QCD in the chiral SU(3) limit from baryon masses on Lattice QCD ensembles

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Outline

Hadron masses from the chiral Lagrangian

- Meson and baryon masses at the one-loop level
- Large-N_c constraints on low-energy constants (LECs)

Chiral extrapolations for Lattice QCD data

- Lattice QCD data on CLS ensembles
- Our global Fit with finite-volume and discretization effects
- Result

Summary and future work

Hadron masses from the chiral Lagrangian

- Meson and baryon masses at the one-loop level
- Large-N_c constraints on low-energy constants (LECs)

The chiral SU(3) Lagrangian

A few terms of the chiral Lagrangian for hadron masses

$$
\begin{aligned}
\mathscr{L} = & -f^2\,\mathbf{tr}\,U_\mu\,U^\mu + \tfrac{1}{2}\,f^2\,\mathbf{tr}\,\chi_+ + 4\,L_6\,(\mathbf{tr}\,\chi_+)^2 + 4\,L_7\,(\mathbf{tr}\,\chi_-)^2 \\
& -8\,L_4\,\mathbf{tr}\,U_\mu\,U^\mu\,\mathbf{tr}\,\chi_+ - 8\,L_5\,\mathbf{tr}\,U_\mu\,U^\mu\,\chi_+ \\
& +\mathbf{tr}\,\bar{B}\,(i\gamma^\mu D_\mu - M_{[8]})\,B \\
& +F\,\mathbf{tr}\,\bar{B}\,\gamma^\mu\gamma_5\,[iU_\mu,\,B] + D\,\mathbf{tr}\,\bar{B}\,\gamma^\mu\gamma_5\,\{iU_\mu,\,B\} + \cdots\,,
\end{aligned}
$$

with

$$
\begin{aligned}
U_\mu &= \tfrac{1}{2}\,u^\dagger\,(\partial_\mu e^{i\Phi/f})\,u^\dagger\,, & u &= e^{i\Phi/2f}\,, \\
D_\mu B &= \partial_\mu B + [\Gamma_\mu,\,B]\,, & \Gamma_\mu &= \tfrac{1}{2}\,u\,(\partial_\mu u^\dagger) + \tfrac{1}{2}\,u^\dagger\,(\partial_\mu u)\,, \\
\chi_\pm &= \tfrac{1}{2}\,(u\,\chi_0\,u \pm u^\dagger\chi_0\,u^\dagger)\,, & \chi_0 &= 2\,B_0\,\mathrm{diag}(m,\,m,\,m_s)\,,
\end{aligned}
$$

The chiral SU(3) Lagrangian

A few terms of the chiral Lagrangian for hadron masses

$$\begin{aligned}\mathscr{L} = & -f^2\,\mathbf{tr}\,U_\mu U^\mu + \tfrac{1}{2} f^2\,\mathbf{tr}\,\chi_+ + 4\,L_6\,(\mathbf{tr}\,\chi_+)^2 + 4\,L_7\,(\mathbf{tr}\,\chi_-)^2 \\ & -8\,L_4\,\mathbf{tr}\,U_\mu U^\mu\,\mathbf{tr}\,\chi_+ - 8\,L_5\,\mathbf{tr}\,U_\mu U^\mu \chi_+ \\ & +\mathbf{tr}\,\bar{B}\,(i\gamma^\mu D_\mu - M_{[8]})\,B \\ & +F\,\mathbf{tr}\,\bar{B}\,\gamma^\mu\gamma_5\,[iU_\mu,\,B] + D\,\mathbf{tr}\,\bar{B}\,\gamma^\mu\gamma_5\,\{iU_\mu,\,B\} + \cdots\,,\end{aligned}$$

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},\ B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}$$

- More LECs for hadron masses up to N^3LO
- Even more LECs for decuplet baryons involved
- For scattering processes, we need additional LECs.

Meson masses at 1 loop with on-shell masses

$$m_\pi^2 = 2\,B_0\,m + \frac{10\,m_\pi^2 - 4\,m_K^2 + 3\,m_\eta^2}{18\,f^2}\,\bar{I}_\pi - \frac{m_\pi^2}{6\,f^2}\,\bar{I}_\eta$$
$$+\frac{m_\pi^2}{f^2}\left\{8\,(m_\pi^2 + 2\,m_K^2)\,(2\,L_6 - L_4) + 8\,m_\pi^2\,(2\,L_8 - L_5)\right\},$$

with chiral logs $\bar{I}_Q = \frac{m_Q^2}{(4\,\pi)^2}\,\ln\frac{m_Q^2}{\mu^2}$, similar for m_K^2 and m_η^2 .

