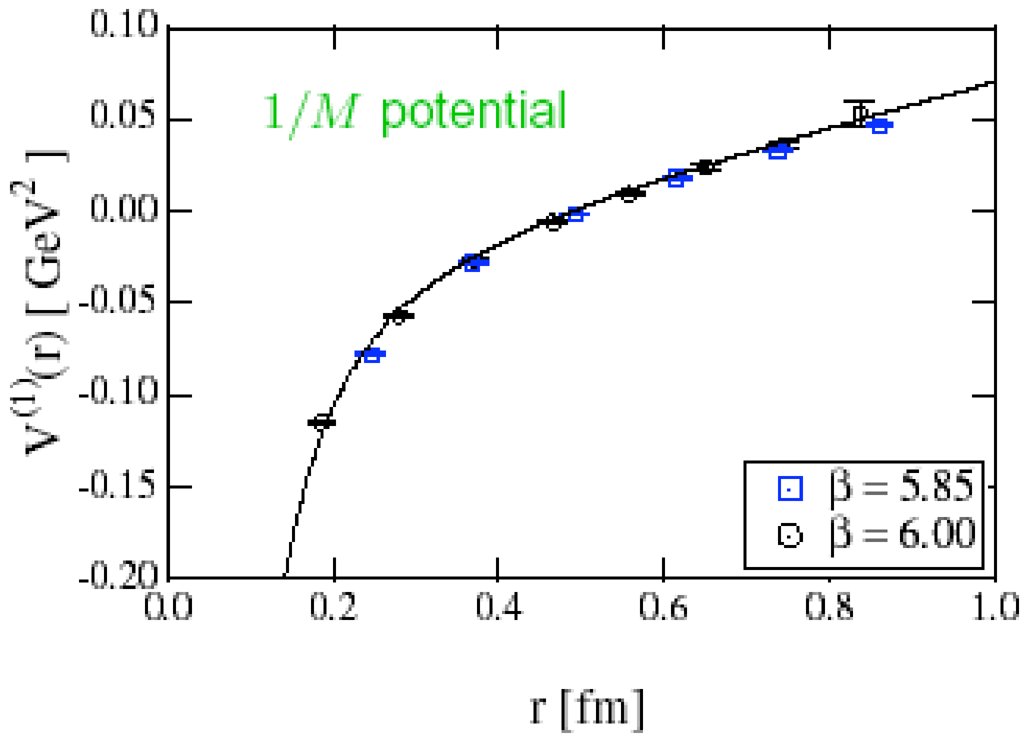
○ Koma Koma Wittig PoS LAT2007(07)111

$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t\ \Big\langle$$



$$\Big\rangle$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N



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$$V^{(1)} = -\frac{1}{2}\int_0^\infty dt\, t \left\langle \overset{\mathbf{E(t)}}{\boxed{\bullet \quad \bullet}} \right\rangle$$

gauge invariant wilson loops can be calculated also in QCD vacuum model and large N

$c_F = 1 + \alpha_s/\pi(13/6 + 3/2\ln m/\mu) + \ldots), d_{sv,vv} = O(\alpha_s^2)$ from NRQCD.

$$V^{(2)}_{\rm SD} = -\frac{r^k}{r^2} c_F\, \epsilon^{kij}\, i \int_0^\infty dt\, t \left\langle \overset{\mathbf{E(t)}}{\underset{\mathbf{B}}{\boxed{\mathtt{i} \quad \mathtt{j}}}} \right\rangle \mathbf{L}_1\cdot\mathbf{S}_2 + (1\leftrightarrow 2)\ |V^{(2)}_{LS}$$

$$-\frac{r^k}{r^2}\left(c_F\, \epsilon^{kij}\, i\int_0^\infty dt\, t \left\langle \boxed{\mathtt{i} \quad \mathtt{j}} \right\rangle - \frac{2c_F-1}{2}\nabla^k V^{(0)}\right)\mathbf{L}_1\cdot\mathbf{S}_1 + (1\leftrightarrow 2)\ |V^{(1)}_{LS}$$

$$-c_F^2\hat{r}_i\hat{r}_j i\int_0^\infty dt\left(\left\langle \boxed{\mathtt{i} \quad \mathtt{j}} \right\rangle - \frac{\delta_{ij}}{3}\left\langle \boxed{\phantom{\mathtt{i} \quad \mathtt{j}}} \right\rangle\right)\left(\mathbf{S_1}\cdot\mathbf{S_2} - 3(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S_2}\cdot\hat{\mathbf{r}})\right)\ |V_T$$

$$+\left(\frac{2}{3}c_F^2 i\int_0^\infty dt\left\langle \boxed{\phantom{\mathtt{i} \quad \mathtt{j}}} \right\rangle - 4\left(d_{sv}+\frac{4}{3}d_{vv}\right)\delta^{(3)}(\mathbf{r})\right)\mathbf{S}_1\cdot\mathbf{S_2}\ |V_S\ -$$

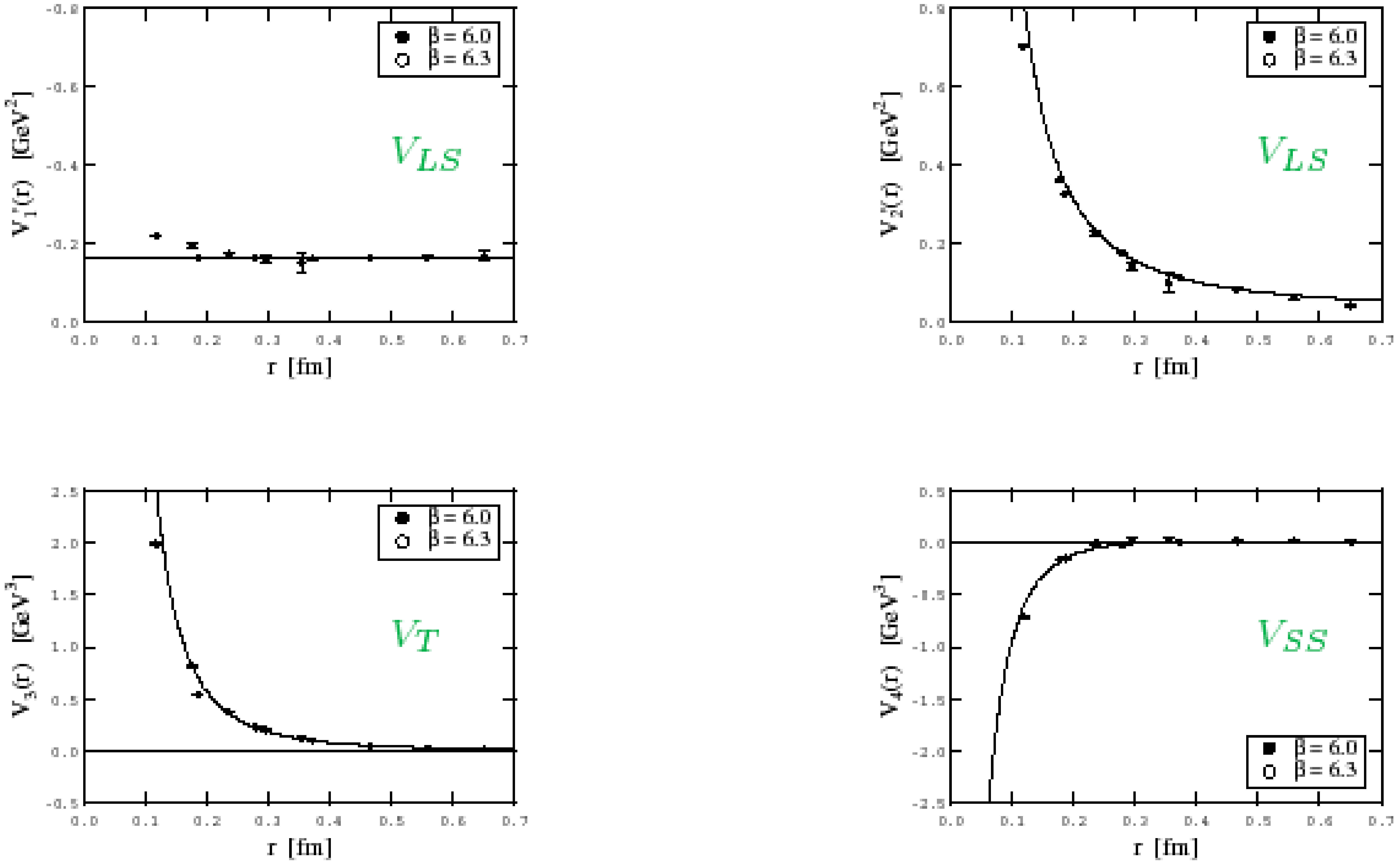
spin dependent 1/m^2 potential

› Pineda Vairo PRD 63 (2001) 054007

› Brambilla Pineda Soto Vairo PRD 63 (2001) 014023

- the potentials contain the contribution of the scale m inherited from NRQCD matching coefficients—> they cance any QM divergences, good UV behaviour
- the flavour dependent part is extracted in the NRQCD matching coefficients
- the nonperturbative part is factorized and depends only on the glue —> only one lattice calculation to get the dynamics and the observables instead of an ab initio calculation of multiple Green functions

Lattice evaluation of the spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

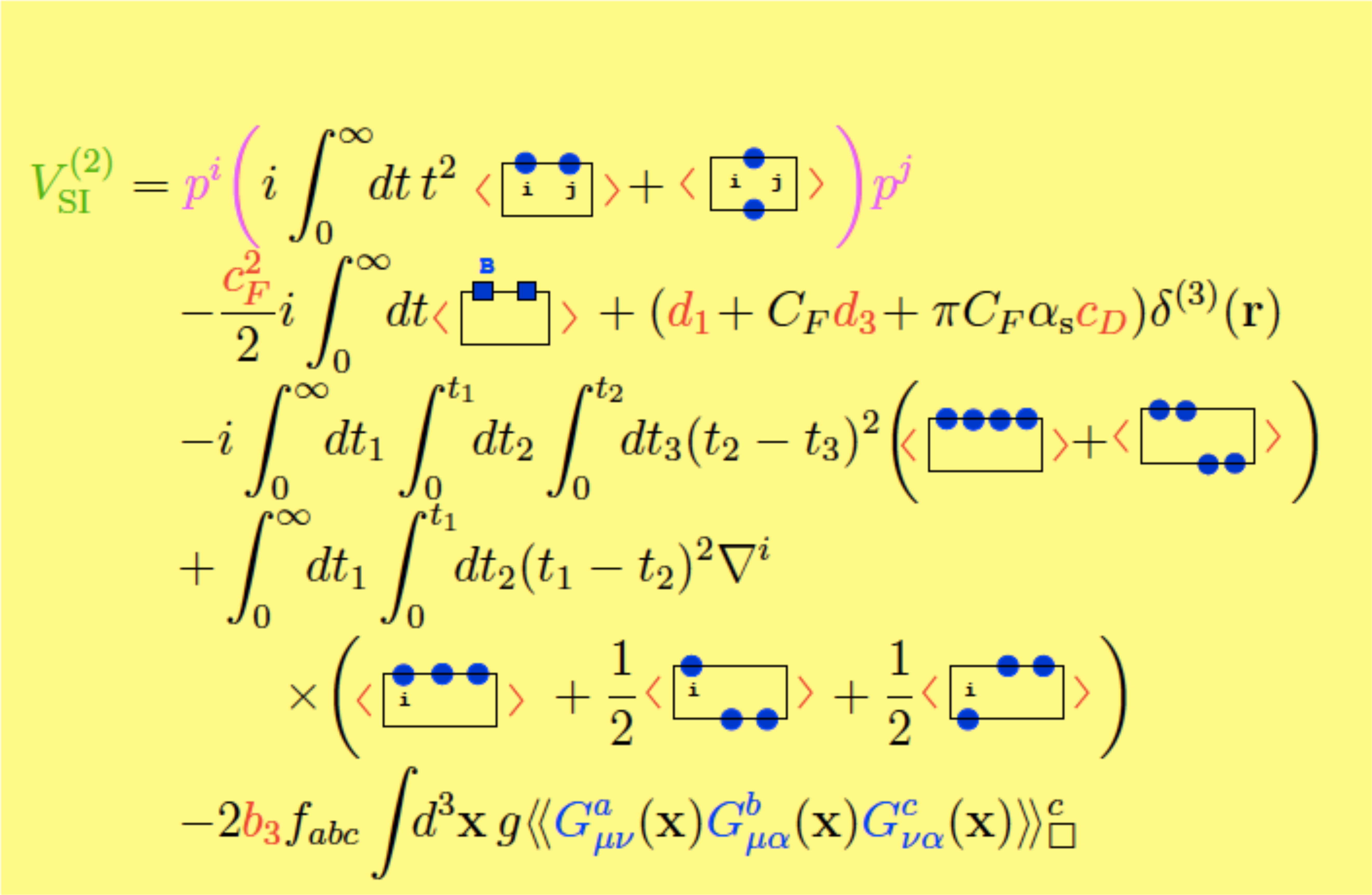
Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, Vairo 2014

spin independent potentials at order 1/m^2

The non-perturbative Potentials

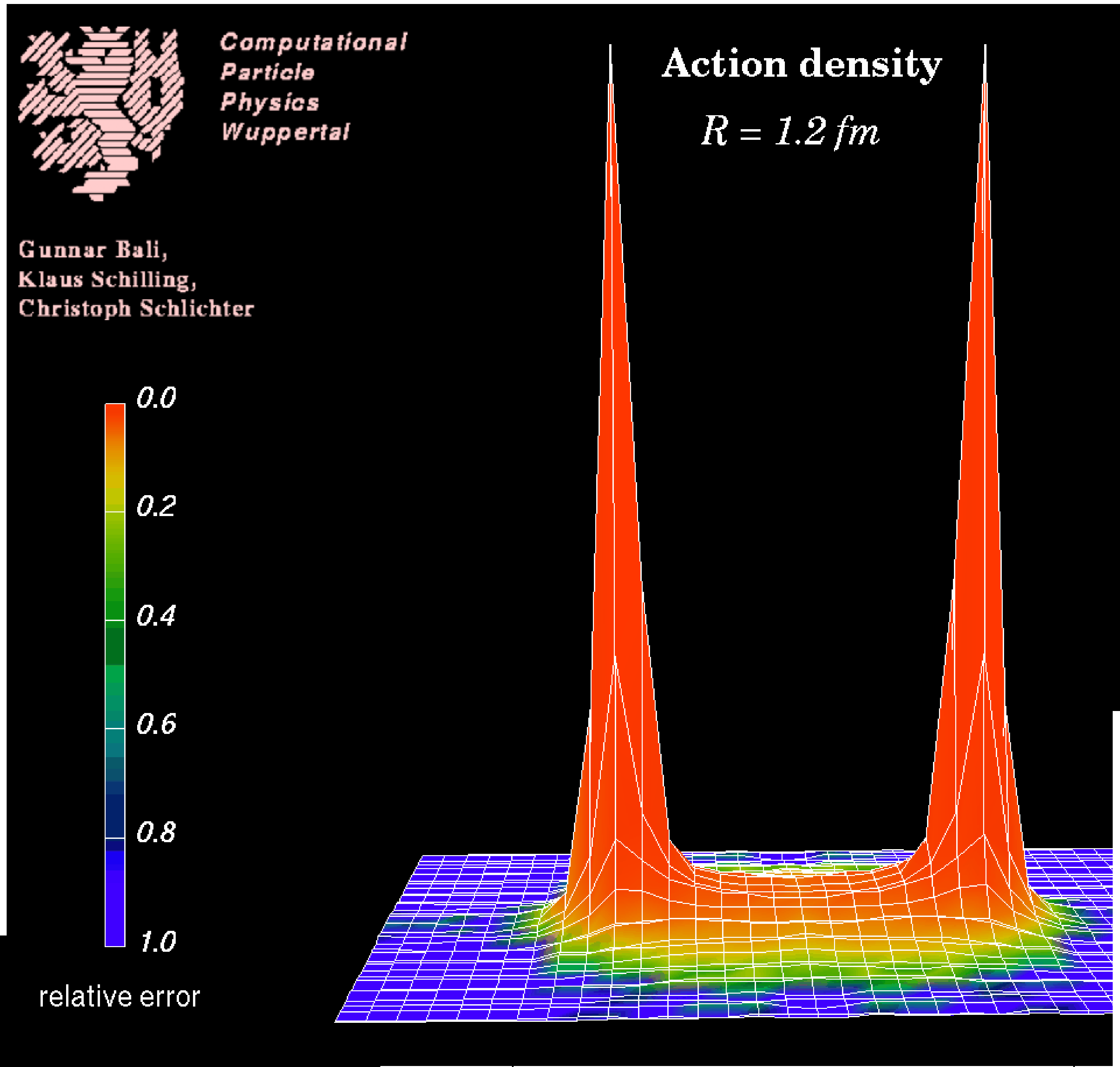


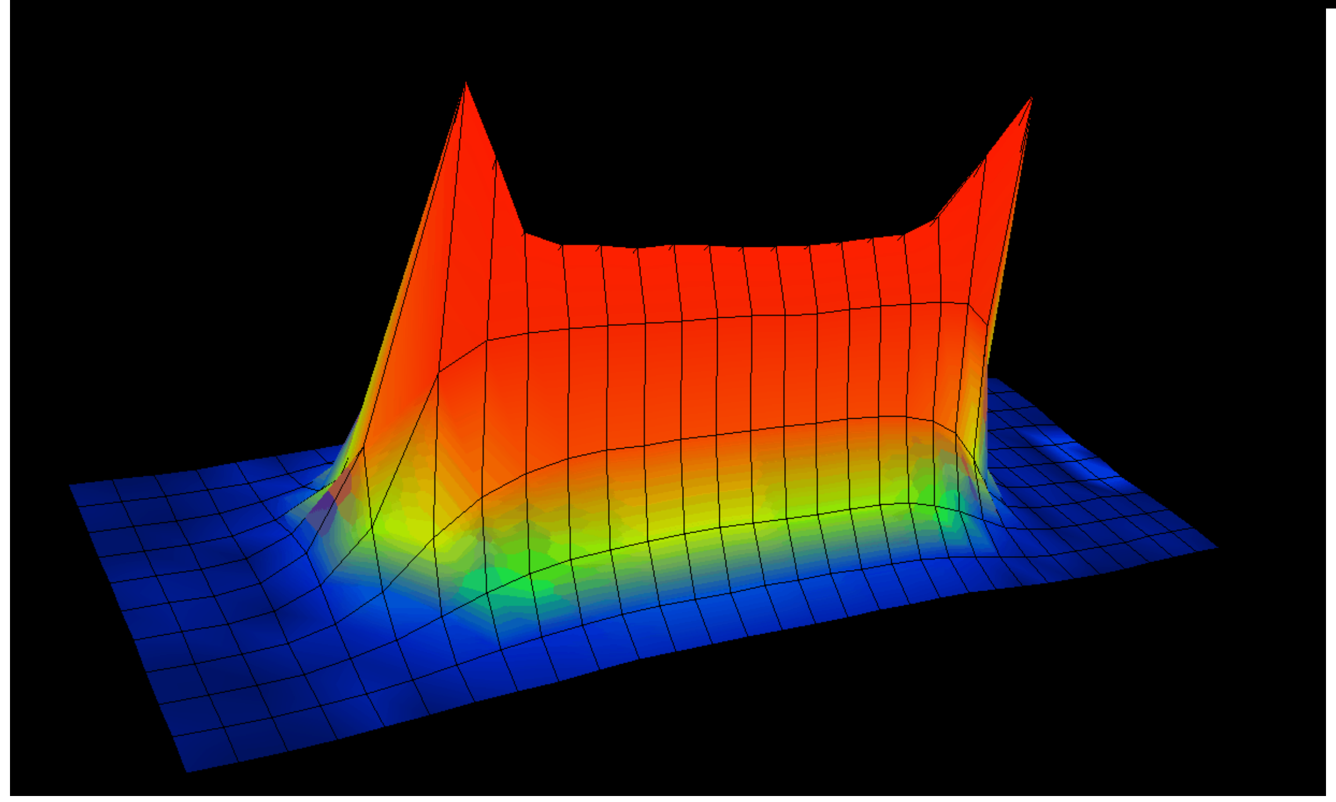
$$
\begin{aligned}
V_{\mathrm{SI}}^{(2)} = {} & p^i \left(i \int_0^\infty dt\, t^2 \, \langle \boxed{\,\mathtt{i}\;\;\mathtt{j}\,} \rangle + \langle \boxed{\,\mathtt{i}\;\;\mathtt{j}\,} \rangle \right) p^j \\
& - \frac{c_F^2}{2} i \int_0^\infty dt \langle \overset{\mathtt{B}}{\boxed{\quad\quad}} \rangle + (d_1 + C_F d_3 + \pi C_F \alpha_{\mathrm{s}} c_D)\delta^{(3)}(\mathbf{r}) \\
& - i \int_0^\infty dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 (t_2 - t_3)^2 \left(\langle \boxed{\quad\quad} \rangle + \langle \boxed{\quad\quad} \rangle \right) \\
& + \int_0^\infty dt_1 \int_0^{t_1} dt_2 (t_1 - t_2)^2 \nabla^i \\
& \qquad \times \left(\langle \boxed{\,\mathtt{i}\quad\,} \rangle + \frac{1}{2} \langle \boxed{\,\mathtt{i}\quad\,} \rangle + \frac{1}{2} \langle \boxed{\,\mathtt{i}\quad\,} \rangle \right) \\
& - 2 b_3 f_{abc} \int d^3\mathbf{x}\, g \langle\!\langle G^a_{\mu\nu}(\mathbf{x}) G^b_{\mu\alpha}(\mathbf{x}) G^c_{\nu\alpha}(\mathbf{x}) \rangle\!\rangle^c_{\Box}
\end{aligned}
$$

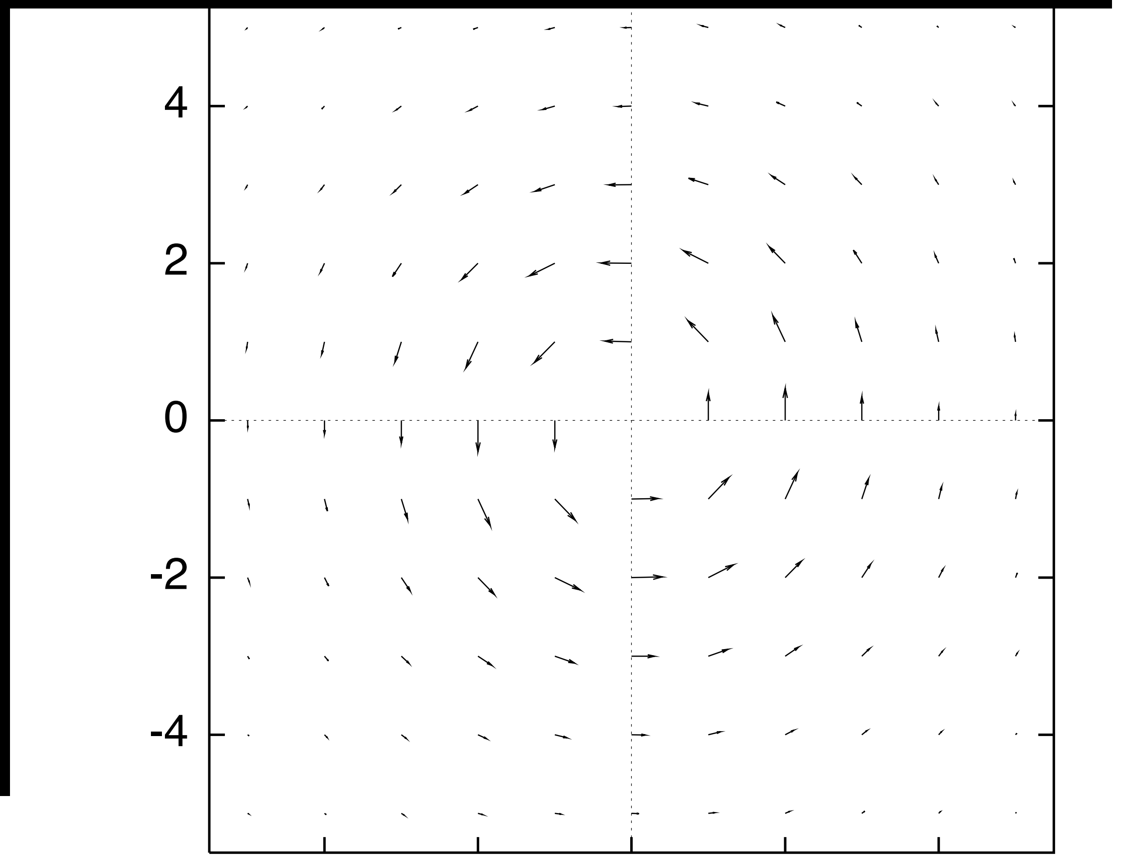
`Brambilla et al. 88 90, Pineda Vairo 00`

Low energy physics factorized in Wilson loops: can be used to probe the confinement mechanism

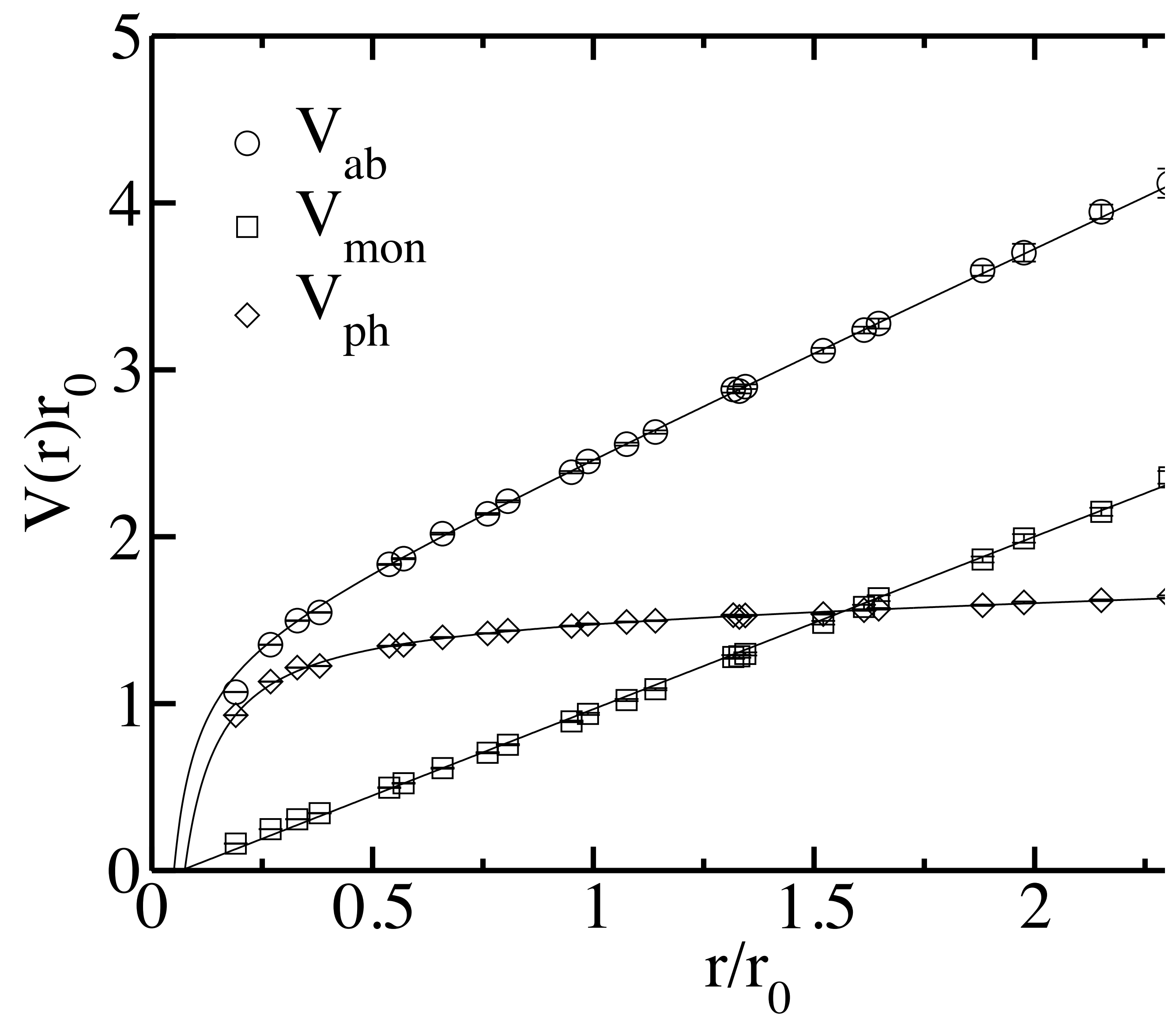
Bali et al







Boryakov et al. 04



Example of low energy QCD model corresponding to a Wilson loop form

Effective string theory and the long-range relativistic corrections to the quark-antiquark potential
N. B, M. Groher, H. Martinez, A. Vairo arXiv:1407.7761

$$\lim_{T\to\infty}\langle W_\Box\rangle = Z\int \mathcal{D}\xi^1\mathcal{D}\xi^2\, e^{iS_{\rm string}(\xi^1,\xi^2)}\,,$$

$$S_{\rm string} = -\sigma\int dt\,dz\,\left(1-\frac{1}{2}\partial_\mu\xi^l\partial^\mu\xi^l\right). \qquad r\Lambda_{\rm QCD}\gg 1$$

one can calculate chromo-electric and magnetic Wilson loop insertions with this identification of the Wilson loop in terms of string correlators and calculate the long range behaviour of all potentials

$$V^{(0)}(r) = \sigma r + \mu - \frac{\pi}{12r} \quad \text{Luescher term}$$

$$V^{(1,0)}(r) = \frac{g^2\Lambda^4}{2\pi\sigma}\ln\left(\sigma r^2\right)+\mu_1\,,$$

$$V^{(2,0)}_{\mathbf{p}^2}(r) = 0\,,$$

$$V^{(2,0)}_{\mathbf{L}^2}(r) = -\frac{g^2\Lambda^4\,r}{6\sigma}\,,$$

$$V^{(2,0)}_{LS}(r) = -\frac{\mu_2}{r}-\frac{c_F^{(1)}g^2\Lambda^2\Lambda'}{\sigma\,r^2}\,,$$

$$V^{(1,1)}_{\mathbf{p}^2}(r) = 0\,,$$

$$V^{(1,1)}_{\mathbf{L}^2}(r) = \frac{g^2\Lambda^4\,r}{6\sigma}\,,$$

$$V^{(1,1)}_{L_2\,S_1}(r) = -\frac{c_F^{(1)}g^2\Lambda^2\Lambda}{\sigma\,r^2}\,,$$

