From Lattice QCD to experimental data with the chiral Lagrangian

Matthias F.M. Lutz

GSI Helmholtzzentrum für Schwerionenforschung GmbH

- ✔ **Sustainable path towards experimental data**
- ✔ **Results from two flavor ensembles**
- ✔ **Global fits and χPT convergence issues**
- ✔ **Summary and outlook**

Chiral extrapolation for QCD with light quarks

✔ Sustainable approach

- use Lattice QCD data where they are 'cheap'
- exploit the nonlinearity of chiral symmetry
  to predict experimental data where taking Lattice QCD data is expensive

✔ the low-energy constants (LEC) of the chiral Lagrangian
that determine the quark-mass dependence of a hadron mass
impact the scattering processes involving the Goldstone bosons and that hadron

- critical challenge: what is the convergence radius of a chiral expansion
- conventional expansion schemes appear often very slow (if at all) convergent

✔ novel expansion scheme in terms of on-shell masses

- pioneered for various hadrons on flavor SU(3) ensembles
- chiral expansion is not necessarily smooth - first order transitions are possible
- revisted for flavor SU(2) chiral expansions

Chiral extrapolation for QCD with up and down quarks

✔ consider $m_{u,d} \simeq 2 - 5$ MeV to be small in QCD

- approximate $\mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ chiral symmetry
- apply χPT in terms of the chiral Lagrangian

✔ how-to power count in the presence of heavy fields?

- controversial how to deal with the $\Delta(1232)$ baryon
- conventional expansion schemes appear very slow (if at all) convergent