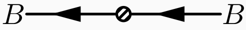
- 1+3 LECs: f, $(2\,L_6 - L_4)$, $(2\,L_8 - L_5)$, and $(3\,L_7 + L_8)$
- Set of 3 nonlinear equations with LECs, quark masses, m, m_s, and on-shell meson masses, m_π^2, m_K^2 and m_η^2
  - μ dependence from $\bar{I}_Q$ is balanced by L_j.
- Different from conventional chiral perturbation theory (χPT)

Towards baryon masses

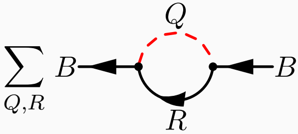
LECs & diagrams for baryon masses

$B \in \{N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega\}$

There are 2 LECs: $M_{[8]}, M_{[10]}$.



There are 5 LECs proportional to m, m_s (or m_π^2, m_K^2, m_η^2).



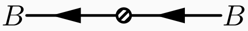
$Q \in \{\pi, K, \eta\}$
$R \in \{N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega\}$

There are 4 LECs: F, D, C, H.

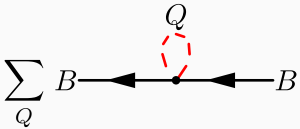
- Loops have on-shell hadron masses [a]
  - Not only meson, but baryon masses as well
  - Different from conventional χPT approach

[a] Lutz,Heo,Guo,Semke [arXiv:1801.06417,1801.10122,nucl-th/0511061]

LECs & diagrams for baryon masses ..cont'd



There are 12 LECs proportional to $m^2, m\, m_s, m_s^2$.



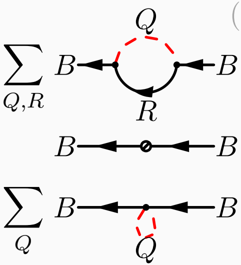
$B \in \{N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega\}$
$Q \in \{\pi, K, \eta\}$

There are 17 LECs proportional to $m\, \bar{I}_Q, m_s\, \bar{I}_Q$.

- The set of 8 nonlinear equations with
  - $\overbrace{2}^{\text{LO}} + \overbrace{5}^{\text{NLO}} + \overbrace{4}^{\text{N}^2\text{LO}} + \overbrace{12+17}^{\text{N}^3\text{LO}} =$ 40 LECs
  - On-shell baryon masses, $M_N, M_\Lambda, M_\Sigma, M_\Xi, M_\Delta, M_{\Sigma^*}, M_{\Xi^*}, M_\Omega$
  - On-shell meson masses, m_π, m_K, m_η
- μ dependence from loops is balanced by LECs.

40 LECs?

Large-N_c constraints on low-energy constants



LECs	Large-N_c LO	Large-N_c NLO
(F, D, C, H) 4	1	2
12	5	8
17	5	11

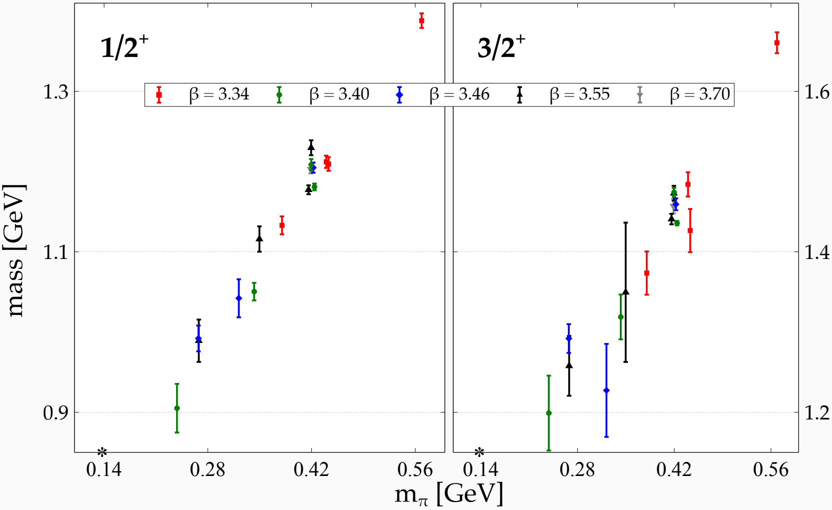
- There were early works on large-N_c analysis. [a] [b]
- Novel large-N_c sum rules have been published by our group. [c]
- What do we do with those?

