Lattice results for the speed of sound in dense QCD-like theories

Etsuko Itou (YITP, Kyoto U./ RIKEN iTHEMS)

Compact Stars in the QCD Phase diagram (CSQCD2024), YITP, Kyoto University. 2024/10/07

Conformal bound (Holography bound)

Conformal bound: A conjecture proposed by A.Cherman et al., 2009

"maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

for a broad class of 4-dim. theories"

A bound on the speed of sound from holography

Aleksey Cherman* and Thomas D. Cohen†
Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, MD 20742-4111

Abhinav Nellore‡
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544

We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of 1/3 in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

All Lattice Monte Carlo results have satisfied this bound for 40years!

Conformal bound (Holography bound)

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"maximal value of c_s^2/c^2 is 1/3 (non-interacting theory)

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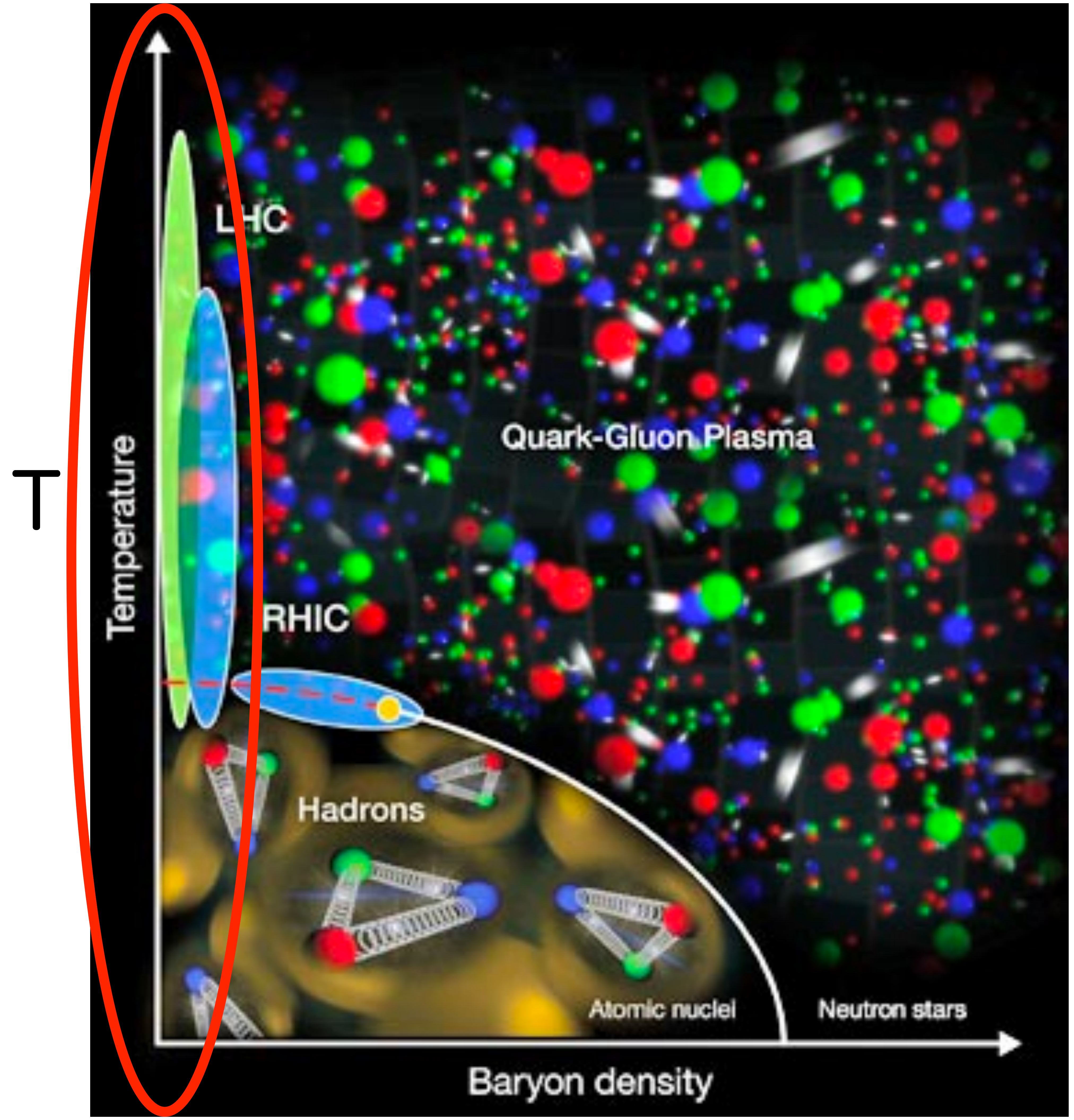
The first counterexample using Lattice Monte Carlo is shown in finite-density QCD-like theory (K.Iida and EI, 2022)

v_s^2 exceeds 1/3 in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

Introduction

expected QCD phase diagram



$\propto \mu$ ©BNL/RHIC

QCD in finite temperature

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi$$

- Lattice gauge theory is only known nonperturbative and gauge invariant regularization method

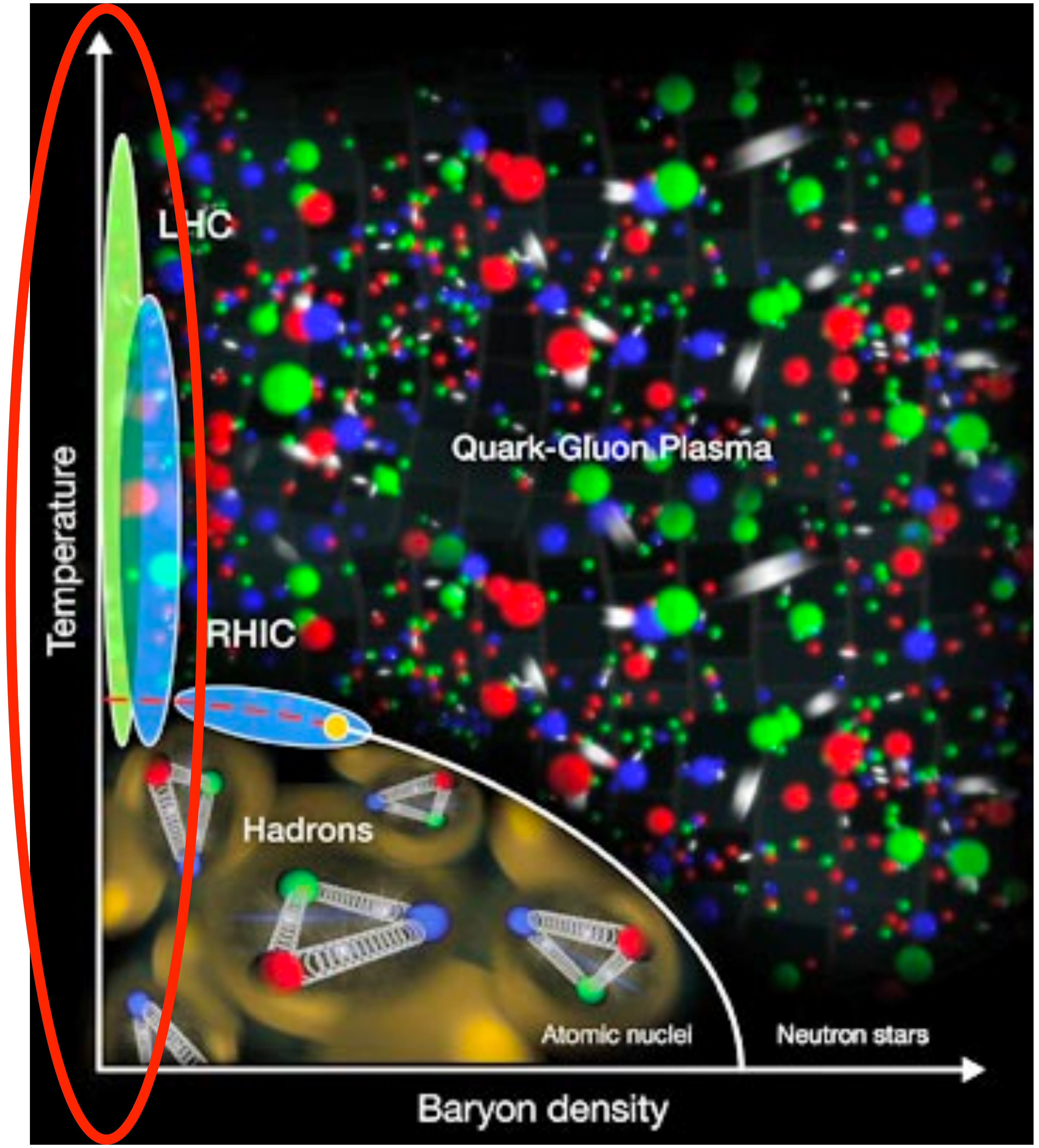
- Finite-T QCD at $\mu = 0$ axis:

  studied by lattice MC and collider experiments

- μ is quark chemical potential; $\mu_B = N_c\mu$

Sound velocity: finite-T transition

EoS and sound velocity at zero-μ



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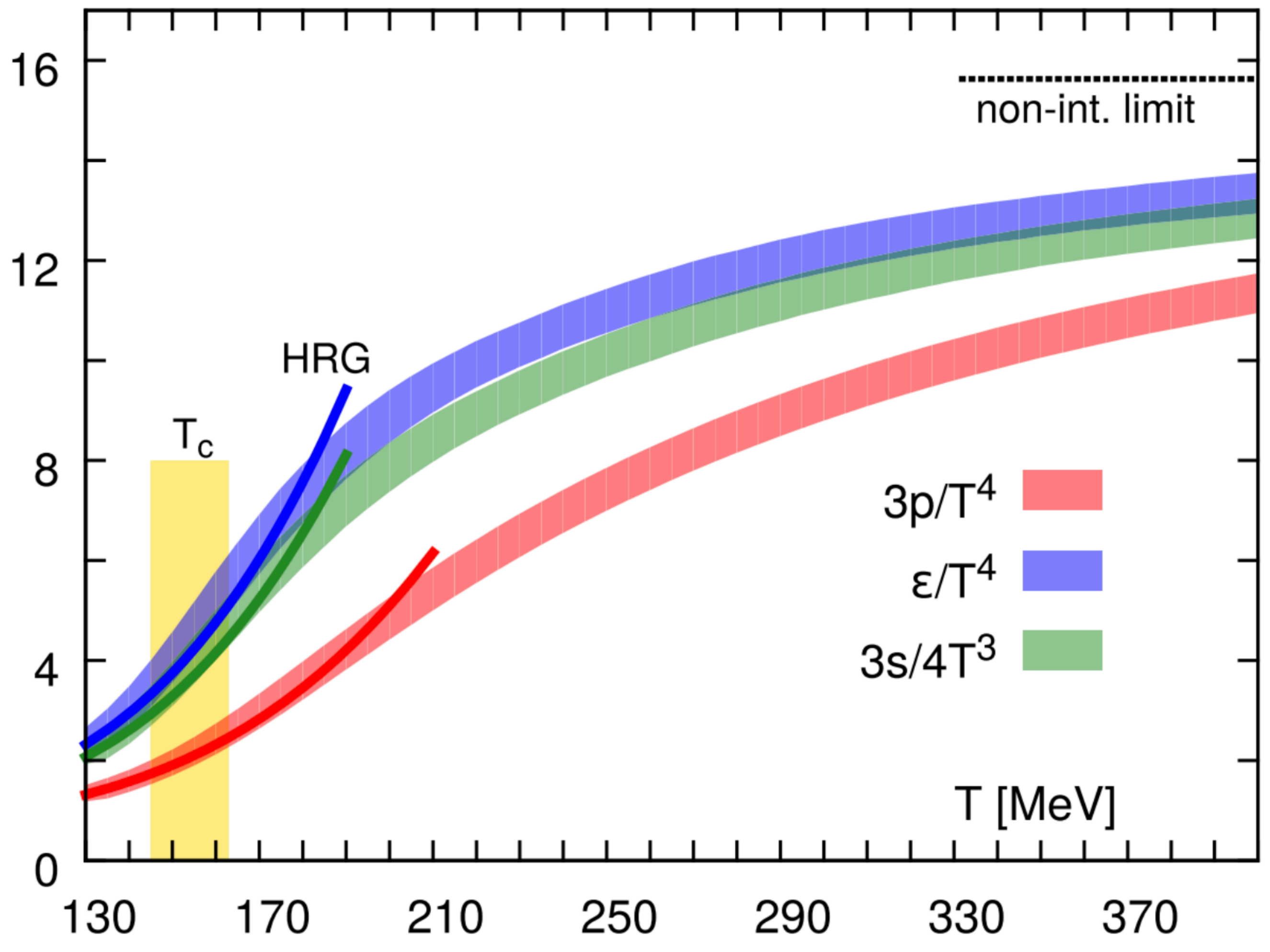
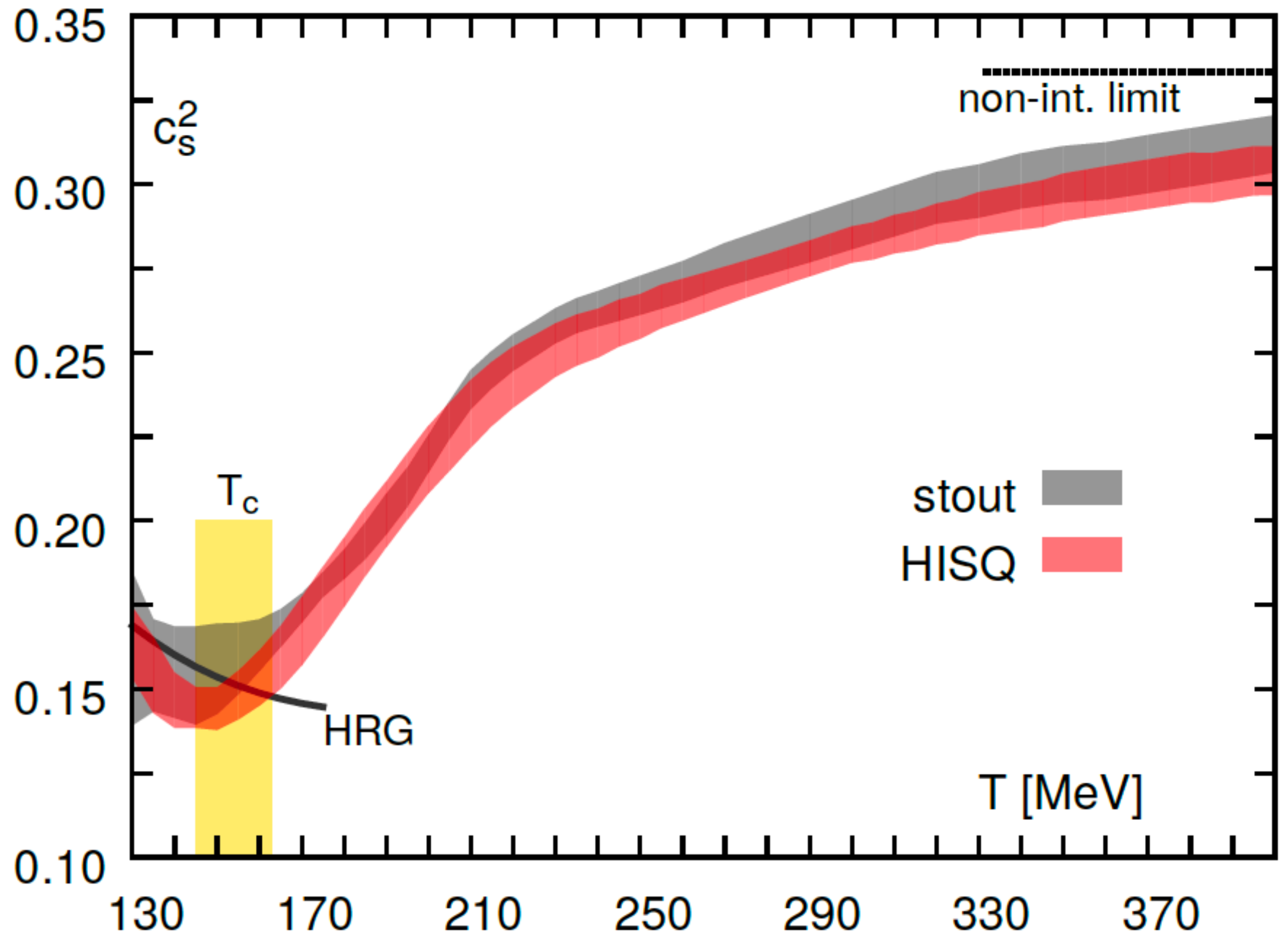
Finite Temperature transition (Nf=2+1 QCD)

EoS (p and ε)

Sound velocity $c_s^2 = \partial p / \partial \epsilon$



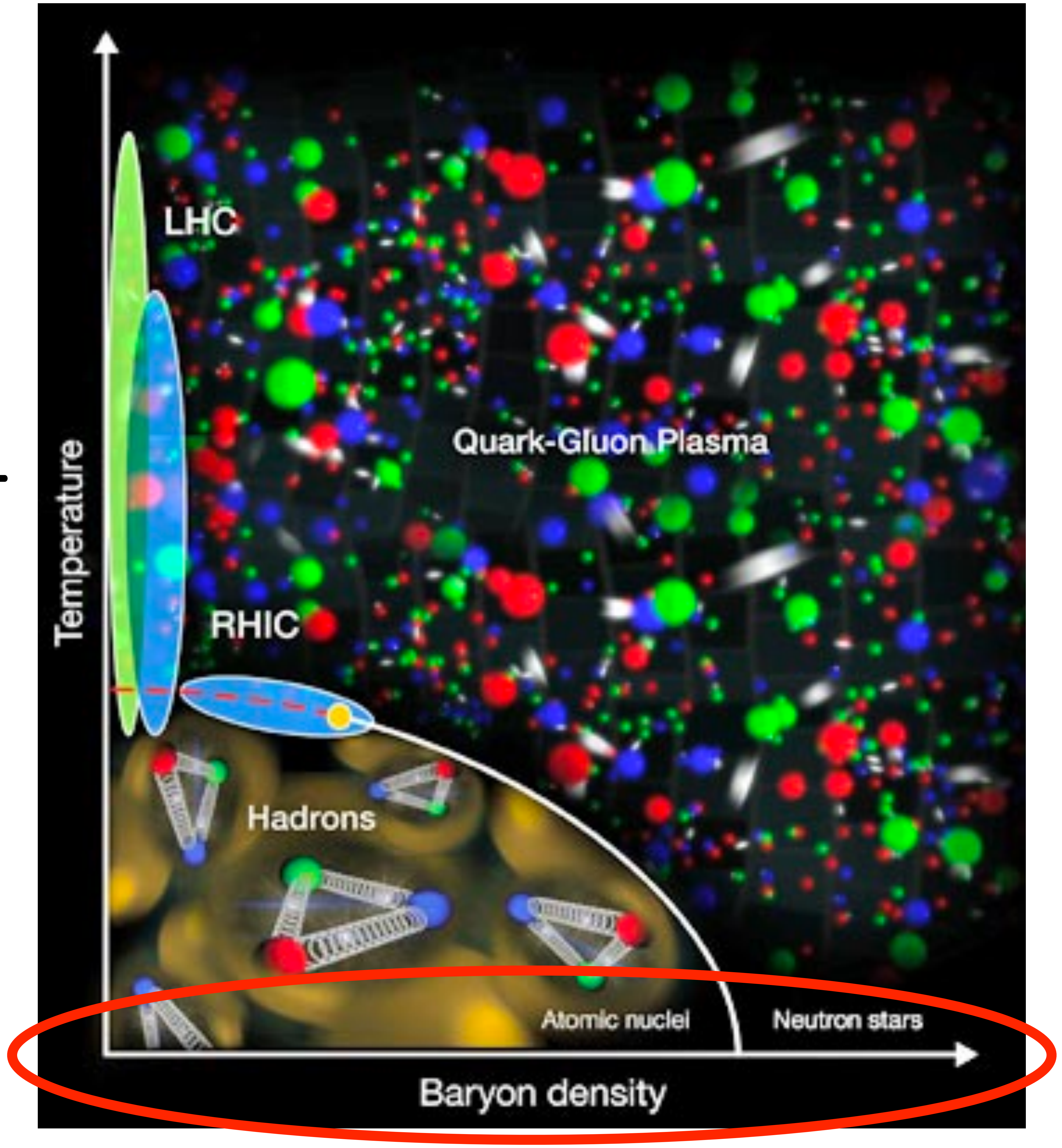


HotQCD (2014)

Introduction

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

Finite density QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- In $\mu \neq 0$ regime, MC simulation suffers from the sign problem

K.Nagata, Finite-density lattice QCD and sign problem: Current status and open problems Prog.Part.Nucl.Phys. 127 (2022) 103991

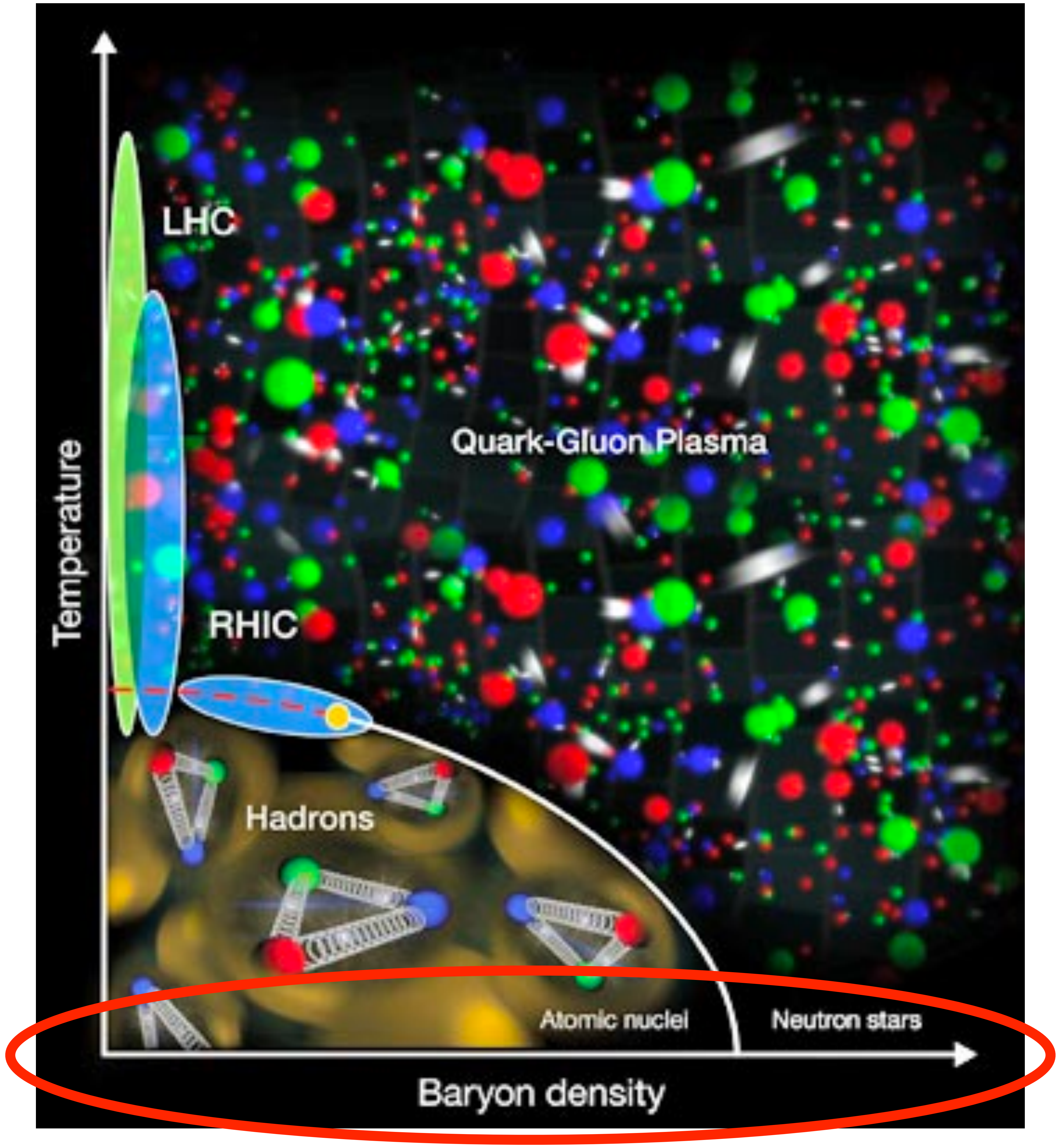
- Sign problem is NP-hard

Troyer and Weise, 2005

Need to change the theory or the algorithm

Sound velocity: finite density regime

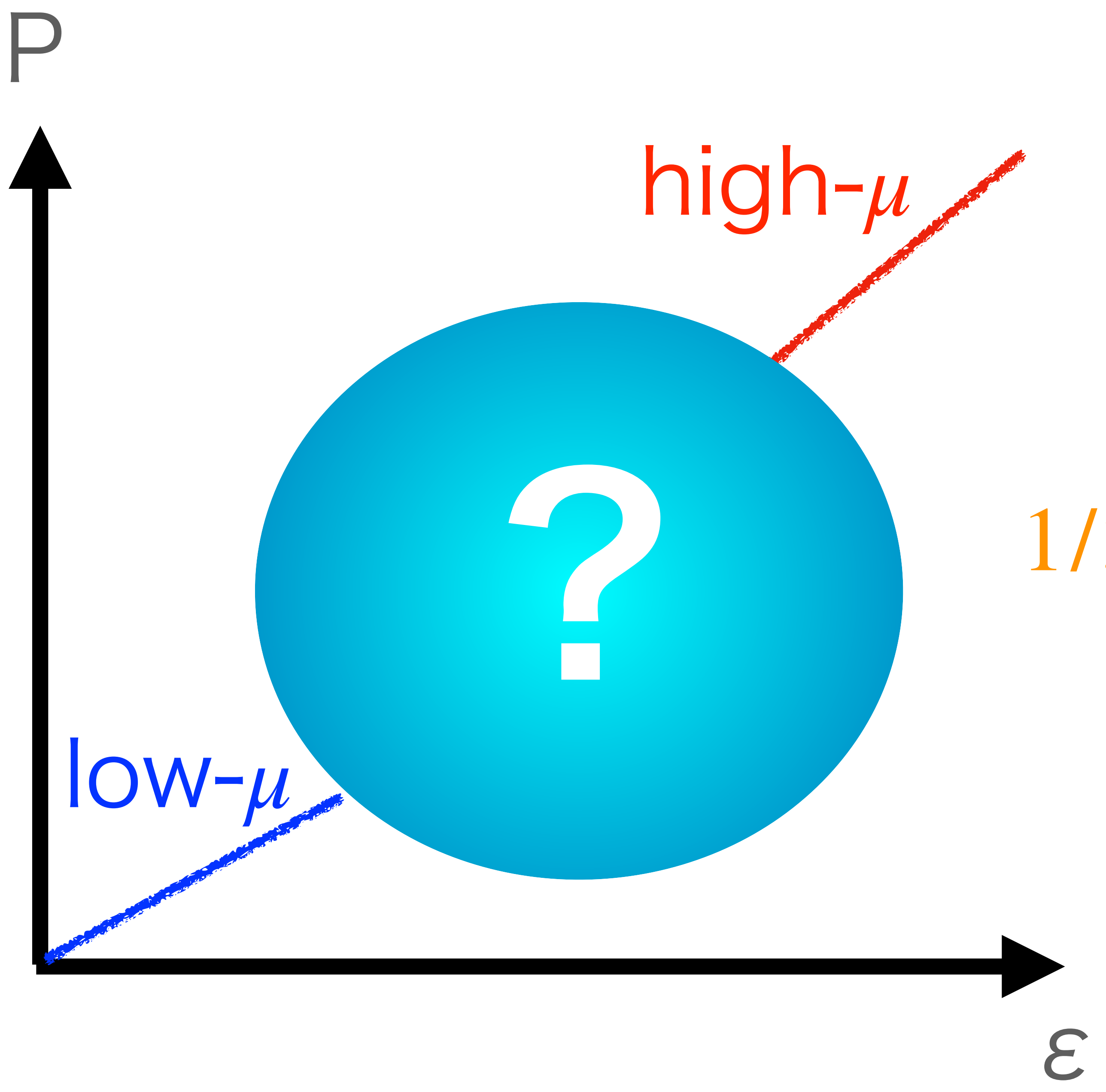
EoS and sound velocity at low-T and high-μ