✔ novel expansion scheme in terms of on-shell masses

- some pioneerig works of our group at GSI

F. Hermsen, T. Isken, MFML and D. Thoma, arXiv:2402.04905

T. Isken, X.-Y. Guo, Y. Heo, C.L. Korpa and MFML, arXiv:2309.09695

U. Sauerwein, MFML, RGE Timmermans, arXiv:2105.06755

MFML, U. Sauerwein, RGE Timmermans, arXiv:2003.10158

X. Guo, Y. Heo, MFML, arXiv:1907.00714

MFML, Y. Heo, X. Guo, arXiv:1907.00237

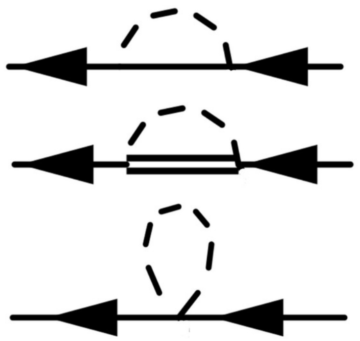
MFML, Y. Heo, X. Guo, arXiv:1801.06417

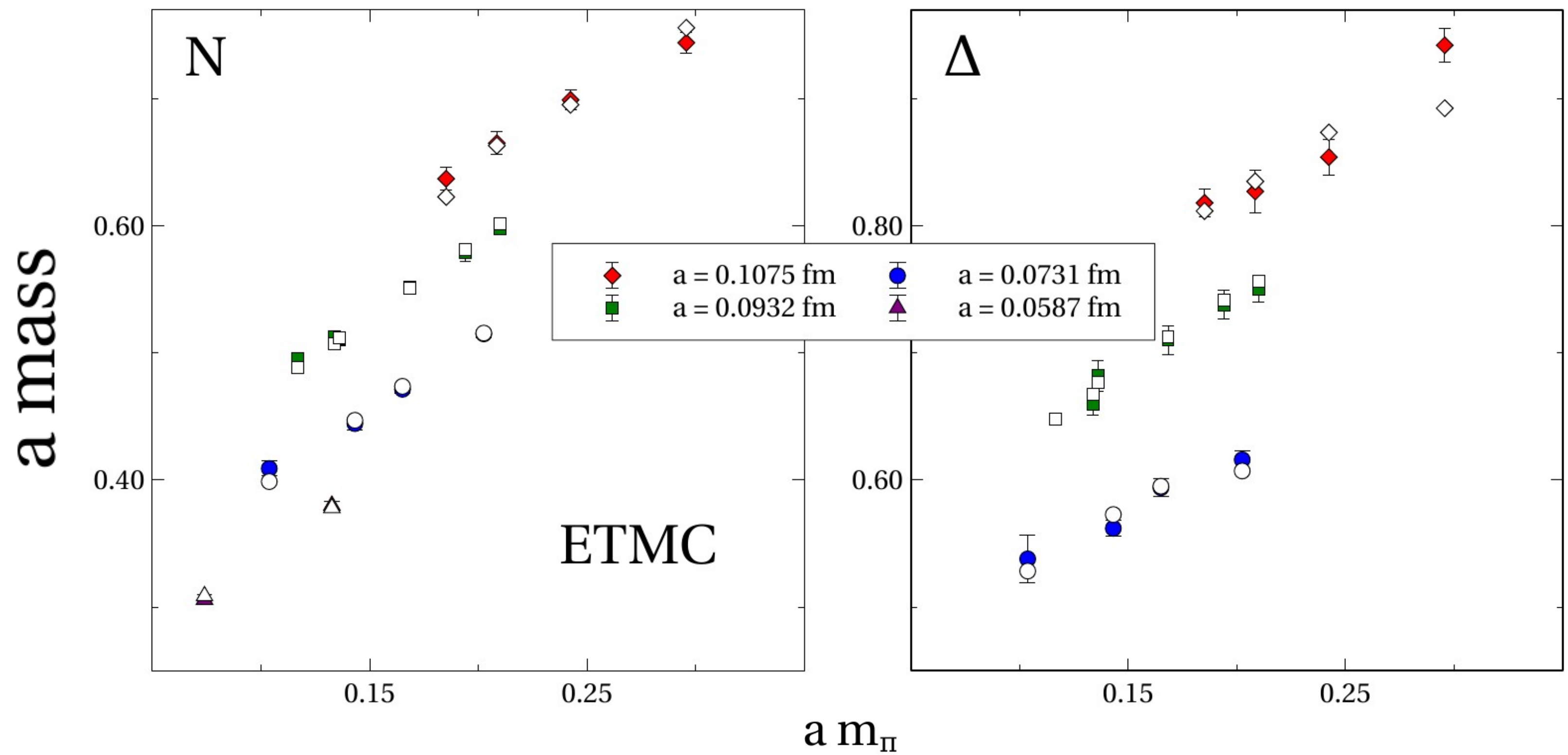
A. Semke, MFML, arXiv:nucl-th/0606027

Chiral extrapolation for QCD with up and down quarks

✔ baryon masses at the one-loop level

- pioneered computation with on-shell masses in loops
- converging chiral expansion of loop contributions
- possibility of parametric phase transitions



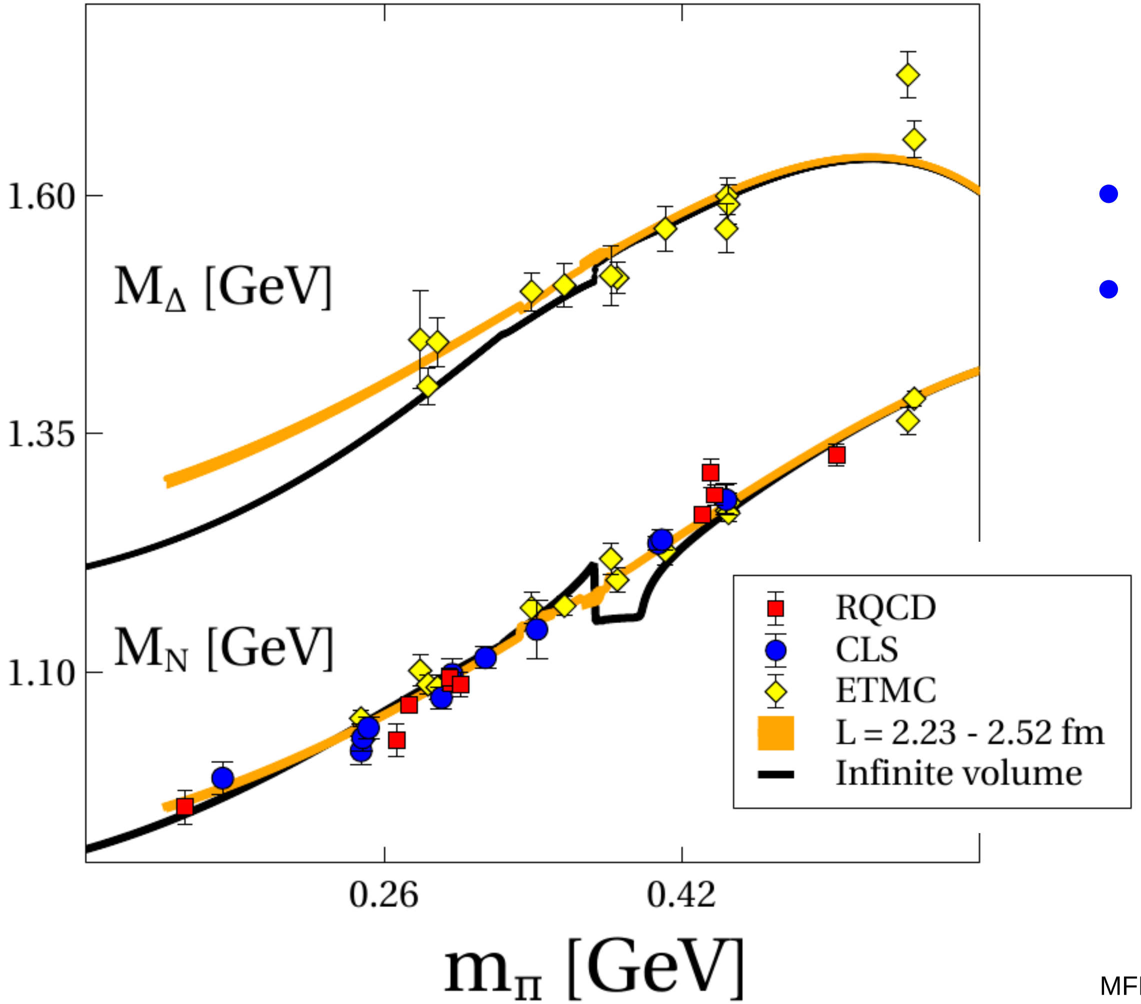


MFML, U. Sauerwein, RGE Timmermans, Eur.Phys.J.C 80 (2020) 9, 844

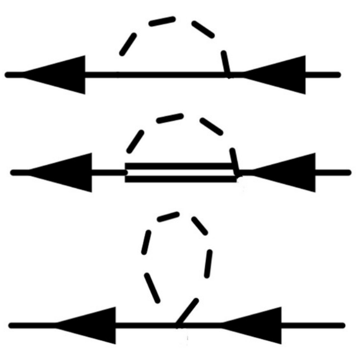
Chiral extrapolation for QCD with up and down quarks

