Framework for phase transitions between the Maxwell and Gibbs constructions

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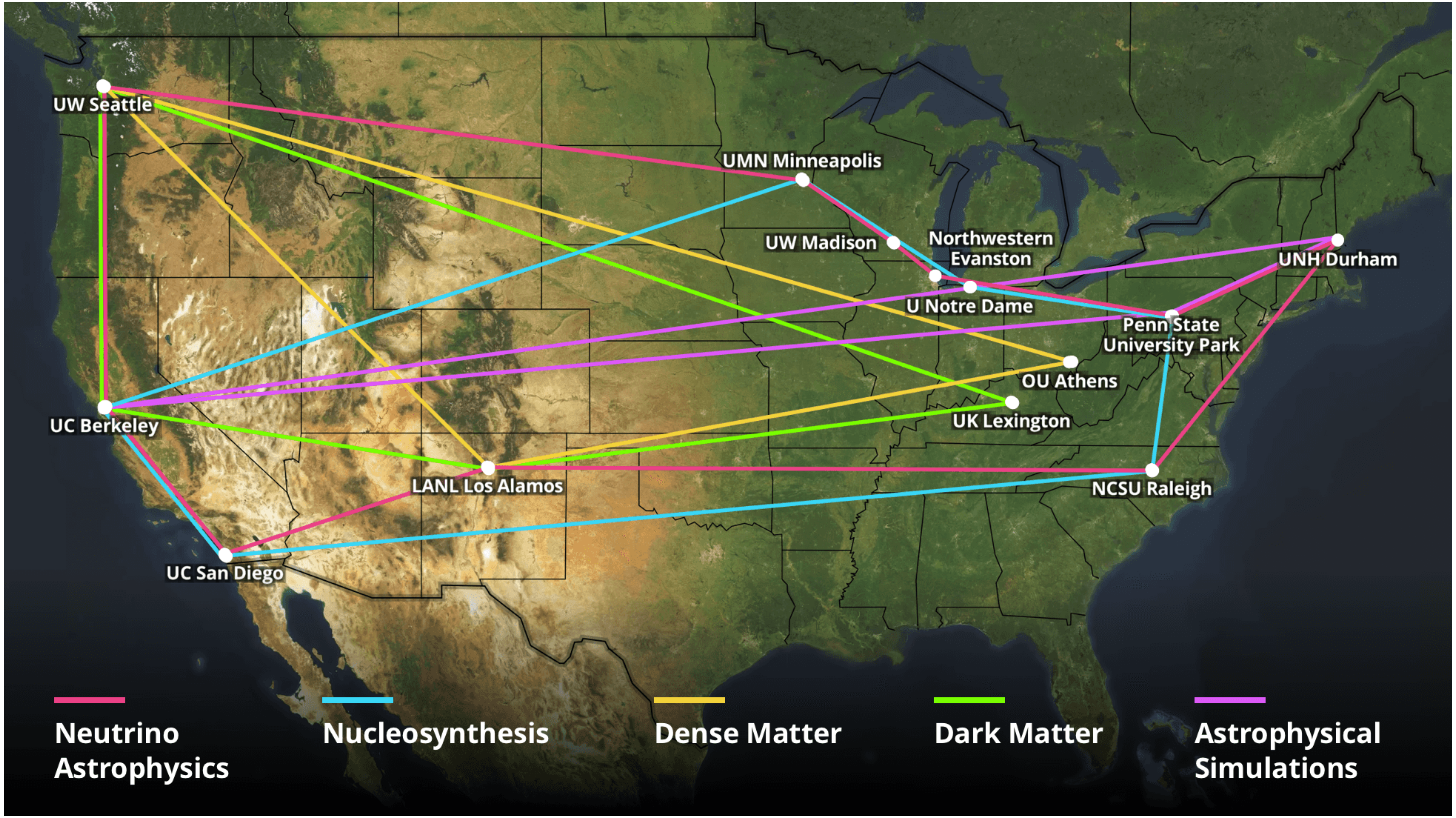
Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

N3AS



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Neutron Star

The densest static environment

The environment around Proto-NS, stellar accretion and X-ray bursts

Hadronic EOS

- Uniform nuclear matter

$$\varepsilon(n_p, n_n) \propto n_p^{5/3} + n_n^{5/3} + \dots$$

$$p(n_p, n_n) = n_p\mu_p + n_n\mu_n - \varepsilon$$

n, p, e^-

$n_{n,p},\ P,\ \mu$

- Nucleon weak β-equilibrium

$$\bar{\nu} + p + e^- \leftarrow n \quad (+N)$$

$$(N+)\ p + e^- \rightarrow n + \nu$$

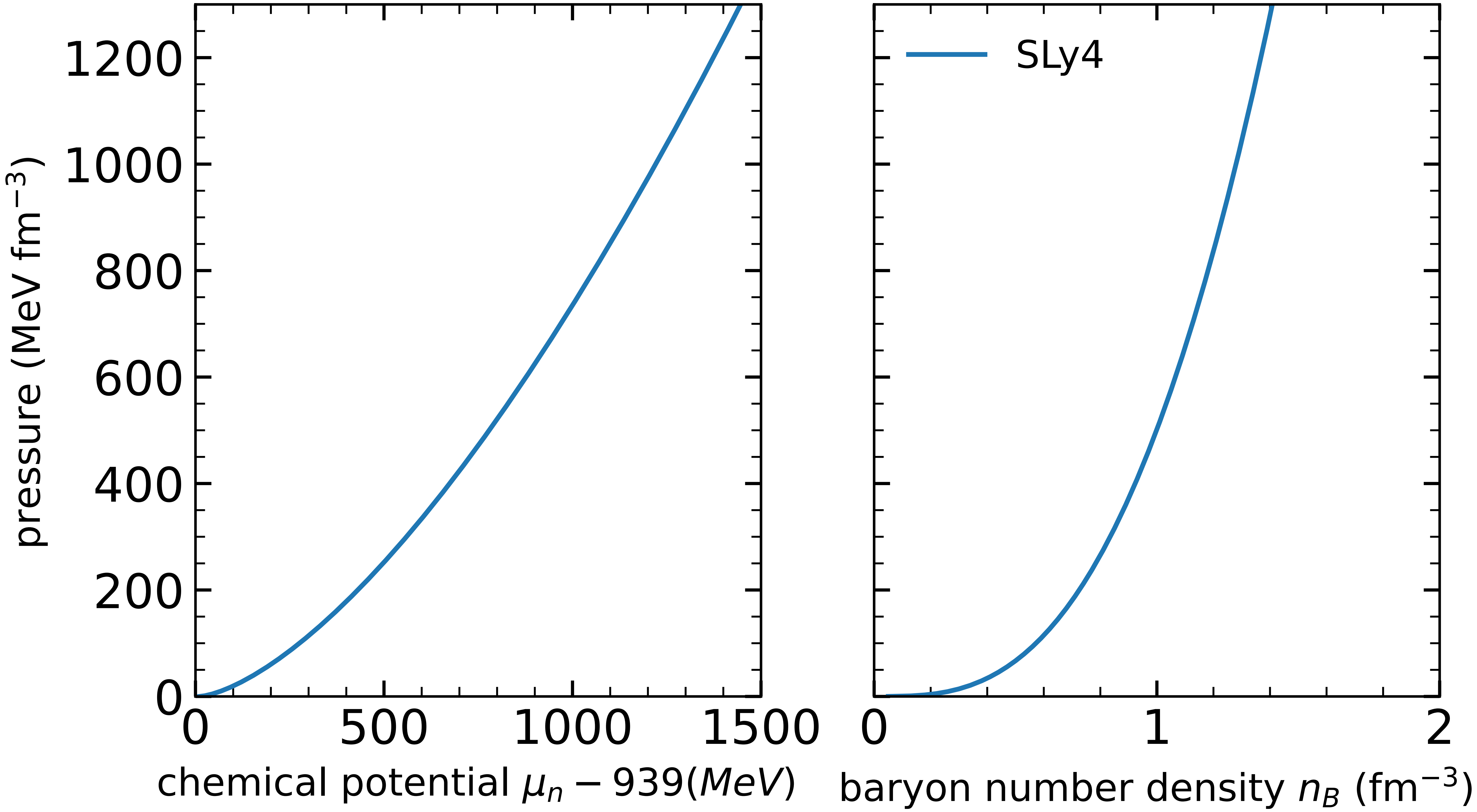
leads to $\mu_p + \mu_e = \mu_n$

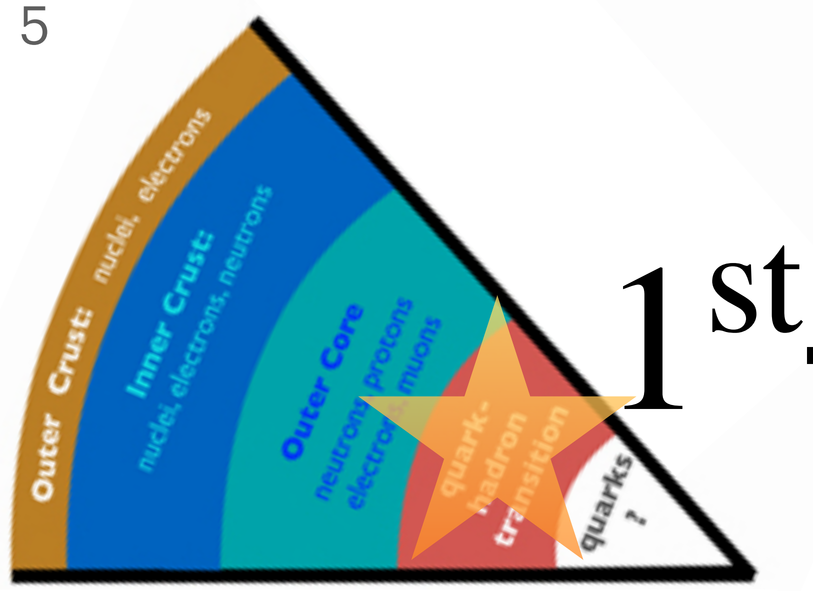
- Baryon number conservation:

$$n_p + n_n = n_B$$

- Local charge neutrality:

$$n_p + n_e = 0$$





1st-order Transition

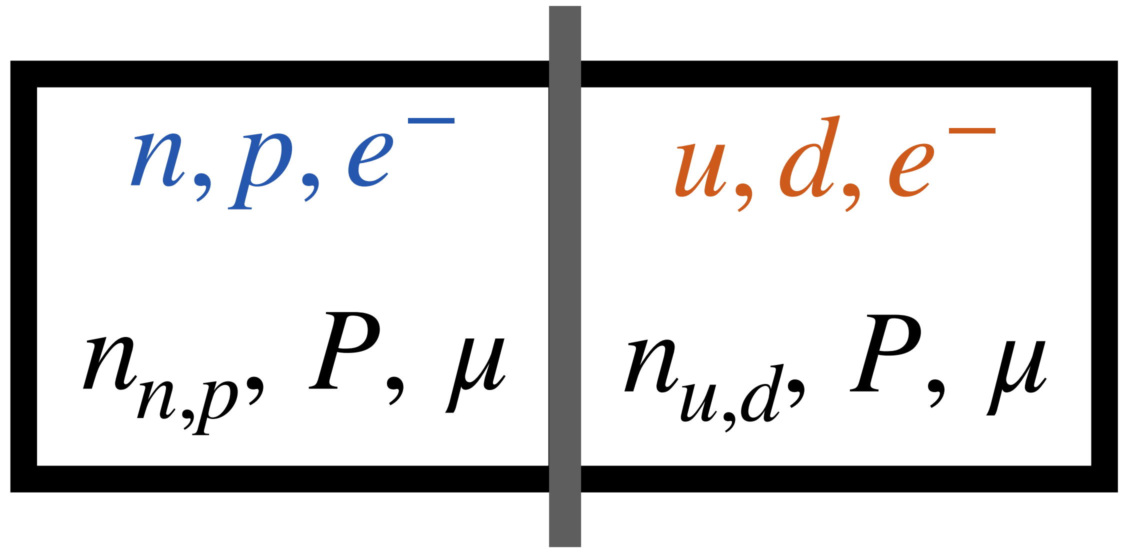
Maxwell construction

- Mechanical equilibrium

  $P_{npe} = P_{ude} = P$

- Strong equilibrium

  $\mu_n = \mu_u + 2\mu_d = \mu$



- (Modified) Urca process

  $\bar{\nu} + u + e^- \leftarrow d \quad (+N)$

  $(N+)\ u + e^- \rightarrow d + \nu$

  leads to $\mu_u + \mu_e = \mu_d = \mu_s$

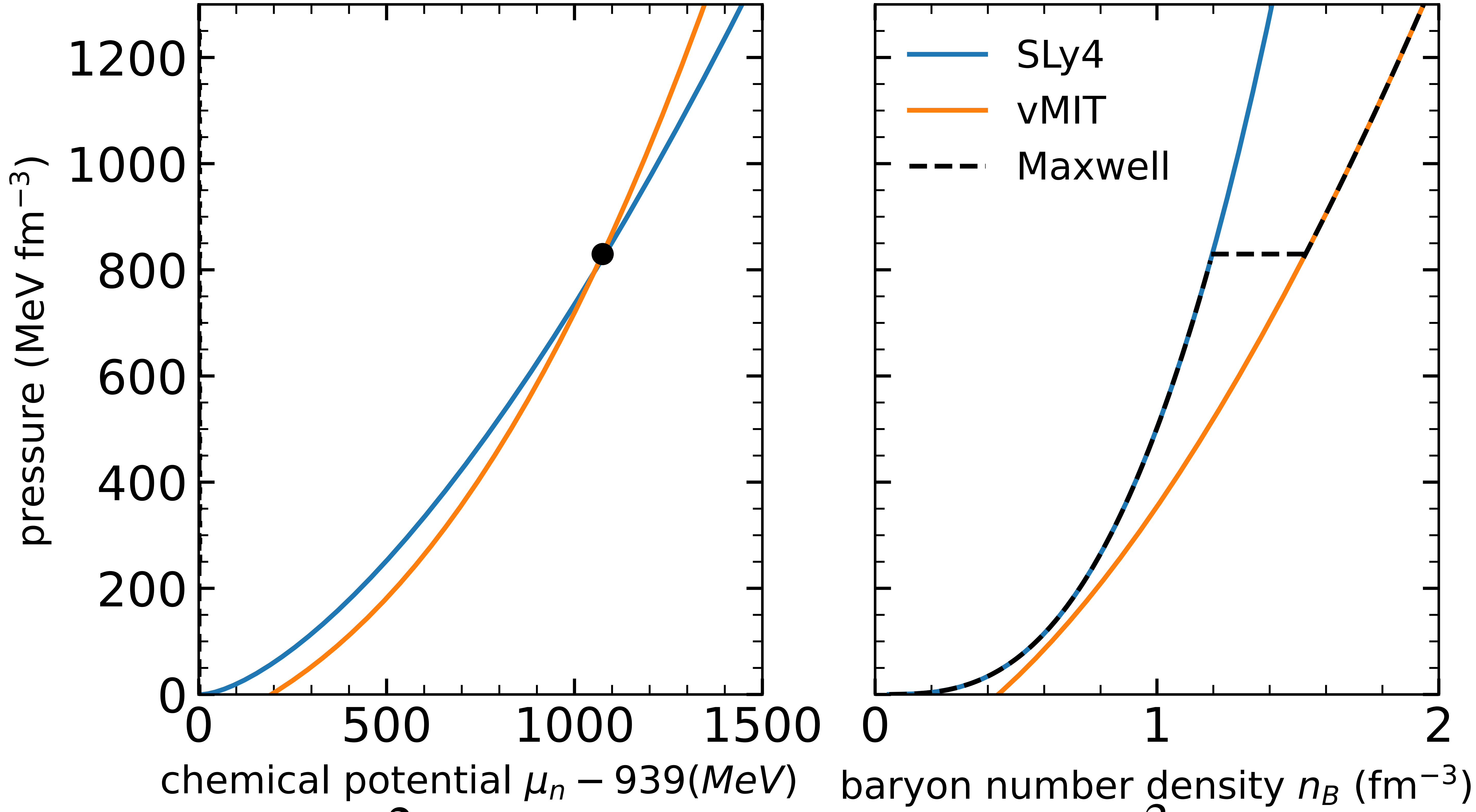
- Baryon number conservation:

  $n_u + n_d + n_s = n_Q = n_B/3$

- Local charge neutrality:

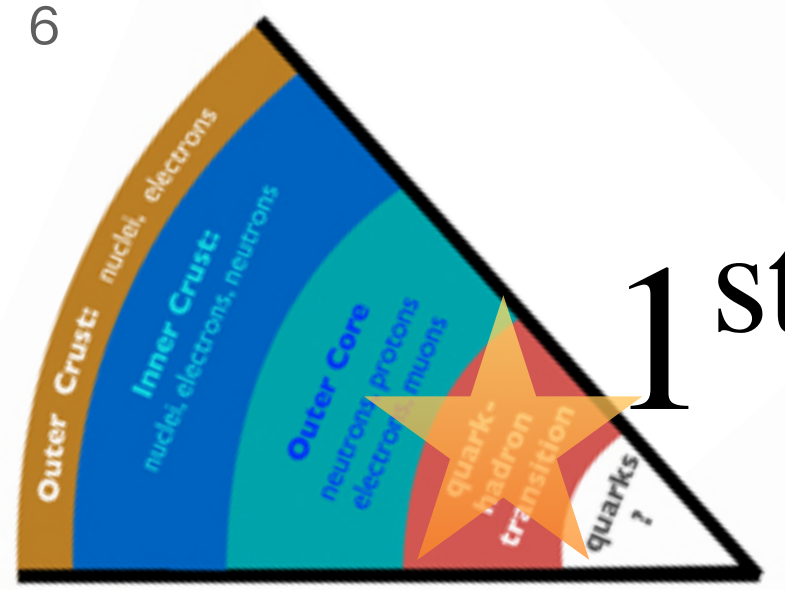
  $$n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$$

  $n_{e,N} = n_p$



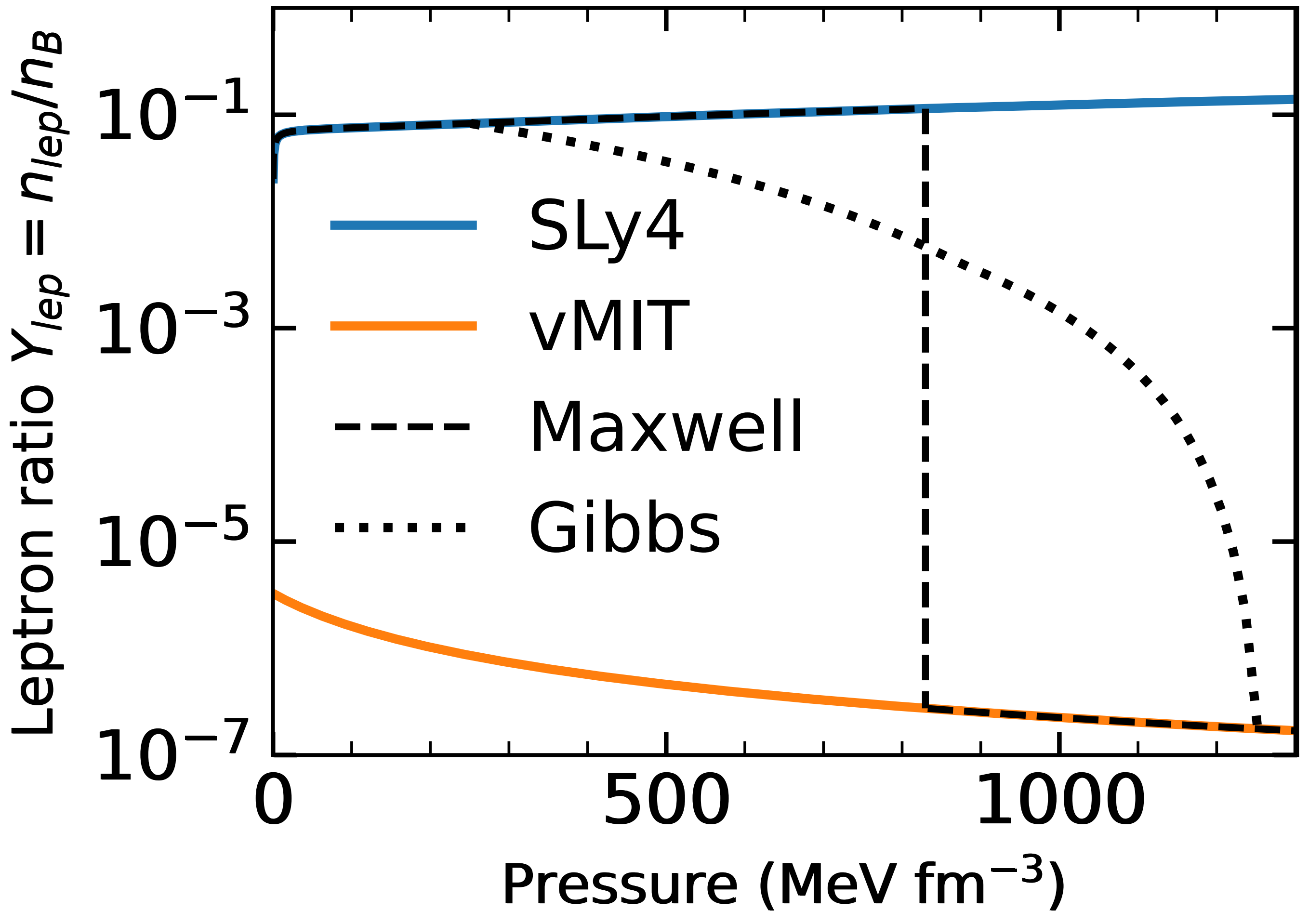
$\mu_u + 2\mu_d = \mu_n$

$3n_Q = n_B$

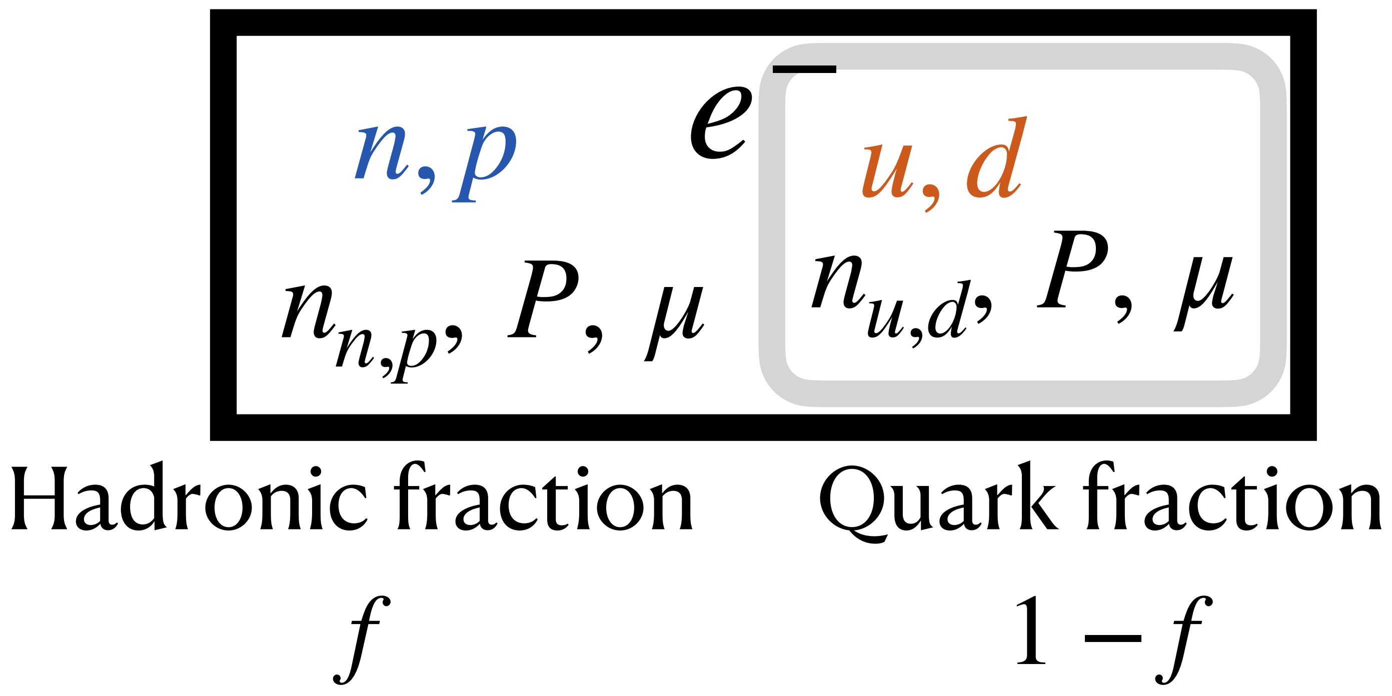


1st-order Transition

Maxwell or Gibbs construction



Gibbs construction



- Local charge neutrality (Maxwell):

$$n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$$

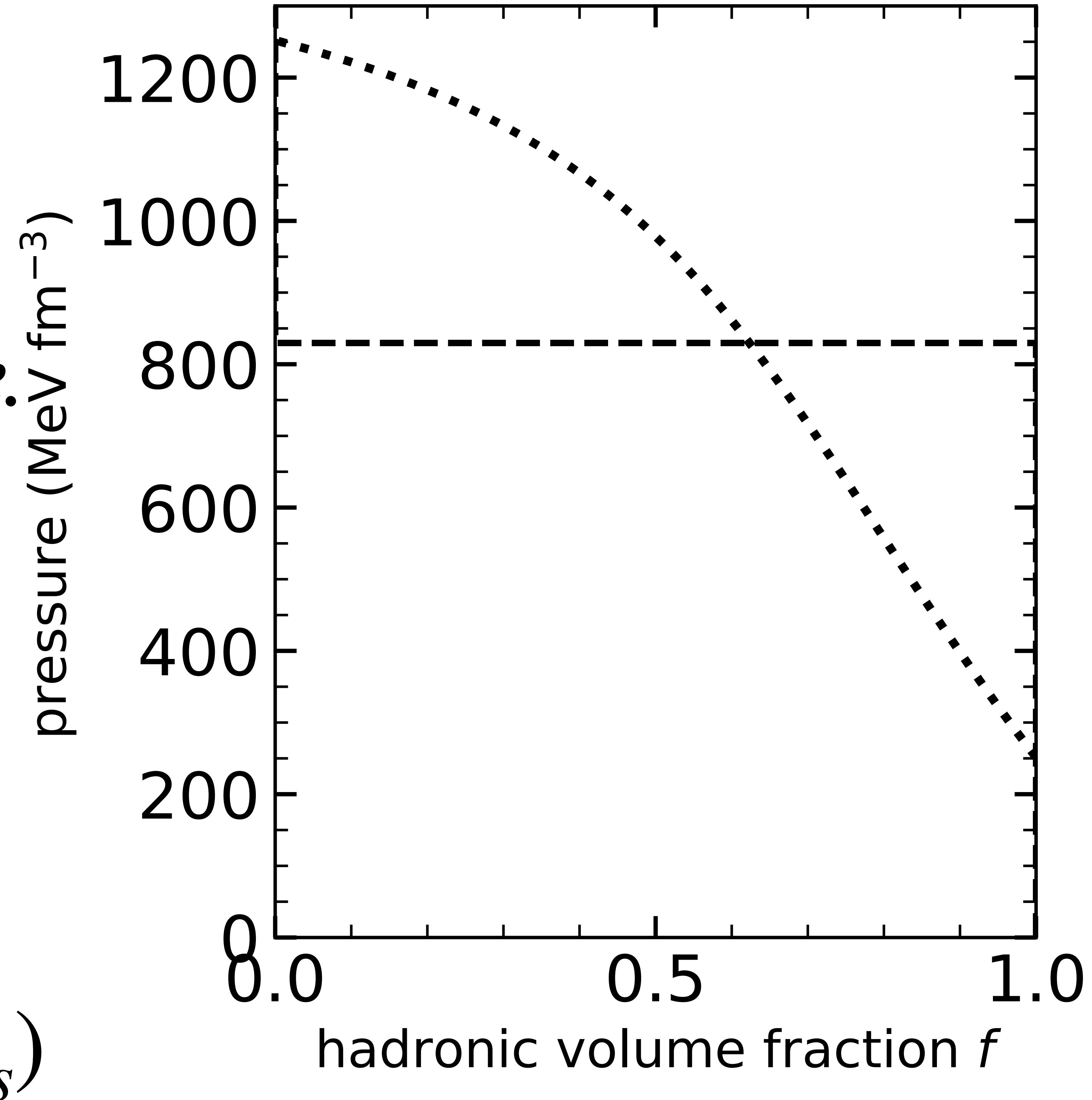
$$n_{e,N} = n_p$$

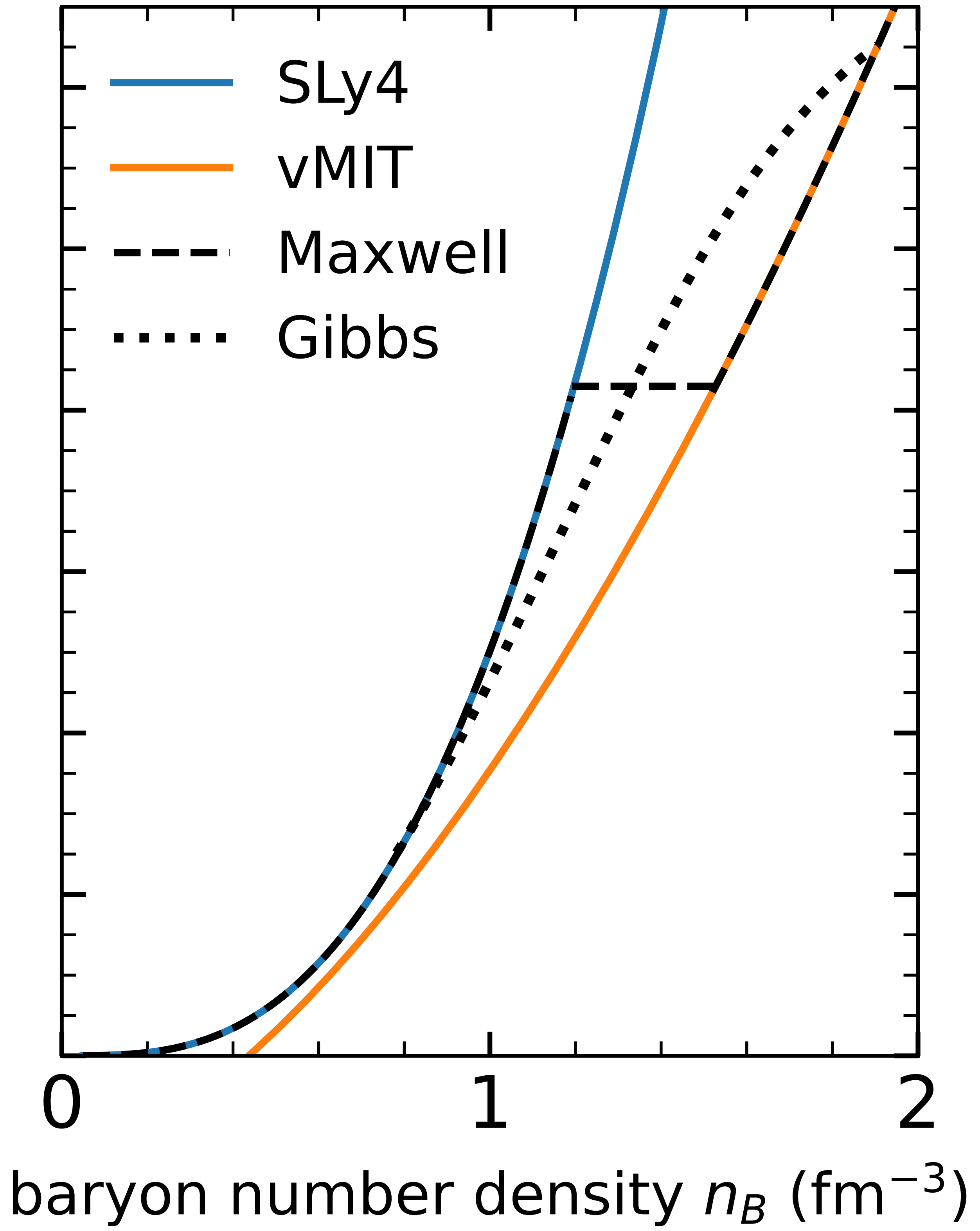
- *Leptons **aren't** balanced at the interface.*
- *Energy **isn't** minimized!*

- Global charge neutrality (Gibbs):

$$n_e = fn_{e,N} + (1-f)n_{e,Q}$$

$$n_B = f(n_p + n_n) + \frac{1-f}{3}(n_u + n_d + n_s)$$



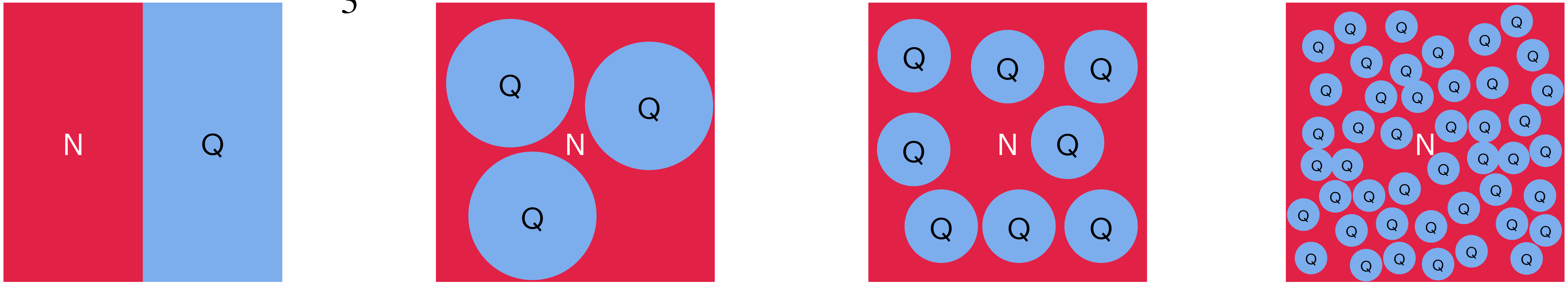


arXiv: 2302.04289

Problem of Gibbs Construction

- e.g. volume fraction $f = 0.5$:

$n_{e,N} = n_p \quad n_{e,Q} = \dfrac{2n_u - n_d - n_s}{3}$ $\qquad\qquad n_e = fn_{e,N} + (1-f)n_{e,Q}$

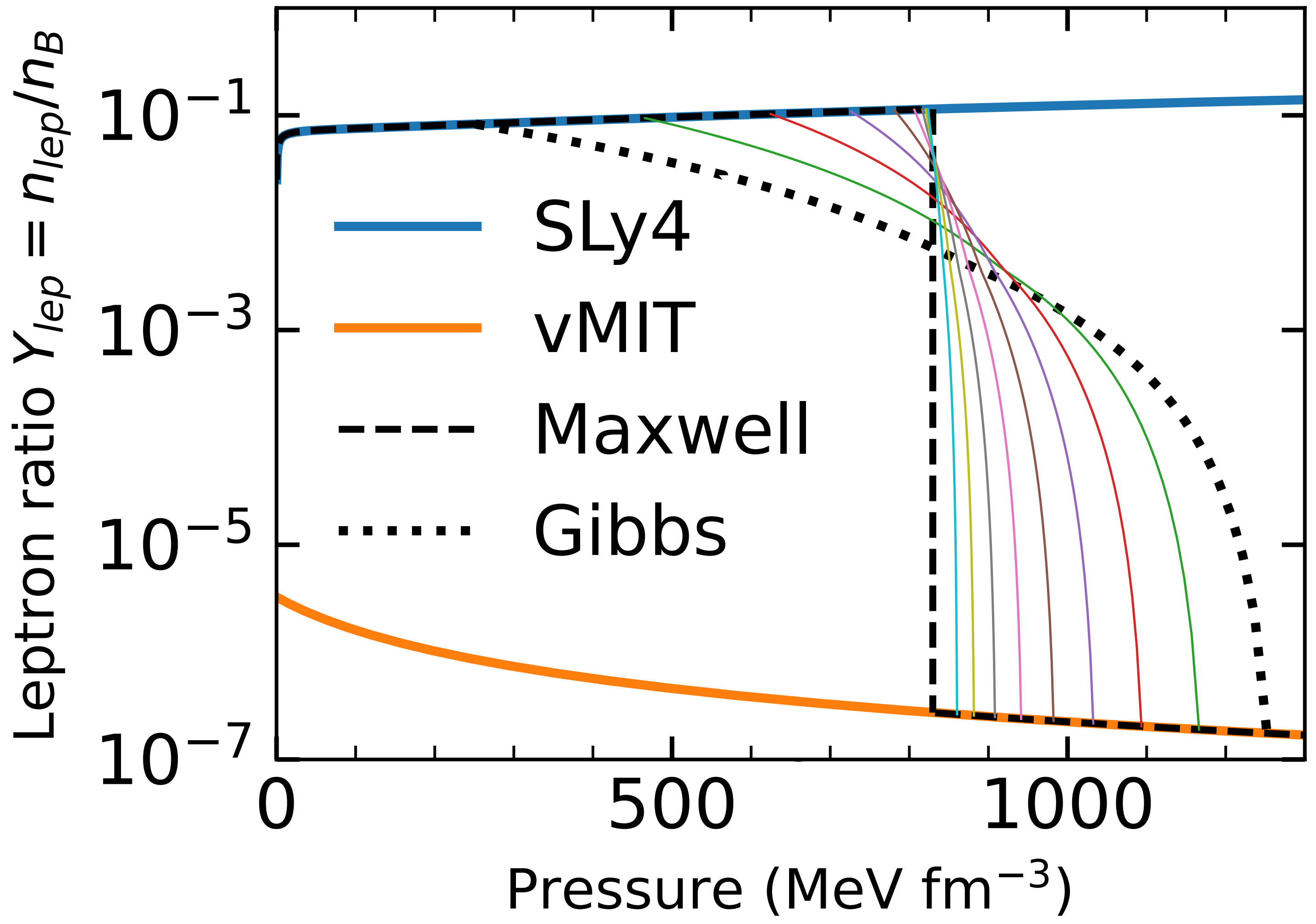


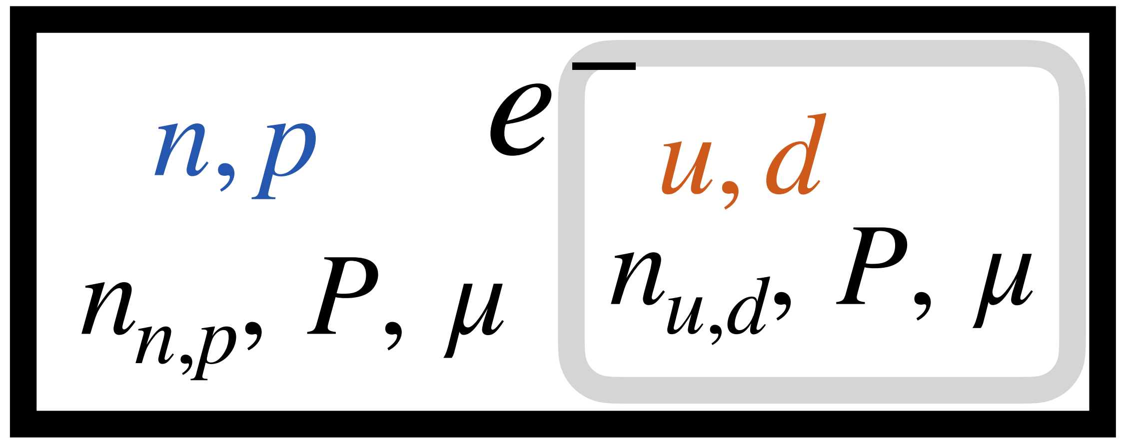
Surface energy increases $\longrightarrow$ $\qquad\qquad$ $\longleftarrow$ Coulomb energy increases

- Gibbs construction assumes infinite mixing leading to infinite boundary.
- Gibbs construction is realistic only when surface tension is negligibly small.
- The amount of boundary reflected on the charge neutrality condition.

Between Maxwell & Gibbs

Partially local & partially global





Global lepton fraction
$1 - g$

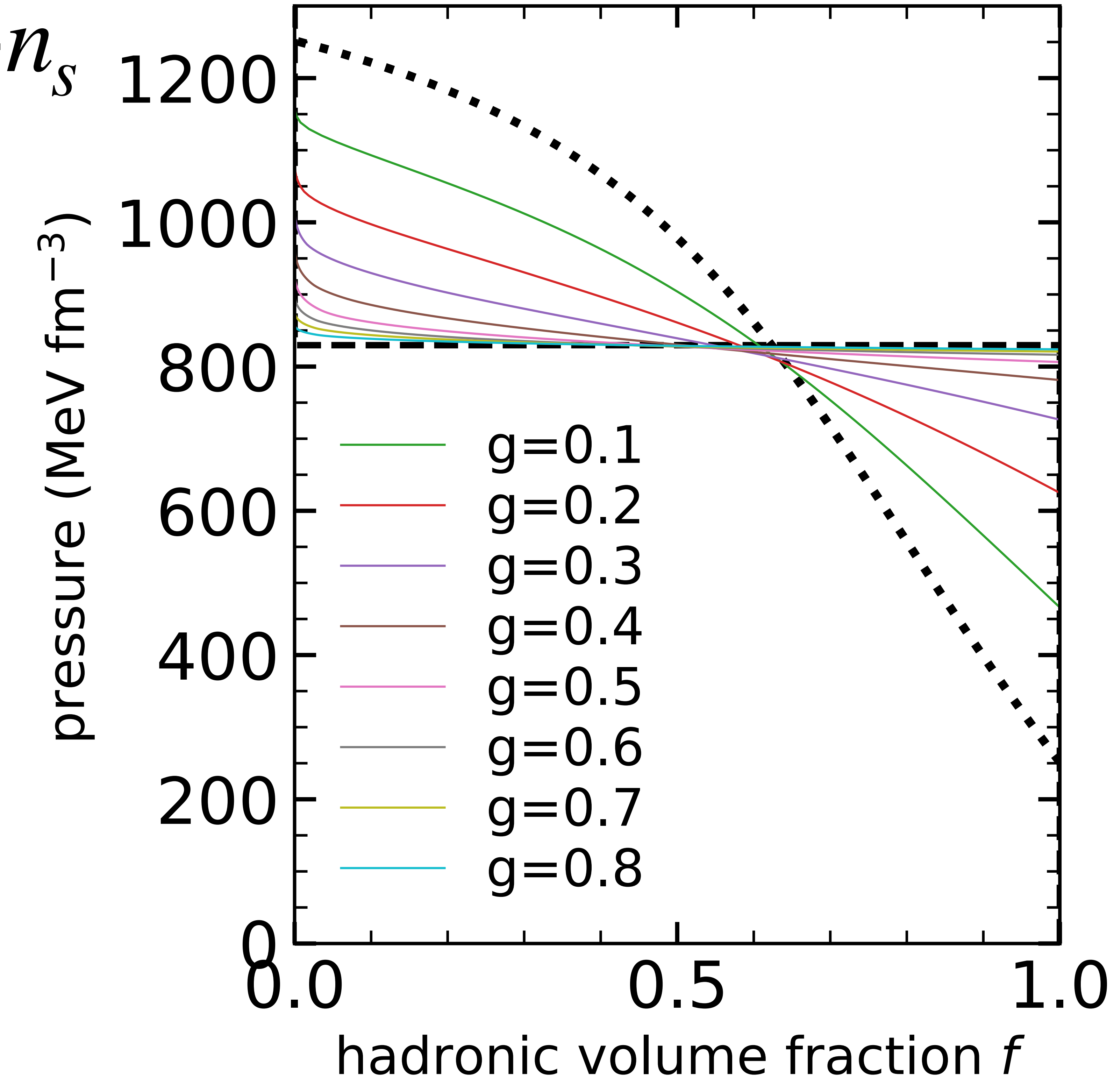
- Locally neutral lepton densities:

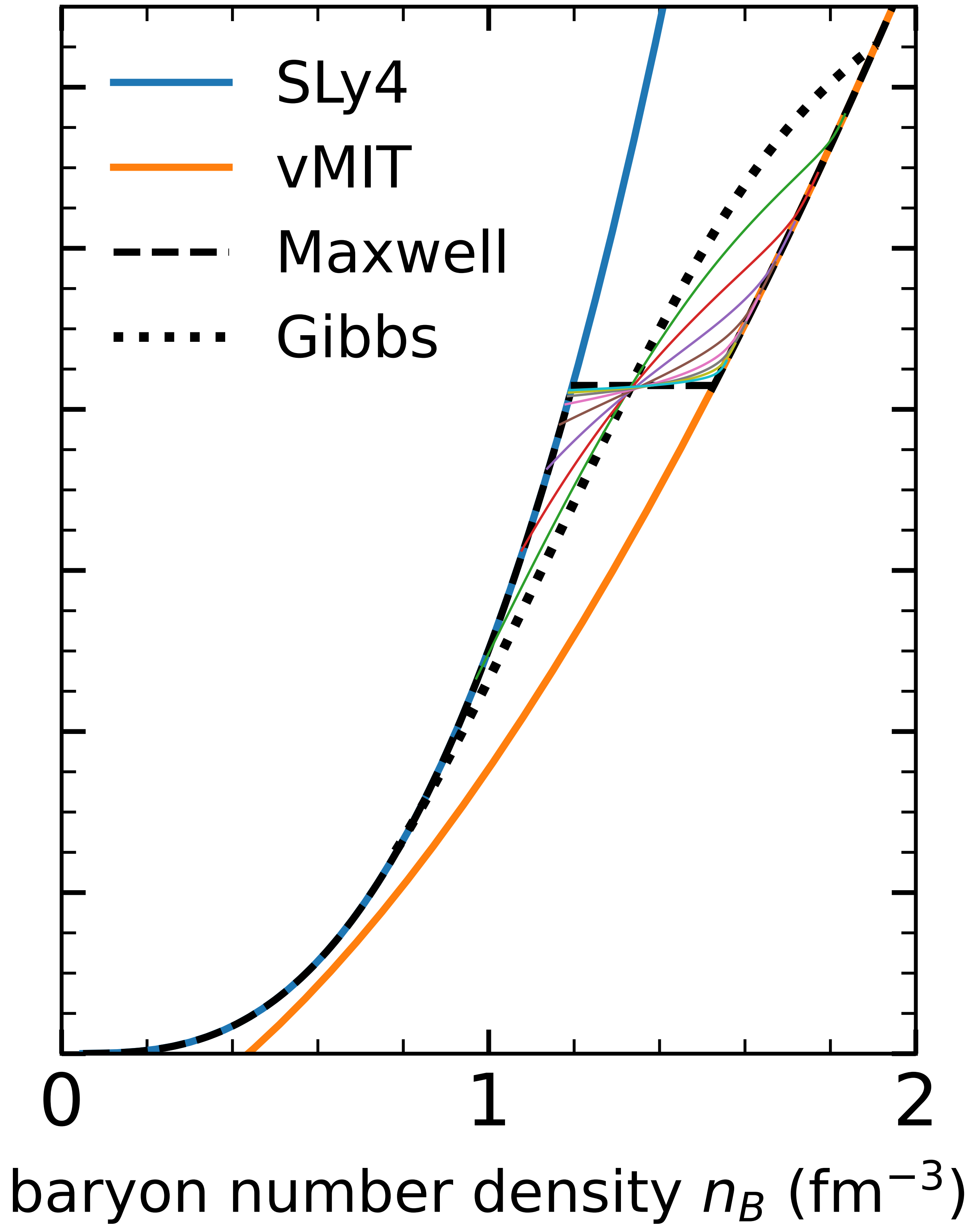
$$n_{e,N} = n_p,\ n_{e,Q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s$$

- Global lepton density, $n_{e,G}$
- Total lepton density:

$$n_e = g(fn_{e,N} + (1-f)n_{e,Q}) + (1-g)n_{e,G}$$

- $g = 0 \ \rightarrow$ Gibbs transition
  $g = 1 \ \rightarrow$ Maxwell transition
- g could be determined by Surface & Coulomb energy.





