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BAYESIAN INFERENCE OF HYBRID NEUTRON STAR EQUATION OF STATE FROM MULTI-MESSENGER ASTRONOMY

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Compact Stars in the QCD phase diagram (CSQCD-2024)

October 7-11, 2024

Bayesian inference from multi-messenger astronomy

Agnostic

- Model-independent
- No evidence to physical quantities

Physics Informed

- Model-dependent
- Quantitative measure of physical parameters

Metamodeling

- Quasi-independent
- Quantitative measure of physical parameters

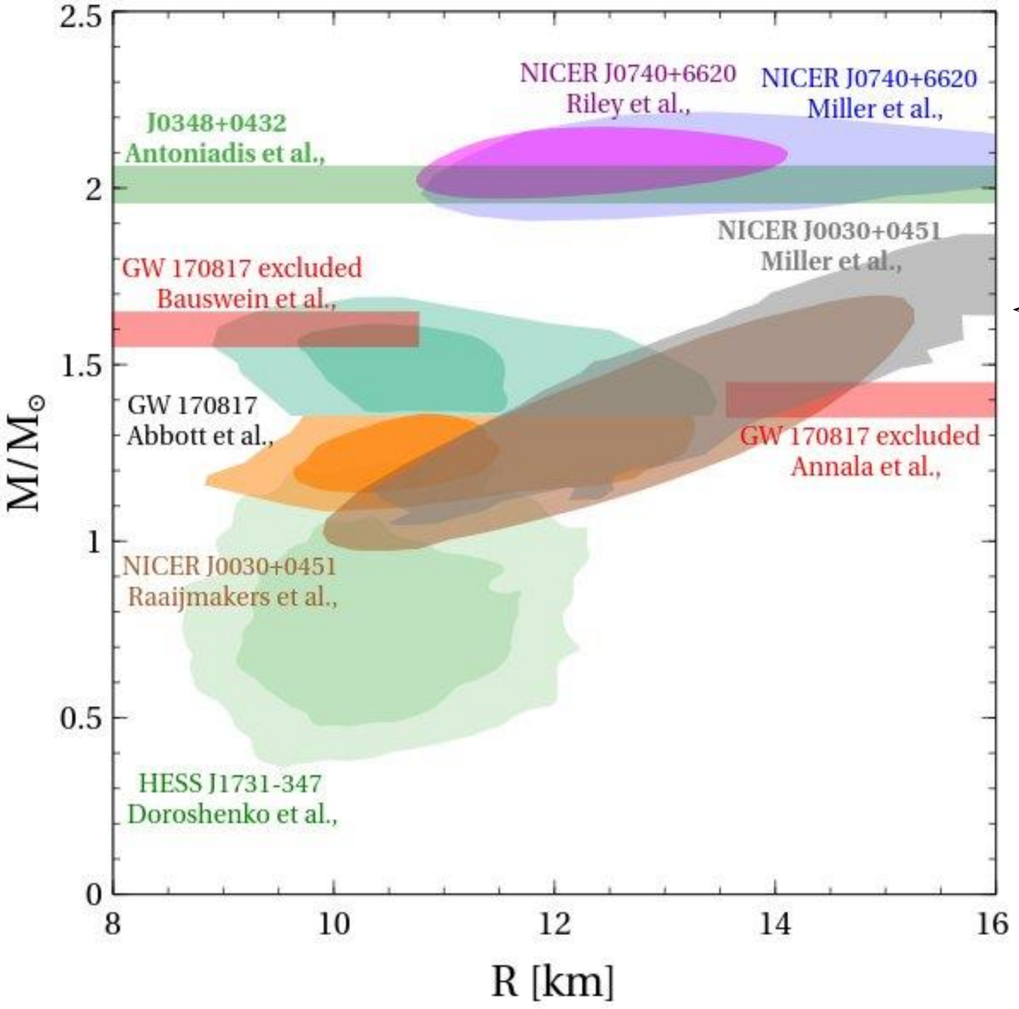
Margueron, Casali & Gulminelli.
PRC97, 025805 and 025806 (2018)

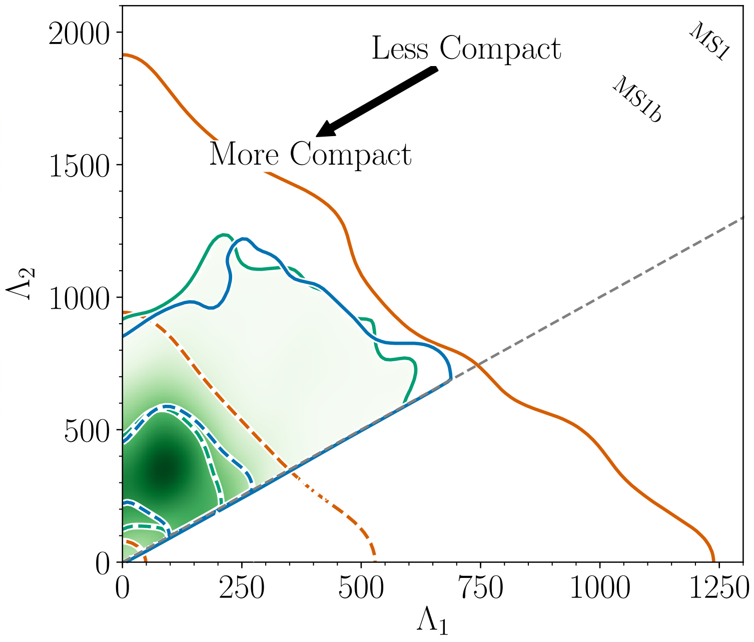
Bayes' theorem:

$$p(H_1 \mid D, I) = \frac{p(D \mid H_1, I)\, p(H_1 \mid I)}{p(D \mid I)}$$

Likelihood: $p(D \mid H_1, I)$; Prior: $p(H_1 \mid I)$; Posterior: $p(H_1 \mid D, I)$; Evidence: $p(D \mid I)$

Bayesian inference from multi-messenger astronomy





$$P\left(E_{MR}\,|\pi_i\right) = \int_{l_2} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho)d\tau + \\ + \int_{l_3} \mathcal{N}(\mu_R, \sigma_R, \mu_M, \sigma_M, \rho)d\tau$$

$$P\left(E_{GW}\,|\pi_i\right) = \int_{l_{22}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \\ + \int_{l_{32}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau + \\ + \int_{l_{23}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau \\ + \int_{l_{33}} \beta(\Lambda_1(\tau), \Lambda_2(\tau))d\tau$$

$$P\left(E_A\,|\pi_i\right) = \Phi(M_i, \mu_A, \sigma_A) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{M_i - \mu_A}{\sqrt{2}\sigma_A}\right)\right]$$

Bayes' theorem:

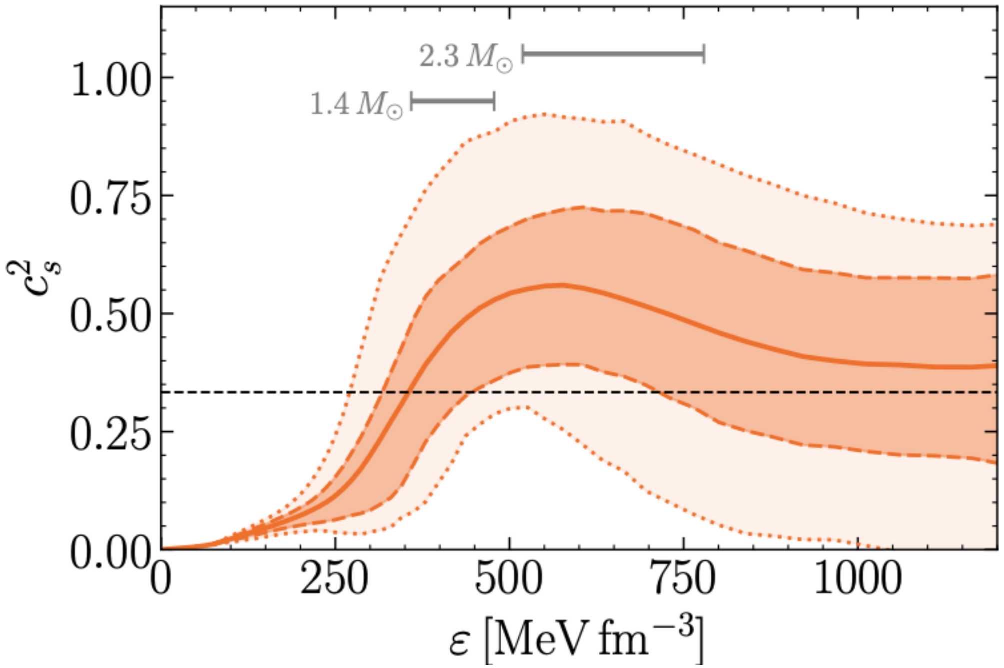
Likelihood Prior

$$p(H_1 \mid D, I) = \frac{p(D \mid H_1, I)\, p(H_1 \mid I)}{p(D \mid I)}$$

Posterior Evidence

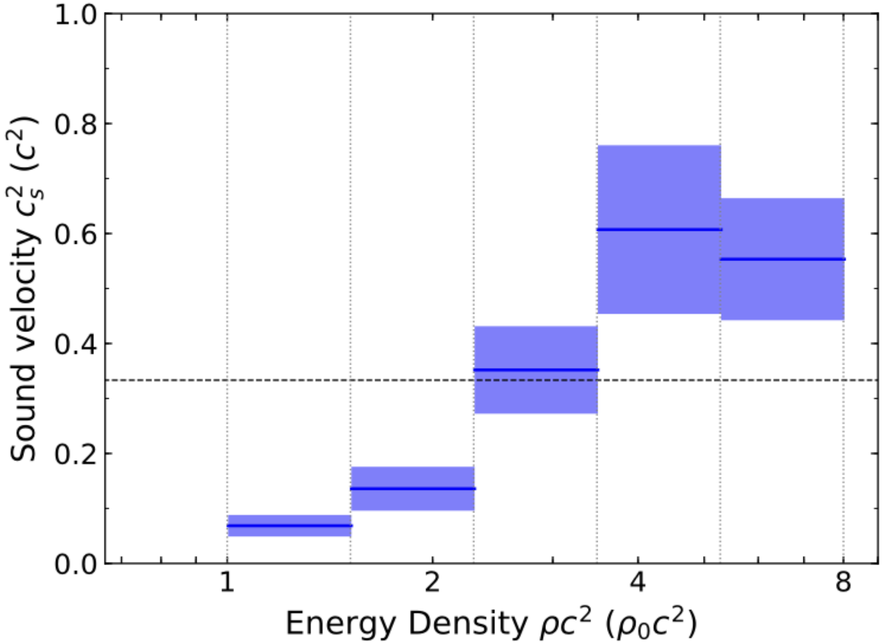
Speed of Sound in Neutron Stars

Brandes, Weise & Kaiser. PRD108, 094014 (2023)



$$c_s^2(\varepsilon,\theta) = \frac{(\varepsilon_{i+1}-\varepsilon)c_{s,i}^2 + (\varepsilon-\varepsilon_i)c_{s,i+1}^2}{\varepsilon_{i+1}-\varepsilon_i}$$

Fujimoto, Fukushima & Murase. PRD101, 054016 (2020)



$$p_i = p_{i-1} + c_{s,i}^2(\rho_i - \rho_{i-1})$$

Annala et al. Nature Phys. 16, 907 (2020)
QM: c_s^2 <= 1/3
HM: max(c_s^2) > 0.5

CSC: c_s^2 >> 1/3!
PhT is before or after the peak?

$\rho_0 \approx 150$ MeV/fm^3

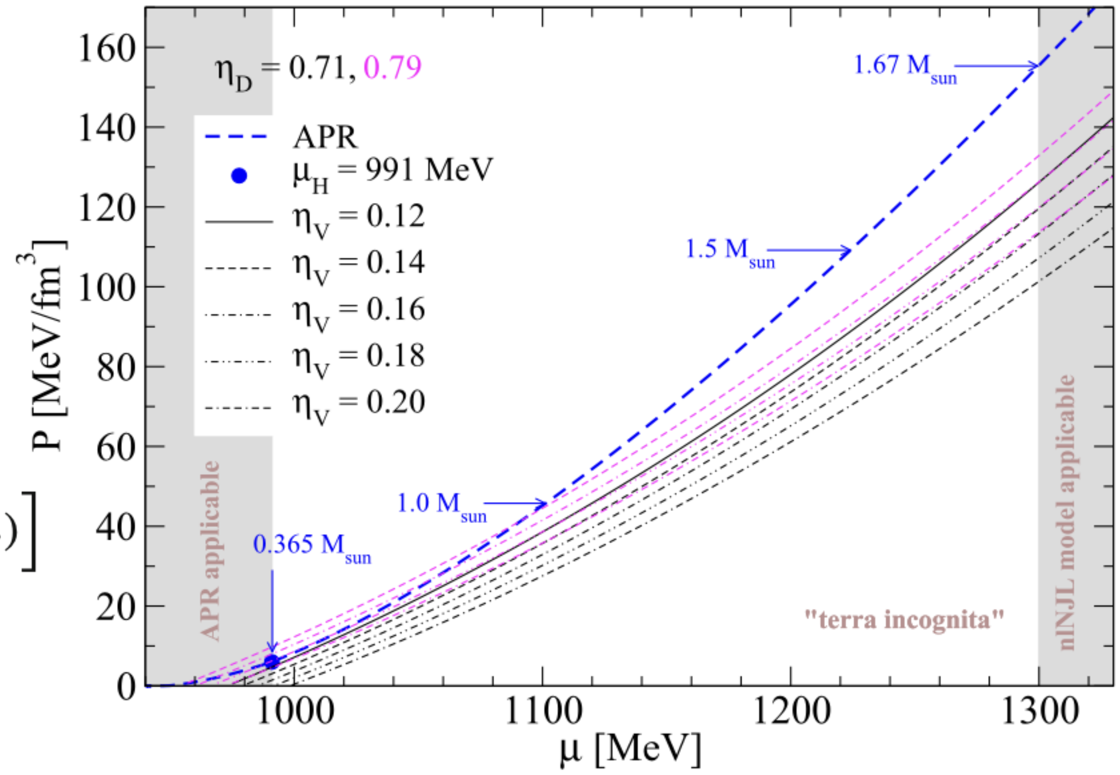
Two-Zone Interpolation of Phase Transition

Color superconducting two-flavor quark matter (nonlocal NJL)

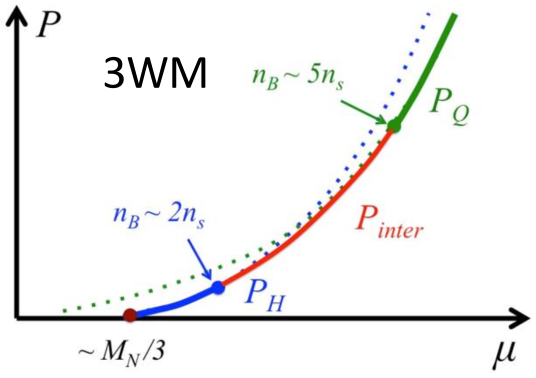
$$\Omega^{\mathrm{MFA}} = \frac{\bar{\sigma}^2}{2G_S} + \frac{\bar{\Delta}^2}{2H} - \frac{\bar{\omega}^2}{2G_V} - \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \ln\det\left[S^{-1}(\bar{\sigma}, \bar{\Delta}, \bar{\omega}, \mu_{fc})\right]$$

$$\eta_D = G_D/G_S \text{ and } \eta_V = G_V/G_S$$

Alvarez-Castillo, Blaschke, Grunfeld & Pagura. PRD99, 063010 (2019)