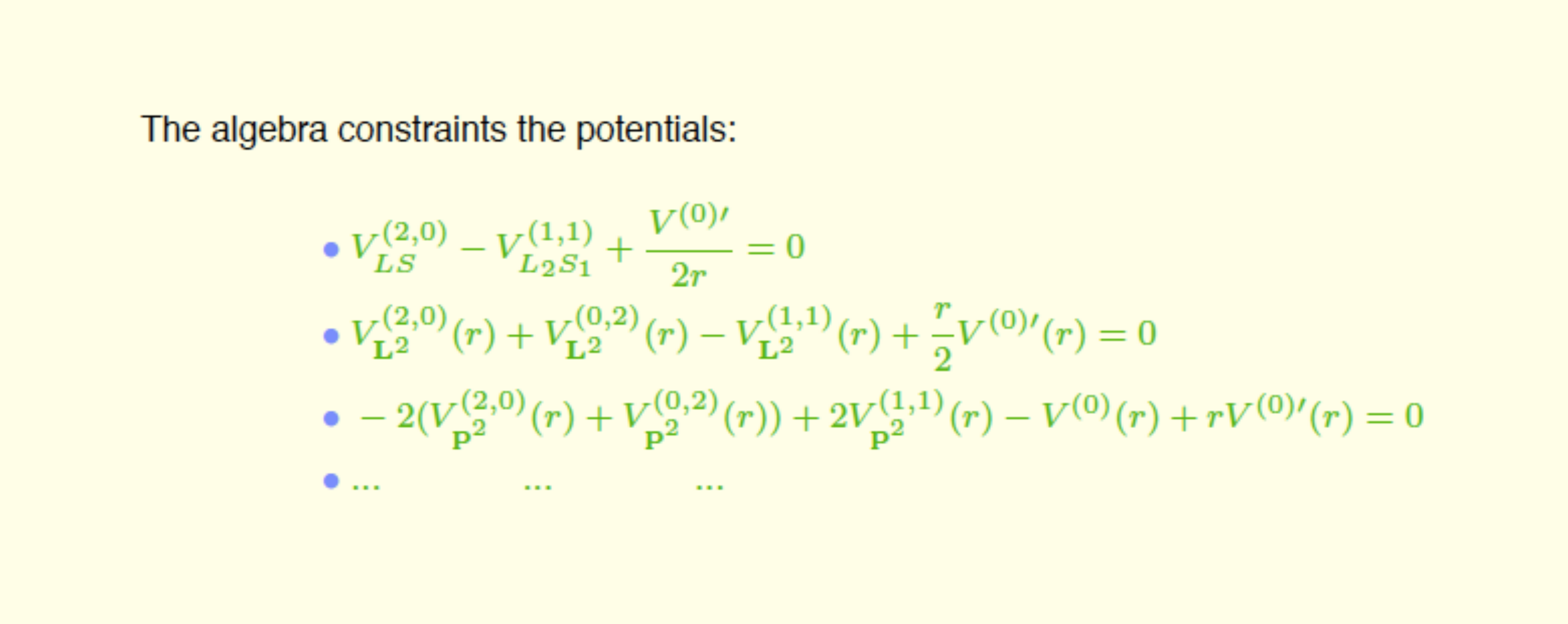
$$V^{(1,1)}_{S^2}(r) = \frac{2\pi^3c_F^{(1)}c_F^{(2)}g^2\Lambda'''^2}{45\sigma^2\,r^5}-4(d_{sv}+d_{vv}C_f)\delta^{(3)}(\mathbf{r})\,,$$

$$V^{(1,1)}_{\mathbf{S}_{12}}(r) = \frac{\pi^3c_F^{(1)}c_F^{(2)}g^2\Lambda'''^2}{90\sigma^2\,r^5}\,,$$

$$V^{(2,0)}_{r}(r) = -\frac{2\,\zeta_3\,g^4\Lambda^8r}{\pi^3\sigma^2}+\mu_3+\frac{\mu_4}{r^2}+\frac{\mu_5}{r^4}+\frac{\pi^3c_F^{(1)\,2}g^2\Lambda'''^2}{60\sigma^2r^5} + \frac{\pi C_f\alpha_{\rm s}c_D^{(1)\prime}}{2}\delta^{(3)}(\mathbf{r})-d_3^{(1)\prime}f_{abc}\int d^3\mathbf{x}\,\lim_{T\to\infty}g\langle\!\langle F^a_{\mu\nu}(x)F^b_{\mu\alpha}(x)F^c_{\nu\alpha}(x)\rangle\!\rangle\,,$$

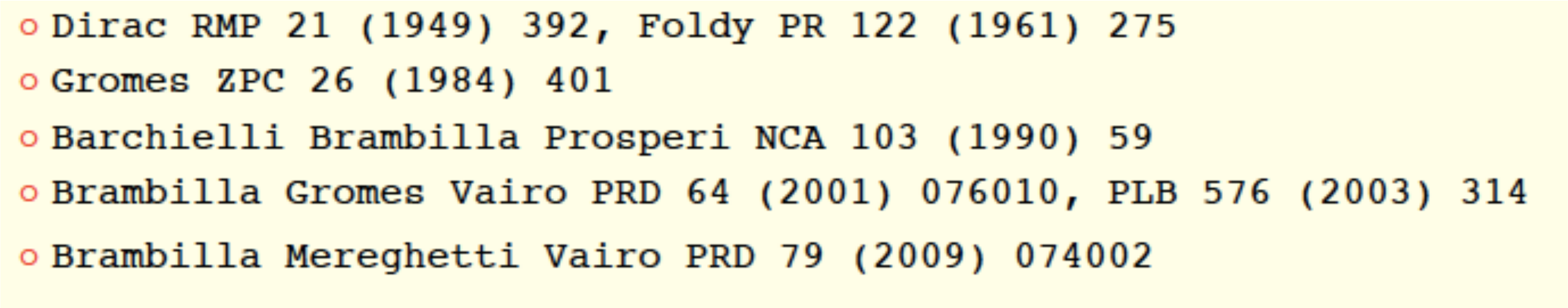
$$V^{(1,1)}_{r}(r) = -\frac{\zeta_3\,g^4\Lambda^8r}{2\pi^3\sigma^2}+(d_{ss}+d_{vs}C_f)\,\delta^{(3)}(\mathbf{r})\,,$$

Poincare invariance in the pNRQCD is realised via relations among the potentials



The algebra constraints the potentials:

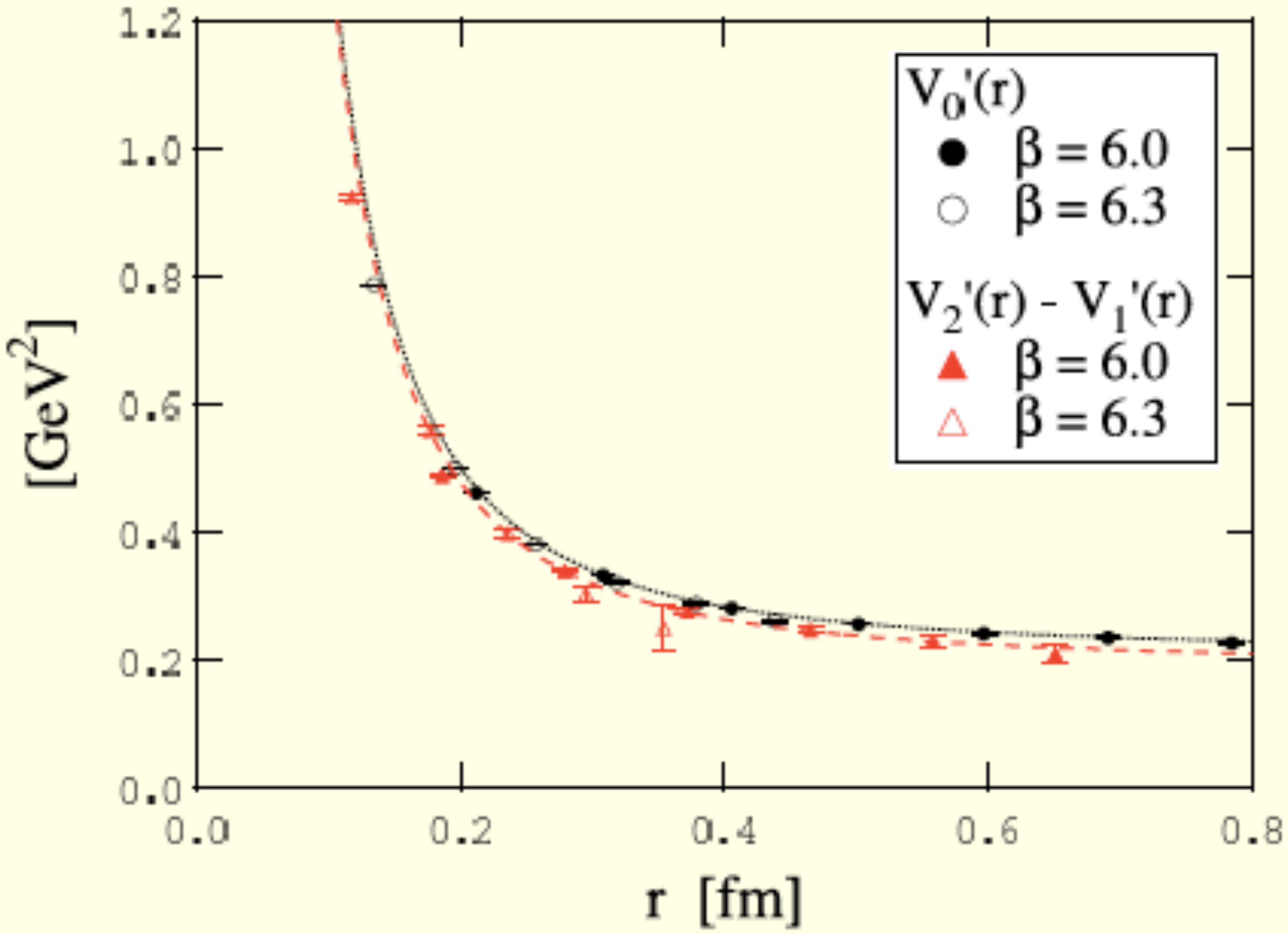
- $V_{LS}^{(2,0)} - V_{L_2S_1}^{(1,1)} + \frac{V^{(0)\prime}}{2r} = 0$
- $V_{\mathbf{L}^2}^{(2,0)}(r) + V_{\mathbf{L}^2}^{(0,2)}(r) - V_{\mathbf{L}^2}^{(1,1)}(r) + \frac{r}{2}V^{(0)\prime}(r) = 0$
- $-2(V_{\mathbf{p}^2}^{(2,0)}(r) + V_{\mathbf{p}^2}^{(0,2)}(r)) + 2V_{\mathbf{p}^2}^{(1,1)}(r) - V^{(0)}(r) + rV^{(0)\prime}(r) = 0$
- ... ...



- Dirac RMP 21 (1949) 392, Foldy PR 122 (1961) 275
- Gromes ZPC 26 (1984) 401
- Barchielli Brambilla Prosperi NCA 103 (1990) 59
- Brambilla Gromes Vairo PRD 64 (2001) 076010, PLB 576 (2003) 314
- Brambilla Mereghetti Vairo PRD 79 (2009) 074002

Constraint on the spin-dependent potentials

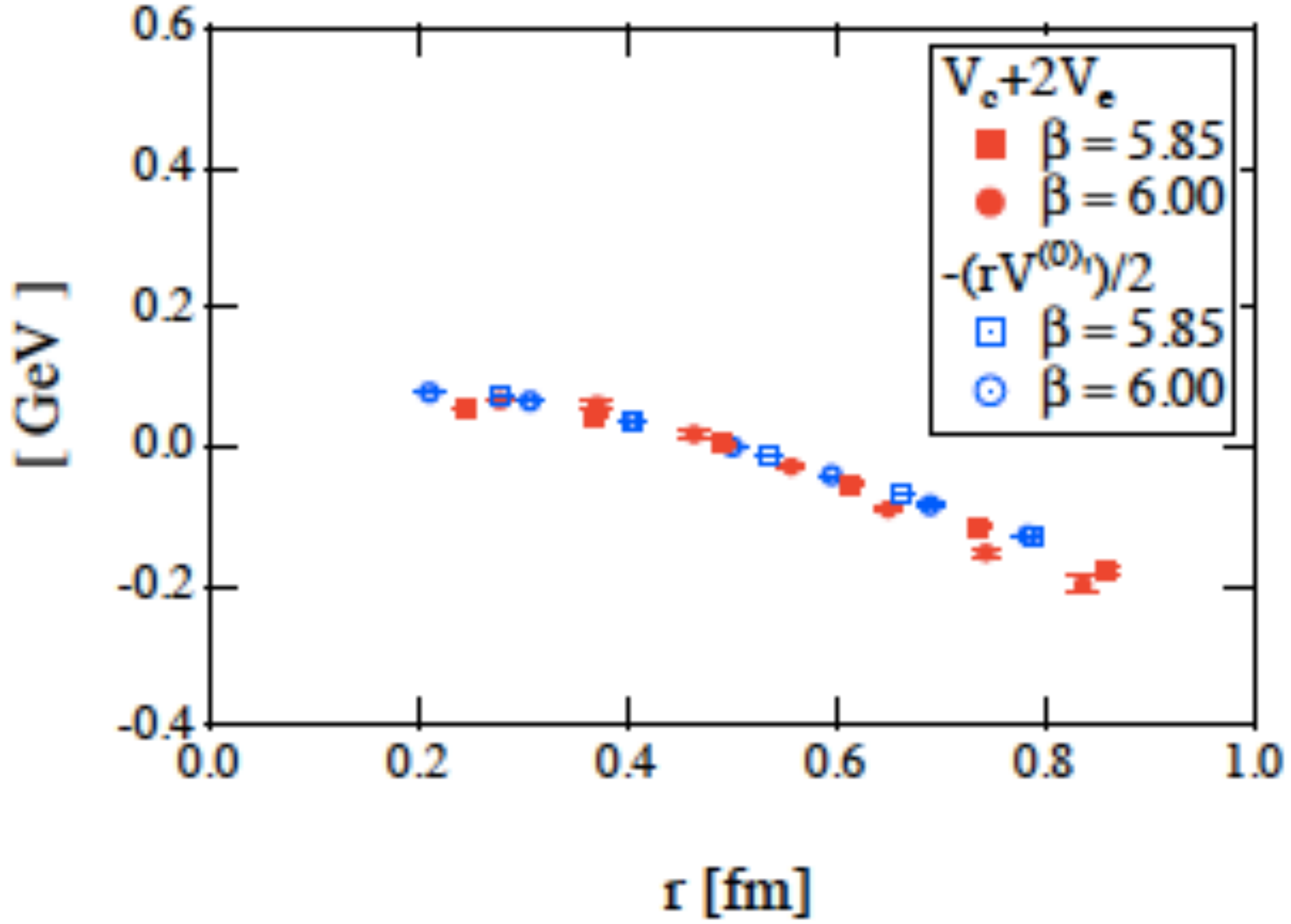
A lattice determination of $V_{LS}^{(2,0)} - V_{L_2S_1}^{(1,1)} + \frac{V^{(0)\prime}}{2r} = 0$



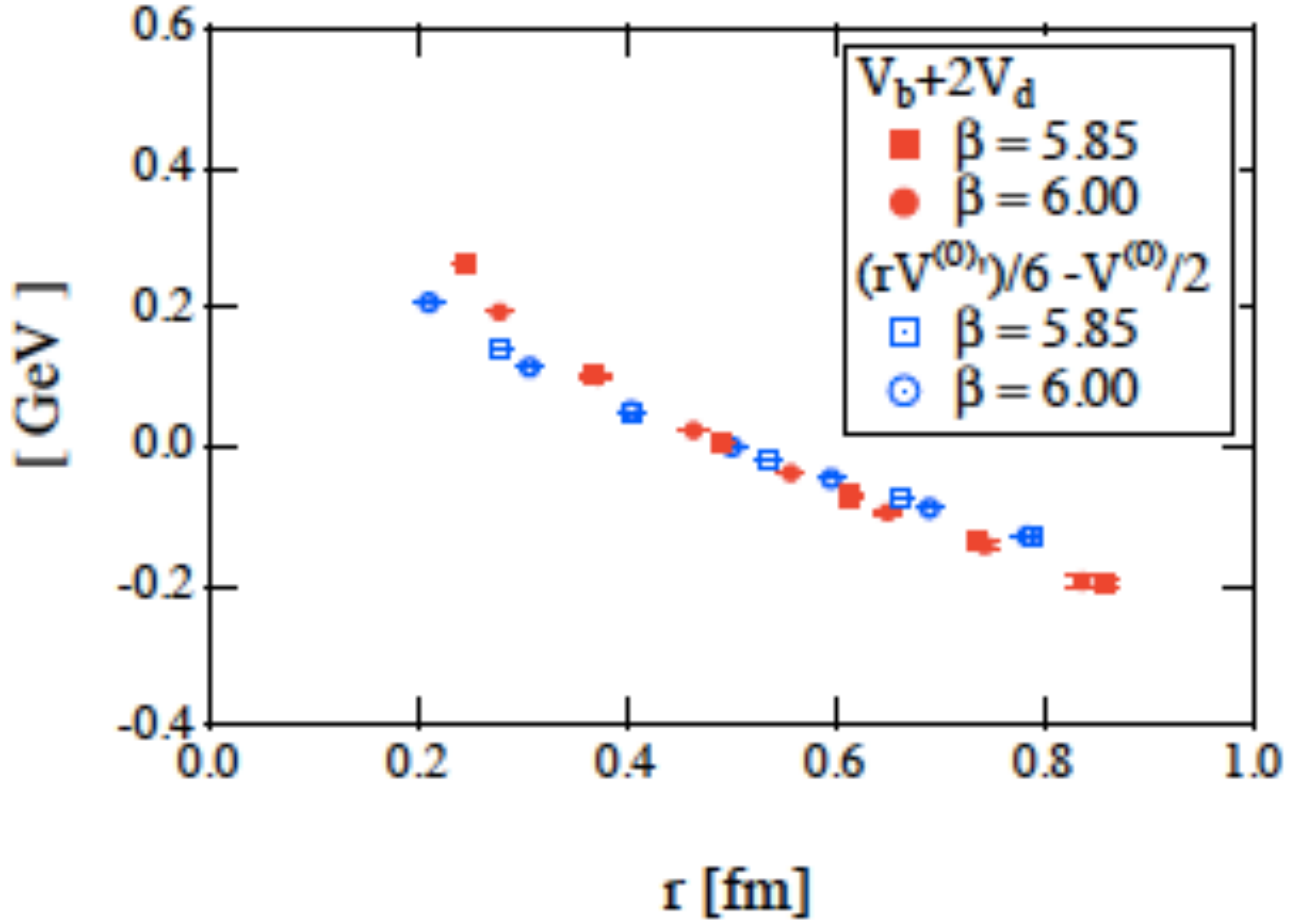
○ M.Koma Y.Koma NPB 769(07)79

A lattice determination of

$$V_{\mathbf{L}^2}^{(2,0)}(r) + V_{\mathbf{L}^2}^{(0,2)}(r)$$
$$-V_{\mathbf{L}^2}^{(1,1)}(r) + \tfrac{r}{2}V^{(0)\prime}(r) = 0$$



$$-2(V_{\mathbf{p}^2}^{(2,0)}(r) + V_{\mathbf{p}^2}^{(0,2)}(r))$$
$$+2V_{\mathbf{p}^2}^{(1,1)}(r) - V^{(0)}(r) + rV^{(0)\prime}(r) = 0$$



○ M.Koma Y.Koma Wittig PoS LAT2007(2007)111

Applications of strongly coupled pNRQCD include: Inclusive quarkonium decays

Imaginary parts of the Potential

$$\mathrm{Im}\, V_s\Big|_{\mathrm{P-wave}} = \Omega_{ij}^{SJ}\, \boldsymbol{\nabla}^i \delta^3(\mathbf{r}) \boldsymbol{\nabla}^j \times \left[3\,\frac{\mathrm{Im}\, f_1(^{2S+1}\mathrm{P}_J)}{m^4} + \frac{\mathcal{E}}{27}\,\frac{\mathrm{Im}\, f_8(^{2S+1}\mathrm{S}_S)}{m^4}\right]$$

where

$$\mathcal{E} = 18 \sum_{n \neq 0} \frac{\langle 0|g\mathbf{E}|n\rangle \cdot \langle n|g\mathbf{E}|0\rangle}{(E_n^{(0)} - E_0^{(0)})^4} = \frac{1}{2}\int_0^\infty dt\, t^3 \, \langle g\mathbf{E}^a(t,\mathbf{0})\Phi_{ab}(t,0;\mathbf{0})g\mathbf{E}^b(0,\mathbf{0})\rangle$$

The correlator
can be calculated
on the lattice

(1) N. Brambilla, H. S. Chung, D. Müller and A. Vairo
Decay and electromagnetic production of strongly coupled quarkonia in pN
JHEP 04 (2020) 095 `arXiv:2002.07462`

(2) N. Brambilla, D. Eiras, A. Pineda, J. Soto and A. Vairo
Inclusive decays of heavy quarkonium to light particles
Phys. Rev. D 67 (2003) 034018 `hep-ph/0208019`

(3) N. Brambilla, D. Eiras, A. Pineda, J. Soto and A. Vairo
New predictions for inclusive heavy quarkonium P-wave decays
Phys. Rev. Lett. 88 (2002) 012003 `hep-ph/0109130`

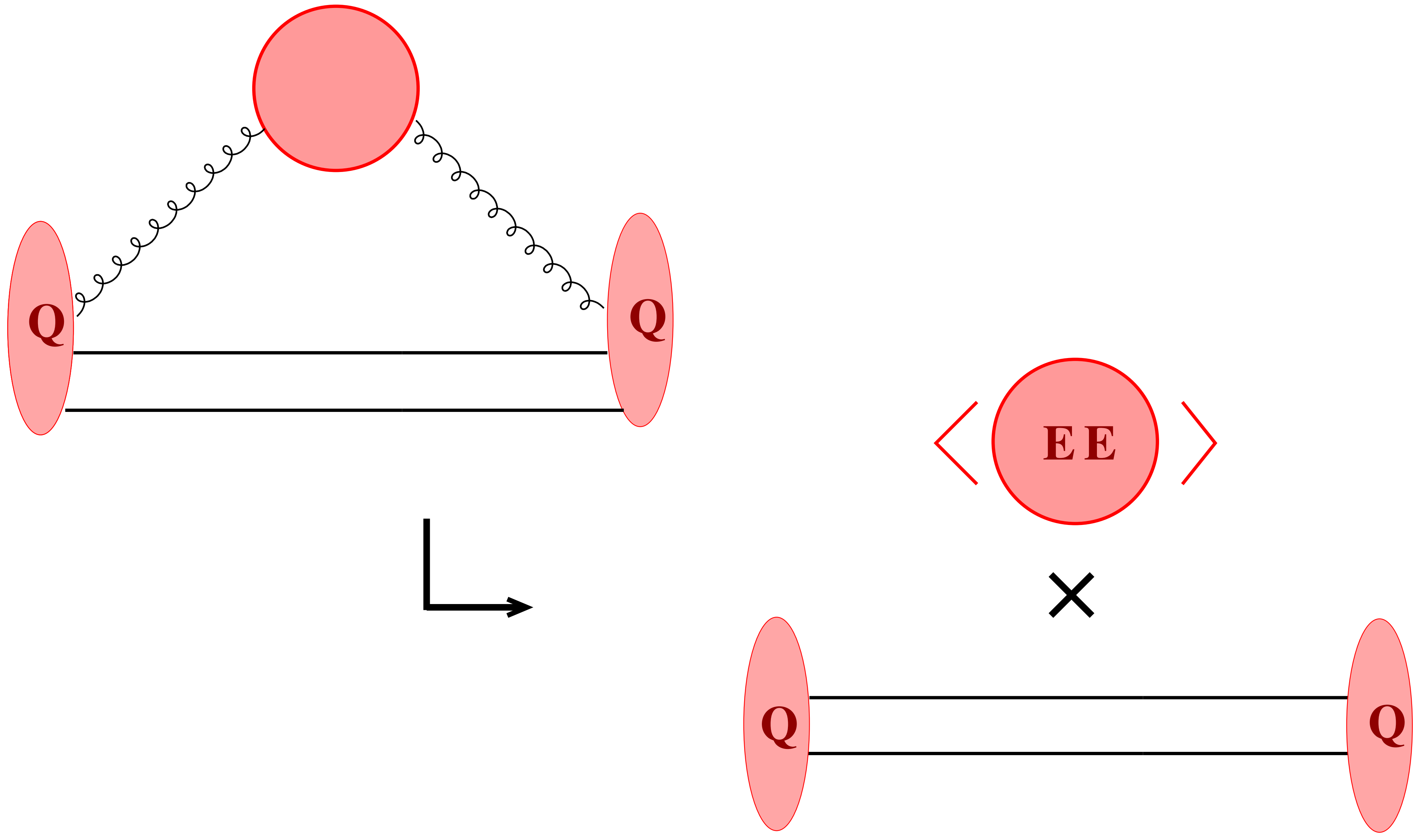
P-wave decays at $\mathcal{O}(mv^5)$

$$\Gamma(\chi_J \to \mathrm{LH}) = \frac{\left|R'(0)\right|^2}{\pi m^4}\left[9\,\mathrm{Im}\,f_1 + \frac{\mathrm{Im}\,f_8}{9}\,\mathcal{E}\right]$$

$$\Gamma(\chi_J \to \gamma\gamma) = 9\,\mathrm{Im}\,f_{\gamma\gamma}\,\frac{\left|R'(0)\right|^2}{\pi m^4} \quad J = 0, 2$$

Brambilla et al. 01, 02, 03

octet matrix element



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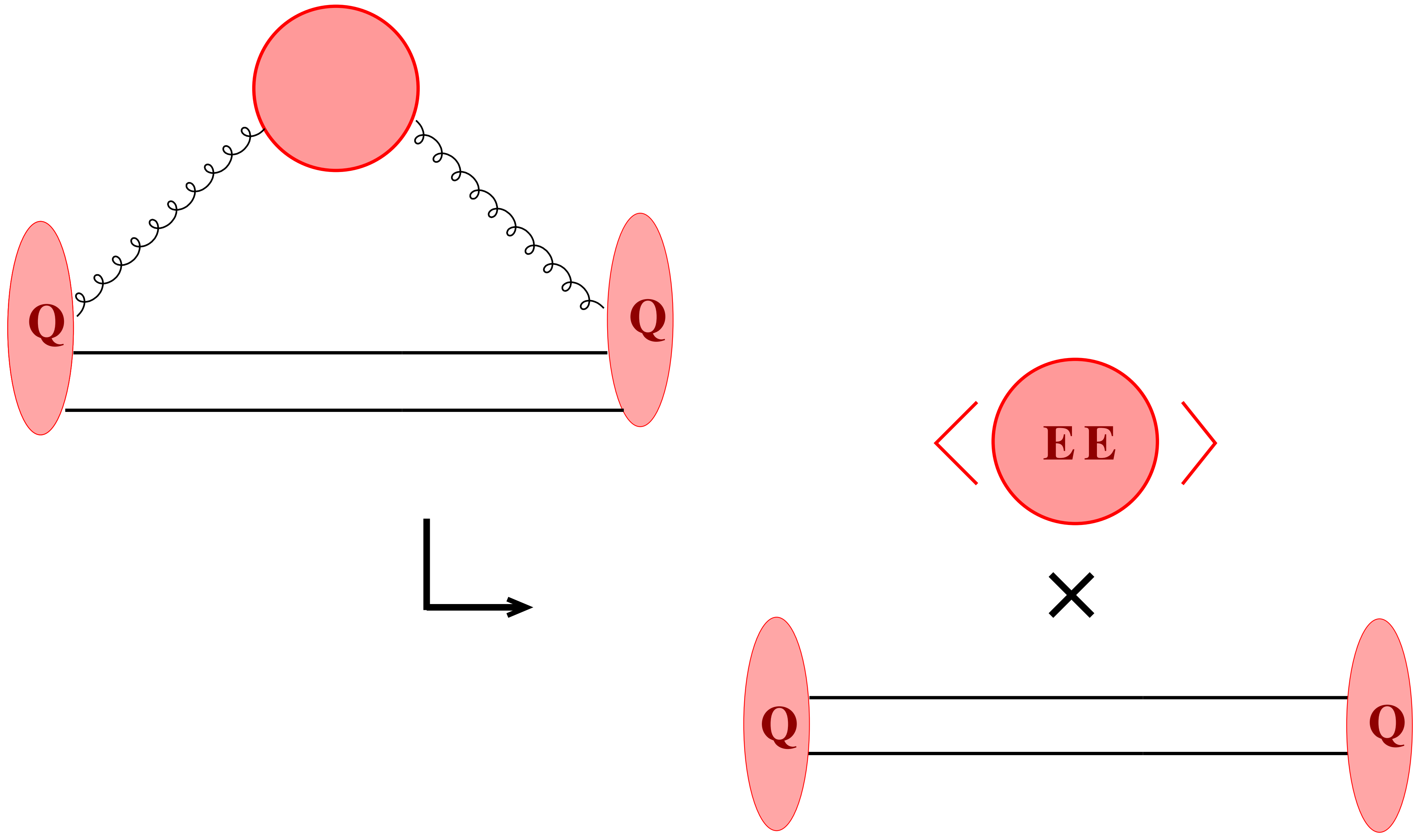

- $\langle\chi|O_8({}^1S_0)|\chi\rangle = \frac{\left|R'(0)\right|^2}{18\pi m^2}\mathcal{E}; \quad \mathcal{E} \equiv \int_0^{\infty} dt\, t^3\, \langle \mathrm{Tr}(g\mathbf{E}(t)\, g\mathbf{E}(0))\rangle$

- *The quarkonium state dependence factorizes.*

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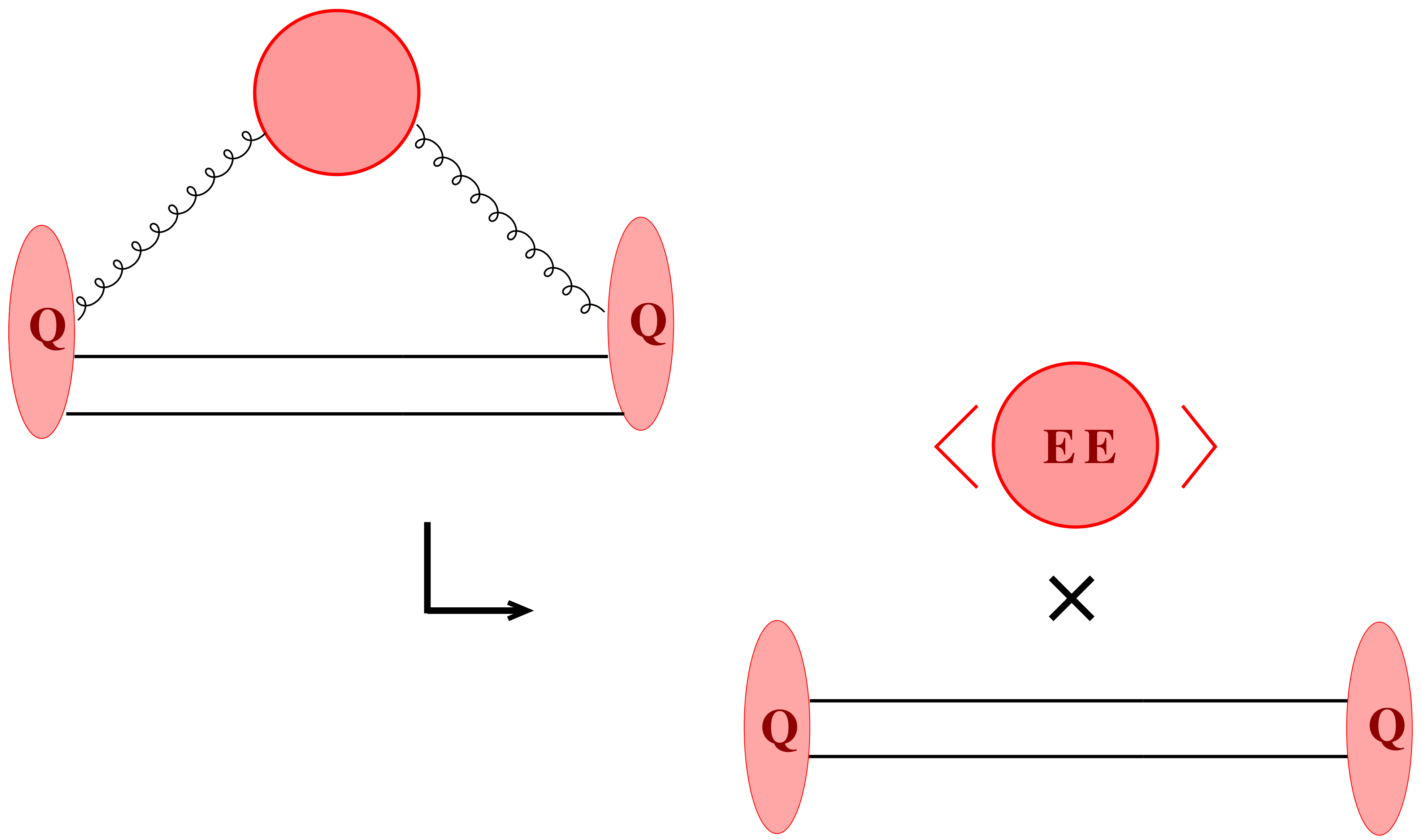