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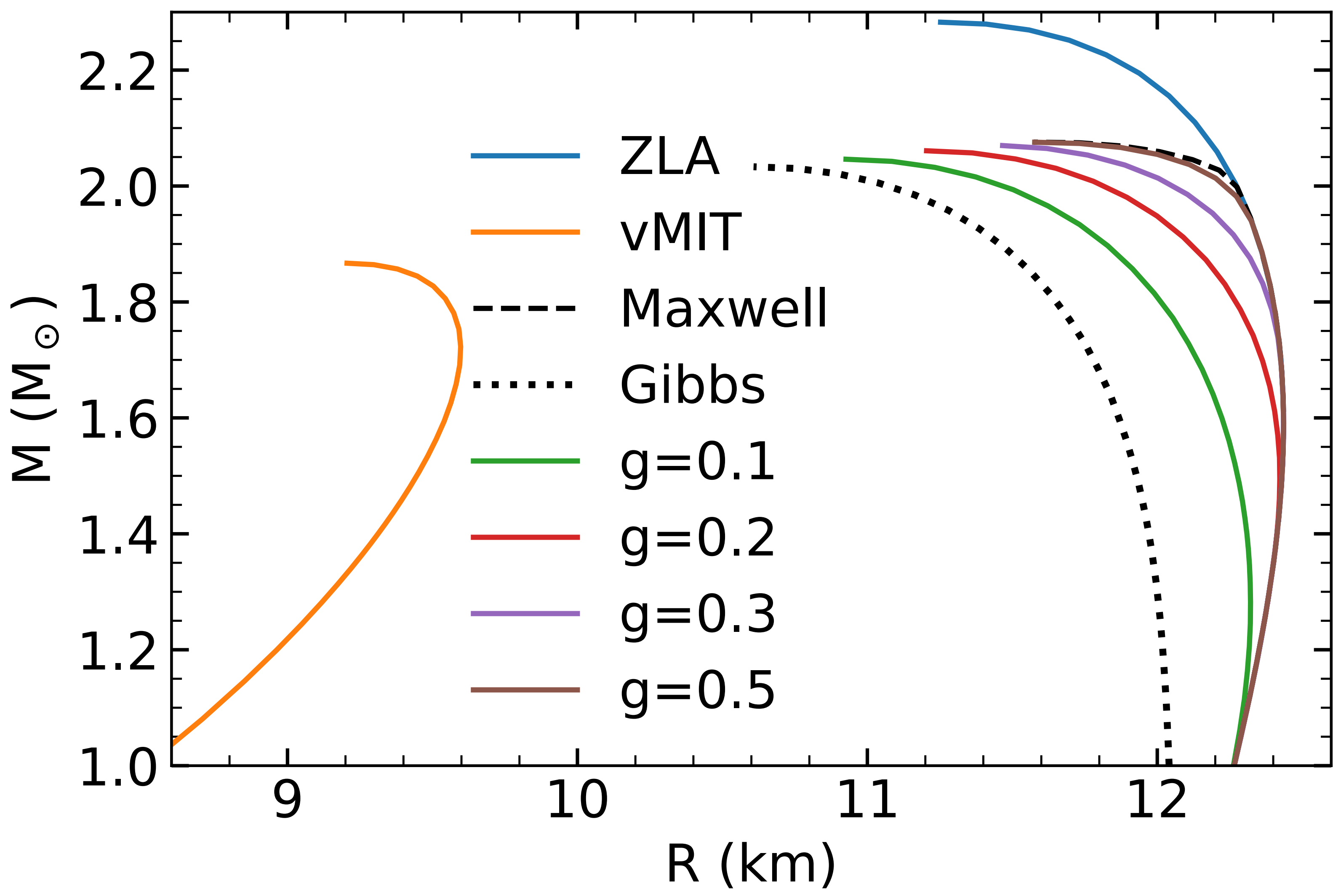
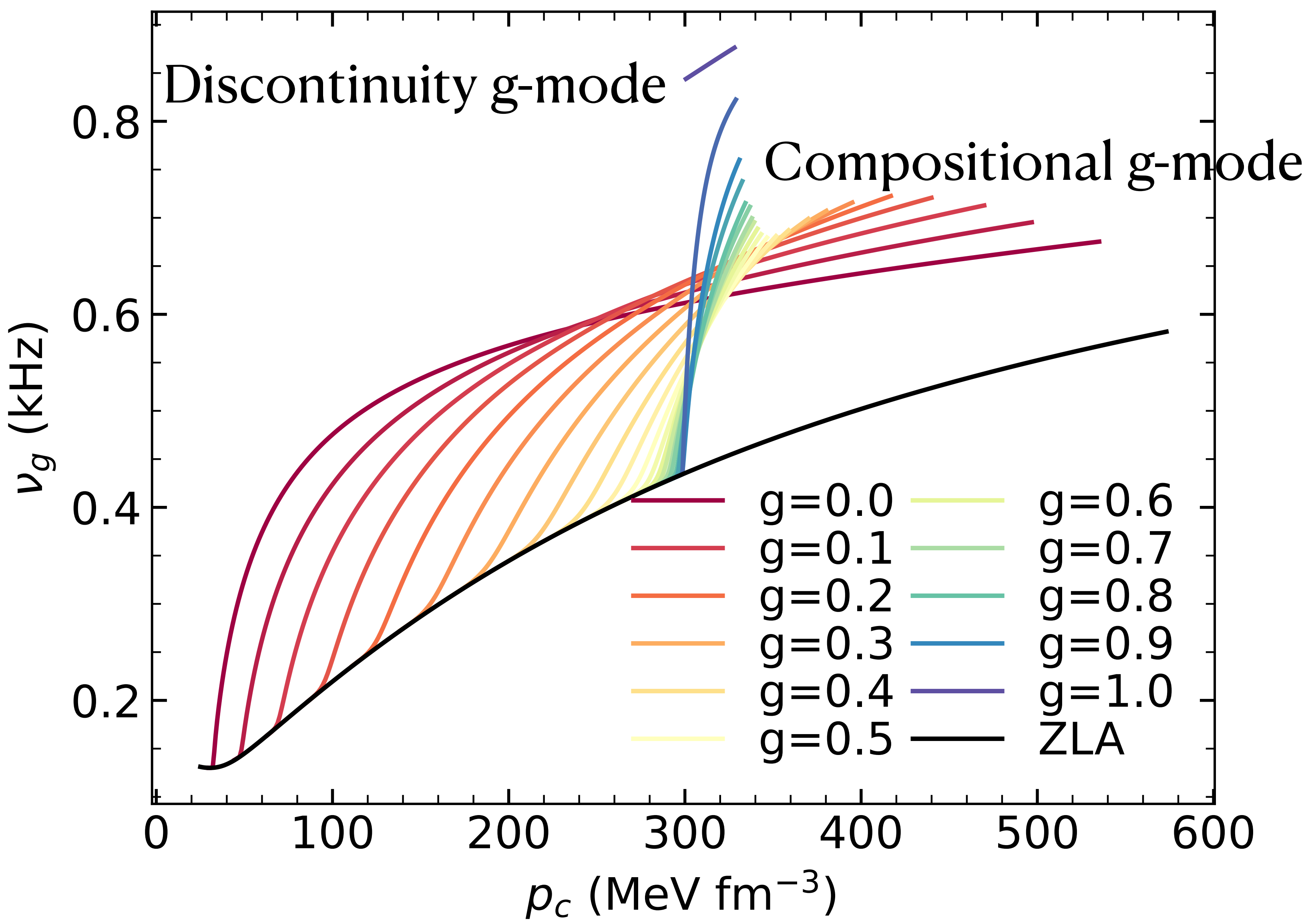
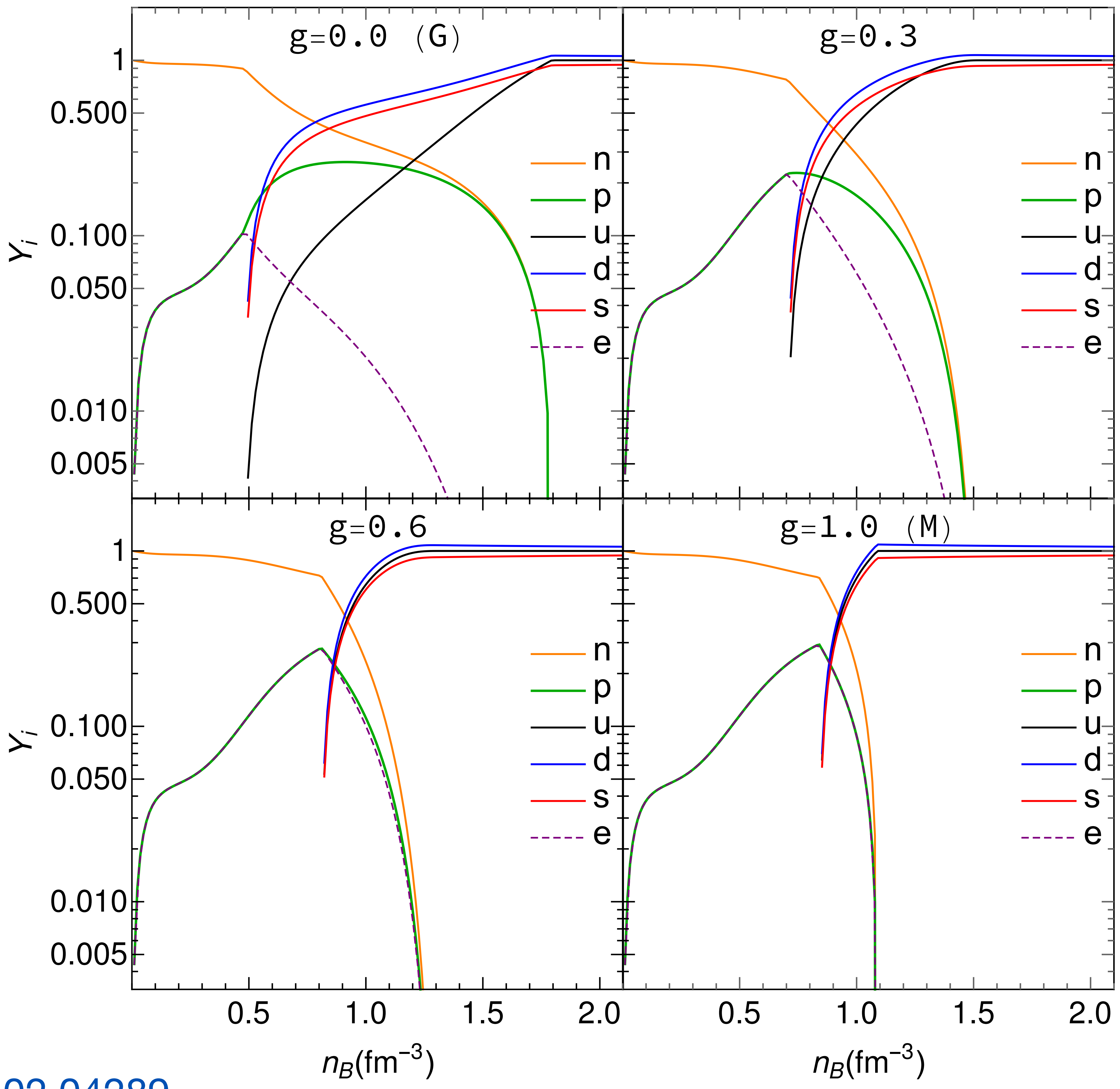
Between Maxwell & Gibbs







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Between Maxwell & Gibbs

Extend to finite temperature:

- Relativistic Fermi integrals, JEL polynomials.
- Introduce anti-particles as,

$\mu_{e^-} = -\mu_{e^+}$

$\mu_{\mu^-} = -\mu_{\mu^+}$

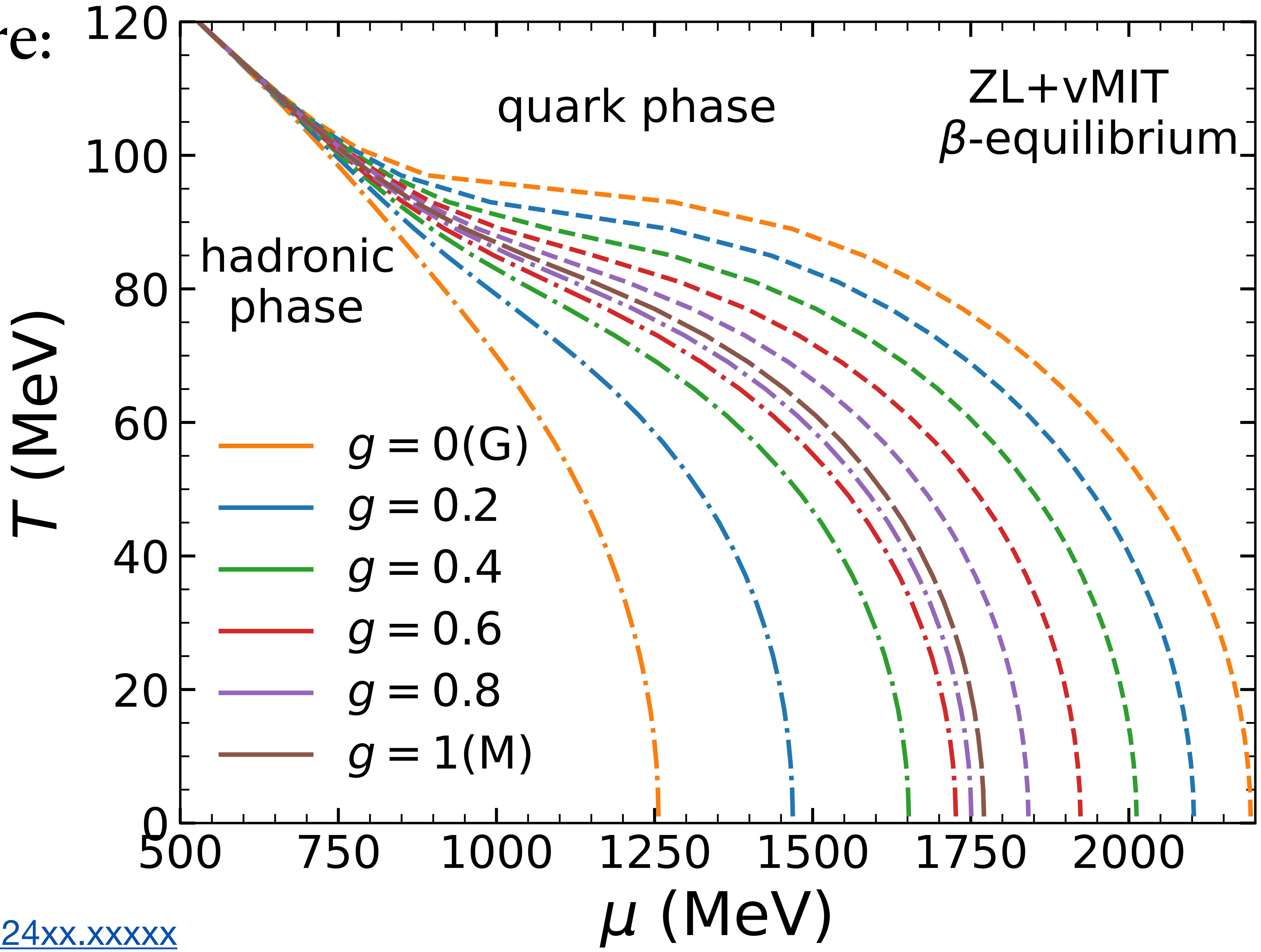
$\mu_{u^-} = -\mu_{u^+}$

$\mu_{d^-} = -\mu_{d^+}$

$\mu_{s^-} = -\mu_{s^+}$

- Add photon contribution,

$\varepsilon_{photon} \propto T^4$



arXiv: 24xx.xxxxx

Between Maxwell & Gibbs

Extend to off-β-equilibrium:

- Ignore β-equilibrium condition,

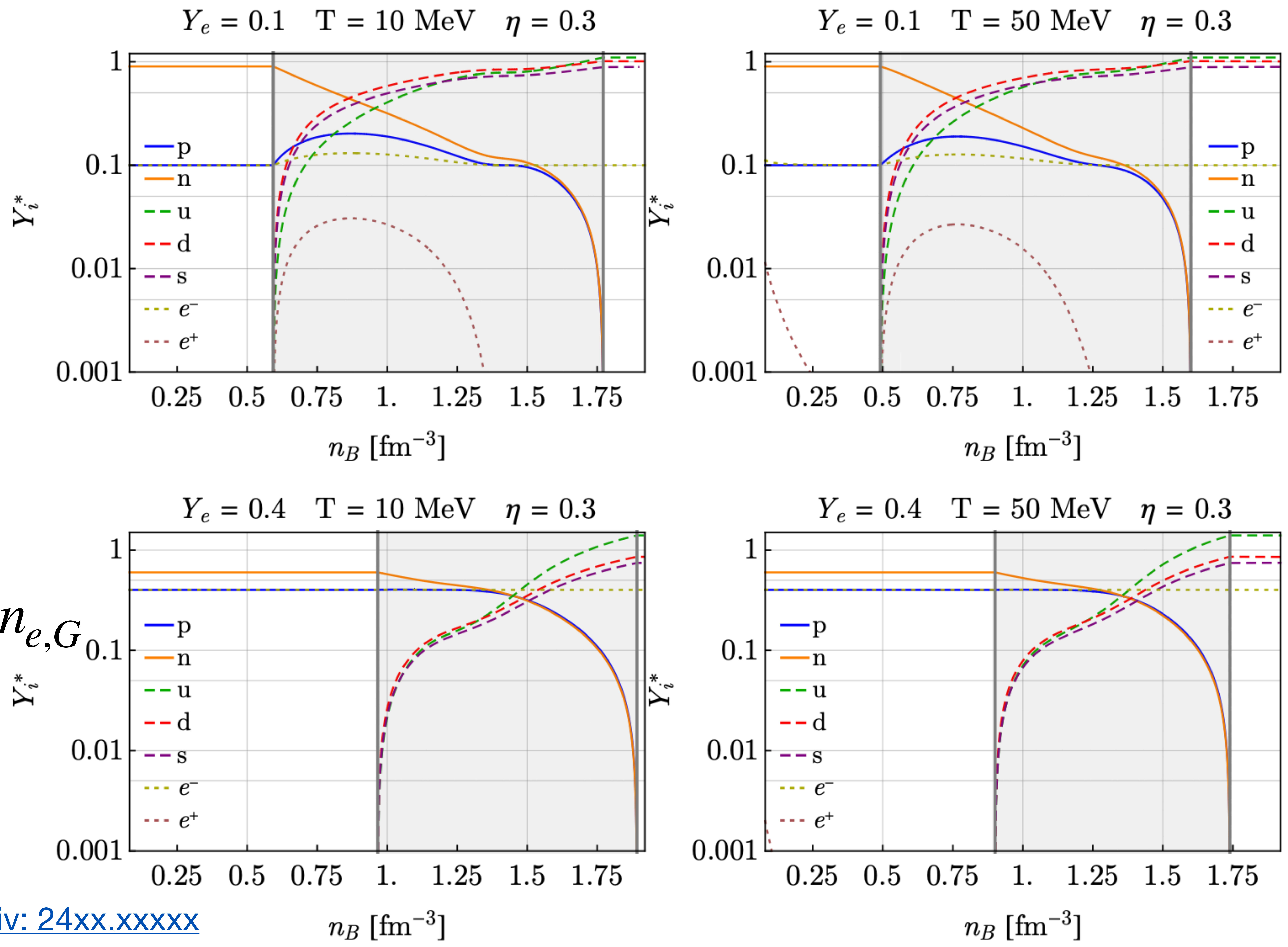
$$\mu_d = \mu_u + g\mu_{e,Q} + (1-g)\mu_{e,G}$$
$$\mu_n = \mu_p + g\mu_{e,N} + (1-g)\mu_{e,G}$$

