



Neutron stars and Constraints for the Equation of State of Dense Matter + MUSES

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⭐ Astrophysics group



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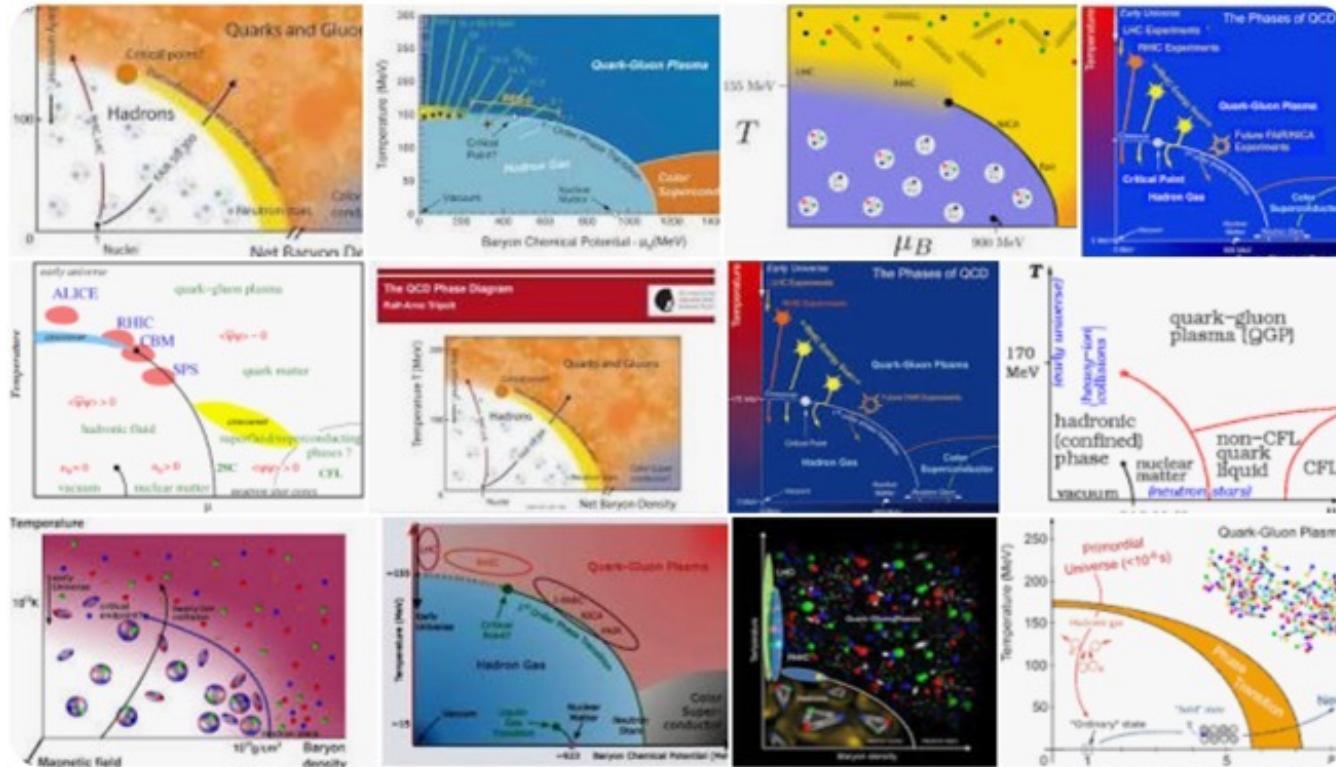
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⭐ QCD Phase Diagrams

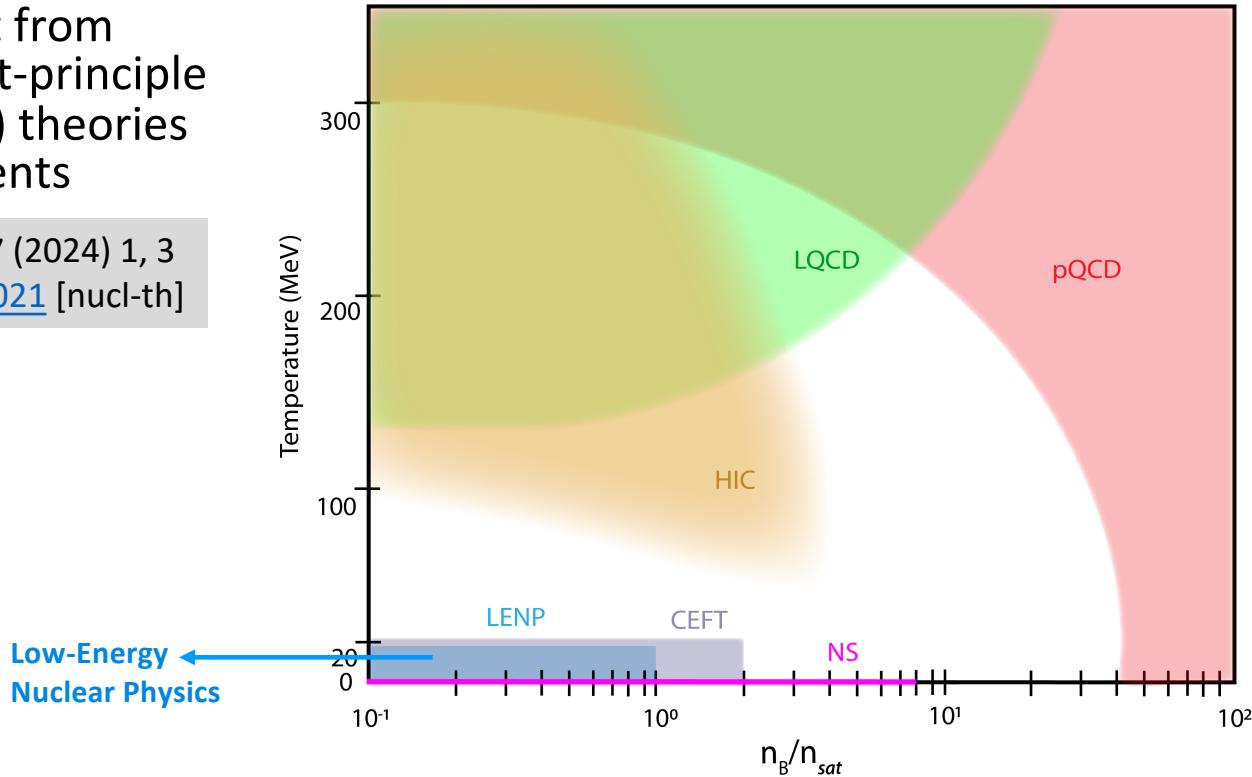


⭐ Our QCD Phase Diagram



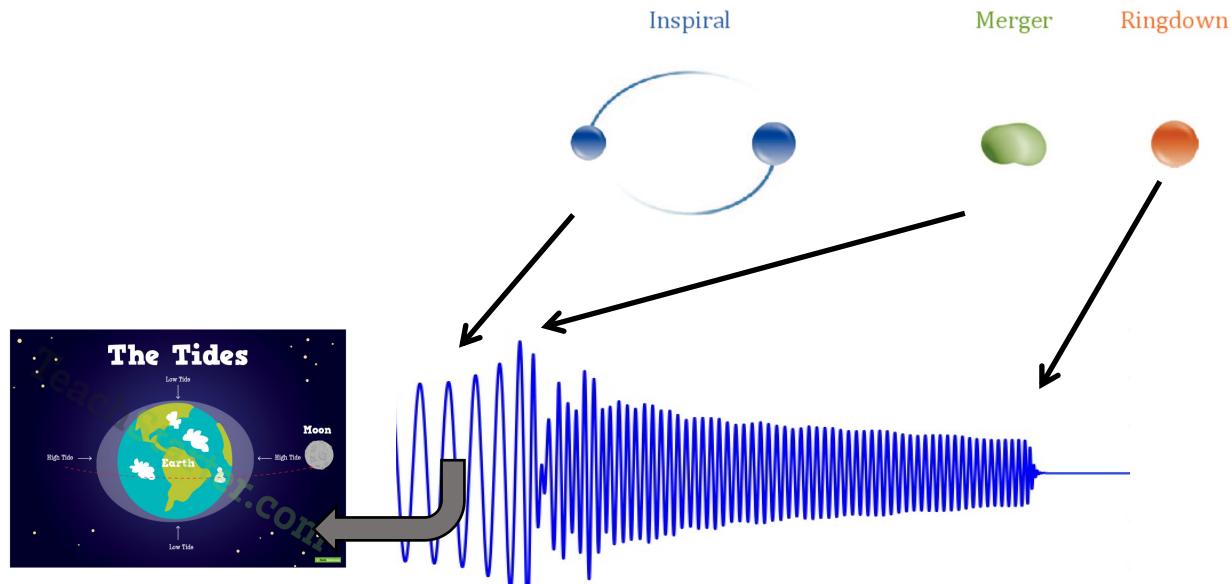
- * Current input from different (first-principle and effective) theories and experiments

Living Rev.Rel. 27 (2024) 1, 3
e-Print: [2303.17021](https://arxiv.org/abs/2303.17021) [nucl-th]