[a] 't Hooft [Nucl.Phys.B 72 (1974) 461] Witten [Nucl.Phys.B 160 (1979) 57-115]
[b] Luty,M-Russell,Dashen,Jenkins,Manohar [arXiv:hep-ph/9310369,9310379,9411234]
[c] Lutz,Semke [arXiv:1012.4365] Heo,Kobdaj,Lutz,Guo [arXiv:1908.11816,1801.06417]

Chiral extrapolations for Lattice QCD data

- We consider Lattice QCD data on CLS ensembles
  - 3 classes: $m_s = m$, $m_s \sim$ const., and $2m + m_s \sim$ const.
- Our global Fit w/ finite-volume and discretization effects
- Result

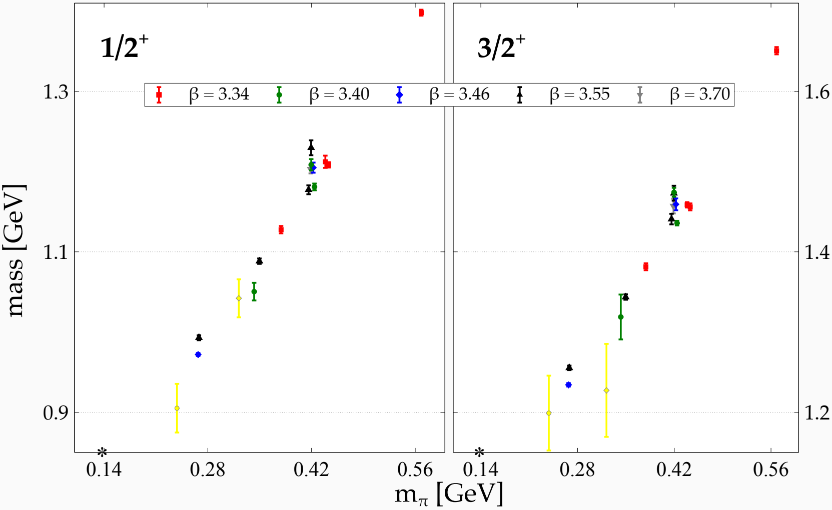
Flavor-symm. ensembles: comparison



From RQCD Collaboration · Bali et al [arXiv:2211.03744]

⇒ Such ensembles are of the utmost importance to SU(3) χPT.

Flavor-symm. ensembles: comparison ..cont'd



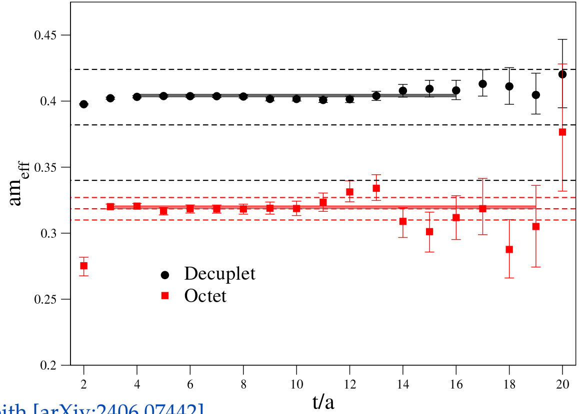
Reevaluated baryon masses on some CLS ensembles

RQCD Collaboration · Bali et al [arXiv:2211.03744]

Hudspith,Lutz,Mohler [arXiv:2404.02769] Lutz,Heo,Hudspith [arXiv:2406.07442]

Flavor-symm. ensembles ..cont'd

Effective mass plots of baryons on the X251 ensemble



Lutz,Heo,Hudspith [arXiv:2406.07442]

Much-improved statistical precision

- A single exponential ansatz for the correlation functions
- On gauge-fixed wall sources

Finite-box effects

Loop contributions evaluated in the finite box [a] [b]

