The effect of Isovector Scalar Meson on Neutron Star Matter Based on a Parity Doublet Model

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Outline of the presentation

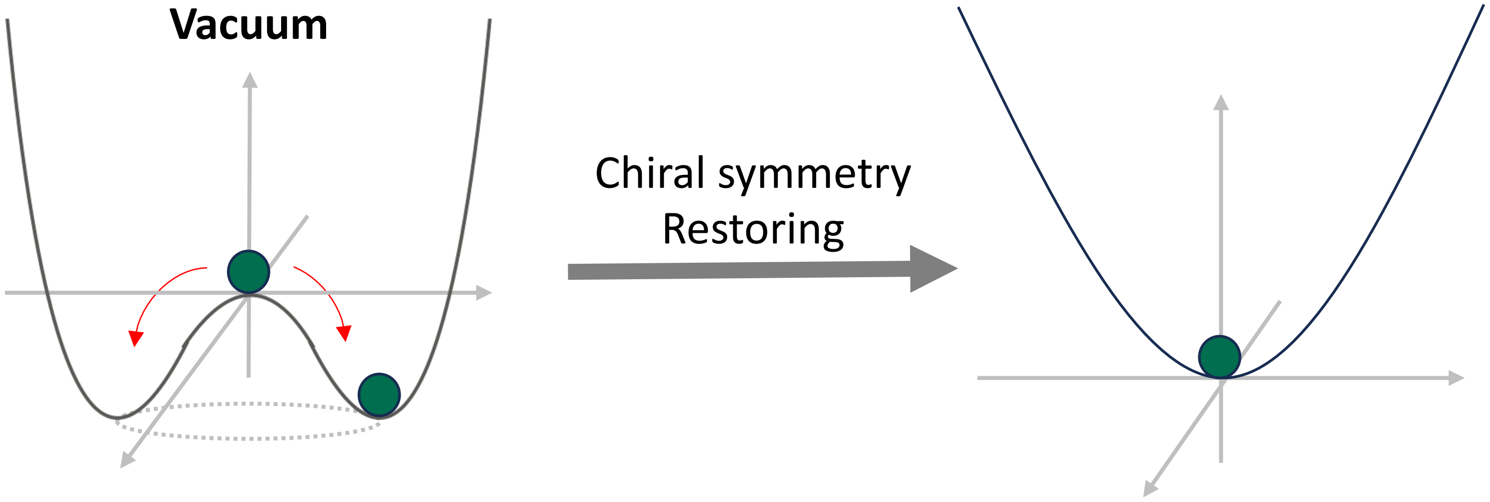
1. **Introduction**

2. **Parity doublet model with isovector scalar meson a_0(980)**

3. **Results**
   a) **Symmetry energy**
   b) **Neutron star Mass – Radius relation**