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* *Bottomonium and charmonium (below threshold) P-wave decays depend on 4 non-perturbative parameters [3 w.f. + 1 corr.].*

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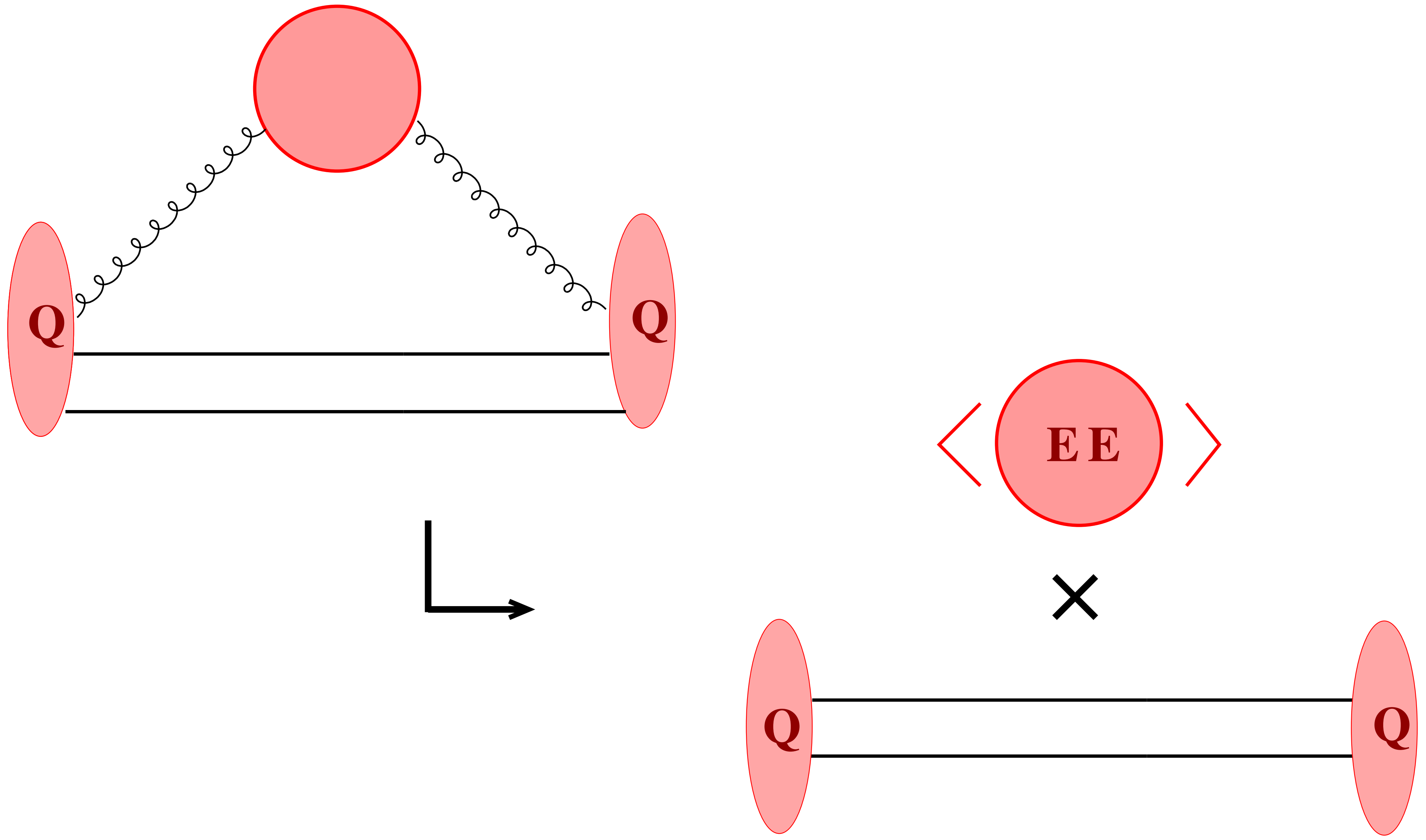

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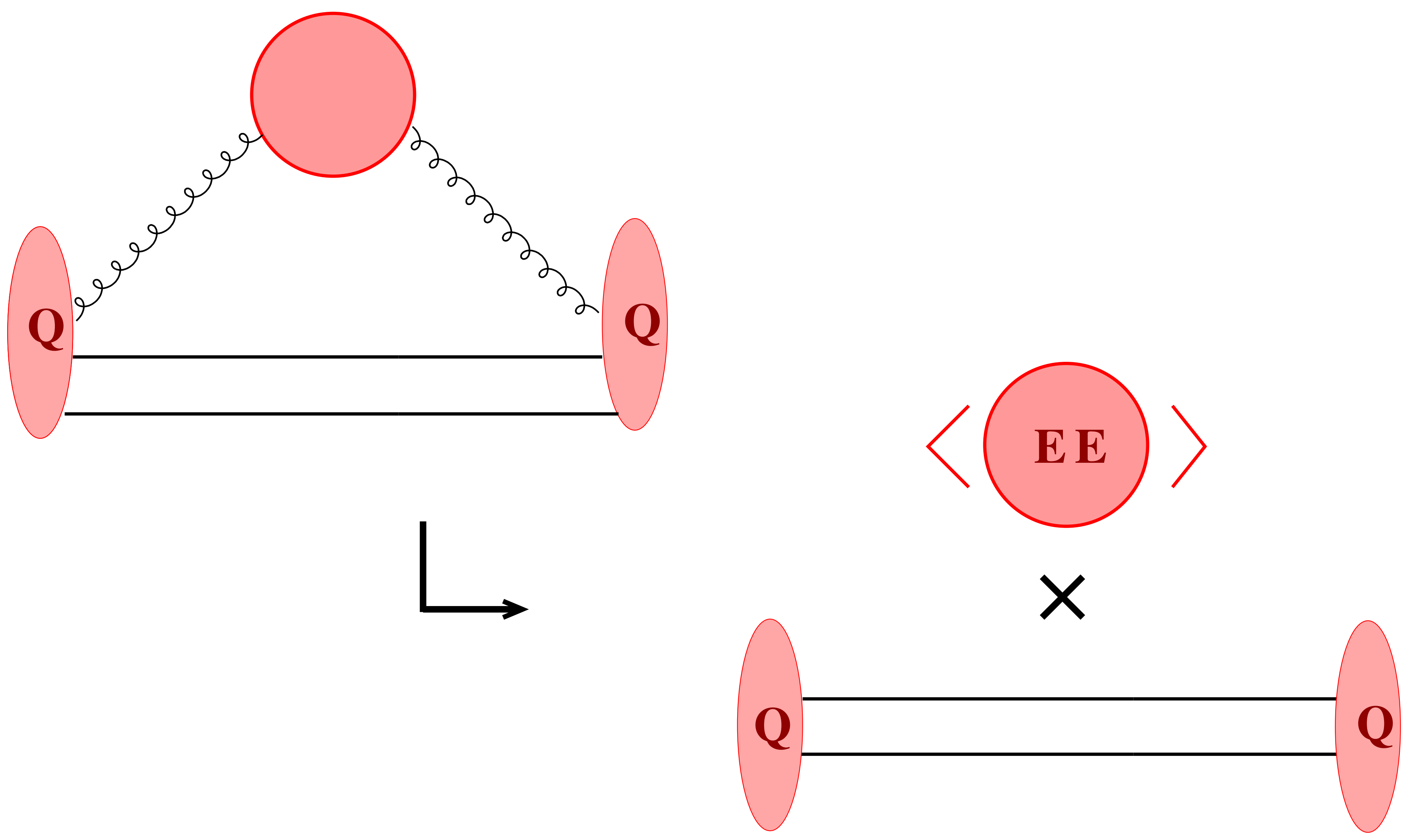

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instead that on 6 nonperturbative parameters with 3 LDMEs in NRQCD

- The IR divergence of $\mathrm{Im}\, f_1$ cancels against the chromoelectric correlator $\mathcal{E}_3$.

$\mathcal{E}_3(\Lambda)$ can be obtained from a least squares fit to the ratios of decay rates $\Gamma(\chi_{c0}(1P) \to \mathrm{LH})/\Gamma(\chi_{c1}(1P) \to \mathrm{LH})$, $\Gamma(\chi_{c1}(1P) \to \mathrm{LH})/\Gamma(\chi_{c2}(1P) \to \mathrm{LH})$, $\Gamma(\chi_{c0}(1P) \to \mathrm{LH})/\ \Gamma(\chi_{c0}(1P) \to \gamma\gamma)$, and $\Gamma(\chi_{c2}(1P) \to \mathrm{LH})/\Gamma(\chi_{c2}(1P) \to \gamma\gamma)$ (from PDG) at leading order in v. In the $\overline{\mathrm{MS}}$ scheme, we obtain

$$\mathcal{E}_3(1\ \text{GeV}) = 2.05^{+0.94}_{-0.65}.$$

$\mathcal{E}_3(\Lambda)$ at different scales follows from the one loop renormalization group improved expression ($\beta_0 = 11N_c/3 - 4T_F n_f/3$)

$$\mathcal{E}_3(\Lambda) = \mathcal{E}_3(\Lambda') + \frac{24C_F}{\beta_0} \log \frac{\alpha_\mathrm{s}(\Lambda')}{\alpha_\mathrm{s}(\Lambda)},$$

The octet matrix element on charmonium state is

$$\langle \chi_{cJ}(1P)|\mathcal{O}_8(^1S_0)|\chi_{cJ}(1P)\rangle = (3.53^{+1.05}_{-1.15}{}^{+1.62}_{-1.12}) \times 10^{-3}\ \text{GeV}^3$$

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P-wave charmonium annihilation widths

From the ratio of widths and the experimental value of the electromagnetic width, we get

$$\Gamma(\chi_{c0}(1P) \to \mathrm{LH}) = 8.3^{+3.0}_{-3.1}\ \text{MeV} \qquad [10.6 \pm 0.6\ \text{MeV from PDG}]$$

$$\Gamma(\chi_{c1}(1P) \to \mathrm{LH}) = 0.42^{+0.06+0.28}_{-0.06-0.22}\ \text{MeV} \qquad [0.552 \pm 0.041\ \text{MeV from PDG}]$$

$$\Gamma(\chi_{c2}(1P) \to \mathrm{LH}) = 1.4^{+0.6}_{-0.6}\ \text{MeV} \qquad [1.60 \pm 0.09\ \text{MeV from PDG}]$$

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$$\Gamma(\chi_{c2}(1P) \to \mathrm{LH}) = 1.4^{+0.6}_{-0.6}\ \text{MeV} \qquad [1.60 \pm 0.09\ \text{MeV from PDG}]$$

P-wave bottomonium annihilation widths

From the ratio of widths and the experimental value of the electromagnetic width, we get

$$\Gamma(\chi_{b0}(nP) \to \mathrm{LH}) = 1.07^{+0.33}_{-0.37}\ \text{MeV}$$
$$\Gamma(\chi_{b1}(nP) \to \mathrm{LH}) = 0.14 \pm 0.06\ \text{MeV}$$
$$\Gamma(\chi_{b2}(nP) \to \mathrm{LH}) = 0.28^{+0.09}_{-0.10}\ \text{MeV}$$

which are almost independent of the principal quantum number $n = 1, 2, 3$.

These are proper predictions that exploit the universality of the chromoelectric correlator.

Applications of pNRQCD include:

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quarkonium production

pNRQCD for hadroproduction

(1) N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Inclusive production of J/ψ, $\psi(2S)$ and Υ states in pNRQCD
JHEP 03 (2023) 242 `arXiv:2210.17345`

(2) N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Production and polarization of S-wave quarkonia in potential Nonrelativistic QCD
Phys. Rev. D 105 (2022) 11, L111503 `arXiv:2203.07778`

(3) N. Brambilla, H. S. Chung and A. Vairo
Inclusive production of heavy quarkonia in pNRQCD
JHEP 09 (2021) 032 `arXiv:2106.09417`

(4) N. Brambilla, H. S. Chung and A. Vairo
Inclusive hadroproduction of P-wave heavy quarkonia in potential Nonrelativistic QCD
Phys. Rev. Lett. 126 (2021) 8 `arXiv:2007.07613`

Applications of pNRQCD include:

quarkonium at finite temperature and non-equilibrium evolution in medium

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **96** (2017) no.3, 034021 [arXiv:1612.07248 [hep-ph]].

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **97** (2018) no.7, 074009 [arXiv:1711.04515 [hep-ph]].

N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, Phys. Rev. D **100** (2019) no.5, 054025 [arXiv:1903.08063 [hep-ph]].

N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, [arXiv:2012.01240 [hep-ph]].

N. Brambilla, M. Escobedo, A. Islam, M. Strickland . A. Tiwari, A. Vairo arXiv 2205.10289

N. Brambilla, M. Escobedo, et al **2302.11826,** 2403.15545

N. Brambilla,J. Ghiglieri, A. Vairo, Peter Petreczky
Phys.Rev.D 78 (2008) 014017 • e-Print: 0804.0993 [hep-ph]

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JHEP 09 (2010) 038 • e-Print: 1007.4156 [hep-ph]

quarkonium production

pNRQCD for hadroproduction

(1) N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Inclusive production of J/ψ, $\psi(2S)$ and Υ states in pNRQCD
JHEP 03 (2023) 242 `arXiv:2210.17345`

(2) N. Brambilla, H. S. Chung, A. Vairo and X. P. Wang
Production and polarization of S-wave quarkonia in potential Nonrelativistic QCD
Phys. Rev. D 105 (2022) 11, L111503 `arXiv:2203.07778`

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Inclusive production of heavy quarkonia in pNRQCD
JHEP 09 (2021) 032 `arXiv:2106.09417`

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Phys. Rev. Lett. 126 (2021) 8 `arXiv:2007.07613`

N. Brambilla, V. Leino, Peter Petreczky, A. Vairo e-Print 2007.10078 [hep-ph],
N. Brambilla, V. Leino, J. Meyer Steudte,P. Petreczky 2206.02861

Lattice calculation of the heavy quark transport coefficient

Applications of pNRQCD include:

quarkonium at finite temperature and non-equilibrium evolution in medium

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **96** (2017) no.3, 034021 [arXiv:1612.07248 [hep-ph]].

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **97** (2018) no.7, 074009 [arXiv:1711.04515 [hep-ph]].

N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, Phys. Rev. D **100** (2019) no.5, 054025 [arXiv:1903.08063 [hep-ph]].

N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, [arXiv:2012.01240 [hep-ph]].

N. Brambilla, M. Escobedo, A. Islam, M. Strickland . A. Tiwari, A. Vairo arXiv 2205.10289

N. Brambilla, M. Escobedo, et al **2302.11826,** 2403.15545

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WE CAN USE THE SAME FRAMEWORK TO ADDRESS XYZ PRODUCTION and PROPAGATION in MEDIUM in BOEFT!

Applications of pNRQCD include:

quarkonium at finite temperature
and non-equilibrium evolution in medium

quarkonium production

WE CAN USE THE SAME FRAMEWORK TO ADDRESS XYZ PRODUCTION
and PROPAGATION in MEDIUM in BOEFT!

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Notice that the lattice cannot access directly these processes!

The EFT act as an intermediate layer to allow use lattice calculation

on the low energy scale

WE CAN USE THE SAME FRAMEWORK TO ADDRESS XYZ PRODUCTION
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Applications of strongly coupled pNRQCD include: Quarkonium Production at LHC

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NRQCD factorization formula for quarkonium production
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Bodwin Braaten Lepage 1995

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long distance matrix elements (LDME) give the probability of a qqbar pair with certain quantum number to evolve into a final quarkonium H

short distance coefficients
partonic hard scattering cross section
convoluted with parton distribution

they are vacuum expectation values of four fermion operators with color singlet and color octet contributions and a projection over quarkonium plus X in the middle

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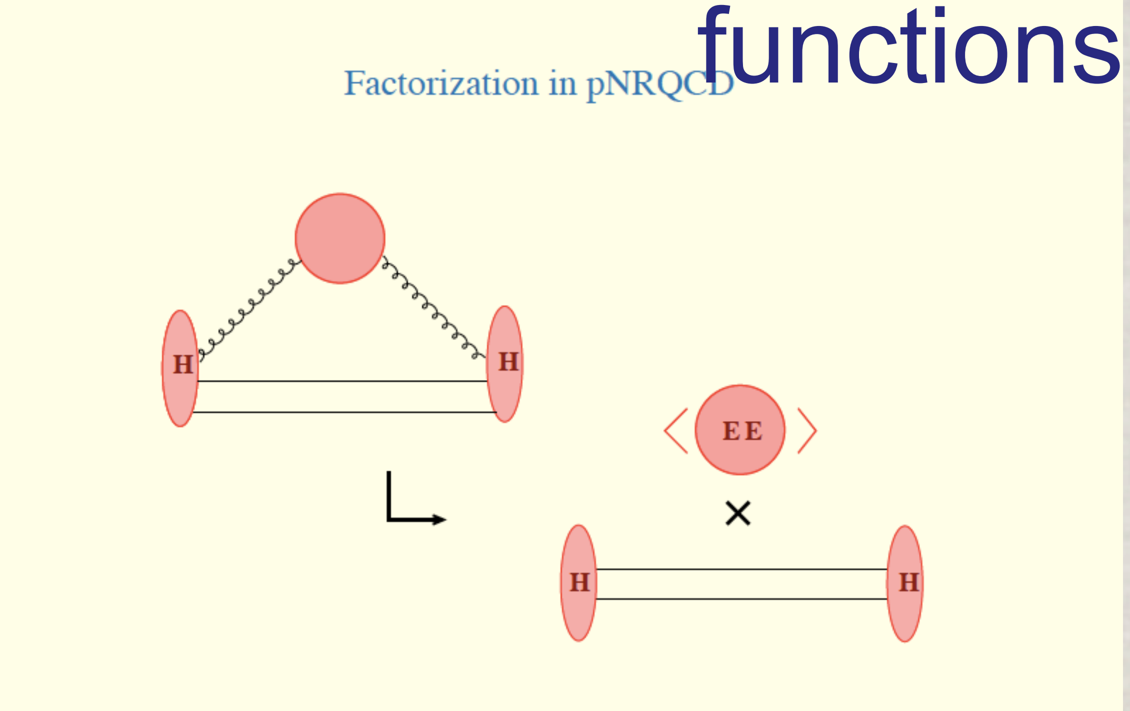
One problem is the proliferation of LDMEs:
nonperturbative objects
that cannot evaluated on the lattice
and should be extracted from the data,
they depend on the considered quarkonium state

Intense work in the theory community, within QCD, NRQCD and SCET,

Qiu, Nayak, Sterman, Butenschon Kniehl , Bodwin , Hee Soh, Chung, J. Lee, Kuang Ta Chao, Y. Q. Ma, Gong Wang, Fleming, Mehen, Yu Jia, Braaten, Lansberg, Leibovich, Rothstein…

Factorization of LDMEs in pNRQCD : the NRQCD LDMEs are factorized in terms of wave functions and universal nonperturbative correlators depending only on the glue



- The number of nonperturbative unknowns is reduced by half
- The nonperturbative unknowns are correlators of gluonic fields that can be calculated on the lattice

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

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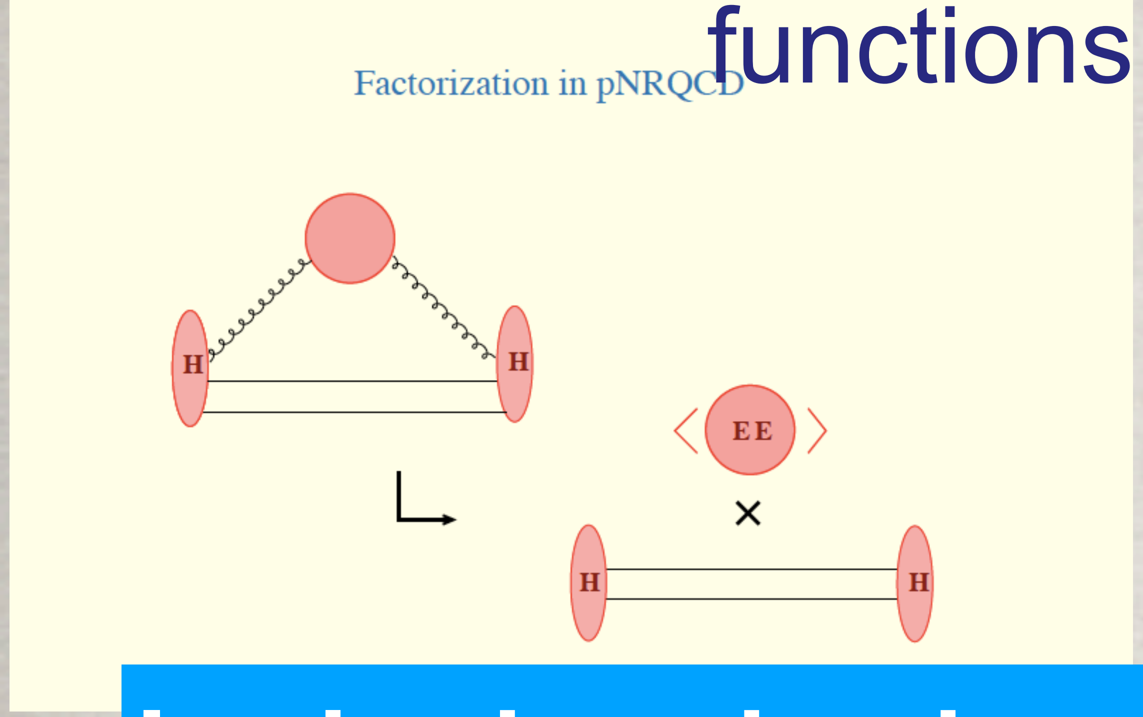
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Inclusive hadroproduction of p wave quarkonia

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle$$
$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle.$$

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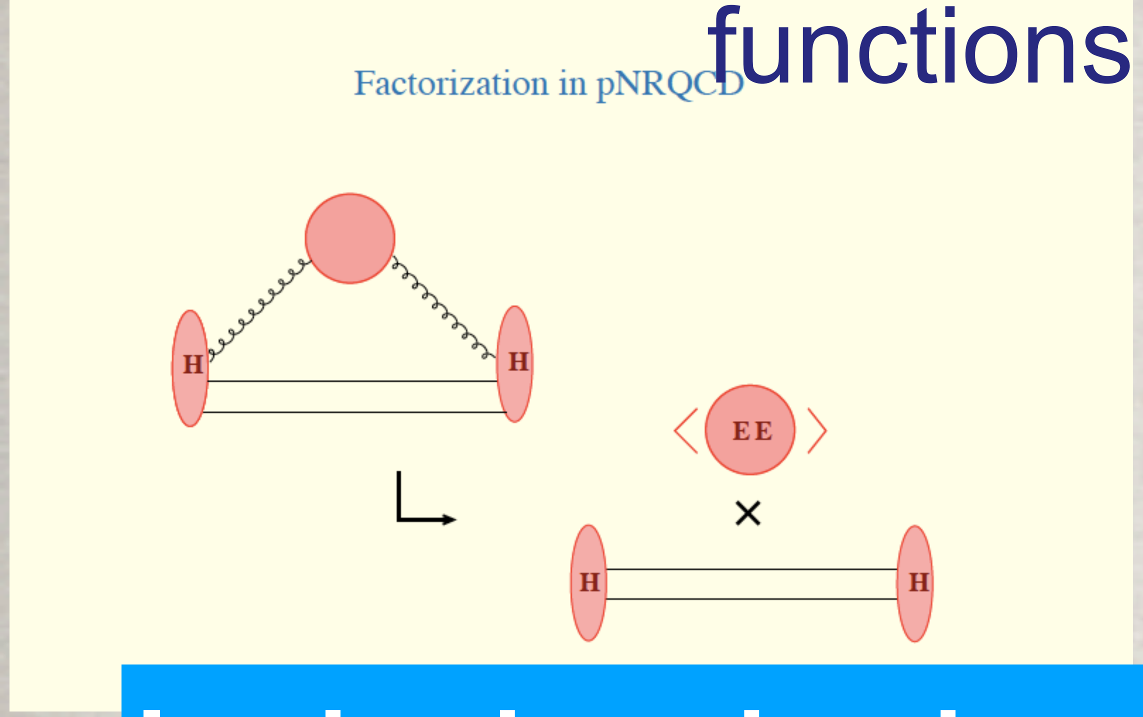

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$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}$$

$$\frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$$

$$+ (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}$$

$$\frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \frac{\mathcal{E}}{9N_c m^2}$$

- The dimensionless correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE with Wilson lines Φ extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c}\int_0^\infty t\,dt \int_0^\infty t'\,dt' \langle\Omega|\Phi_\ell^{\dagger ab}\Phi_0^{\dagger da}(0,t)gE^{d,i}(t)gE^{e,i}(t')\Phi_0^{ec}(t',0)\Phi_\ell^{bc}|\Omega\rangle$$

- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

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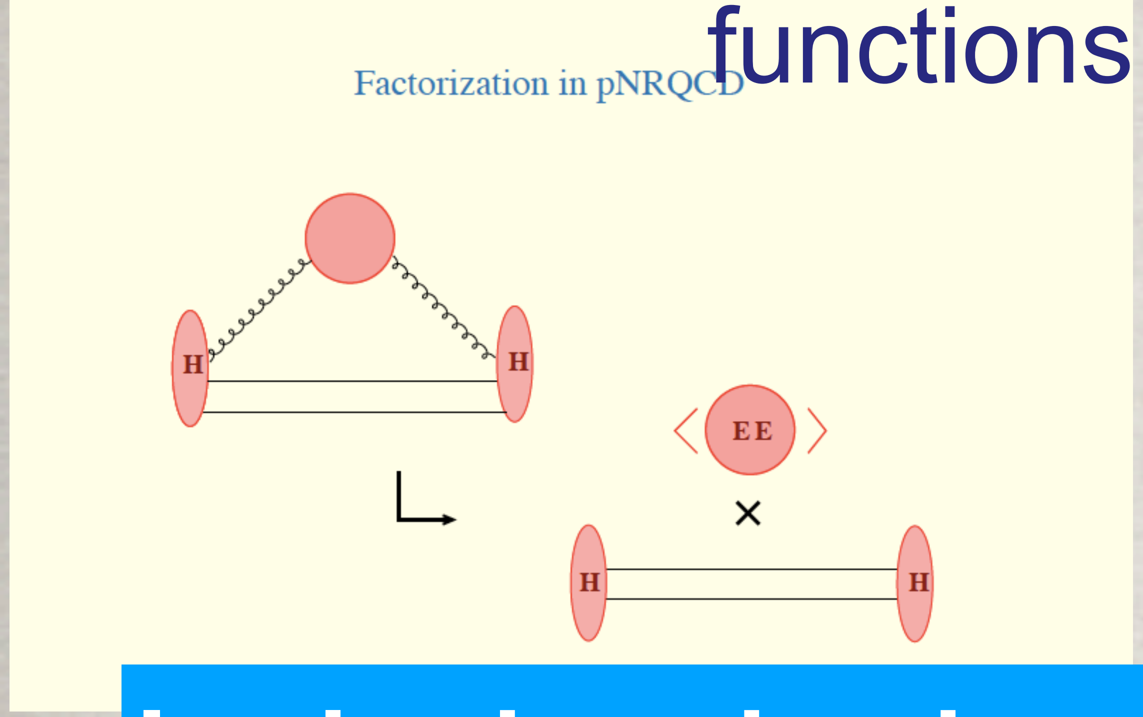

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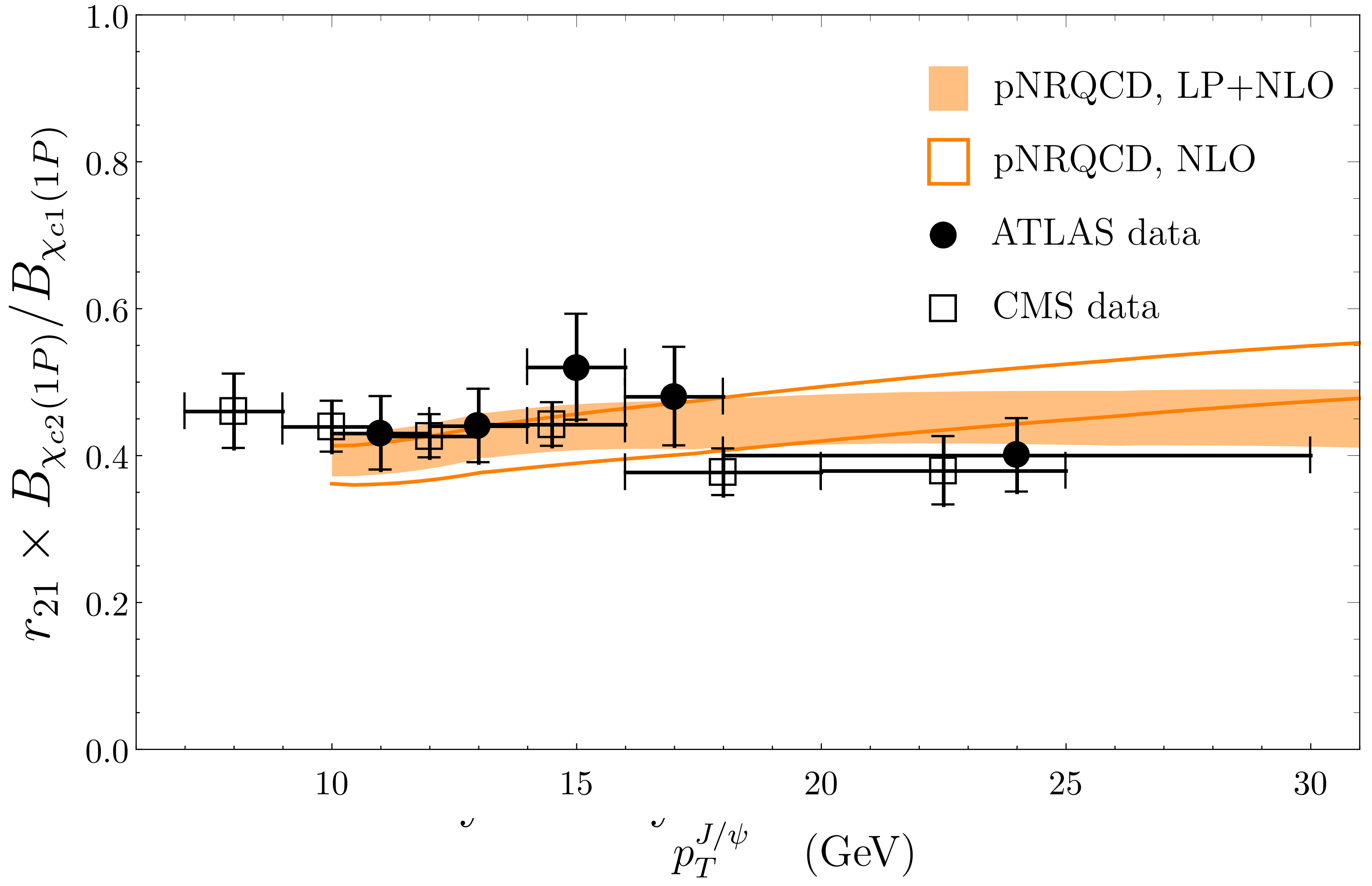
- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

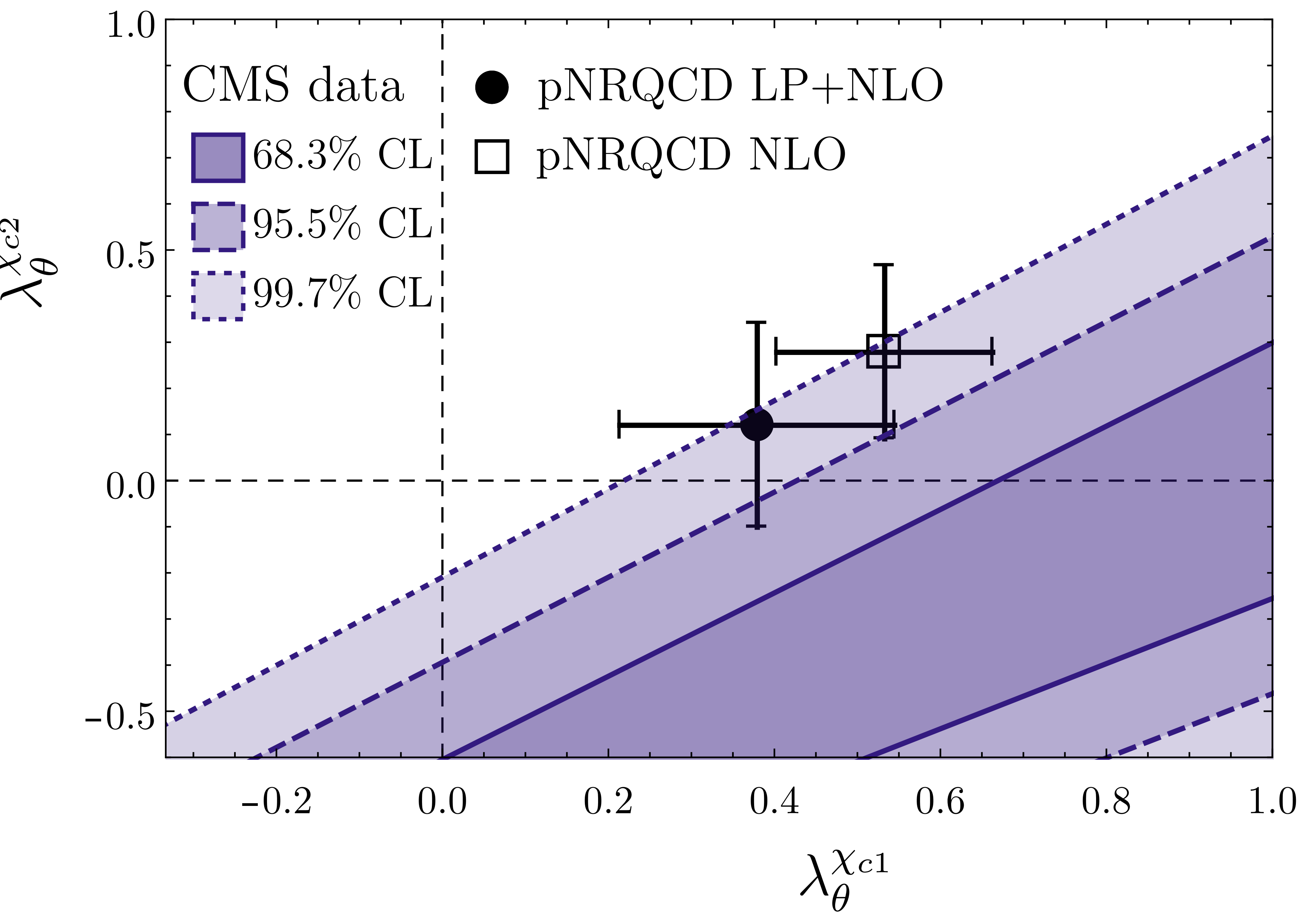
$\mathcal{E}$ is a universal quantity that ***does not depend on quark flavor or radial excitation***. Determination of $\mathcal{E}$ directly leads to ***determination of all*** χ_{cJ} ***and*** $\chi_{bJ}(nP)$ ***cross sections, as well as*** h_c ***and*** h_b ***production rates***.