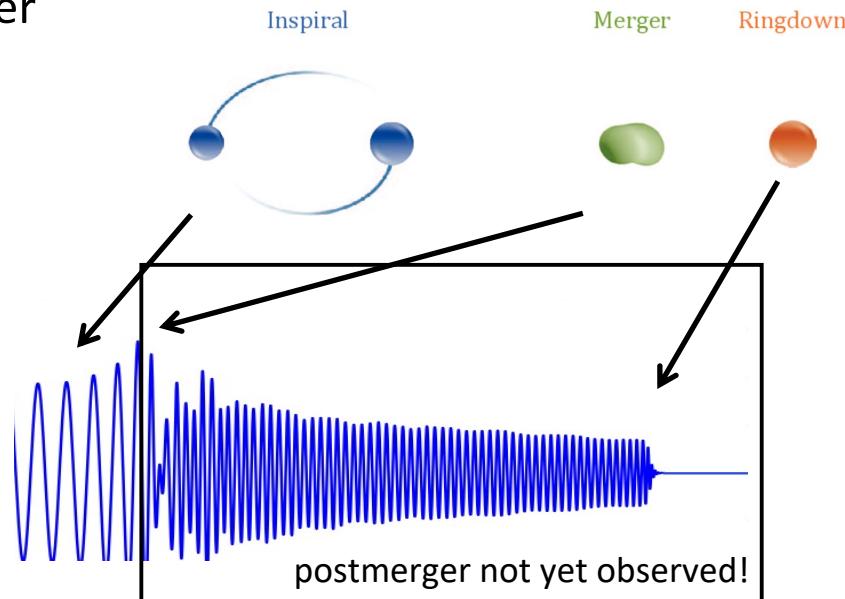
★ Gravitational Wave Data

- * Several measurements from neutron-star mergers but only GW170817 provided electromagnetic counterparts and a relevant measurement of the tidal deformability



★ Gravitational Wave Data

- * Several measurements from neutron-star mergers but only GW170817 provided electromagnetic counterparts and a relevant measurement of the tidal deformability
- * Without the post-merger (hot) part



⭐ Chiral Effective Field Theory

- * EoS computed up to N3LO in many-body perturbation theory (with three-body forces up to N2LO) for $n_B \lesssim 2n_{sat}$
- * Provides E_{sym} and slope parameter L at n_{sat}

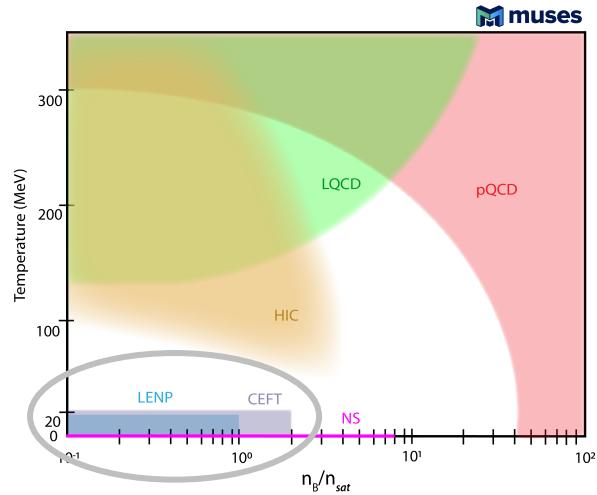
Ann.Rev.Nucl.Part.Sci. 71 (2021) 403-432

e-Print: [2101.01709](https://arxiv.org/abs/2101.01709)

- * Can be used to study the liquid-gas phase transition for isospin-symmetric nuclear matter from a finite-temperature calculation up to $T \sim 25$ MeV

Phys.Rev.C 95 (2017) 3, 034326

e-Print: [1612.04309](https://arxiv.org/abs/1612.04309)



⭐ Lattice QCD

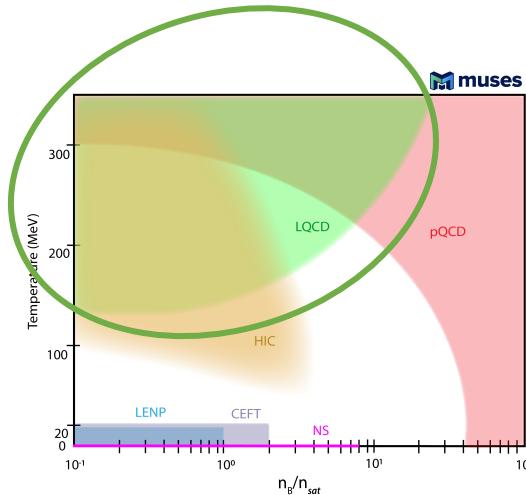
- * EoS up to $\mu_B/T=3.5$ obtained from Taylor expansion

Phys.Rev.Lett. 126 (2021) 23, 232001
e-Print: [2102.06660](https://arxiv.org/abs/2102.06660)

- * BSQ susceptibilities
- * Partial pressures (with hadronic phase treated as ideal resonance gas)
- * Pseudo phase-transition line
- * Limits on the critical point location $\mu_B \gtrsim 300$ MeV and $T_c \lesssim 132$ MeV.

Phys.Rev.Lett. 125 (2020) 5, 052001
e-Print: [2002.02821](https://arxiv.org/abs/2002.02821)

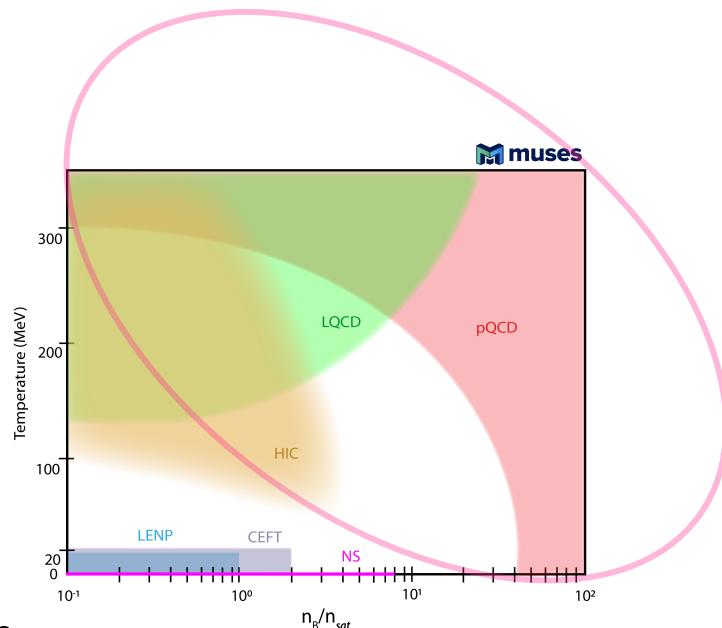
Phys.Rev.Lett. 123 (2019) 6, 062002
e-Print: [1903.04801](https://arxiv.org/abs/1903.04801)



⭐ Perturbative QCD

- * Resummed perturbative QCD EoS calculated to N3LO using HTL perturbation theory in agreement with lattice for $T \gtrsim 2 T_c$ at $\mu_B = 0$

JHEP 08 (2011) 053 e-Print: [1103.2528](#)



- * The curvature of the QCD phase transition line
- * Application at high density: starting at $n_B \sim 40 n_{sat}$ from N3LO calculation

Phys. Rev. D 104 (2021) 7, 074015 e-Print: [2103.07427](#)

(and extrapolations to lower densities)

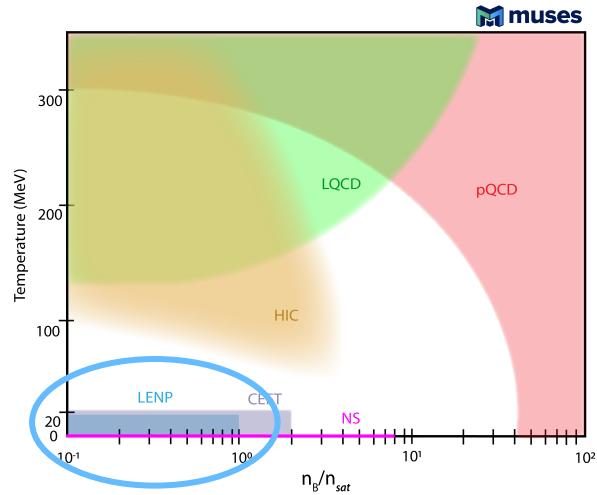
- * Transport coefficients at finite T and μ_B

★ Low-Energy Nuclear Physics

- * Isospin symmetric matter at n_{sat}

Saturation density, n_{sat} (fm^{-3})	0.17 ± 0.03
	$0.148 - 0.185$
	0.148 ± 0.0038
Binding energy per nucleon, B/A (MeV)	-15.677
	-16.24
Compressibility, K_∞ (MeV)	240 ± 20
	210 - 270
	251 - 315

Phys. Rev. C 89 (2014) 4, 044316
e-Print: [1404.0744](https://arxiv.org/abs/1404.0744)



- * Hyperon and Δ -baryon potentials at n_{sat}
- * Symmetry energy E_{sym} and derivative L at ans around n_{sat}
- * Heavy-ion collision measurements of neutron skin
- * Liquid-gas critical point