©BNL/RHIC

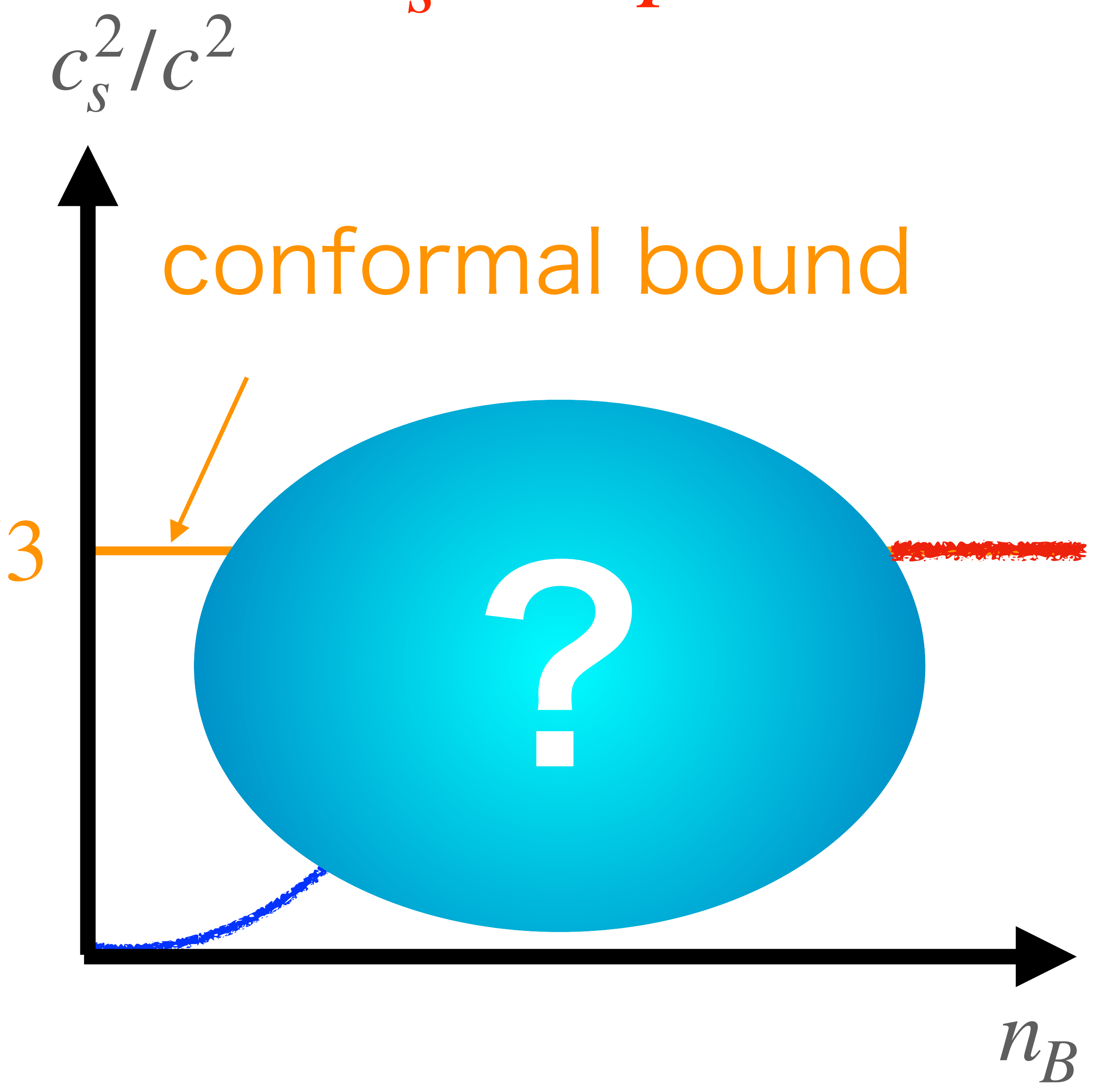
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p/\partial \epsilon$



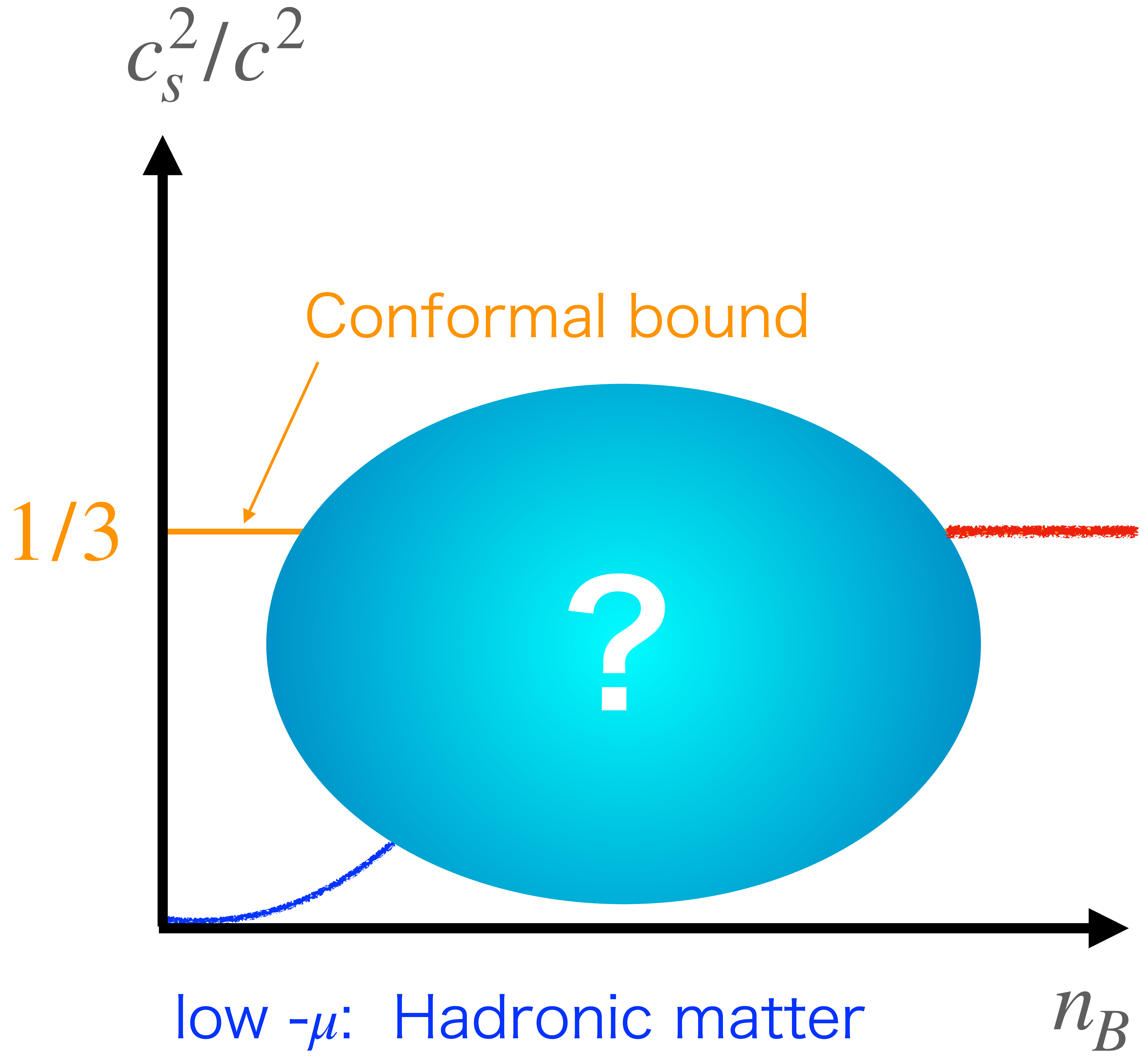
low -μ ($n_B \lesssim 2n_0$): Hadronic matter

high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

Prediction by phenomenology and effective models

Sound velocity has a peak?



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests c_s^2 peaks at $n_B = 1 - 10n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model c_s^2 peaks at $n_B = 1 - 5n_0$

  McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation (work for any # of color)

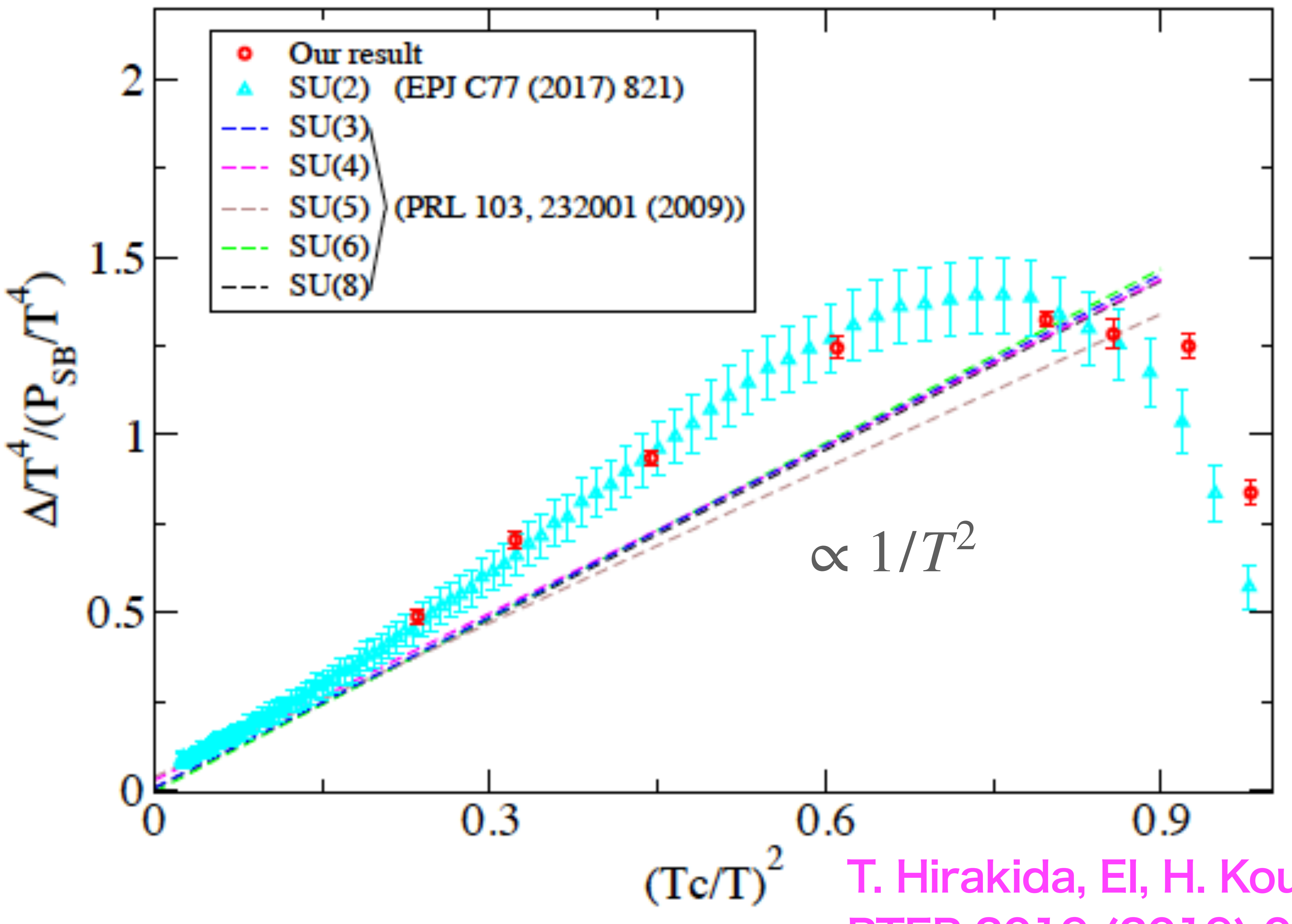
  Kojo (2021), Kojo and Suenaga (2022)

→ Lattice study on 2color dense QCD the sign problem is absent!!

2color QCD $\approx$ 3color QCD at $\mu = 0$

EoS shows very similar at least quenched QCD case

Trace anomaly ($\Delta = (\epsilon - 3p)$) of pure SU(Nc) gauge theories with several Nc



T. Hirakida, EI, H. Kouno
PTEP 2019 (2019) 033B01

Finite density QCD

$$\mathscr{L} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi$$

3color QCD: a=1 - 8
2color QCD: a=1 - 3

Lattice MC for 2color QCD in finite-density regimes gives a hint for dense-QCD

Our 2color QCD projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
  Phase diagram by Lattice simulation (T=80MeV)
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253
  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
  Scale setting of Lattice simulation
- K.Ishiguro, K.Iida, EI, PoS, Lattice 2021
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01
  Velocity of sound by Lattice simulation (T=80MeV)
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023) and
  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, PoS, Lattice 2022
  Mass spectrum by Lattice simulation
- K.Murakami, K.Iida, EI, JHEP 02 (2024) 152
  Hadron potential w/ finite μ by Lattice simulation
- K.Iida, EI, K.Murakami, D. Suenaga, e-Print: 2405.20566 [hep-lat] to appear JHEP
  Phase diagram and EoS by Lattice simulation (T=40MeV)

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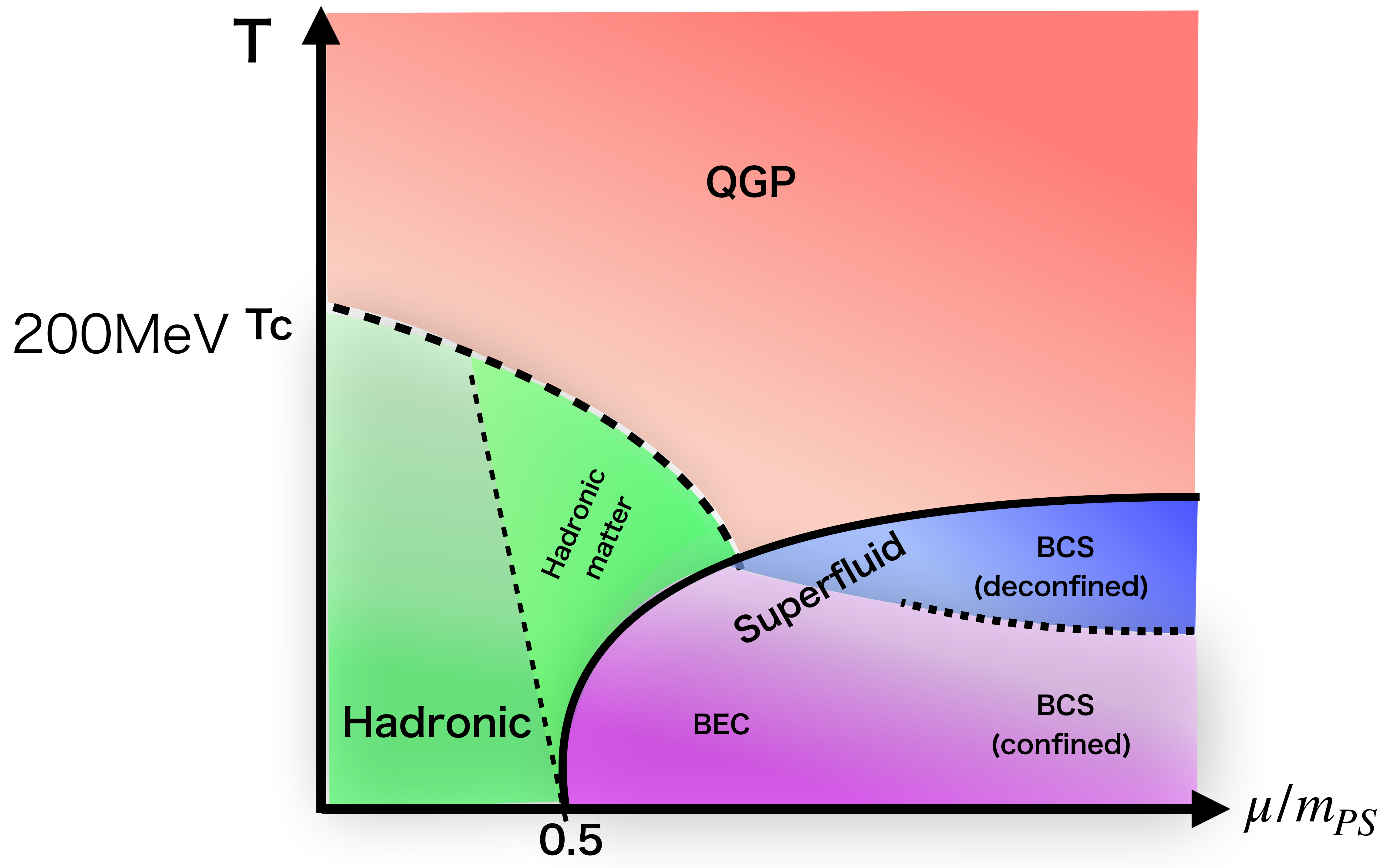
2color QCD phase diagram

(1) K.Iida, EI, K.Murakami, D.Suenage arXiv: 2405.20566 [hep-lat] to appear JHEP
(2) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(3) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(4) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Definition of phases

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181

	Hadronic	Hadronic-matter	QGP	Superfluid BEC	Superfluid BCS
$\langle \lvert L \rvert \rangle$	zero	zero	non-zero		
$\langle qq \rangle$	zero	zero	zero	non-zero	$\propto \Delta(\mu)\mu^2$
$\langle n_q \rangle$		non-zero		non-zero	$n_q/n_q^{\text{tree}} \approx 1$



(1) If $\langle qq \rangle \neq 0$, then it is superfluid phase

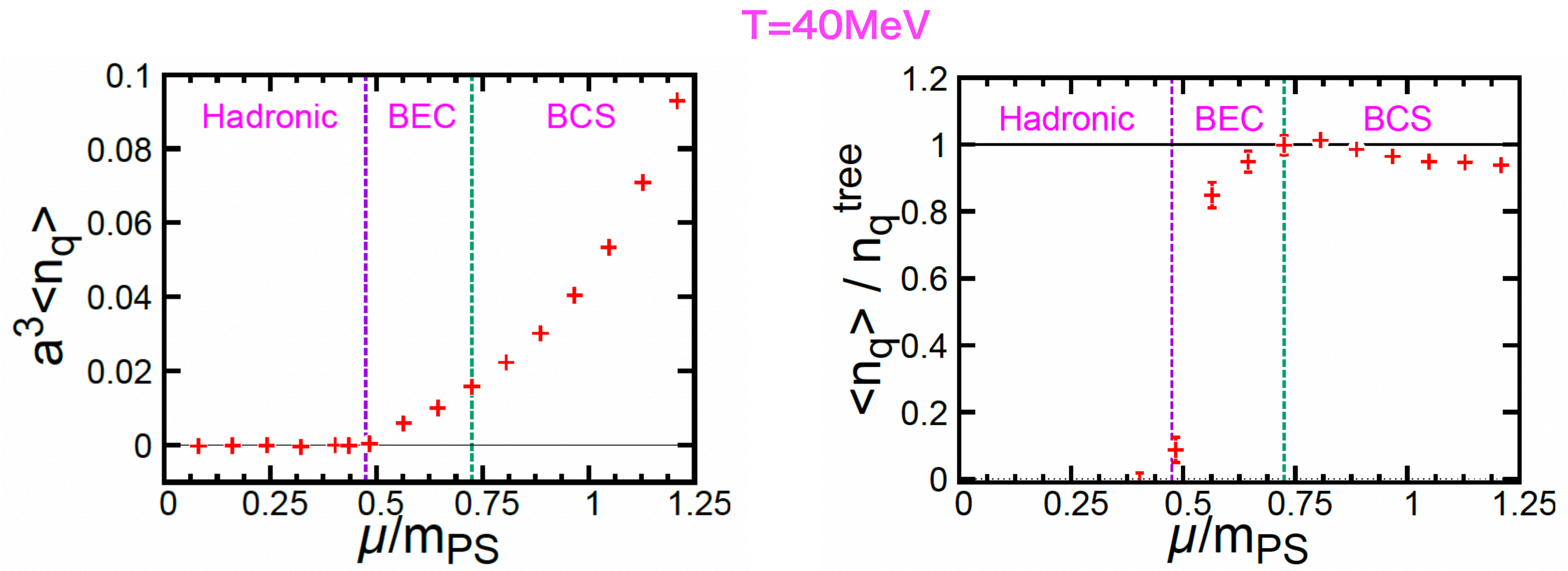
(2) If $\langle n_q \rangle$ becomes consistent with the one of free quark theory, then it is BCS phase

free theory result on the lattice:

$$n_q^{\text{tree}}(\mu) = \frac{4N_c N_f}{N_s^3 N_\tau} \sum_k \frac{i \sin \tilde{k}_0 [\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos \tilde{k}_\nu]^2 + \sum_\nu \sin^2 \tilde{k}_\nu}$$

The other regime of superfluid phase, we call such a regime as BEC phase

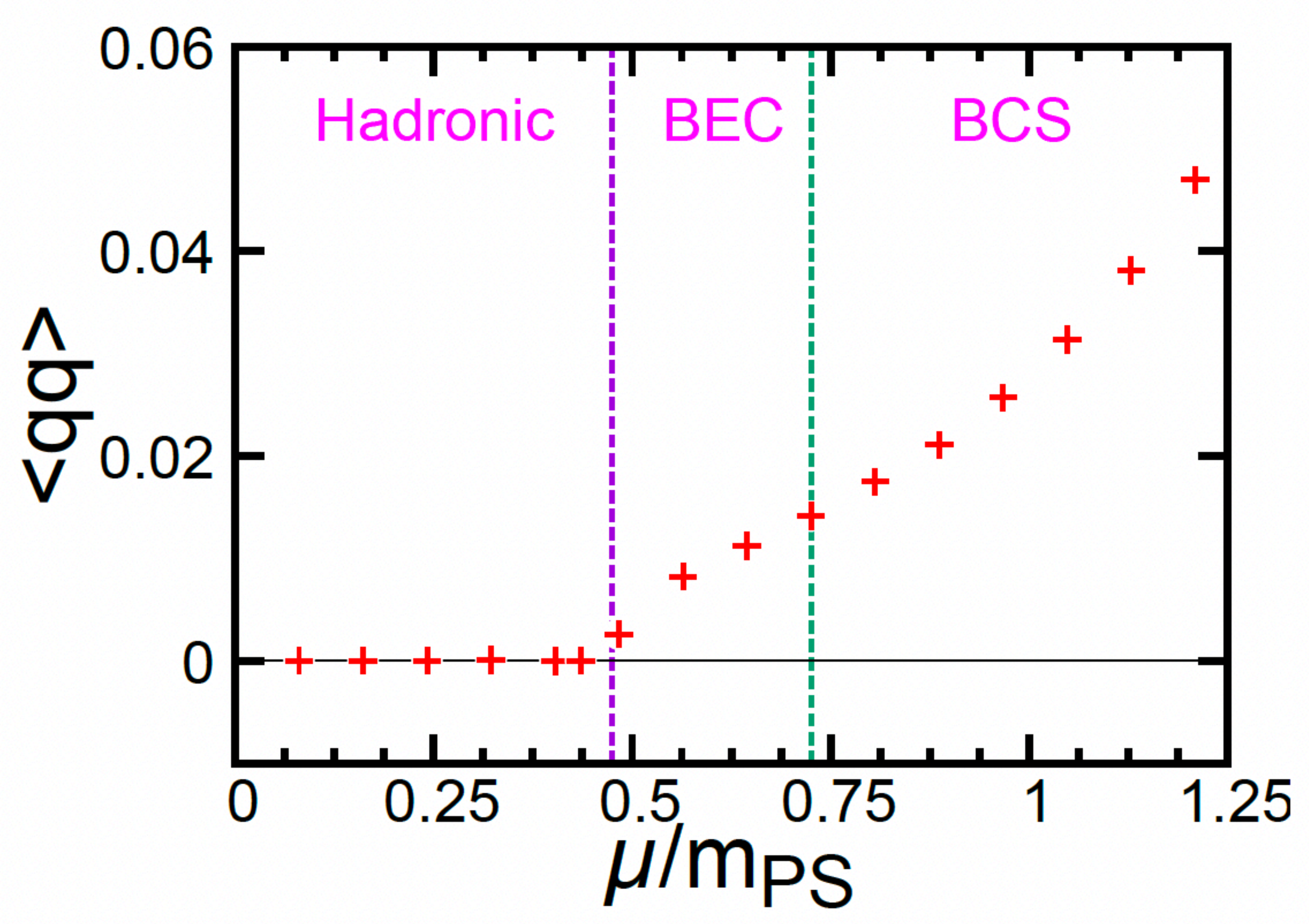
Quark number density



At $\mu/m_{PS} \approx 0.73$, the BEC-BCS crossover.