->good description of data at ATLAS and CMS

N. B. Chung Muller Vairo 2002.07462, N.B. Chung Vairo 2007.07613

fix non pert coefficient here-> predict the rest





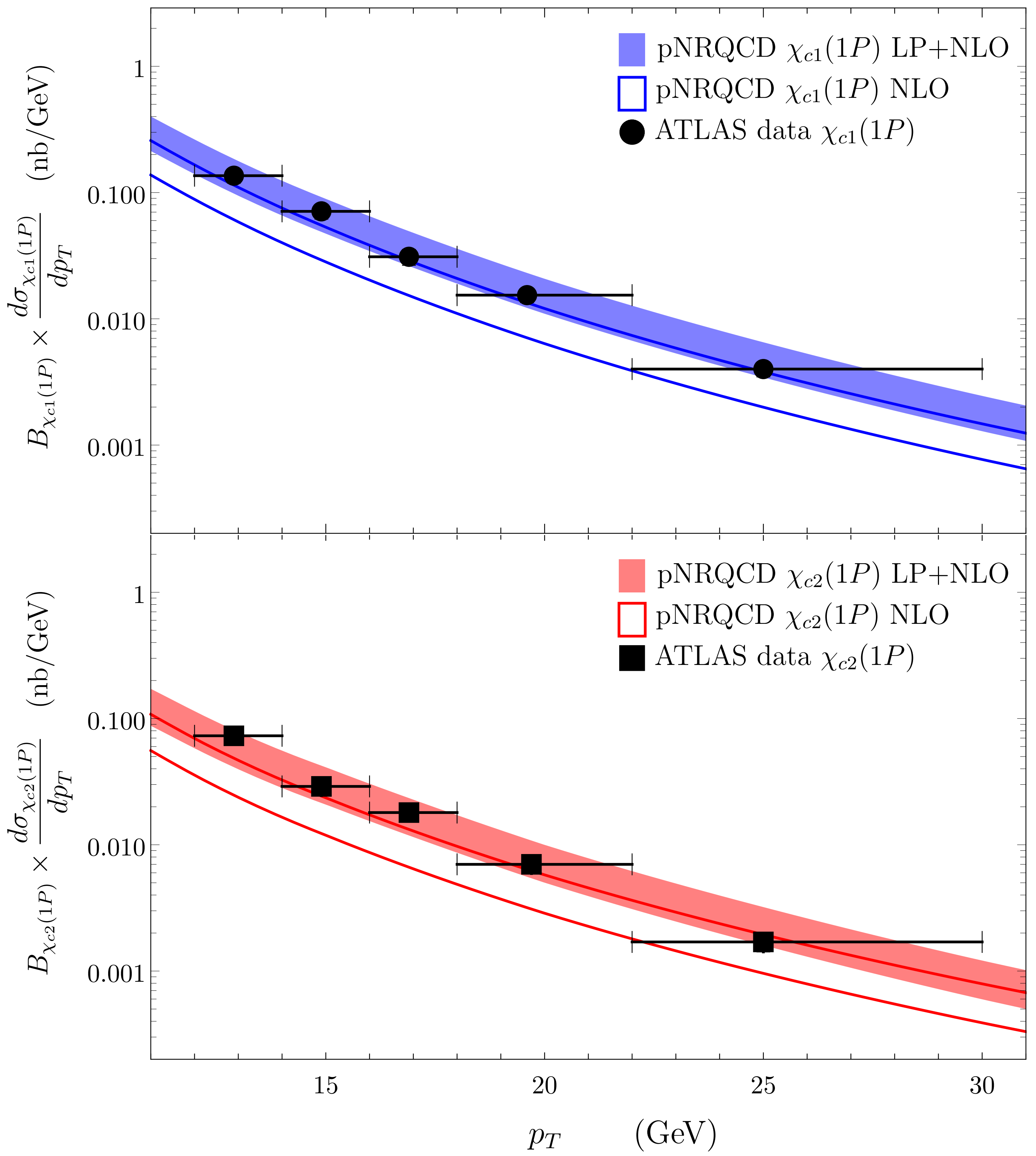
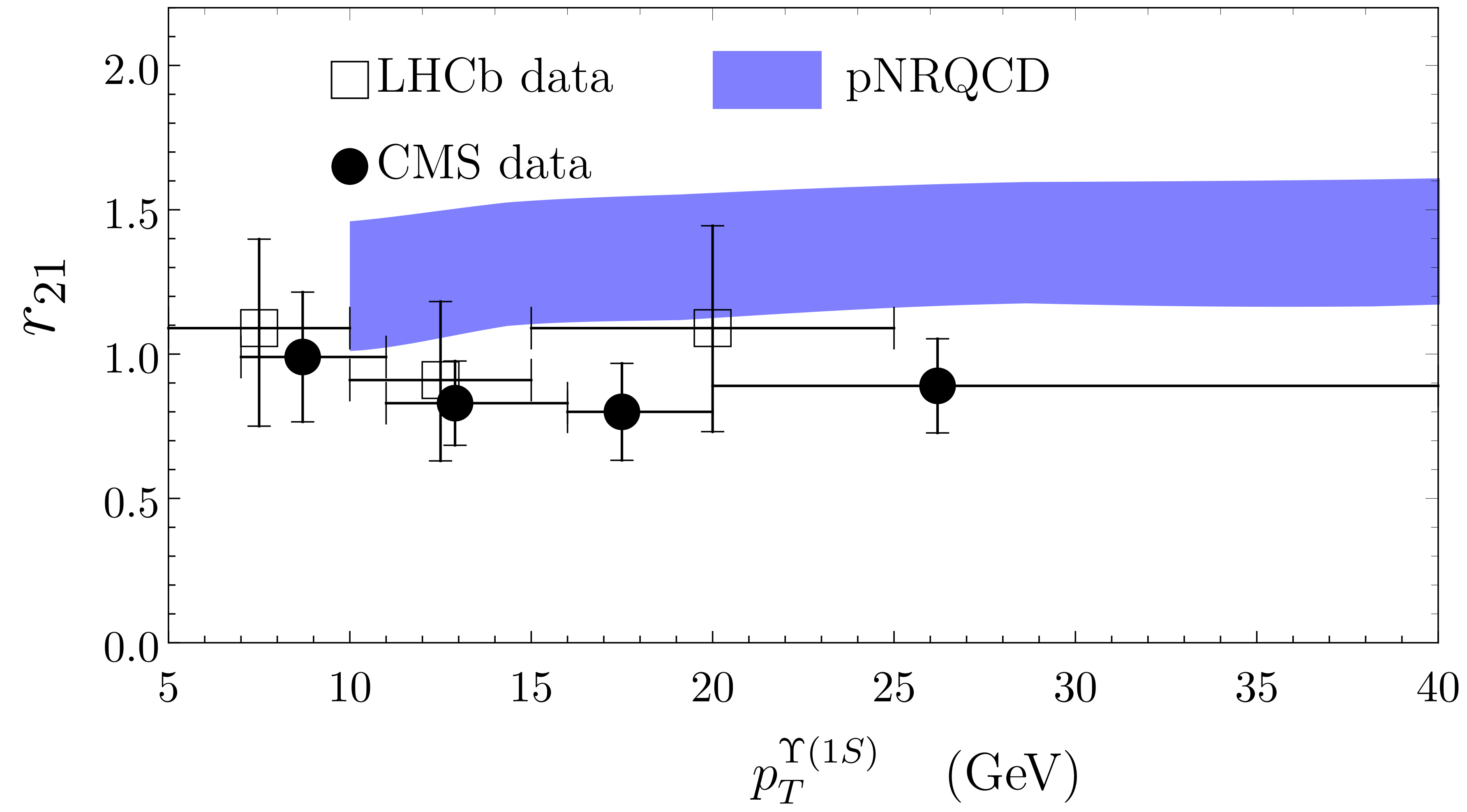


Figure 4. Production cross sections of the $\chi_{c1}(1P)$ and $\chi_{c2}(1P)$ at the LHC center of mass energy $\sqrt{s} = 7$ TeV and in the rapidity range $|y| < 0.75$ compared with ATLAS measurements [41].



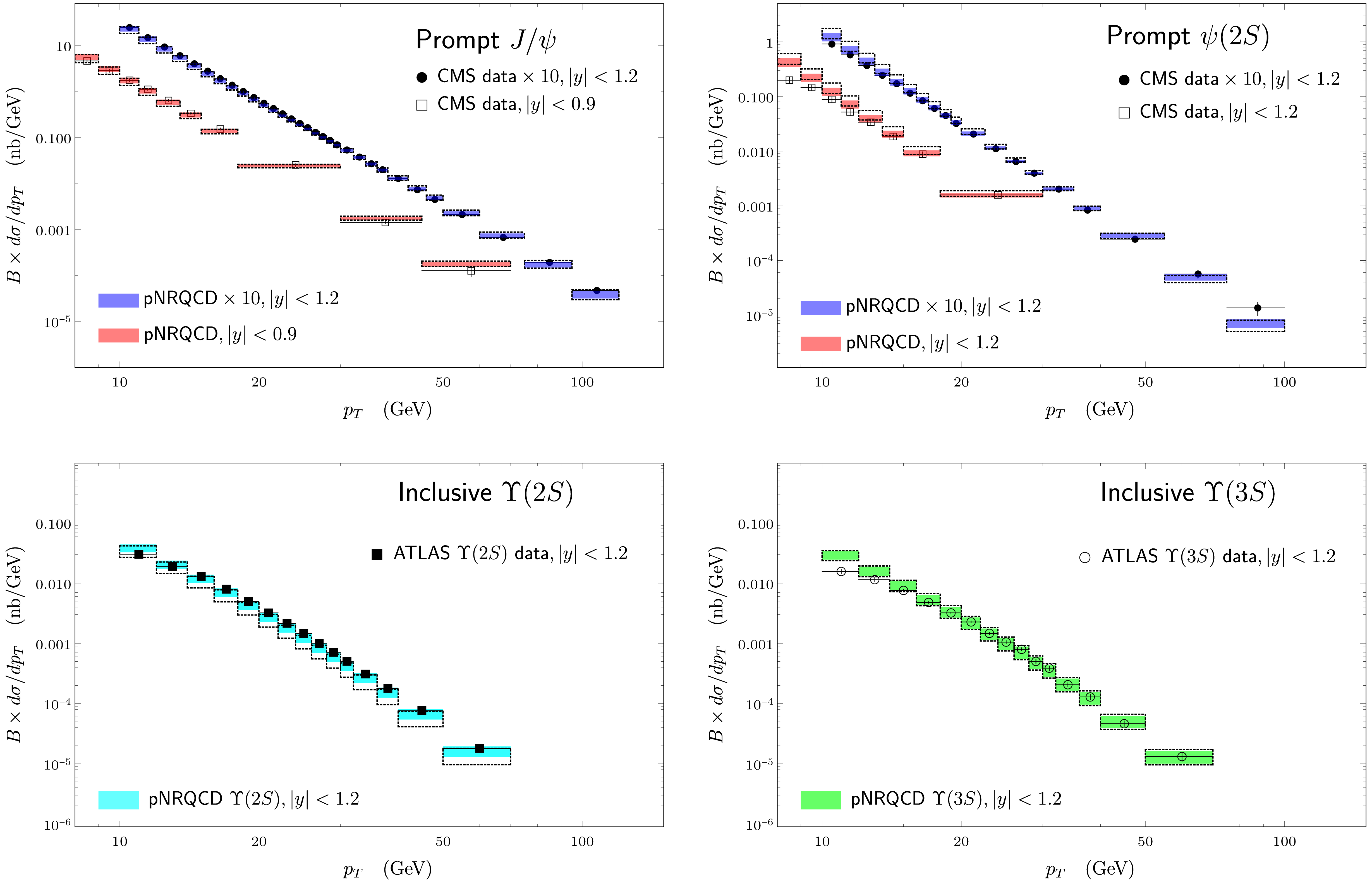
- e-Print: 2106.09417 , N.B., Hee Sok Chung, A. Vairo

We applied the same procedure to the study of S-wave quarkonium production->

3 octet LDMEs—> 3 nonperturbative correlators independent of the flavour that could be could be calculates on the lattice

- :2210.17345

$\sigma(pp \to J/\psi + X), \sigma(pp \to \psi(2S) + X)$ and $\sigma(pp \to \Upsilon(nS) + X)$

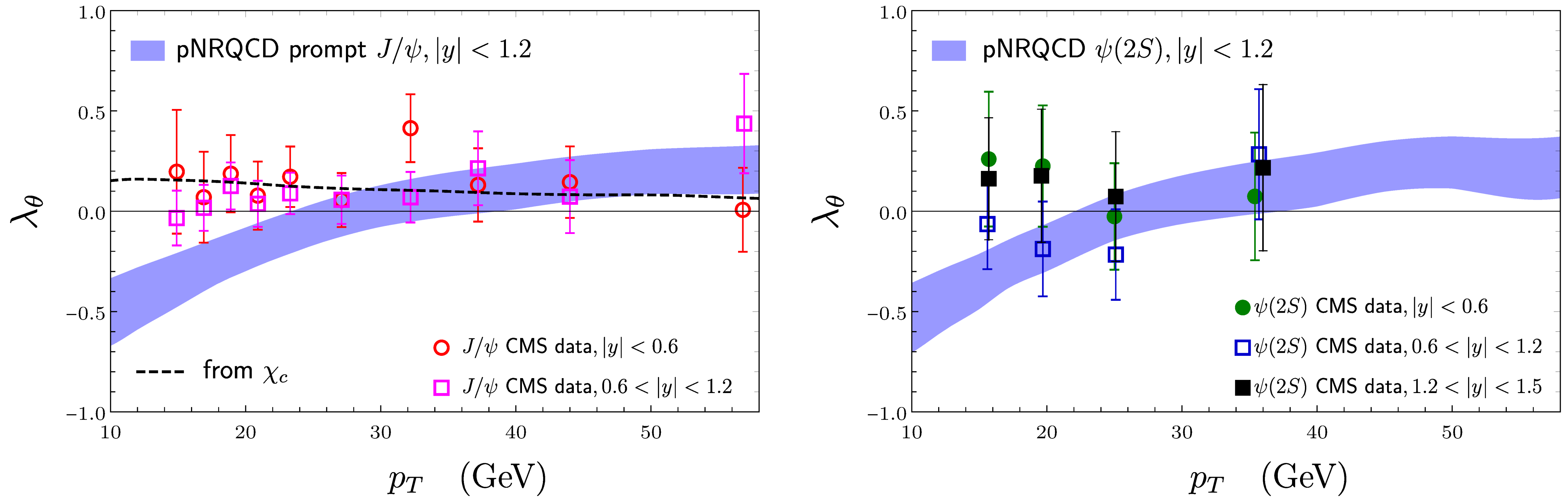


@ center of mass energy $\sqrt{s} = 7$ TeV.

○ CMS coll JHEP 02 (2012) 011, PRL 114 (2015) 191802

ATLAS coll PRD 87 (2013) 052004

Polarization of J/ψ and $\psi(2S)$



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○ CMS coll PRL 110 (2013) 081802, PLB 727 (2013) 381

What about the XYZ?

What about the XYZ?

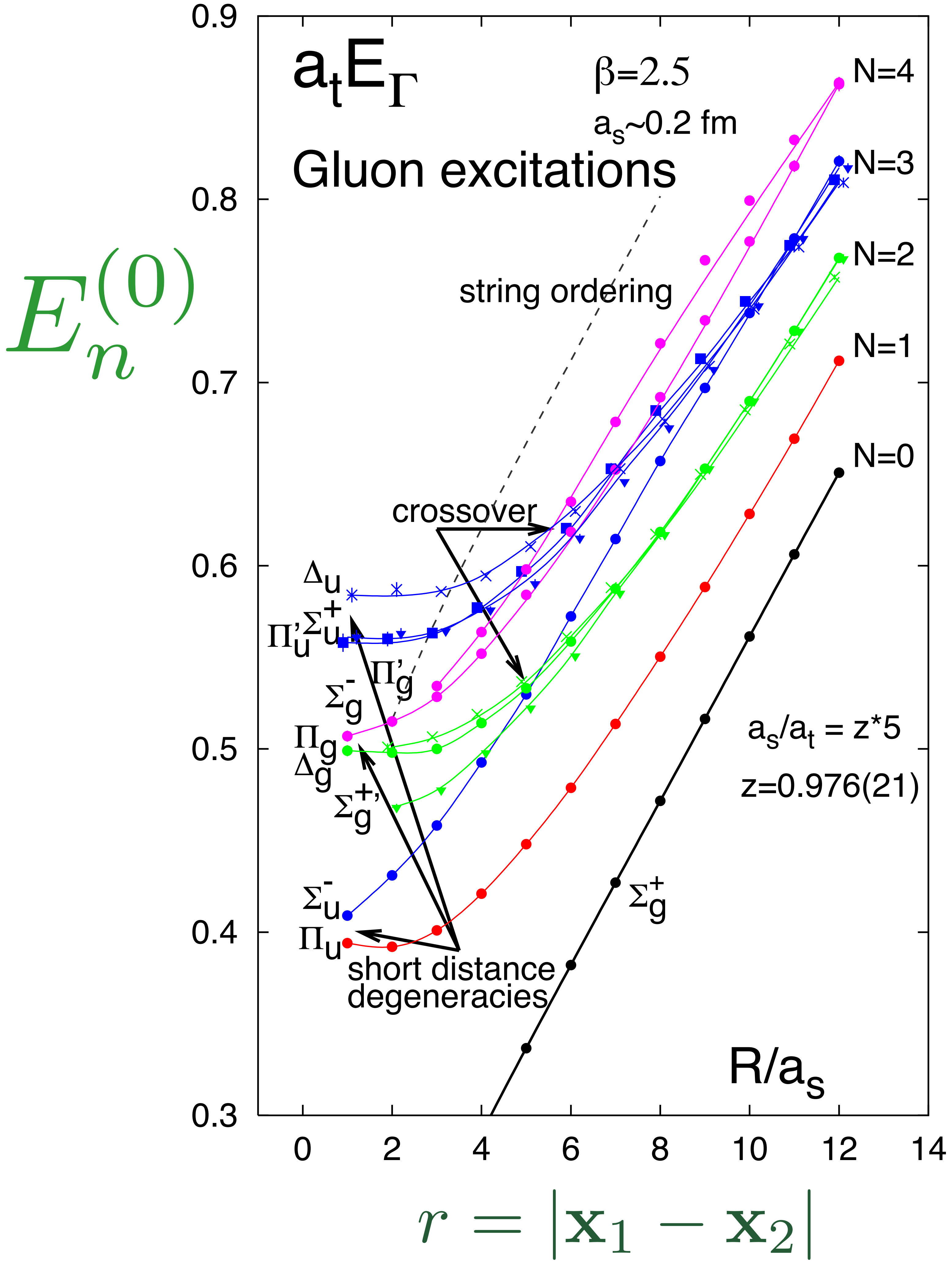
let us start with the hybrids

Exotics: Hybrids

In QCD there is not only the QQbar static energy, the usual confinement potential but there is a tower of static energies!

Exotics: Hybrids

Lattice Spectrum of NRQCD hybrid static energies E^0_n



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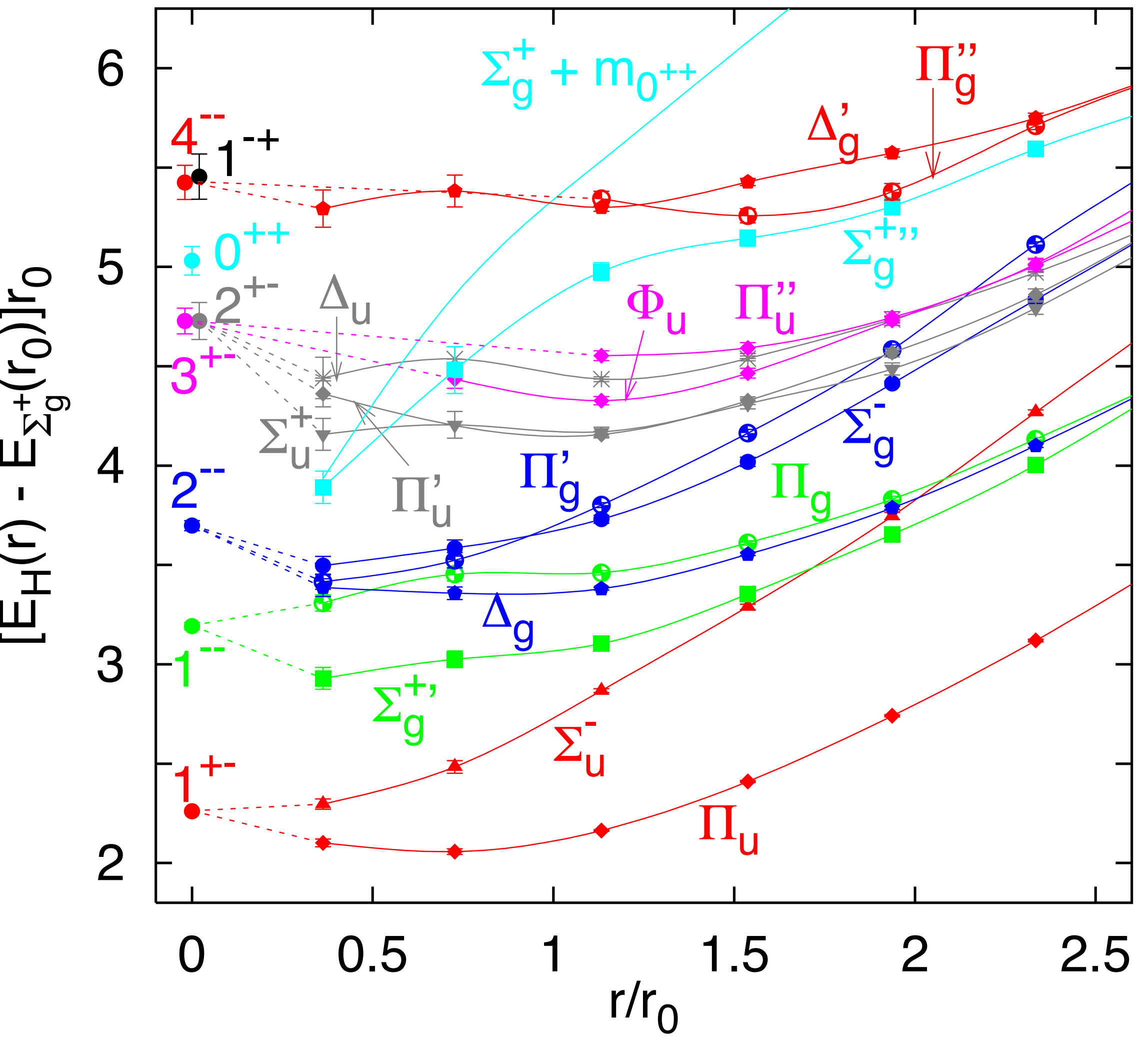
Juge Kuti Mornigstar 98-06

Schlosser, Wagner 2111.00741, Bali Pineda 2004

○ Capitani Philipsen Reisinger Riehl Wagner PRD 99 (2019) 034502

Hybrids static energies at short distances



The BOEFT characterises the hybrids static energy for short distance

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.

the hybrid static energy can be written as a (multipole) expansion in r:

octet potential

non perturbative coefficient

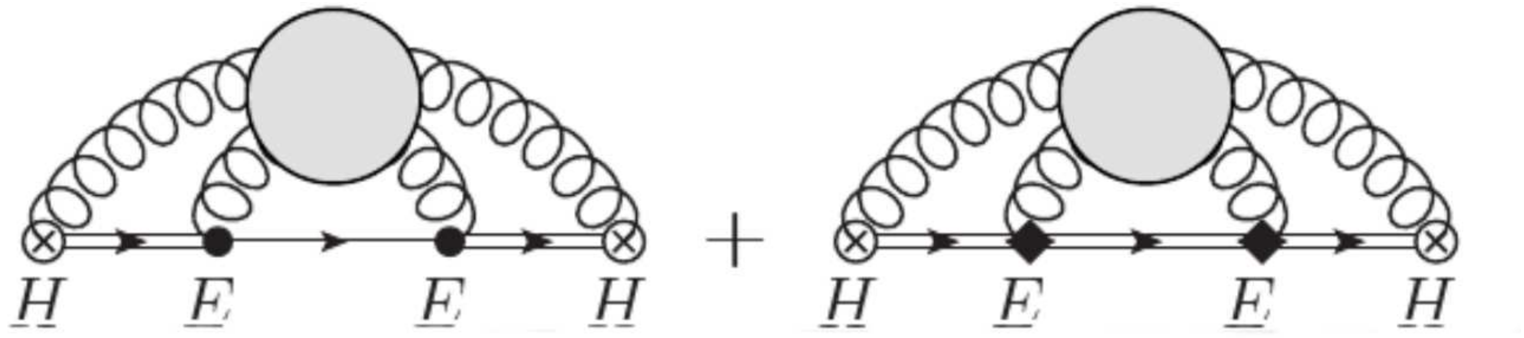
$$E_g = \frac{\alpha_s}{6r} + \Lambda_g + a_g r^2 + \dots$$

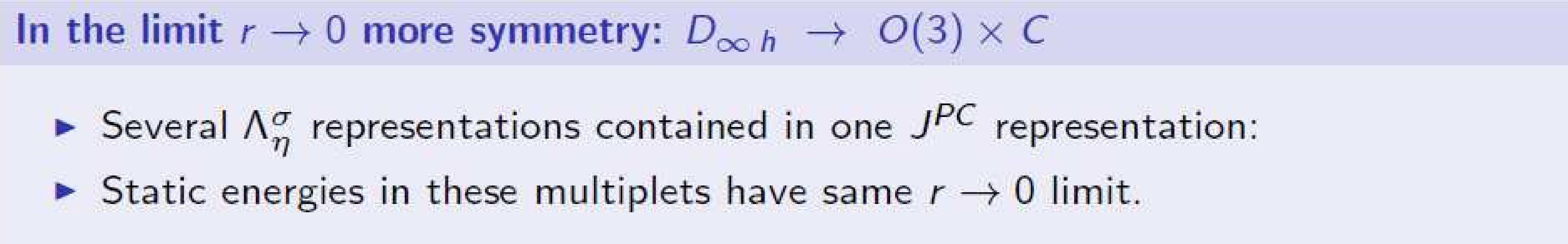
Λ_g is the gluelump mass: $\Lambda_g = \lim_{T\to\infty} \frac{i}{T} \ln\langle H^a(T/2)\phi_{ab}^{\rm adj}(T/2,-T/2)H^b(-T/2)\rangle$

calculated on the lattice

Foster Michael PRD 59 (1999) 094509
Bali Pineda PRD 69 (2004) 094001
Lewis Marsh PRD 89 (2014) 014502

a_g can be expressed as field correlators (single line = singlet, double line = octet), e.g.,

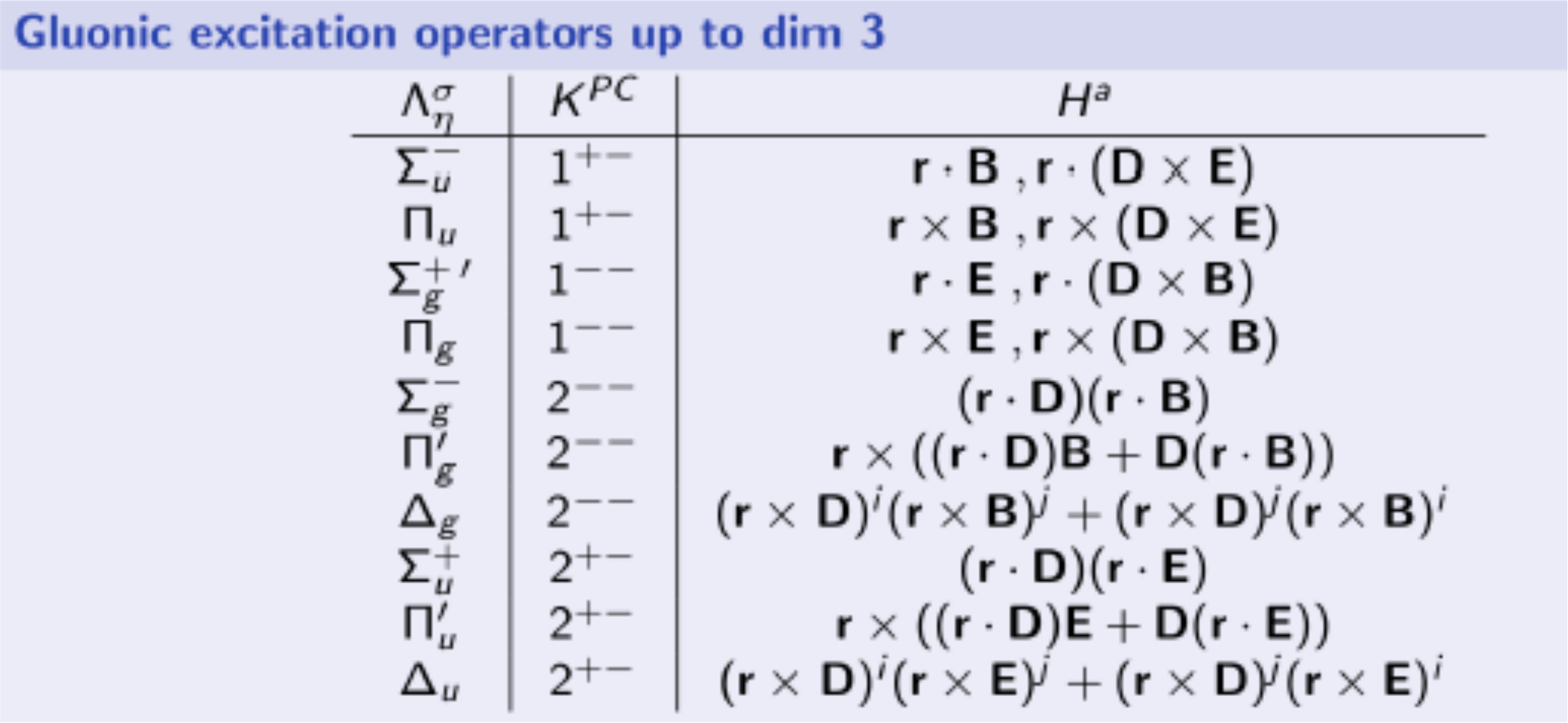




In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π_g', Δ_g; Σ_u^+, Π_u', Δ_u are degenerate.