★ Heavy-Ion Collisions

- * Particle yields for π^\pm , K^\pm , p/\bar{p} , $\Lambda/\bar{\Lambda}$, $\Xi^-/\bar{\Xi}^+$ and $\Omega^-/\bar{\Omega}^+$... can indicate e.g. deconfinement

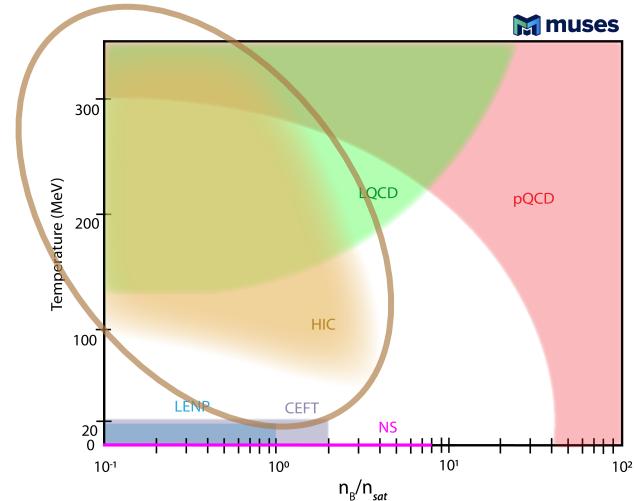
Phys.Lett.B 728 (2014) 216-227 e-Print: [1307.5543](#)

Phys.Rev.C 77 (2008) 044908 e-Print: [0705.2511](#)

- * Fluctuation observables, such as cumulants of particle multiplicity distributions, can relate to thermodynamic susceptibilities, used to e.g. exclude a critical point until a given μ_B

PoS FACESQCD (2010) 017 e-Print: [1106.3887](#)

- * Flow harmonics *Acta Phys.Polon.Supp.* 16 (2023) 1, 1-A48 e-Print: [2209.04957](#)
- * Hanbury Brown–Twiss (HBT) interferometry

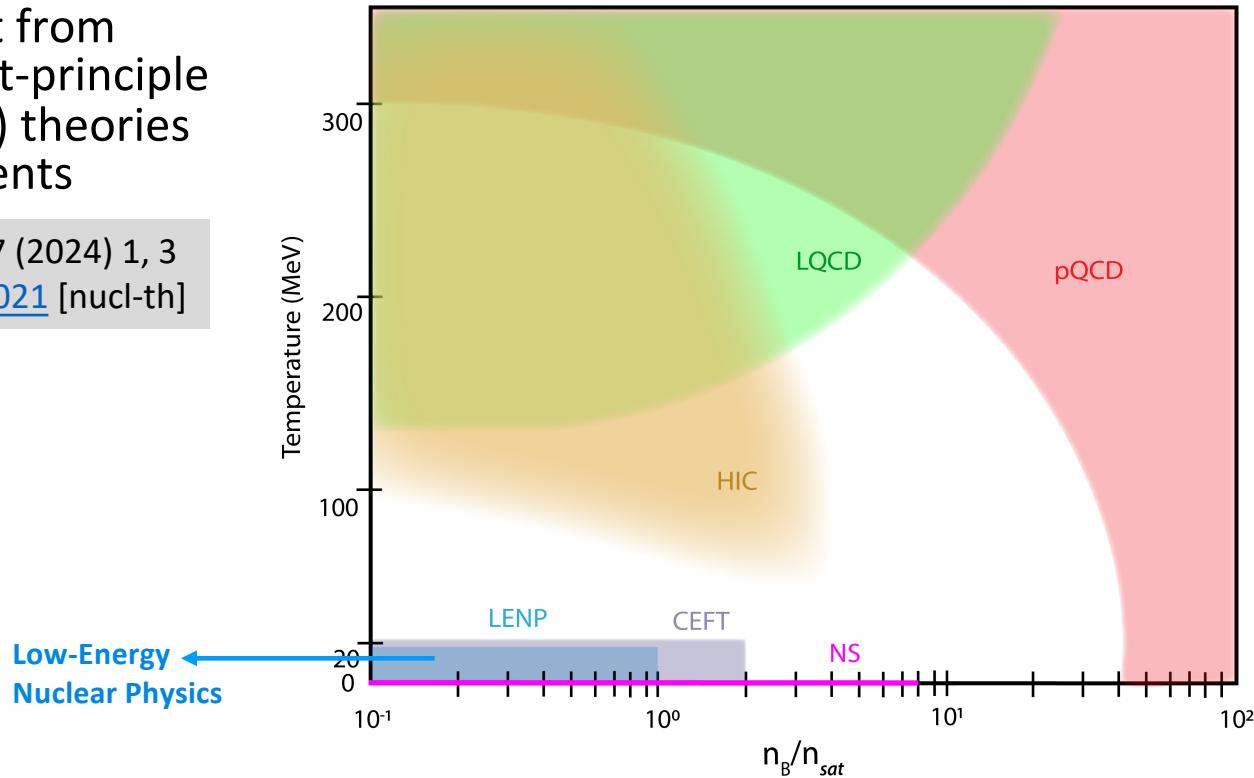


⭐ Our QCD Phase Diagram

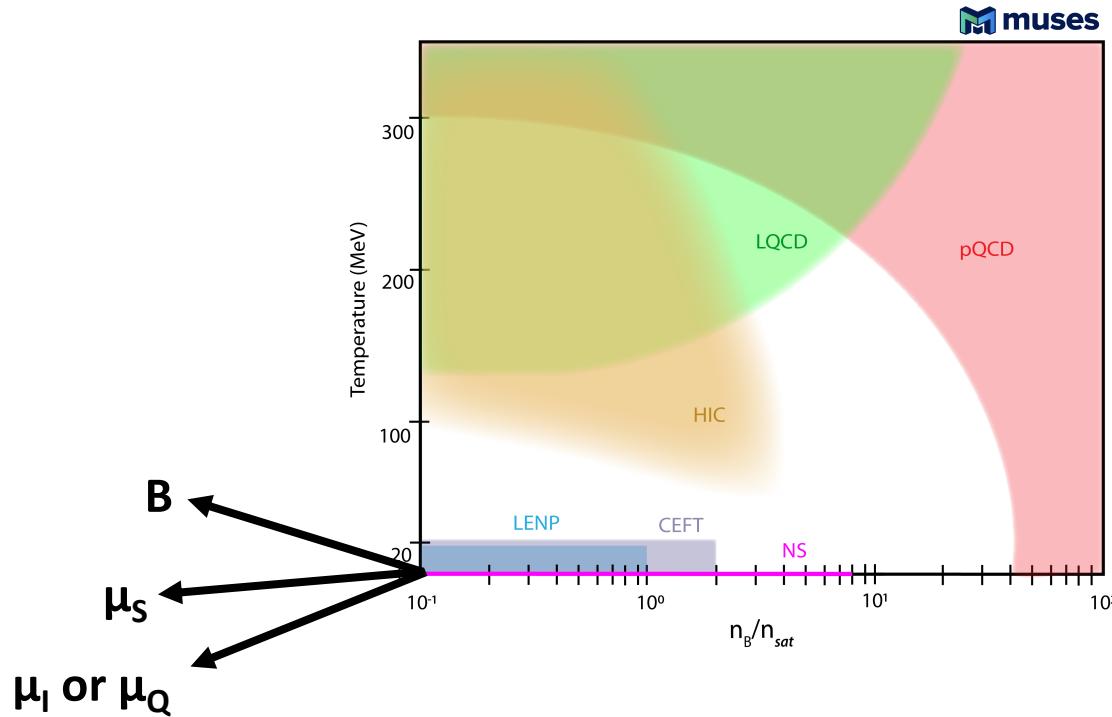


- * Current input from different (first-principle and effective) theories and experiments