✔ baryon masses at the one-loop level

- accurate reproduction of baryon masses on lattice ensembles
- predict parametric phase transition at large pion masses



- smooth results in finite box
- parametric phase transition in the infinite volume



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Matrix elements of the axial current

$$\langle N(\bar p)|\, A^\mu_i(0)\,|N(p)\rangle = \bar u_N(\bar p)\left(\gamma^\mu\, G_A(q^2) + \tfrac{q^\mu}{2\,M_N}\, G_P(q^2)\right)\gamma_5\;\tfrac{\tau_i}{2}\, u_N(p)$$

- compute $G_A(q^2)$ and $G_P(q^2)$ from the chiral Lagrangian

$$\begin{aligned}
\mathcal{L} = {} & \bar N\big(i\,\not{D} - M\big)\,N + 2\,\zeta_N\,\bar N\,\chi_+\,\big(i\,\not{D} - M\big)\,N \\
& + 4\,b_\chi\,\bar N\,\chi_+\,N + 4\,c_\chi\,\bar N\,\chi_+^2\,N + g_A\,\bar N\,\gamma^\mu\,\gamma_5\,i\,U_\mu\,N \\
& + 4\,g_\chi\,\bar N\,\gamma^\mu\,\gamma_5\,\chi_+\,i\,U_\mu\,N + \frac{g_R}{2}\,\bar N\,\gamma^\mu\,\gamma_5\,[D^\nu\,,\,F^-_{\mu\nu}]\,N \\
& - 4\,g_S\,\bar N\,U_\mu\,U^\mu\,N - g_T\,\bar N\,i\,\sigma^{\mu\nu}\,\big[U_\mu\,,\;U_\nu\big]\,N \\
& - g_V\,\Big(\bar N\,i\,\gamma^\mu\,\{U_\mu,\,U_\nu\}\,D^\nu N + \text{h.c.}\Big) + \cdots\,,
\end{aligned}$$

$$\begin{aligned}
U_\mu &= \tfrac{1}{2}\,u^\dagger\,\Big((\partial_\mu\,e^{i\,\pi/f}) - \big\{i\,a_\mu,\,e^{i\,\pi/f}\big\}\Big)\,u^\dagger\,, \qquad u = e^{i\,\pi/(2\,f)}\,, \\
\Gamma_\mu &= \tfrac{1}{2}\,u^\dagger\,\big[\partial_\mu - i\,a_\mu\big]\,u + \tfrac{1}{2}\,u\,\big[\partial_\mu + i\,a_\mu\big]\,u^\dagger\,, \\
D_\mu\,N\ &= \partial_\mu N + \Gamma_\mu\,N\,, \qquad \chi_+ = 2\,B_0\,m\,\cos\big(\pi/f\big) + \cdots\,,
\end{aligned}$$

F. Hermsen, T. Isken, MFML, D. Thoma, Phys.Rev.D 109 (2024) 11, 114029

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& + 4\,g_\chi\,\bar N\,\gamma^\mu\,\gamma_5\,\chi_+\,i\,U_\mu\,N + \frac{g_R}{2}\,\bar N\,\gamma^\mu\,\gamma_5\,[D^\nu\,,\,F^-_{\mu\nu}]\,N \\
& - 4\,g_S\,\bar N\,U_\mu\,U^\mu\,N - g_T\,\bar N\,i\,\sigma^{\mu\nu}\,\big[U_\mu\,,\;U_\nu\big]\,N \\
& - g_V\,\Big(\bar N\,i\,\gamma^\mu\,\{U_\mu,\,U_\nu\}\,D^\nu N + \mathrm{h.c.}\Big) + \cdots\,,
\end{aligned}$$

- further terms involving the isobar fields
- determine the low-energy constants (LEC):
  $M, f, g_A, g_S, g_V, g_T \cdots$ from Lattice QCD data

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✔ axial-vector formfactor at the one-loop level