(Here, m_{PS} denotes the pion mass at $\mu = 0$. $m_{PS} \approx 750\text{MeV}$)

Diquark condensate (T=40MeV)



- The order parameter of the superfluidity
- According to the ChPT, the critical μ is

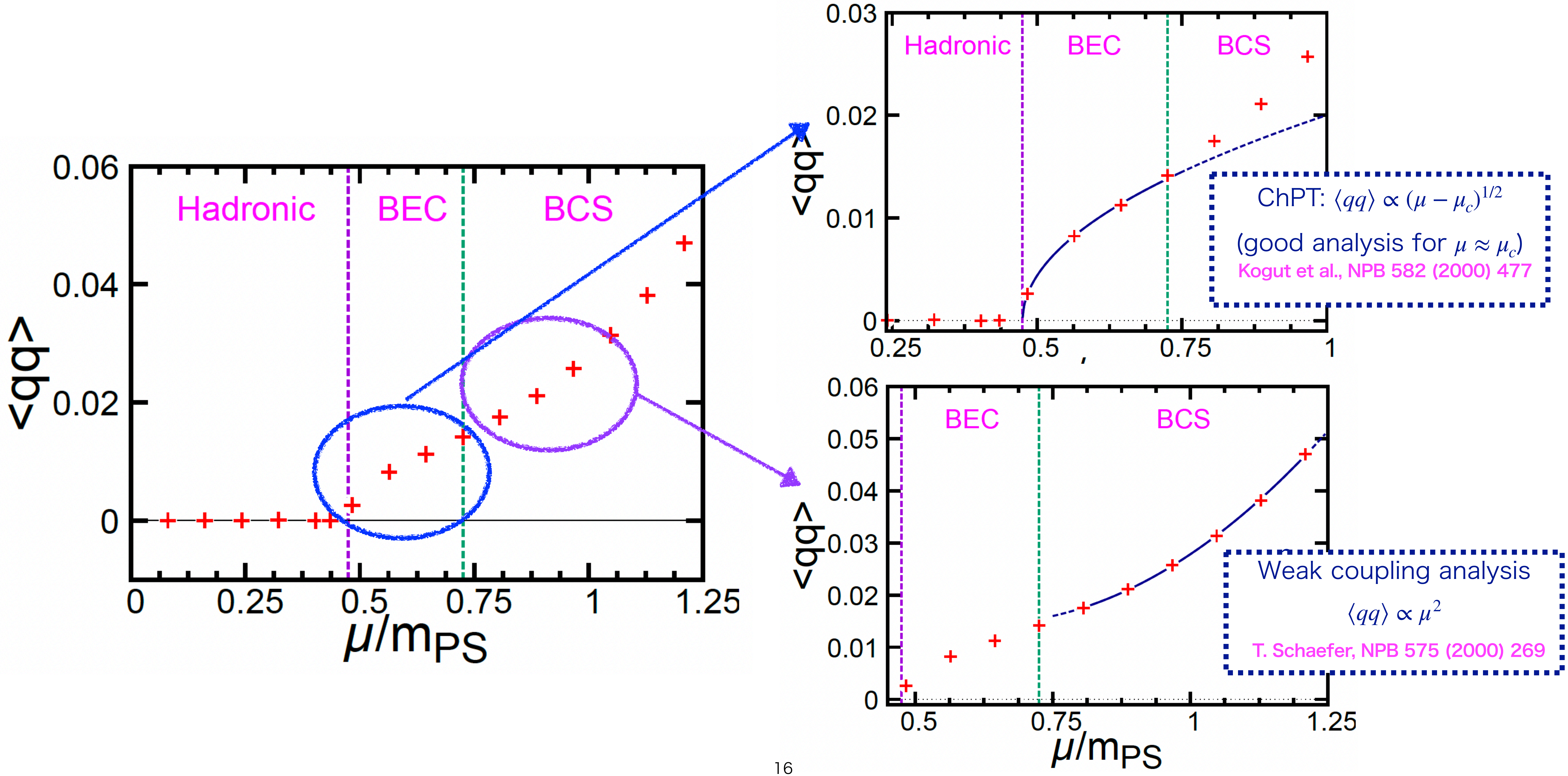
  $\mu/m_{\mathrm{PS}} = 0.5$

- $0 \leq \mu/m_{\mathrm{PS}} \lesssim 0.5$: hadronic phase

  $0.5 \lesssim \mu/m_{\mathrm{PS}} \lesssim 0.73$: BEC phase

  $0.73 \lesssim \mu/m_{\mathrm{PS}}$: BCS phase

Diquark condensate (T=40MeV)



Confine or deconfine in high density?

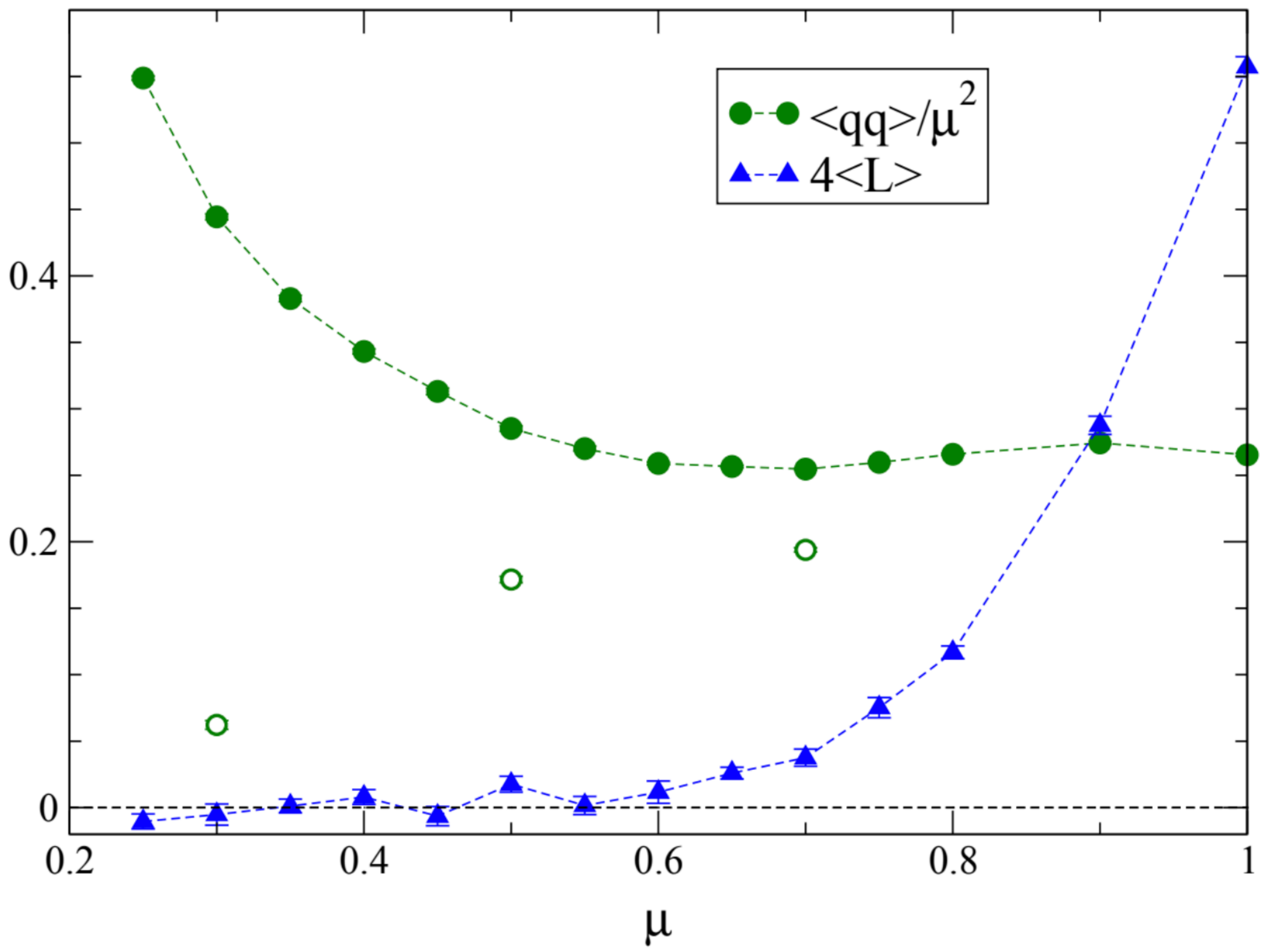


Fig. 10. The Polyakov loop L and the superfluid diquark condensate $\langle qq\rangle/\mu^2$ for $j = 0.04$ as function of μ. Open symbols show $\langle qq\rangle/\mu^2$ extrapolated to $j = 0$.

- Pioneer paper by S.Hands et al. (2006)

Deconfinement in dense 2-color QCD

Simon Hands[1], Seyong Kim[2], and Jon-Ivar Skullerud[3]

[1] Department of Physics, University of Wales Swansea, Singleton Park, Swansea SA2 8PP, UK
[2] Department of Physics, Sejong University, Gunja-Dong, Gwangjin-Gu, Seoul 143-747, Korea
[3] School of Mathematics, Trinity College, Dublin 2, Ireland

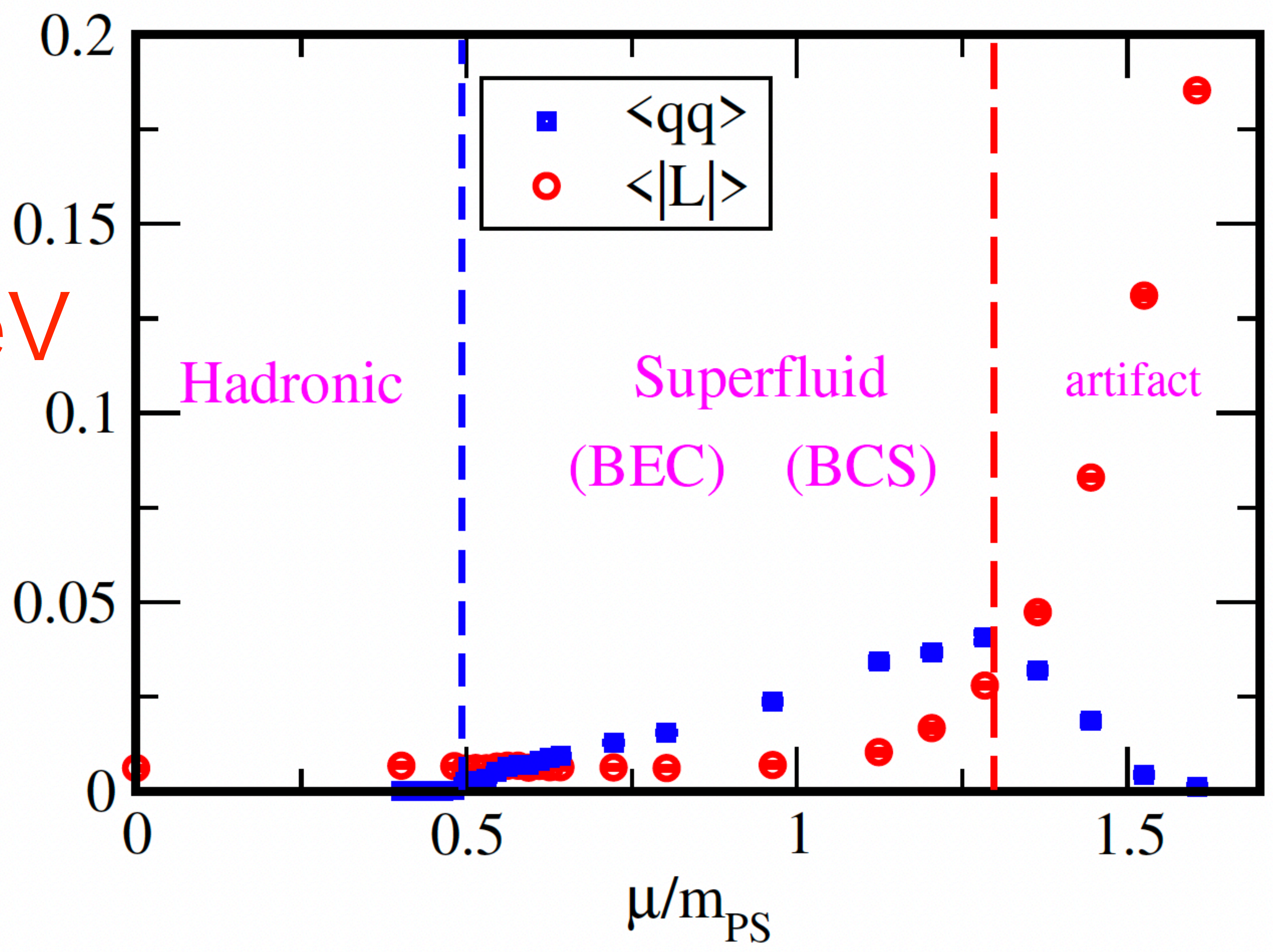
- diquark condensate: $\langle qq\rangle \propto \mu^2$

  Polyakov loop increases at T=45MeV

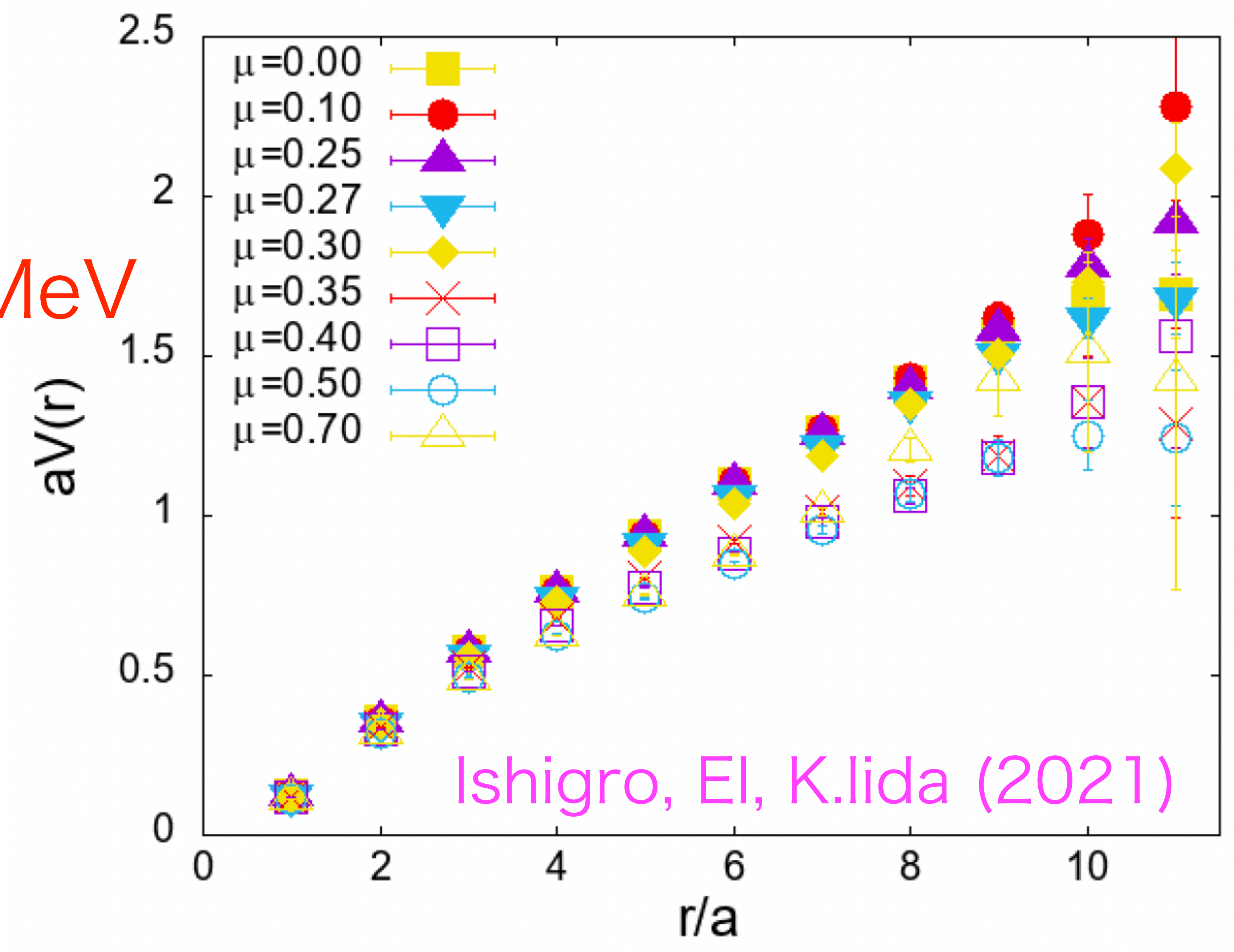
- Superfluid w/ deconfinement

Confine or deconfine in high density?

T=80MeV

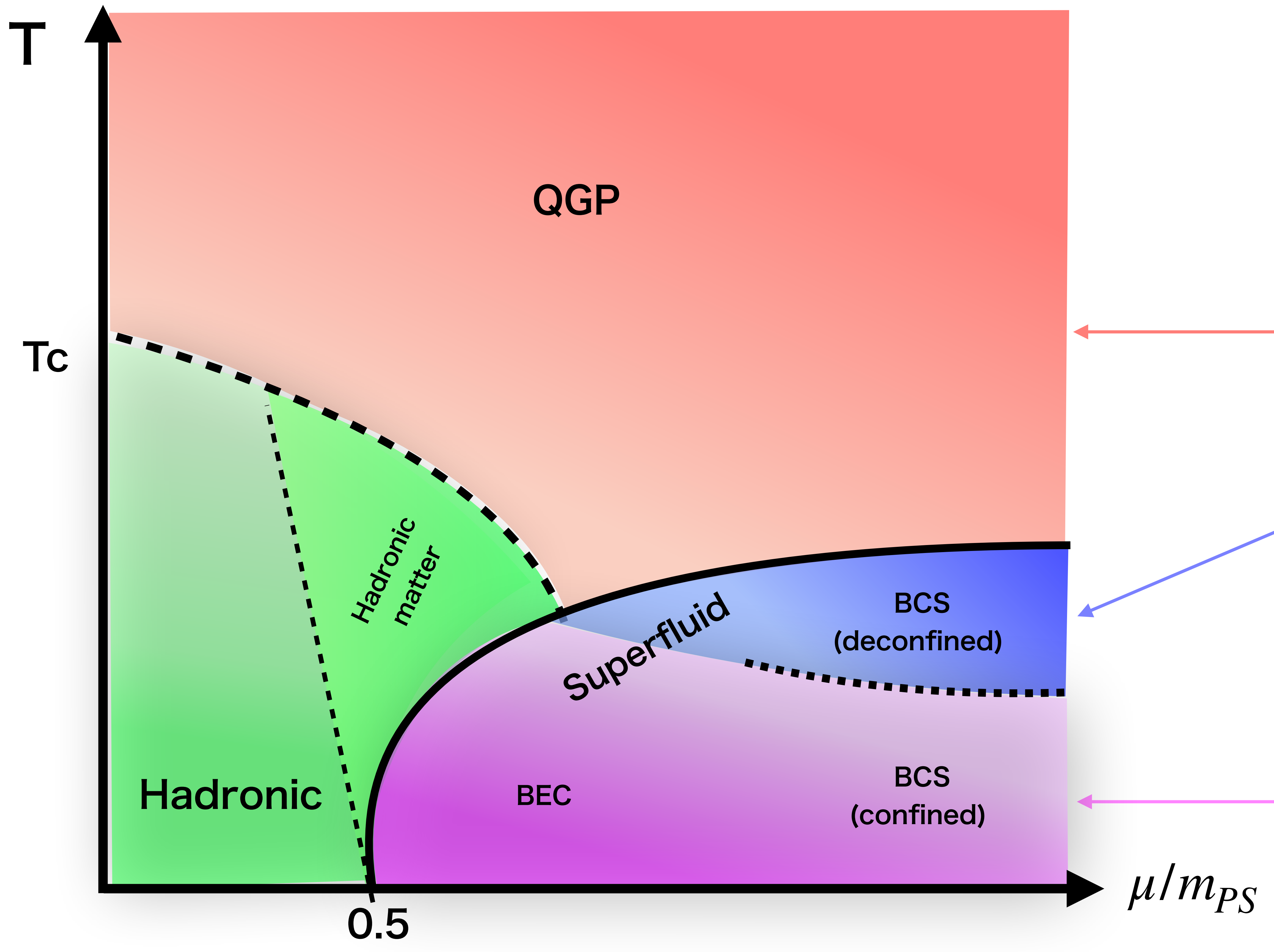


T=40MeV



- We conclude:
  confinement occurs even at T=80MeV
- $q\bar{q}$ potential at T=40MeV also show a linear potential
- Recently, S.Hands's and Russian groups also updated. They conclude T~90-100MeV is the critical T for deconfinement
- In 2color QCD, even in high-density $\mu \approx$ 1GeV, the confinement occurs. Hadronic superfluidity not quarkionic superfluidity?

Current status on 2color QCD phase diagram



At least 4 independent groups are studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

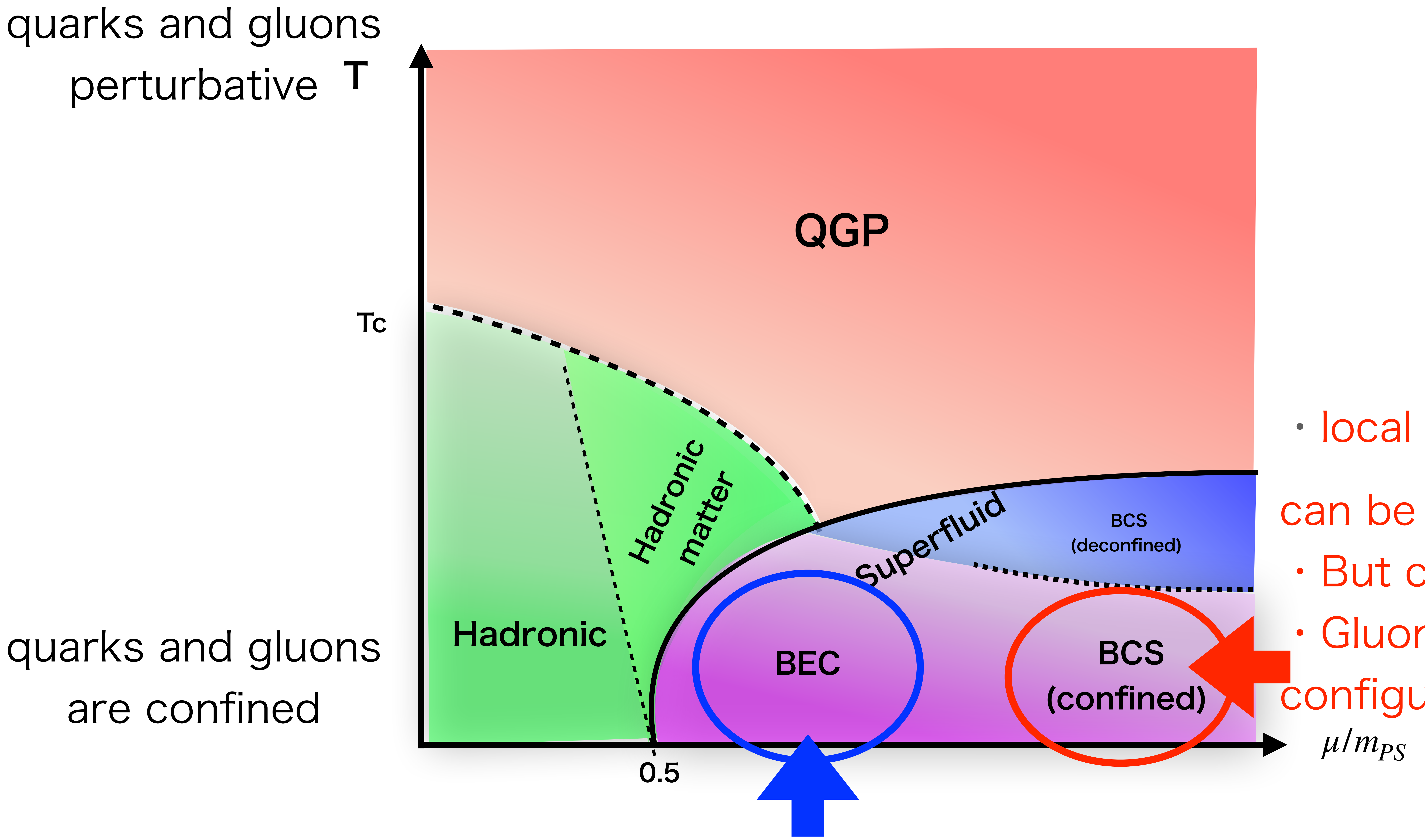
T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)

- Even $T \approx 100\mathrm{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq T_{SF}$ (superfluid/QGP) : constraint from 't Hooft anomaly matching

T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

- 2color QCD phase diagram has been determined by independent works!

Short summary for phase diagram



- local quantities, $\langle n_q \rangle$, $\langle qq \rangle$, can be described by free theory
- But confinement remains.
- Gluon has nontrivial instanton configuration

Predictions by ChPT works very well!

Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01
K.Iida, EI, K.Murakami, D.Suenaga, e-Print: 2405.20566 to appear JHEP

Equation of state

- trace anomaly: $\epsilon - 3p = \frac{1}{N_s^3}\left(a\frac{d\beta}{da}|_{LCP}\langle\frac{\partial S}{\partial \beta}\rangle_{sub.} + a\frac{d\kappa}{da}|_{LCP}\langle\frac{\partial S}{\partial \kappa}\rangle_{sub.} + a\frac{\partial j}{\partial a}\langle\frac{\partial S}{\partial j}\rangle \right)$

  No renormalization for μ

  $\langle \cdot \rangle_{sub.} = \langle \cdot \rangle_{\mu,T} - \langle \cdot \rangle_{\mu=0,T}$

  Zero at $j \to 0$

- pressure: $p(\mu) = \int_{\mu_o}^{\mu} n_q(\mu')d\mu'$

Early works for EoS in dense 2color QCD
Hands et al. (2006)
Hands et al. (2012), T~47MeV (coarse lattice)
Astrakhantsev et al. (2020), T~140MeV

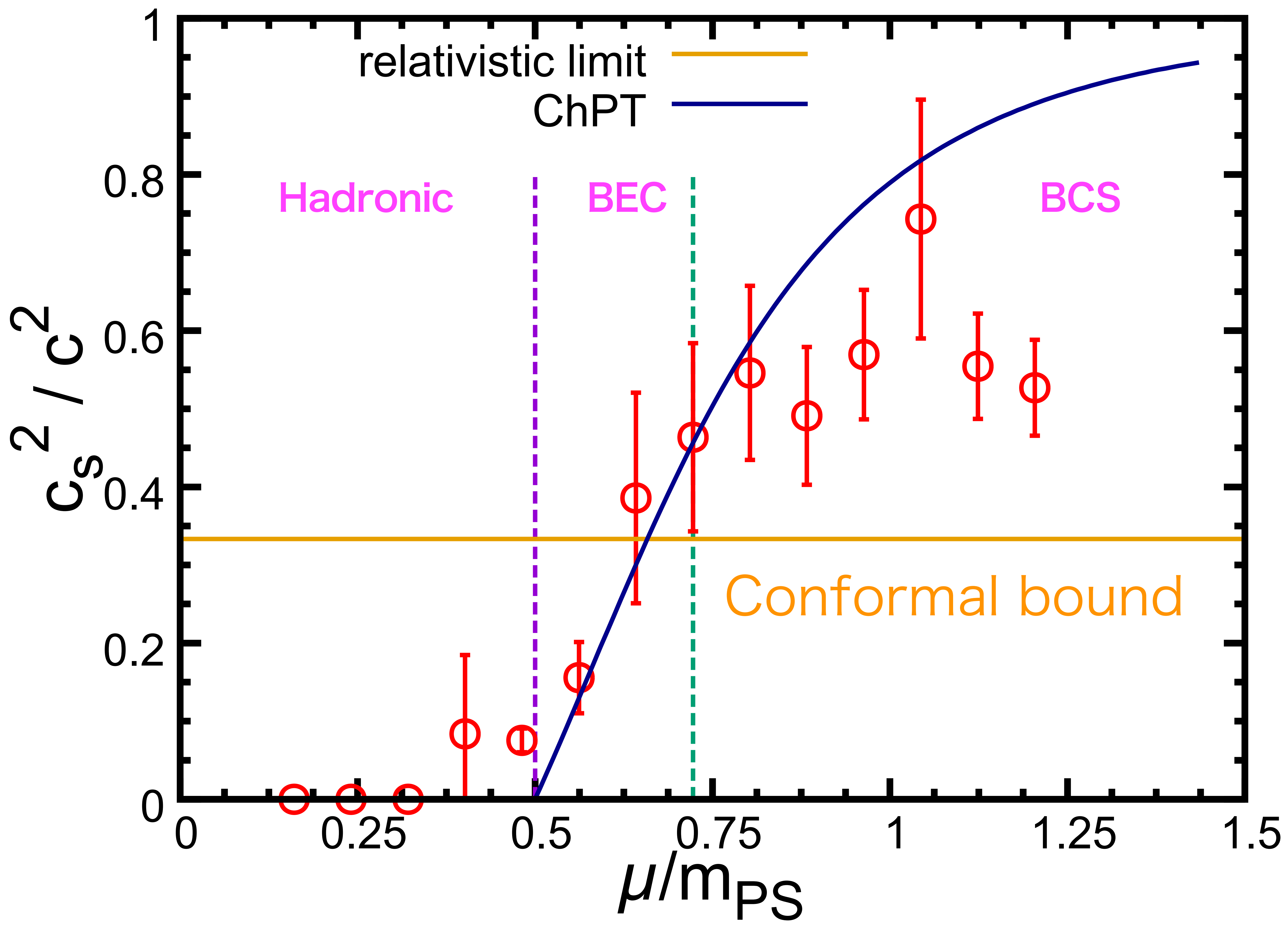
Our work
Nonperturbatively calculate beta fn.