- And replace it with,

$$n_e = n_B Y_e = g(f n_{e,N} + (1-f)n_{e,Q}) + (1-g)n_{e,G}$$

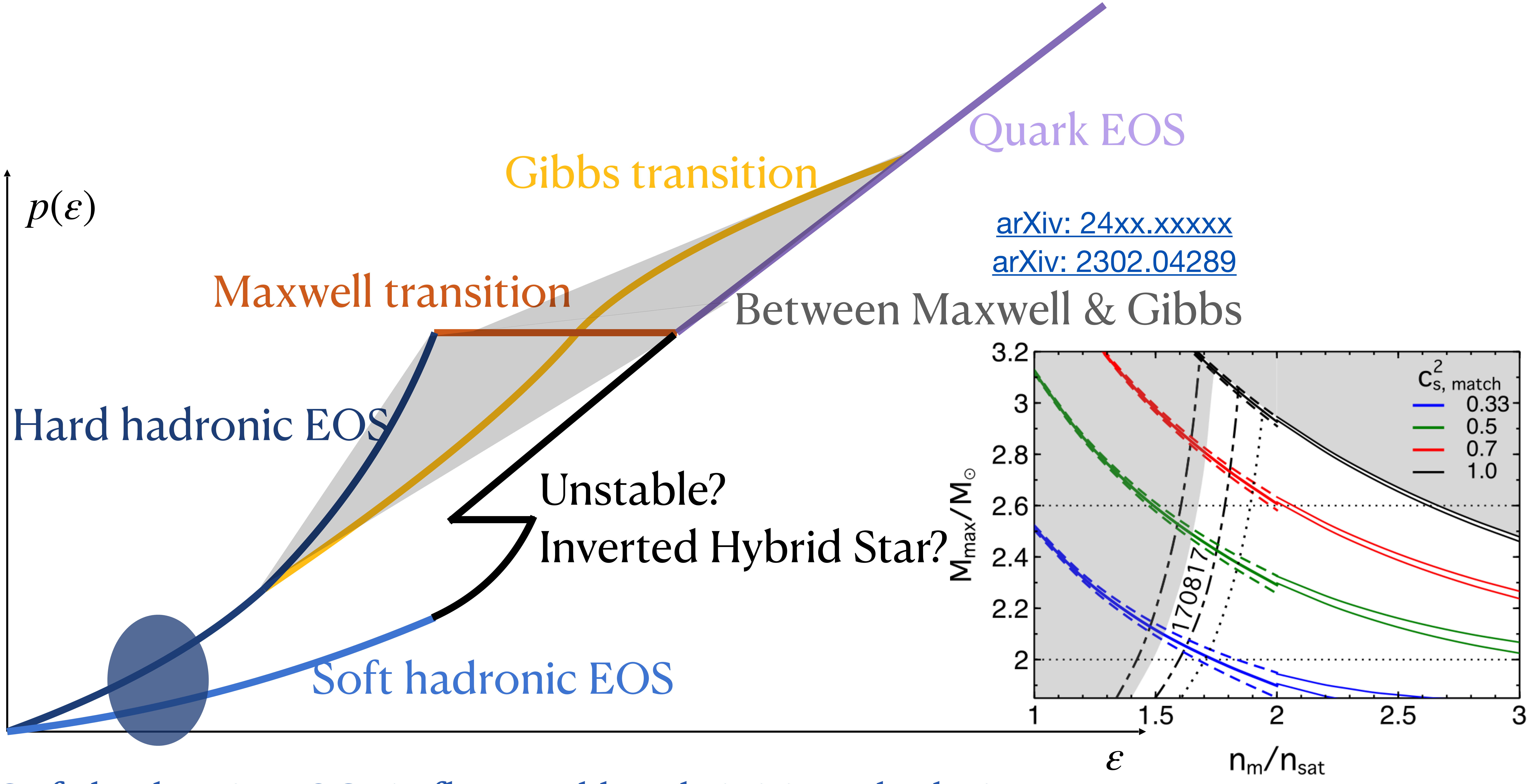
- The final EOS is,

$$\varepsilon = \varepsilon(n_B, Y_e, T)$$
$$p = p(n_B, Y_e, T)$$



arXiv: 24xx.xxxxx

Hadron-quark Transition in Neutron Star Core

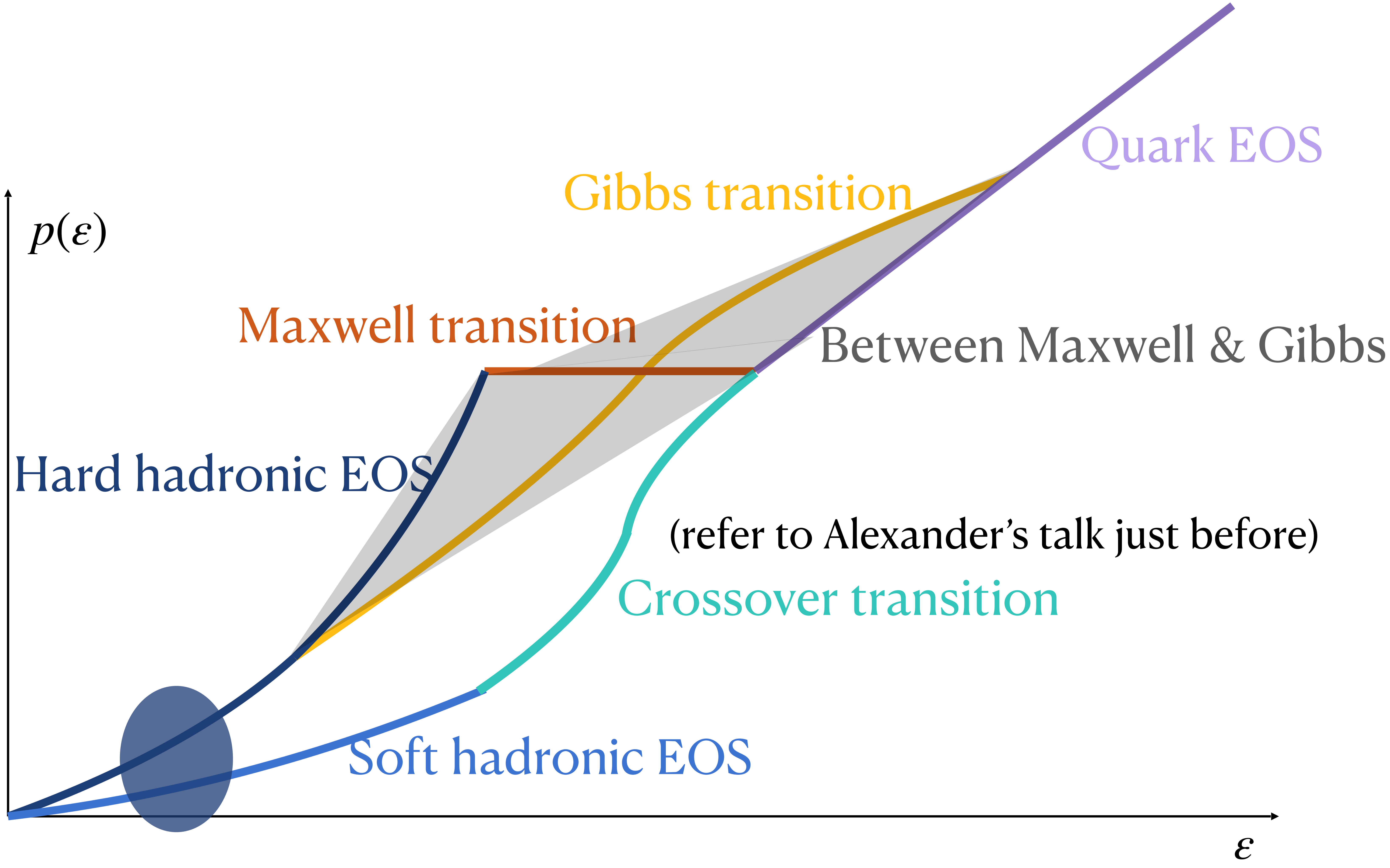


arXiv: 2009.06441

Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

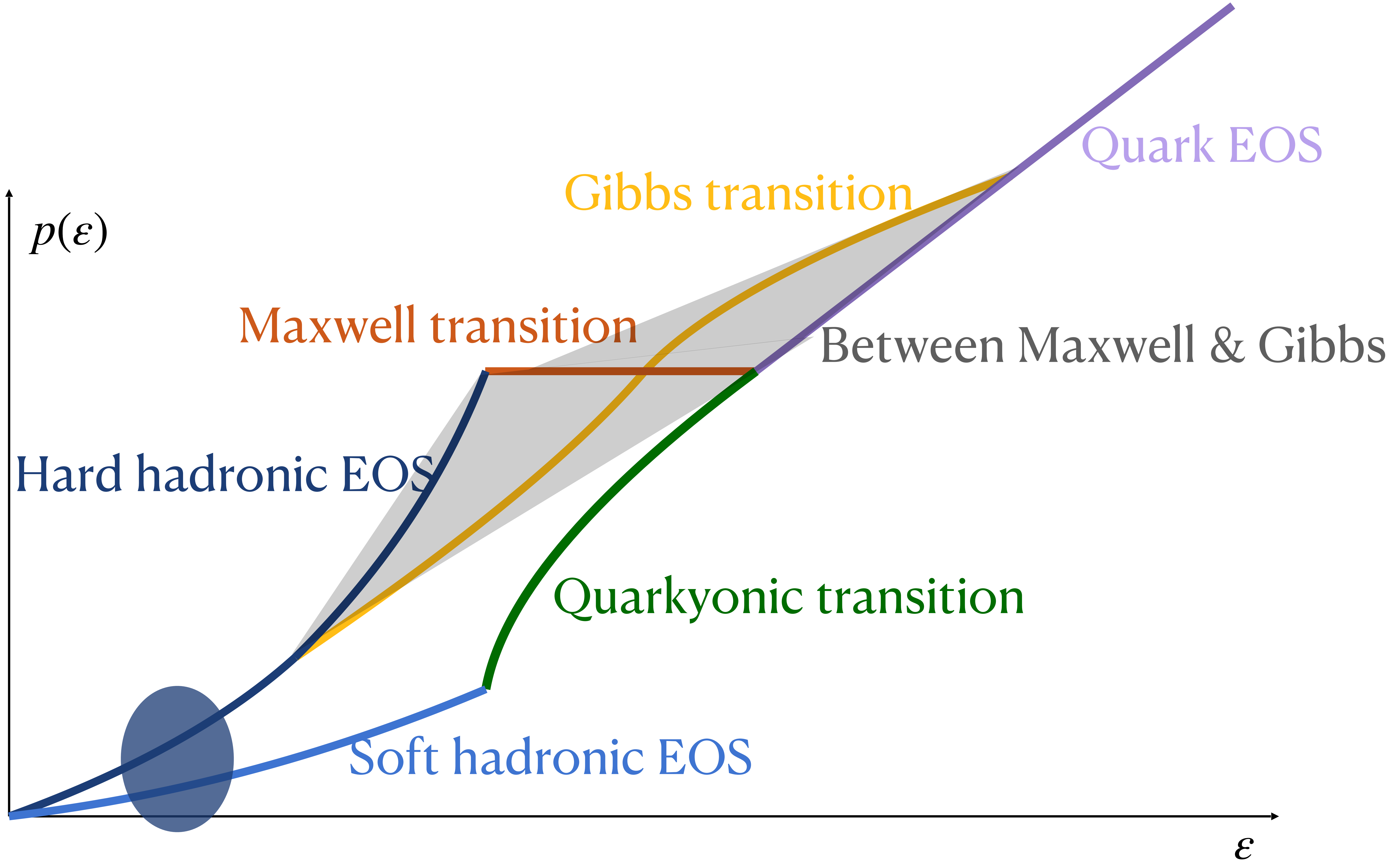
arXiv: 2406.05267 arXiv: 1808.02858

Hadron-quark Transition in Neutron Star Core



Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

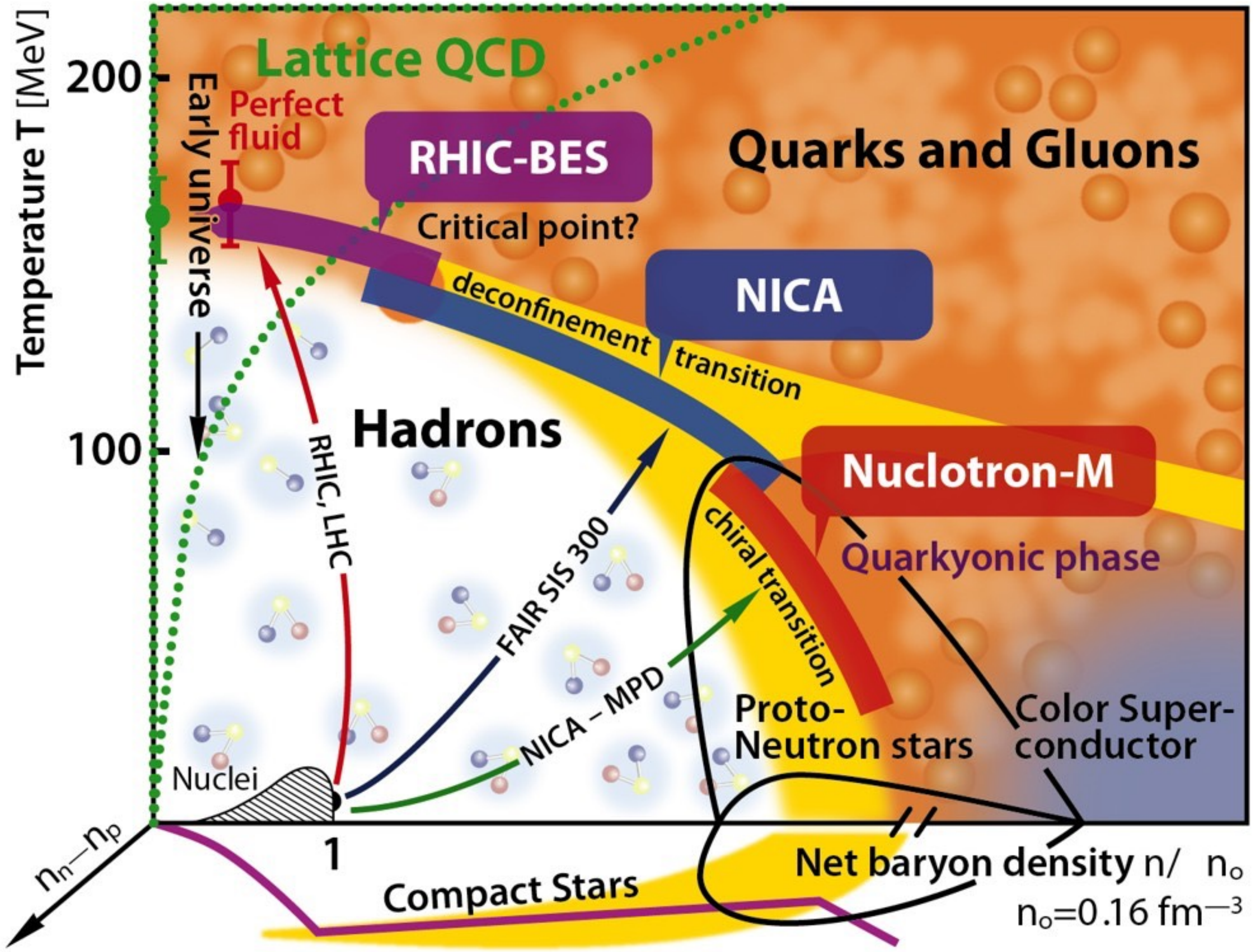
Hadron-quark Transition in Neutron Star Core



Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

Quarkyonic Matter

- The hypothetical phase between hadronic matter and deconfined quark matter (David Blaschke 2008).



https://nica.jinr.ru/physics.php

Sanjay and McLerran 2018

Dynamical realization:

K. Jeong et. al. 2020
T. Kojo et. al. 2021
Y. Fujimoto et. al. 2023

Extend isospin, flavor, finite T:

Zhao & Lattimer 2020
S. Sen et. al. 2021
D. Duarte et. al. 2021
J. Margueron et. al. 2021

Include better hadronic EOS:

G. Cao et. al. 2021
A. Kumar et. al. 2022
C. Xia et. al. 2023

Asymptotic Free

Gross, Wilczek and Politzer 1973

- QCD beta function:

$$\beta(\alpha_s) = q^2 \frac{\partial \alpha_s}{\partial q^2} = -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 - \cdots$$

where $\alpha_s = \frac{g^2}{4\pi}, \beta_0 = \frac{33 - 2N_f}{12\pi} > 0, \beta_1 = \frac{153 - 19N_f}{24\pi^2} > 0$