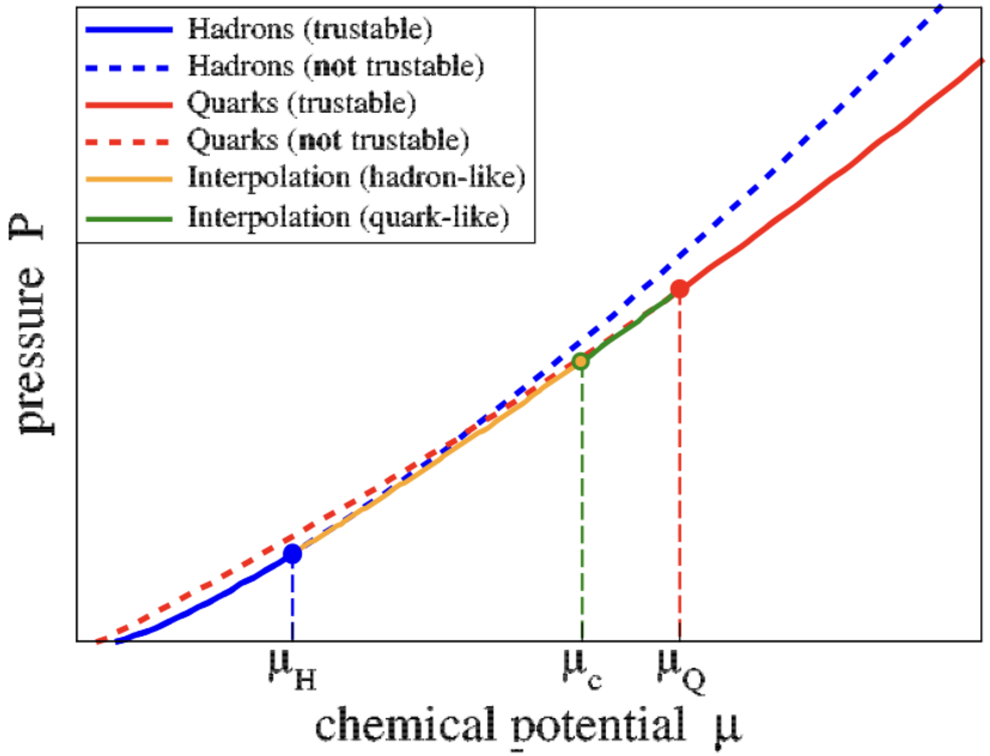


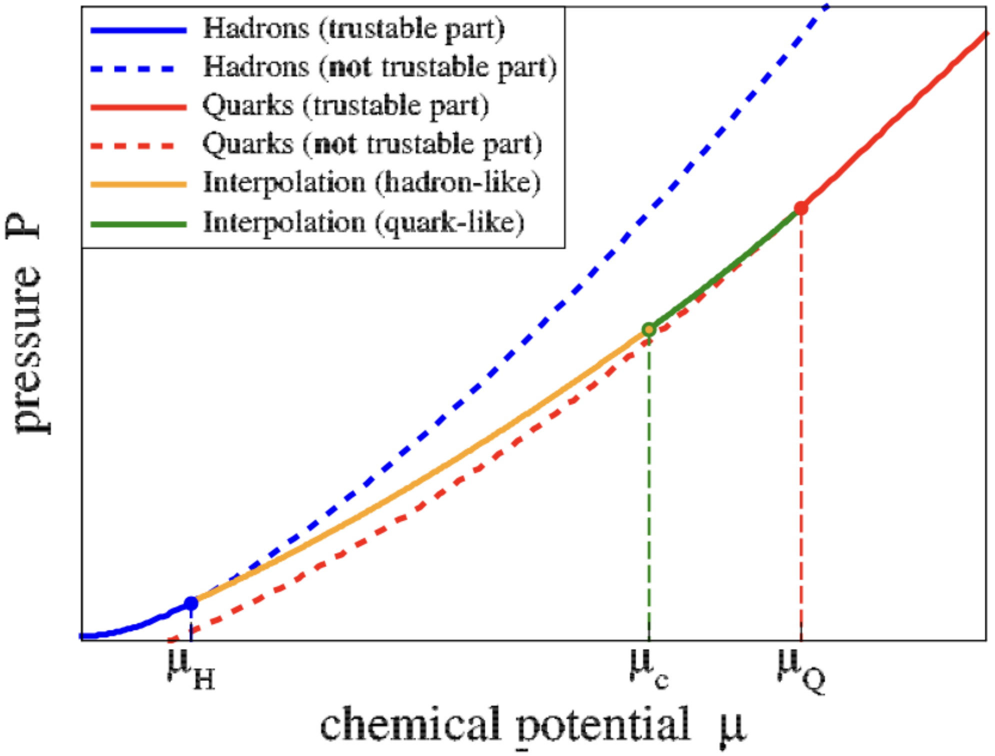
Ayriyan et al. EPJA57, 318 (2021)



Kojo, EPJA52, 51 (2016)

TWO-ZONE INTERPOLATION OF PHASE TRANSITION

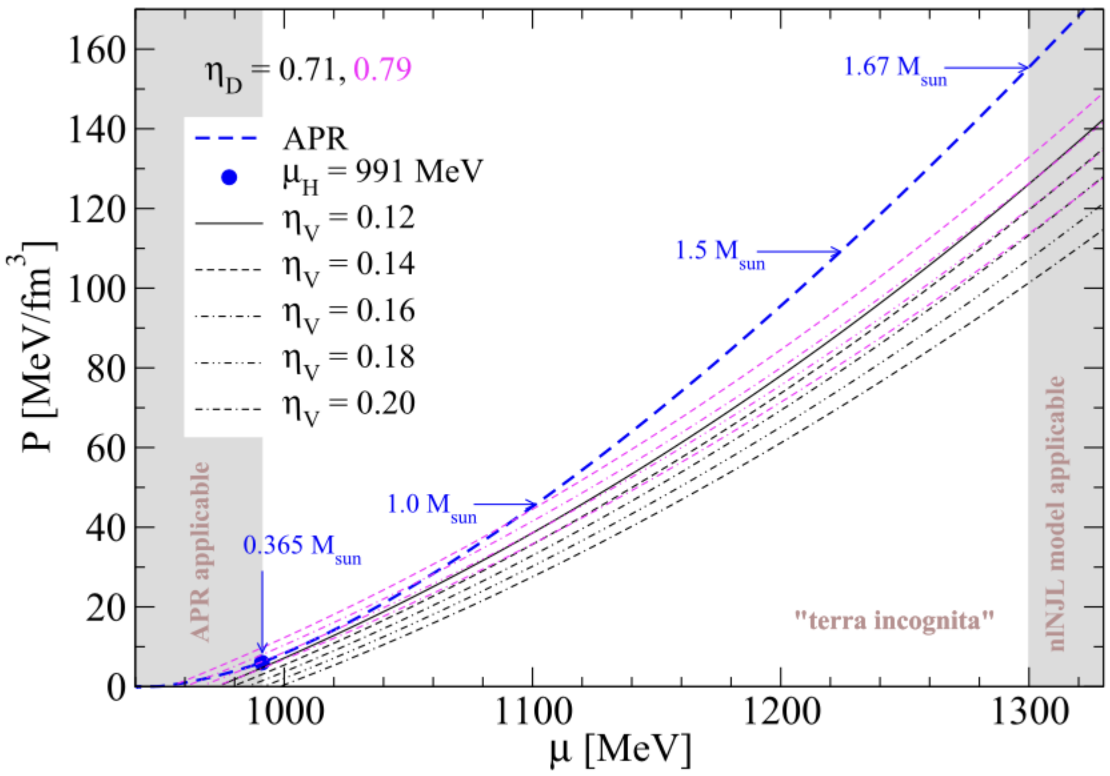


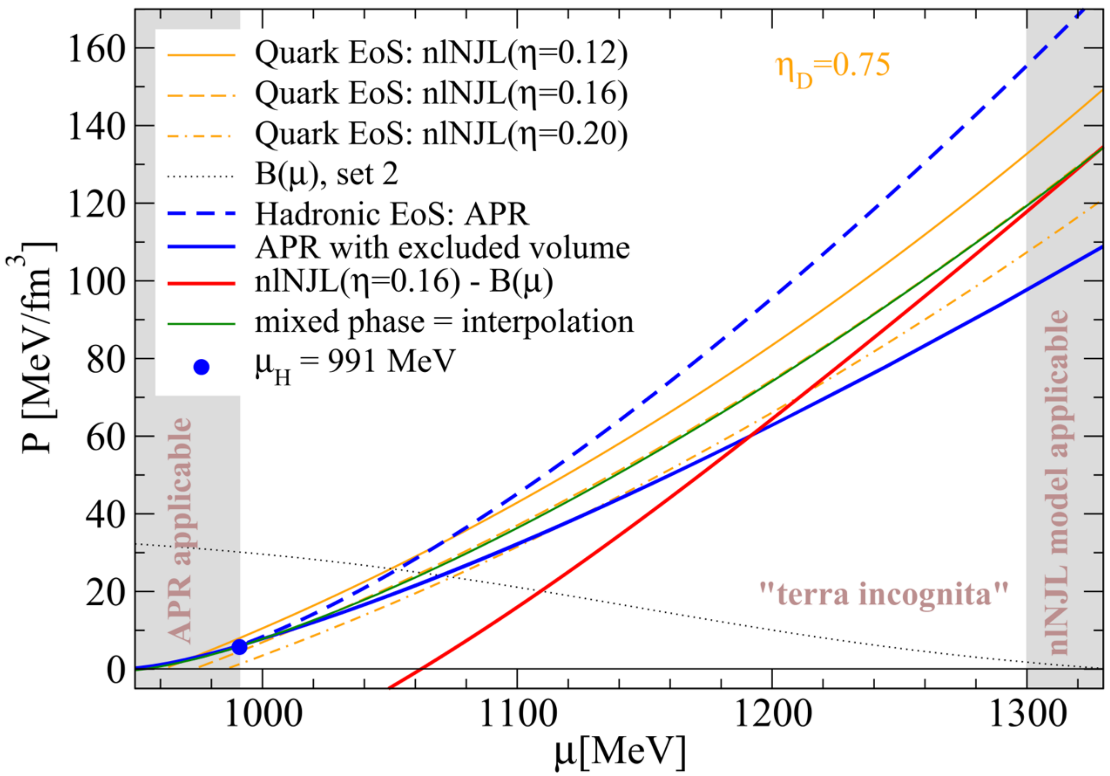


$$\begin{cases} P_\eta(\mu) = a_\eta\,(\mu-\mu_H)^2 + b_\eta\,(\mu-\mu_H) + c_\eta, & \mu \le \mu_c, \\ P_\rho(\mu) = a_\rho\left(\mu-\mu_Q\right)^2 + b_\rho\left(\mu-\mu_Q\right) + c_\rho, & \mu \ge \mu_c, \end{cases}$$