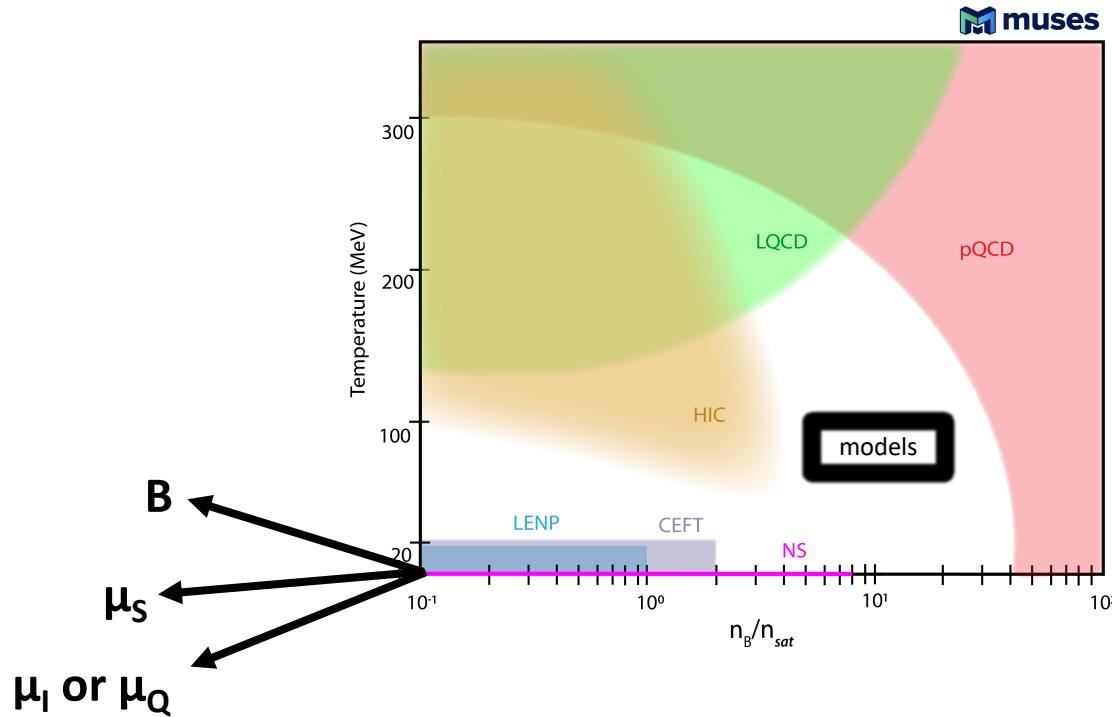
Living Rev.Rel. 27 (2024) 1, 3
e-Print: [2303.17021](https://arxiv.org/abs/2303.17021) [nucl-th]



⭐ What about More Dimensions?

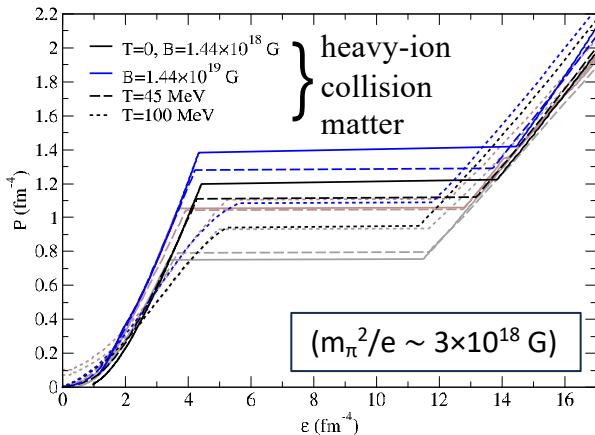


⭐ What about More Dimensions?



★ 5D ~~Phase Diagrams~~ Curves

- * Curves for the CMF model (with quark deconfinement)



Phys.Rev.D 108 (2023) 6, 063011
e-Print: [2304.02454](https://arxiv.org/abs/2304.02454)

- * Neutron-star matter also shown for comparison in different colors

$B=1.44 \times 10^{18} \text{ G}$ for neutron-star matter

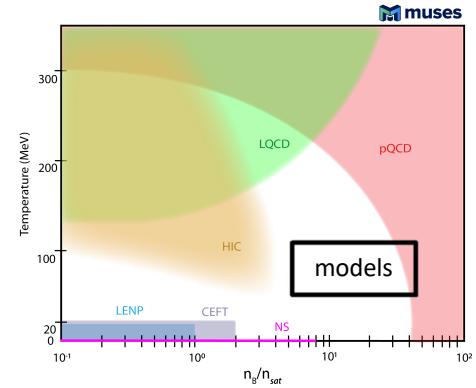
$B=1.44 \times 10^{19} \text{ G}$ for neutron-star matter

- * (Stronger) phase transition takes place at larger ϵ and μ_B for larger B in CMF model
- * (Weaker) phase transition takes place at lower μ_B for larger T
- * Phase transition takes place at larger μ_B and is stronger for heavy-ion collision matter (for any T and B) in CMF model



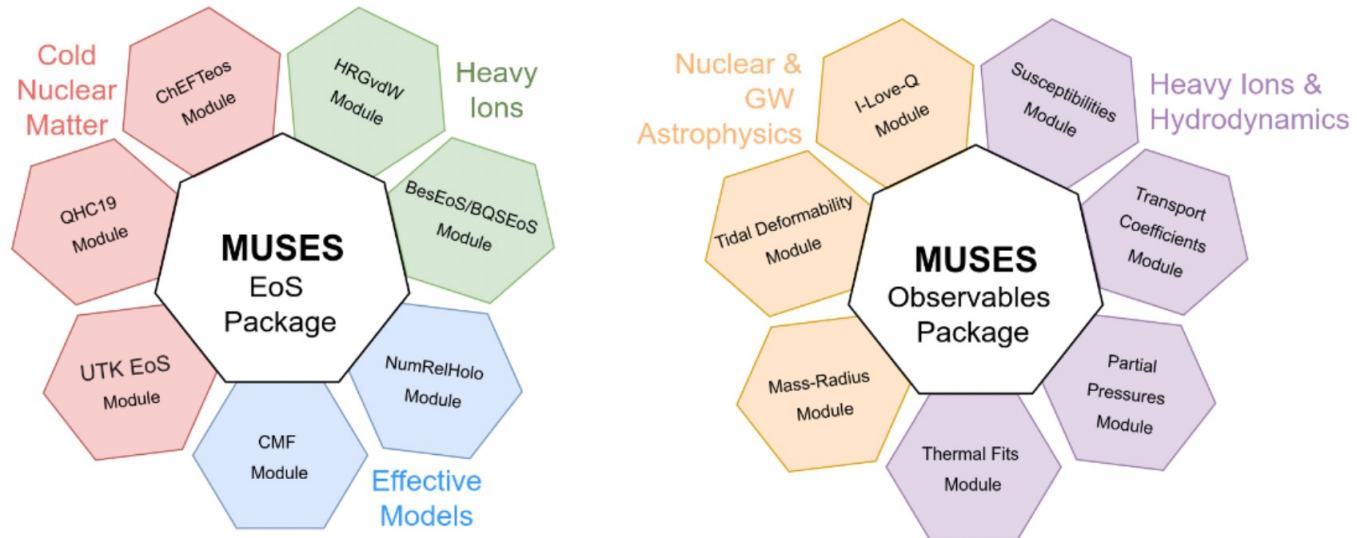
muses Cyberinfrastructure

- * Modular Unified Solver of the Equation of State
- * Modular: different theories/models (modules) for the user to pick from **and modify**
- * Unified: different modules are merged together to ensure maximal coverage of the phase diagram
- * Developers: physicists + computer scientists working together to develop optimized software that generates EoS's over large ranges of temperature and chemical potentials to cover the whole phase diagram, together with observables
- * Users: interested scientists from different communities, who provide input to the cyberinfrastructure





muses Modules



+ Lepton Module, Synthesis Module, Interpolator Module, ...



muses Alpha Release (September 2024)

- * We invited interested people to test these modules and provide feedback
- * Includes a first set of modules (open-source, but still preliminary)
- * soon to be release to entire community ...



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- * Includes a first set of modules (open-source, but still preliminary)
 - ✓ BQS EOS: 4D lattice QCD with alternative expansion scheme in μB
 - ✓ ISING-TEXS EOS: 2D Critical behavior into lattice QCD alternative expansion
 - ✓ NUMRELHOLO: 2D AdS/CFT correspondence based EoS



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 - ✓ NUMRELHOLO: 2D AdS/CFT correspondence based EoS
- ✓ CMF: 3D Chiral EoS with different orders for deconfinement
- ✓ CEFT: 2D EoS for interacting nucleons and pions
- ✓ UTK os Crust DFT: 2D EoS including nuclei
- ✓ Lepton module, Synthesis module, CompOSE outputs



muses Alpha Release (September 2024)

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 - ✓ Transport coefficients: thermal conductivity, baryon conductivity & diffusion, shear & bulk viscosities, ...



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- * Includes a first set of modules (open-source, but still preliminary)
 - ✓ Transport coefficients: thermal conductivity, baryon conductivity & diffusion, shear & bulk viscosities, ...
 - ✓ QLIMR module: quadrupole moment, tidal Love number, moment of inertia, mass, and radius of slowly rotating neutron star
 - ✓ Flavor equilibration for weak β -equilibrium: Urca rates, relaxation rates, damping time, bulk viscosity.
 - ✓ Susceptibilities