- Keep only the first term on the right-hand side,

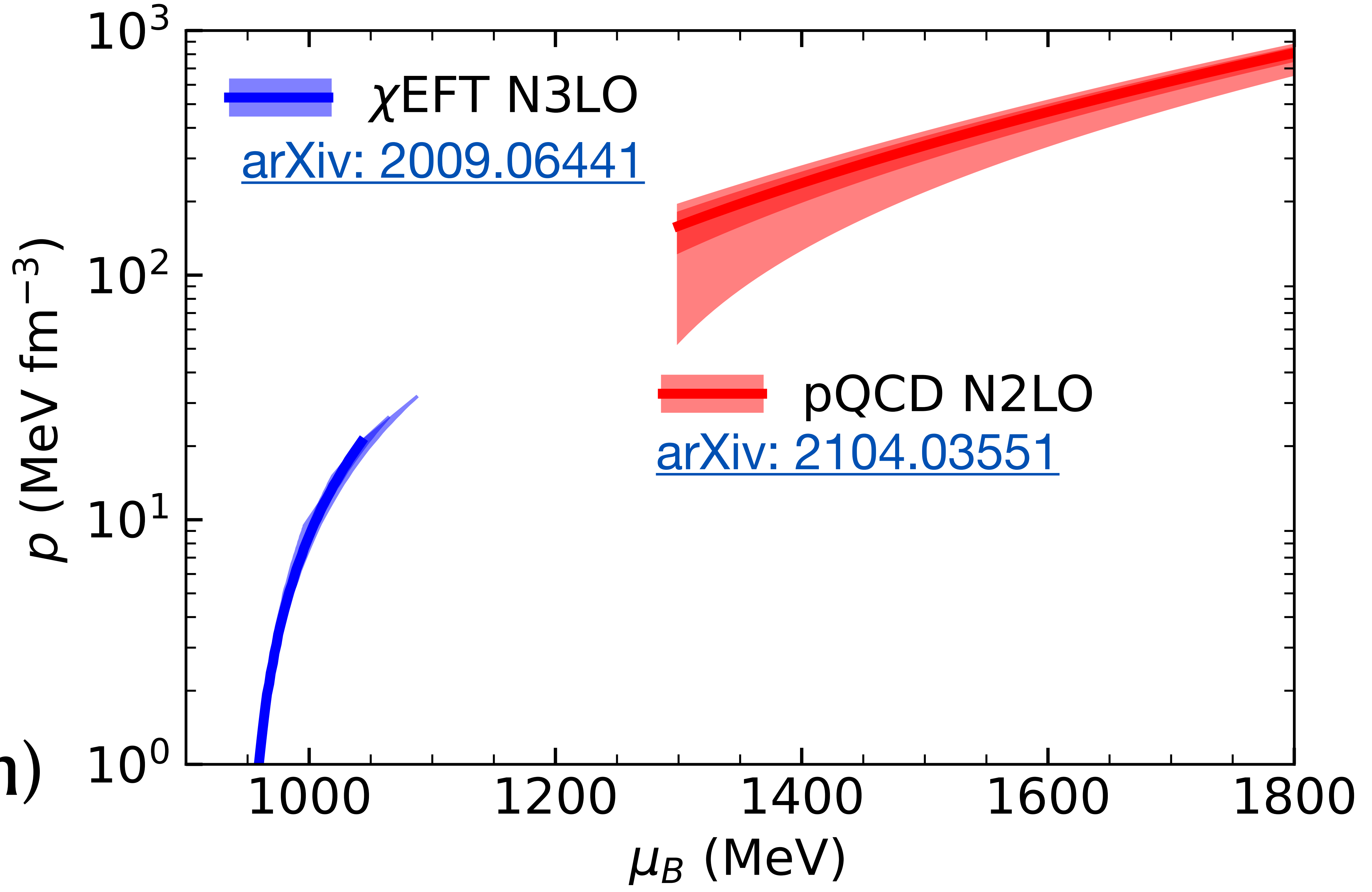
$$\alpha_s \approx \frac{1}{\beta_0 \log q^2/\Lambda_{QCD}^2}$$

therefore $\lim_{q>>\Lambda_{QCD}} \alpha_s(q) \to 0$

- Perturbative QCD:

QCD Lagrangian (quark-gluon coupling)

+ Analytical method (vacuum and ring diagram)

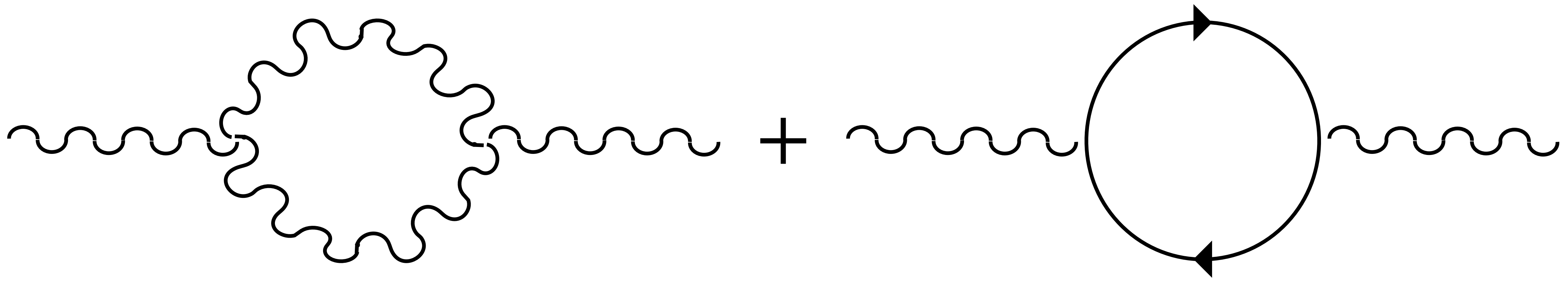


Speculation from large N_c

McLerran & Pisarski 2007

- Confinement due to screening of gluons

$$m^2_{Debye} \approx \Pi = g^2 \left[(N_c + \frac{N_f}{2})\frac{T^2}{3} + \frac{N_f \mu^2}{2\pi^2} + \cdots \right]$$

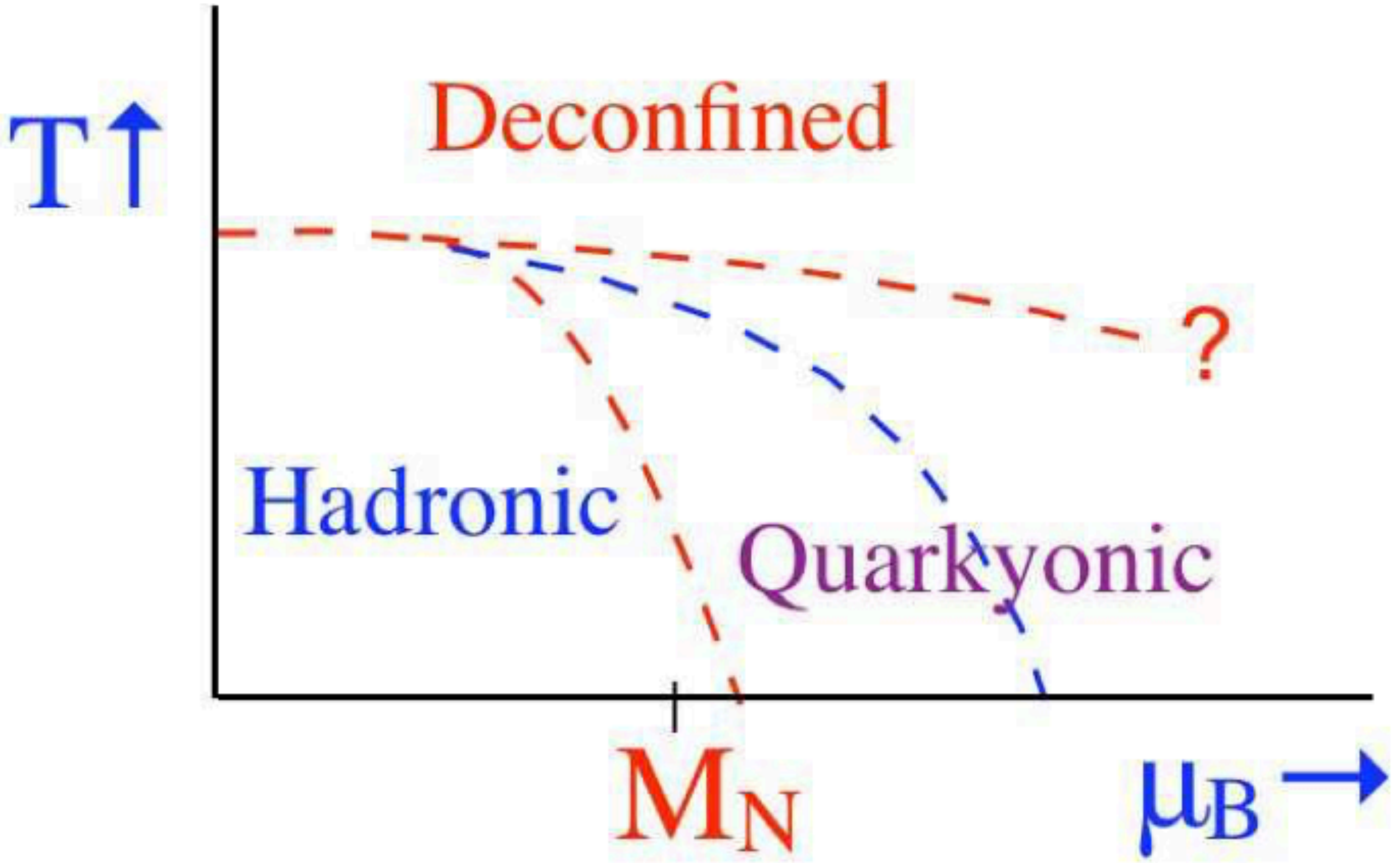


- Large m_{Debye} $\longrightarrow$ stronger screening$\longrightarrow$weaker long-range interactions$\longrightarrow$deconfinement

- Large N_c limit: $N_c \to \infty$ while fixing $\lambda_{'tHooft} = g^2 N_c$ and N_f:

$m^2_{Debye} \propto T^2$ for high temperature;

$m^2_{Debye} \propto \frac{\mu^2}{N_c} \to 0$ for high chemical potential.



- Asymptotic free + Confinement (at the same time) ????

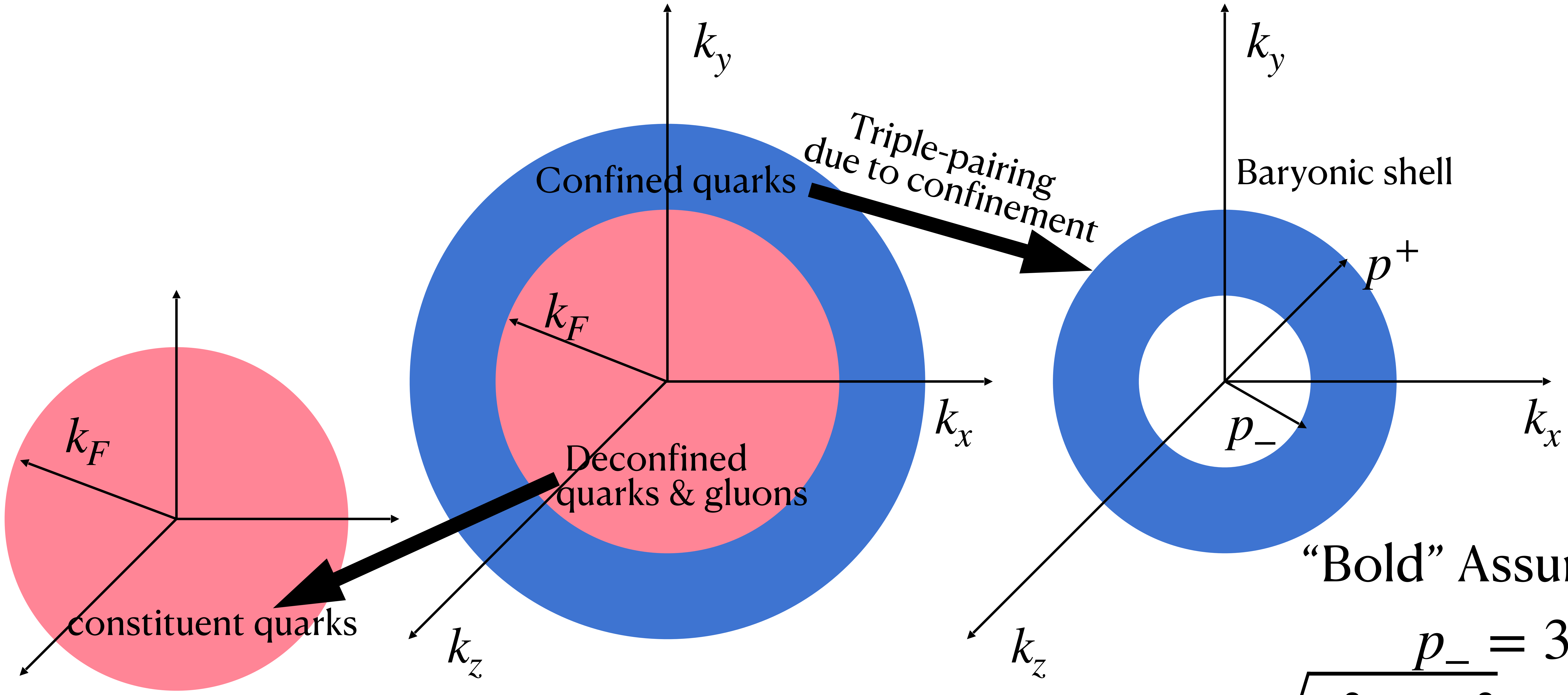
Quark + Baryon = Quarkyonic matter

(refer to Larry's talk tomorrow for more details)

Quarkyonic Matter Momentum Space

Nucleons are degenerate with quarks (quark-hadron duality)

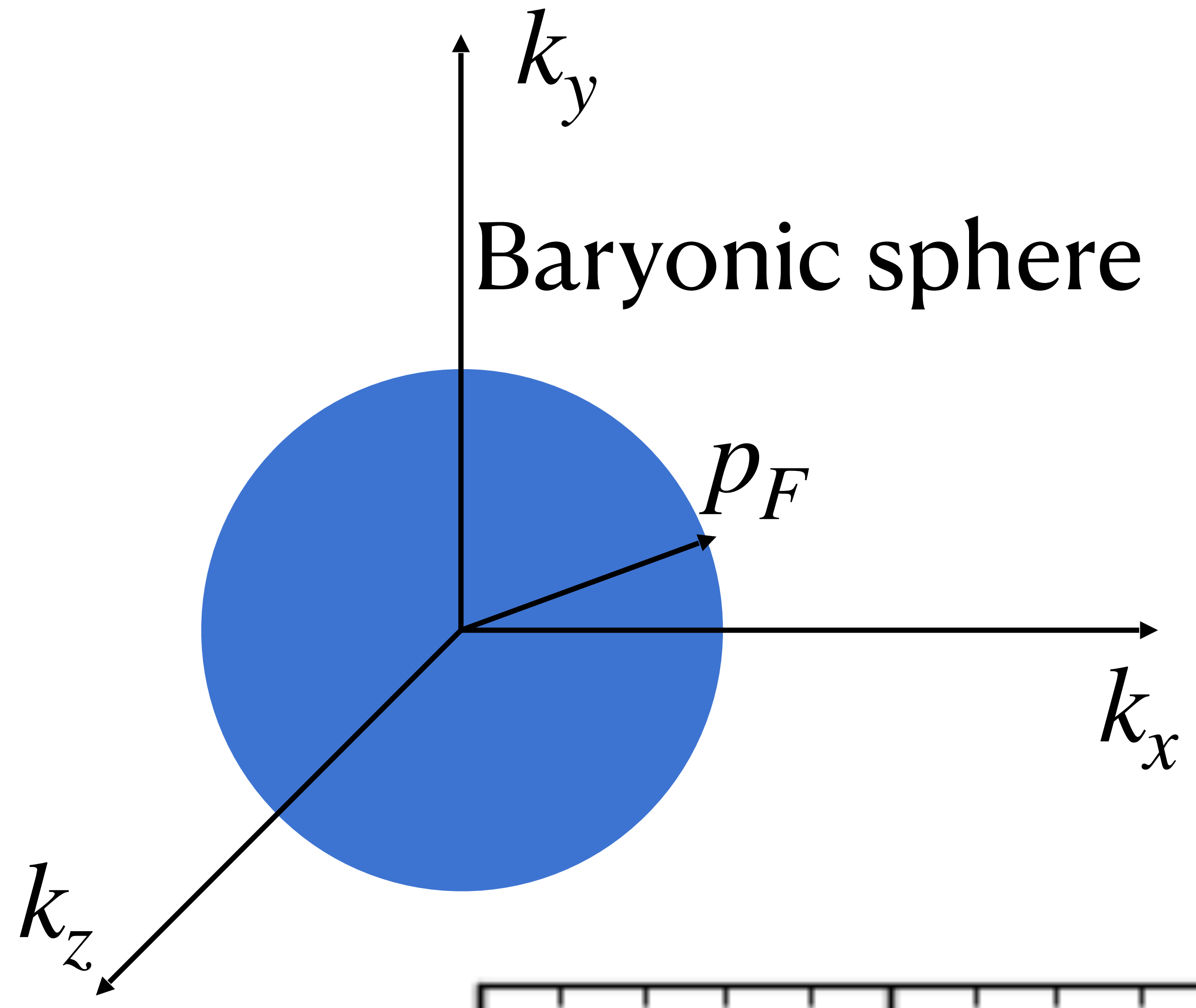
- Perturbative quarks = quarks deep inside Fermi sphere
- Baryons = triple-pair of quarks near Fermi surface