Ayriyan et al. EPJA57, 318 (2021)

Two-Zone Interpolation of Phase Transition





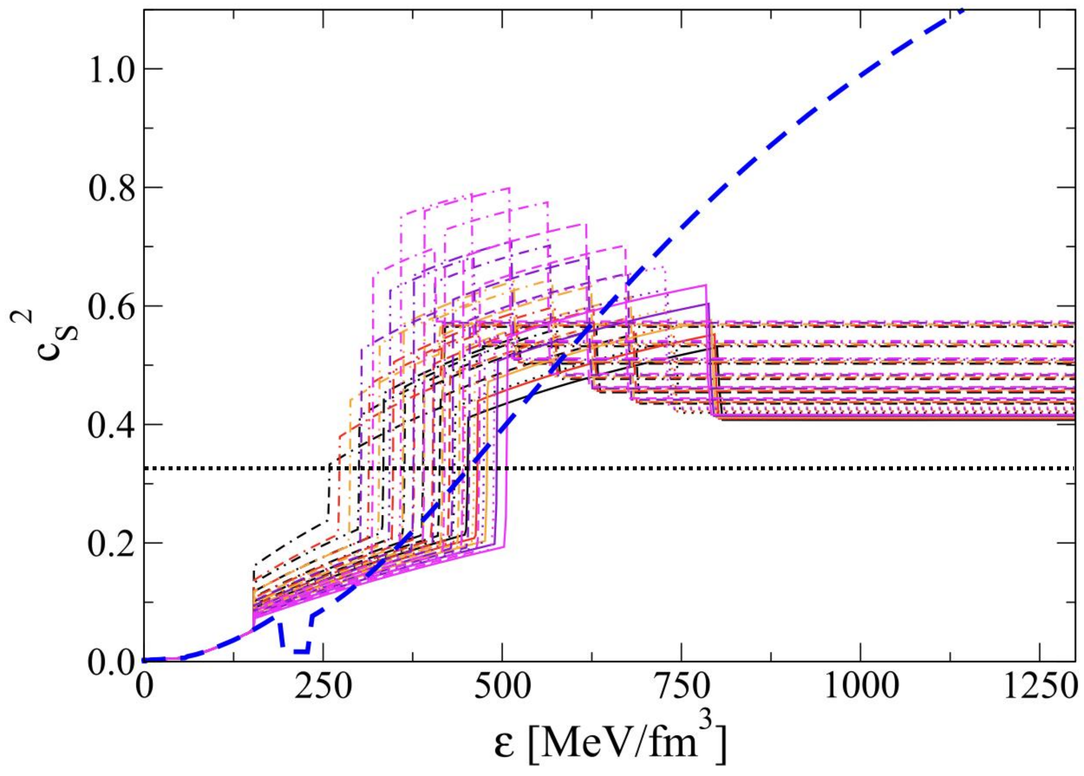
Two-zone interpolation builds a <u>mixed phase</u> construction between
(1) stiffened hadronic equation of state due to excluded volume (<u>quark Pauli blocking - substructure effects at high density</u>) and
(2) quark equation of state with correction for density-dependent bag pressure (<u>confining effects at low density</u>).

$$P_\eta(\mu) = a_\eta\,(\mu - \mu_H)^2 + b_\eta\,(\mu - \mu_H) + c_\eta,\ \ \mu \le \mu_c$$

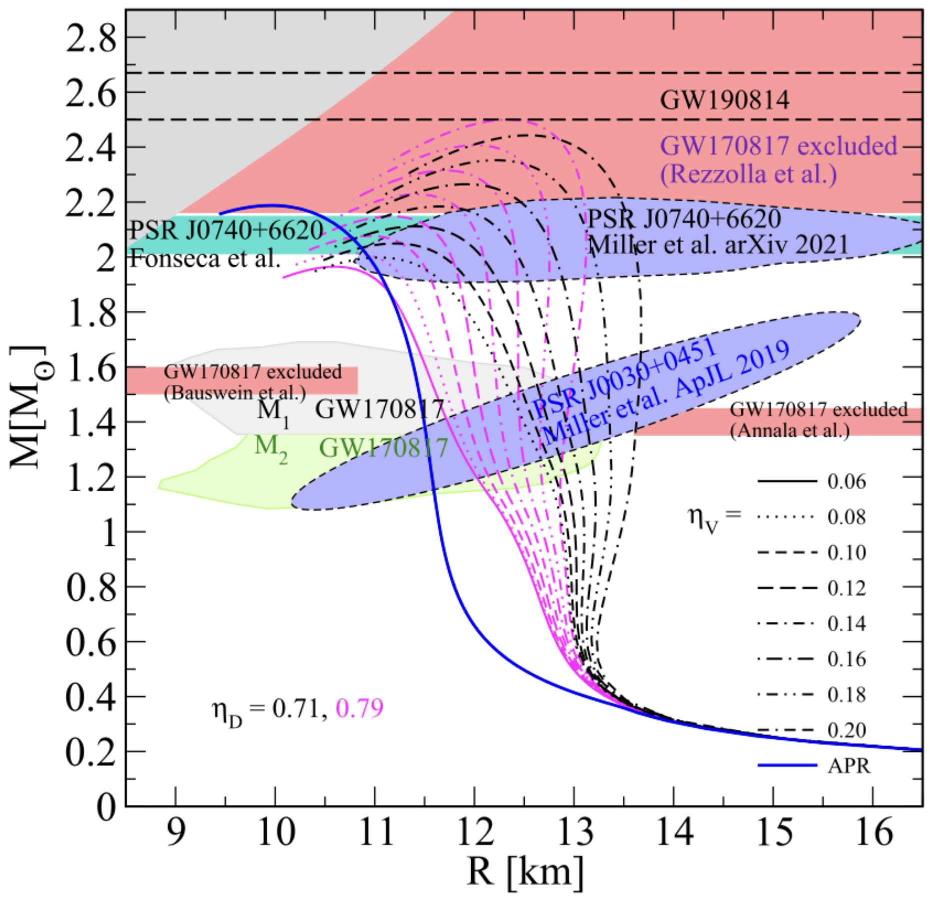
$$P_\rho(\mu) = a_\rho\left(\mu - \mu_Q\right)^2 + b_\rho\left(\mu - \mu_Q\right) + c_\rho,\ \mu \ge \mu_c$$

Ayriyan et al. EPJA57, 318 (2021)