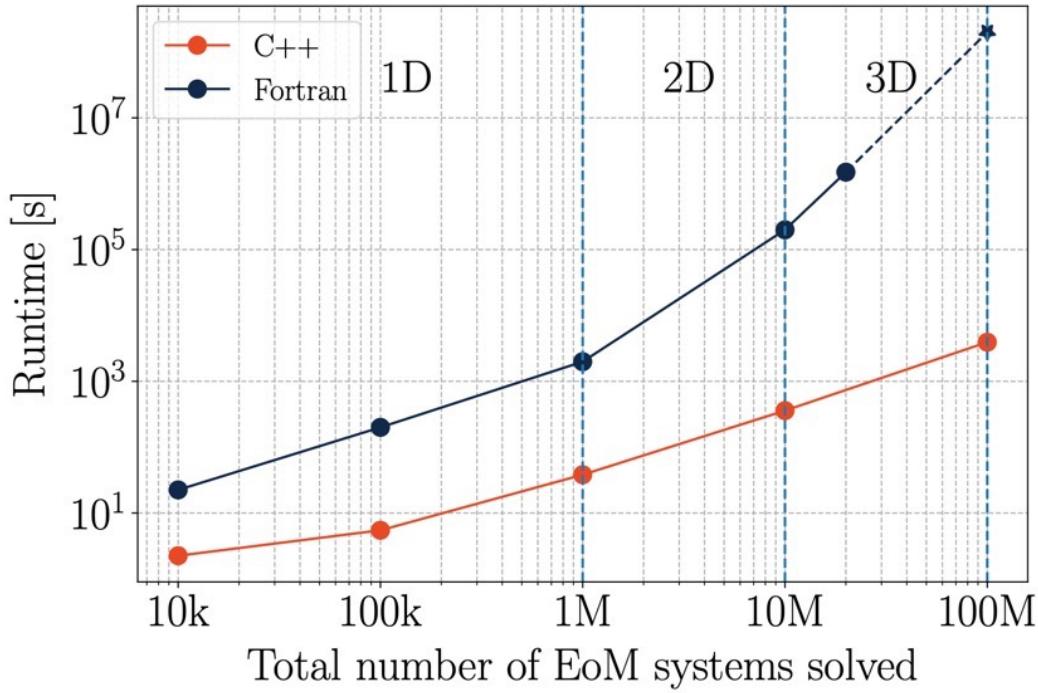


muses

Beta Release: October 2024

- * After Beta release we will provide in-person/online workshops and schools
 - * Online tutorials tools ...
 - * Stay tuned
-
- * Coming in 2025: more dimensions for EoS's, pasta phases, Thermal-FIST module, more interpolating functions, fully parametrized EoS's ...

⭐ Results for CMF++

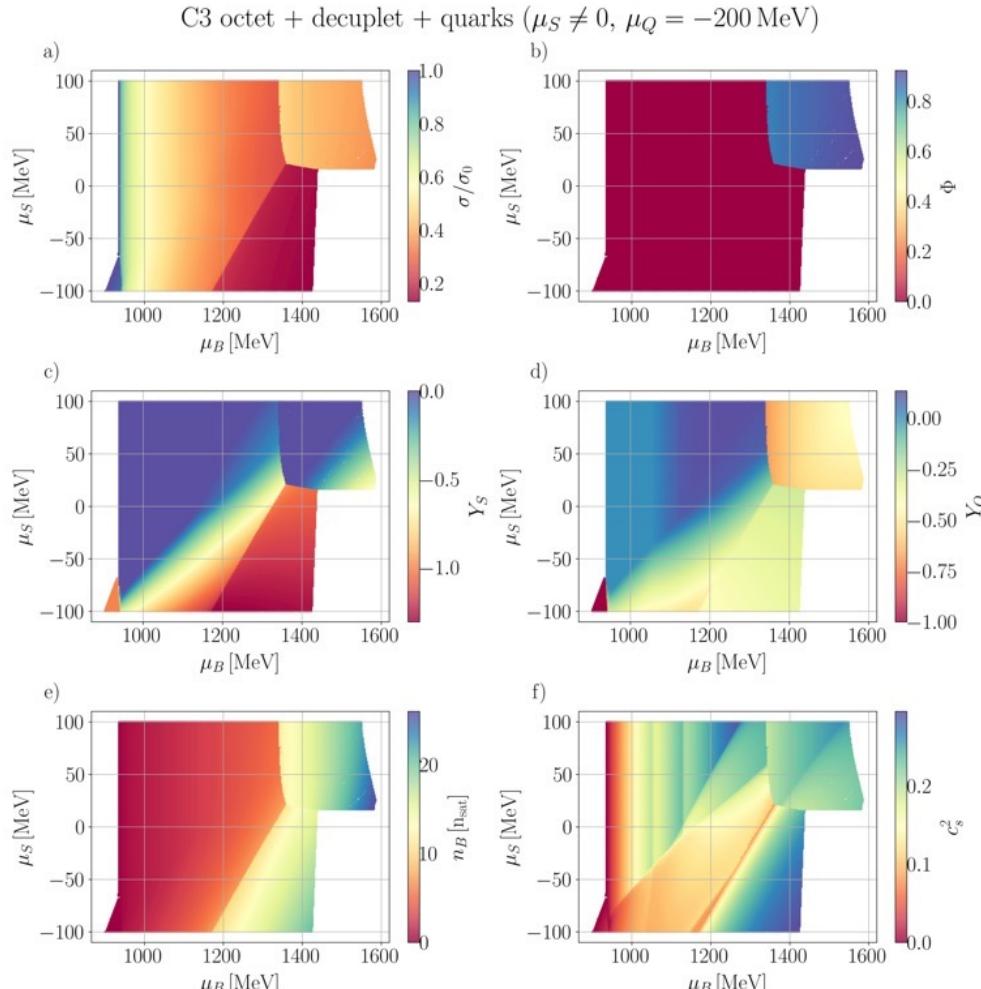




Results for CMF++

e-Print: [2409.06837](#)

- a) scalar meson field σ normalized by vacuum
- b) deconfinement field Φ
- c) strangeness fraction
- d) charge fraction
- e) baryon density
- f) speed of sound squared

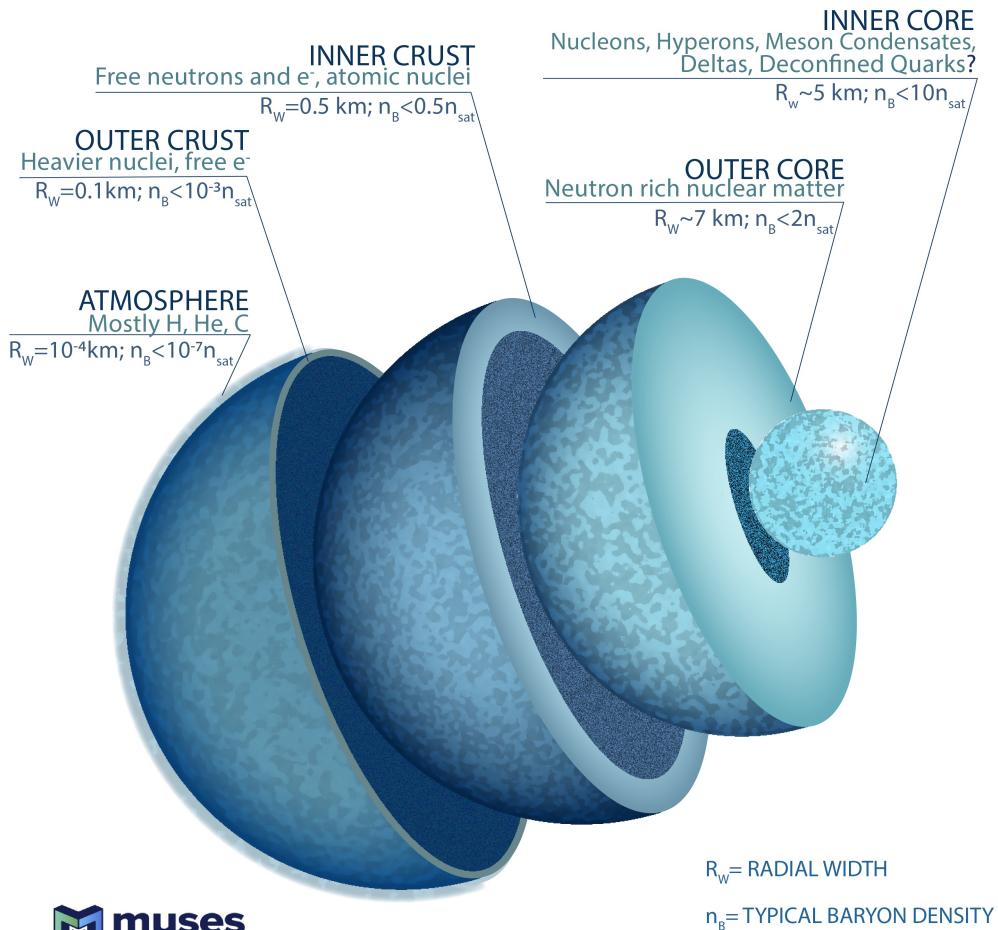


⭐ Additional Slides

★ Neutron Stars

- * Mostly made up of dense matter (beyond saturation density)
- * With inner core (beyond 2x saturation density) containing exotic matter