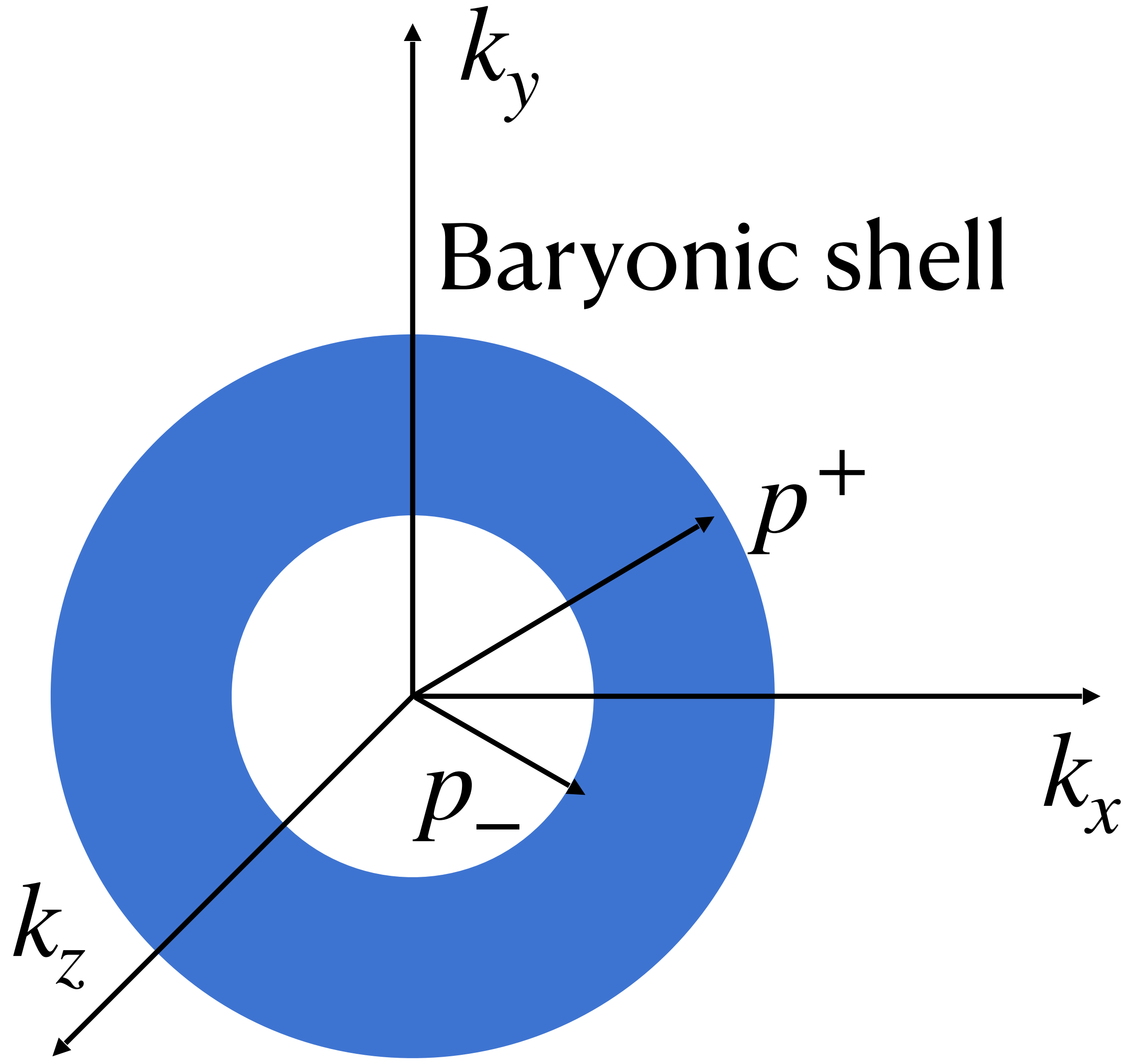
"Bold" Assumption:

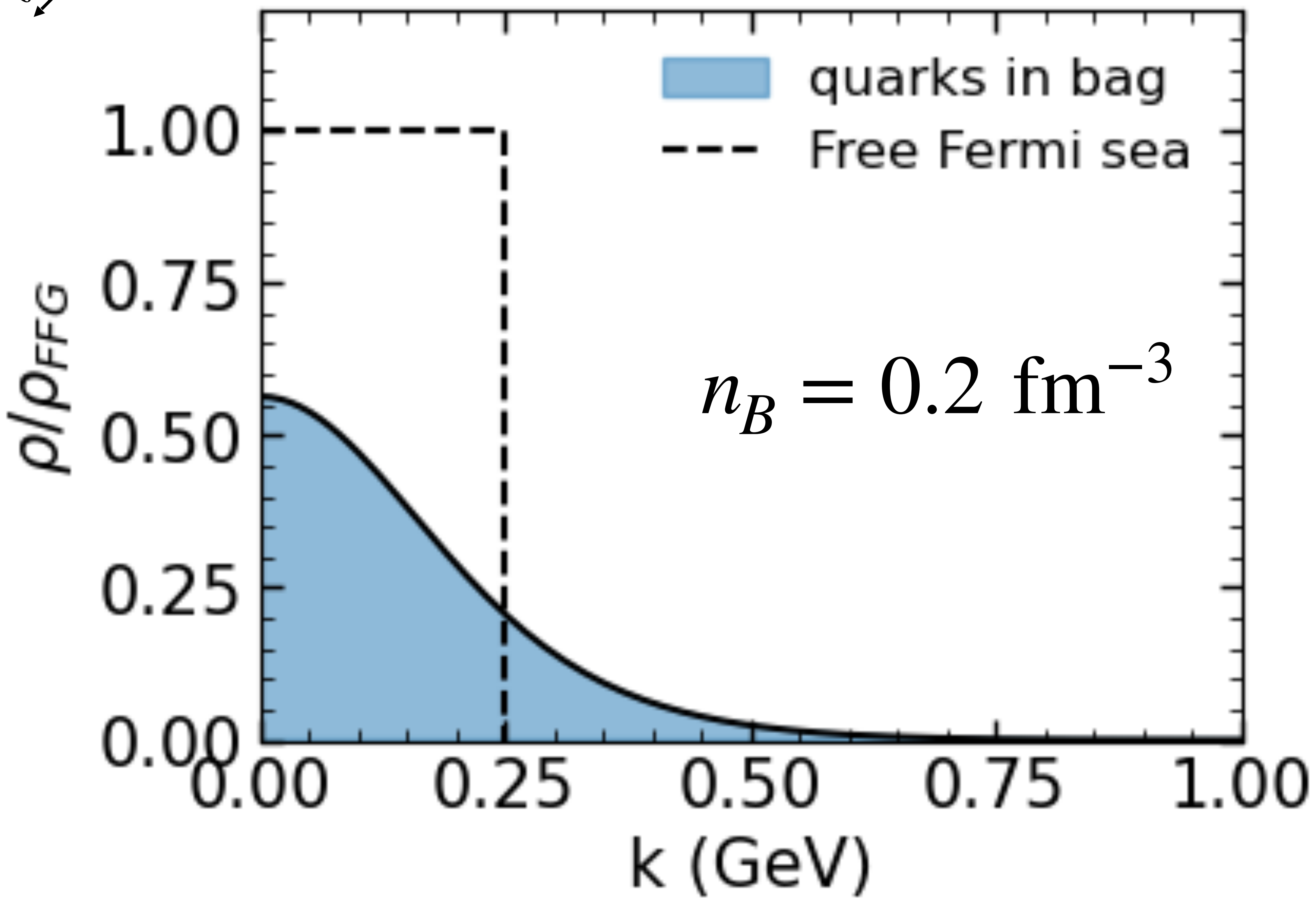
$$p_- = 3k_F$$

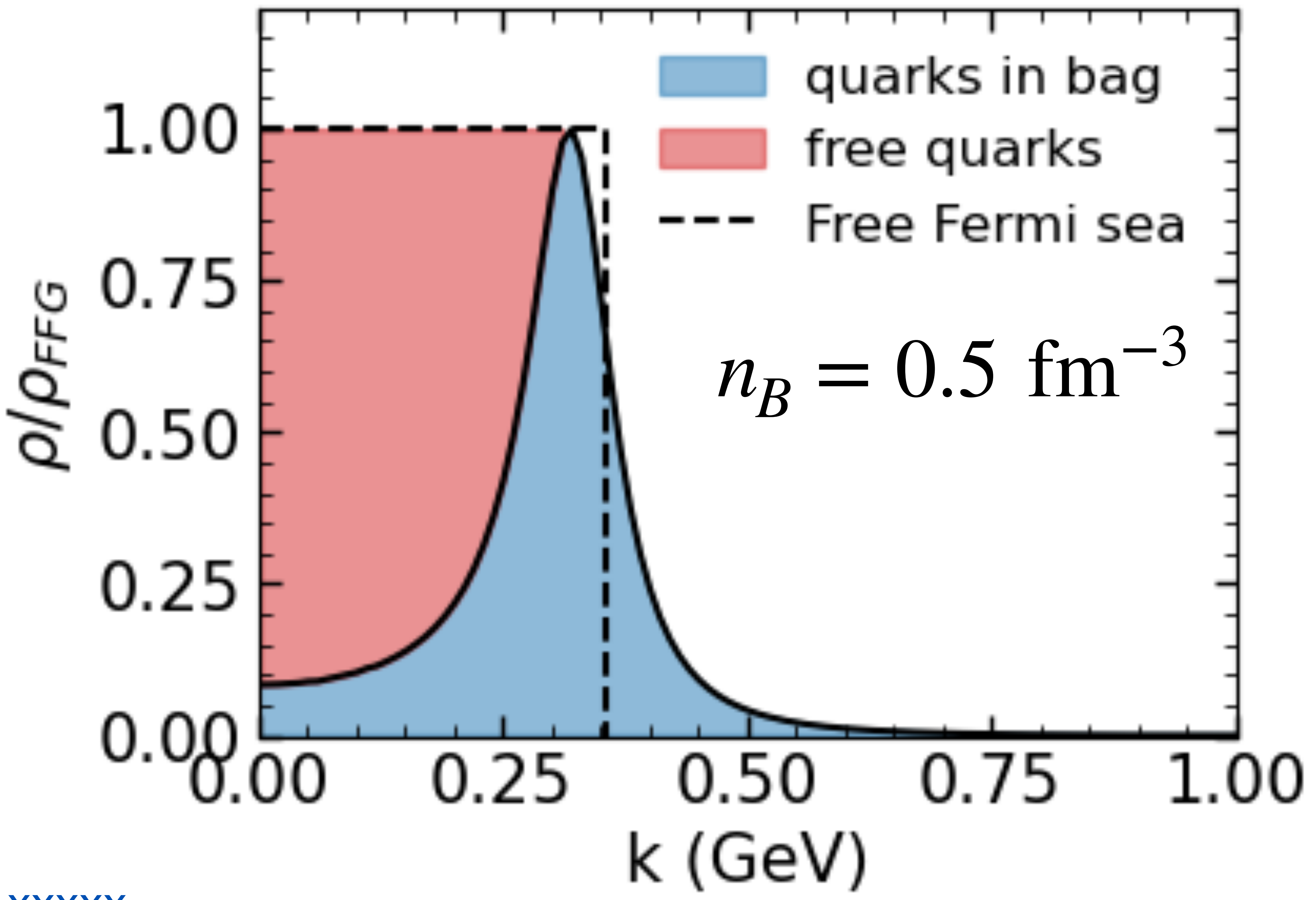
or $\sqrt{p_-^2 + m_N^2} = 3\sqrt{k_F^2 + m_Q^2}$