[a] Lutz,Heo,Guo [arXiv:1801.06417,2301.06837]
[b] Lutz,Bavontaweepanya,Kobdaj,Schwarz [arXiv:1401.7805]

- From tadpoles,

$$\bar{I}_Q = \frac{m_Q^2}{(4\pi)^2}\,\ln\frac{m_Q^2}{\mu^2} + \frac{1}{4\pi^2}\sum_{\vec{n}\in\mathbb{Z}^3}^{\vec{n}\neq 0}\frac{m_Q}{L|\vec{n}|}\,K_1(m_Q\,L|\vec{n}|)\,,$$

- From bubbles,

$$\Delta\bar{I}_{QR}(M_B) = \frac{1}{8\pi^2}\sum_{\vec{n}\in\mathbb{Z}^3}^{\vec{n}\neq 0}\left(\int_0^1 \mathrm{d}z\,K_0\big(L|\vec{n}|\,\mu(z)\big) - \frac{2\,m_Q\,K_1(m_Q\,L|\vec{n}|)}{L|\vec{n}|\,(M_R^2 - m_Q^2)}\right),$$

$$\mu^2(z) = z\,M_R^2 + (1-z)\,m_Q^2 - z\,(1-z)\,M_B^2\,,$$

where $V = L^3$ and $K_n(x)$ is the modified Bessel function of 2nd kind.

Discretization effects

We implement its effects [a] by the spurion-field approach [b].

[a] Lutz,Heo,Guo,Hudspith [arXiv:2301.06837,2406.07442]
[b] Aoki et al [arXiv:hep-lat/0509049]

- $O(a)$-improved Wilson quark action [b]
- We use $O(a^2)$-dependent leading order LECs.

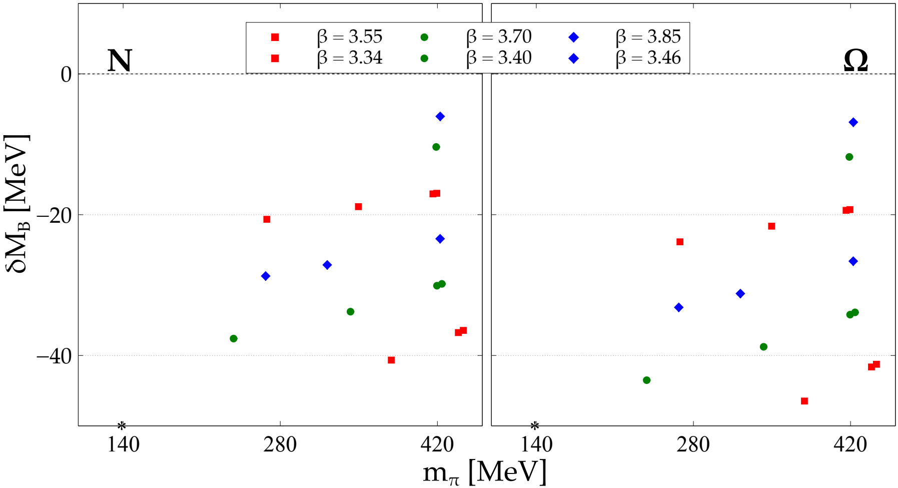
$$
\begin{aligned}
M_{[8]} &\to M_{[8]} + a^2\gamma_{M_8}\,, & M_{[10]} &\to M_{[10]} + a^2\gamma_{M_{10}}\,,\\
b_0 &\to b_0 + a^2\gamma_{b_0}\,, & d_0 &\to d_0 + a^2\gamma_{d_0}\,,\\
b_D &\to b_F + a^2\gamma_{b_D}\,, & d_D &\to d_D + a^2\gamma_{d_D}\,,\\
b_F &\to b_F + a^2\gamma_{b_F}\,,
\end{aligned}
$$

where a depends upon the 6 sets of ensembles at fixed values of β.

$$\Longrightarrow\; a_{\rm CLS}^{\beta=3.34}\,,\, a_{\rm CLS}^{\beta=3.40}\,,\, a_{\rm CLS}^{\beta=3.46}\,,\, a_{\rm CLS}^{\beta=3.55}\,,\, a_{\rm CLS}^{\beta=3.70}\,,\, a_{\rm CLS}^{\beta=3.85}$$