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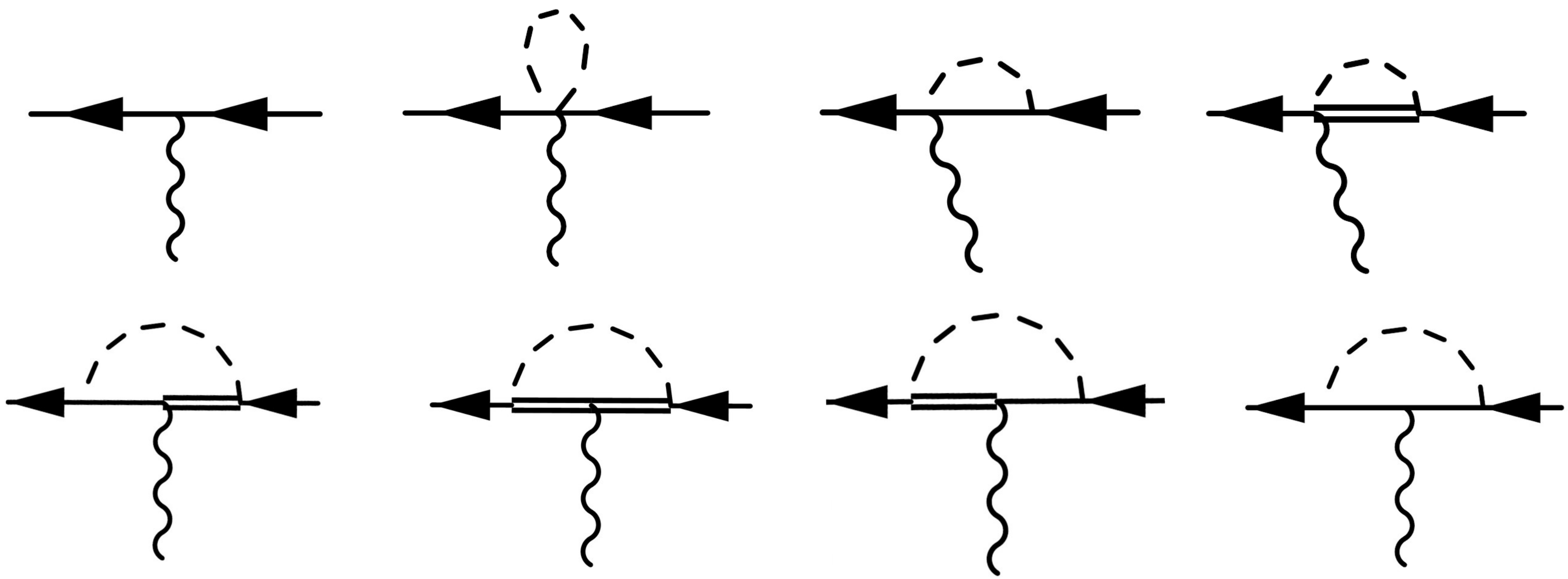

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U. Sauerwein, MFML, RGE Timmermans, Phys.Rev.D 105 (2022) 5, 054005

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- compute $G_A(q^2)$ and $G_P(q^2)$ from the chiral Lagrangian

$$\begin{aligned} G_A(t) &= g_A\, Z_N + 4\, g_\chi^+\, m_\pi^2 + g_R\, t \\ &+ \frac{g_A}{f^2} \left\{ J^A_\pi(t) + J^A_{\pi N}(t) + J^A_{N\pi}(t) \right\} + \frac{g_A^3}{4\, f^2}\, J^A_{N\pi N}(t) + \frac{5\, h_A\, f_S^2}{9\, f^2}\, J^A_{\Delta\pi\Delta}(t) \\ &+ \frac{f_S}{3\, f^2} \left\{ J^A_{\pi\Delta}(t) + J^A_{\Delta\pi}(t) \right\} + \frac{2\, g_A\, f_S^2}{3\, f^2} \left\{ J^A_{N\pi\Delta}(t) + J^A_{\Delta\pi N}(t) \right\} + \mathcal{O}\left(Q^4\right)\ , \end{aligned}$$

$$\begin{aligned} \frac{t - m_\pi^2}{4\, M_N^2}\, G_P(t) &= -g_A \Big(Z_N + Z_\pi + f_\pi/f - 2 \Big) - \ m_\pi^2\, (4\, g_\chi^+ + g_\chi^-) - g_R\, (t - m_\pi^2) \\ &+ \frac{g_A}{f^2} \left\{ J^P_\pi(t) + J^P_{\pi N}(t) + J^P_{N\pi}(t) \right\} + \frac{g_A^3}{4\, f^2}\, J^P_{N\pi N}(t) + \frac{5\, h_A\, f_S^2}{9\, f^2}\, J^P_{\Delta\pi\Delta}(t) \\ &+ \frac{f_S}{3\, f^2} \left\{ J^P_{\pi\Delta}(t) + J^P_{\Delta\pi}(t) \right\} + \frac{2\, g_A\, f_S^2}{3\, f^2} \left\{ J^P_{N\pi\Delta}(t) + J^P_{\Delta\pi N}(t) \right\} + \mathcal{O}\left(Q^4\right)\ , \end{aligned}$$

- how to derive the various loop expressions?

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$$\lim_{m\to 0}\left[G_A(t) + \frac{t}{4\, M_N^2}\, G_P(t)\right] = 0 \ \to \qquad \lim_{m\to 0}\left[J^A_{\cdots} + J^P_{\cdots}\right] = 0\,,$$

- how to derive the various loop expressions?
- without violating the chiral Ward identities of QCD
- but subtracting power-counting violating terms

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A novel set of scalar basis loop functions

- express $G_A(q^2)$ and $G_P(q^2)$ in terms of a complete set of basis functions
- a renormalization scale dependent scalar tadpole

$$\bar{I}_\pi = \frac{m_\pi^2}{16\,\pi^2}\,\log\frac{m_\pi^2}{\mu^2} \sim Q^2 \qquad \text{but} \quad \bar{\mathrm{I}}_{\mathrm{N}} = \bar{\mathrm{I}}_\Delta = 0$$