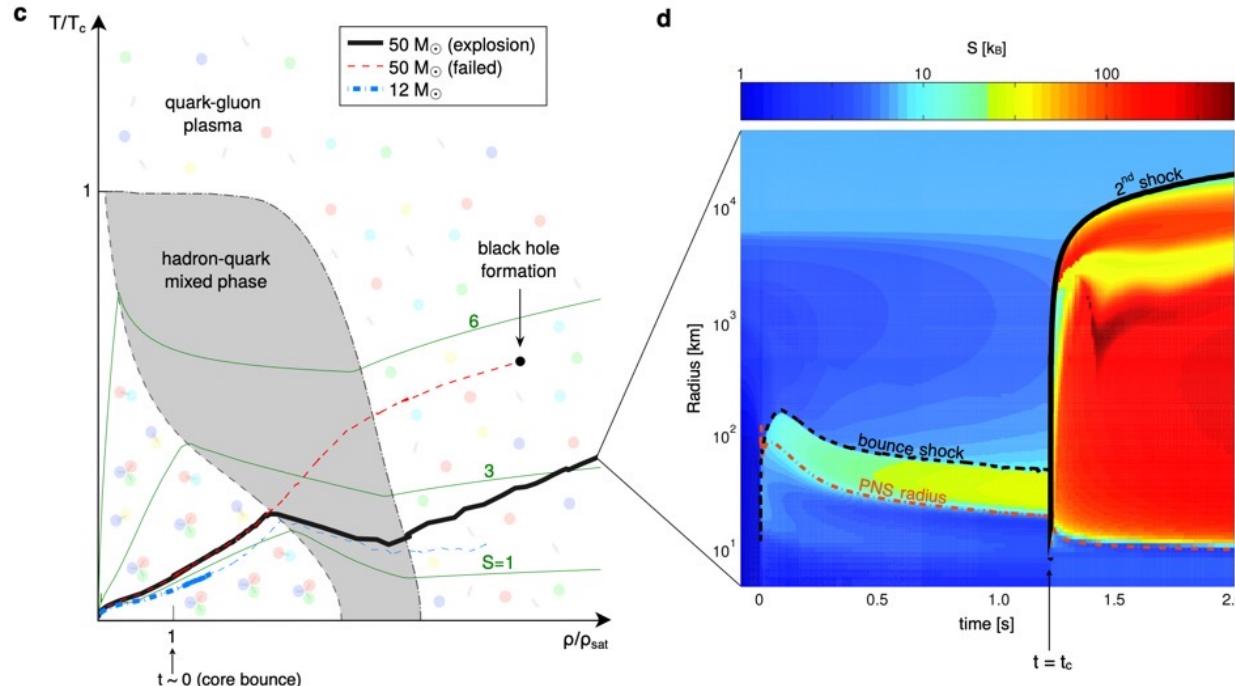
Living Rev.Rel. 27 (2024) 1, 3
e-Print: [2303.17021](https://arxiv.org/abs/2303.17021) [nucl-th]



★ Supernovae

Nature Astron. 2 (2018) 12, 980-986
e-Print: [1712.08788](https://arxiv.org/abs/1712.08788)

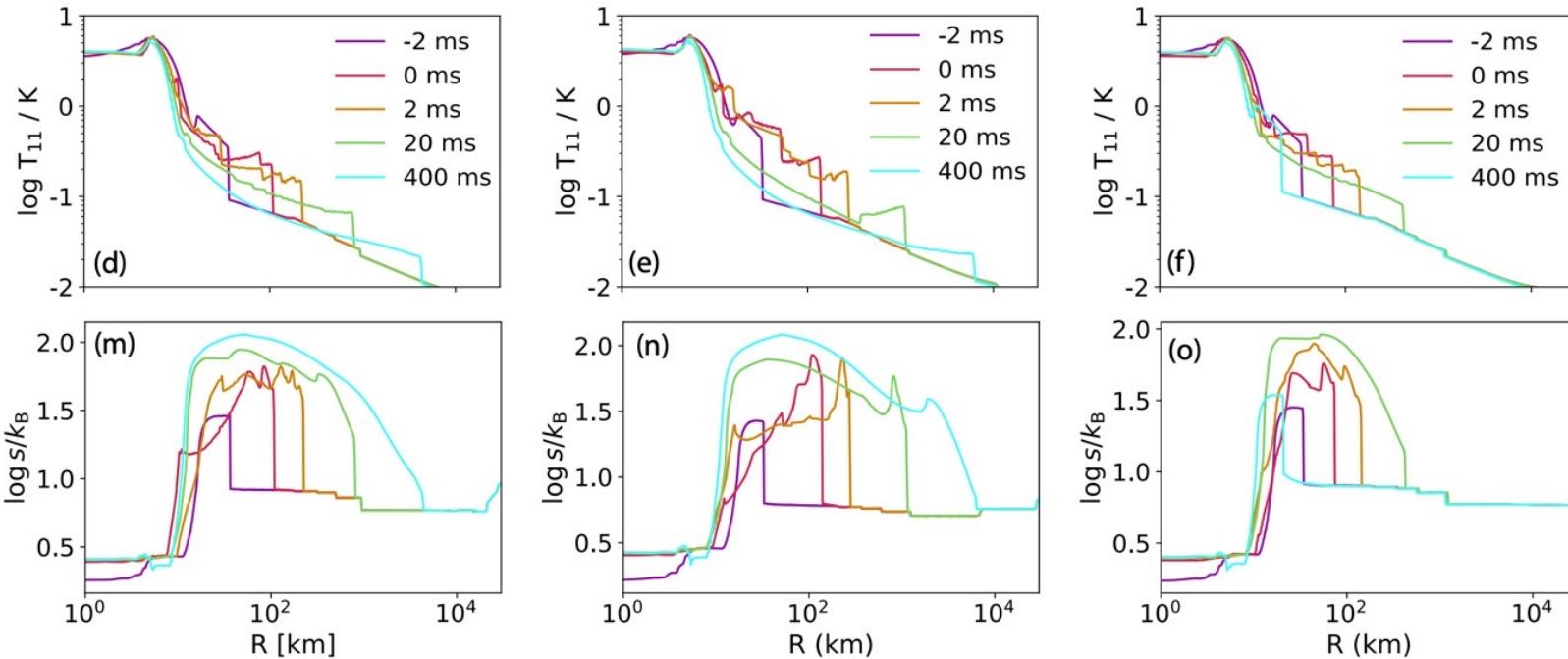
- * Dense matter reaching temperatures of few tens of MeV and S/B>2



⭐ Supernovae

Mon.Not.Roy.Astron.Soc. 516 (2022) 2, 2554-2574
e-Print: [2204.10397](https://arxiv.org/abs/2204.10397) [astro-ph.HE]

- * Dense matter reaching temperatures of few tens of MeV and S/B>2

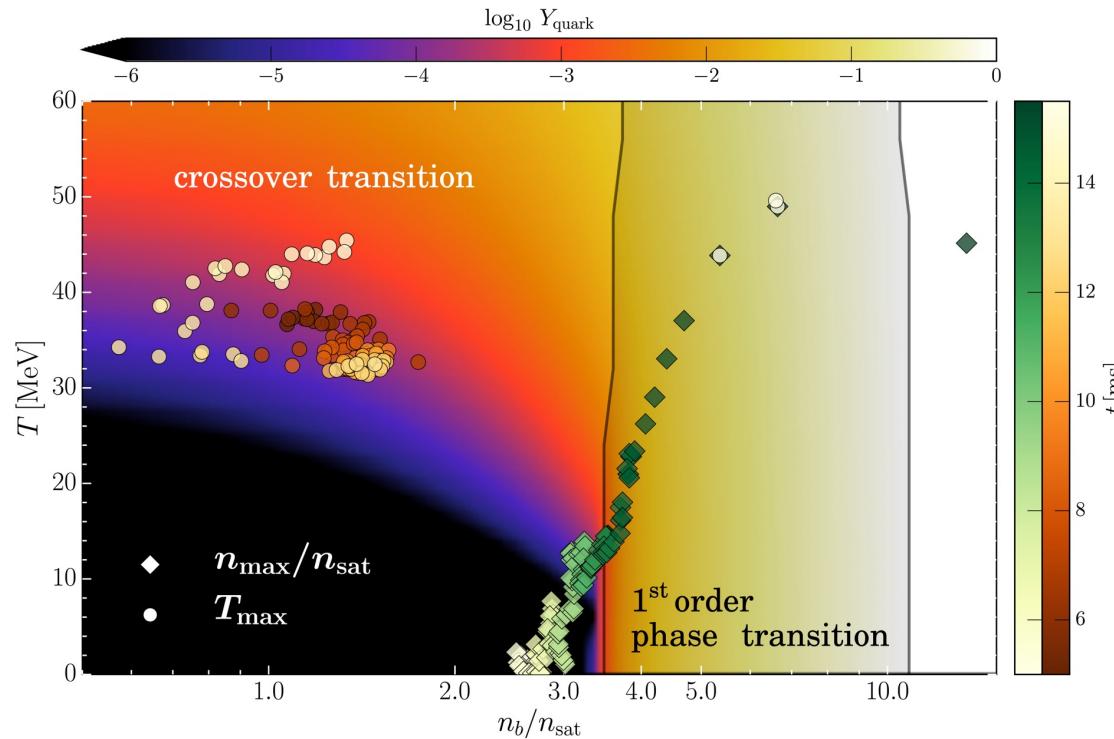




Neutron-Star Mergers

Phys.Rev.Lett. 122 (2019) 6, 061101 e-Print: [1807.03684](https://arxiv.org/abs/1807.03684)