Two-Zone Interpolation of Phase Transition

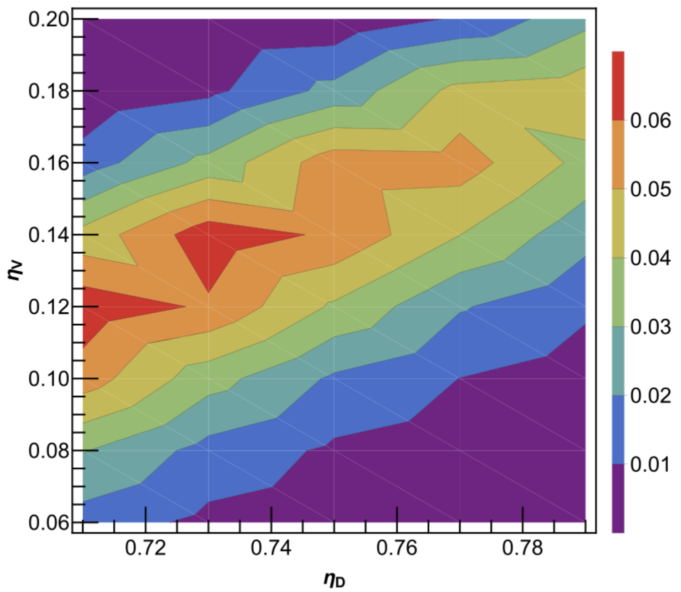


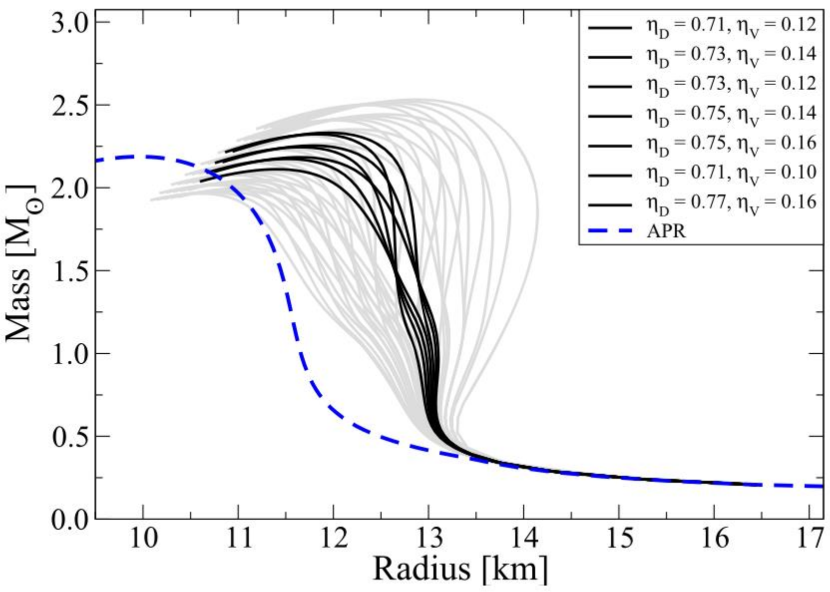
Squared speed of sound peak in the mixed phase is at 0.45

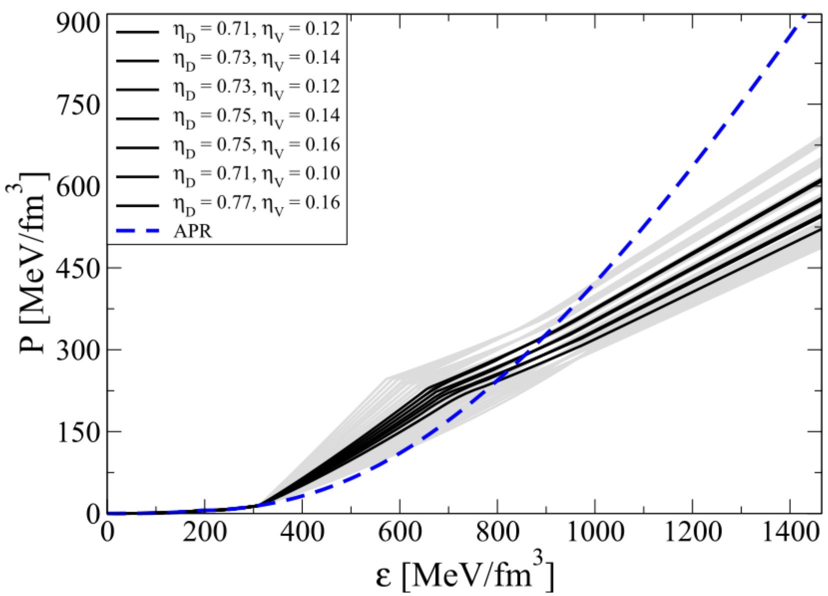


Ayriyan et al. EPJA57, 318 (2021)

Two-Zone Interpolation of Phase Transition







The BI results show that the most probable equations of state lie along the proportionality line between η_V and η_D.

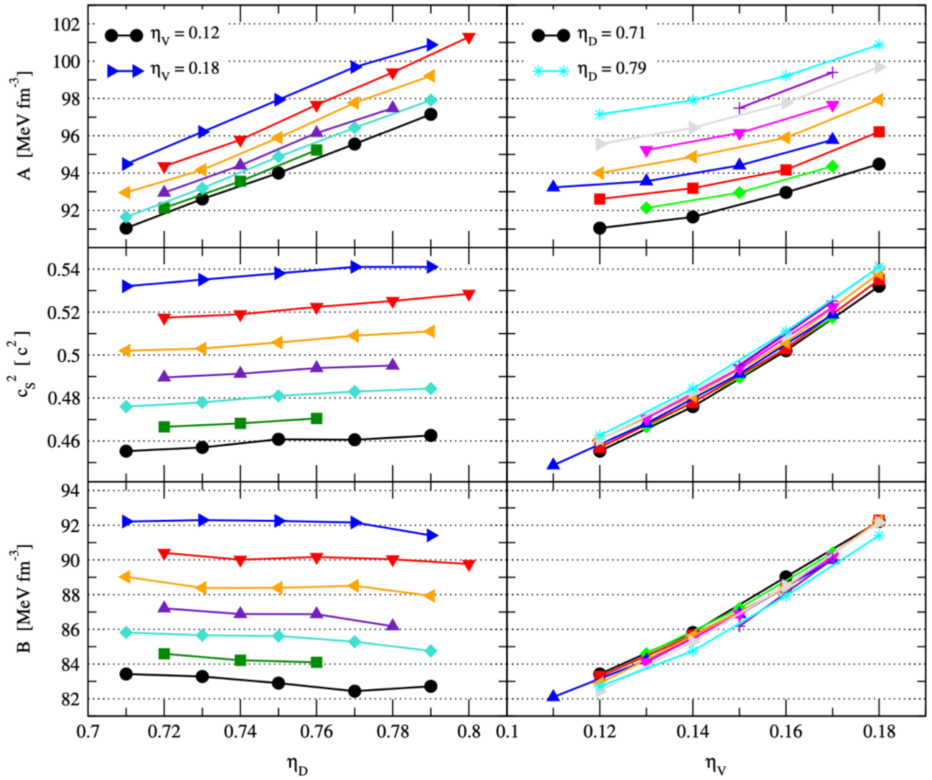
Ayriyan et al. EPJA57, 318 (2021)