4. **Summary & future work**

Chiral symmetry restoration and the origin of hadron mass



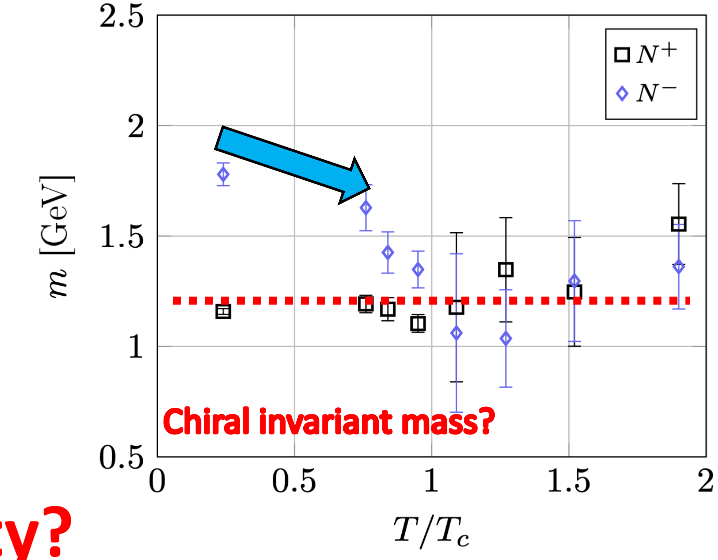
$< \bar{q}q > \neq 0$

$< \bar{q}q > = 0$

Chiral symmetry restoration happens at high T and/or density

e.g. Finite-T lattice calculation

G. Aarts et al. (2018)



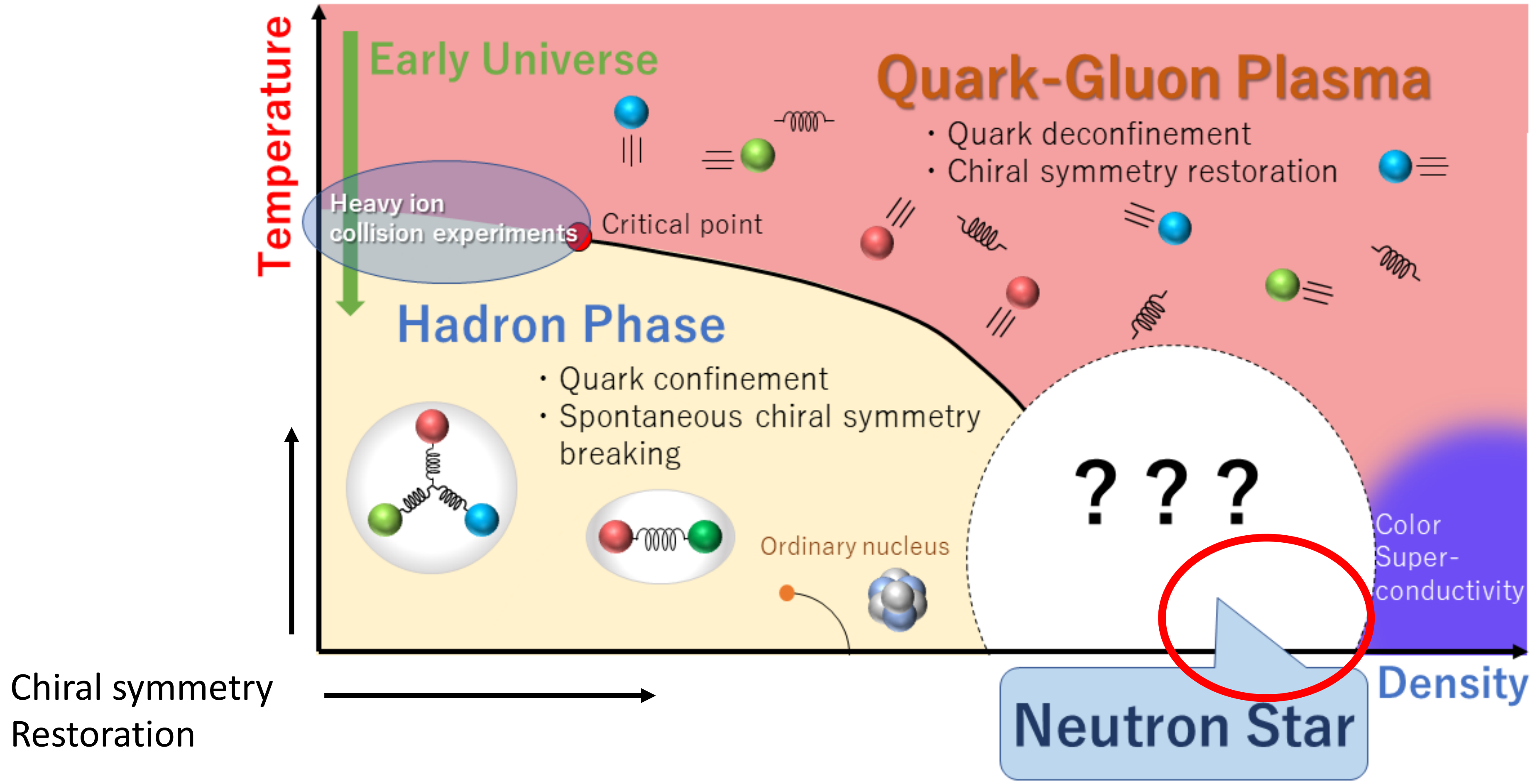
Spontaneous breaking of chiral symmetry generates the mass of hadrons:
e.g. In linear sigma model,

$$m_N \propto < \bar{q}q > \longrightarrow 0 \ ?$$

What about finite density?