Discretization effects ..cont'd

To octet and decuplet masses on flavor-symm. ensembles



Lutz,Heo,Hudspith [arXiv:2406.07442]

Towards our Fit strategy

Our global Fit strategy for LECs

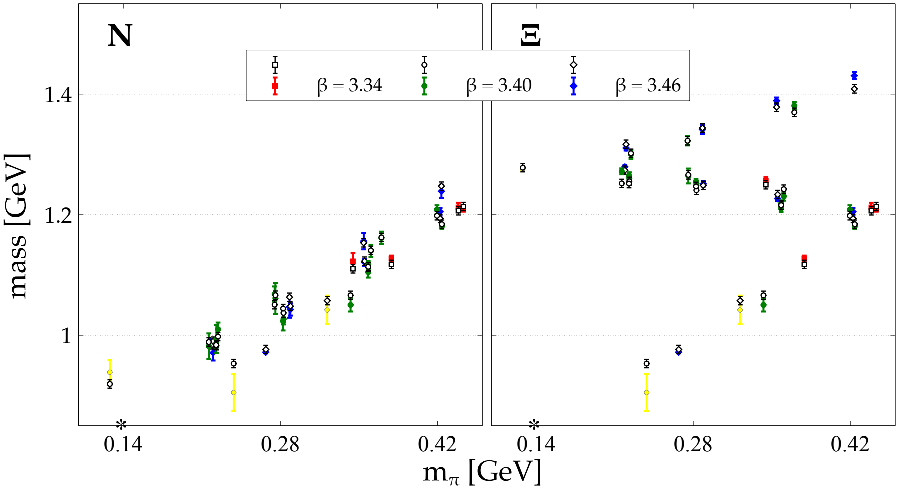
- We consider Lattice QCD data points on CLS ensembles. [a] [b]
  - m_π & m_K are inputs and chosen smaller than 0.55 GeV.
  - m_η is determined as a solution of coupled nonlinear equations, together with m & m_s.
  - 348 data points for baryon $(N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*, \Omega)$ masses, which are solutions of our set of coupled nonlinear equations.
- Unconventional scale setting is performed in terms of the empirical values of isospin-averaged 8 baryon masses.
- We assume residual systematic error of 7 MeV in the baryon masses at N^{3}LO, based on the flavor-SU(3) chiral Lagrangian.
- $44 = 30_{\text{LEC}} + 14_{\text{Lattice}}$ Fit parameters
  - Large-N_c relations for LECs enter only at N^{3}LO
  - Lattice parameters, together with a few of LECs

[a] RQCD Collaboration · Bali et al [arXiv:2211.03744]

[b] Hudspith,Lutz,Mohler [arXiv:2404.02769] Lutz,Heo,Hudspith [arXiv:2406.07442]

Result

Some octet masses on 3 coarsest lattices

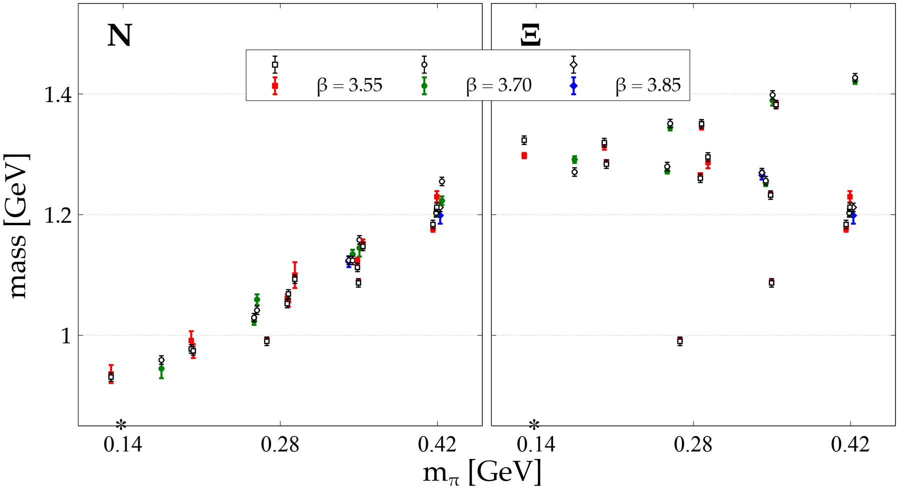


χ^2/dof = 0.988, our Fit: white symbols w/ systematic error of 7 MeV

More in Lutz,Heo,Hudspith [arXiv:2406.07442]