$a\frac{d\beta}{da} = -0.3521, \quad a\frac{d\kappa}{da} = 0.02817$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

Sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$), T=80MeV ($16^4$ lattices)

K.Iida and EI, PTEP 2022 (2022) 11, 111B01



Chiral Perturbation Theory (ChPT)

$$c_s^2/c^2 = \frac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$$: no free parameter!!

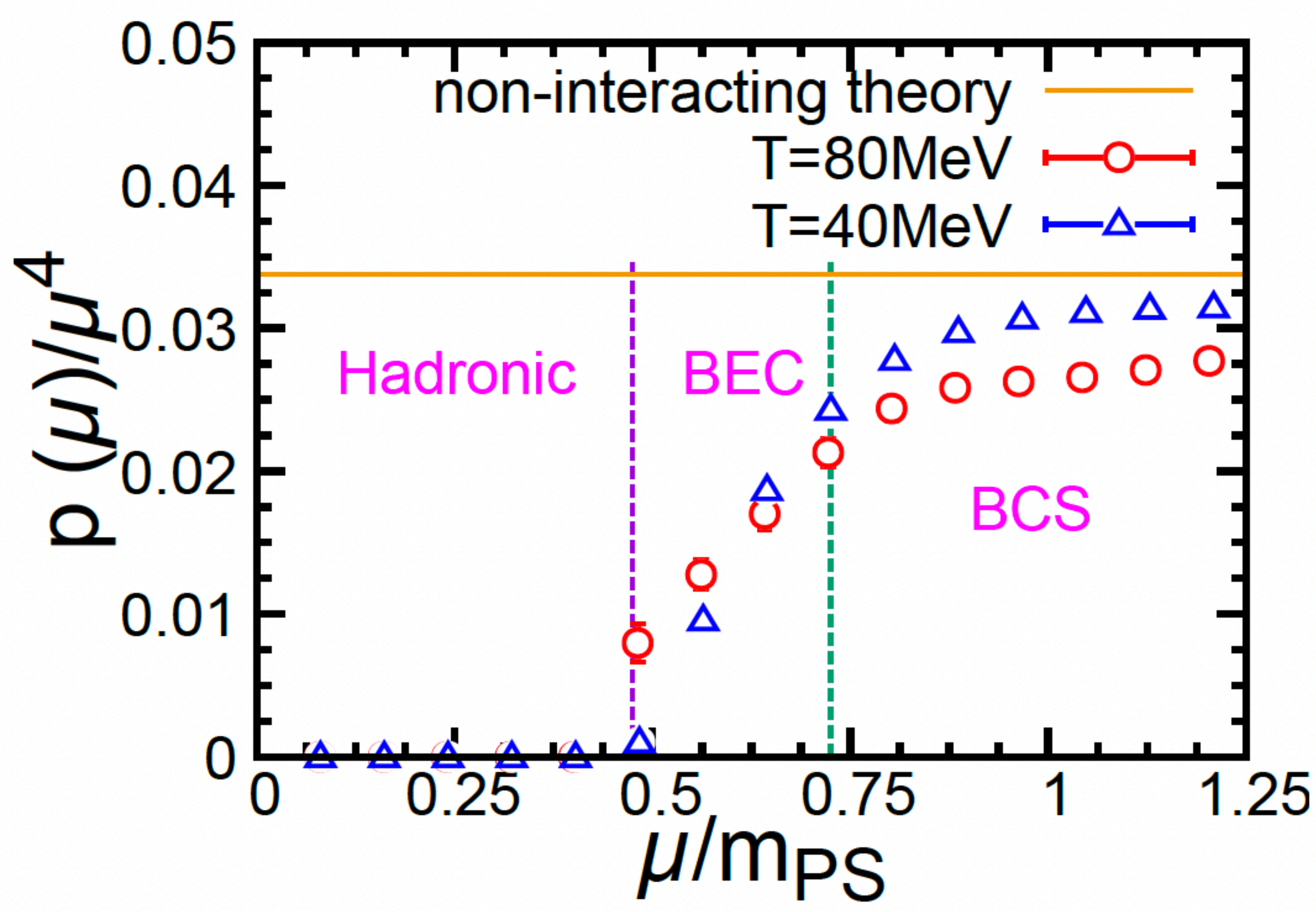
Son and Stephanov (2001) : 3color QCD with isospin μ

Hands, Kim, Skullerud (2006) : 2color QCD with real μ

- In BEC phase, our result is consistent with ChPT.
- c_s^2/c^2 exceeds the conformal bound
- We calculated $\left.\frac{\partial p}{\partial e}\right|_{T=\mathrm{const.}}$

  The sound velocity squared is $\left.\frac{\partial p}{\partial e}\right|_{s=\mathrm{const.}}$
- Studying the $T \to 0$ limit is important

T dependence of EoS

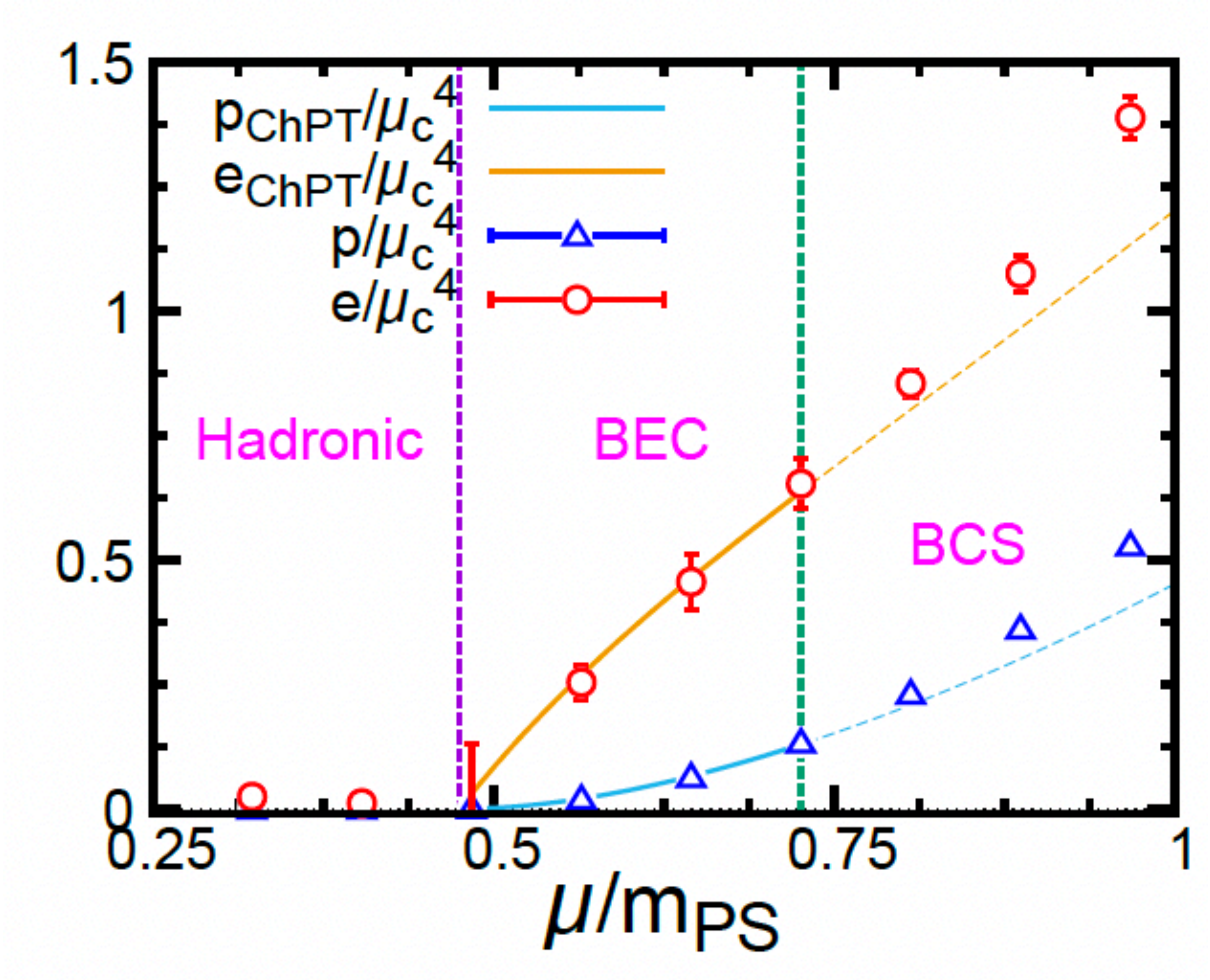


- p increases more rapidly near the critical point at lower-T
- In high-μ, the data approaches the Stefan-Boltzmann limit (=non-interacting theory)

$$p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$$

- Our largest data of p at T=40MeV reaches at 93% of p_{SB}

EoS and consistency with ChPT result in BEC



- ChPT prediction (valid for near μ_c)

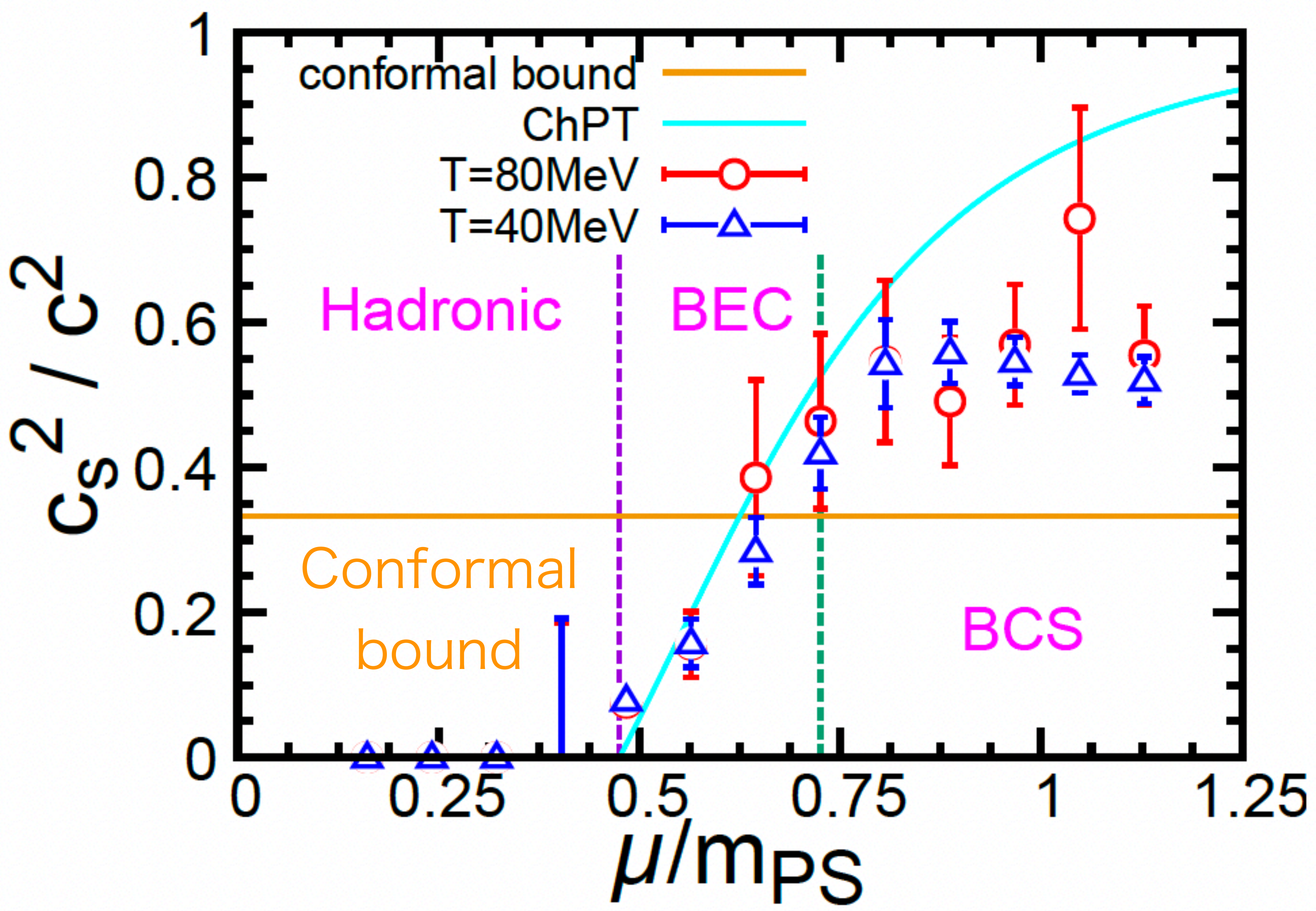
$$p_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)^2$$

$$e_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)\left(1 + 3\frac{\mu_c^2}{\mu^2}\right)$$

- We obtain the pion decay constant(F) from fit of p : F=51.1(5) MeV
from fit of e : F=56.7(7) MeV
cf.) F=60.8(1.6) by fitting of $\langle n_q \rangle$ at 140MeV (different mass, staggered fermion)

N. Astrakhantsev et al. (2020)

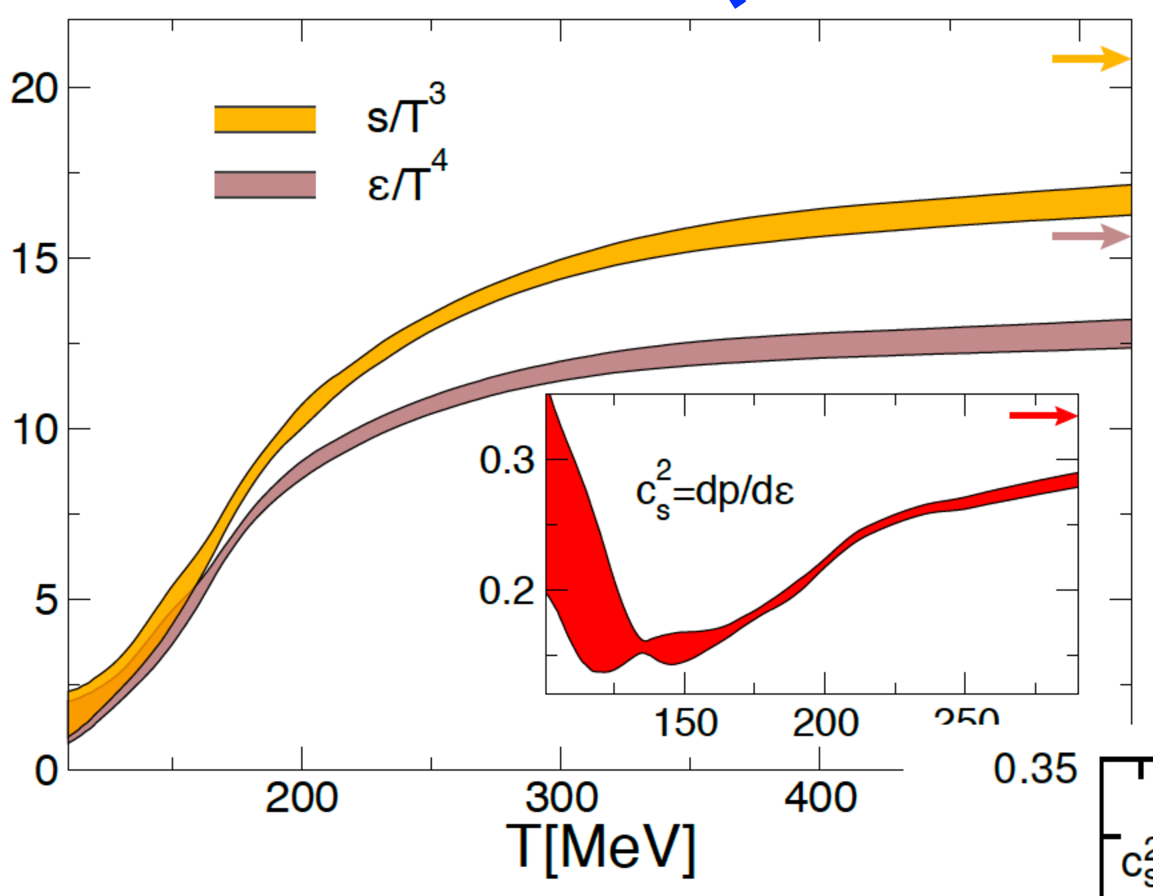
Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



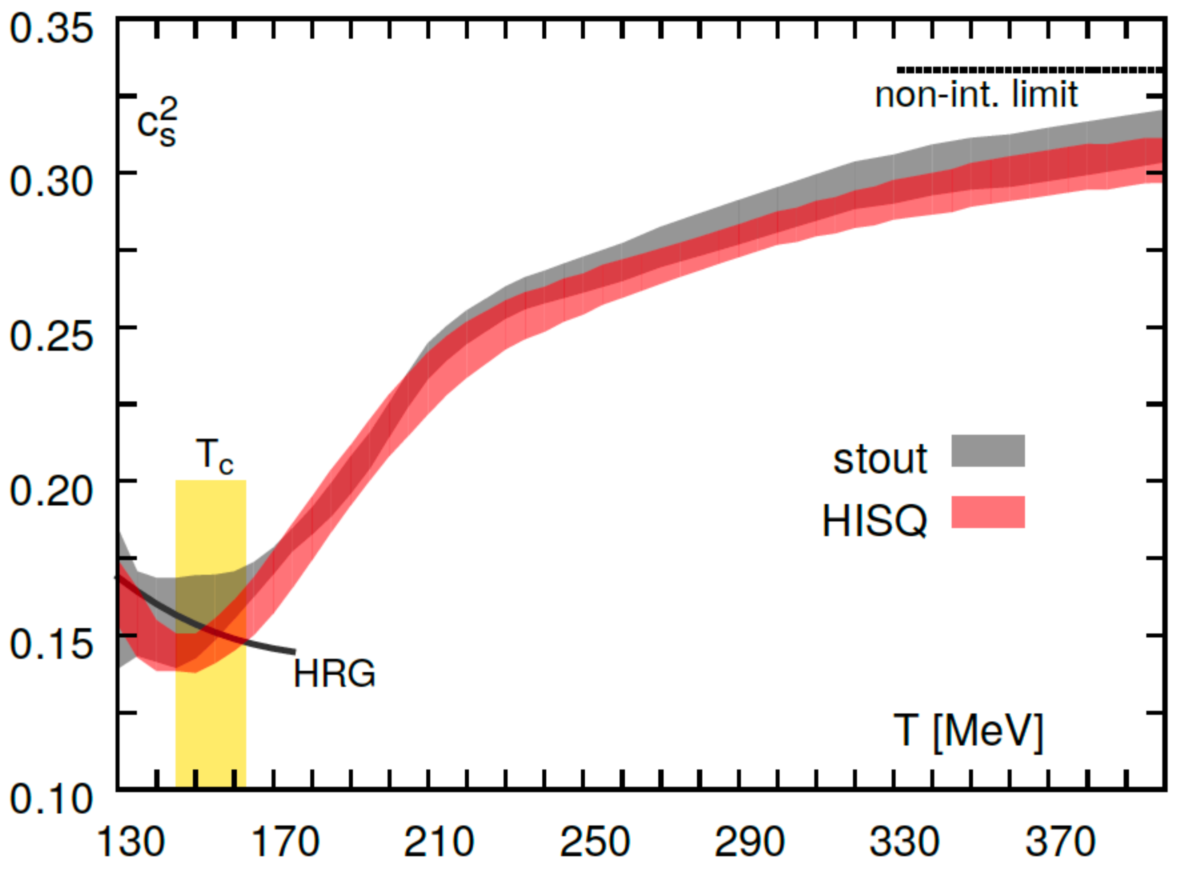
- T-dependence of the sound velocity is negligible!
- In BEC phase, our result is consistent with ChPT
- It exceeds the conformal bound
- Confirmed by the data with small statistical errors!!

Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)



Borsanyi et al. (2013)

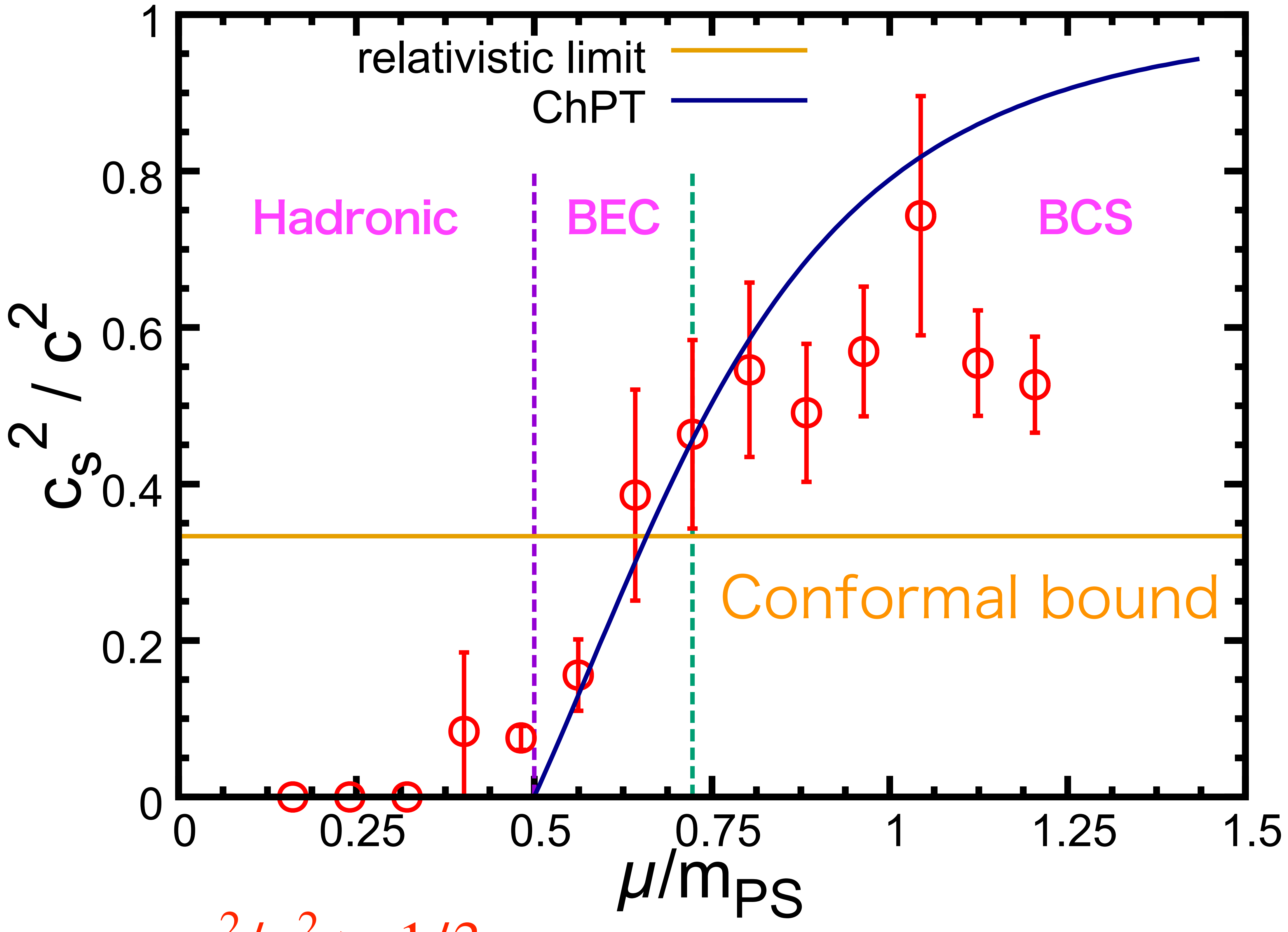


HotQCD (2014)

- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

Finite Density transition (Nf=2 2color QCD)

Iida and Itou arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories.