Neutron star: laboratory for highly dense matter

Neutron star = highly (iso-spin) asymmetric matter



Study of neutron star …

Implication to the chiral invariant mass of nucleon!

Parity Doublet Model (PDM)

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- **Parity doublet model (PDM) models considers the parity doubling of nucleons using linear**

$$m_{\pm j} = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2(\sigma - ja)^2 + 4m_0^2} + \pm(g_1-g_2)(\sigma - ja)\right]$$

$$\mathcal{L} = \sum_{\alpha j} \bar{N}_{\alpha j}(i\partial\!\!\!/ - m_{\alpha j})N_{\alpha j}$$

$$(p, n) : j = (+, -)$$

$$+\mathcal{L}^{L\sigma M}(M) + \mathcal{L}^{HLS}(\omega, \rho)$$

- **The nucleon mass in this model is given by**

$$m_N \sim m_{\bar{q}q} + m_0$$

$$N_{ir} = \frac{1+\gamma_5}{2}N_i$$

$$N_{il} = \frac{1-\gamma_5}{2}N_i,$$

$$(i = 1, 2)$$

PDM with isovector scalar meson $a_0(980)$

- To study the asymmetric matter like neutron star, it is important to consider the isovector scalar meson which mediate the attractive force in the isovector channel

	Isoscaler	Isovector
Scaler	σ	$\vec{a}_0$
Pseudoscaler	η	$\vec{\pi}$

$a_0(980)$: 0^{++}
Lightest isovector scalar

- We construct a PDM with $a_0(980)$ meson => **U(2) PDM**

$$M = (\sigma + i\vec{\tau}\cdot\vec{\pi}) - (\vec{\tau}\cdot\vec{a}_0 + i\eta)$$

In asymmetric matter,

$$\vec{a_0} \sim \bar{q}\vec{\tau}q \to \bar{q}\tau_3 q \sim \bar{u}u - \bar{d}d \neq 0$$

a0 meson does not appear in symmetry matter

Parity Doublet Model (PDM)

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The mesonic Lagrangian is based on the extended linear sigma model

$$
\begin{aligned}
\mathcal{L}^{L\sigma M}(M) = & \frac{1}{4}\mathrm{tr}\left[\partial_\mu M \partial^\mu M^\dagger\right] \\
& + \frac{\bar{\mu}^2}{4}\mathrm{tr}[M^\dagger M] \\
& - \frac{\lambda_{41}}{8}\mathrm{tr}[(M^\dagger M)^2] + \frac{\lambda_{42}}{16}\{\mathrm{tr}[M^\dagger M]\}^2 \\
& + \frac{\lambda_{61}}{12}\mathrm{tr}[(M^\dagger M)^3] + \frac{\lambda_{62}}{24}\mathrm{tr}[(M^\dagger M)^2]\mathrm{tr}[M^\dagger M] + \frac{\lambda_{63}}{48}\{\mathrm{tr}[M^\dagger M]\}^3 \\
& + \frac{m_\pi^2 f_\pi}{4}\mathrm{tr}[M + M^\dagger] \\
& + \frac{K}{8}\{\det M + \det M^\dagger\}
\end{aligned}
$$

Taking mean field approximation,

$$\sigma(x) \to \sigma, \qquad \pi(x) \to 0, \qquad \eta(x) \to 0.$$

$$\vec{a_0}(x) \to a\delta_{i3}, \qquad a_0^{i=3} \equiv a.$$

Parity Doublet Model (PDM)