Hadronic to quarkyonic

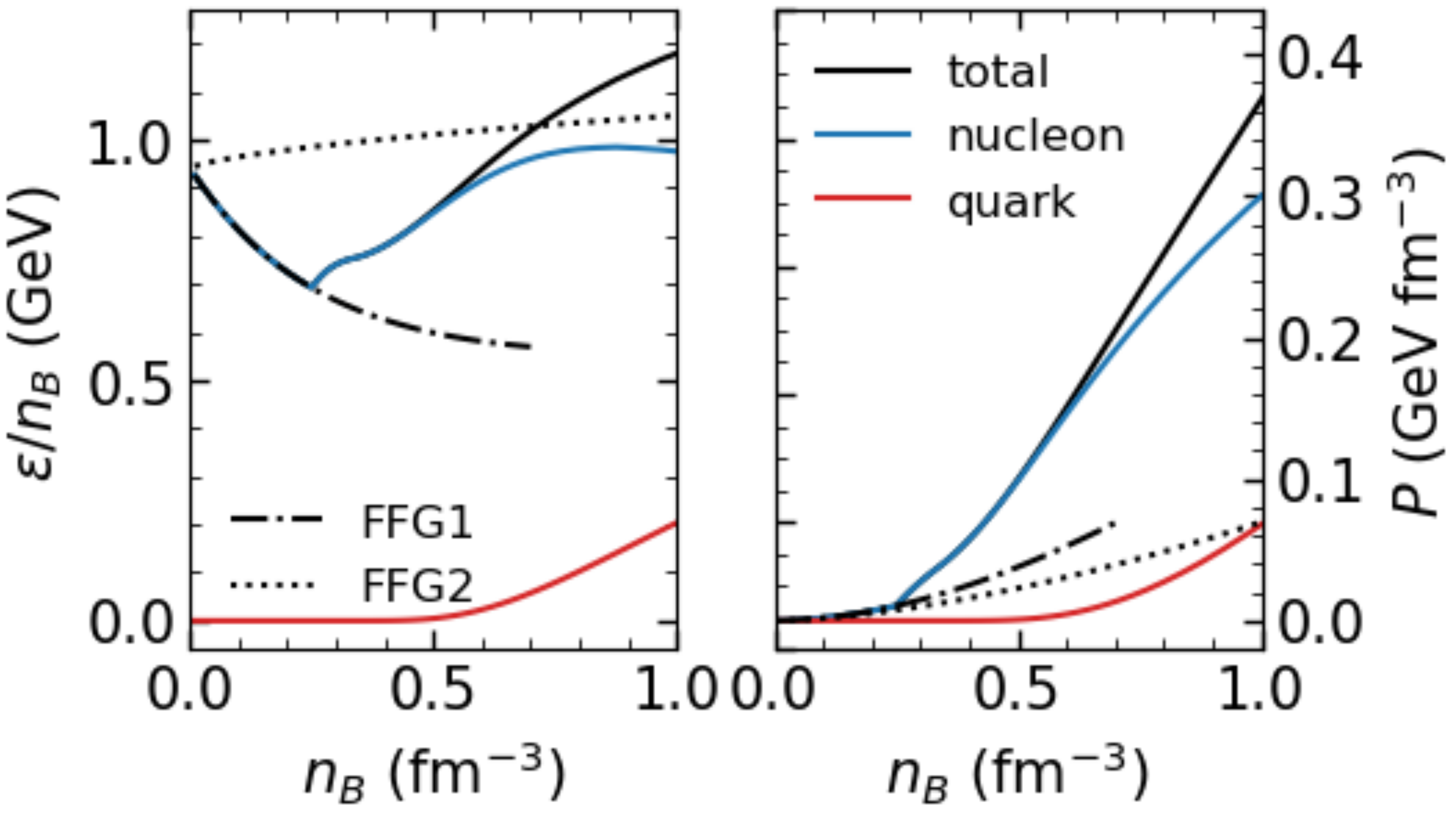






arXiv: 24xx.xxxxx

Quarkyonic EOS



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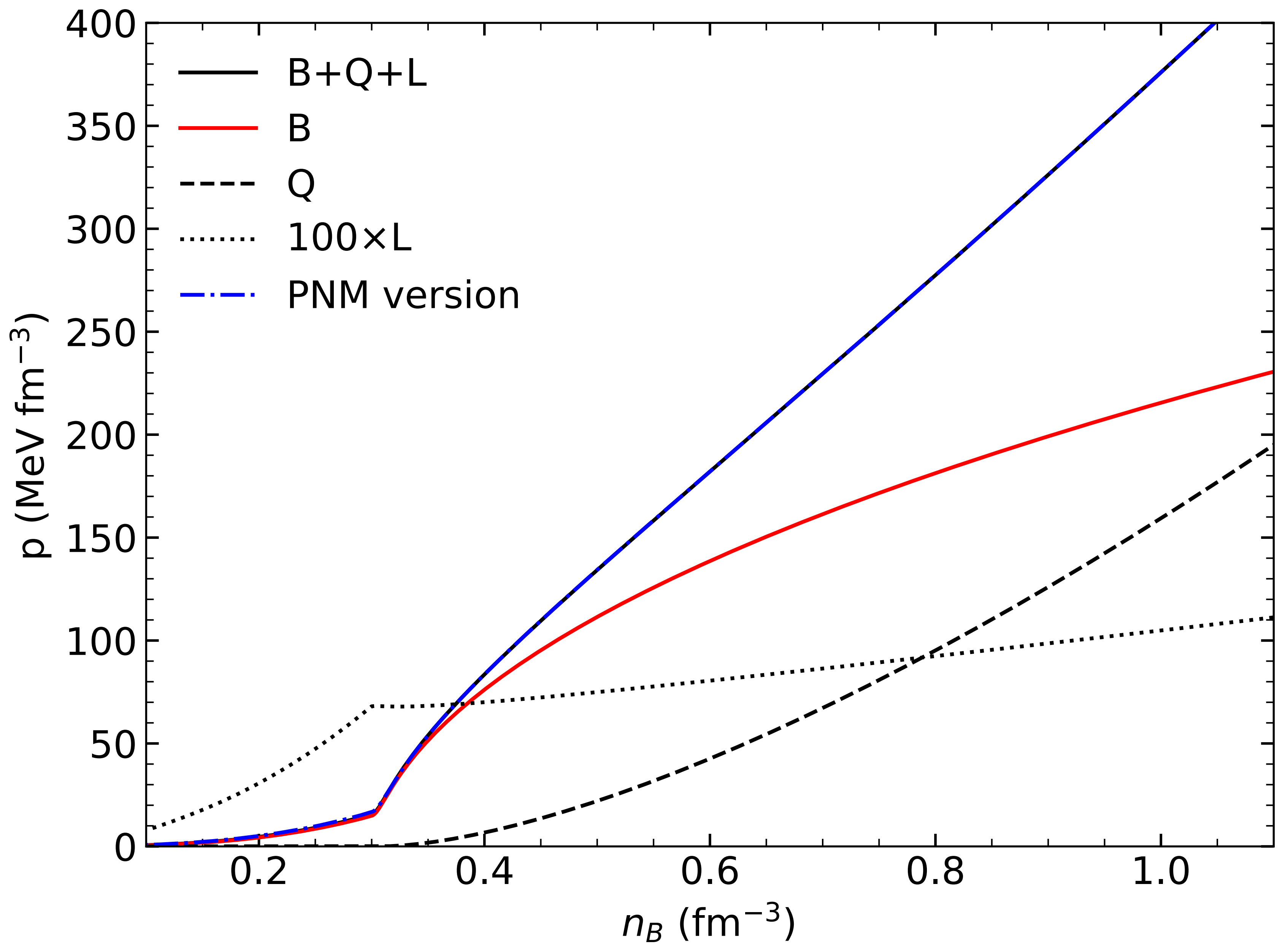
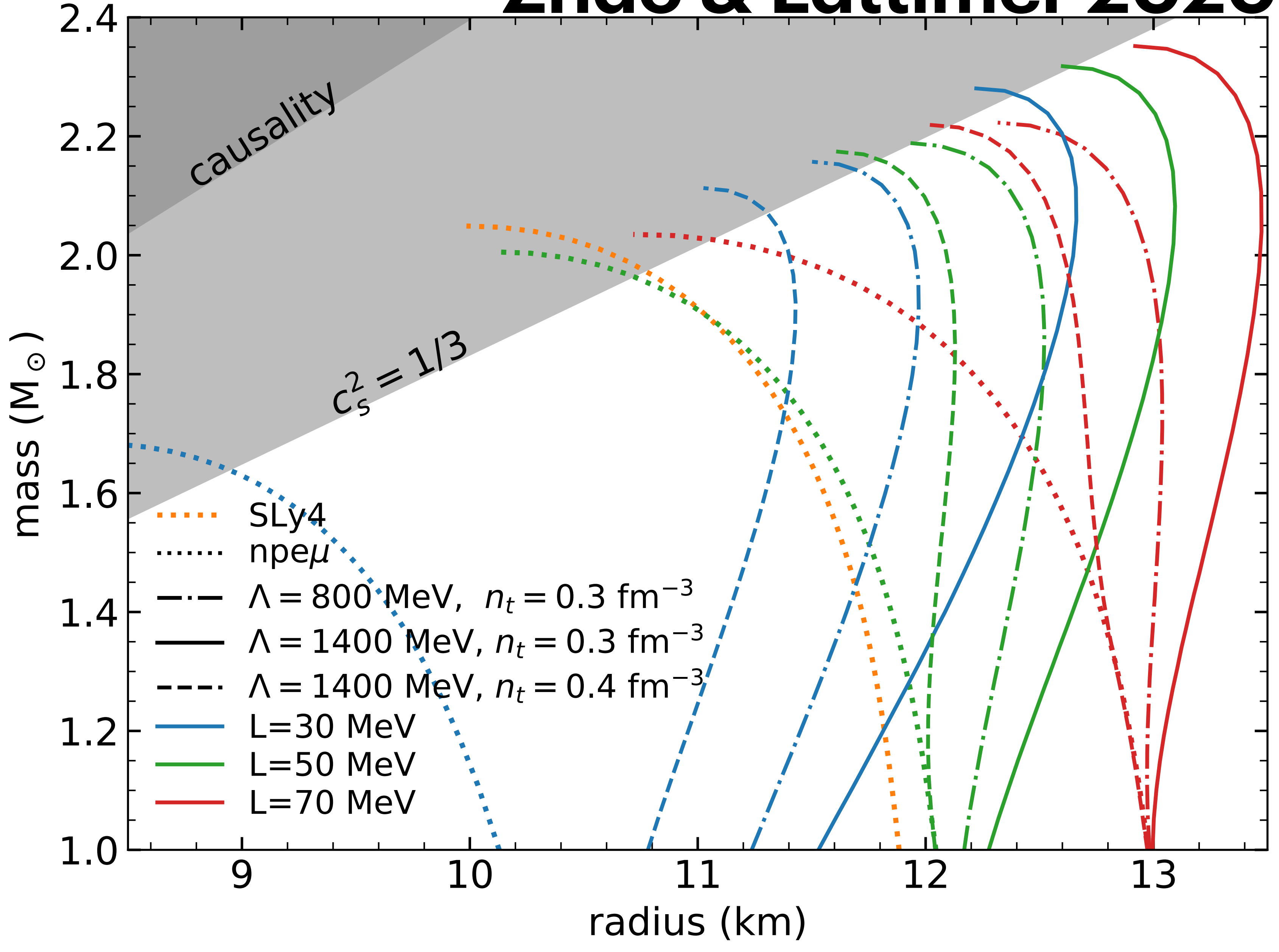


Zhao & Lattimer 2020



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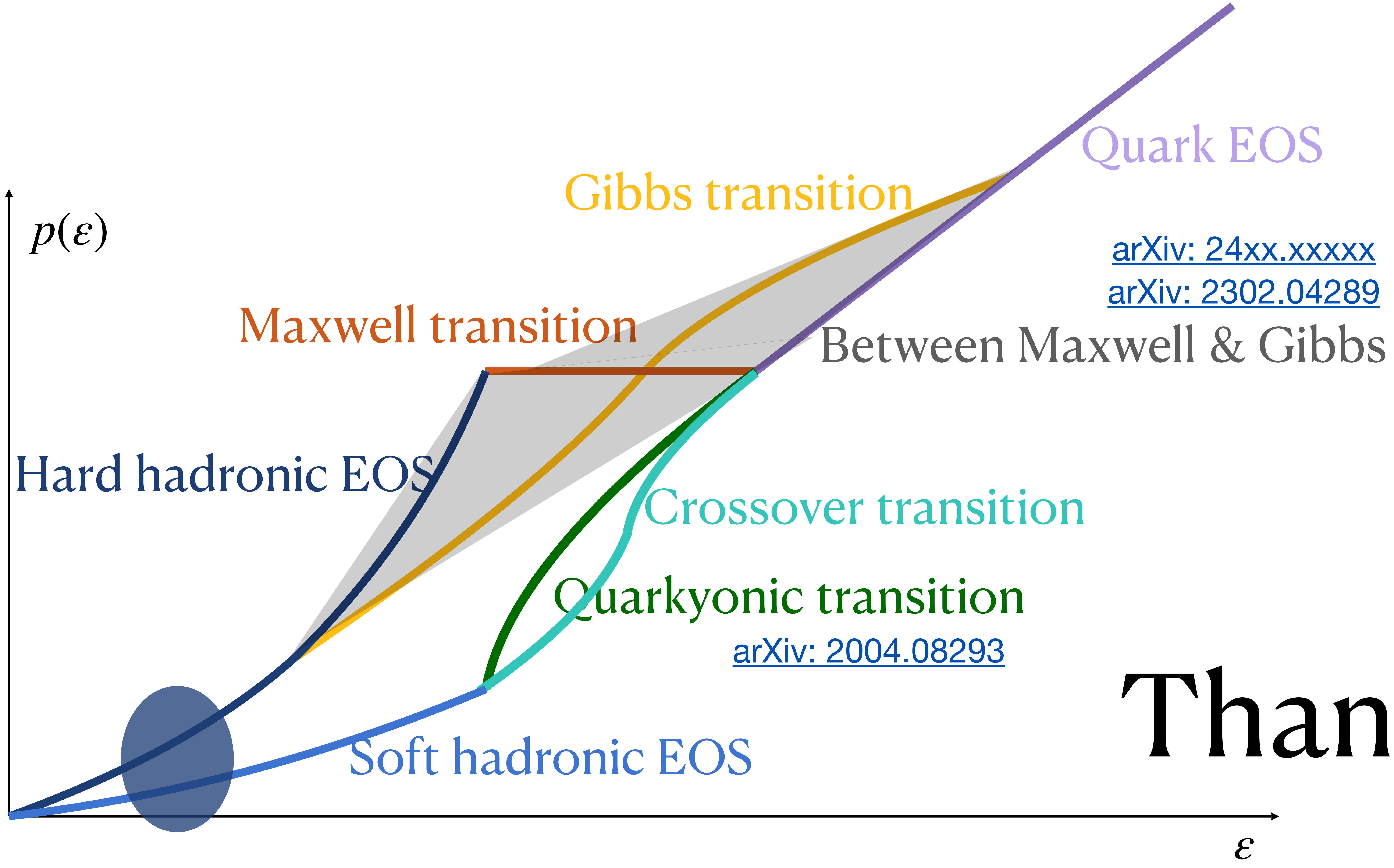

Thank you!

Soft hadronic EOSs is flavored by ab-initio calculation, nuclear experiments & neutron star merger observation.

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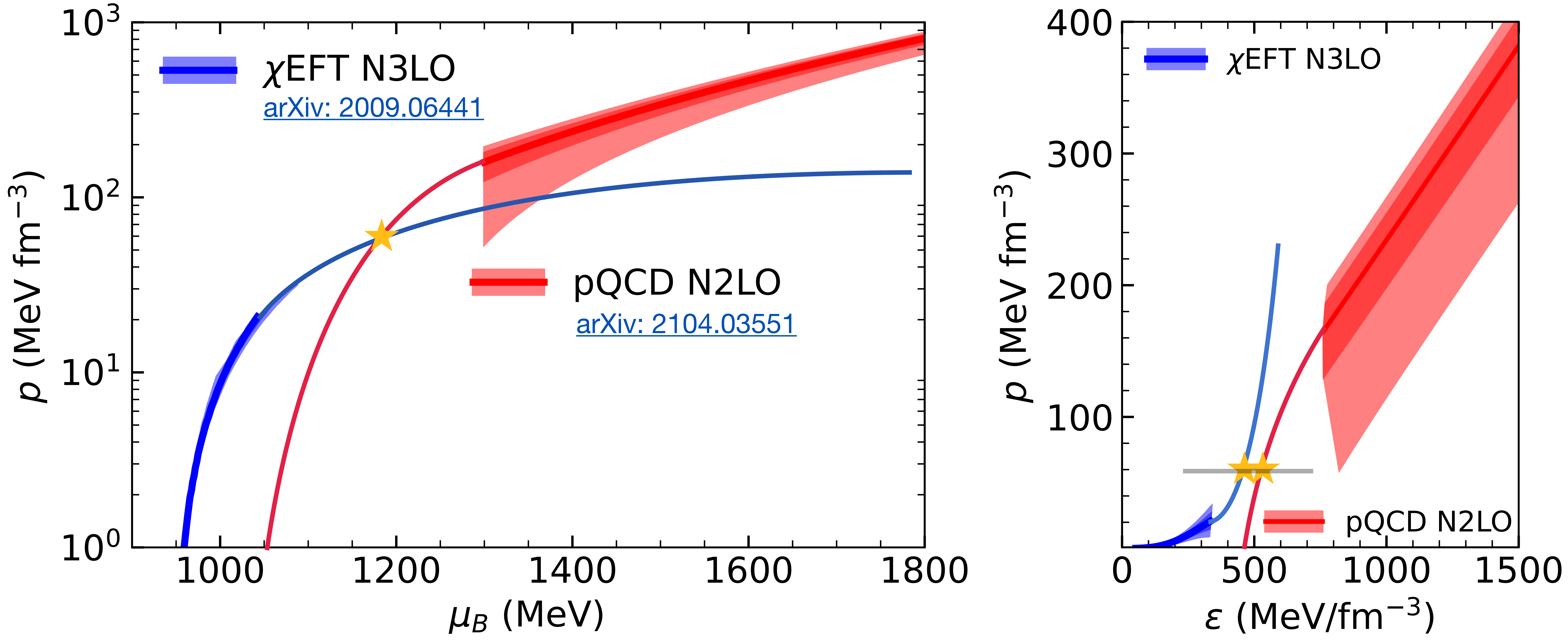
Back up slides

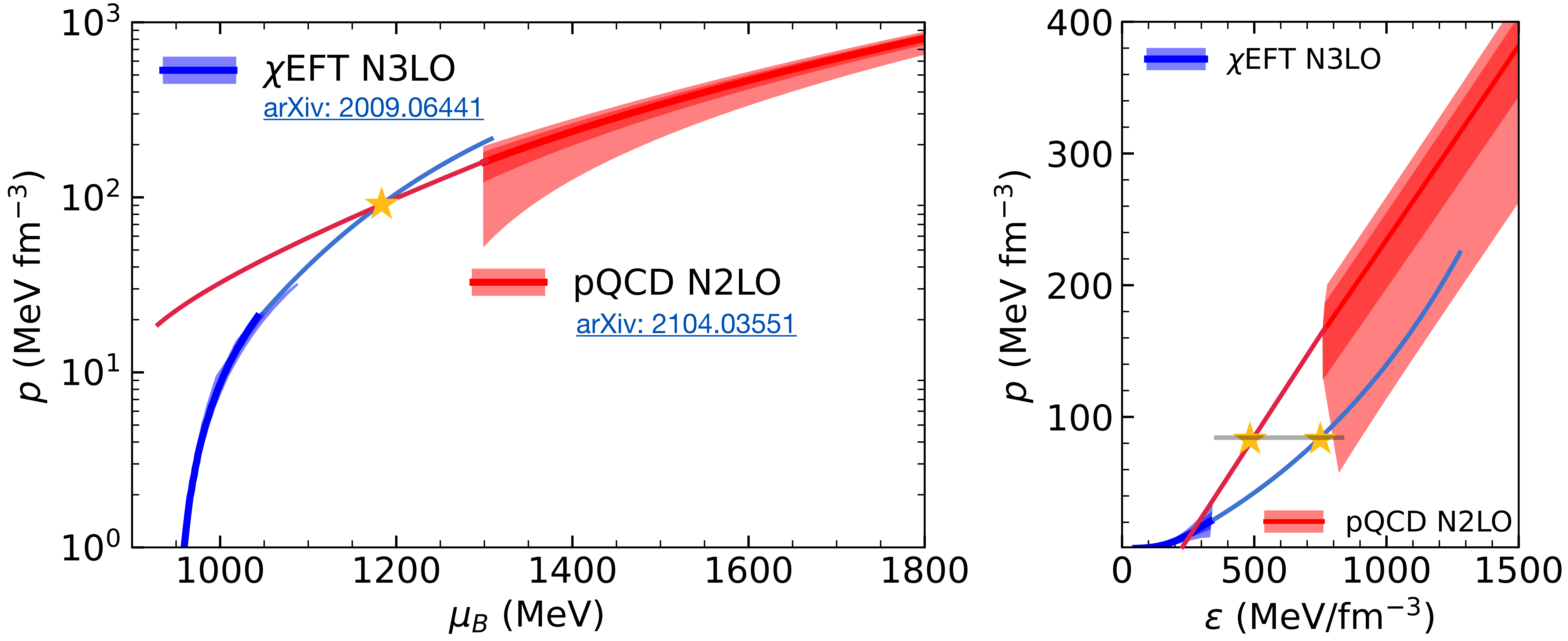
Maxwell Construction

Hybrid Neutron Stars



Maxwell Construction

Inverted Hybrid Star



Crossover Construction

Smooth interpolation