- finite and scale-independent bubble and triangle loop functions

$$\bar{I}_{\pi R} = \frac{\gamma_N^R - 2}{16\,\pi^2} - \int_0^1 \frac{dv}{16\,\pi^2}\,\log\frac{F_{L\pi R}(0,v)}{M_R^2} \sim Q\,,$$

$$F_{L\pi R}(u,v) = m_\pi^2 + u\left(M_L^2 - m_\pi^2 - (1-u)\,M_N^2\right) + v\left(M_R^2 - m_\pi^2 - (1-v)\,M_N^2\right)$$
$$+\,u\,v\left(2\,M_N^2 - t\right),$$

$$\bar{I}^{(m,n)}_{L\pi R}(t) = -\frac{\gamma^{(m,n)}_{L\pi R}}{16\,\pi^2\,M^2} + \int_0^1\int_0^{1-u}\frac{dv\,du\,u^m\,v^n}{16\,\pi^2\,F_{L\pi R}(u,v)} \sim Q^0\,,$$

- where only $\bar{I}^{(n,0)}_{L\pi R}(t)$ and $\bar{I}^{(0,n)}_{L\pi R}(t)$ with $L, R \in \{N, \Delta\}$ and n=0,1, $\cdots$ are in the basis

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A novel set of scalar basis loop functions

- power-counting for bubble and triangle loop functions

$$\bar I_{\pi R} = \frac{\gamma_N^R - 2}{16\,\pi^2} - \int_0^1 \frac{dv}{16\,\pi^2}\,\log\frac{F_{L\pi R}(0,v)}{M_R^2} \sim Q \quad \text{for} \quad \mathrm{m}_\pi/\Delta \sim \mathrm{Q}^0$$

$$\text{but} \quad \bar{\mathrm{I}}_{\pi\Delta}(\mathrm{M}_\mathrm{N}^2) \sim \frac{\mathrm{m}_\pi^2}{\Delta\,\mathrm{M}} \sim \mathrm{Q}^2 \quad \text{for} \quad \mathrm{m}_\pi/\Delta \sim \mathrm{Q}$$

$$\bar I^{(m,n)}_{L\pi R}(t) = -\frac{\gamma^{(m,n)}_{L\pi R}}{16\,\pi^2\,M^2} + \int_0^1\int_0^{1-u} \frac{dv\,du\,u^m\,v^n}{16\,\pi^2\,F_{L\pi R}(u,v)} \sim Q^0 \quad \text{for} \quad \mathrm{m}_\pi/\Delta \sim \mathrm{Q}^0$$

$$\text{but} \quad \bar{\mathrm{I}}^{(\mathrm{m,n})}_{\mathrm{N}\pi\Delta}(\mathrm{t}) \sim \frac{\mathrm{m}_\pi}{\Delta\,\mathrm{M}^2} \sim \mathrm{Q} \quad \text{for} \quad \mathrm{m}_\pi/\Delta \sim \mathrm{Q}$$

$$\text{but} \quad \bar{\mathrm{I}}^{(\mathrm{m,n})}_{\Delta\pi\Delta}(\mathrm{t}) \sim \frac{\mathrm{m}_\pi^2}{\Delta^2\,\mathrm{M}^2} \sim \mathrm{Q}^2 \quad \text{for} \quad \mathrm{m}_\pi/\Delta \sim \mathrm{Q}$$

- discriminate two domains $m_\pi/\Delta \sim Q$ and $m_\pi/\Delta \sim Q^0$

  with $\Delta = M_\Delta - M_N\Big|_{m_{u,d}\to 0}$ and $M = M_N\Big|_{m_{u,d}\to 0}$ and $\gamma_N^\Delta = \frac{\Delta}{M}\,\log\frac{M+\Delta}{\Delta}$

- Lattice QCD data ask for controll of both domains $\Delta \sim 300$ MeV

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A novel set of scalar basis loop functions

- explicit form of subtraction terms: some examples

$$\bar{I}^{(m,n)}_{L\pi R}(t) = -\frac{\gamma^{(m,n)}_{L\pi R}}{16\,\pi^2\,M^2} + \int_0^1\int_0^{1-u} \frac{dv\,du\,u^m\,v^n}{16\,\pi^2\,F_{L\pi R}(u,v)} \sim Q^0\,,$$

$$\gamma^{(n,m)}_{N\pi N} = 0\,, \qquad \gamma^{(0,0)}_{\Delta\pi N} = \gamma^{(0,0)}_{N\pi\Delta} = \frac{1}{a}\,\log(1+a) + \log\frac{1+a}{a}\,,$$

$$\gamma^{(1,0)}_{\Delta\pi N} = \gamma^{(0,1)}_{N\pi\Delta} = \frac{1+a}{3\,a} - \frac{1}{3\,a^2}\,\log(1+a) - \frac{a}{3}\,\log\frac{1+a}{a}\,,$$

$$\gamma^{(2,0)}_{\Delta\pi N} = \gamma^{(0,2)}_{N\pi\Delta} = \frac{-2+a+a^2-2\,a^3}{10\,a^2} + \frac{1}{5\,a^3}\,\log(1+a) + \frac{a^2}{5}\,\log\frac{1+a}{a}\,,$$

$$\gamma^{(0,0)}_{\Delta\pi\Delta} = \log\frac{1+a}{a}\,,$$

$$\gamma^{(1,0)}_{\Delta\pi\Delta} = \gamma^{(0,1)}_{\Delta\pi\Delta} = \frac{1}{2}\left(1 - a\,\log\frac{1+a}{a}\right),$$