Result ..cont'd

Some octet masses on 3 finest lattices

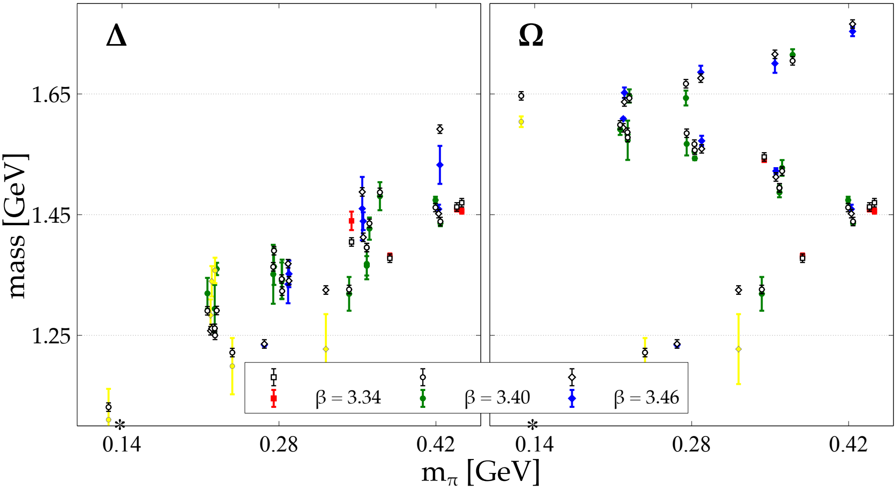


χ^2/dof = 0.988, our Fit: white symbols w/ systematic error of 7 MeV

More in Lutz,Heo,Hudspith [arXiv:2406.07442]

Result ..cont'd

Some decuplet masses on 3 coarsest lattices

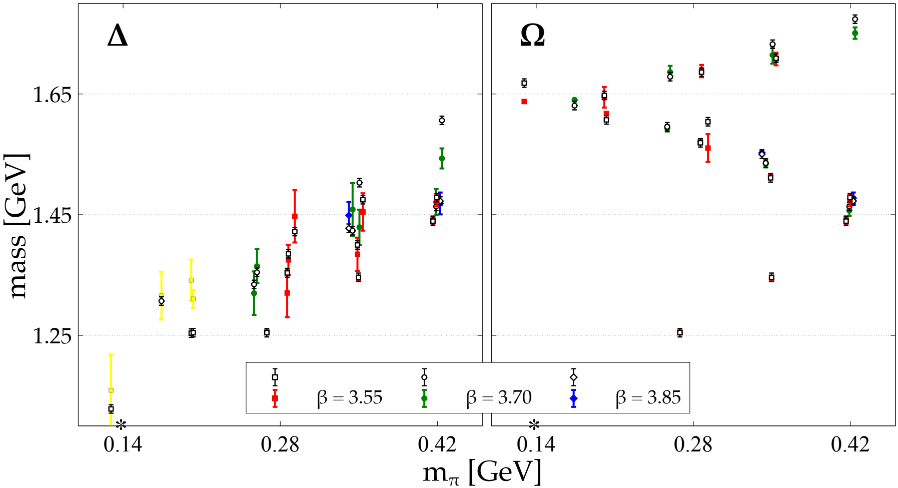


χ^2/dof = 0.988, our Fit: white symbols w/ systematic error of 7 MeV

More in Lutz,Heo,Hudspith [arXiv:2406.07442]

Result ..cont'd

Some decuplet masses on 3 finest lattices



χ^2/dof = 0.988, our Fit: white symbols w/ systematic error of 7 MeV

More in Lutz,Heo,Hudspith [arXiv:2406.07442]

LECs in the meson sector

	MILC[1]	HPQCD[2]	previous Fit[3]	current Fit[4]
$10^3\,(2\,L_6 - L_4)$	0.04(24)	0.23(17)	0.0411(3)	0.0296(41)
$10^3\,(2\,L_8 - L_5)$	−0.20(11)	−0.15(20)	0.0826(12)	−0.0769(51)
$10^3\,(L_8 + 3\,L_7)$			−0.4768(4)	−0.3145(28)
m_s/m			26.15(1)	27.62(4)
f [MeV]			92.4*	82.35(68)