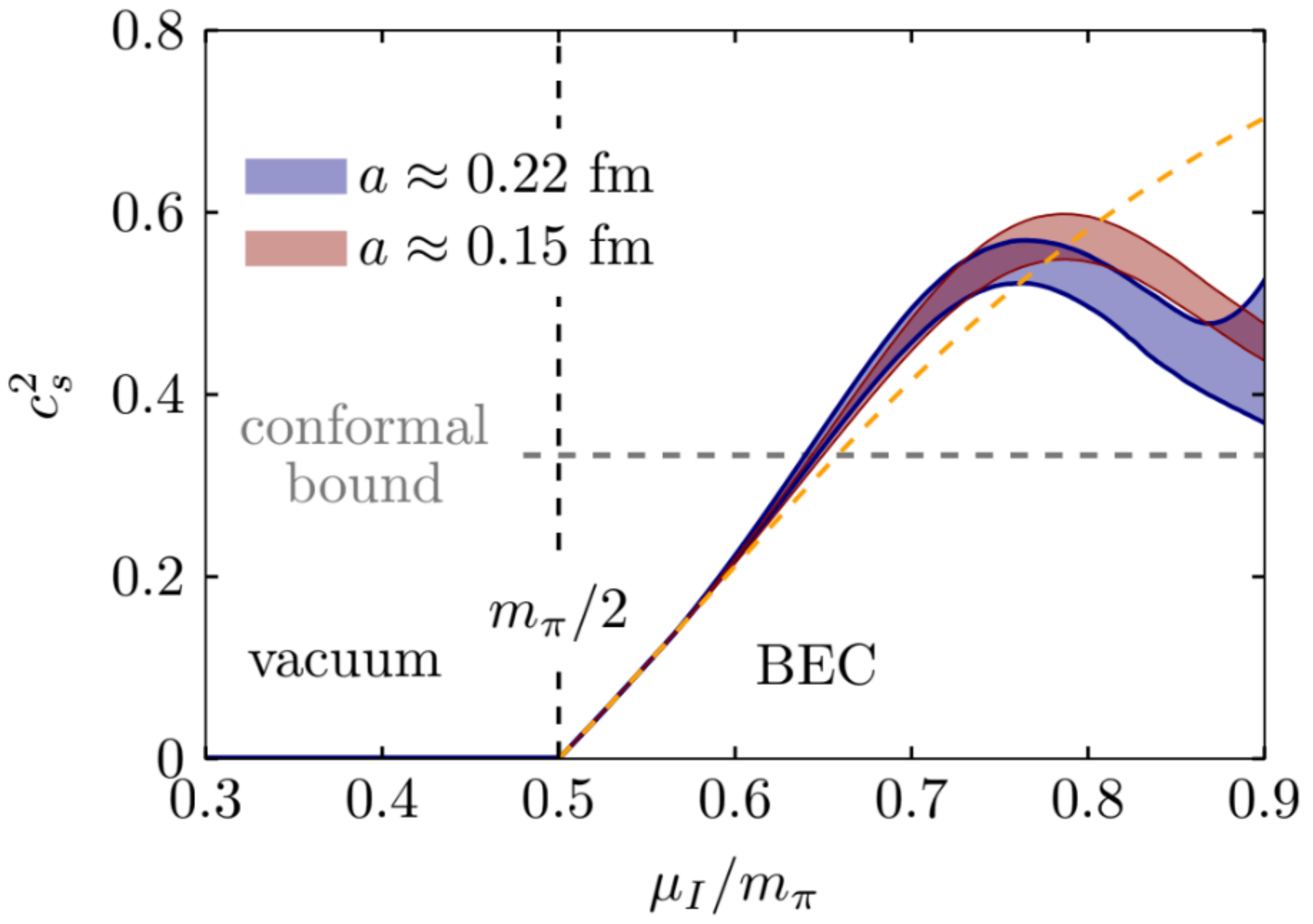
Lattice MC for 3 color QCD with isospin chemical potential

3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

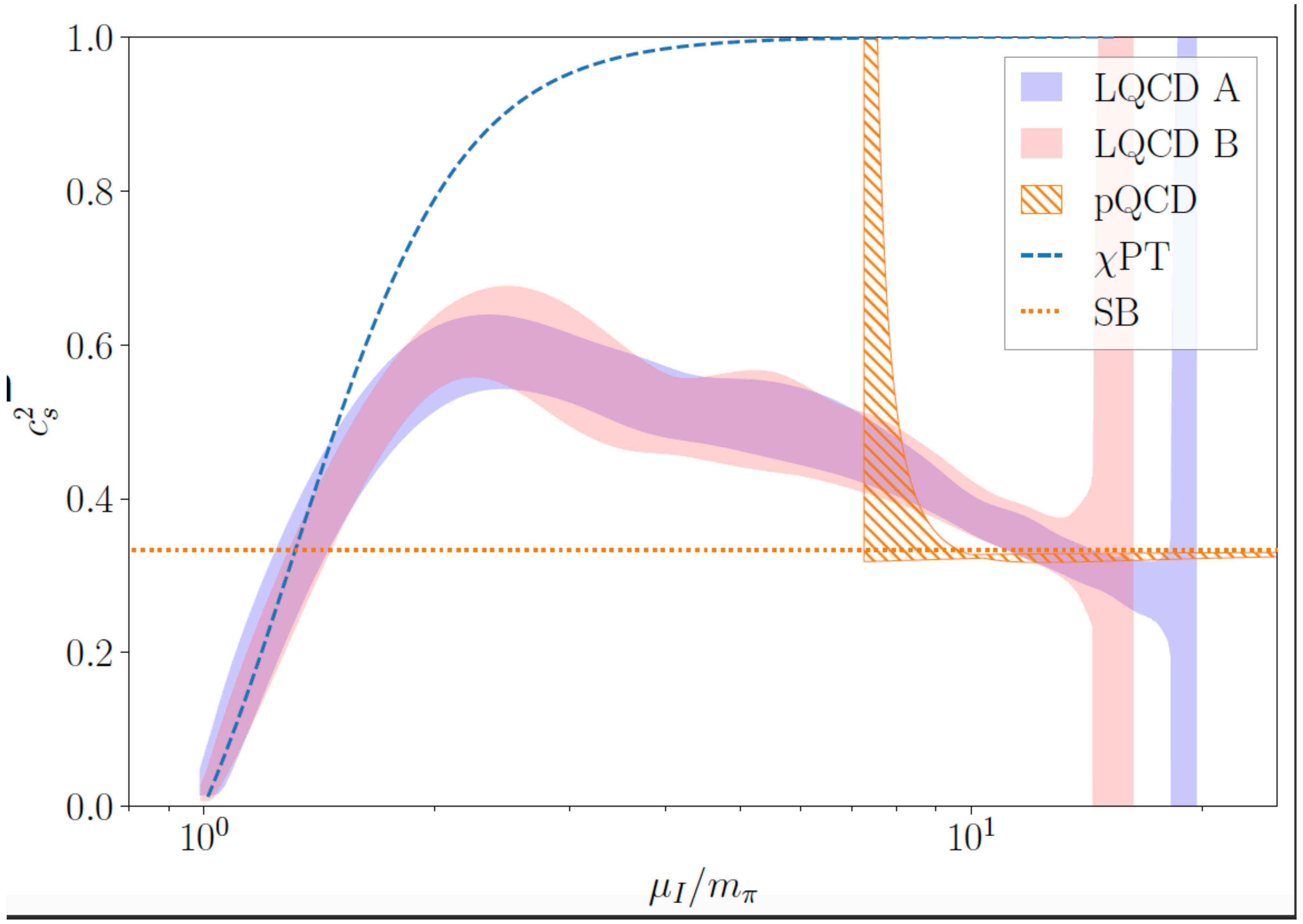
B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

R. Abbott et al. arXiv:2307.15014

Result with spline interpolation



New algorithm for n-point fn. calc.



Conformal bound (Holography bound)?

conjecture : $c_s^2/c^2 \leq 1/3$ is valid for a broad class of 4-dim. theories

A bound on the speed of sound from holography

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We show that the squared speed of sound v_s^2 is bounded from above at high temperatures by the conformal value of $1/3$ in a class of strongly coupled four-dimensional field theories, given some mild technical assumptions. This class consists of field theories that have gravity duals sourced by a single scalar field. There are no known examples to date of field theories with gravity duals for which v_s^2 exceeds $1/3$ in energetically favored configurations. We conjecture that $v_s^2 = 1/3$ represents an upper bound for a broad class of four-dimensional theories.

Several strong evidences of $c_s^2/c^2 > 1/3$ are found in finite density QCD-like theory using Lattice Monte Carlo

Counterexamples of conformal bound

PHYSICAL REVIEW D **94,** 106008 (2016)

Breaking the sound barrier in holography

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It has been conjectured that the speed of sound in holographic models with UV fixed points has an upper bound set by the value of the quantity in conformal field theory. If true, this would set stringent constraints for the presence of strongly coupled quark matter in the cores of physical neutron stars, as the existence of two-solar-mass stars appears to demand a very stiff equation of state. In this article, we present a family of counterexamples to the speed of sound conjecture, consisting of strongly coupled theories at finite density. The theories we consider include $\mathcal{N} = 4$ super Yang-Mills at finite R-charge density and nonzero gaugino masses, while the holographic duals are Einstein-Maxwell theories with a minimally coupled scalar in a charged black hole geometry. We show that for a small breaking of conformal invariance, the speed of sound approaches the conformal value from above at large chemical potentials.

N=4 SYM at finite density

Evidence against a first-order phase transition in neutron star cores: impact of new data

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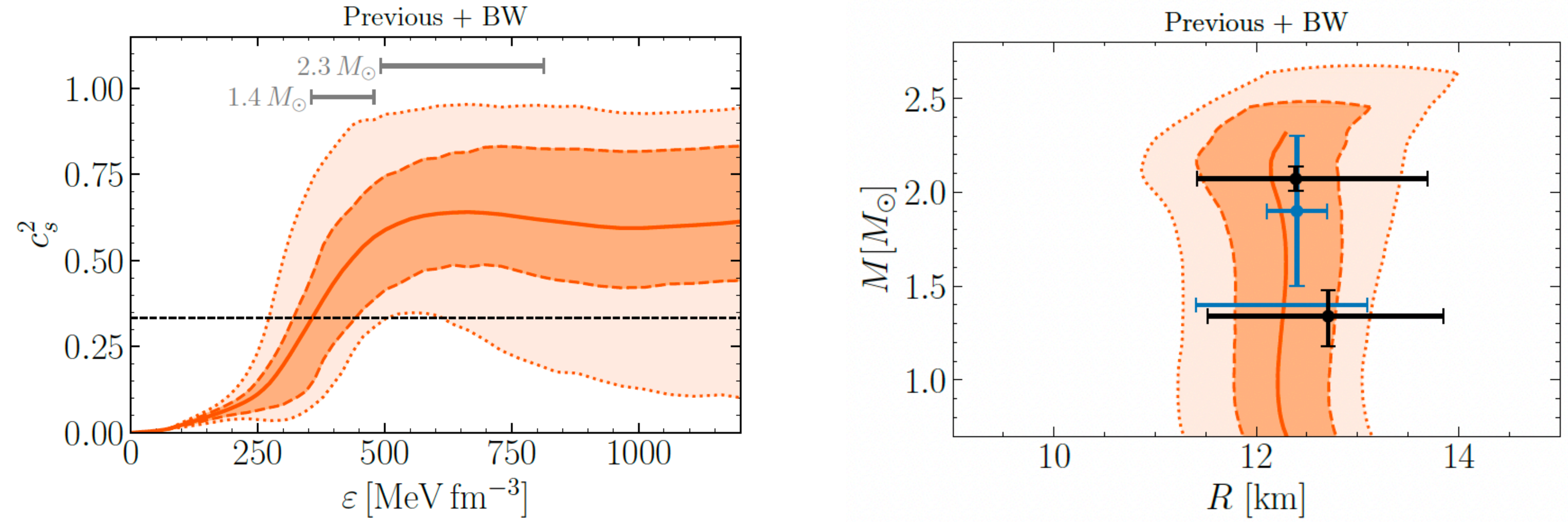
(Dated: June 13, 2023)

With the aim of exploring the evidence for or against phase transitions in cold and dense baryonic matter, the inference of the sound speed and equation-of-state for dense matter in neutron stars is extended in view of recent new observational data. The impact of the heavy (2.35 $M_\odot$) black widow pulsar PSR J0952-0607 and of the unusually light supernova remnant HESS J1731-347 is inspected. In addition a detailed re-analysis is performed of the low-density constraint based on chiral effective field theory and of the perturbative QCD constraint at asymptotically high densities, in order to clarify the influence of these constraints on the inference procedure. The trace anomaly measure, $\Delta = 1/3 - P/\varepsilon$, is also computed and discussed. A systematic Bayes factor assessment quantifies the evidence (or non-evidence) of a phase transition within the range of densities realised in the core of neutron stars. One of the consequences of including PSR J0952-0607 in the data base is a further stiffening of the equation-of-state, resulting for a typical 2.1 solar-mass neutron star in a reduced central density of less than five times the equilibrium density of normal nuclear matter. The evidence against the occurrence of a first-order phase transition in neutron star cores is further strengthened.

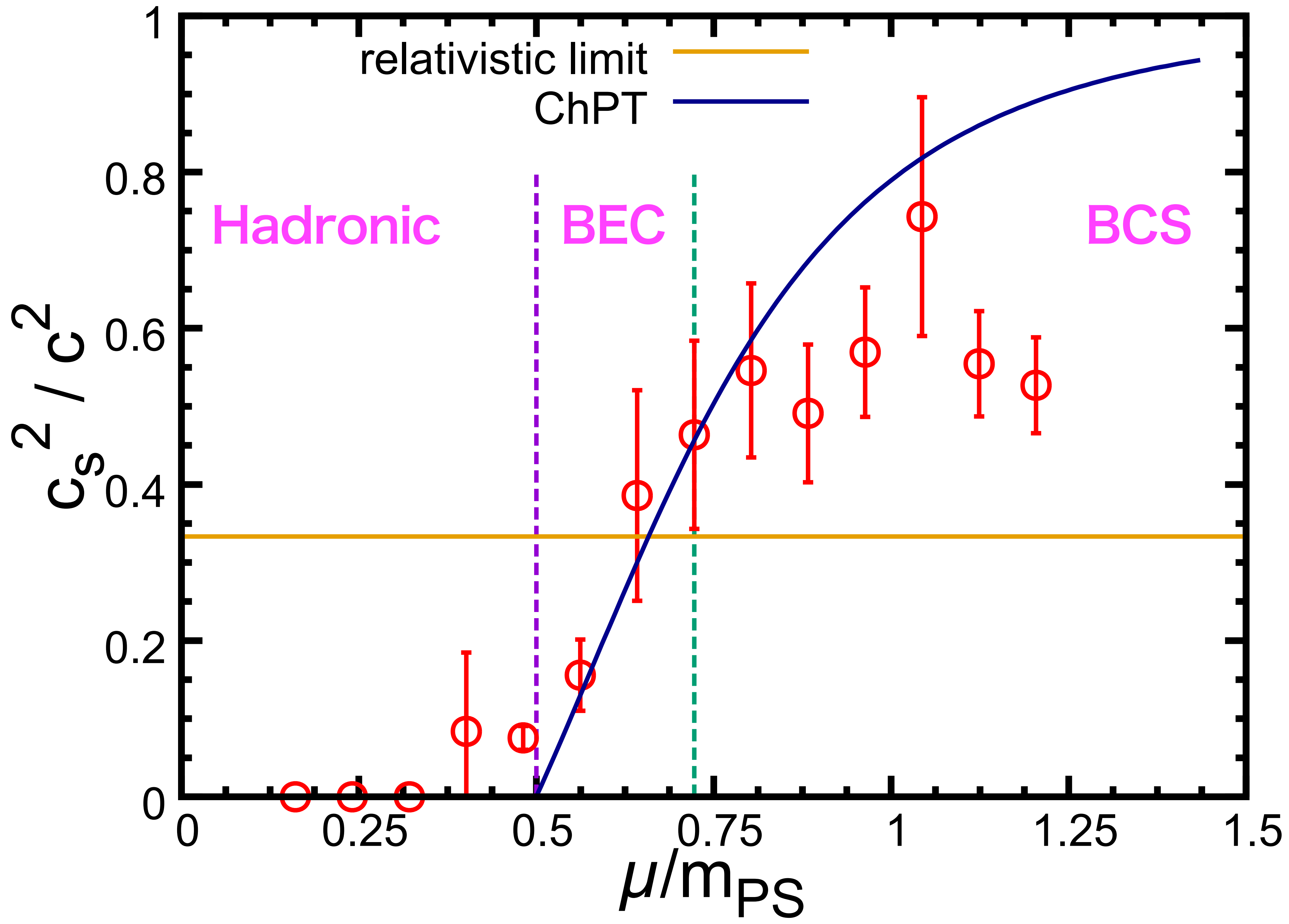
Bayesian analyses of recent observation data of neutron star



arXiv:2306.06218

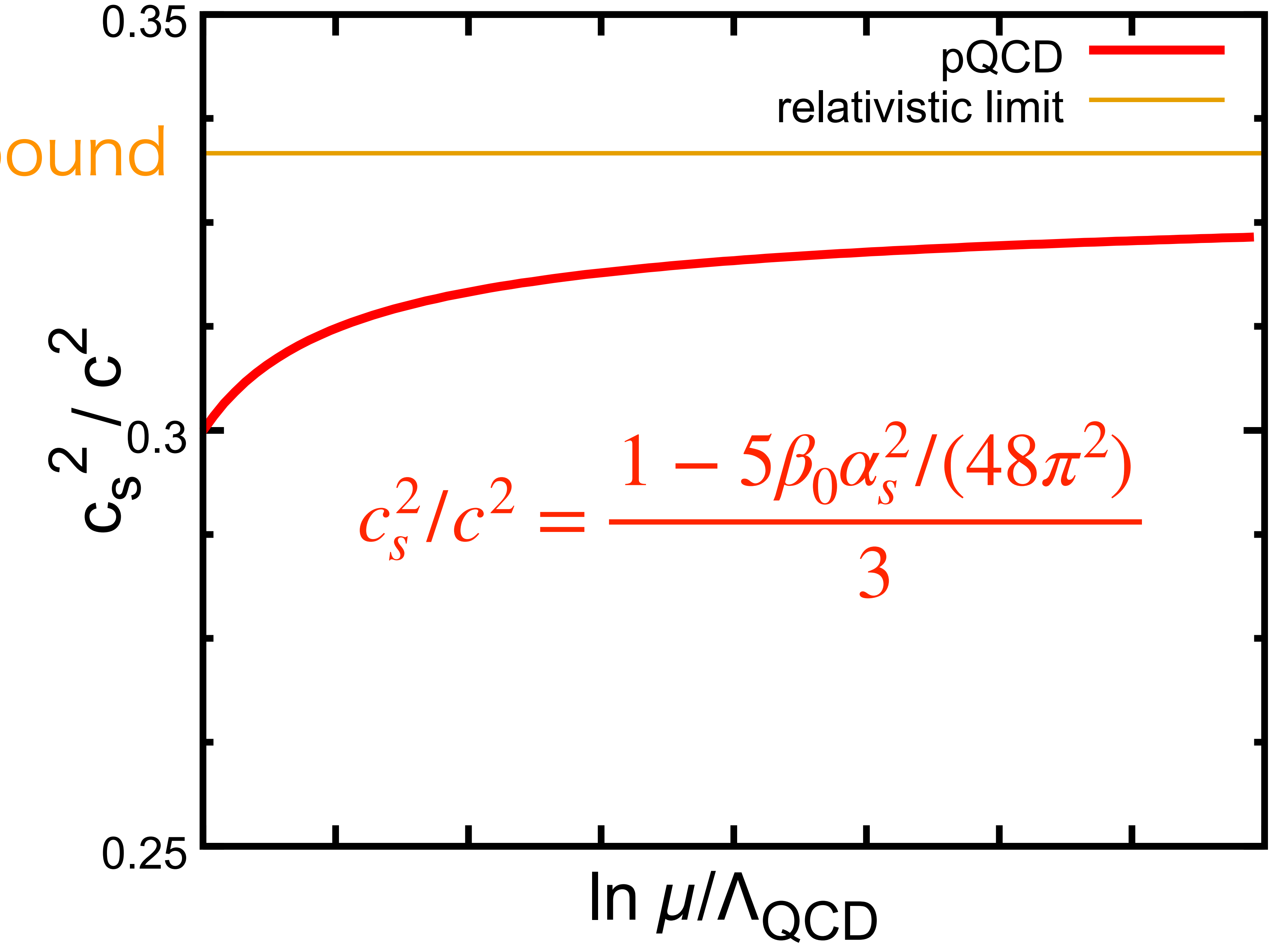
Further high density?

Kojo, Baym, Hatsuda (2021)



pQCD prediction
(Ultra high-density regime)

Conformal bound



- Upper bound of chemical potential in lattice simulation comes from $a\mu \ll 1$
  (Here, we take $a\mu \leq 0.8$)
- To study high-density, the lighter mass / finer lattice spacing are needed

Summary and future work

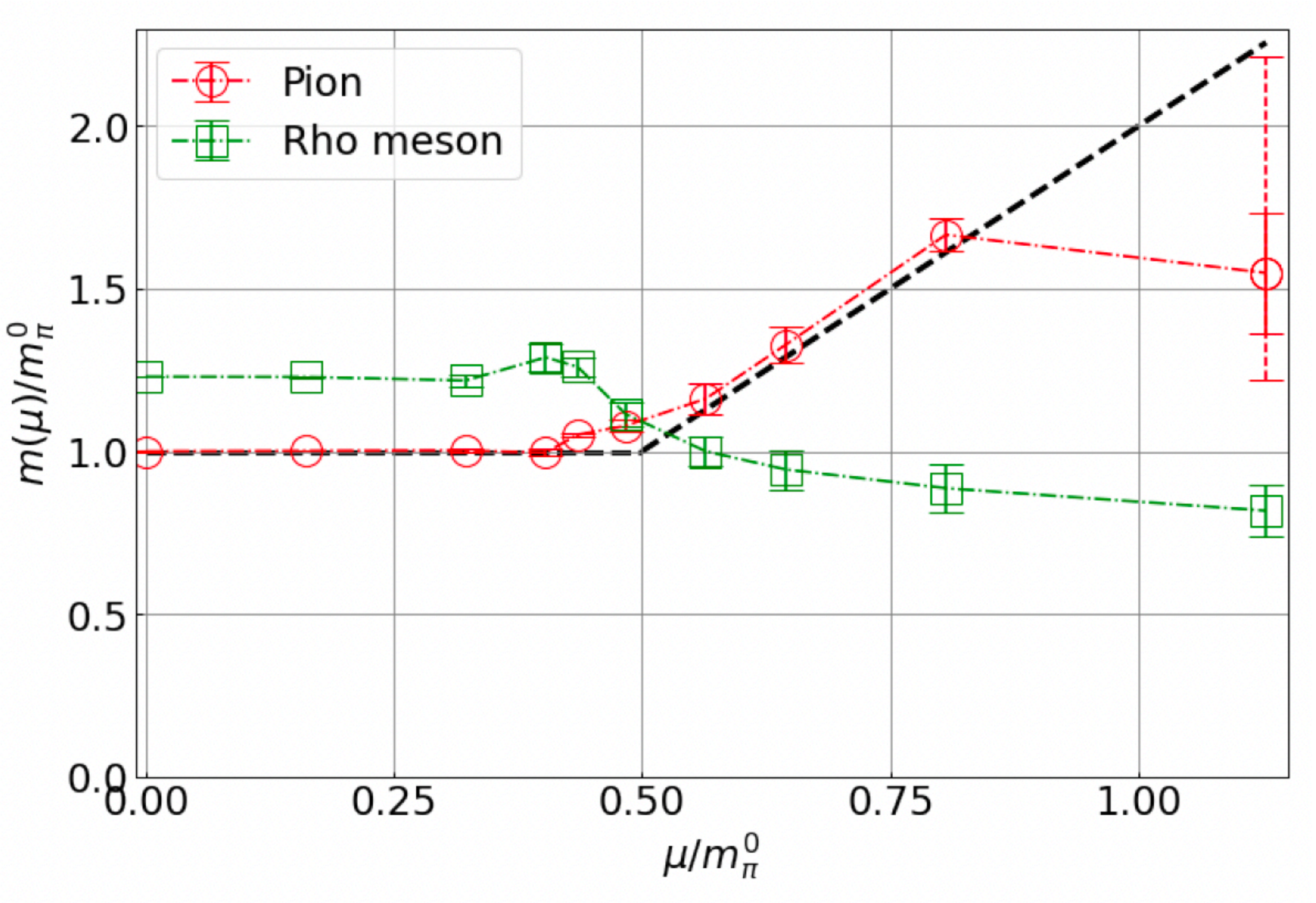
- Lattice numerical simulation for QCD-like theory w/o the sign problem has been ongoing
  2color finite-density QCD , 3color w/ isospin chemical potential

- Sound velocity exceeds the conformal bound in finite-μ QCD-like theory
  Several lattice results have been obtained in the last few years

- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), strong coupling with trace anomaly? (McLerran,Fukushima, Fujimoto et al.), others?
    Effective model analyses combined with the lattice results are also ongoing
  - Lattice study on hadron interaction potential
    => extended HAL QCD method in finite density
    => mass spectrum in superfluid phase
  - independent of the color dof?

cf.) D.Suenaga (Thu), Y.Fujimoto (Fri), Minato and Fukushima...

Mass spectrum in superfluid phase



K.Murakami, D.Suenaga, K.Iida, EI,
PoS LATTICE2022 (2023) 154

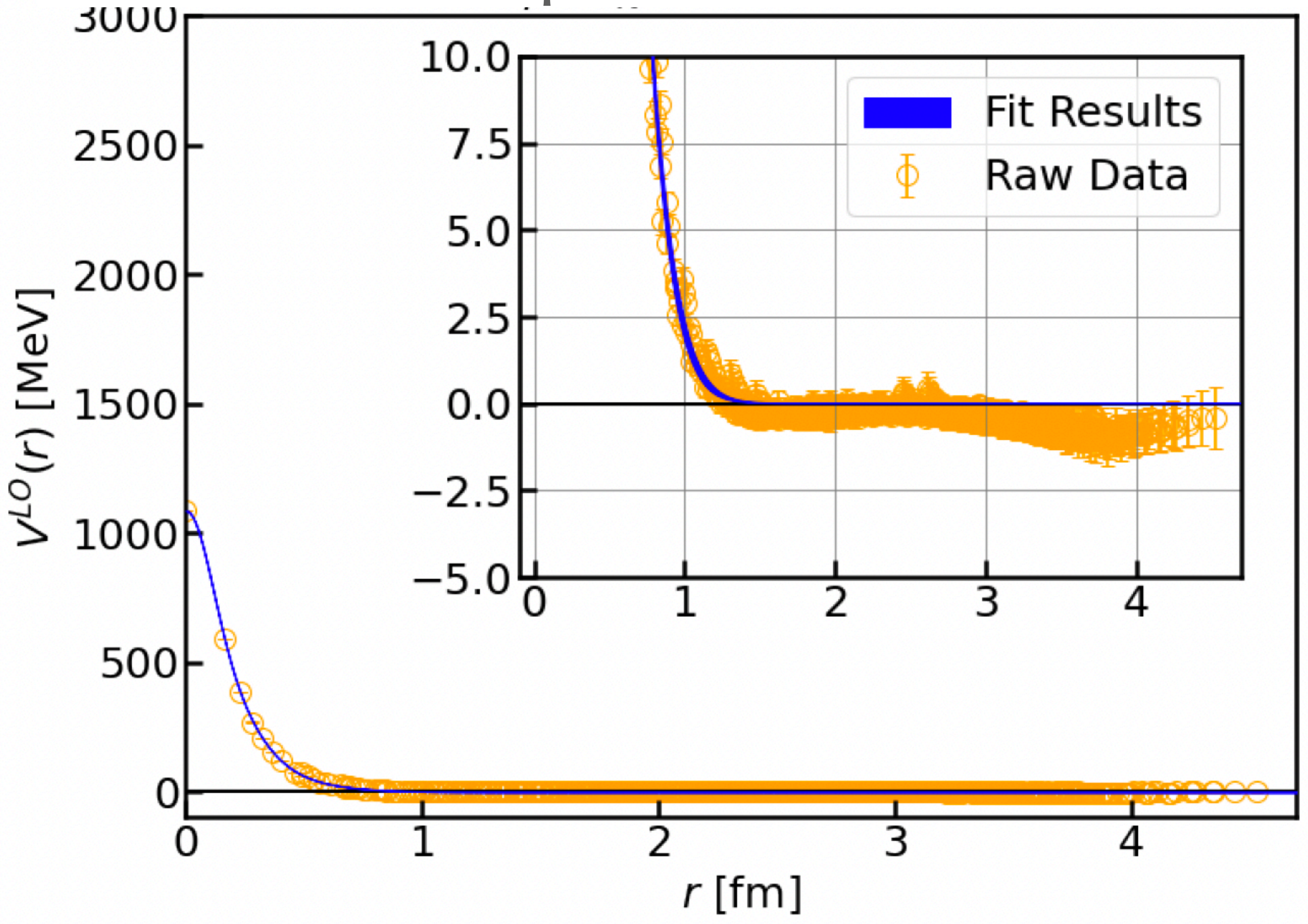
- It is observed that the order of hadron spectra are changed in superfluid phase
- rho meson becomes lighter than pion
- Such a changing is also predicted in 3color QCD Hatsuda-Lee(1992)

Hadron potential

- In hadronic phase, pion and diquark potential are equivalent because of extended flavor symmetry.