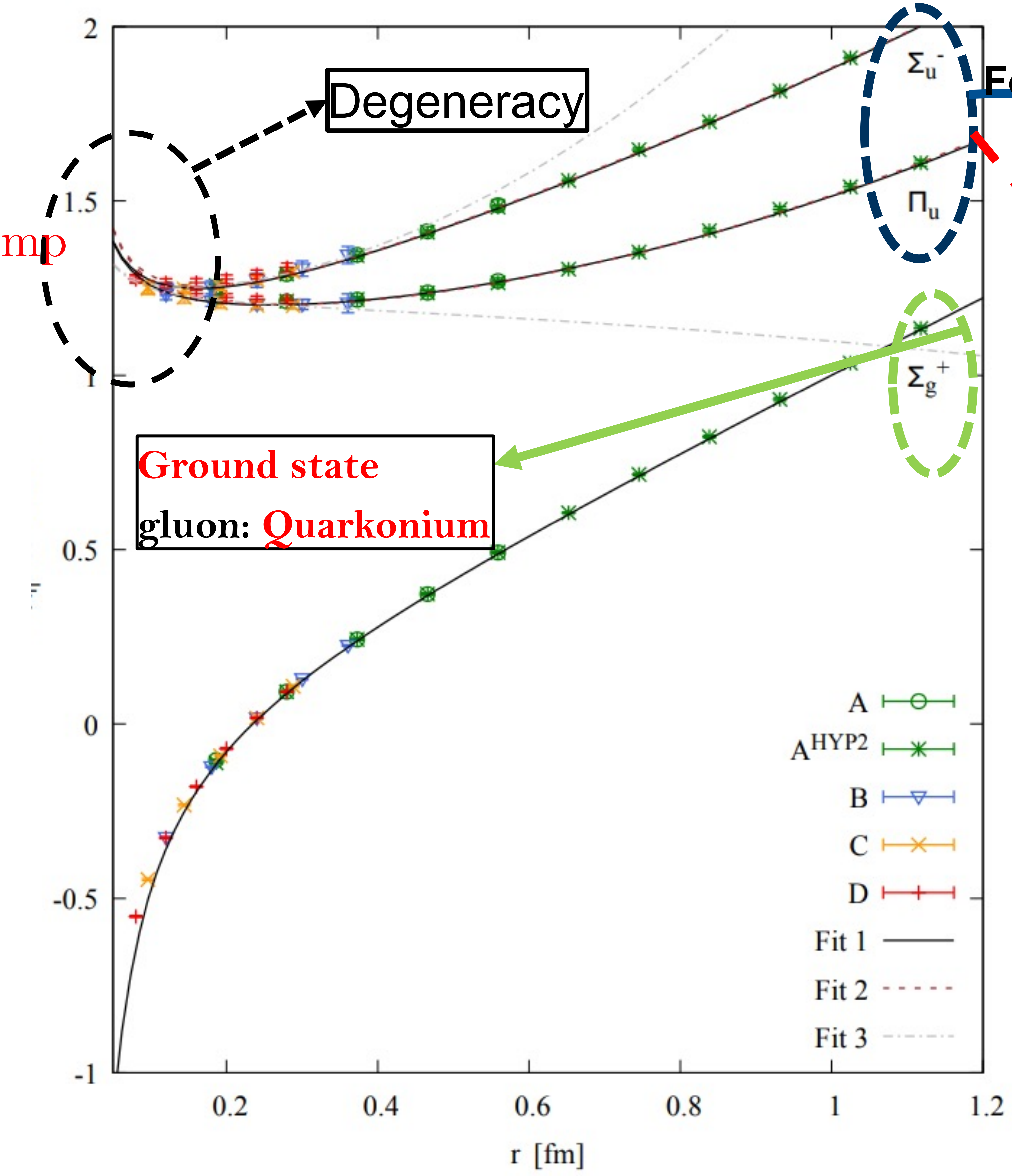


Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
Δ_u	2^{+-}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{E})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{E})^i$

BOEFT for HYBRIDS

1^{+-} gluelump



Degeneracy

Ground state

gluon: **Quarkonium**

Focus on the first hybrid static energy and construct BOEFT In this case

Schlosser and Wagner Phys. Rev. D. 105, (2022)

The first hybrid static energy excitation

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016

$$P^{i\dagger}_{\kappa\lambda} O^a(\mathbf{r},\mathbf{R},t) H^{ia}_{\kappa}(\mathbf{R},t) = Z_\kappa \Psi_{\kappa\lambda}(\mathbf{r},\mathbf{R},t)$$

$$\mathcal{L}_{\text{BOEFT for }1+-} = \int d^3r \sum_{\lambda\lambda'} \mathrm{Tr}\left\{\Psi^\dagger_{1+-\lambda}\left(i\partial_0 - V_{1+-\lambda\lambda'}(r) + \hat{r}^{i\dagger}_\lambda \frac{\boldsymbol{\nabla}^2_r}{m} \hat{r}^i_{\lambda'}\right)\Psi_{1+-\lambda'}\right\}$$

- $\lambda = \pm 1, 0;\quad \hat{r}^i_0 = \hat{r}^i$ and $\hat{r}^i_{\pm 1} = \mp\left(\hat{\theta}^i \pm i\hat{\phi}^i\right)/\sqrt{2}.$
- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \frac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \frac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

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fitted from the lattice hybrids static energies

The LO e.o.m. for the fields $\Psi^\dagger_{1+-\lambda}$ are a set of coupled Schrödinger equations:

$$i\partial_0 \Psi_{1+-\lambda} = \left[\left(-\frac{\boldsymbol{\nabla}^2_r}{m} + V^{(0)}_{1+-\lambda}\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C^{\text{nad}}_{1+-\lambda\lambda'}\right]\Psi_{\kappa\lambda'}$$

The eigenvalues $\mathcal{E}_N$ give the masses M_N of the states as $M_N = 2m + \mathcal{E}_N$.

$$\hat{r}^{i\dagger}_\lambda \left(\frac{\boldsymbol{\nabla}^2_r}{m}\right)\hat{r}^i_{\lambda'} = \delta_{\lambda\lambda'}\frac{\boldsymbol{\nabla}^2_r}{m} + C^{\text{nad}}_{1+-\lambda\lambda'}$$

with $C^{\text{nad}}_{1+-\lambda\lambda'} = \hat{r}^{i\dagger}_\lambda \left[\dfrac{\boldsymbol{\nabla}^2_r}{m}, \hat{r}^i_{\lambda'}\right]$ called the nonadiabatic coupling.

BOEFT for E_{Π_u} and $E_{\Sigma_u^-}$ hybrids

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- $V_{1+-\lambda\lambda'} = V^{(0)}_{1+-\lambda\lambda'} + \dfrac{V^{(1)}_{1+-\lambda\lambda'}}{m} + \dfrac{V^{(2)}_{1+-\lambda\lambda'}}{m^2} + \cdots$
- For the static potential: $V^{(0)}_{1+-\lambda\lambda'} = \delta_{\lambda\lambda'} V^{(0)}_{1+-\lambda}$, with $V^{(0)}_{1+-0} = E_{\Sigma_u^-}$, $V^{(0)}_{1+-\pm 1} = E_{\Pi_u}$.

fitted from the lattice hybrids static energies

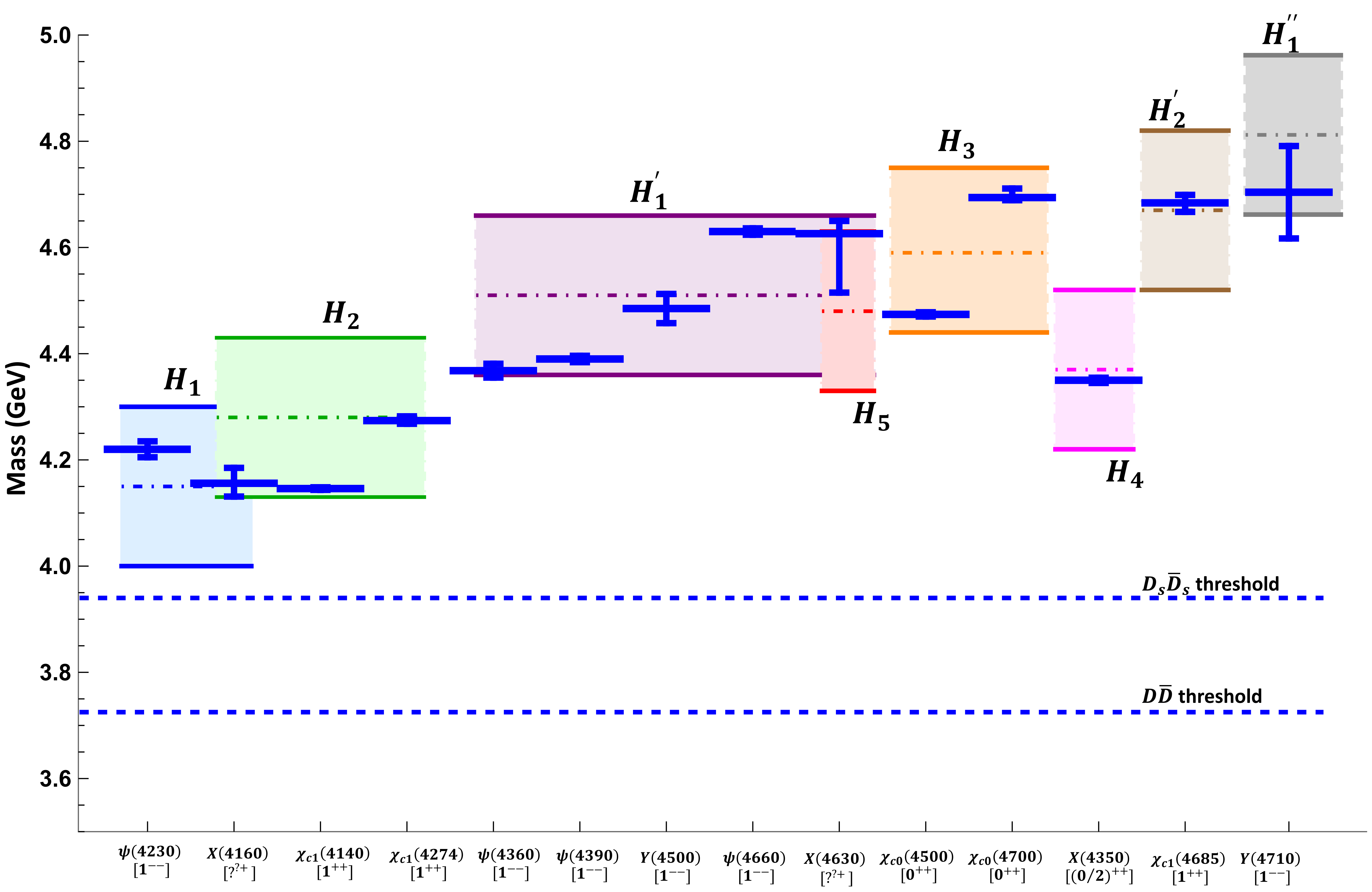
$$\left[-\frac{1}{mr^2}\,\partial_r r^2 \partial_r + \frac{1}{mr^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E^{(0)}_{\Sigma} & 0 \\ 0 & E^{(0)}_{\Pi}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma} \\ \psi^{(N)}_{-\Pi}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma} \\ \psi^{(N)}_{-\Pi}\end{pmatrix}$$

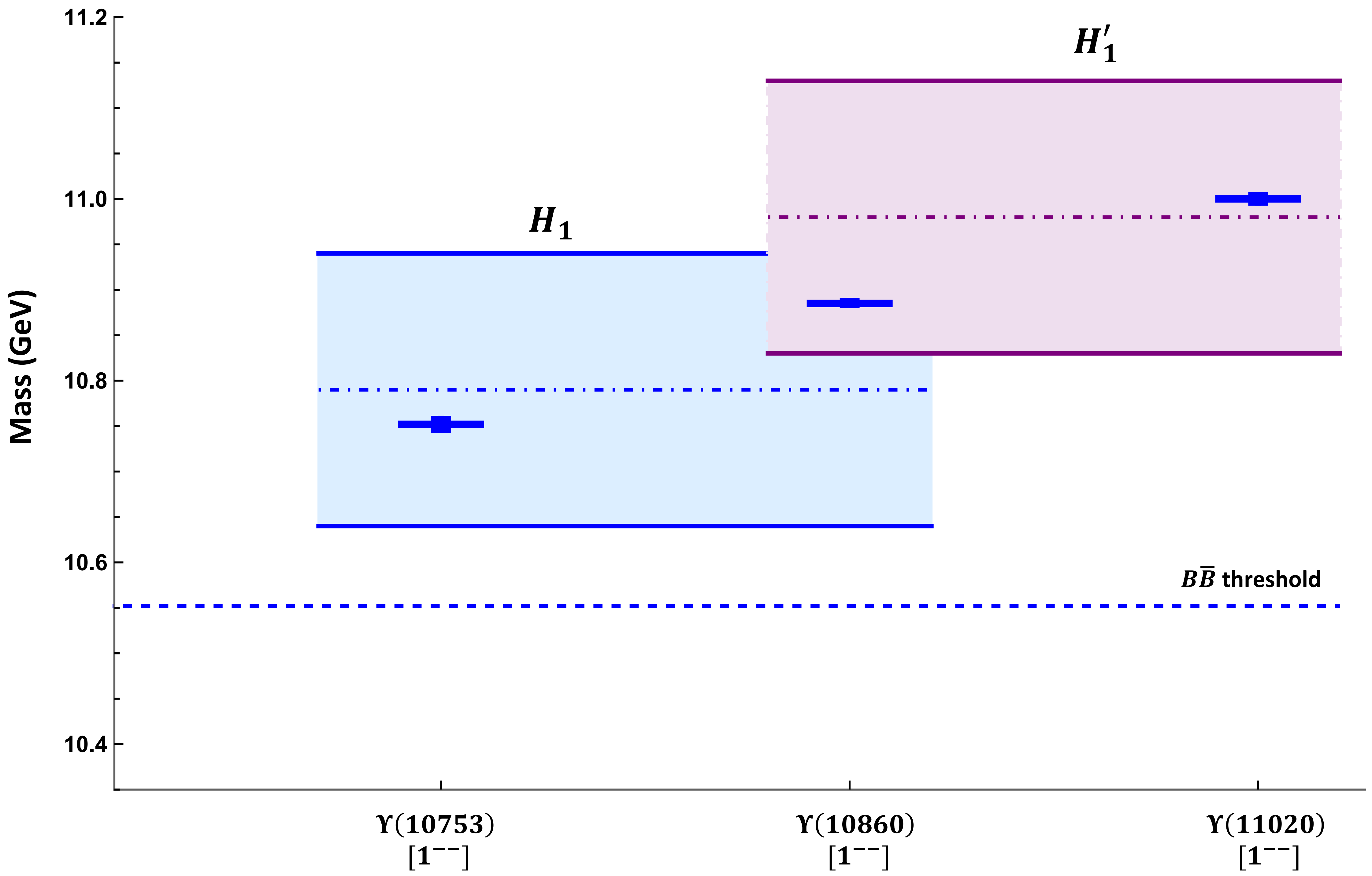
$$\left[-\frac{1}{mr^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{mr^2} + E^{(0)}_{\Pi}\right]\psi^{(N)}_{+\Pi} = \mathcal{E}_N\, \psi^{(N)}_{+\Pi}$$

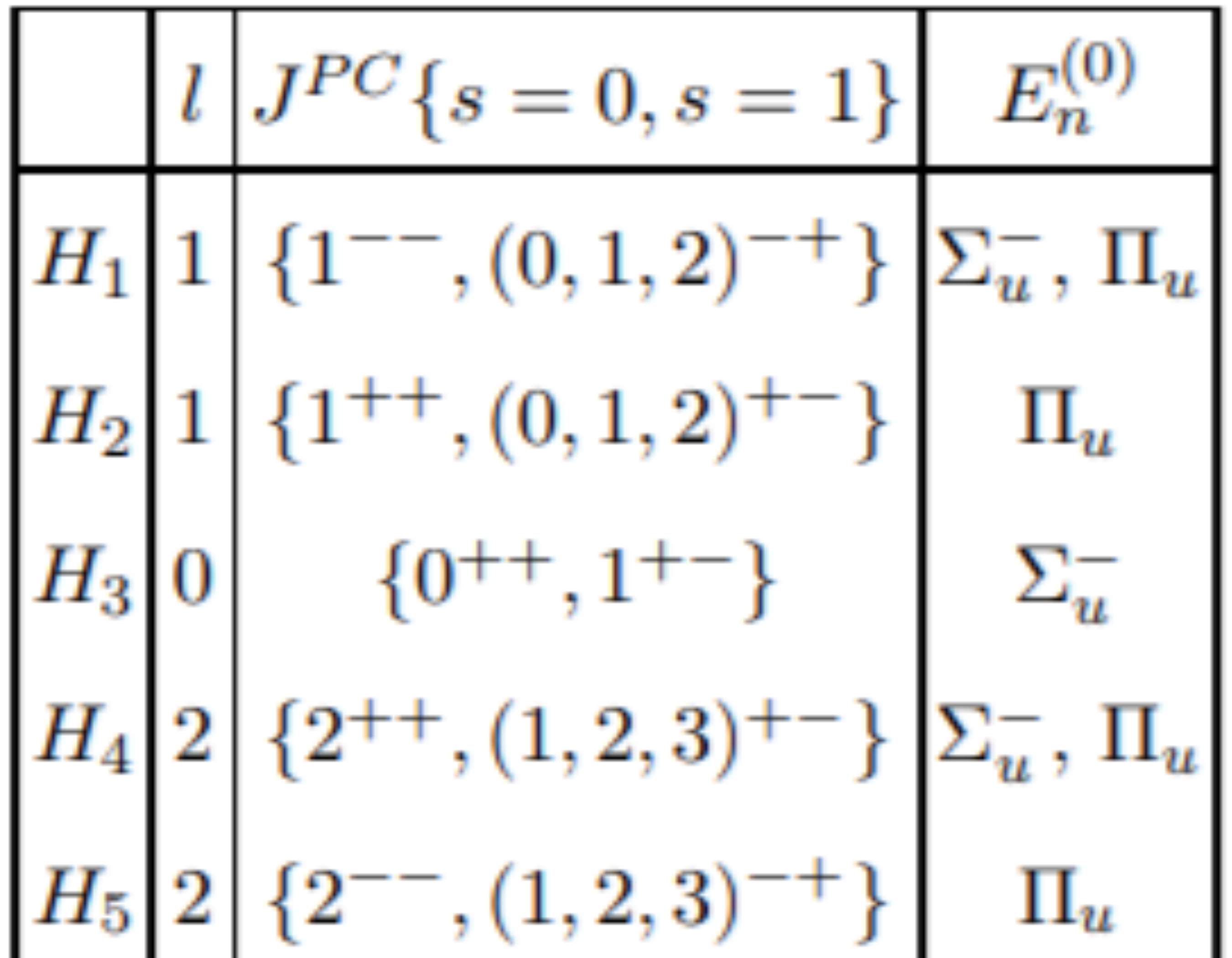
Mixing remove the degeneration among opposite parity states: ->Lambda doubling existing also in molecular physics

- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$

Hybrid multiplets as predicted by BOEFT (coloured rectangles) compared to the neutral isoscalar states observed in charmonium/bottomonium sector (crosses)







	l	$J^{PC}\{s=0,s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--},(0,1,2)^{-+}\}$	Σ_u^-,Π_u
H_2	1	$\{1^{++},(0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++},1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++},(1,2,3)^{+-}\}$	Σ_u^-,Π_u
H_5	2	$\{2^{--},(1,2,3)^{-+}\}$	Π_u

Note: Band in the mass value for each multiplet is due to the error (150 Mev) on the gluelump mass measured on the lattice

N. B. W. K. Lai, A- Mohapatra, A. Vairo 2212.09187

Spin dependent interactions

The **BOEFT** gives a prescription to calculate the **hybrids spin dependent potentials at order 1/m and 1/m^2**

1/m

$$V^{(1)}_{1+-\,\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V_{SK}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + V_{SK\,b}(r)\left[\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}^{\dagger}_{\lambda}\right)\left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j}_{\lambda'}\right)\cdot\boldsymbol{S} + \left(r^{i}\boldsymbol{K}^{ij}\hat{r}^{j\dagger}_{\lambda}\right)\cdot\boldsymbol{S}\left(\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_{\lambda'}\right)\right]$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$$

$$S_{12} = 12(\mathbf{S}_1\cdot\hat{\mathbf{r}})(\mathbf{S}_2\cdot\hat{\mathbf{r}}) - 4(\mathbf{S}_1\cdot\mathbf{S}_2)$$

1/m^2

$$V^{(2)}_{1+-\,\lambda\lambda'\,\mathrm{SD}}(\boldsymbol{r}) = V^{(2)}_{LS\,a}(r)\left(\hat{r}^{i\dagger}_{\lambda}\boldsymbol{L}\,\hat{r}^{i}_{\lambda'}\right)\cdot\boldsymbol{S} + V^{(2)}_{LS\,b}(r)\hat{r}^{i\dagger}_{\lambda}\left(L^{i}S^{j} + S^{i}L^{j}\right)\hat{r}^{j}_{\lambda'} + V^{(2)}_{S^2}(r)\boldsymbol{S}^2\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,a}(r)S_{12}\delta_{\lambda\lambda'} + V^{(2)}_{S_{12}\,b}(r)\hat{r}^{i\dagger}_{\lambda}\hat{r}^{j}_{\lambda'}\left(S^{i}_{1}S^{j}_{2} + S^{i}_{2}S^{j}_{1}\right)$$

$\left(K^{ij}\right)^{k} = i\epsilon^{ikj}$ is the angular momentum of the spin one gluons

$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

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$\boldsymbol{L}$ is the orbital angular momentum of the heavy-quark-antiquark pair.

Features:

- New spin structures with respect to the quarkonium case: all terms at order 1/m and two terms at order 1/m^2

Differently from the quarkonium case, the hybrid potential gets a first contribution already at order $\Lambda^2_{\mathrm{QCD}}/m_h$. The corresponding operator does not contribute at LO to matrix elements of quarkonium states as its projection on quark-antiquark color singlet states vanishes. Hence, spin splittings are remarkably less suppressed in heavy quarkonium hybrids than in heavy quarkonia.

Hybrid spin dependent potentials at order 1/m and 1/m^2

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Features:

- The nonperturbative part in V_i (r) depend on nonperturbative gluonic correlators non local in time not yet calculated on the lattice: six unknowns, the octet perturbative part can be calculated in perturbation theory
- The only flavor dependence is carried by the perturbative NRQCD matching coefficients

Hybrid spin dependent potentials at order 1/m and 1/m^2

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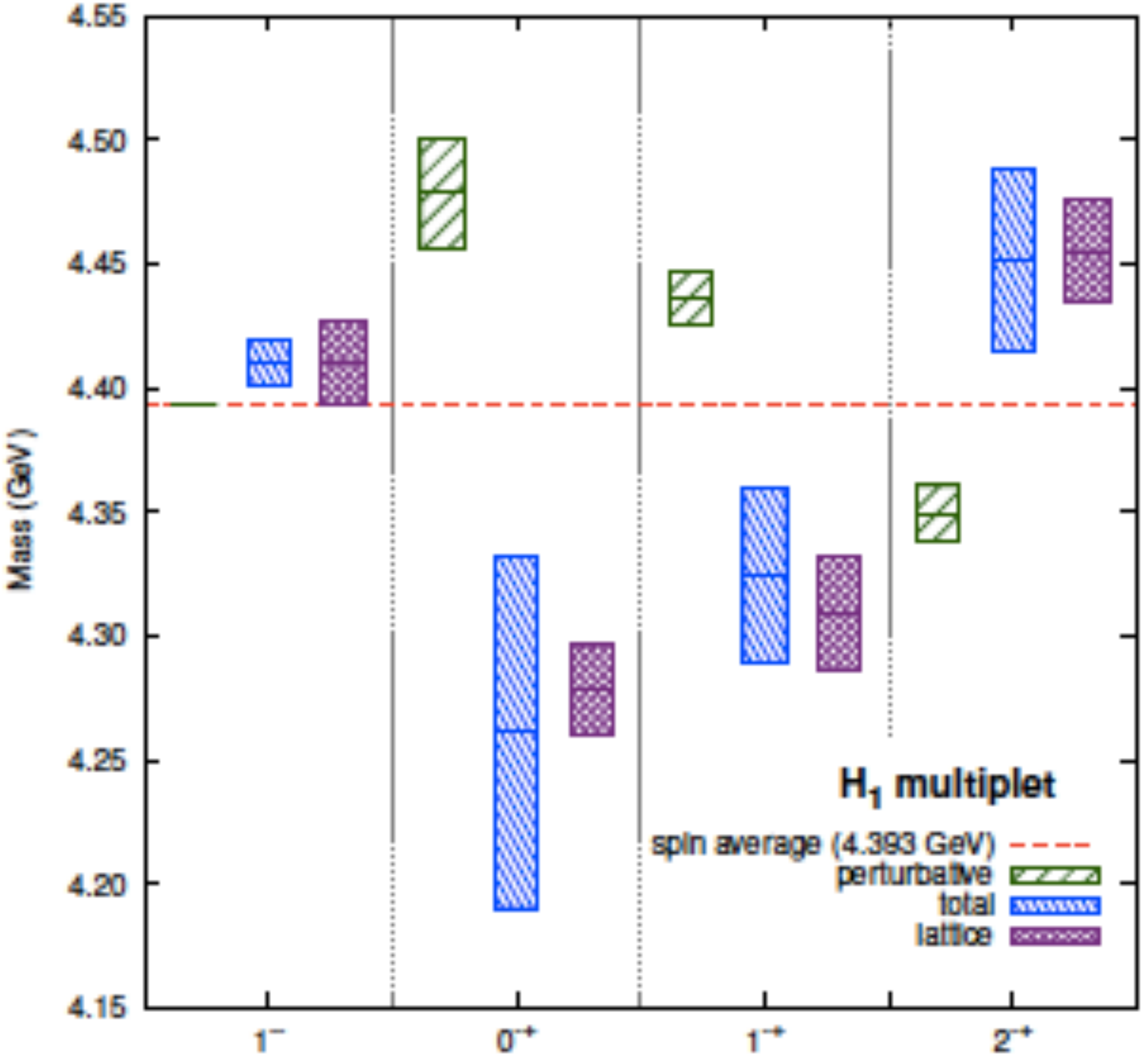
USE LATTICE CALCULATION OF THE CHARMONIUM SPIN MULTIPLETS TO EXTRACT the 6 UNKNOWNs and PREDICT THE BOTTOMONIUM SPIN MULTIPLETS, learn also about the DYNAMICS

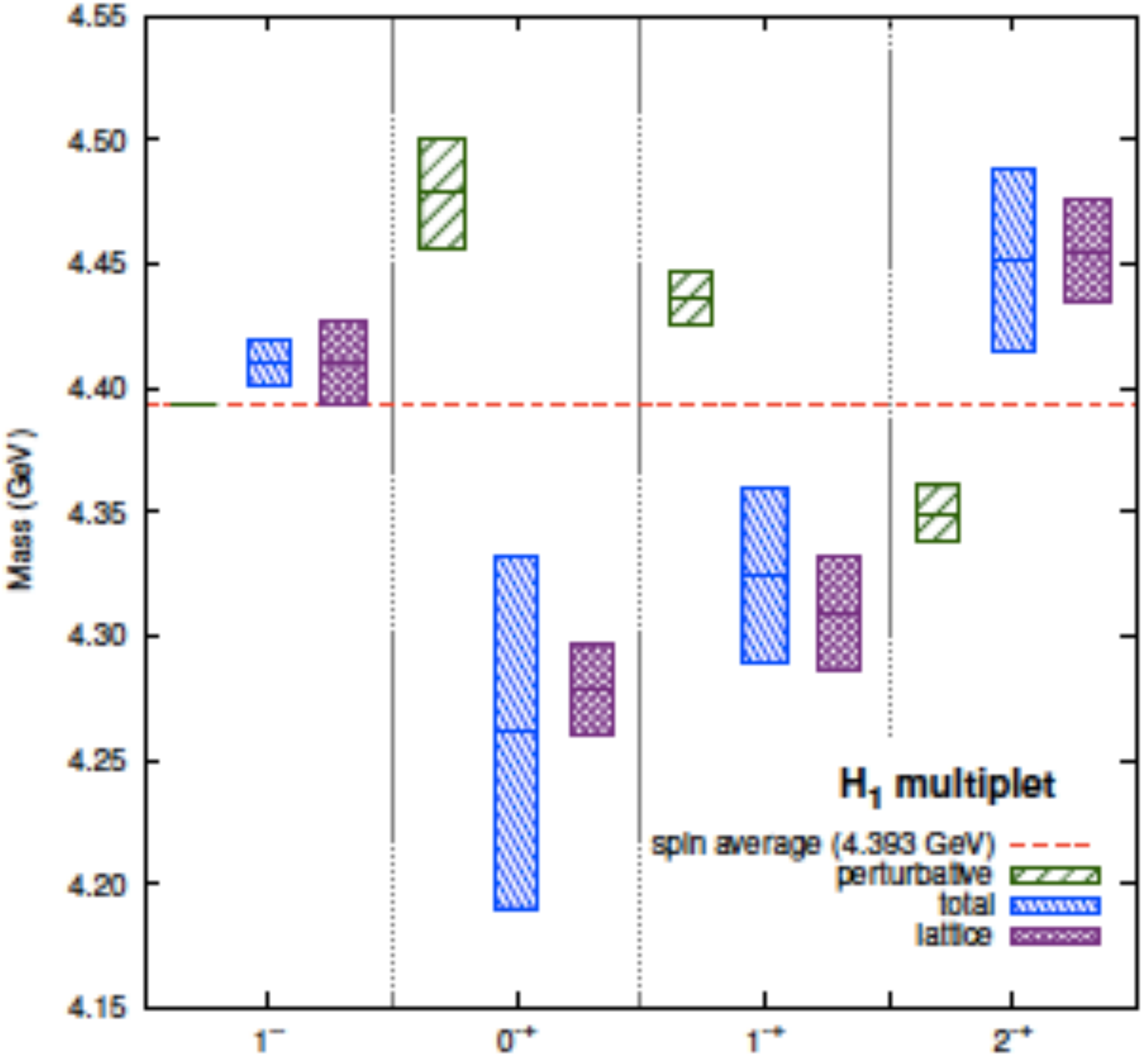
Charmonium Hybrids Multiplets H_1

lattice data from (violet) from
G. K. C. Cheung, C. O'Hara, G. Moir, M. Peardon, S. M. Ryan, C. E. Thomas, and D. Tims (Hadron Spectrum), JHEP **12**, 089 (2016), arXiv:1610.01073 [hep-lat].
with a pion of about 240 MeV