$$\gamma^{(2,0)}_{\Delta\pi\Delta} = \gamma^{(0,2)}_{\Delta\pi\Delta} = \frac{1}{6}\left(1 - 2\,a + 2\,a^2\,\log\frac{1+a}{a}\right),$$

$$a = \frac{\Delta}{M}\left(2 + \frac{\Delta}{M}\right),$$

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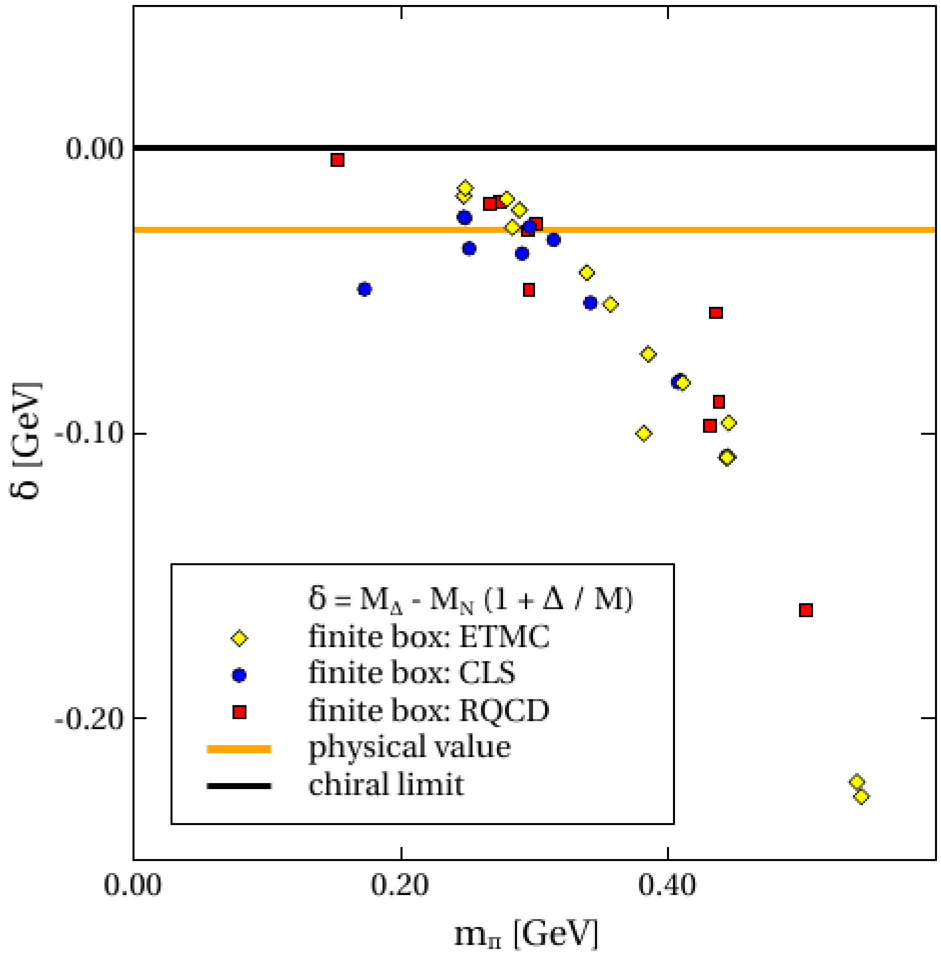
Novel expansion parameters

- $m_{u,d} \sim m_\pi^2 \sim Q^2$ $\qquad M_N \sim Q^0$
- $\delta = M_\Delta - M_N\,(1 + \Delta/M) \sim Q^2$ $\qquad \Delta/M = r \sim Q^0$
- with M and $M + \Delta$ from M_N and M_Δ in the chiral limit



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Some explicit examples of loop functions

- contributions to $G_A(t)$

$$\bar{J}^A_{N\pi N}(t) = \bar{I}_\pi + m_\pi^2\, \bar{I}_{\pi N} + \mathcal{O}(Q^4)\,,$$

$$\bar{J}^A_{\Delta\pi\Delta}(t) = \frac{2}{3}\left(2\, r\, t\, \alpha^A_{81} + \frac{5}{9}\, m_\pi^2\, \alpha^A_{82} - \frac{10}{3}\, \delta\, M_N\, \alpha^A_{83}\right) \bar{I}_{\pi\Delta} + \frac{4}{3}\, t\, \alpha^A_{91}\, M_N^2 \left(\bar{I}^{(2,0)}_{\Delta\pi\Delta}(t) + \bar{I}^{(0,2)}_{\Delta\pi\Delta}(t)\right) + \mathcal{O}(Q^4)\,,$$

- contributions to $G_P(t)$

$$\bar{J}^P_{N\pi N}(t) = -\bar{I}_\pi - m_\pi^2\, \bar{I}_{\pi N} - 4\, m_\pi^2\, M_N^2 \left(\bar{I}^{(2,0)}_{N\pi N}(t) + \bar{I}^{(0,2)}_{N\pi N}(t)\right) + \mathcal{O}(Q^4)\,,$$

$$\bar{J}^P_{\Delta\pi\Delta}(t) = -\frac{2}{3}\left(2\, r\, t\, \alpha^P_{81} + \frac{5}{9}\, m_\pi^2\, \alpha^P_{82} - \frac{10}{3}\, \delta\, M_N\, \alpha^P_{83}\right) \bar{I}_{\pi\Delta} - \frac{4}{3}\left[t\, \alpha^P_{91} + \frac{1}{3}\, m_\pi^2\, \alpha^P_{92}\right] M_N^2 \left(\bar{I}^{(2,0)}_{\Delta\pi\Delta}(t) + \bar{I}^{(0,2)}_{\Delta\pi\Delta}(t)\right) + \mathcal{O}(Q^4)\,,$$