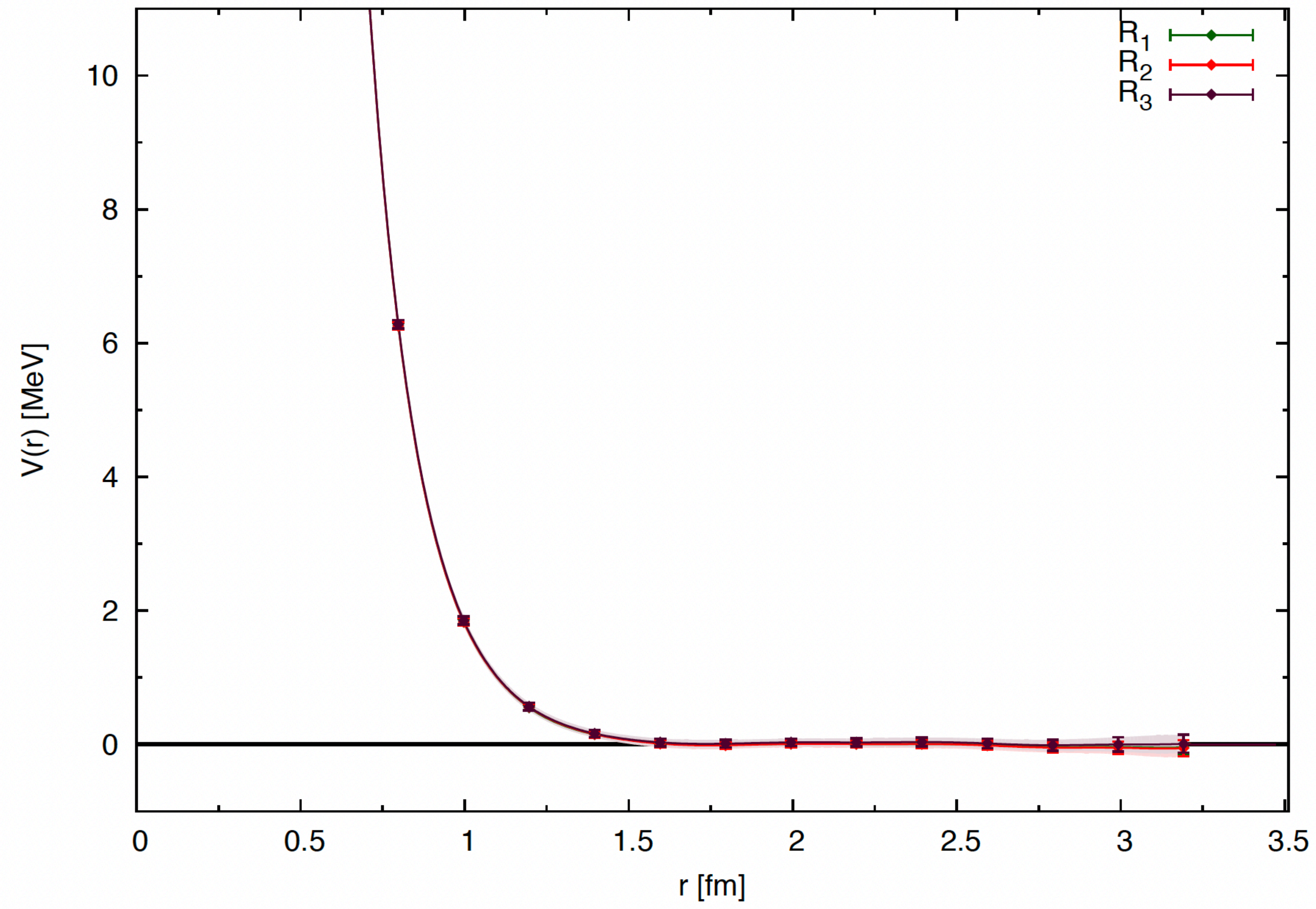
- Pion potential for 2color and 3color QCD are qualitatively same

Diquark-diquark potential in hadronic phase of 2color QCD



K.Murakami, K.Iida, EI, JHEP 11 (2023) 231

I=2 $\pi\pi$ potential of 3color QCD



T.Kurth et al.(HAL QCD coll.), JHEP12(2013)015

backup

Two problems at low-T high-μ QCD

- Sign problem (at $\mu \neq 0$ $S_E[U]$ takes complex value)

➡ Reduce the color dof, 2color QCD
quarks becomes pseudo-real reps.
The sign problem is absent from 2color QCD with even Nf

- Onset problem in low-T, high-μ (e.g. $\mu_q > m_\pi/2, \;\; m_N/3$),
It comes from the phase transition to superfluid phase(SSB of baryon sym.)

Kogut et al. NPB642 (2002)18

➡ Add an explicit breaking term of the sym., then take $j \to 0$ limit

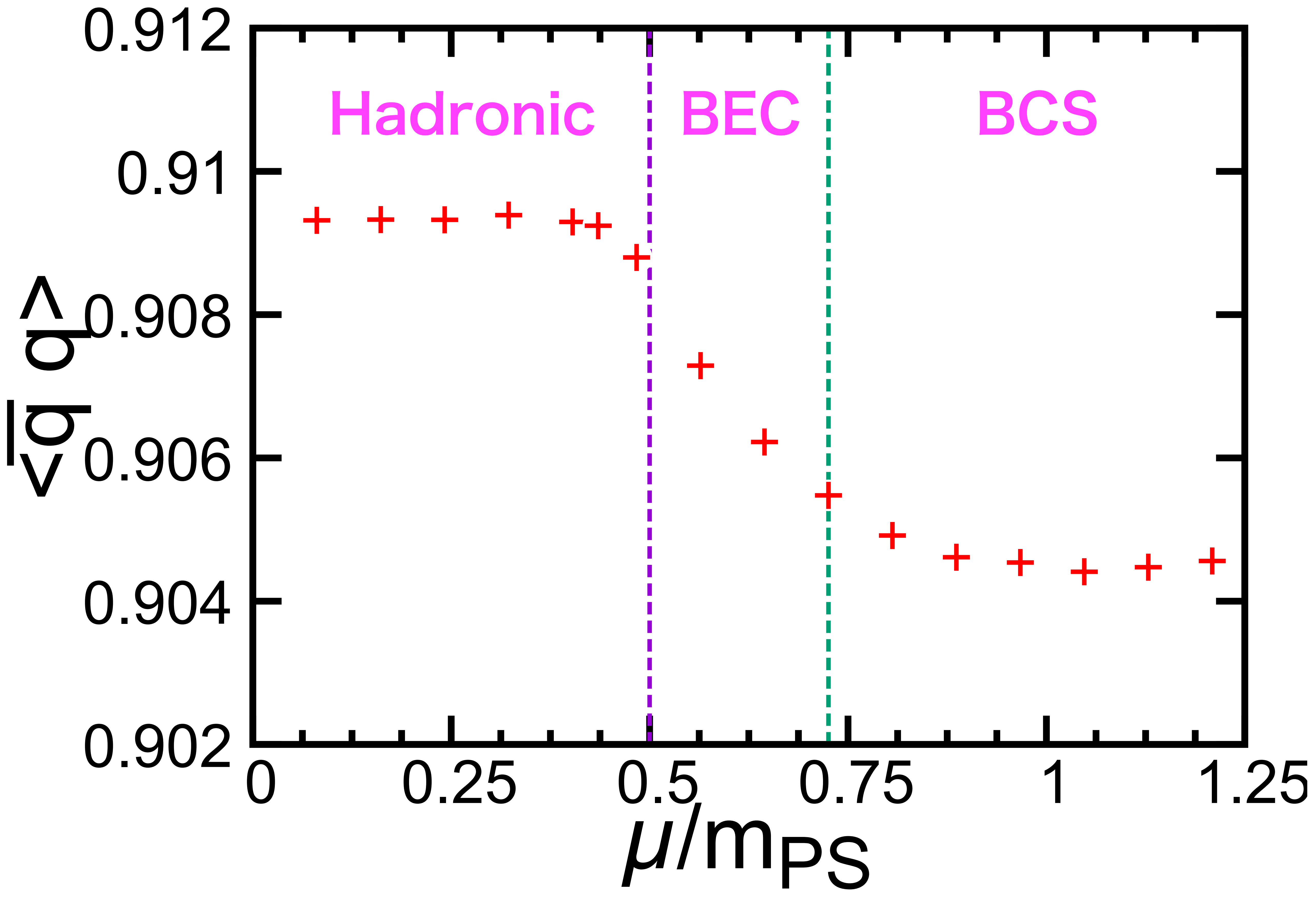
$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

HMC simulations for whole T-μ regime are doable!

(j->0 extrapolation is taken in all plots today)

Chiral condensate (T=40MeV)

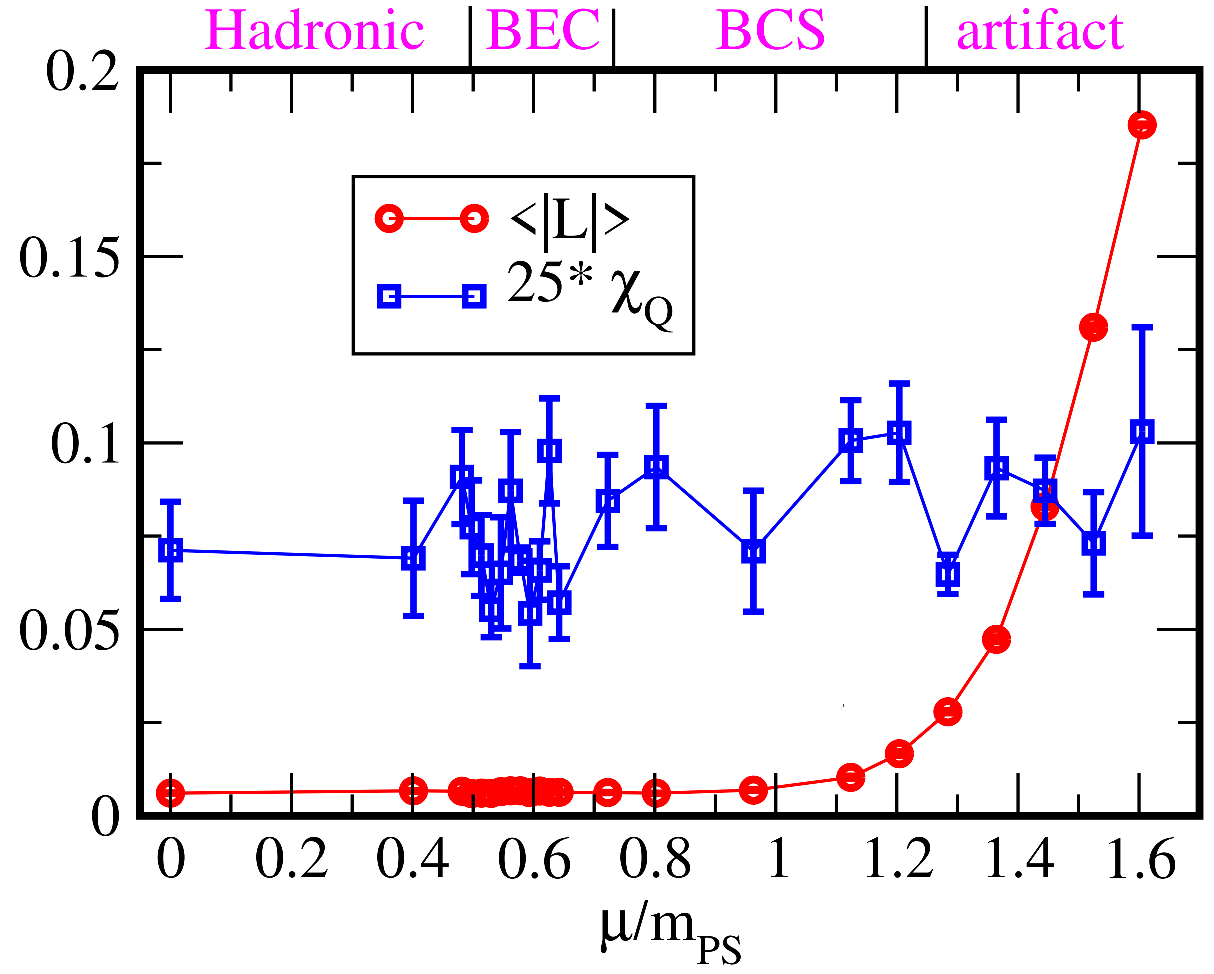
K.Iida, EI, T.-G. Lee: JHEP2001(2020)181



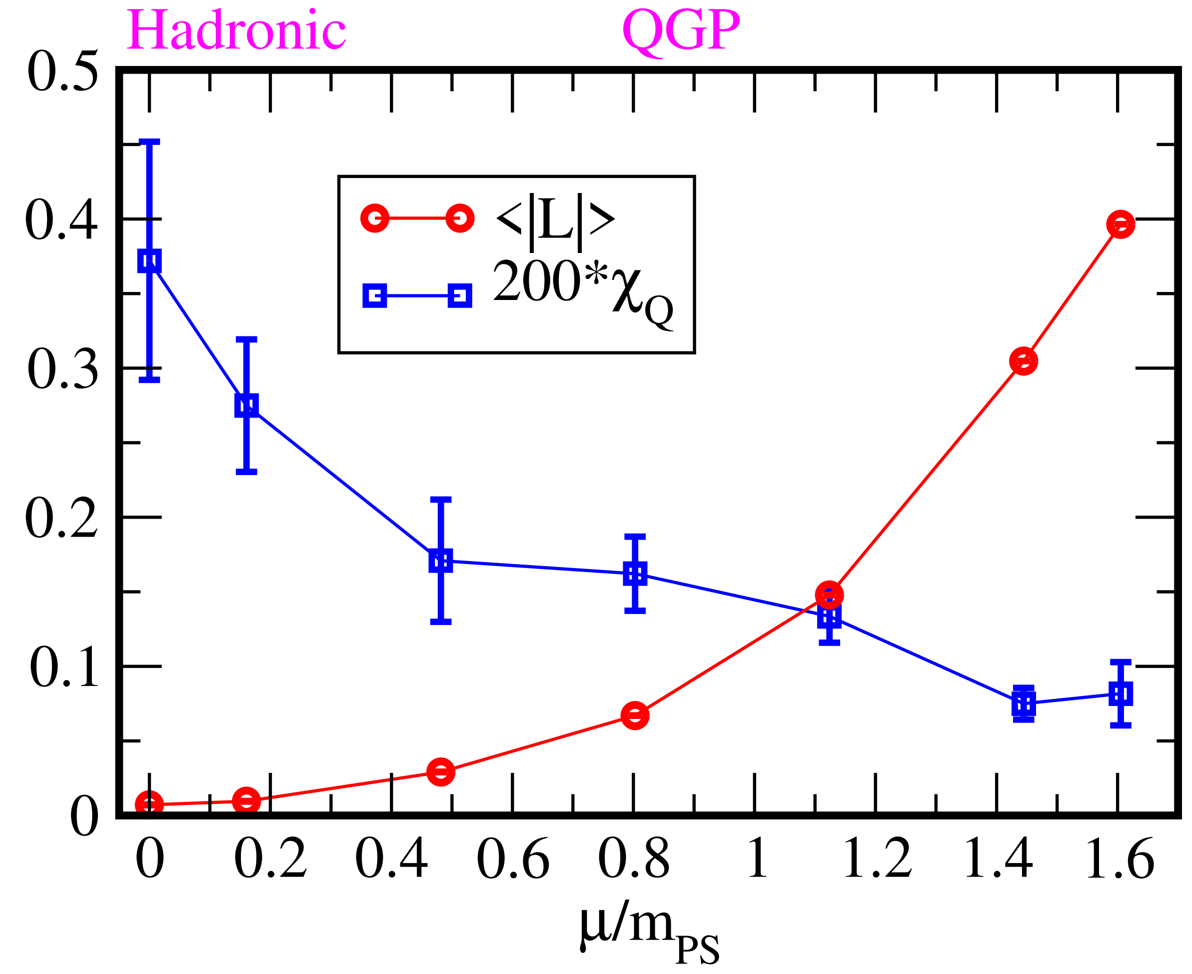
- We use Wilson fermion chiral sym. is broken (additive renormalization is needed)
- But it seems that chiral sym. becomes restored in high density
- Results using the staggered fermion also shows the similar behavior [N. Astrakhantsev et al.(2020)]

Topological susceptibility χ_Q

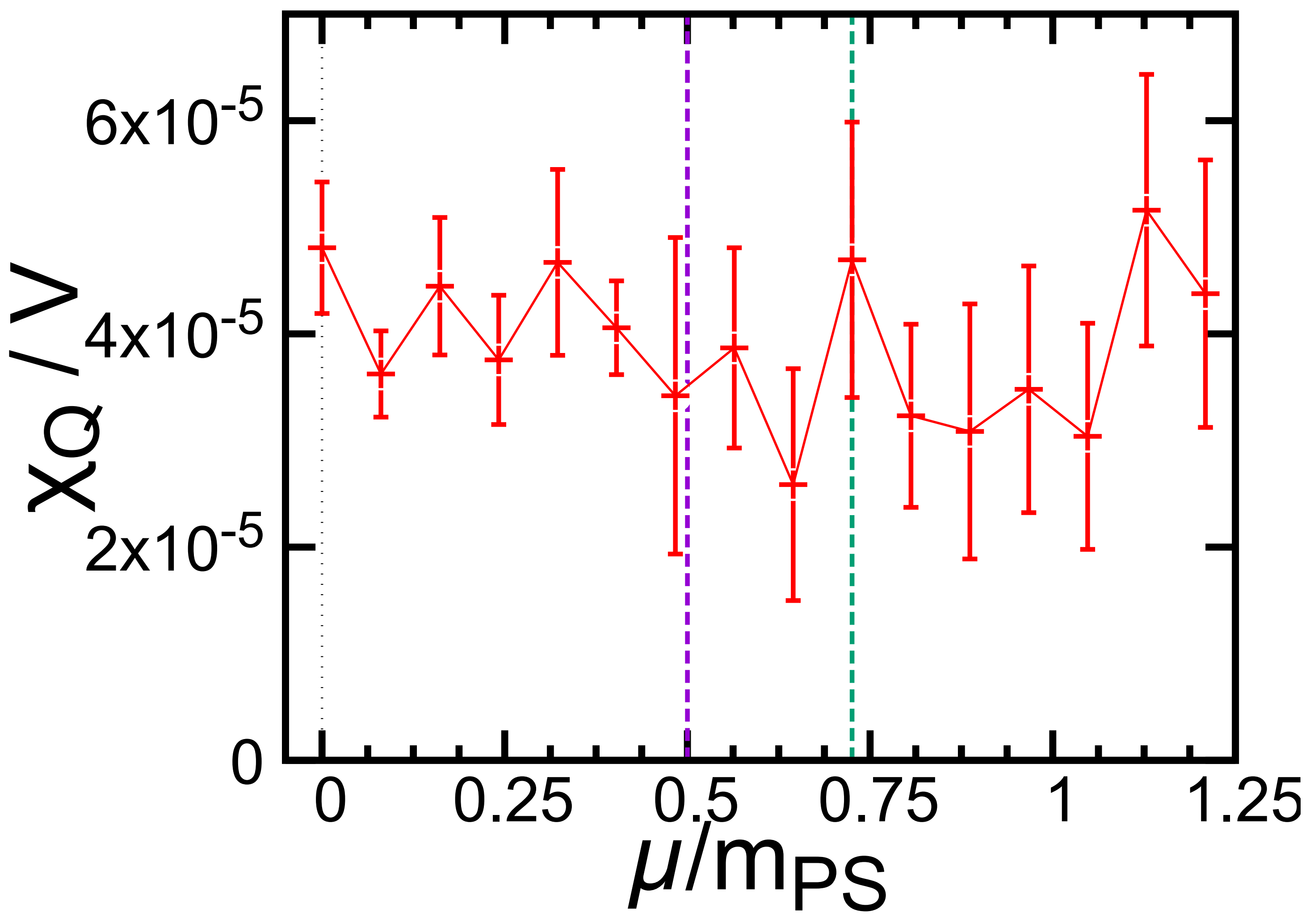
T=80MeV



T=160MeV



T=40MeV

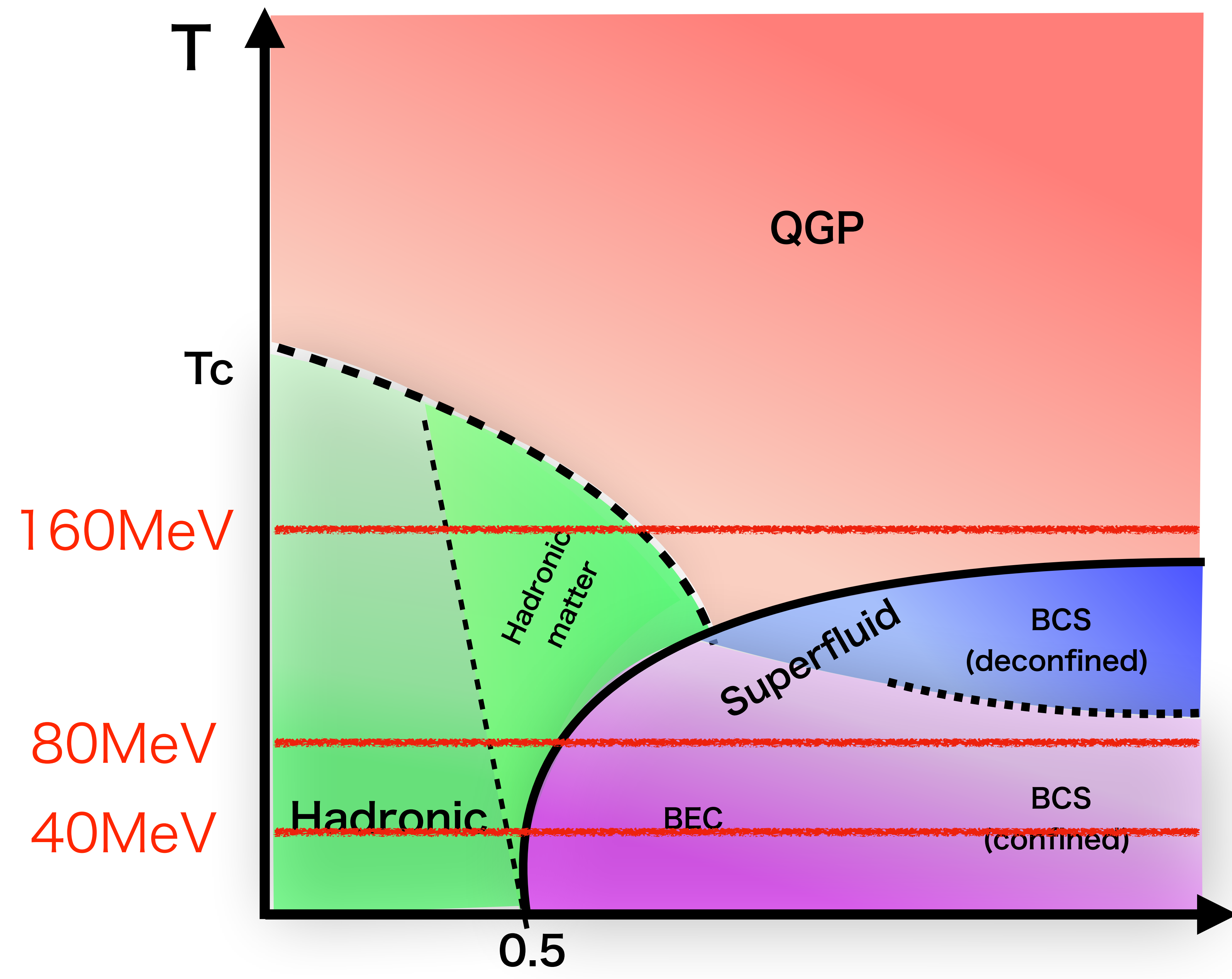


- j->0 extrapolation is done
- $\chi_Q > 0$ even in high density

Is it related with confinement?

cf.) Kawaguchi-Suenaga(2023)

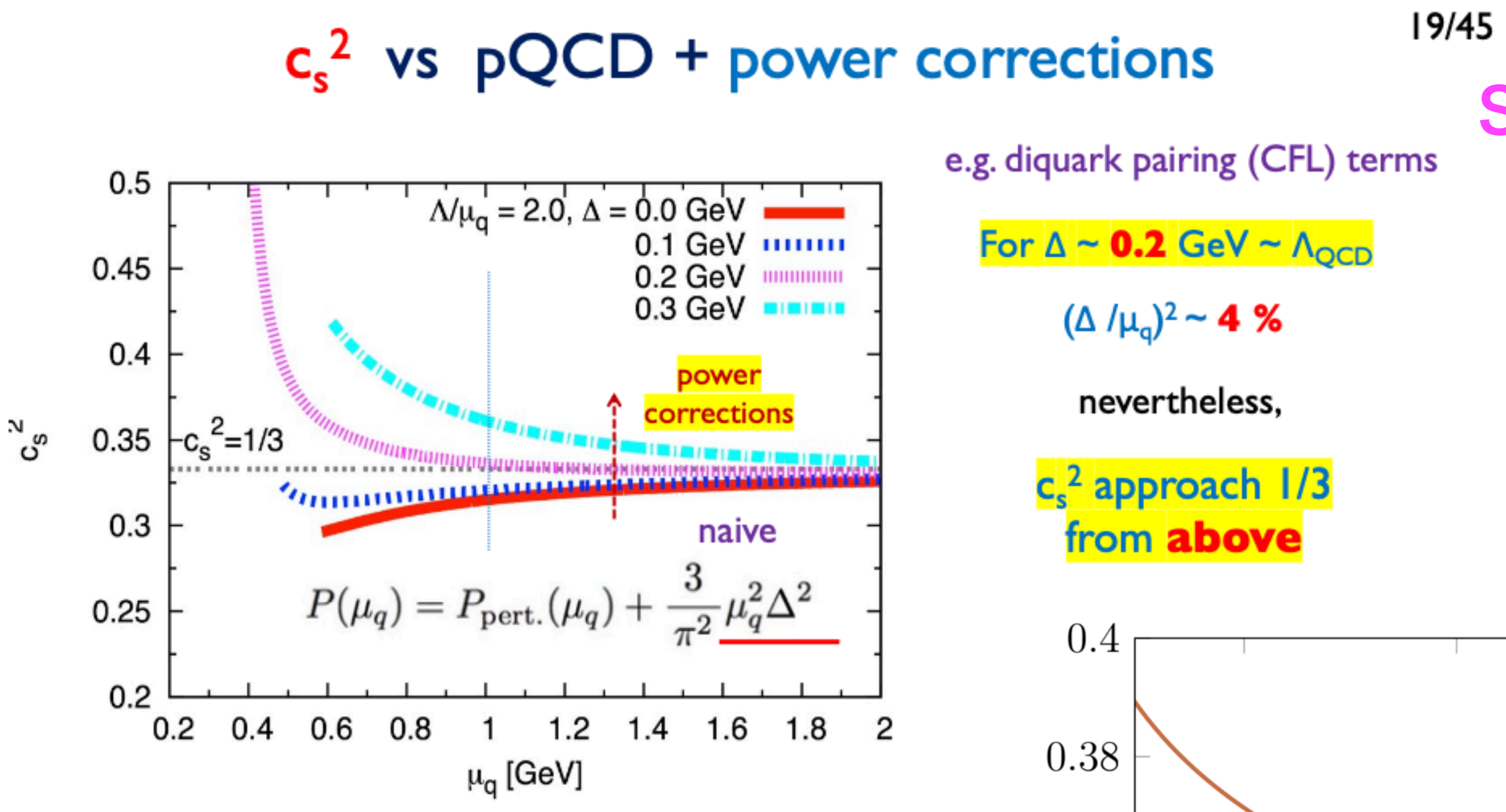
If quark mass is heavy then $\chi_Q > 0$?