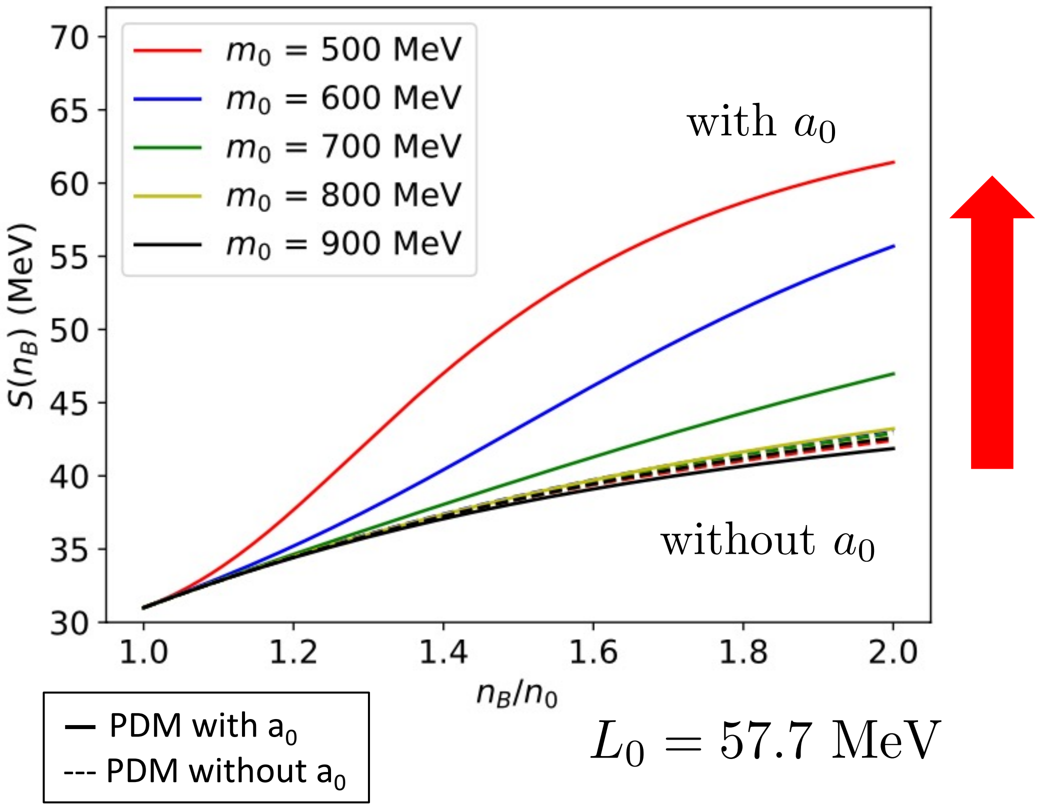
The vector meson is included using hidden local symmetry (HLS).

Under mean field approximation, the vector meson Lagrangian is written in a more familiar form:

$$\mathcal{L}^{HLS} = -\, g_{\omega NN} \sum_{\alpha j} \bar{N}_{\alpha j} \gamma^0 \omega N_{\alpha j} - g_{\rho NN} \sum_{\alpha j} \bar{N}_{\alpha j} \gamma^0 \frac{\tau_3}{2} \rho N_{\alpha j} + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \underline{\lambda_{\omega\rho} g_{\omega NN}^2 g_{\rho NN}^2 \omega^2 \rho^2}$$

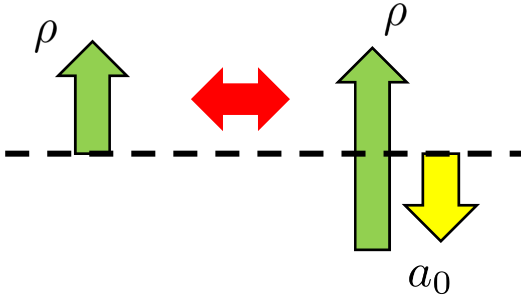
$$\omega_\mu(x) \to \omega \delta_{\mu 0}, \qquad \rho_\mu^i(x) \to \rho \delta_{\mu 0} \delta_{i3},$$