CONSTANT SPEED OF SOUND AND NLNJL

$$P(\mu) = A\left(\frac{\mu}{\mu_x}\right)^{1+\beta} - B$$

$$\varepsilon(\mu) = A\beta\left(\frac{\mu}{\mu_x}\right)^{1+\beta} + B$$

$$n_B(\mu) = A\frac{1+\beta}{\mu_x}\left(\frac{\mu}{\mu_x}\right)^{\beta}$$



$$A = a_1\eta_D + b_1\eta_V^2 + c_1$$

$$c_s^2 = a_2\eta_D + b_2\eta_V^2 + c_2$$

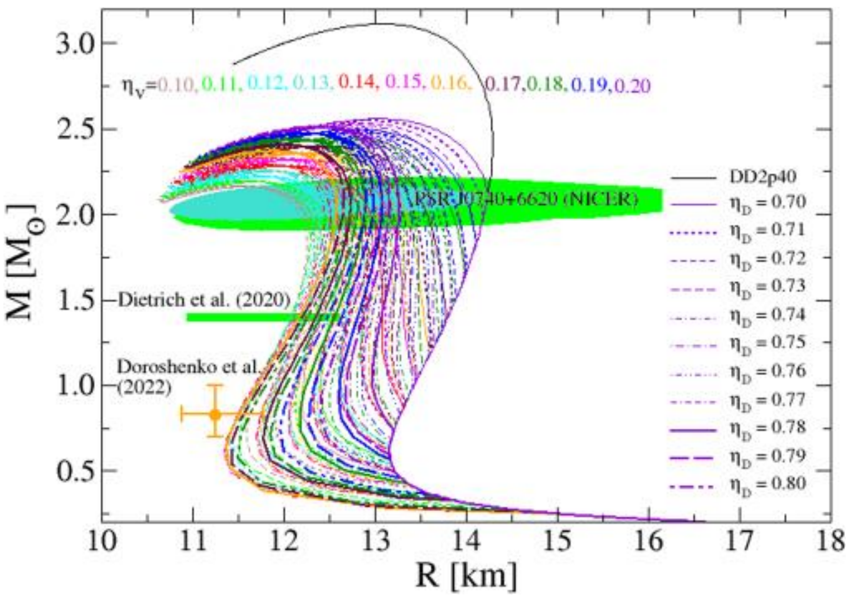
$$B = a_3\eta_D + b_3\eta_V^2 + c_3$$

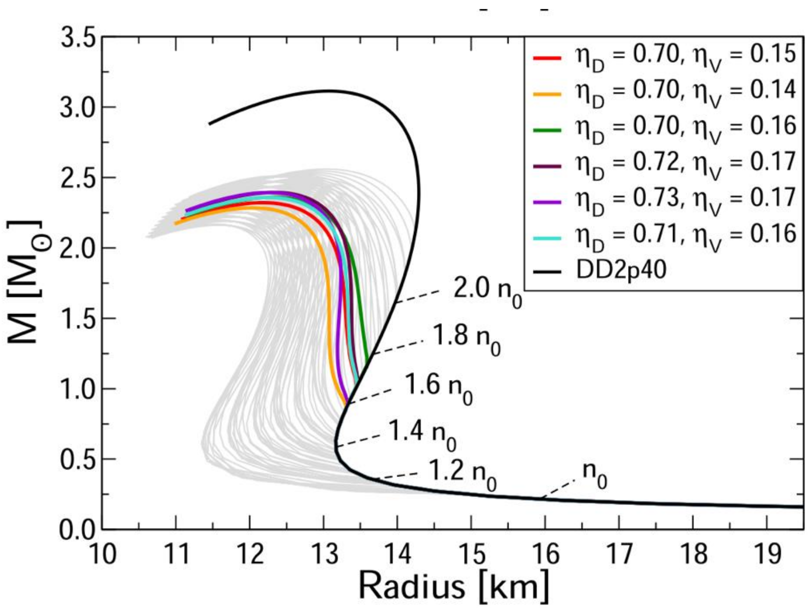
i	parameter	unit	a_i	b_i	c_i
1	A	MeV fm^{-3}	80.66330	199.80900	30.57520
2	c_s^2	c^2	0.11205	4.31830	0.31244
3	B	MeV fm^{-3}	-10.42990	502.99800	83.46230

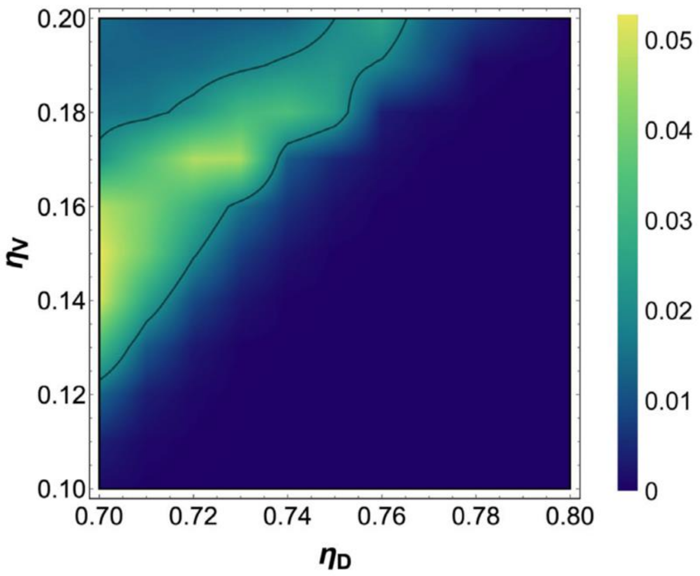
η_D	η_V	A [MeV/fm^3]	c_s^2 [c^2]	B [MeV/fm^3]	χ^2
0.70	0.15	91.484	0.488	87.209	0.039
0.71	0.12	91.053	0.456	83.425	0.022
0.71	0.14	91.649	0.476	85.815	0.032
0.71	0.16	92.963	0.502	89.021	0.047
0.71	0.18	94.481	0.532	92.214	0.075
0.72	0.13	92.132	0.467	84.592	0.026
0.72	0.15	92.954	0.490	87.209	0.038
0.72	0.17	94.366	0.517	90.408	0.058
0.73	0.12	92.612	0.457	83.280	0.021
0.73	0.14	93.190	0.478	85.658	0.031
0.73	0.16	94.170	0.503	88.385	0.048
0.73	0.18	96.211	0.535	92.290	0.073
0.74	0.11	93.236	0.449	82.095	0.017
0.74	0.13	93.563	0.468	84.217	0.026
0.74	0.15	94.410	0.491	86.884	0.039
0.74	0.17	95.780	0.519	90.011	0.061
0.75	0.12	94.000	0.461	82.899	0.044
0.75	0.14	94.875	0.481	85.614	0.031
0.75	0.16	95.894	0.506	88.391	0.056
0.75	0.18	97.934	0.538	92.249	0.078
0.76	0.13	95.235	0.470	84.101	0.027
0.76	0.15	96.153	0.494	86.873	0.039
0.76	0.17	97.660	0.522	90.172	0.063
0.77	0.12	95.556	0.461	82.437	0.021
0.77	0.14	96.433	0.483	85.287	0.032
0.77	0.16	97.770	0.509	88.512	0.074
0.77	0.18	99.685	0.541	92.155	0.085
0.78	0.15	97.485	0.495	86.179	0.042
0.78	0.17	99.340	0.525	90.034	0.065
0.79	0.12	97.604	0.464	82.718	0.020
0.79	0.14	97.912	0.484	84.755	0.033
0.79	0.16	99.216	0.511	87.929	0.053
0.79	0.18	100.878	0.541	91.415	0.084
0.80	0.17	101.116	0.528	89.766	0.070

Shahrbaf, Antić, Ayriyan, Blaschke & Grunfeld, PRD107, 054011 (2023)

Constant speed of sound and nlNJL

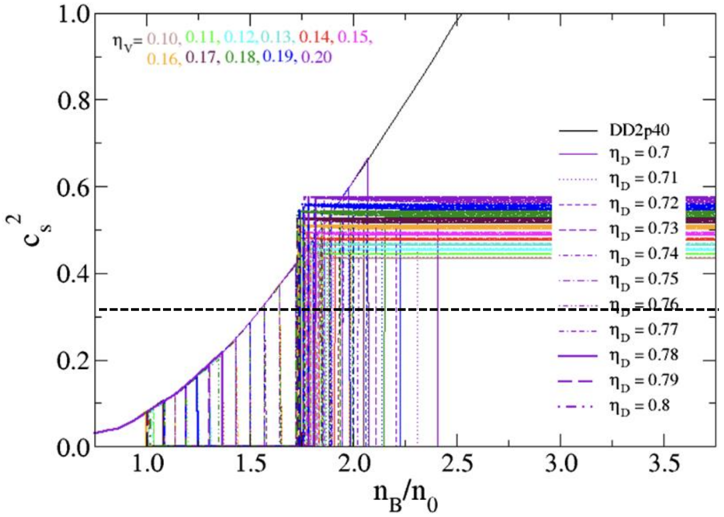






1. BI with multimessenger astronomy data selects EoS along a line of proportionality between η_V and η_D!
2. CSC matter leads to the need for a low density onset.
3. Squared speed of sound is above 0.45.

Shahrbaf, Antić, Ayriyan, Blaschke & Grunfeld, PRD107, 054011 (2023)



CSC MATTER WITH CONFORMAL LIMIT

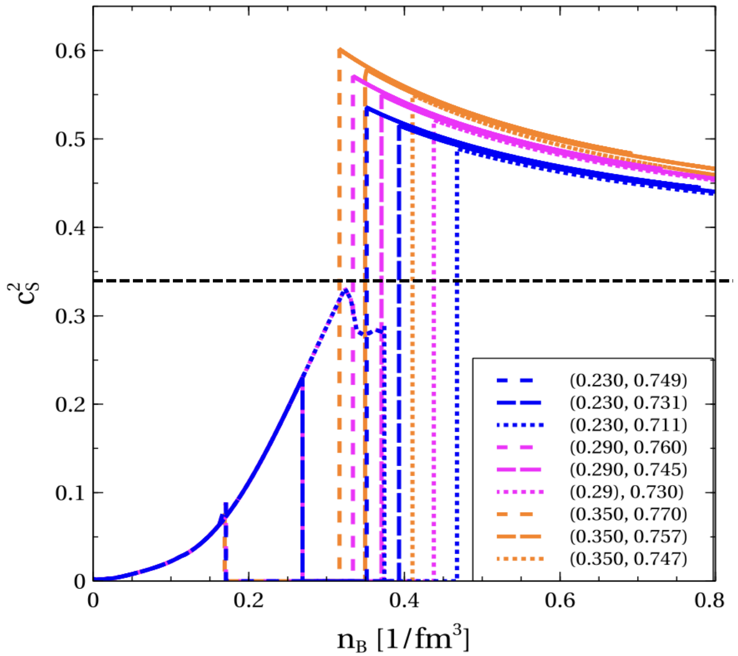
$$p = \frac{A_4 \mu^4}{2\pi^2} + \frac{\Delta^2 \mu^2}{\pi^2} - B$$