- * Dense matter reaching temperatures of many tens of MeV



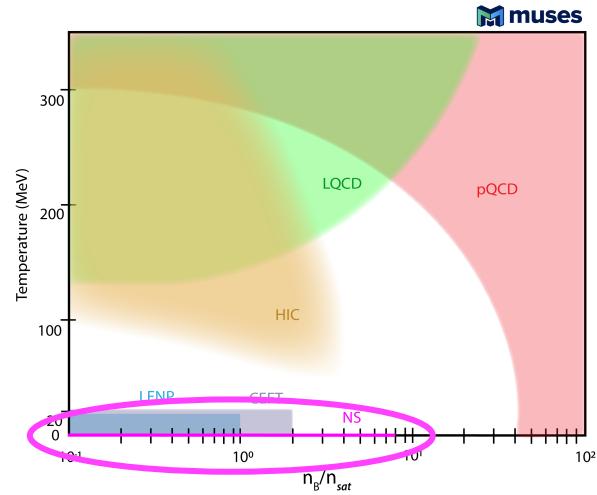
★ Astrophysics

- * Neutron-star maximum mass

Neutron Star	$M_{max} (M_\odot)$
PSR J0740+6620	$\geq 2.08 \pm 0.07$
PSR J0348+0432	$\geq 2.01 \pm 0.04$

- * Masses and radii from NICER

Neutron Star	$M (M_\odot)$	Radius (km)
PSR J0030+0451	$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$
PSR J0740+6620	$2.072^{+0.067}_{-0.066}$	$12.39^{+1.30}_{-1.98}$
PSR J0030+0451	$1.44^{+0.15}_{-0.14}$	$13.02^{+1.24}_{-1.06}$
PSR J0740+6620	$2.08^{+0.07}_{-0.07}$	$13.7^{+2.6}_{-1.5}$
PSR J0437+4715	$1.418^{+0.037}_{-0.037}$	$11.36^{+0.95}_{-0.63}$



- * More neutron star masses and radii (quiescent low-mass X-ray binaries), tidal deformability from gravitational waves, cooling data, ...

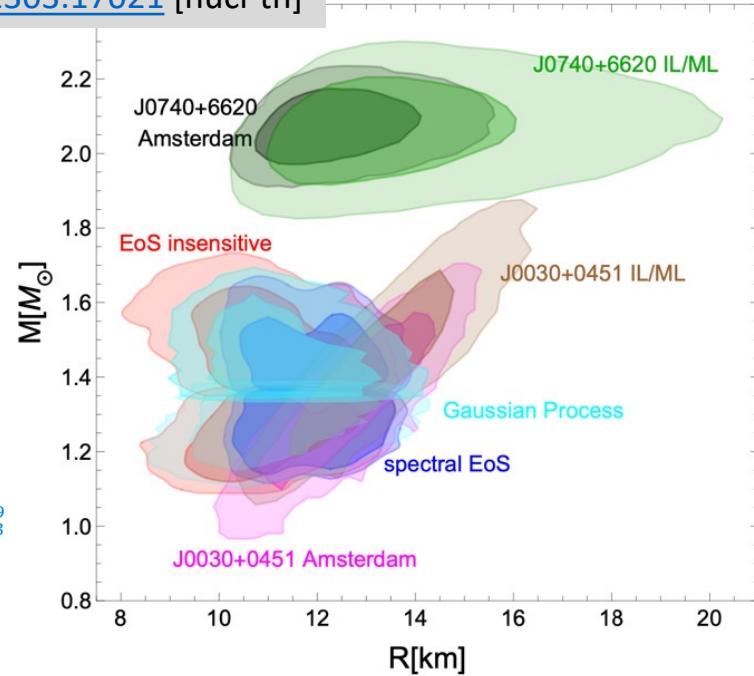
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MUSES EoS Module: Neutron Stars & Mergers

Theory/Model: Crust DFT, χ EFT, CMF

| α -release |

Details

- System has a long lifetime.
- Weak decay: $s \rightarrow u + W^-$.
- Strangeness is most likely not in equilibrium.
- Electrically neutral for stability
 $Y_Q + Y_{\text{lep}} = 0$.

Needs

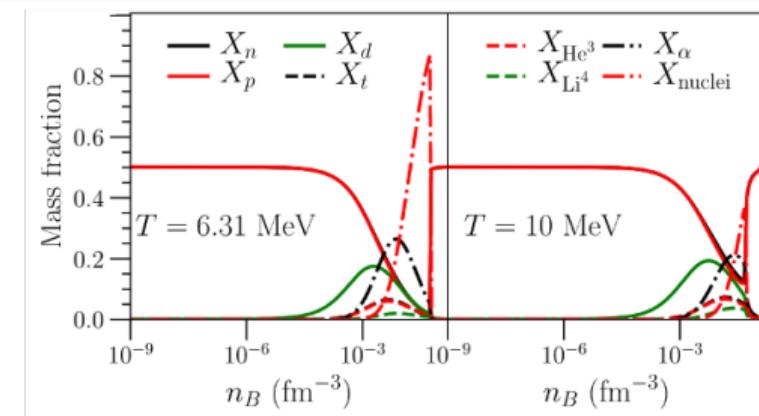
- Standard EoS:
 $(P, s, \varepsilon, \rho_B, c_s^2, \mu_i, Y_i)$.
- Lepton EoS.
- EoS at $T=0$ for neutron stars.
- EoS at finite- T for mergers.
- Ranges:
 $0 < n_B < 20?$ n_{sat} ,
 $0 < T < 100?$ MeV.
- Variable proton fraction for mergers.

Crust DFT EoS Module (Holt's/Steiner's groups: Satyajit R., Zidu L.)

Range: $T \sim 0$ MeV; $\mu_B < 1000$ MeV; $10^{-12} < n_B (\text{fm}^{-3}) < 2$

| α -release |

UTK or Crust density functional theory (DFT) EoS includes nuclei and nucleonic degrees of freedom based on a phenomenological fit of free energy density to nuclear experiments & astronomical observations.



X du, A. Steiner, J Holt, PRC 110 (2022)

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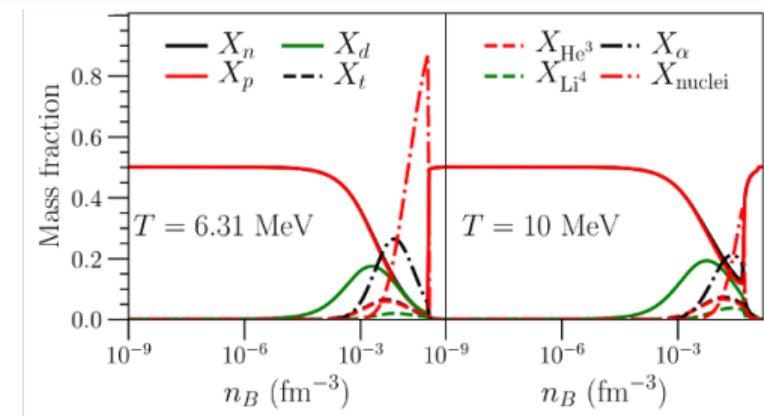
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Outlook

- Addition of finite T EoS.
- Extension to strangeness degrees of freedom.
- Machine learned emulator.



X du, A. Steiner, J Holt, PRC 110 (2022)

χ EFT EoS Module (Holt's group: David F.)

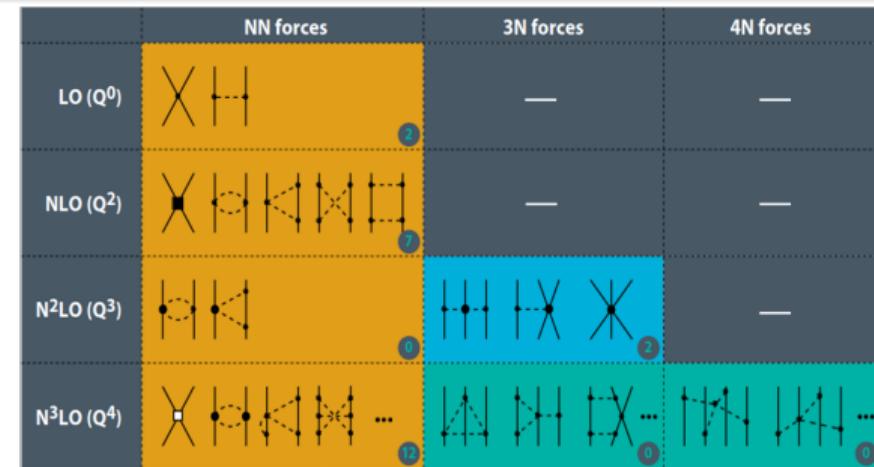
Range: $T \sim 0$ MeV; $\mu_B < 1000$ MeV; $0 < Y_p < 0.5$

| α -release |

Interacting nucleons and pions within chiral effective field theory (χ EFT) fitted to nucleon scattering data and boundstate potential.

Status before MUSES

- Fortran 77 proprietary code.
- Spaghetti code and not properly documented.
- Antique integration and interpolating routines.



χ EFT EoS Module (Holt's group: David F.)