○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1



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with a pion of about 240 MeV

height of the boxes is an estimate of the uncertainty:
estimated by the parametric size of higher order corrections, m alpha_s^5 for the perturbative part, powers of Lambda_qcd/m for the nonperturbative part, plus the statistical error on the fit

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

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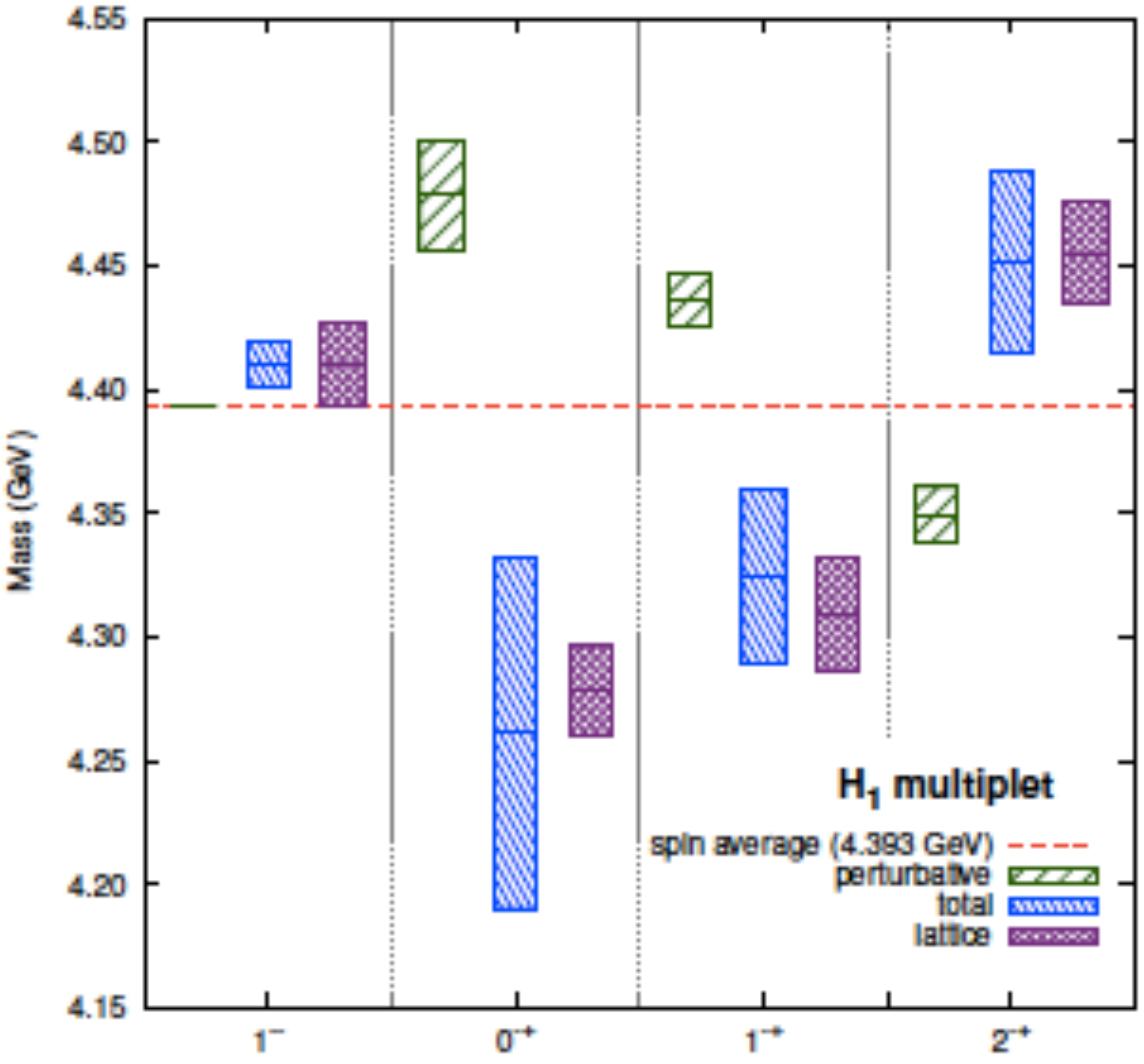

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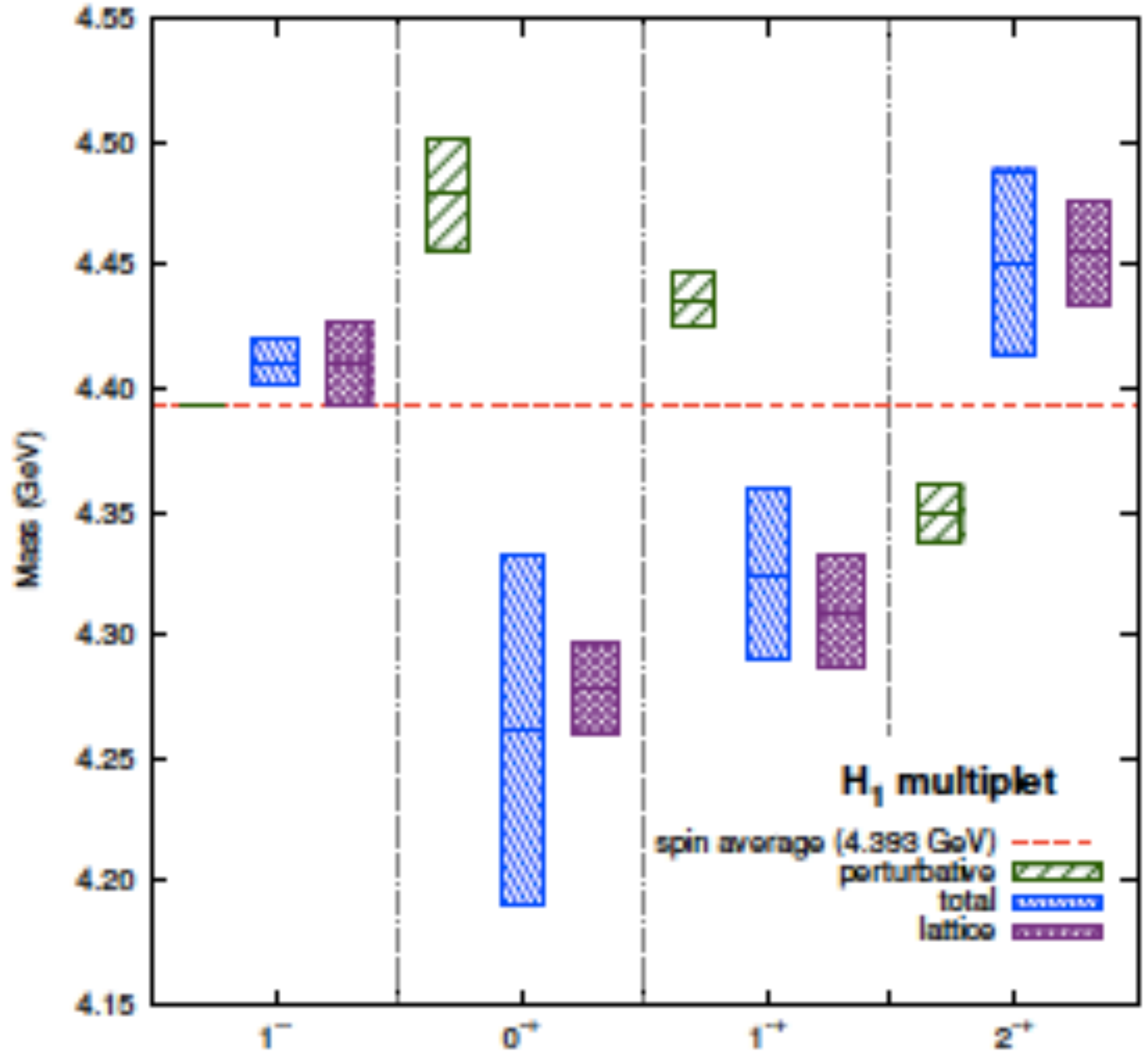
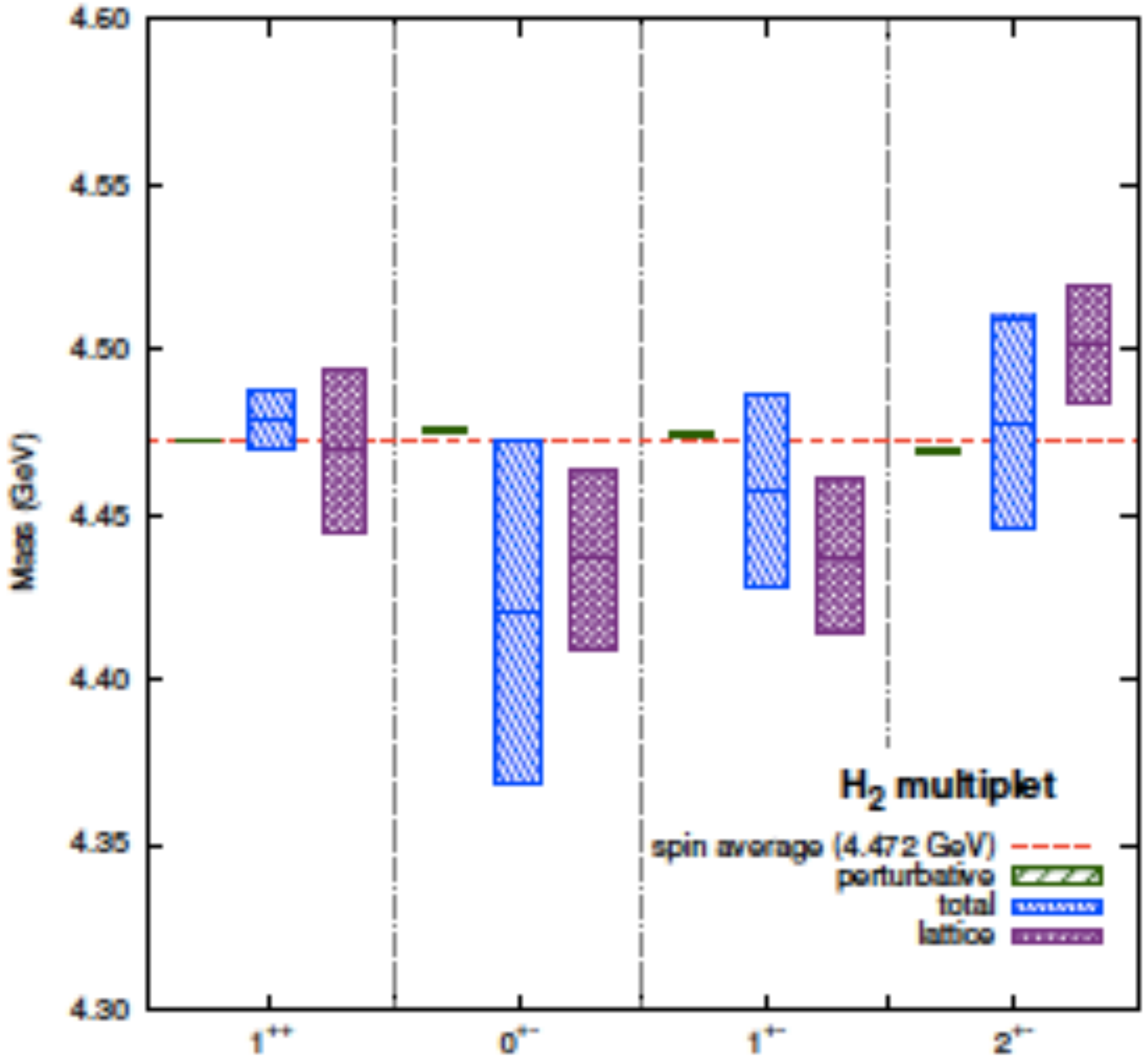
the perturbative part produces a pattern opposite to the lattice and to ordinary quarkonia —> discrepancy can be reconciled thanks to the nonperturbative parts, especially the one at order 1/m which goes like Lambda^2/m and is parametrically larger than the perturbative contribution at order m v^4

which is interesting as
some models
are taking
the spin interaction
from perturbation theory
with a constituent gluon

- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Charmonium Hybrids Multiplets H_1 and H_2





H_1 and H_2 corresponds to l=1 and are negative and positive parity resp. The mass splitting between H_1 and H_2 is a result of lambda-doubling

H_3 and H_4 are also calculated

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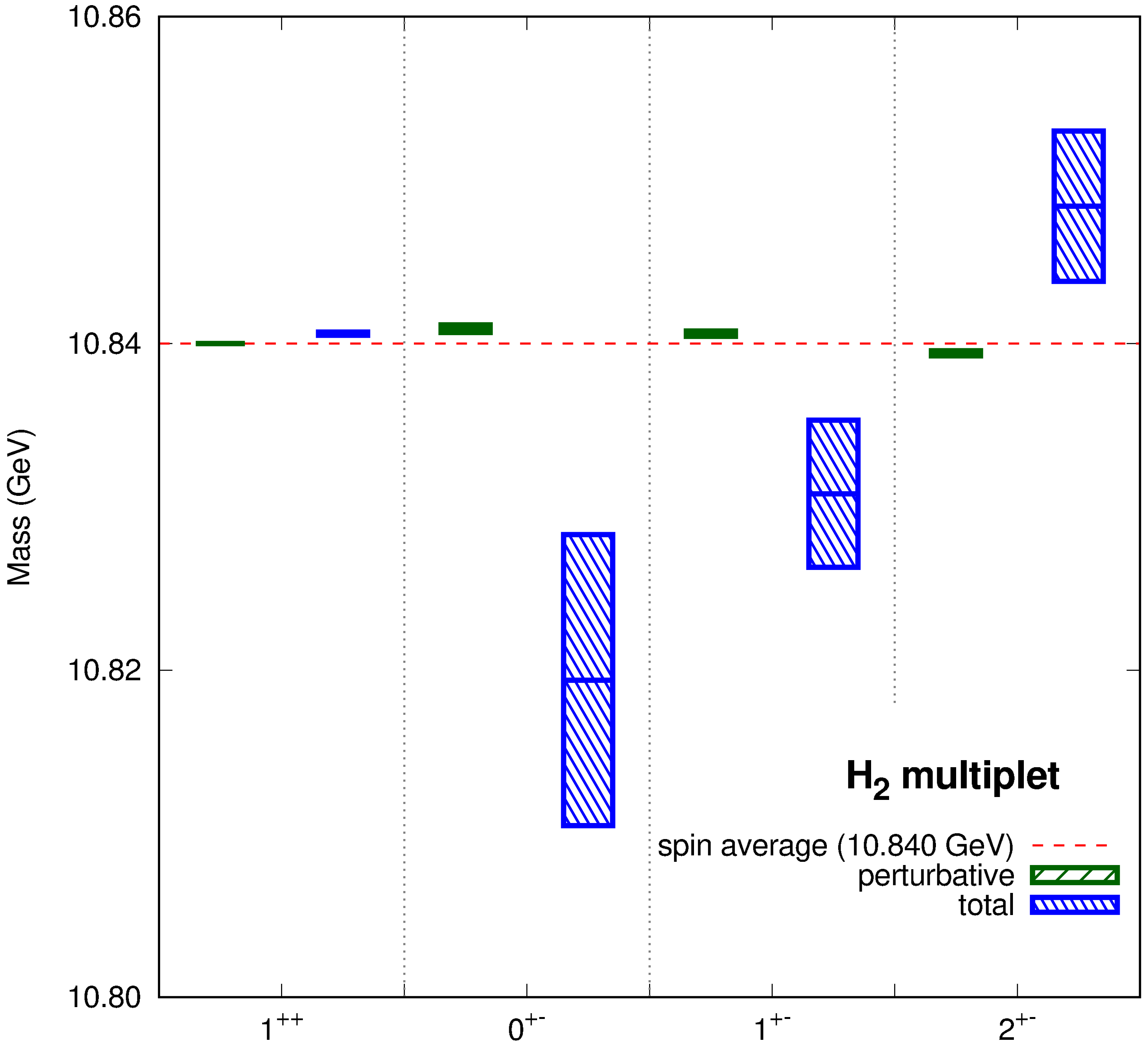
here you find predictions for all H multiplets

Bottomonium hybrid spin splittings

thanks to the BOEFT factorizatio we can fix the nonperturbative unknowns from a charmonium hybrid calculationthe nonperturbative low energy unknownsdo not depend on the flavor: we can predict the bottomonium hybrids splin splittings



and also the other H multiplets

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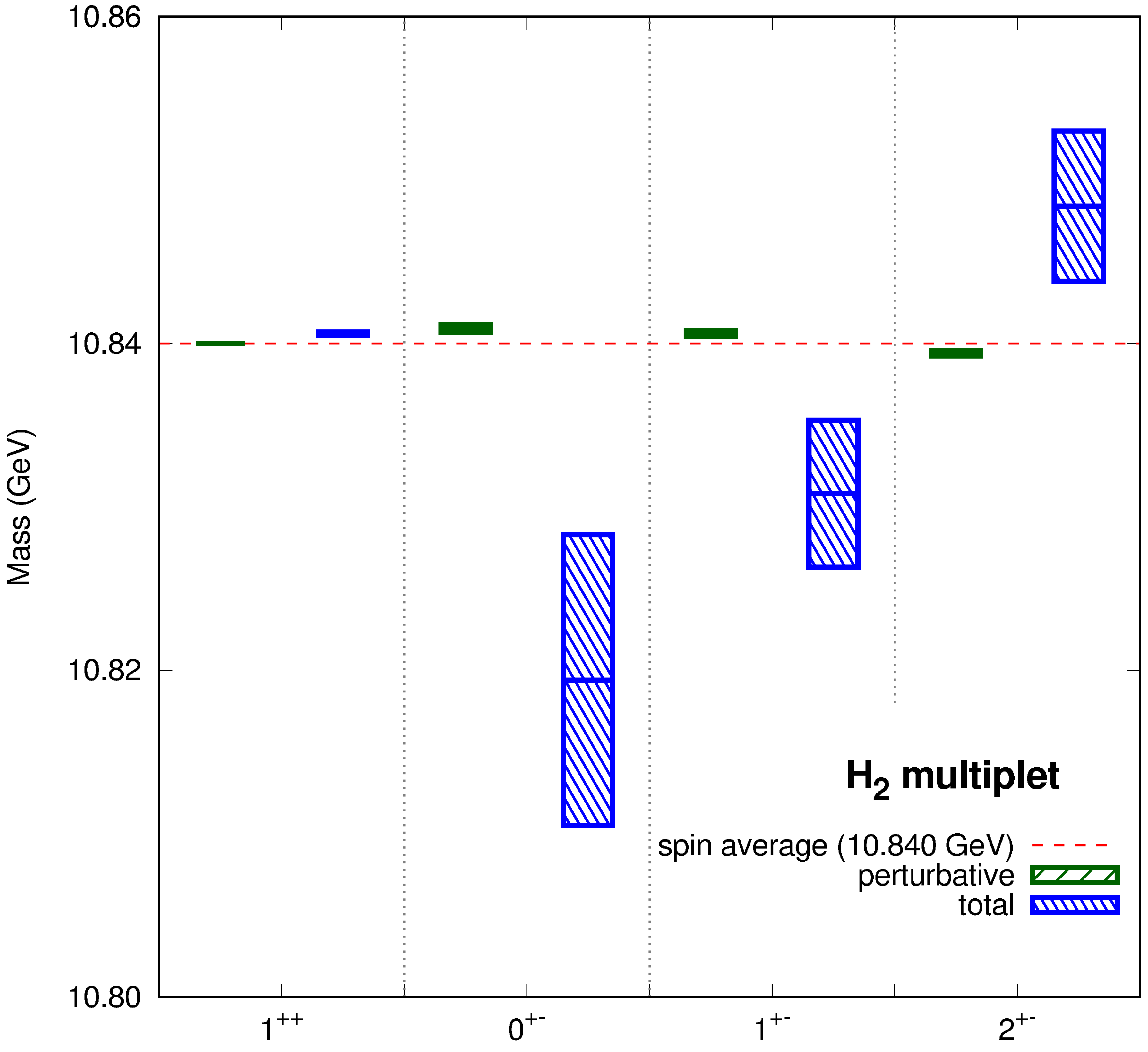
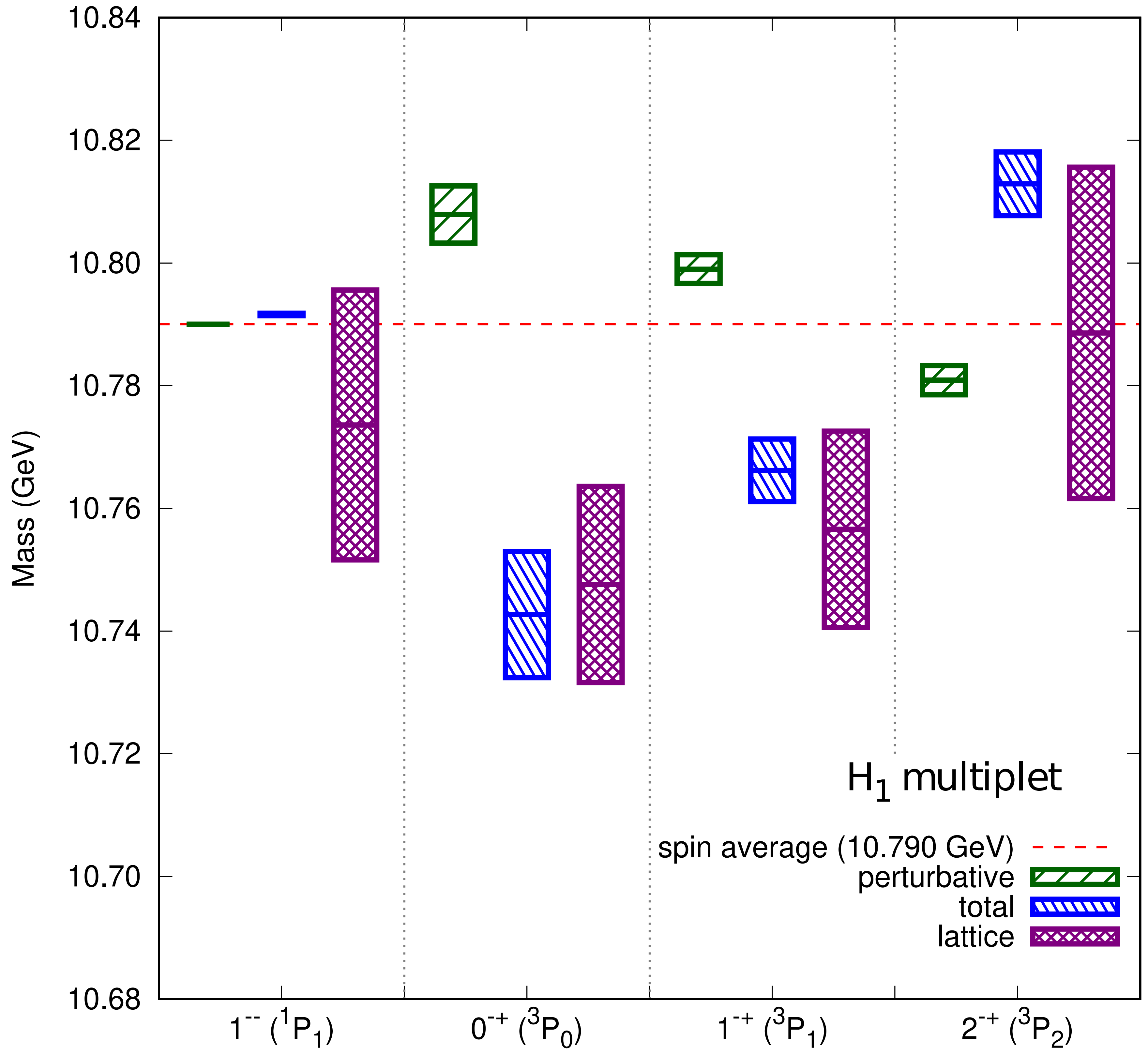


and also the other H multiplets

○ Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017

Comparison of our prediction to the existing lattice data on H1

Bottomonium H_1 hybrid spin splittings



blue BOEFT predictions (more precise), violet actual lattice calculation

○ Ryan et al arXiv:2008.02656 [2+1 flavors, $m_\pi = 400$ MeV]
unpublished plot by J. Segovia and J. Tarrus

Spin effects

->difficult to insert in models

->this spin structure has huge impact in phenomenology : larger spin multiplets separation than in quarkonium

->less spin symmetry in decays due to quarkonium-hybrids mixing via a spin operator at 1/m

Oncala & Soto, Phys. Rev. D. 96, (2017)

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Oncala & Soto, Phys. Rev. D. 96, (2017)

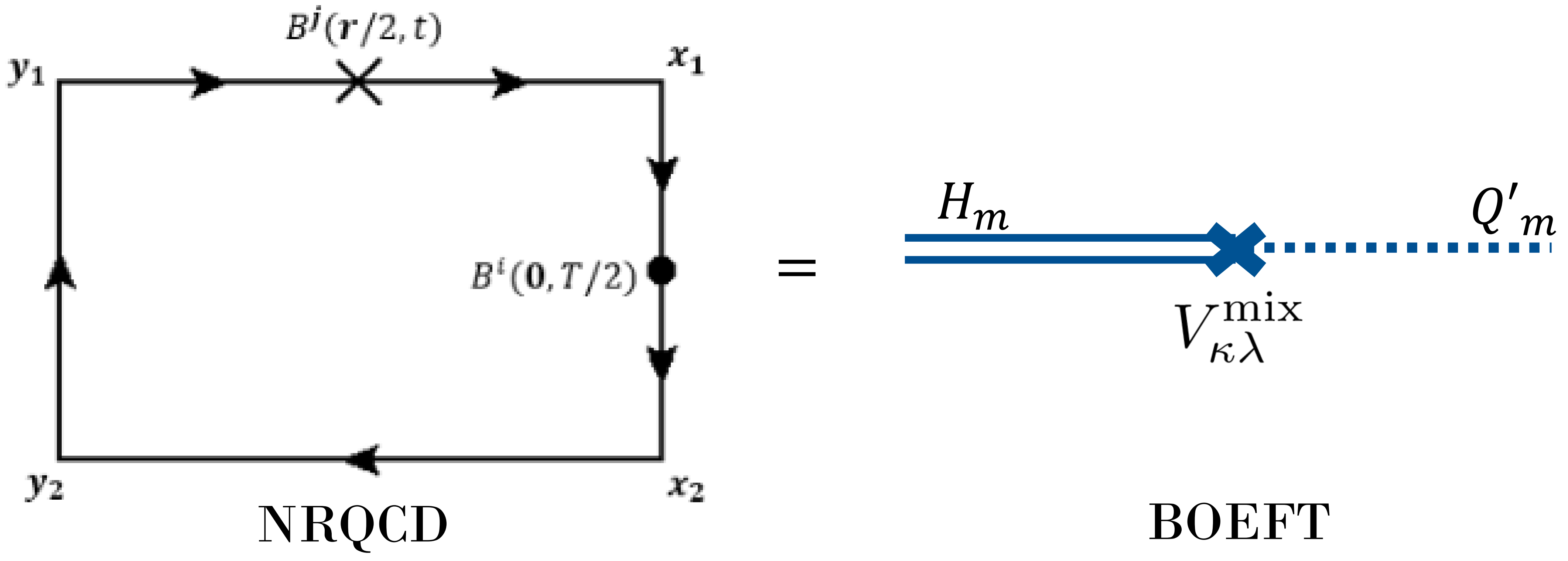
- Hybrid states in the same energy range and same quantum #'s as quarkonium can mix.

  Mixing impact spectrum and decay properties of hybrid. Implications on hybrid interpretation for exotics.