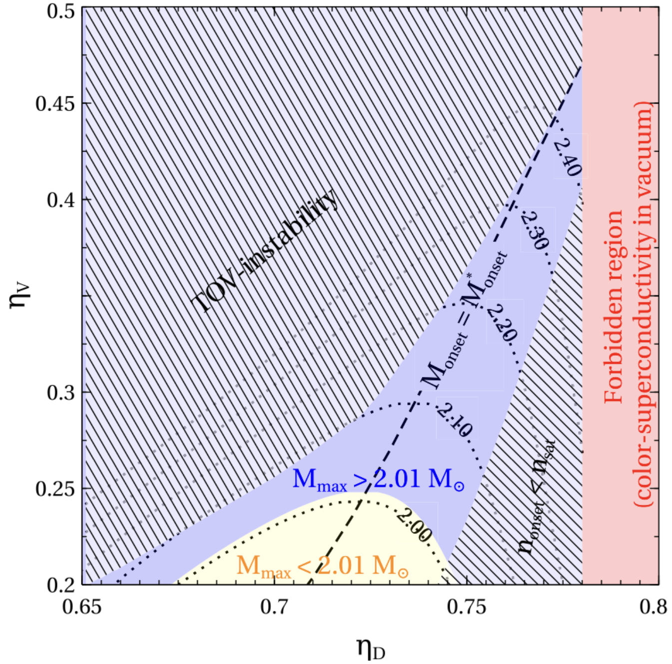
$$A_4 = a_1 + b_1 \eta_V + c_1 \eta_V^2 + \left(d_1 + \frac{e_1}{\eta_V}\right) \eta_D,$$

$$\Delta = (a_2 + b_2 \eta_V + c_2 \eta_V^2) \sqrt{d_2 + e_2 \eta_V + \eta_D},$$

$$B = a_3 + b_3 \eta_V + c_3 \eta_V^2 + d_3 \eta_D + e_3 \eta_D^2.$$

i	Units	a_i	b_i	c_i	d_i	e_i
1		0.757	−1.955	1.799	−0.063	0.046
2	MeV	300.7	8.534	−308.2	−0.235	1.458
3	MeV/fm³	72.018	170.8	−241.0	512.7	−626.6





Gärtlein et al. PRD108, 114028 (2023)

A_4

η_V	η_D = 0.4	0.5	0.6	0.7	0.8
0.1	0.738	0.781	0.819	0.857	0.898
0.2	0.501	0.521	0.540	0.555	0.571
0.3	0.369	0.379	0.389	0.394	0.403
0.4	0.281	0.289	0.296	0.299	0.303

Δ (MeV)

η_V	η_D = 0.4	0.5	0.6	0.7	0.8
0.1	162.6	185.4	209.5	231.0	248.0
0.2	201.9	220.7	238.9	256.9	273.0
0.3	214.0	230.4	245.7	262.2	275.1
0.4	219.1	233.0	246.1	260.1	271.7

B (MeV/fm³)

η_V	η_D = 0.4	0.5	0.6	0.7	0.8
0.1	192.5	185.3	168.2	138.8	95.0
0.2	204.0	196.9	178.6	148.9	105.4
0.3	208.1	200.8	181.9	152.4	108.6
0.4	210.1	202.5	183.1	153.1	108.8

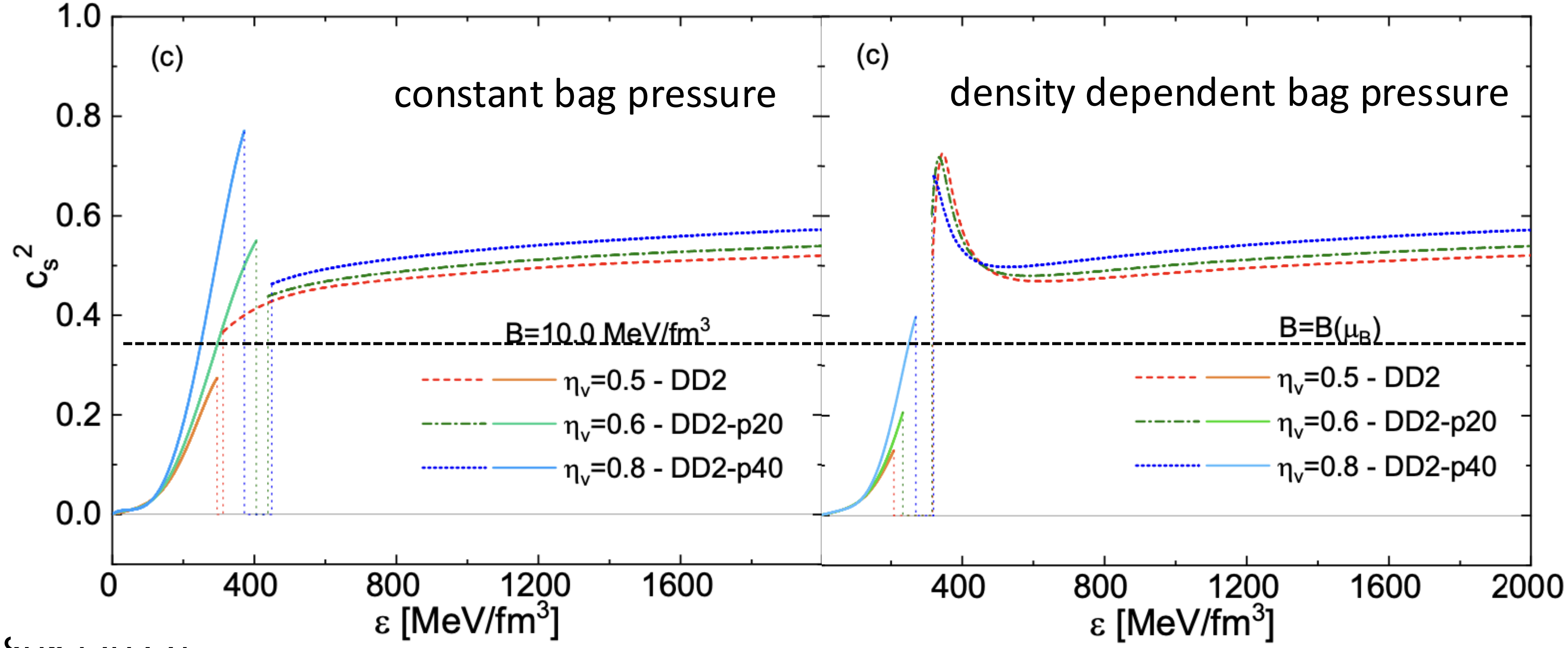
IMPACT OF A BAG CONSTANT

$$j_S^f(x) = \int d^4z \; g(z) \; \bar{\psi}(x + \frac{z}{2}) \; \Gamma_f \; \psi(x - \frac{z}{2}) \, ,$$

$$j_D^a(x) = \int d^4z \; g(z) \; \bar{\psi}_C(x + \frac{z}{2}) \; i\gamma_5 \tau_2 \lambda_a \; \psi(x - \frac{z}{2}) \, ,$$

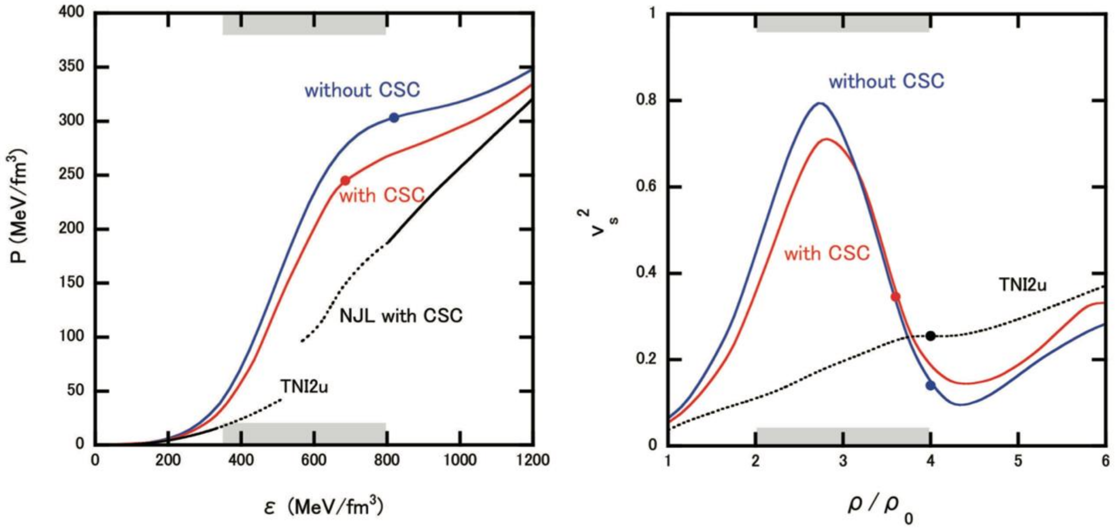
$$j_V^\mu(x) = \bar{\psi}(x) \; i\gamma^\mu \; \psi(x)$$