Range: $T \sim 0$ MeV; $\mu_B < 1000$ MeV; $0 < Y_p < 0.5$

| α -release |

Current status

- High resolution $T = 0$ that agrees with the **Fortran** code for 2D EoS (ρ_B and Y_I).
- Freedom to choose the underlying parameters which was hardcoded into the **Fortran** code.
- 3 times faster than legacy **Fortran** code.
- Able to incorporate multiple χ EFT potentials (currently uses N3LO-414 and N3LO-450).
- Execution via Docker and from Calculation Engine (tutorial notebook available).

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Outlook

- Add extension at finite- T (up to 30 MeV).
- Include a wider variety of χ EFT potentials.
- Provide uncertainty quantification.

CMF EoS Module (Dexheimer's and Hostler's groups)

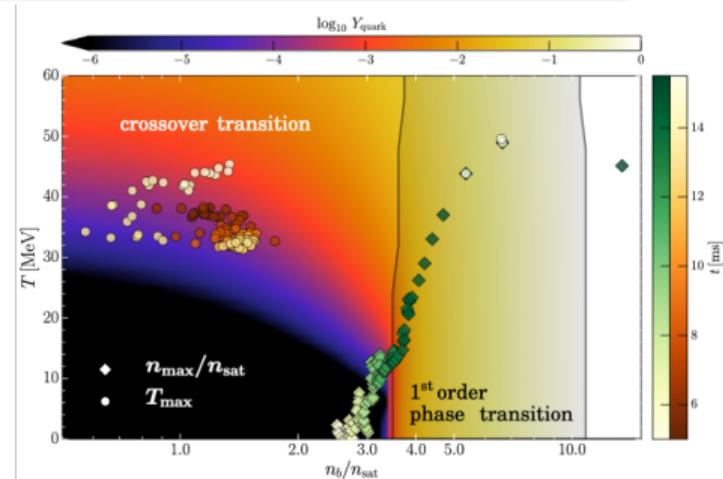
Nikolas C. C., Mateus R. P., Jeff P., Rajesh K.

| α -release |

We created an open-source optimized modular modern C++ code to compute multidimensional EoS tables using the Chiral Mean-field (CMF) model.

Status before MUSES

- Fortran 77 proprietary code.
- Spaghetti code between non- and magnetic cases, not properly documented.
- Antique root solving and integration routines.



CMF EoS Module (Dexheimer's and Hostler's groups:)

Nikolas C. C., Mateus R. P., Jeff P., Rajesh K.

| α -release |

Current status

- High resolution zero temperature that agrees with the previous **Fortran** code for all particles.
- More thorough check for EoS stability in the new code.
- CMF++ runtime is more than 4 orders of magnitude faster than legacy **Fortran**.
- Execution via Docker and from Calculation Engine (tutorial notebook available).

CMF EoS Module (Dexheimer's and Hostler's groups:)

Nikolas C. C., Mateus R. P., Jeff P., Rajesh K.

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Outlook

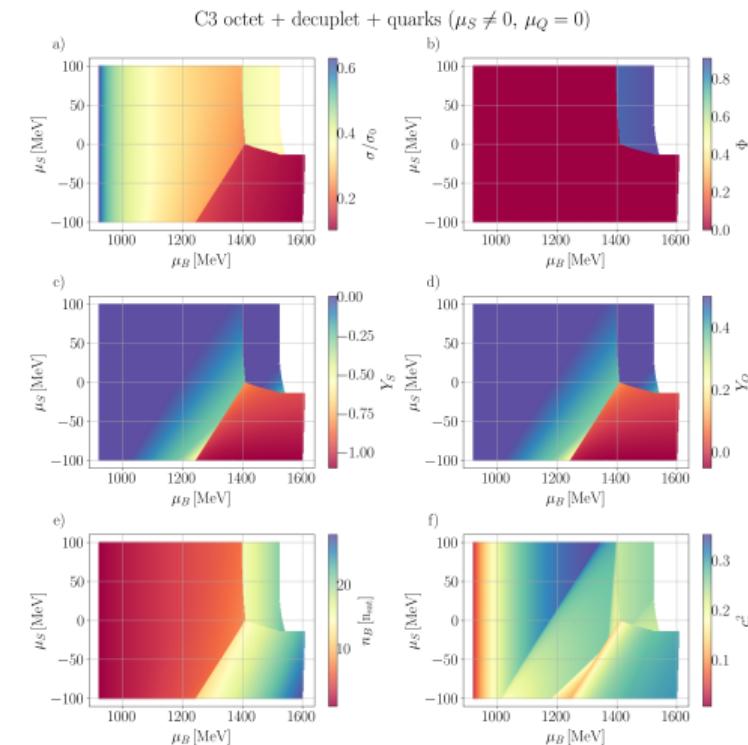
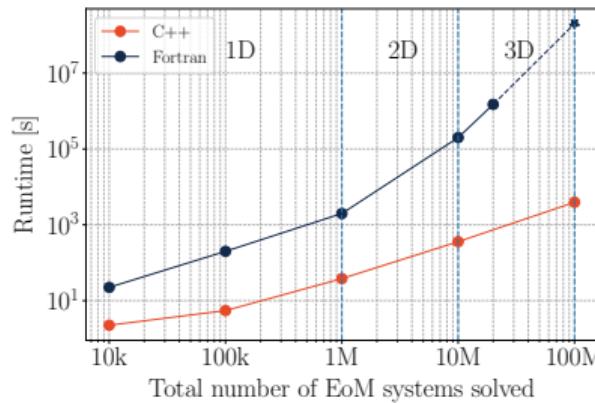
- Couple to flavor equilibration module.
- Zero temperature magnetic case.
- Finite temperature non- and magnetic case.
- Add thermal meson interactions.

CMF Module: Preprint

Phase Stability in the 3-Dimensional Open-source Code for the Chiral mean-field Model

Highlights

- Improved run time.
- Extension to 3D (μ_B, μ_Q, μ_S).
- Stability analysis.



MUSES EoS Module: Heavy-ion Collisions

Theory/Model: HRG, BQS (lattice), 2D Ising T.Ex.S, 4D T.Ex.S, Holography

| α -release | except HRG, 4D T.Ex.S

Details

- System is described in terms of hydrodynamic simulations.
- Short lifetime, the system is not in equilibrium.
- Strangeness conserved $Y_S = 0$, charge fraction $Y_Q = 0.4$.

Needs

- To take into account local fluctuations, 4D EoS is needed.
- Free parameters: T, μ_B, μ_S, μ_Q , thermodynamic variables ($P, s, \varepsilon, n_B, c_s^2$).
- 1st and 2nd order derivatives of pressure with respect to chemical potentials.
- Inclusion of critical point.
- Transport coefficients.

MUSES Observable Module: Neutron Stars & Mergers

QLIMR Module (Yunes's group: Carlos C.)

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Given an EoS, solves the Tolmann-Oppenheimer-Volkoff (TOV) equations

$$\frac{dp}{dr} = -\frac{G\varepsilon m}{c^2 r^2} \left[1 + \frac{p}{\varepsilon}\right] \left[1 + \frac{4\pi r^3 p}{mc^2}\right] \left[1 - \frac{2Gm}{c^2 r}\right]^{-1}$$
$$\frac{dm}{dr} = \frac{4\pi r^2 \varepsilon}{c^2}$$

plus Hartle Thorne method and computes:

- Q: quadrupole moment Q of NS
- L(Λ): tidal Love number (tidal deformability)
- I: moment of inertia
- M: mass of NS (+ δM to correct for rotation)
- R: radius of NS (+ δR to correct for rotation too)
- Local function $f(R)$

MUSES Observable Module: Neutron Stars & Mergers

(Alford/Dexheimer/Most/Yunes's groups: Mateus. R. P., Alexander H., Alexander C.)

Lepton module: β -equilibrium and charge neutral matter

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- Given an EoS, computes:
 - Charged lepton densities necessary to ensure charge neutrality

Flavor equilibration: β -equilibrium: $\mu_n - \mu_p - \mu_e = 0$

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- Given an EoS, computes:
 - Urca rates $n \rightarrow p + e + \bar{\nu}_e; p + e \rightarrow n + \nu_e$, equilibrium charge fractions, relaxation rates, damping time, susceptibilities, bulk viscosity.

Adapter modules to obtain a standard EoS for NS & mergers simulation tools

- Compatibility with CompOSE to provide 1D/2D/3D EoS for NS.
- Ensuring compatibility with merger simulations.

MUSES Observable Module: Neutron Stars & Mergers

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Outlook:

- Nuclear structure (pasta phases).
- Testing more interpolation functions.

MUSES Observable Module: Heavy-ion Collisions

Susceptibilities & hadronic species contributions

- Susceptibilities from lattice QCD will be computable.
- Using HRG model, one can study the breakdown of different hadron families.
- Partial pressure and analogous relations for susceptibilities.

Transport coefficients from Holographic module

- Thermal conductivity, baryon conductivity & diffusion.
- Shear & bulk viscosities, HQ drag force & Langevin diffusion coefficients.
- Jet quenching parameter.

Freeze-out physics

- T and μ_B at chemical freeze-out can be fitted from exp. data with HRG.
- Will be incorporated with Thermal-FIST.

MUSES Equation of State Workflows in the Zero-Temperature Limit

M. R. Pelicer, R.K. et al., under preparation (preliminary)

| α -release |