Ex. $H_1\left[1^{--}\right](4155) \leftrightarrow c\bar{c}\left[1^{--}\right](3S)$

Effect on decay: $H_m \leftrightarrow Q_m^{'} \rightarrow (\eta_c, J/\psi, \cdots) + (\gamma, \cdots)$

- Mixing potential $V_{\kappa\lambda}^{\rm mix}$: determined from matching NRQCD and BOEFT at $O(1/m)$



Expression after matching:

$$\boxed{V_{|\lambda|}^{\rm mix} = -\frac{g c_F}{2 m_Q}\, {}^{(0)}_{\lambda}\langle 1|B^j\left(\mathbf{r}/2, 0\right)|0\rangle^{(0)} P_\lambda^j,}$$

Above expression can be computed on lattice if we identify:

$$|0\rangle^{(0)} = |\Sigma_g^+\rangle$$

$$|1\rangle_{\lambda=0}^{(0)} = |\Sigma_u^-\rangle, |1\rangle_{|\lambda|=1}^{(0)} = |\Pi_u\rangle$$

BOEFT calculation of semi inclusive hybrids decays to quarkonium: H —> Quarkonium (S) plus X (anything light)

$$\Gamma_{H\to S} \;=\; -2\,\langle H|\mathrm{Im}\Delta V|H\rangle.$$

we calculated spin conserving and spin flipping decays
they are same size

Decay to open threshold states not accounted

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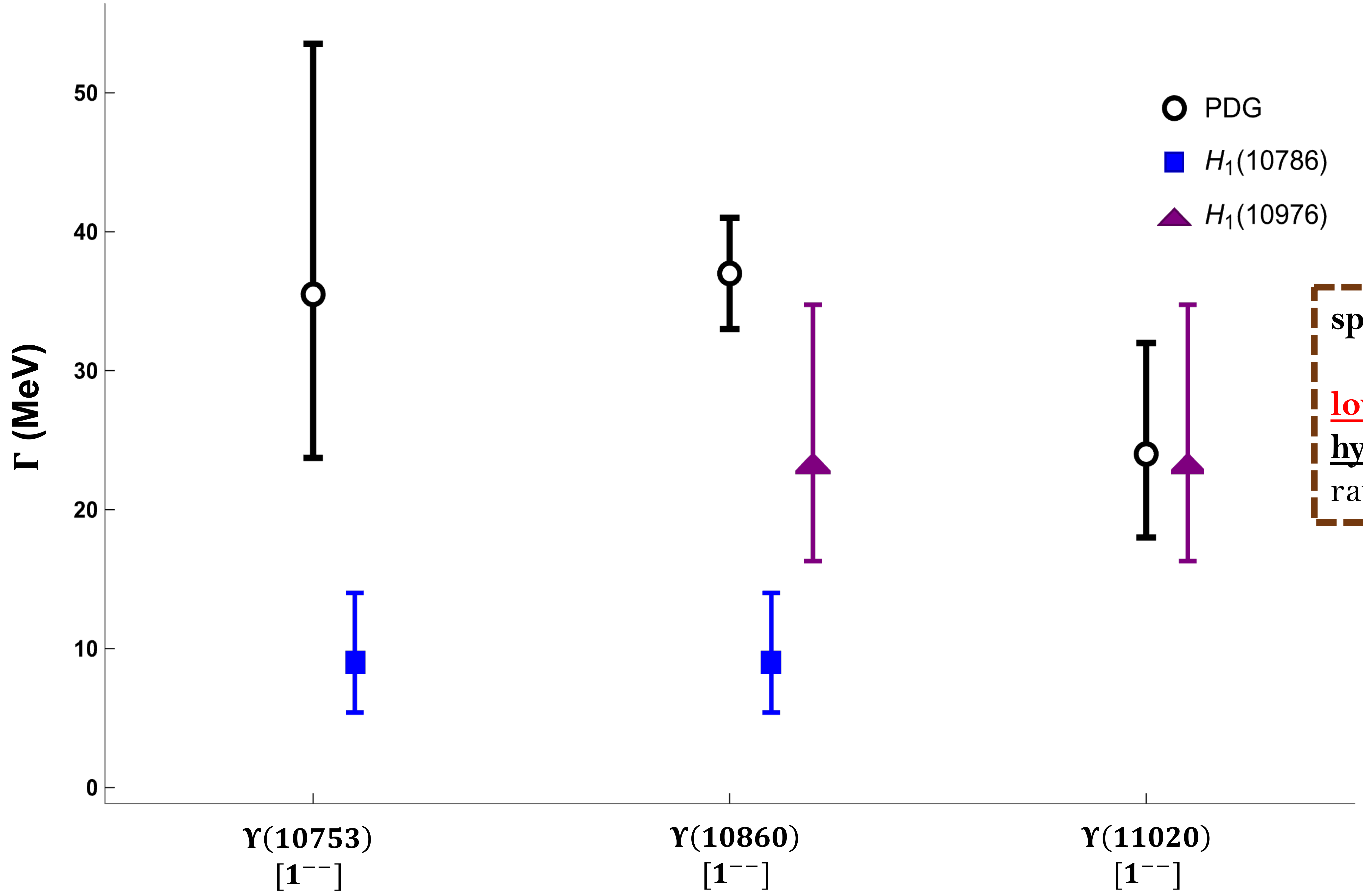
Decay to open threshold states not accounted

- Comparison: bottom exotic states with corresponding bottomonium hybrid state:



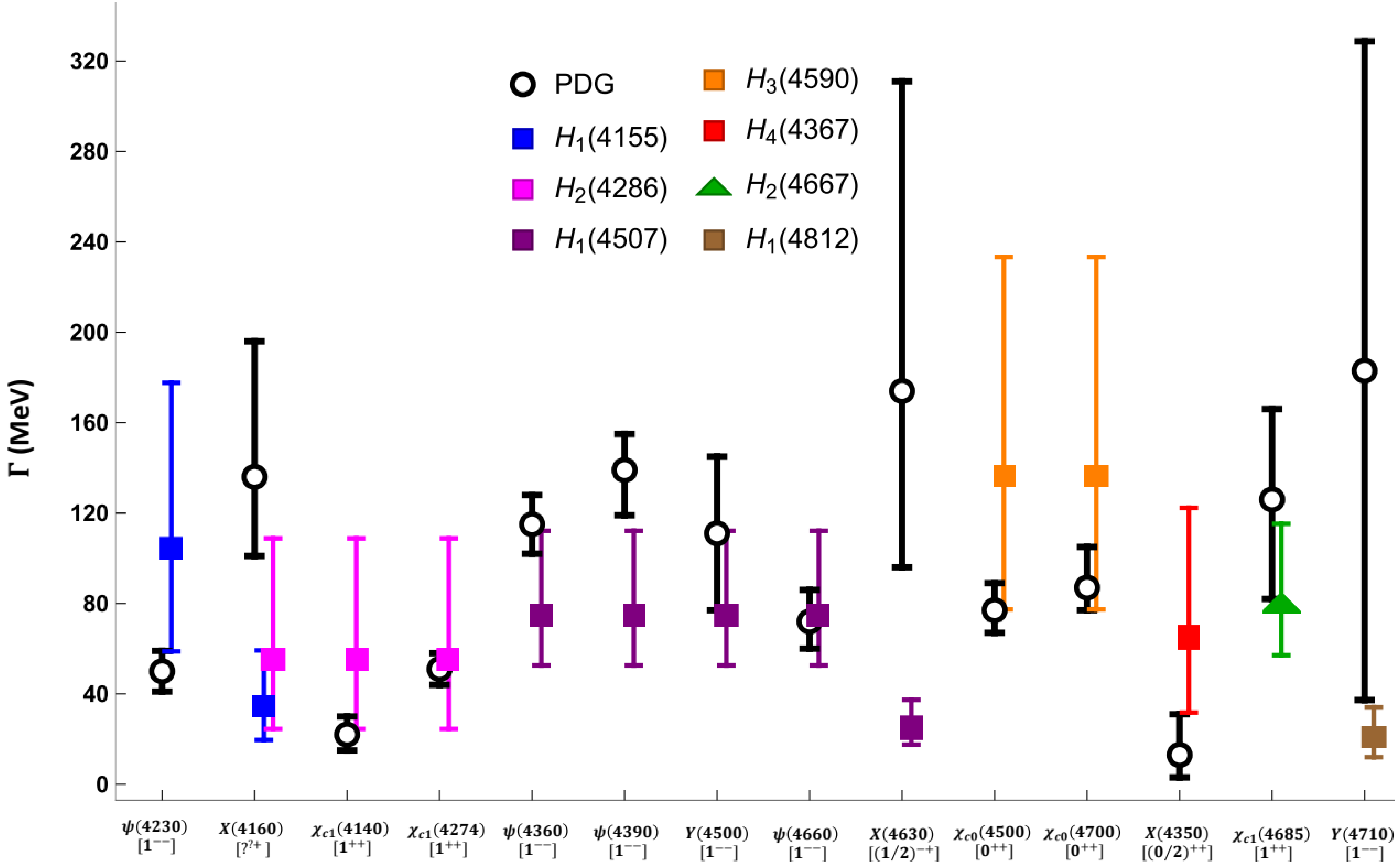
spin-conserving + spin-flipping decays

=

lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

BOEFT calculation of semi inclusive hybrids decays to quarkonium

Comparison: charm exotic states with corresponding charmonium hybrid state:



$$H_m \rightarrow Q_n + X$$

spin-conserving + spin-flipping decays

=

lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Hybrid: Summary

Brambilla, Lai, AM, Vairo arXiv:2212.09187

- Hybrids ($Q\bar{Q}g$): Color singlet state of color octet $Q\bar{Q}$ + gluon. (Q = c, b)

✓ **Isoscalar neutral mesons (Isospin=0)**

✓ Candidates for hybrids based on **mass, quantum numbers,** and **decays** to **quarkonium**:

Charm sector:

➢ $\boldsymbol{X(4160)}$: could be **charm hybrid** $\boldsymbol{H_1[2^{-+}](4155)}$.

➢ $\mathbf{X(4630)}$: could be **charm hybrid** $\boldsymbol{H_1[(1/2^{-+})](4507)}$.

➢ $\boldsymbol{\psi(4390)}$: could be **charm hybrid** $\boldsymbol{H_1[1^{--}](4507)}$.

➢ $\boldsymbol{\psi(4710)}$: could be **charm hybrid** $\boldsymbol{H_1[(1^{--})](4812)}$.

➢ $\mathbf{X(4630)}$: could be **charm hybrid** $\boldsymbol{H_1[(1/2^{-+})](4507)}$.

➢ $\boldsymbol{\chi_{c1}(4685)}$: could be **charm hybrid** $\boldsymbol{H_2[(1^{++})](4667)}$.

Bottom sector:

➢ $\boldsymbol{\Upsilon(10753)}$: could be **bottom hybrid** $\boldsymbol{H_1[(1^{--})](10786)}$.

DISCLAIMER!!!

All the above interpretation can differ accounting for decays to meson-pair threshold states and hybrid-quarkonium mixing.

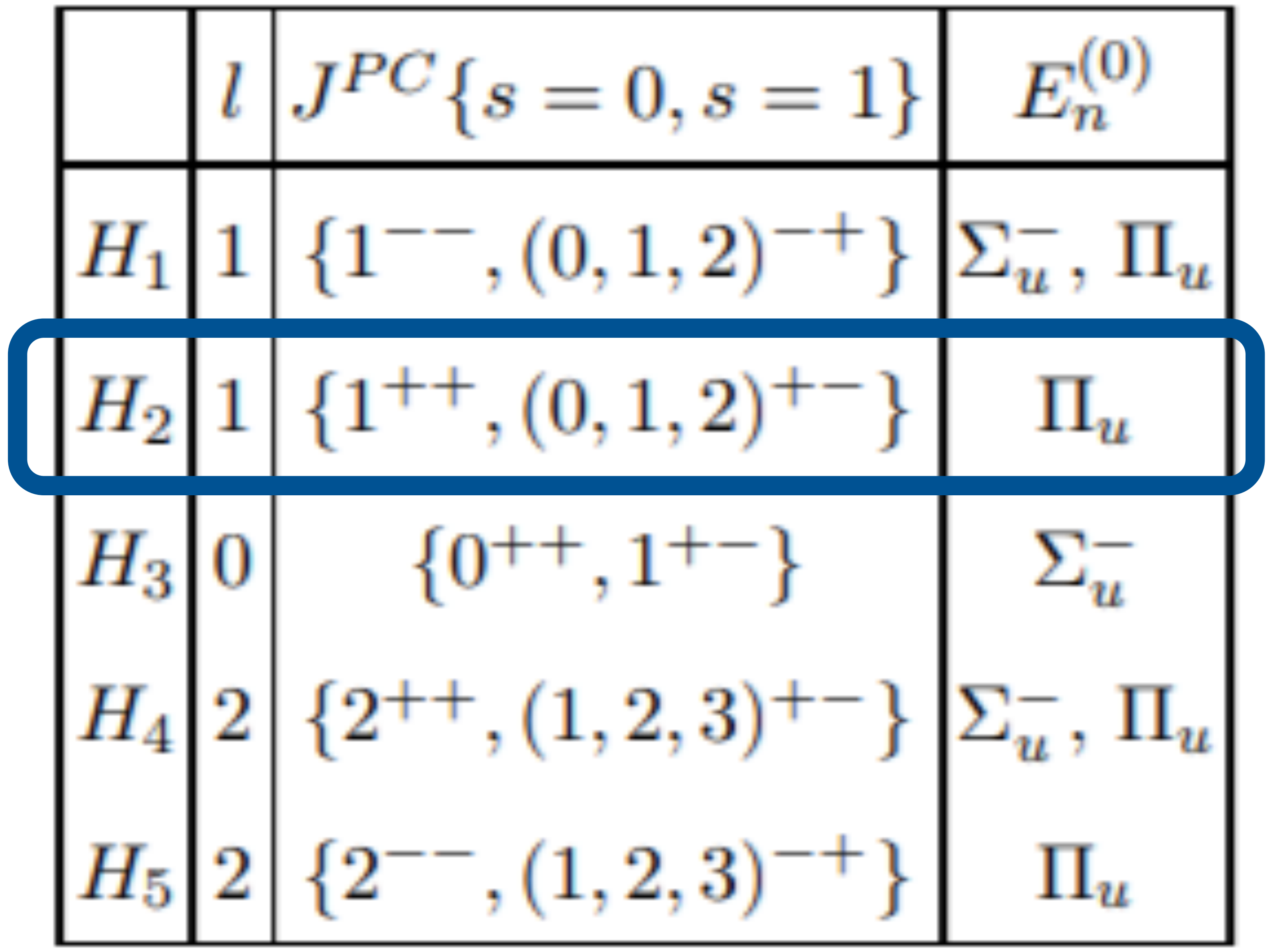
Hybrid Decays

Hybrid decays to meson-pair threshold states:

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden! $H_m \nrightarrow D^{(*)}\bar{D}^{(*)}$

Kou & Pene, Phys Lett B 631 (2005) Page, Phys Lett B 407 (1997) Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)

Born Oppenheimer quantum numbers for hybrids and ground state meson pair **does allow for decay to two s-wave mesons**. Bruschini 2306.17120



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Most quarkonium hybrids can decay into pair of s-wave mesons !

forbidden for decay into pair of s-wave mesons

Recent lattice computation for $c\bar{c}$ hybrid 1^{-+} **decay** to

$D_1\bar{D}$: 258(133) MeV

$D^*\bar{D}$: 88(18) MeV

$D^*\bar{D}^*$: 150(118) MeV

Shi et al 2306.12884

One Born–Oppenheimer Effective Theory to rule them all: hybrids, tetraquarks, pentaquarks, doubly heavy baryons and quarkonium

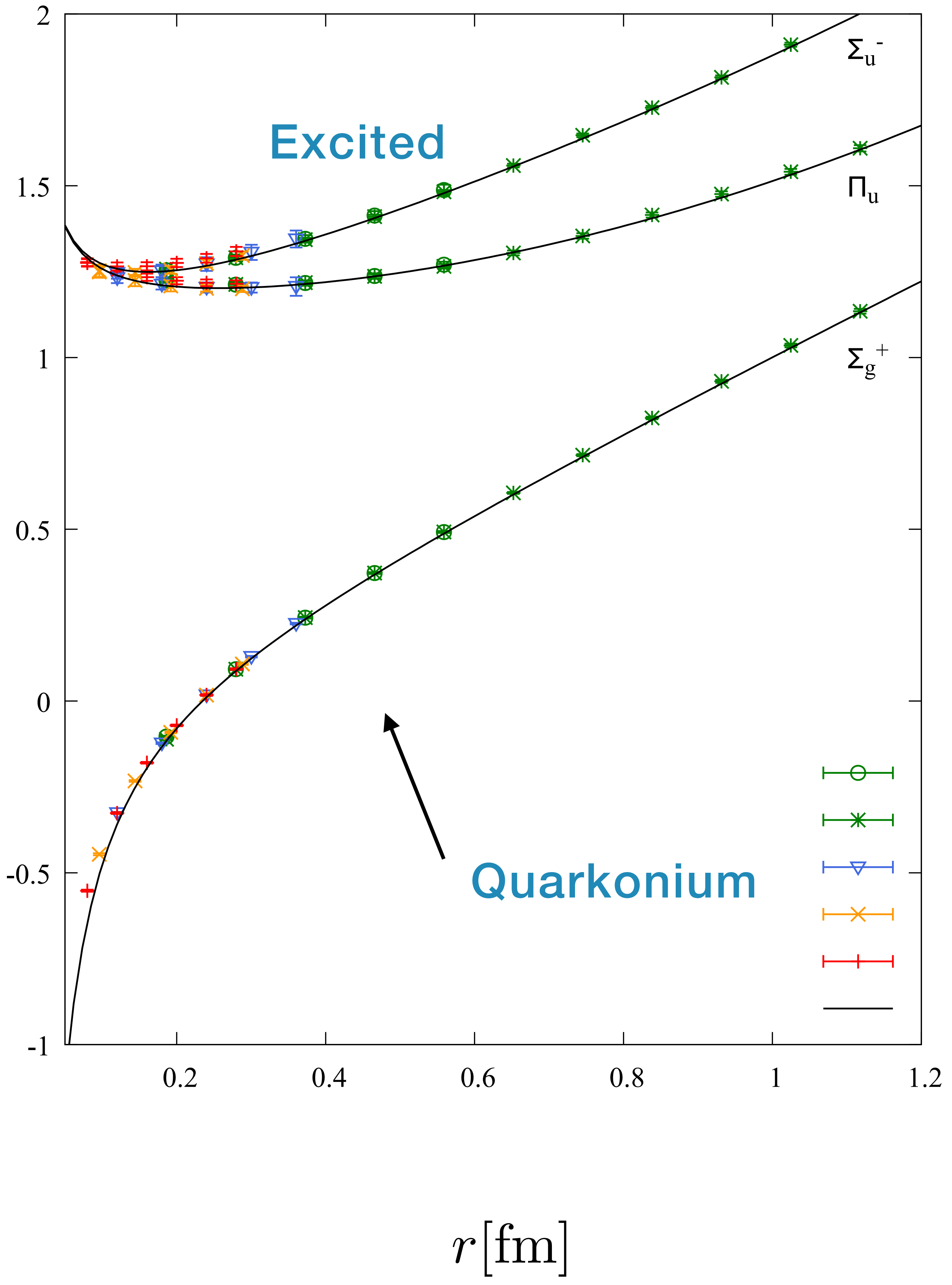
arXiv:2408.04719

Matthias Berwein,[1] Nora Brambilla,[1, 2, 3] Abhishek Mohapatra,[1, *] and Antonio Vairo[1]

The discovery of XYZ exotic states in the hadronic sector with two heavy quarks, represents a significant challenge in particle theory. Understanding and predicting their nature remains an open problem. In this work, we demonstrate how the Born–Oppenheimer (BO) effective field theory (BOEFT), derived from Quantum Chromodynamics (QCD) on the basis of scale separation and symmetries, can address XYZ exotics of any composition. We derive the Schrödinger coupled equations that describe hybrids, tetraquarks, pentaquarks, doubly heavy baryons, and quarkonia at leading order, incorporating nonadiabatic terms, and present the predicted multiplets. We define the static potentials in terms of the QCD static energies for all relevant cases. We provide the precise form of the nonperturbative low-energy gauge-invariant correlators required for the BOEFT: static energies, generalized Wilson loops, gluelumps, and adjoint mesons. These are to be calculated on the lattice and we calculate here their short-distance behavior. Furthermore, we outline how spin-dependent corrections and mixing terms can be incorporated using matching computations. Lastly, we discuss how static energies with the same BO quantum numbers mix at large distances leading to the phenomenon of avoided level crossing. This effect is crucial to understand the emergence of exotics with molecular characteristics, such as the $\chi_{c1}(3872)$. With BOEFT both the tetraquark and the molecular picture appear as part of the same description.

XYZ are a formidable opportunity to learn more about the fundamental strong force!

confinement force



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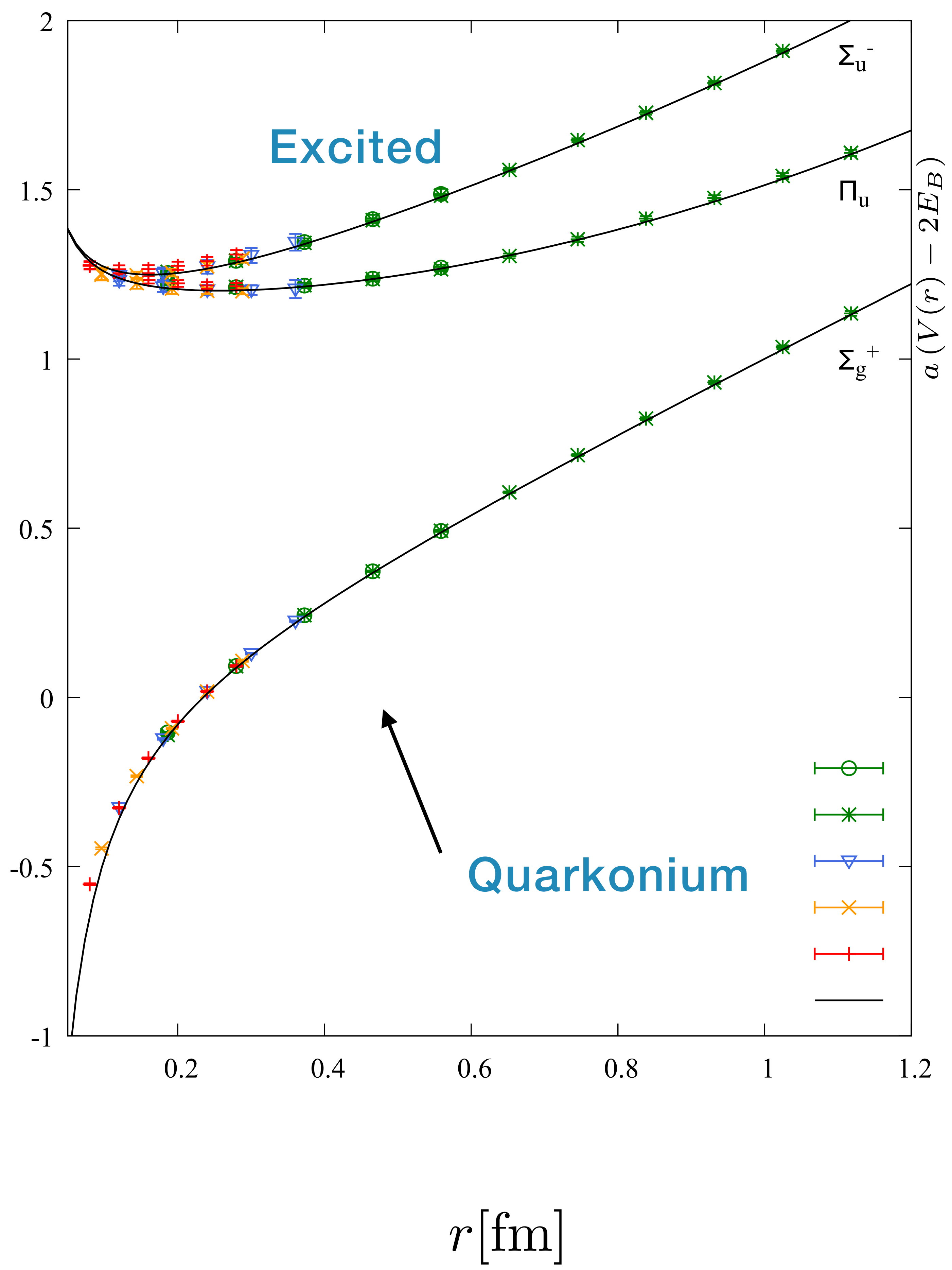
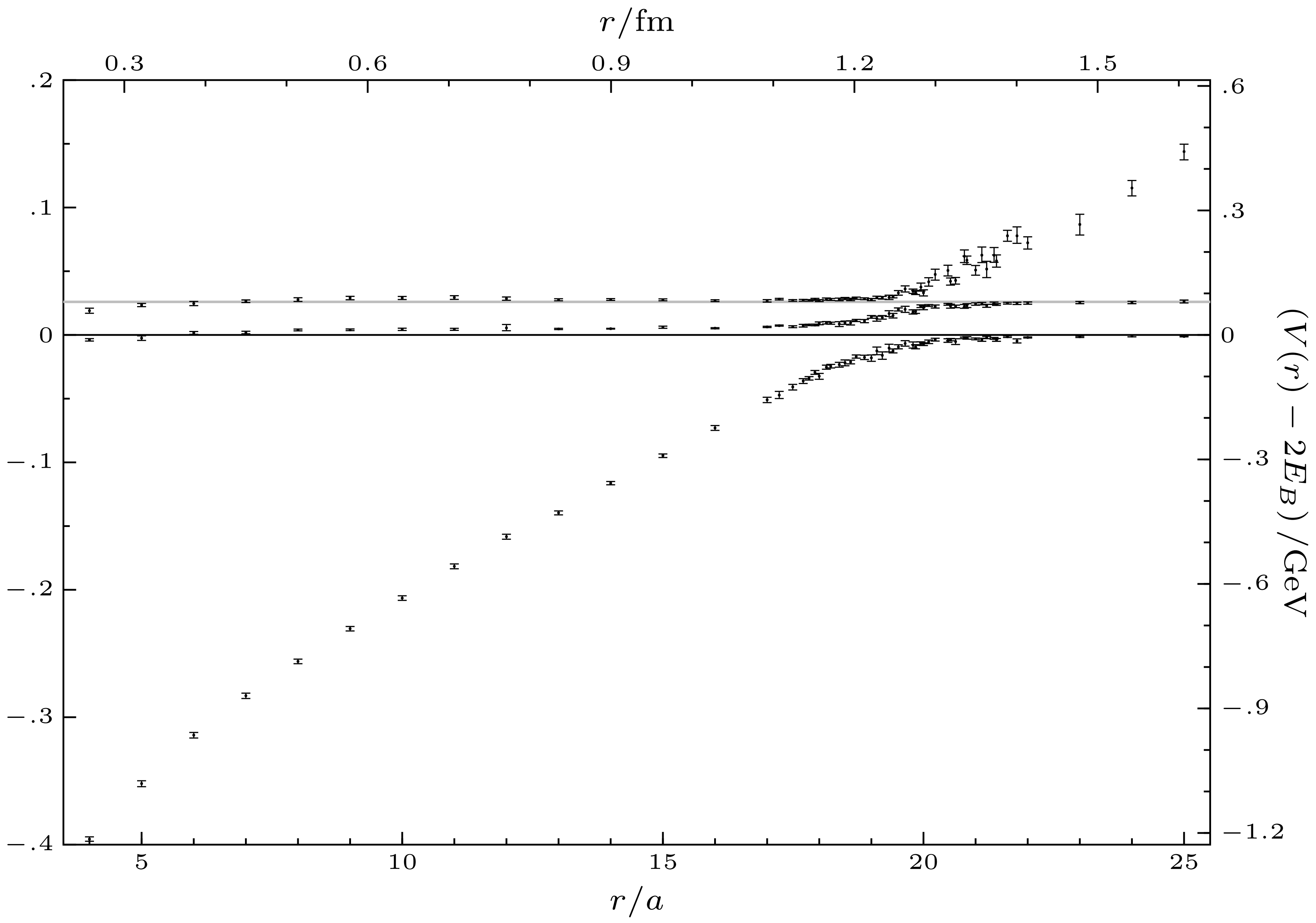
confinement force

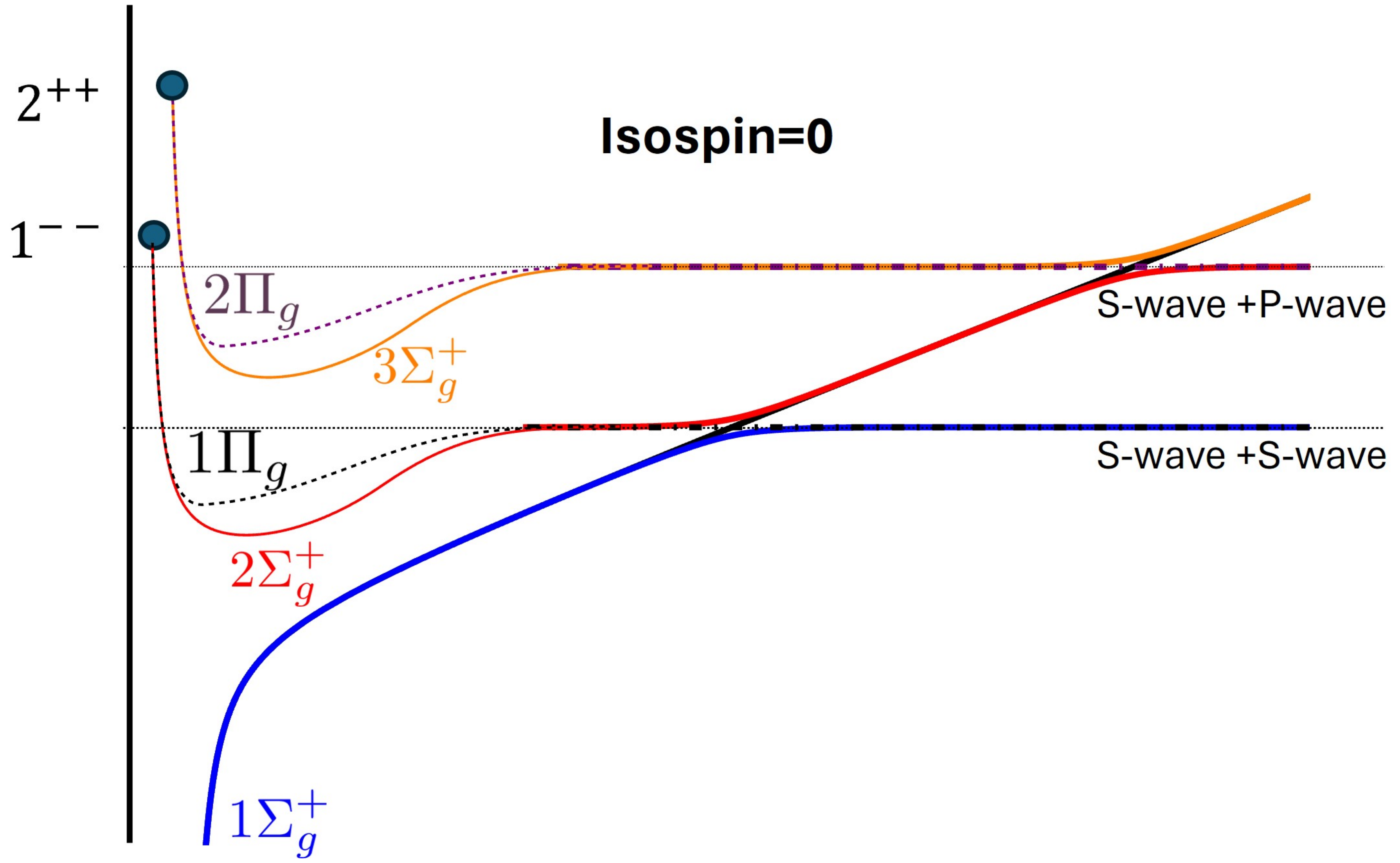




tetraquarks and
heavy-light overlap at
large distance

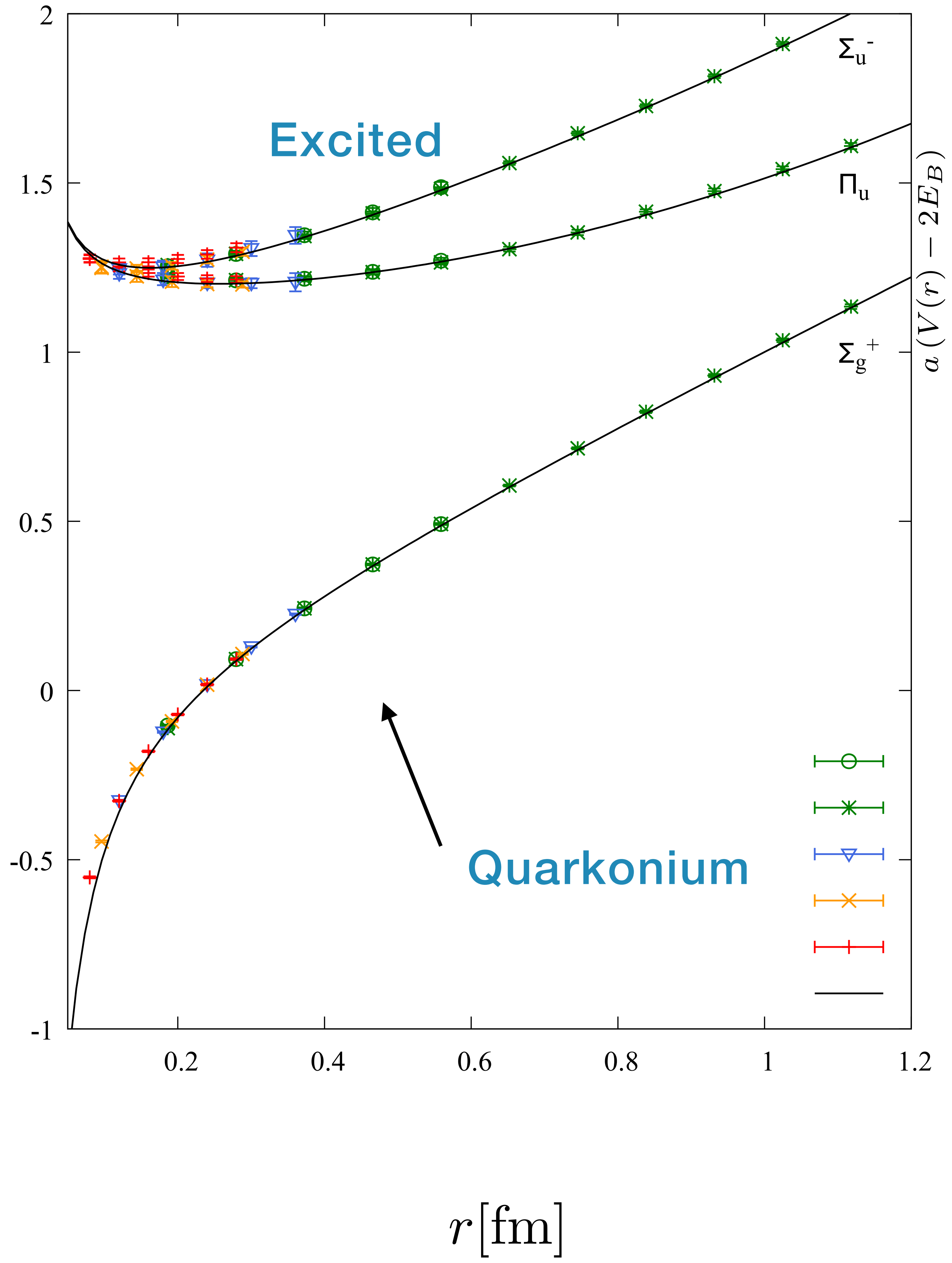
avoided level crossing
between quarkonium
and tetraquarks