[1] Bazavov et al [arXiv:1012.0868] [2] Dowdall,Davies,Lepage,McNeile [arXiv:1303.1670]
[3] Lutz,Heo,Guo [arXiv:2301.06837] [4] Lutz,Heo,Hudspith [arXiv:2406.07442]

- FLAG values: $m_s/m = 27.42(12)$ and $f = 80.3(6.0)$ MeV [a]
- We do not fit to the quark-mass ratios, because they are not given by the Regensburg group [b].

[a] Aoki et al [arXiv:2111.09849]
[b] RQCD Collaboration · Bali et al [arXiv:2211.03744]

What is the difference between our 2 Fits?

Further LECs from baryon masses

	previous Fit[5]	current Fit[6]
f [MeV]	92.4*	82.35(68)
F	0.51*	0.4852(73)
D	0.72*	0.4855(85)
C	1.44*	0.9740(415)
H	2.43*	1.8390(188)
M [MeV]	804.3(1)	840.3(15.7)
$M + \Delta$[MeV]	1115.2(1)	1091.2(13.8)

[5] Lutz,Heo,Guo [arXiv:2301.06837] [6] Lutz,Heo,Hudspith [arXiv:2406.07442]

Important features of our current Fit

- The updated baryon masses on flavor-symm. ensembles
- f as part of our global Fit parameters
- No use of $C = 2\,D$ and $H = 9\,F - 3\,D$ anymore

Further LECs from baryon masses ..cont'd

- 11 left by the NLO Large-N_c relations from 17 for tadpoles

	current Fit		current Fit
$g_0^{(S)}$ [GeV^{-1}]	$-6.2128(1.5691)$	$g_0^{(V)}$ [GeV^{-2}]	$5.0764(740)$
$g_1^{(S)}$ [GeV^{-1}]	$-3.4537(3612)$	$g_1^{(V)}$ [GeV^{-2}]	$-4.0551(576)$
$g_D^{(S)}$ [GeV^{-1}]	$0.9229(6918)$	$g_D^{(V)}$ [GeV^{-2}]	$6.1611(1525)$
$g_F^{(S)}$ [GeV^{-1}]	$-3.8802(6657)$	$g_F^{(V)}$ [GeV^{-2}]	$1.3456(1208)$
$h_1^{(S)}$ [GeV^{-1}]	$-3.3828(7795)$	$h_1^{(V)}$ [GeV^{-2}]	$4.1857(2760)$
$h_2^{(S)}$ [GeV^{-1}]	$0.$	$h_2^{(V)}$ [GeV^{-2}]	$7.7346(2270)$
$h_3^{(S)}$ [GeV^{-1}]	$-5.6282(5798)$	$h_3^{(V)}$ [GeV^{-2}]	$-1.5500(765)$
$h_4^{(S)}$ [GeV^{-1}]	$-6.3879(6177)$	$h_5^{(S)}$ [GeV^{-1}]	$-4.0818(3811)$
$h_6^{(S)}$ [GeV^{-1}]	$6.3879(6177)$		

More in Lutz,Heo,Hudspith [arXiv:2406.07442]

⇒ They play an important role in the meson-baryon scattering.

Summary and future work

Towards hadron spectroscopy

- Many LECs that are relevant for scattering processes can be determined via Lattice QCD data for hadron masses.
  - 1-loop contributions in terms of their on-shell masses
  - Baryon octet and decuplet masses at N^3LO
  - Quark-mass dependence taken into account
- Those can be used in systematic coupled-channel computations.
  - A generalized potential approach would be required for dealing with left- and right-hand cuts. [a]
  - Chiral long-range forces, such as u-ch. exchanges, and more 1 loops, e.g. triangle and box diagrams, were studied for the open-charm system already. [b]
  - Such work will be a challenge to baryon systems.

[a] Lutz,Gasparyan,Danilkin,Epelbaum [arXiv:1003.3426,1009.5928,1212.3057]
[b] Lutz,Guo,Heo,Korpa,Isken [arXiv:2209.10601,2309.09695]

Thanks to Matthias F.M. Lutz and Renwick J. Hudspith

Thank you