- do not expand in Δ/M — very poor convergence : e.g. at $\Delta/M \simeq 0.34$

$$\alpha^A_{nm} \to 1 \ \text{and} \ \alpha^{\rm P}_{\rm nm} \to 1 \quad \text{as} \qquad \Delta \to 0$$

$$\alpha^A_{81} = \alpha^P_{81} \simeq 1.55\,, \qquad \alpha^A_{83} = \alpha^P_{83} \simeq 1.40\,,$$

$$\alpha^A_{82} \simeq 2.78\,, \qquad \alpha^P_{82} \simeq 3.48$$

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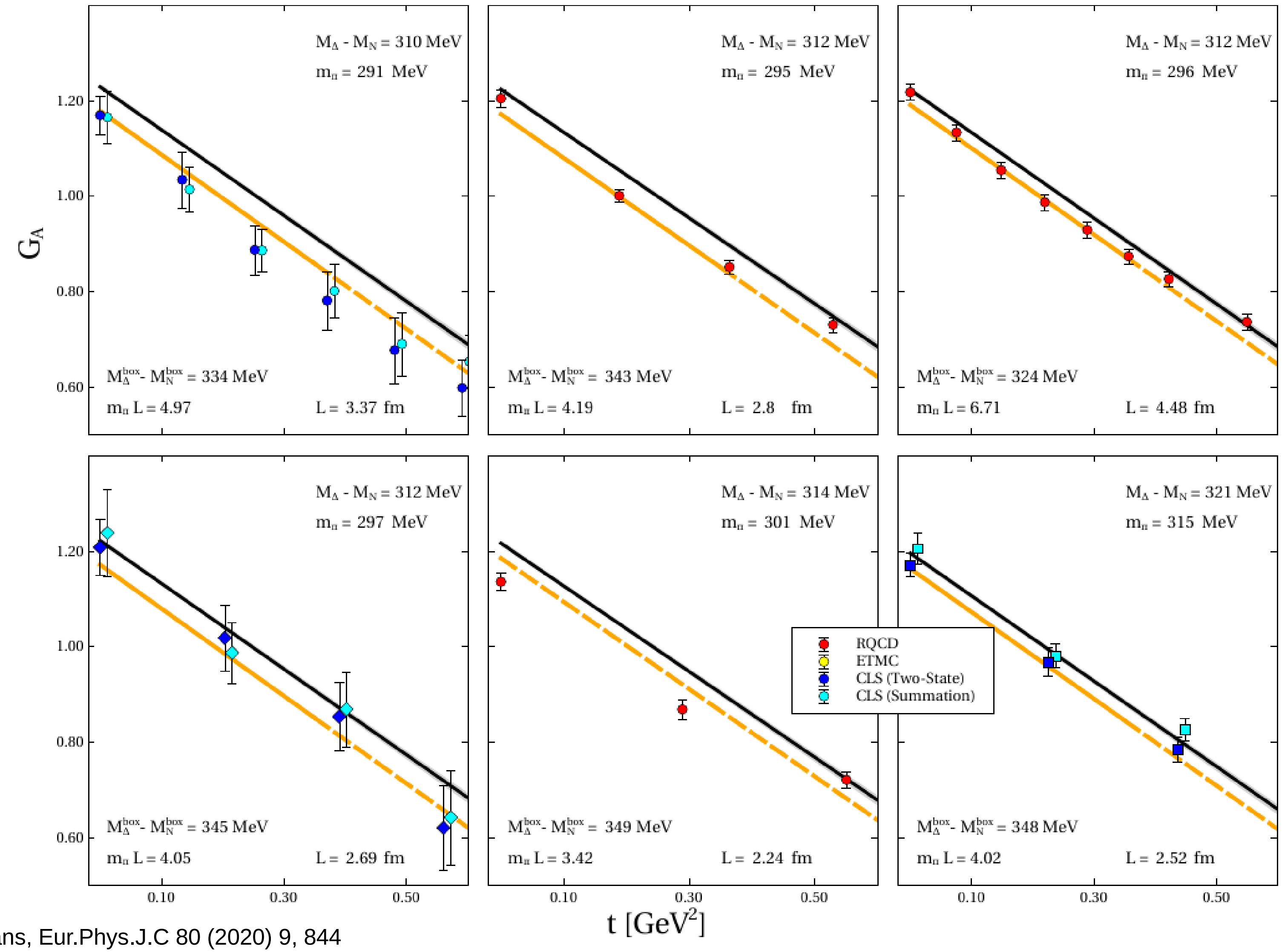
Global fit to lattice data from ETMC, CLS and RQCD

- consider masses (M_N, M_Δ) and axial-vector form factor
- 99 data points with $\chi^2/N_{df} \simeq 1.40$

large impact of box size

- finite-box (orange)

  infinite-box (black)
- mass of Δ matters!