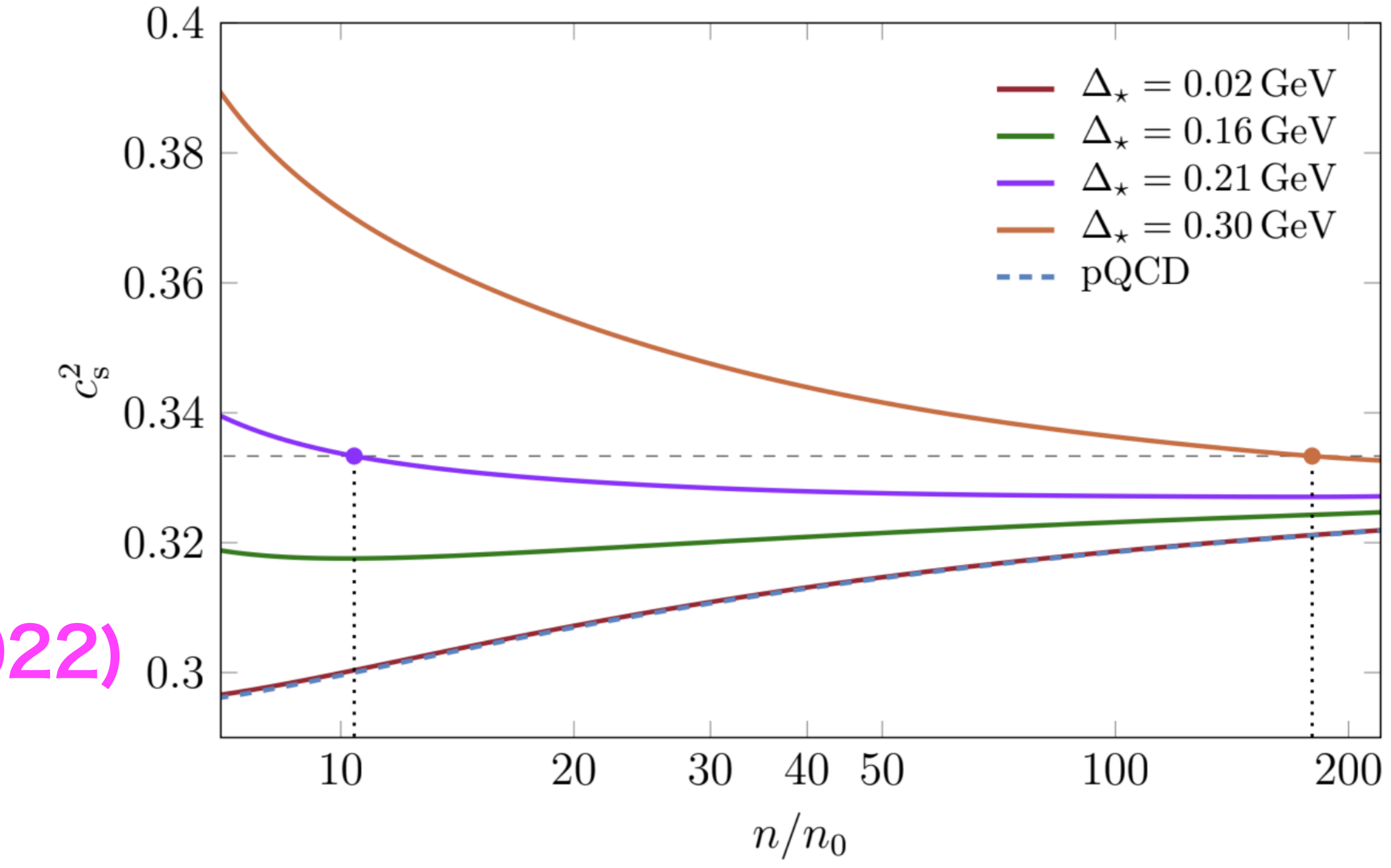


Further high density?

pQCD + power correction due to diquark gap

Slide by Kojo (2019)

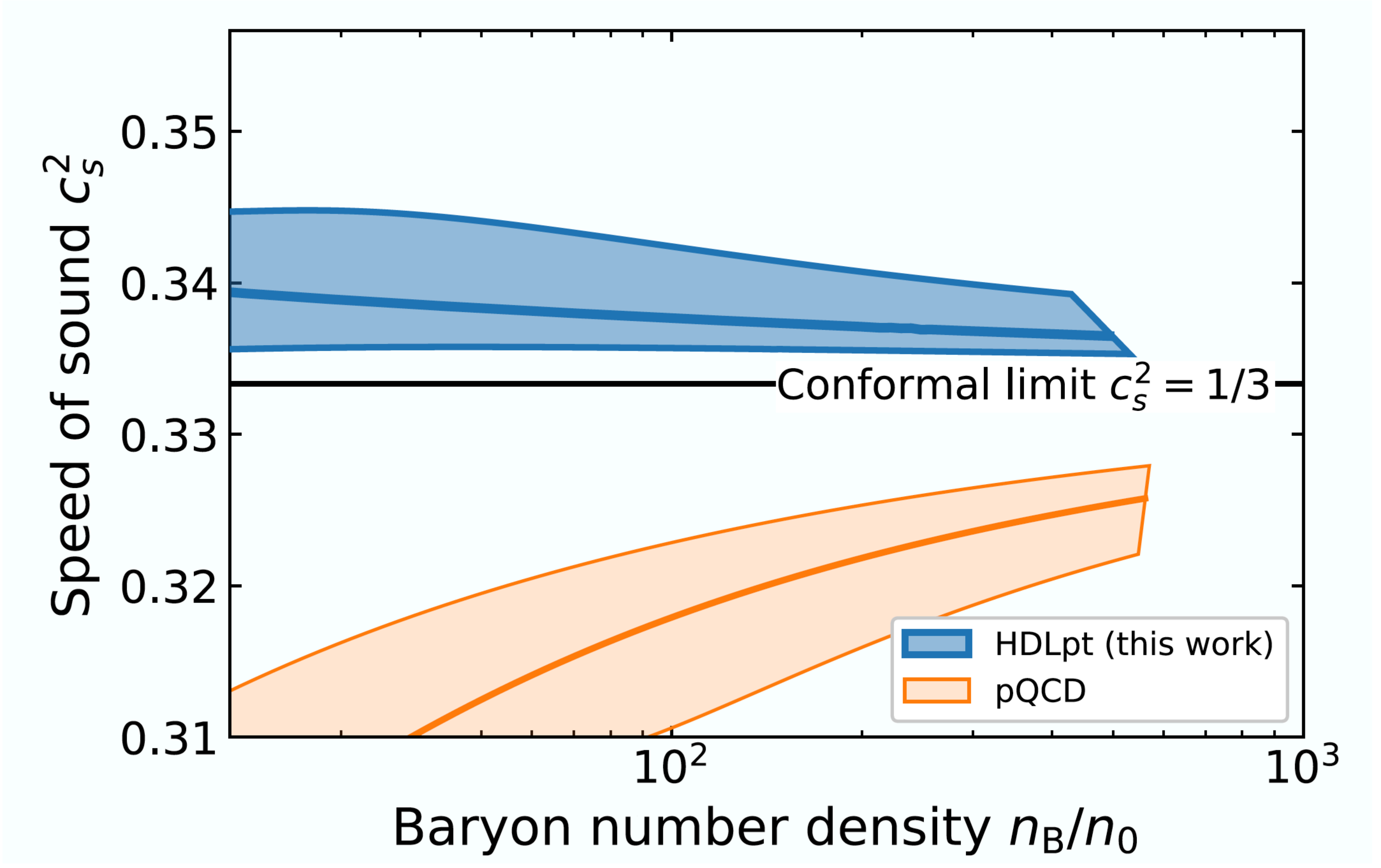




fRG analysis
Braun, Geissel, Schallmo(2022)

Hard thermal loop resummation

Fujimoto and Fukushima(2021)



Implementation QC2D with diquark source term

$$S_F^{cont.} = \underbrace{\int d^4x \bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x)}_{\text{QCD}} + \underbrace{\mu \hat{N}}_{\text{Number op.}} \underbrace{- \frac{j}{2}(\bar{\psi}_1 K \bar{\psi}_2^T - \psi_2^T K \psi_1)}_{\text{diquark source}}$$

construct a single bilinear form of fermion fields

$$S_F = (\bar{\psi}_1 \;\; \bar{\varphi}) \begin{pmatrix} \Delta(\mu) & J\gamma_5 \\ -J\gamma_5 & \Delta(-\mu) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix} \equiv \bar{\Psi}\mathcal{M}\Psi$$

Here, $\Psi = \begin{pmatrix} \psi_1 \\ \varphi \end{pmatrix}$

$$\bar{\varphi} = -\bar{\psi}_2^T C \tau_2, \quad \varphi = C^{-1}\tau_2 \bar{\psi}_2^T$$

$\mathcal{M}$ has non-diagonal components, calculations of det[M] and inverse of M are hard…

$$\mathcal{M}^\dagger \mathcal{M} = \begin{pmatrix} \Delta^\dagger(\mu)\Delta(\mu) + |\bar{J}|^2 & 0 \\ 0 & \Delta^\dagger(-\mu)\Delta(-\mu) + |J|^2 \end{pmatrix}$$

J ($=j\kappa$) term lifts the eigenvalue of Dirac op.

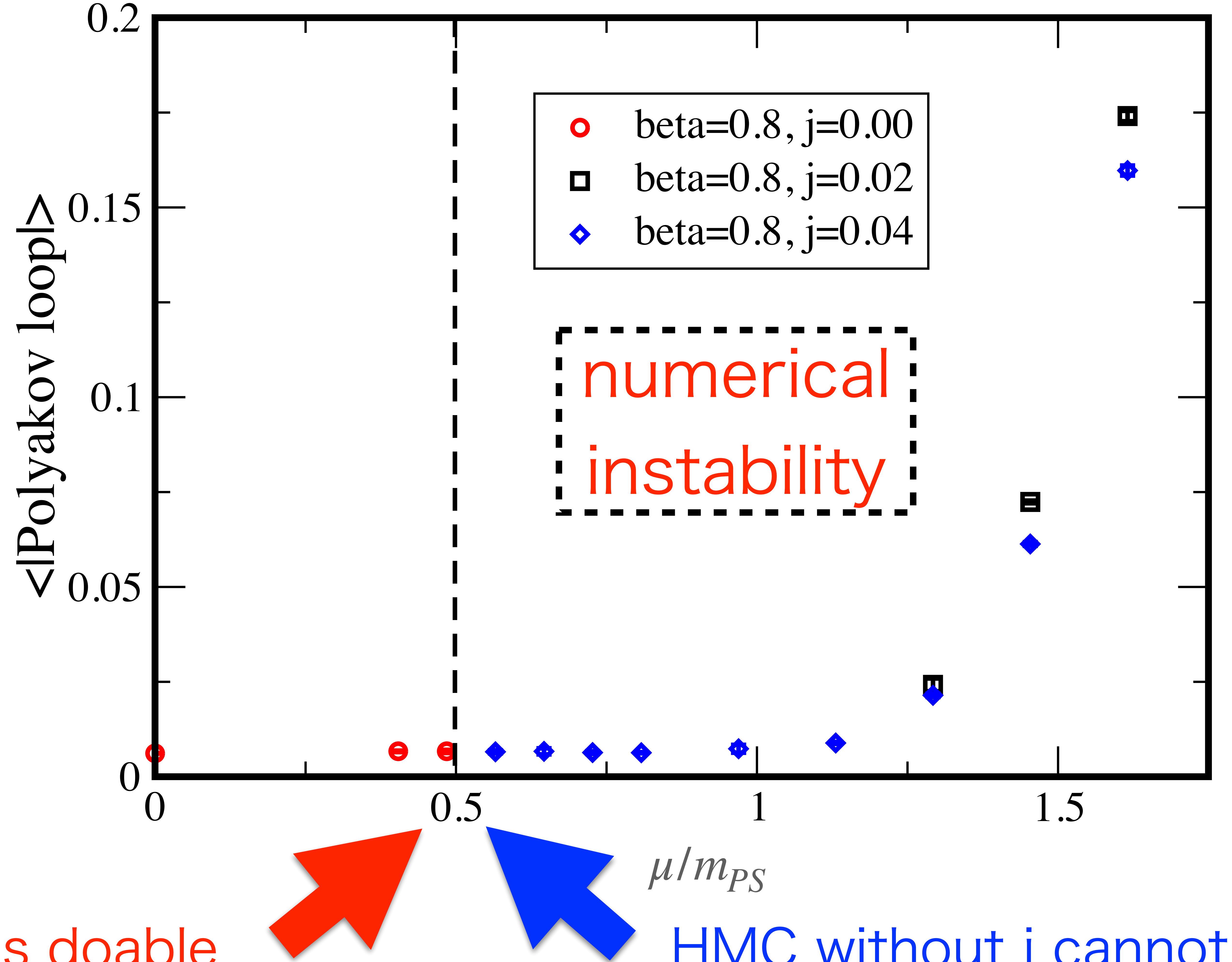
Note that Ψ denotes 2-flavor, $\det \mathcal{M}$ gives Nf=2 action

$\det \mathcal{M}^\dagger \mathcal{M}$ is 4-flavor theory

RHMC algorithm

HMC calculation w or w/o diquark source term

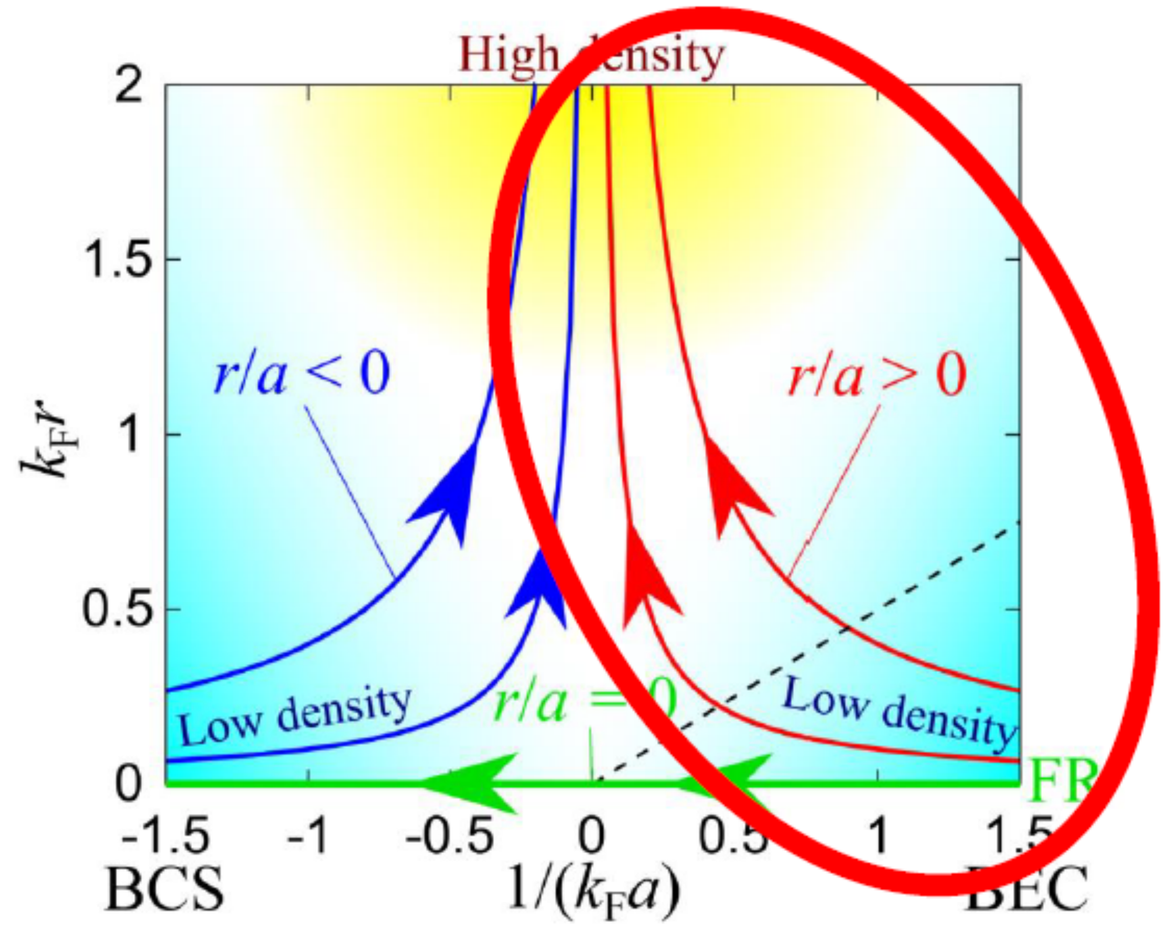
According to chiral perturbation theory,
the hadronic-superfluid phase transition occurs at $\mu/m_{PS} \sim 0.5$

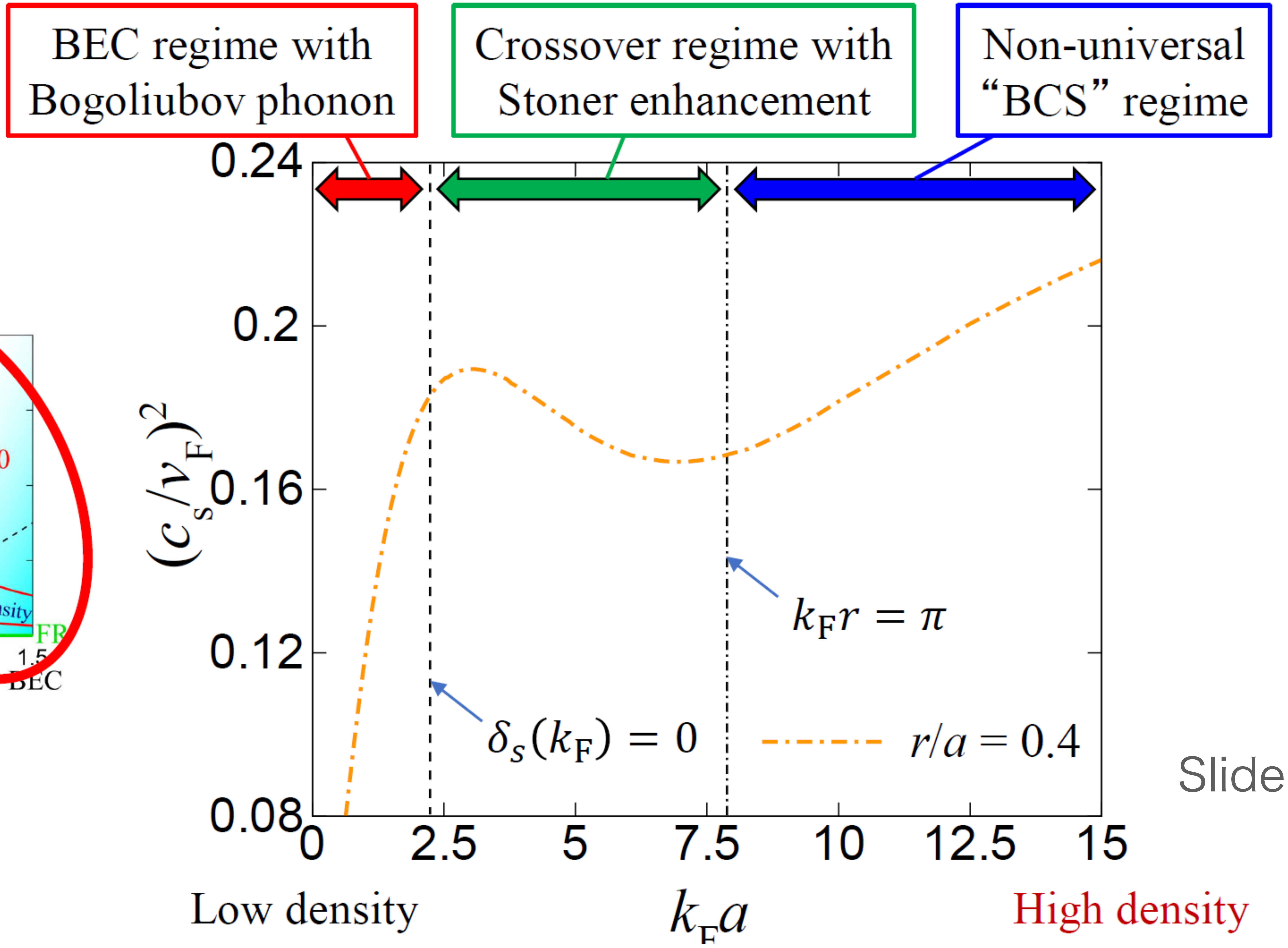


HMC without j is doable
(minimum MC step ~ 1/800)

HMC without j cannot run even with
a tiny MC step(~1/1000)