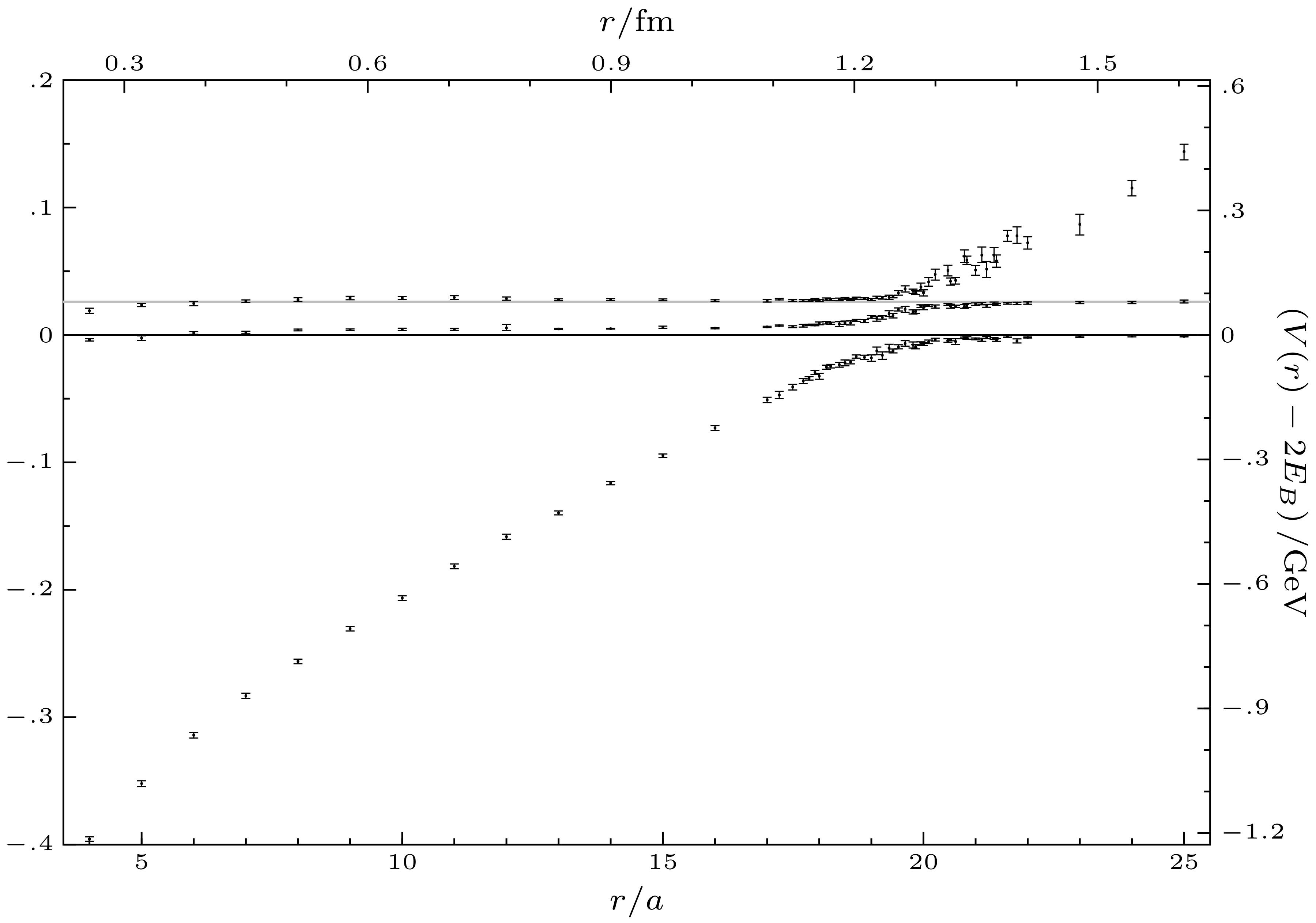
Quarkonium, Tetraquarks
and heavy-light pairs



XYZ are a formidable opportunity to learn more about the fundamental strong force!

confinement force





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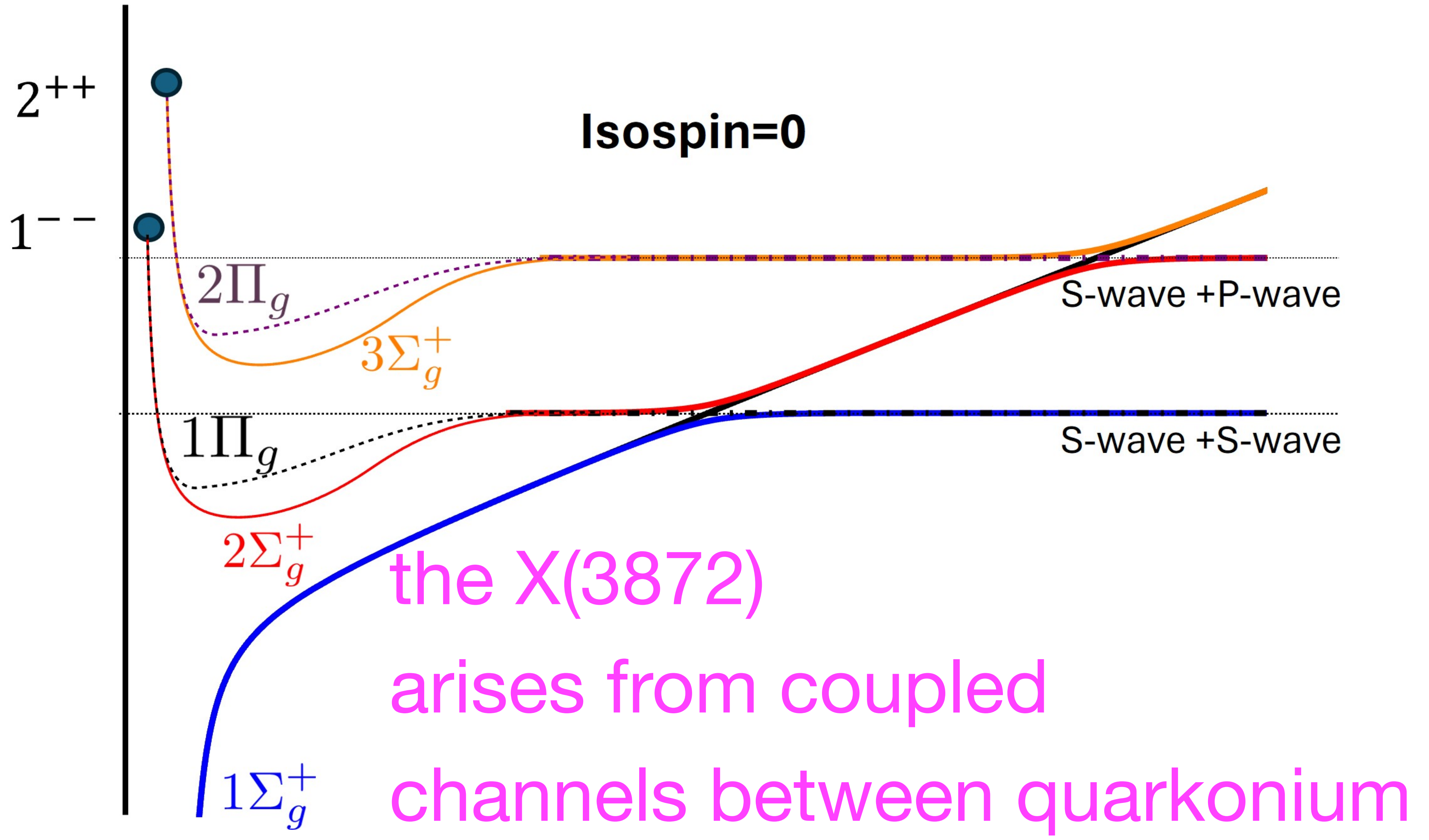


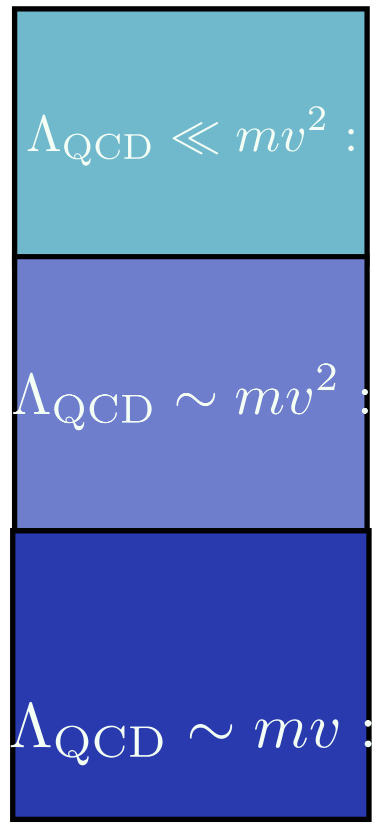
the X(3872)
arises from coupled
channels between quarkonium
and the two first tetraquarks

Our ability to achieve precision calculations and control strongly interacting systems is closely linked to bridging perturbative methods with nonperturbative tools, notably numerical lattice gauge theories

Low energy (nonperturbative) factorized effects depend on the size of the physical system

The EFT factorizes the low energy nonperturbative part.

Depending on the physical system:



local condensates

$\Lambda_{\rm QCD} \ll mv^2$: $\langle F^2(0) \rangle$ $\Upsilon(1S)$, ...

non local condensates

$\Lambda_{\rm QCD} \sim t$ $\Lambda_{\rm QCD} \sim mv^2$: $\langle F(t) F(0) \rangle$ annihilations, short range $c\bar{c}$, $b\bar{b}$, gluelumps, ...

$\Lambda_{\rm QCD} \sim loop$ $\langle e^{ig \oint dx A(x)} \rangle$ long range $c\bar{c}$, $b\bar{b}$, hybrids, glueballs, ...

$\Lambda_{\rm QCD} \sim mv$:

Wilson loops

The more extended the physical object, the more we probe the non-perturbative vacuum.

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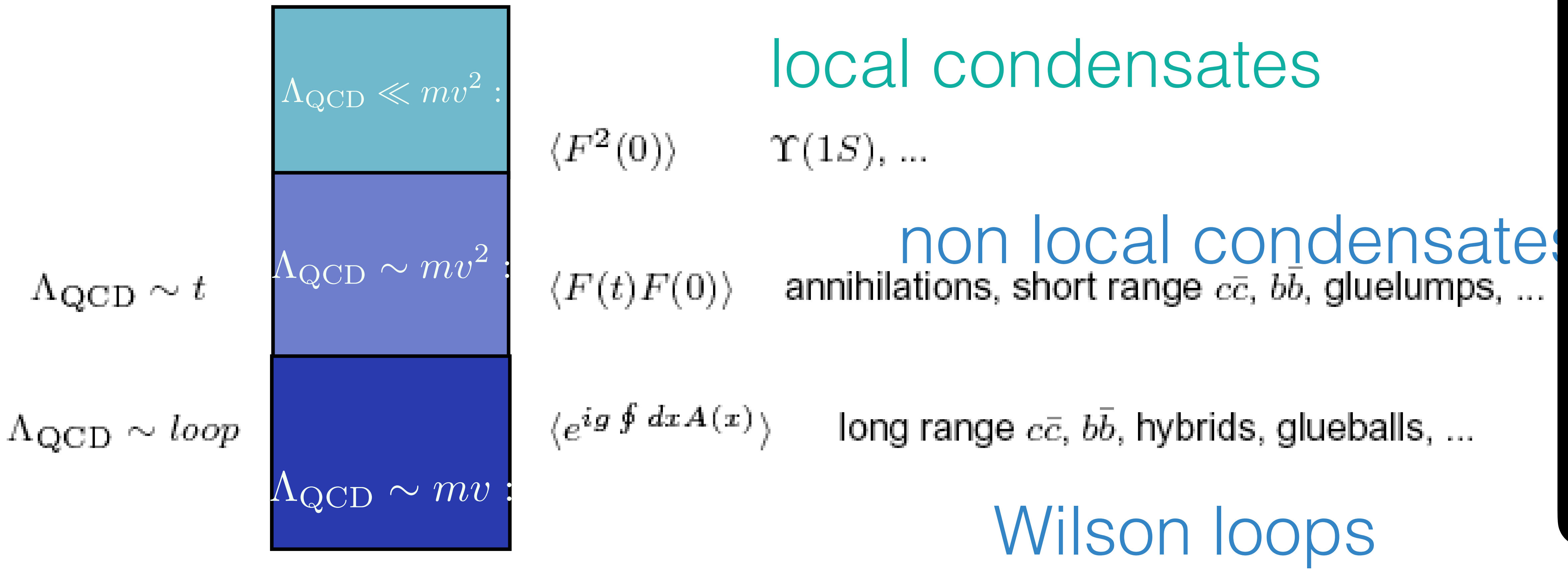

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GAIN:

Inside the EFT:Model independent predictions, power counting

Lattice Calculation of only few nonperturbative objects, universal and depending only on the glue—> at variance with the state dependent calculation of each single observable with the full dynamics!

Inside the EFT: flexible phenomenological applications, understanding of the underlying degrees of freedom and dynamics

CHALLENGE:

Need techniques to reduce noise and improve convergence to continuum for calculation of chromelectric and chomomagnetic fields—> Gradient flow

Avoid change of scheme between continuum and lattice (cutoff) regularization> Gradient flow (composite operators renormalisation in cutoff scheme is painful)

problem of slow convergence to continuum—> cured in gradient flow!

As TUMQCD Lattice collaboration we are addressing theseproblems

Outlook

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Allow us to make calculations with unprecented precision and to systematically factorize short from long range contributions —> **in particular we can explore new characteristics of the strong force**

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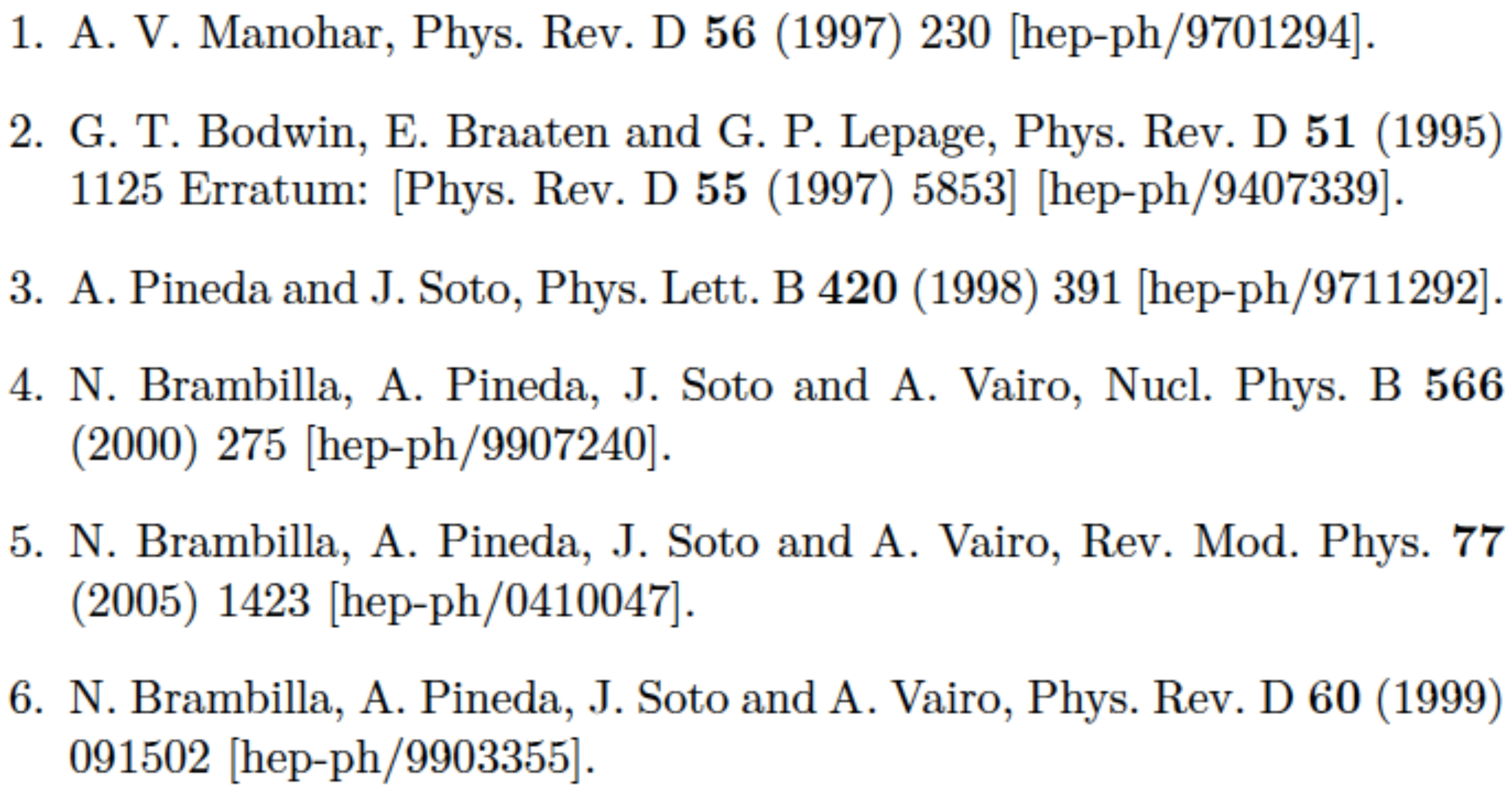
Combining pNREFTs and other EFTs (Chiral, HTL..) , lattice calculations and new concepts (open quantum systems) we can address relevant contemporary problems:

- The XYZ world
- Quarkonium production: we can factorize the LDMEs in low energy correlators to be calculated on the lattice!
- Quarkonium potential and spectrum at finite temperature: pNRQCD +HTL , new paradigm on suppression
- Nonequilibrium evolution of quarkonium in QGP: pNRQCD + HTL+open quantum systems+ lattice: Linblad eqs
- Dark matter pairs in early universe: pNREFT + HTL+open quantum systems: cross section and evolution
- Applications to Jets, neutrinos, cosmology, quantum information

Backup

BASED on CLASSIC PAPERS



1. A. V. Manohar, Phys. Rev. D **56** (1997) 230 [hep-ph/9701294].
2. G. T. Bodwin, E. Braaten and G. P. Lepage, Phys. Rev. D **51** (1995) 1125 Erratum: [Phys. Rev. D **55** (1997) 5853] [hep-ph/9407339].
3. A. Pineda and J. Soto, Phys. Lett. B **420** (1998) 391 [hep-ph/9711292].
4. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. B **566** (2000) 275 [hep-ph/9907240].
5. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Rev. Mod. Phys. **77** (2005) 1423 [hep-ph/0410047].
6. N. Brambilla, A. Pineda, J. Soto and A. Vairo, Phys. Rev. D **60** (1999) 091502 [hep-ph/9903355].

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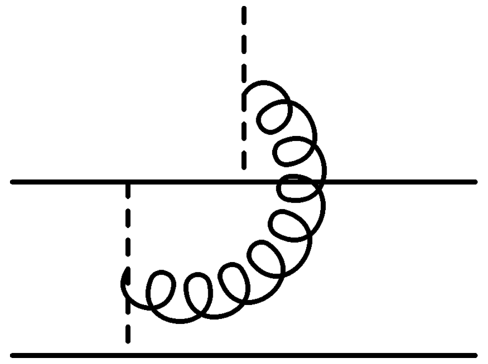
Some building blocks for the calculation:

$$V_A$$

The first contributing diagrams are of the type:

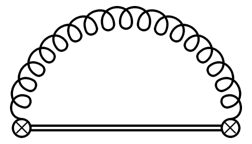


Therefore

$$V_A(r,\mu) = 1 + \mathcal{O}(\alpha_s^2)$$

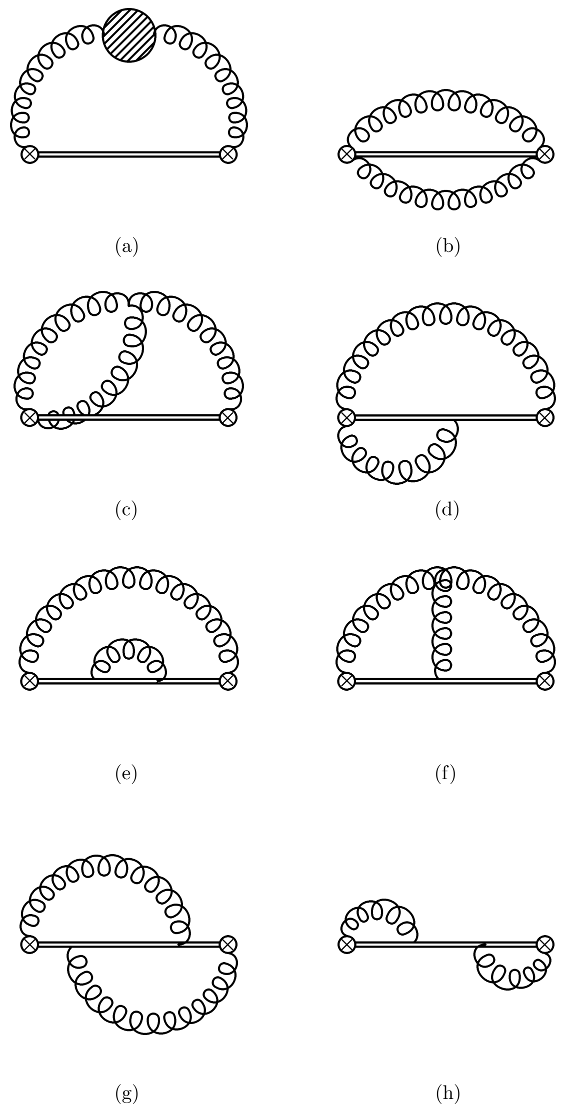
Chromoelectric field correlator: $\langle E(t)E(0)\rangle$

Is known at two loops.



LO

now known NNLO
Kniehl et al. 2021



NLO

○ Eidemüller Jamin PLB 416 (1998) 415

Static octet potential

$$\lim_{T\to\infty} \frac{i}{T} \ln \frac{\left\langle \bullet\!\boxed{}\!\bullet \right\rangle}{\langle \phi^{\mathrm{adj}}_{ab} \rangle} = \frac{1}{2N}\frac{\alpha_s}{r}(1 + \#\alpha_s + \#\alpha_s^2 + \#\alpha_s^3 + \#\alpha_s^3 \ln \mu r + \dots)$$

Is known at three loops.

○ Anzai Prausa A.Smirnov V.Smirnov Steinhauser PRD 88 (2013) 054030

POINCARE' INVARIANCE IN NREFTS

Poincare’ Invariance in NREFTs

EFTs preserve all the invariances of the fundamental QFT.
Therefore NREFTs are constrained by the Poincaré invariance of the fundamental QFT, although Lorentz invariance is apparently broken by the nonrelativistic expansion.

It has been suggested, even before the establishing of EFTs, that Poincaré invariance provides non trivial constraints on the form of the potentials.

○ Dirac RMP 21 (1949) 302

Within NREFTs these constraints may be implemented in a rigorous setting. They allow to fix some of the matching coefficients/potentials of the NREFT to all orders and nonperturbatively without computing them. In QCD, these constraints can be tested against lattice determinations.

Poincare’ invariance gives the same constraints as reparameterization invariance (relations among the matching coefficients of the bilinear fermion terms in NRQCD) PLUS new relations, among the coefficients of the 4 fermions terms in NRQCD and among the potentials in pNRQCD

○ Brambilla Gromes Vairo PRD 64 (2001) 076010, PLB 576 (2003) 314
Berwein Brambilla Hwang Vairo PRD 99 (2019) 094008

Heinonen, Hill, Solon PRD86 (2012) 094020

Poincare' algebra

For any Poincare invariant theory the generators H, P , J and K of time translation, space translation, rotations and Lorentz boosts satisfy the Poincare algebra:

$$\begin{aligned}
[\mathbf{P}^i, \mathbf{P}^j] &= 0 \\
[\mathbf{P}^i, H] &= 0 \\
[\mathbf{J}^i, \mathbf{P}^j] &= i\epsilon_{ijk}\mathbf{P}^k \\
[\mathbf{J}^i, H] &= 0 \\
[\mathbf{J}^i, \mathbf{J}^j] &= i\epsilon_{ijk}\mathbf{J}^k \\
[\mathbf{P}^i, \mathbf{K}^j] &= -i\delta_{ij}H \\
[H, \mathbf{K}^i] &= -i\mathbf{P}^i \\
[\mathbf{J}^i, \mathbf{K}^j] &= i\epsilon_{ijk}\mathbf{K}^k \\
[\mathbf{K}^i, \mathbf{K}^j] &= -i\epsilon_{ijk}\mathbf{J}^k
\end{aligned}$$

Once P and J are written in terms of the EFTs fields, and H and K have been matched, the algebra constraints the matching coefficients of H, which include the potentials, and K.

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\end{aligned}
$$

—>The Poincare algebra imposes the following constraints on the potentials

$$V_{LS}^{(1)}(r) - V_{LS}^{(2)}(r) + \frac{1}{2r}V^{(0)\prime}(r) = 0$$

$$4V_{L^2}^{(1)}(r) - 2V_{L^2}^{(2)}(r) + rV^{(0)\prime}(r) = 0$$

$$4V_{p^2}^{(1)}(r) - 2V_{p^2}^{(2)}(r) + V^{(0)}(r) - rV^{(0)\prime}(r) = 0$$

The constraints are generic and do not depend on the dynamical content of the EFT. They are satisfied by any potential defined in an EFT and derived from a relativistic QFT.

Once P and J are written in terms of the EFTs fields, and H and K have been matched, the algebra constraints the matching coefficients of H, which include the potentials, and K.

LDMEs in pNRQCD

The pNRQCD factorization formulas for P-wave quarkonium em production are

$$\begin{aligned}
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]};\text{em})|\Omega\rangle =&(2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\left[1+\frac{2}{3}\frac{i\mathcal{E}_2}{m}+O\left(v^2\right)\right]\\
\langle\Omega|\mathcal{T}^{\chi_{QJ}}({}^3P_J^{[8]};\text{em})|\Omega\rangle =&(2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\frac{4}{3}\frac{\mathcal{E}_1}{m}\\
\langle\Omega|\mathcal{P}^{\chi_{QJ}}({}^3P_J^{[1]};\text{em})|\Omega\rangle =&(2J+1)\frac{3N_c}{2\pi}|R'(0)|^2\left[m\varepsilon-\frac{2}{3}\mathcal{E}_1+O\left(v^3\right)\right]
\end{aligned}$$

$R'(0)$ is the derivative of the radial wavefunction at the origin, and ε the binding energy.

$$\mathcal{E}_n=\frac{1}{2N_c}\int_0^\infty dt\,t^n\langle\Omega|gE^{i,a}(t)\Phi^{ab}(0,t)gE^{i,b}(0)|\Omega\rangle$$

$\Phi^{ab}(0,t)$ is a Wilson line in the adjoint representation connecting $(t,\mathbf{0})$ with $(0,\mathbf{0})$.

Chromoelectric correlators for electromagnetic production

The wavefunctions at the origin may be computed solving the equation of motion of pNRQCD with potentials determined from lattice QCD or via phenomenological models.

The correlators can be fitted on data for $\chi_{c0}(1P) \to \gamma\gamma$, $\chi_{c2}(1P) \to \gamma\gamma$ and $\sigma(e^+e^- \to \chi_{c1}(1P) + \gamma)$ ($= 17.3^{+4.2}_{-3.9} \pm 1.7$ fb at $\sqrt{s}$ =10.6 GeV from Belle).

- `Belle coll PRD 98 (2018) 092015`

The correlators are universal: they do not depend neither on the flavor of the heavy quark nor on the quarkonium state:

$$\mathcal{E}_1 = -\,0.20^{+0.14}_{-0.14} \pm 0.90 \text{ GeV}^2$$

$$i\mathcal{E}_2 = 0.77^{+0.98}_{-0.86} \pm 0.85 \text{ GeV}$$

The universal nature of the correlators allows to use them to compute cross sections (and decay widths) for quarkonia with different principal quantum number and bottomonia.

LDMEs in pNRQCD

The pNRQCD factorization formulas for P-wave quarkonium hadroproduction are

$$\begin{aligned}
\langle\Omega|\mathcal{O}^{h_Q}({}^1P_1^{[1]})|\Omega\rangle =& 3\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\\
\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle =& 3\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{1}{9N_cm^2}\mathcal{E}\\
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle =& (2J+1)\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\\
\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle =& (2J+1)\times\frac{3N_c}{2\pi}|R^{(0)\prime}(0)|^2\frac{1}{9N_cm^2}\mathcal{E}
\end{aligned}$$

$R^{(0)\prime}(0)$ is the derivative of the radial wavefunction at the origin at leading order in v.
LDMEs are polarization summed in the case of χ_{QJ} states.

The above expressions imply (at leading order in v) the universality of the ratios

$$\frac{m^2\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]})|\Omega\rangle}{\langle\Omega|\mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]})|\Omega\rangle}=\frac{m^2\langle\Omega|\mathcal{O}^{h_Q}({}^1S_0^{[8]})|\Omega\rangle}{\langle\Omega|\mathcal{O}^{h_Q}({}^1P_1^{[1]})|\Omega\rangle}=\frac{\mathcal{E}}{9N_c}$$

$$\mathcal{E}=\frac{3}{N_c}\int_0^\infty dt\,t\int_0^\infty dt'\,t'\,\langle\Omega|\Phi_\ell^{\dagger ab}\Phi^{\dagger ad}(0;t)gE^{d,i}(t)gE^{e,i}(t')\Phi^{ec}(0;t')\Phi_\ell^{bc}|\Omega\rangle$$

$$\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle = \frac{1}{2N_c m^2}\,\frac{3|R_V^{(0)}(0)|^2}{4\pi}\,\mathcal{E}_{10;10}(\mu)$$

$$\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle = \frac{1}{6N_c m^2}\,\frac{3|R_V^{(0)}(0)|^2}{4\pi}\,c_F^2(\mu)\,\mathcal{B}_{00}(\mu)$$

$$\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle = \frac{1}{18N_c}\,\frac{3|R_V^{(0)}(0)|^2}{4\pi}\,\mathcal{E}_{00}$$

$$\begin{aligned}\mathcal{E}_{10;10} &= \Big| d^{dac}\int_0^\infty dt_1\, t_1 \int_{t_1}^\infty dt_2\, gE^{b,i}(t_2) \\ &\qquad \times \Phi_0^{bc}(t_1;t_2) gE^{a,i}(t_1)\Phi_0^{df}(0;t_1)\Phi_\ell^{ef}|\Omega\rangle\Big|^2 \\ \mathcal{B}_{00} &= \Big|\int_0^\infty dt\, gB^{a,i}(t)\Phi_0^{ac}(0;t)\Phi_\ell^{bc}|\Omega\rangle\Big|^2 \\ \mathcal{E}_{00} &= \Big|\int_0^\infty dt\, gE^{a,i}(t)\Phi_0^{ac}(0;t)\Phi_\ell^{bc}|\Omega\rangle\Big|^2\end{aligned}$$