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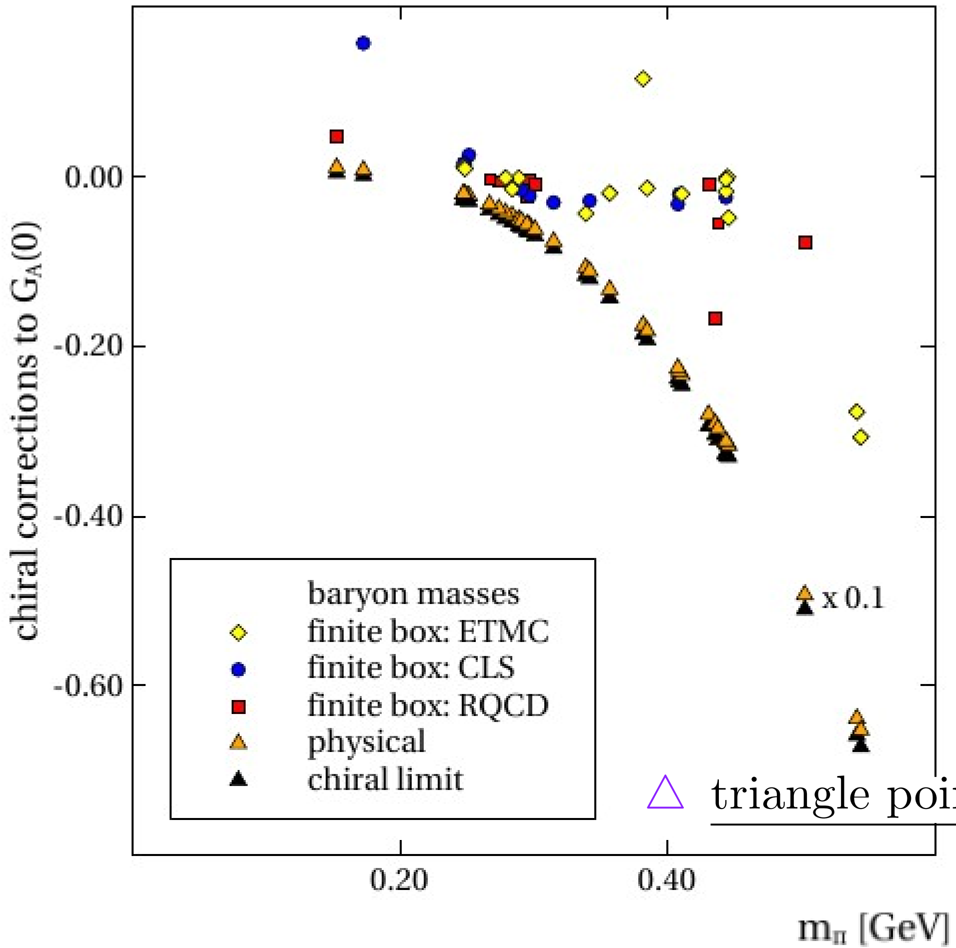
Chiral corrections for the axial form factor

- $\delta = M_\Delta - M_N\,(1 + \Delta/M) \sim Q^2$

  with M and $M + \Delta$ from M_N and M_Δ in the chiral limit
- consider different assumptions for M_N and M_Δ (one-loop level)
- convergence only with on-shell M_N and M_Δ in loop expressions



△ triangle points - no convergence!

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Global fit to lattice data from ETMC, CLS and RQCD

✔ Global fit to results on flavor SU(2) ensembles

- consider nucleon and isobar masses and axial-vector form factor
- use novel form of one-loop expressions according to [1]
  for the first time also induced pseudo-scalar form factor
- in total 124 lattice data points with $\chi^2/N_{dof} \sim 1.077$
- consider all data points with $m_\pi < 525$ MeV

✔ How much strangeness is needed?

- huge discrepancies with empirical values
- $G_A(0) = 1.2732(23)$
- $\sigma_{\pi N} = 58(5)$ MeV
- $f_\pi = (92.21 \pm 0.14)$ MeV

	Fit results
$G_A(0)$	$1.2284(^{+0.0021}_{-0.0059})$
$\langle r_A^2 \rangle$ [fm^2]	$0.2014(^{+0.0032}_{-0.0035})$
g_P	$8.2521(^{+0.039}_{-0.039})$
$\sigma_{\pi N}$ [MeV]	$42.22(^{+0.02}_{-0.05})$
$\sigma_{\pi \Delta}$ [MeV]	$35.27(^{+0.01}_{-0.06})$
f_π [MeV]	$84.96(^{+0.29}_{-0.82})$

[1] F. Hermsen, T. Isken, MFML and D. Thoma, arXiv:2402.04905

Summary and Outlook

✔ Chiral extrapolation of hadron masses and form factors

- chiral expansion with up and down quarks is well convergent
  iff set up with on-shell hadron massses
- quantitative reproduction of two-flavor lattice data sets (ETMC, RQCD, CLS)
- but significant tension with empirical data
- predict low-energy constants of the chiral Lagrangian

✔ Precision extrapolation results for baryons ?

- should use ensembles at physical strange quark mass
  form factor have a sizeable dependence on the isobar and strange-quark mass
- need more high statistic data on the isobar

✔ First global fits to flavor SU(3) ensembles of CLS : talk by Yonggoo

- available data is suitable for a chiral analyses
- use future Lattice data on baryon masses to constrain meson-baryon scattering