Example of cond.mat. model





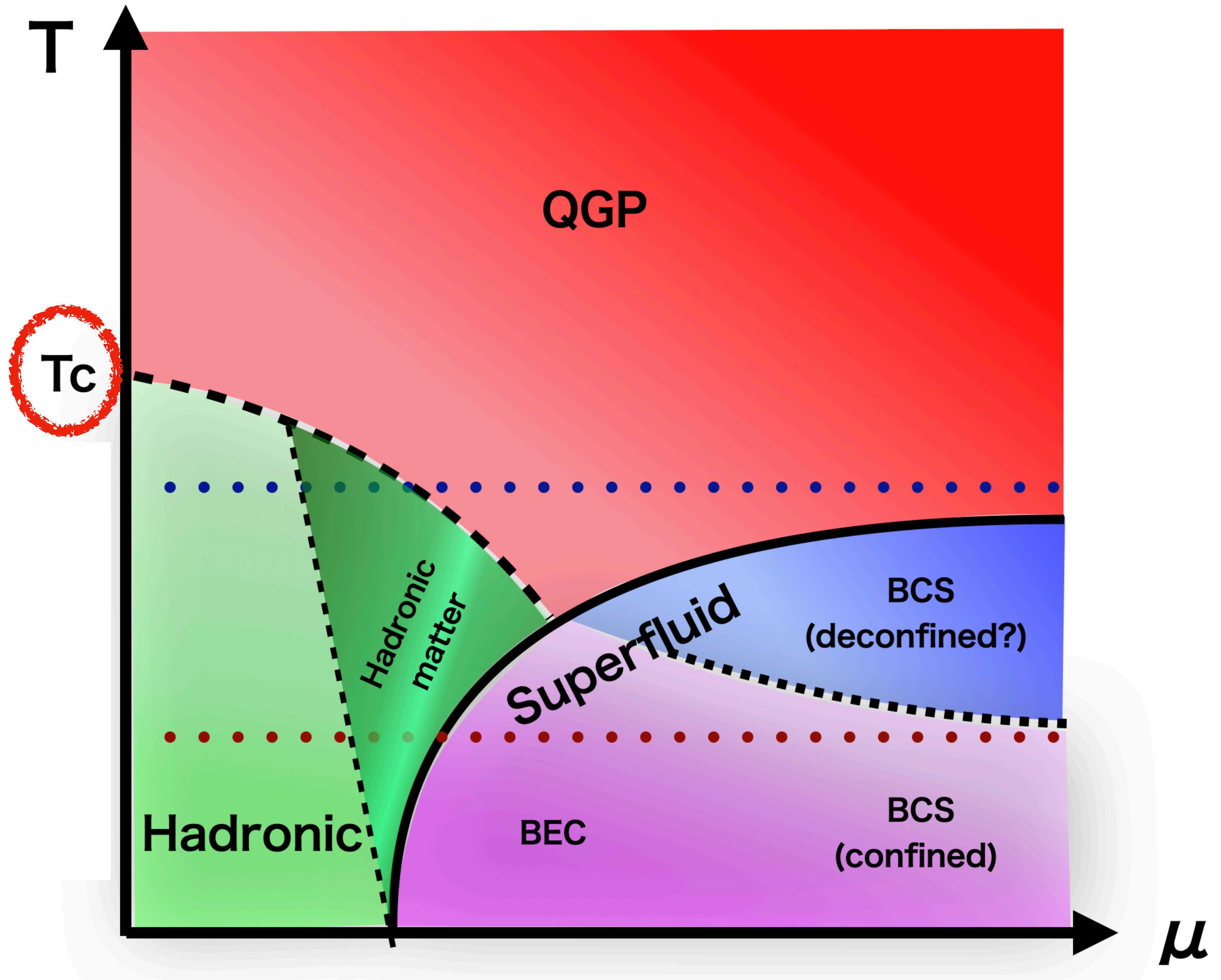
Slide by H.Tajima

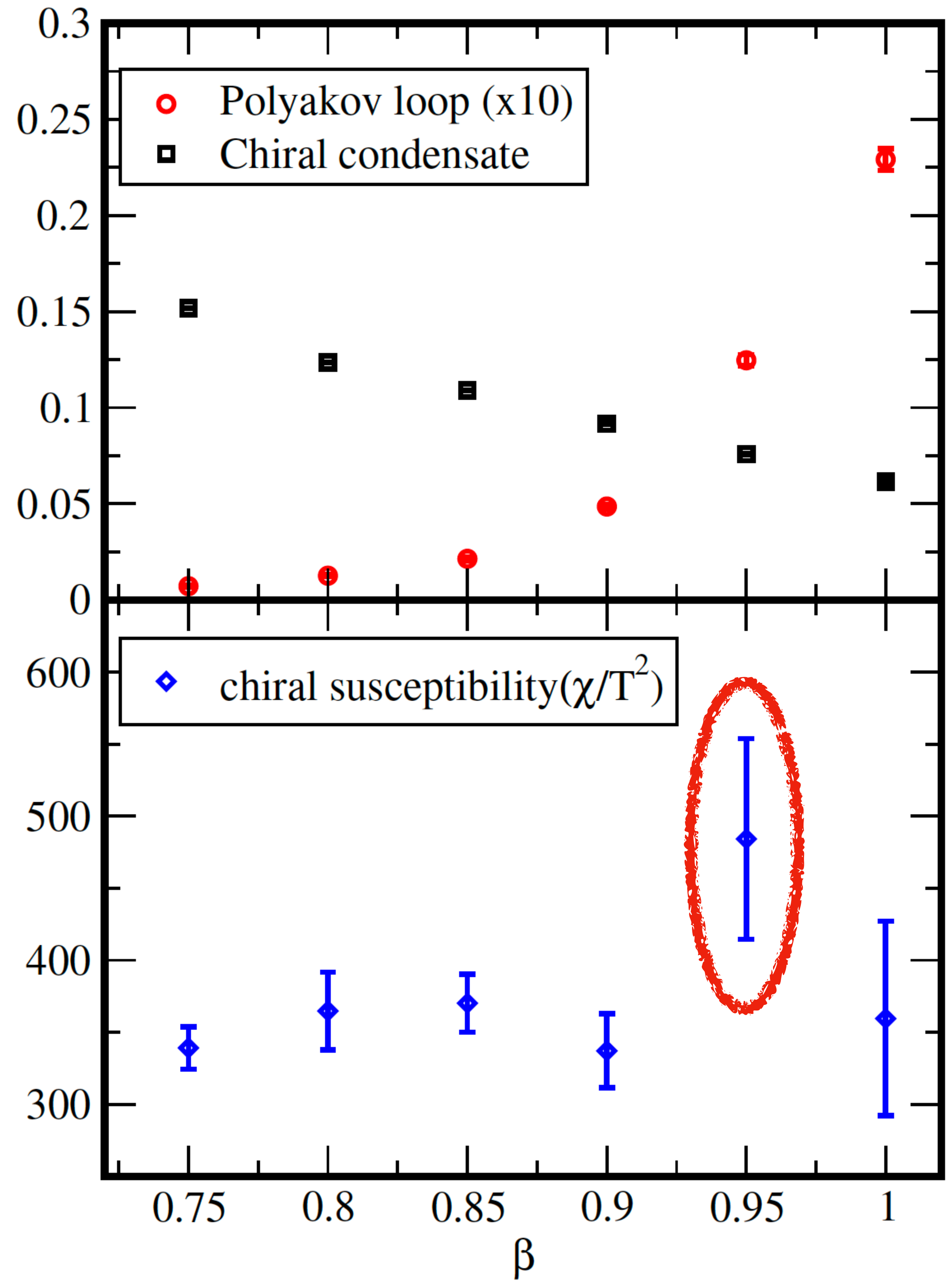
Phase diagram

Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0

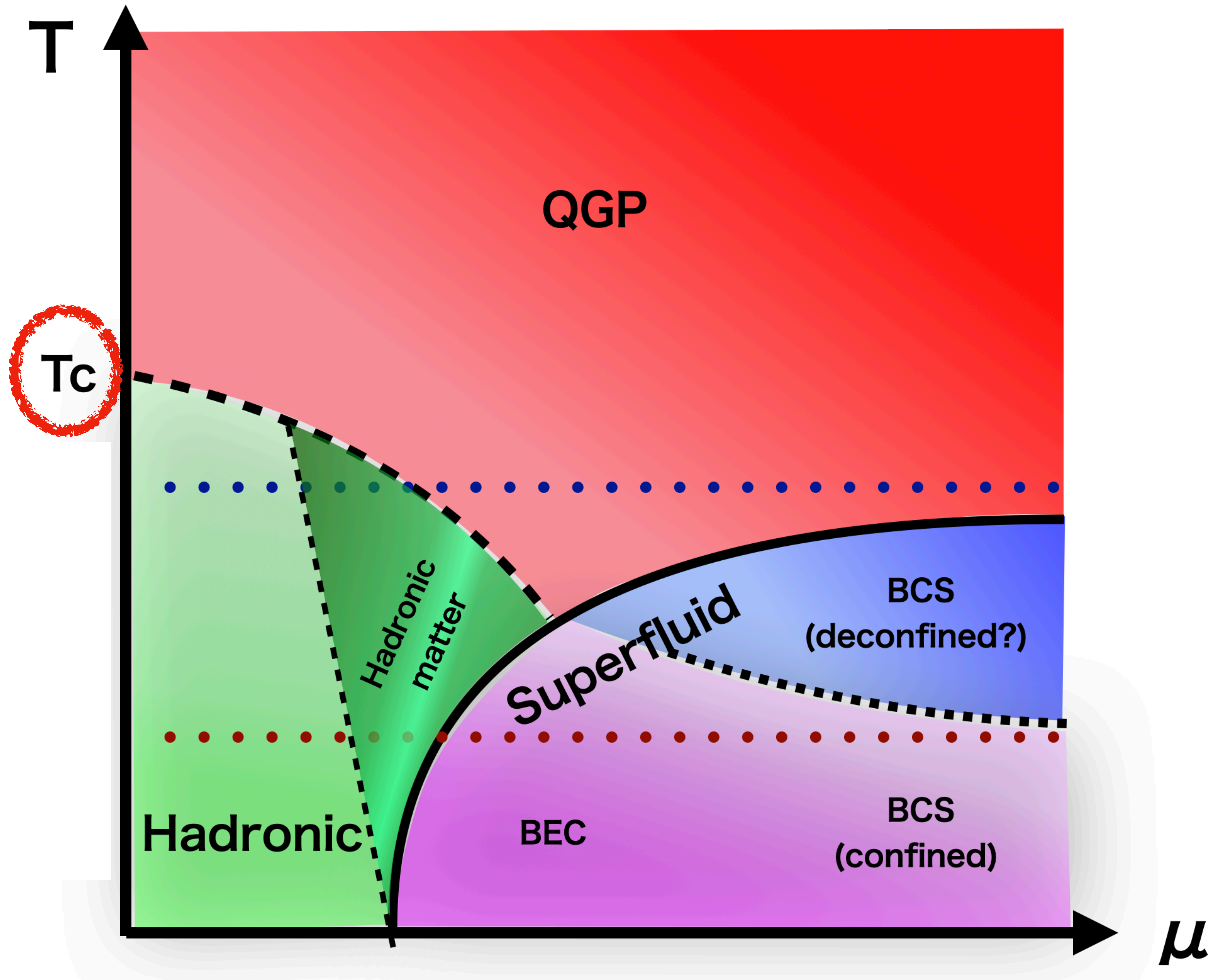
- Tc at $\mu = 0$ from chiral susceptibility





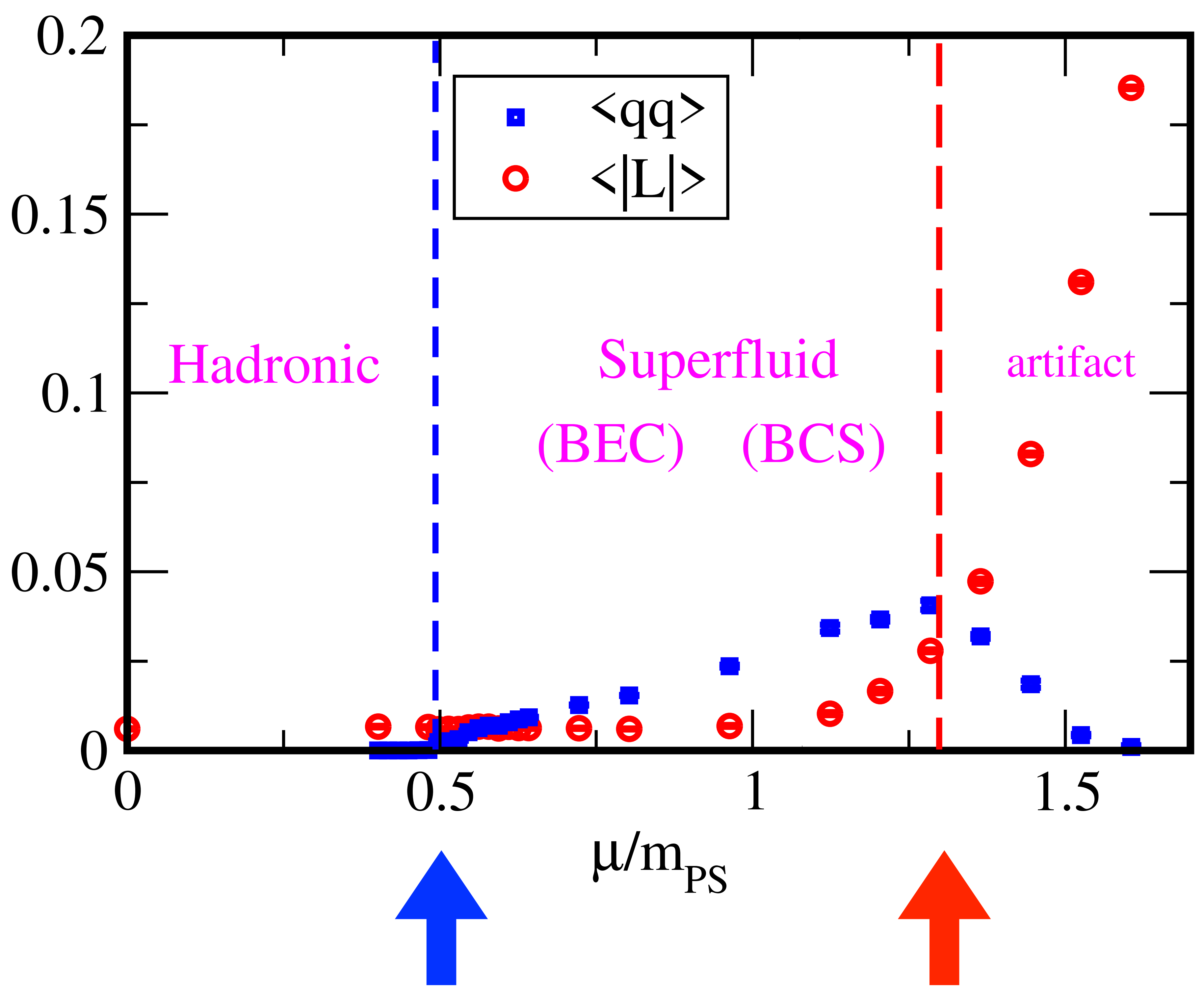
Scale setting at $\mu = 0$

K.Iida, EI, T.-G. Lee: PTEP 2021 (2021) 1, 013B0



- Tc at $\mu = 0$ from chiral susceptibility
- Assume Tc=200MeV
  Tc is realize Nt=10, $\beta = 0.95$ (a=0.1[fm])
- Find relationship between β (lattice bare coupling) and a (lattice spacing)
  In finite density simulation, a=0.1658[fm]

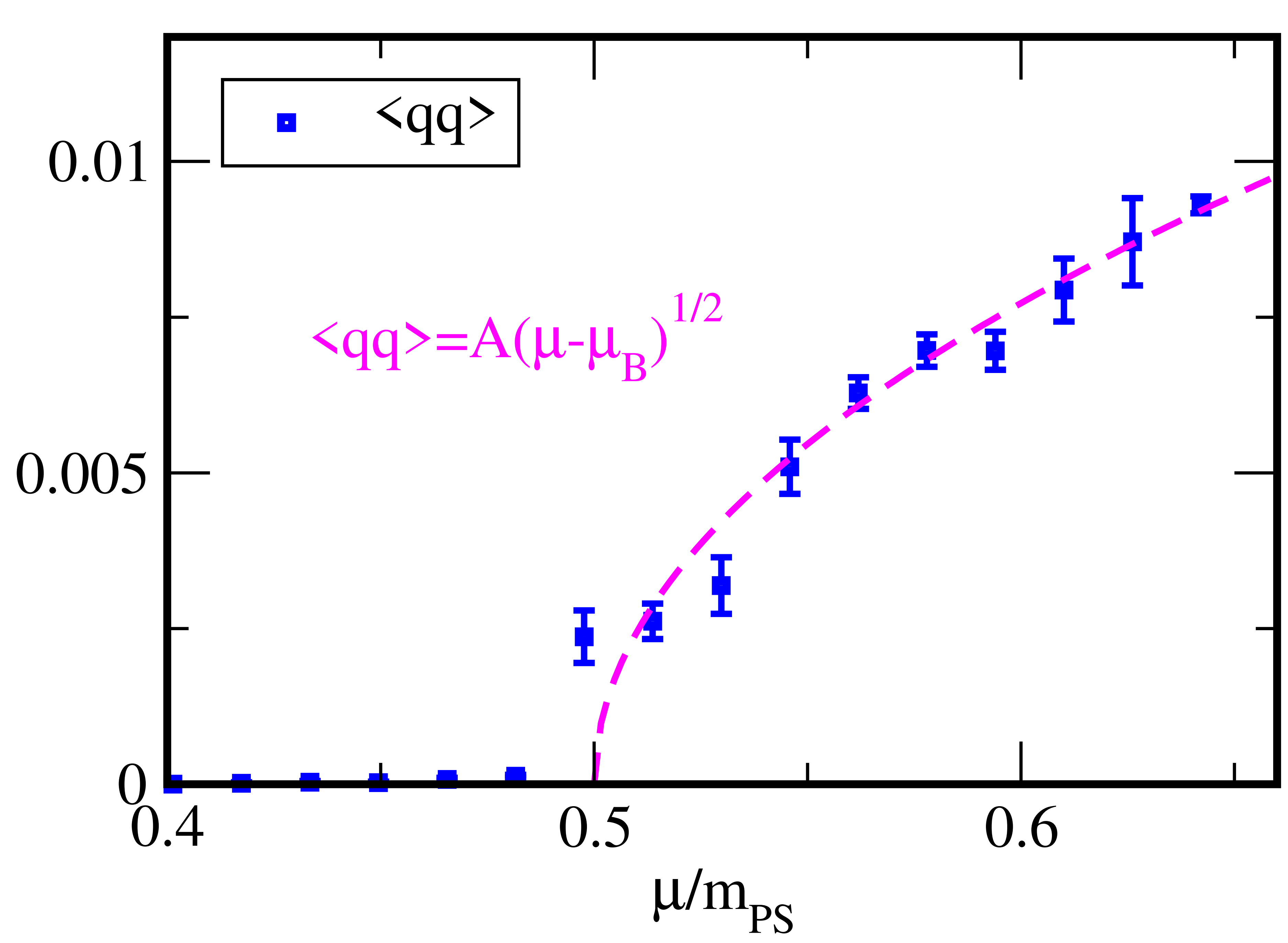
Order parameters in j=0 limit



$\mu_B/m_{\rm PS} \simeq 0.50$

$\mu/m_{\rm PS} \simeq 1.28$

$(\mu_D/m_{\rm PS} \simeq 1.44)$

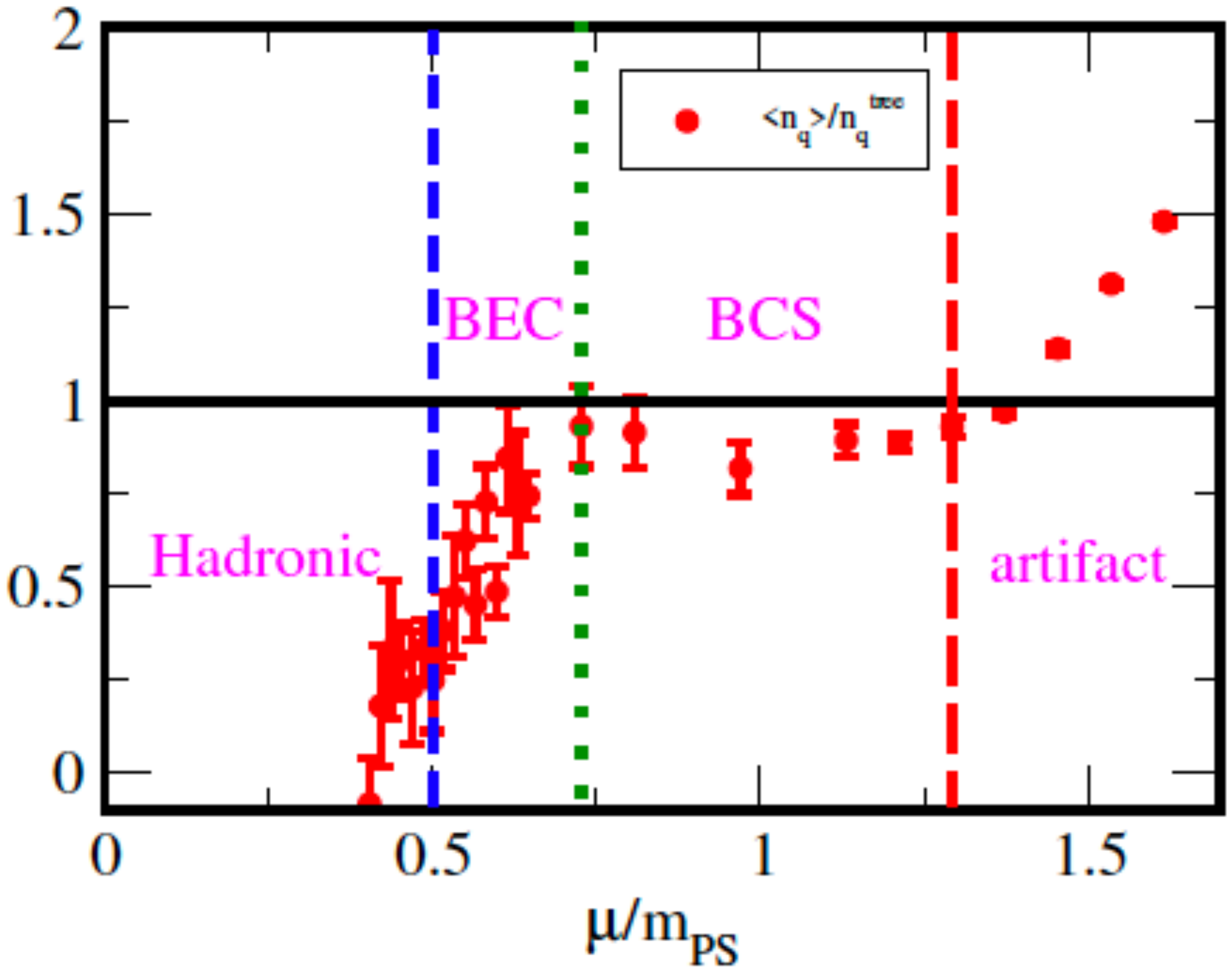


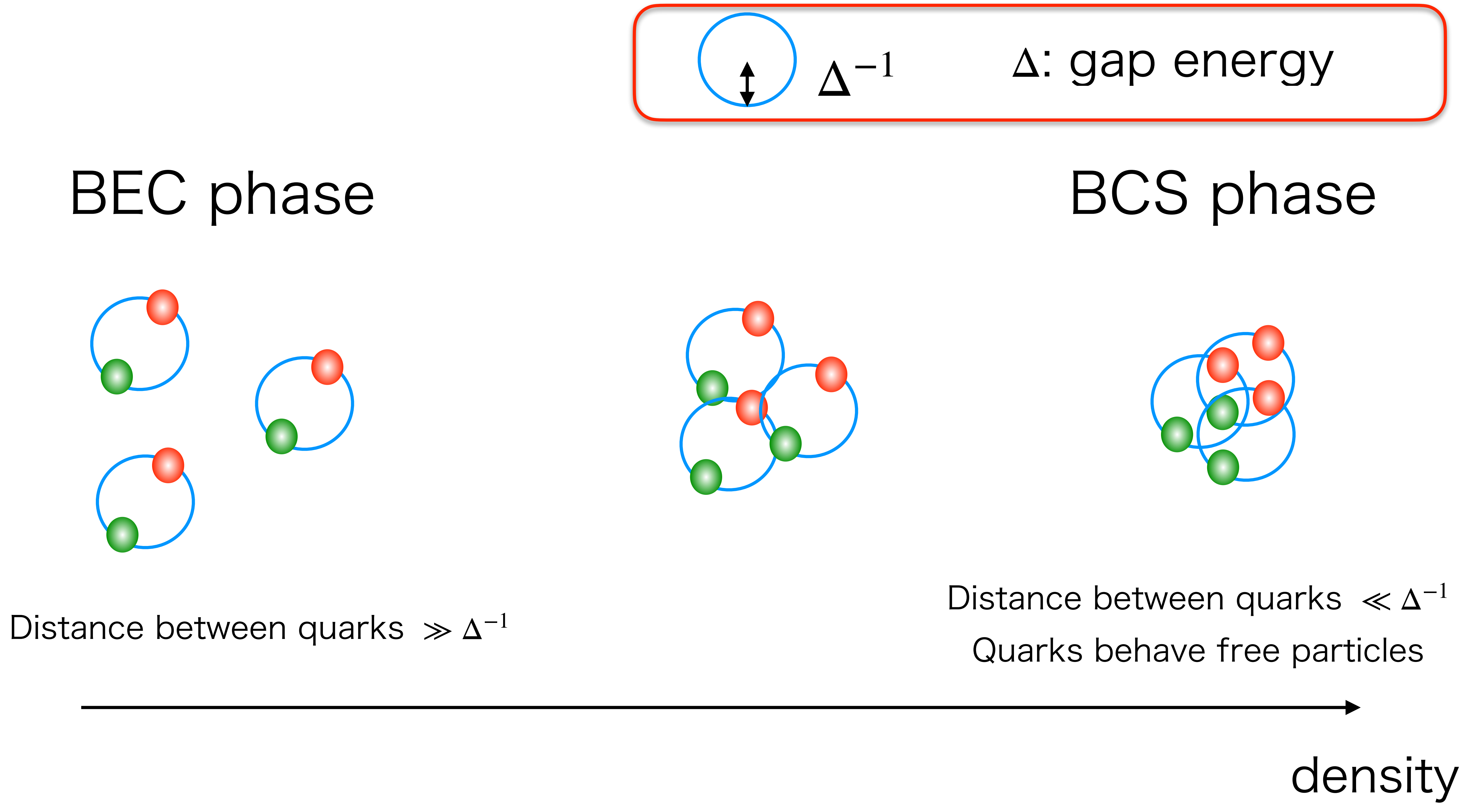
Scaling law of order param.
is consistent with ChPT.

Ref.) Kogut, Stephanov, Toublan, Verbaarshot, Zhitnitsly
NPB 582 (2000) 477

At T=0.39Tc, we find the BCS with confined phase until $\mu \lesssim 1152 MeV$.

BEC/BCS crossover





Number density of free particle

$$n_q^{\mathrm{tree}}(\mu) = \frac{4N_c N_f}{N_s^3 N_\tau} \sum_k \frac{i \sin \tilde{k}_0 [\sum_i \cos k_i - \frac{1}{2\kappa}]}{[\frac{1}{2\kappa} - \sum_\nu \cos \tilde{k}_\nu]^2 + \sum_\nu \sin^2 \tilde{k}_\nu}$$

J->0 extrapolation

Diquark condensate has a strong j dependence

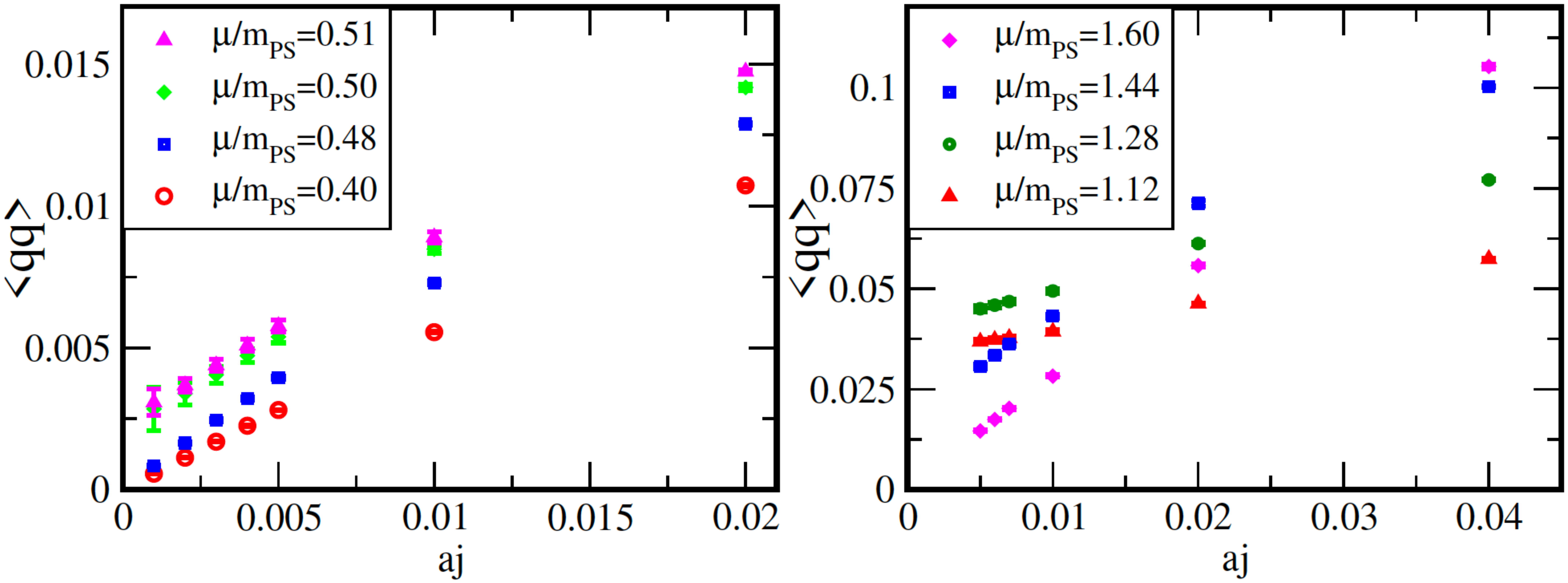


Figure 5. The j-dependence of the diquark condensate for several μ/m_{PS}.

J->0 extrapolation

Chiral condensate and n_q have a mild j-dependence