$$g(\vec{p}) = \exp[-\vec{p}^{\,2}/\Lambda_0^2] \; .$$

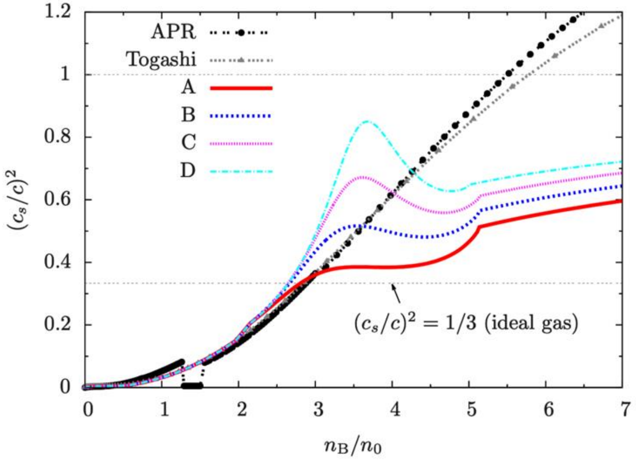


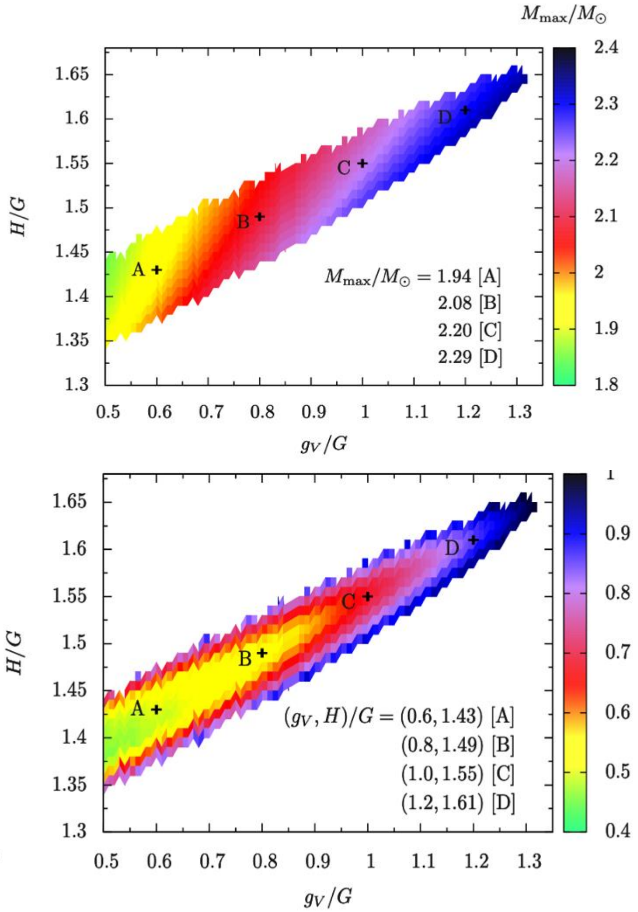
Contrera et al. PRC105, 045808 (2022)

Speed of Sound in Hadron-Quark Crossover



Masuda, Hatsuda & Takatsuka, EPJA52, 65 (2016)

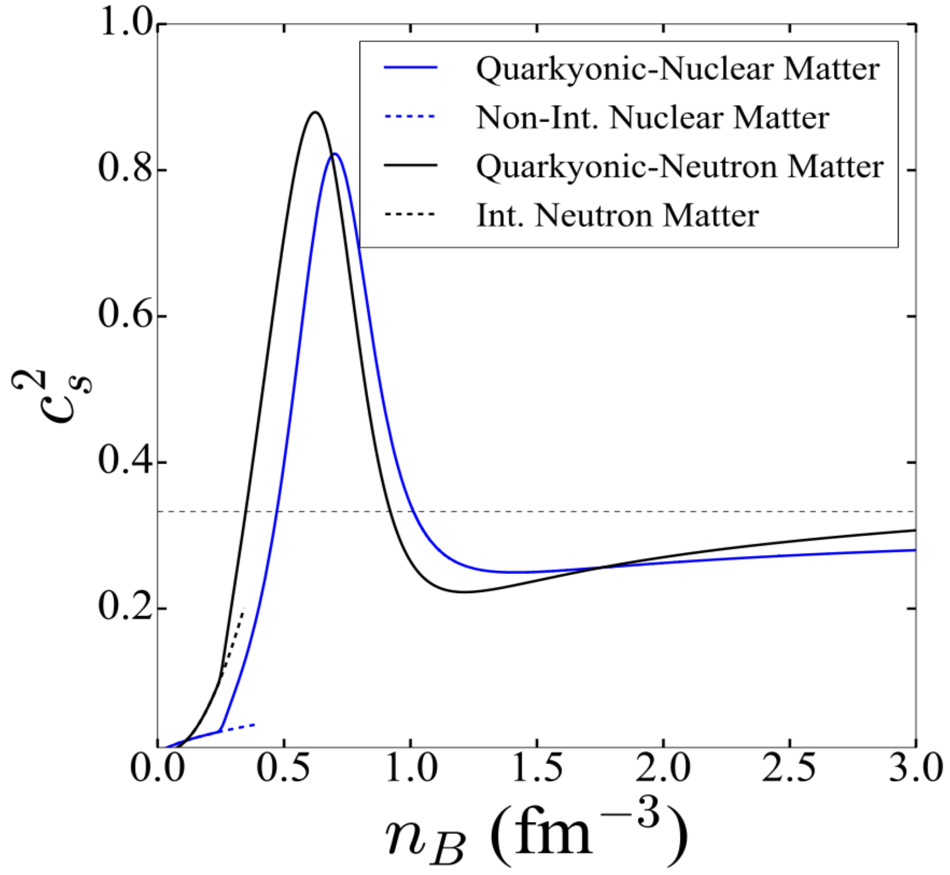


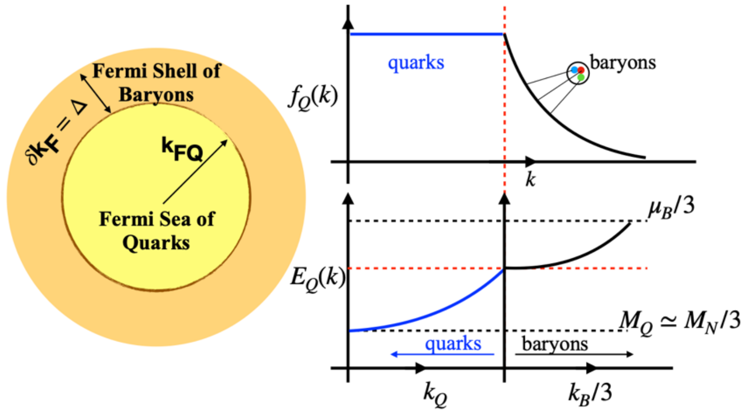


Parameter sets A-D: (gV , H)/G = (0.6, 1.43) (set A), (0.8, 1.49) (set B), (1.0, 1.55) (set C) and (1.2, 1.61) (set D)

Baym et al., ApJ885, 42 (2019)

Speed of Sound in Quarkyonic Matter





$$n_B = \frac{2}{3\pi^2}\left(k_{\rm FB}^3 - (k_{\rm FB}-\Delta)^3 + k_{\rm FQ}^3\right)$$

$$k_{\rm FQ} = \frac{(k_{\rm FB}-\Delta)}{N_c}\,\Theta(k_{\rm FB}-\Delta)$$

$$\epsilon(n_B) = 4\int_{N_c k_{\rm FQ}}^{k_{\rm FB}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_n^2} + 2\times N_c\int_0^{k_{\rm FQ}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+M_q^2}$$

McLerran & Reddy, PRL122, 122701 (2019)

CONCLUSIONS

- Bayesian inference based on multi-messenger astronomy data supports a high peak in the speed of sound ($\max(c_s^2) > 0.45$) at ε=350-600 MeV/fm^3
- Conclusion that high sound speeds ($c_s^2 > 1/3$) indicates hadronic phase is made without considering CSC matter
- Agnostic approaches clearly cannot account for CSC matter
- The phase transition before the peak can be provided by a bag constant dependent on density

CONCLUSIONS

- The high peak in the speed of sound may be related to the presence of quark matter in the form of a mixed phase, taking into account the density-dependent bag constant, crossover, or a quarkyonic form
- CSC matter can be well approximated by constant speed of sound model
- Probably peak in c_S^2 given by agnostic approach with use of multi-messenger astronomy indicates existence quark matter in core of neutron stars

THANKS