Symmetry energy $S(n_B)$



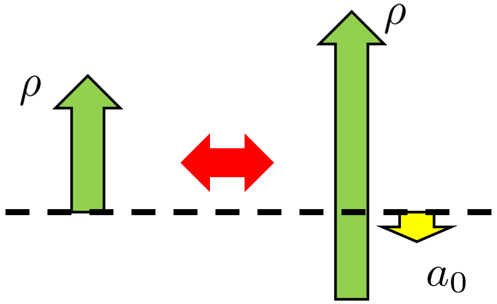
$L_0 = 57.7$ MeV

$S_0 = 31$ MeV



We determine the ρ coupling by **fitting saturation properties**, in the a_0 model the ρ coupling is stronger to fit S_0=31 MeV

$S(n_B > n_0)$



When $n_B > n_0$, the repulsive force of ρ become larger and attractive force of a_0 become smaller

Unified equation of state (EoS) of Neutron star

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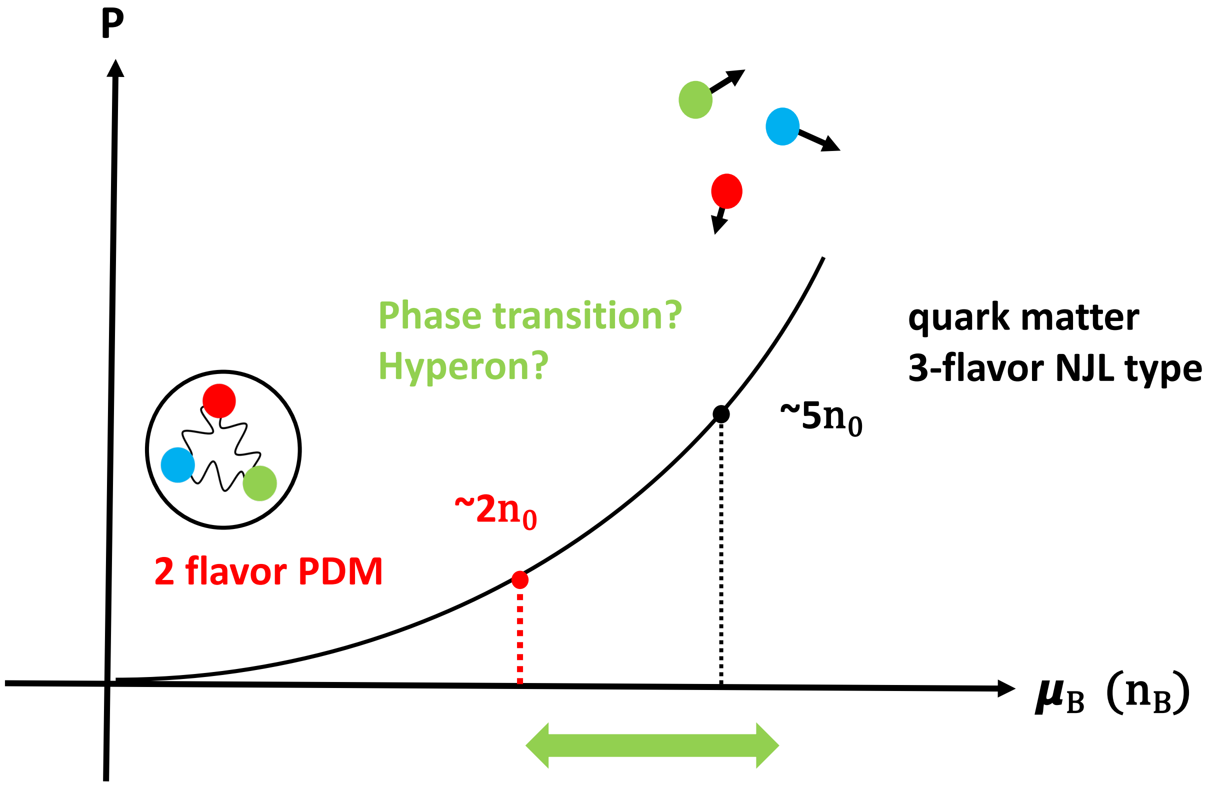
$$P_I(\mu_B) \equiv \sum_{i=0}^{5} c_i \mu_B^i$$



Parity doublet model (PDM) is considered in low density

PDM consider **two opposite parity baryons** which degenerate to non-zero mass m_0 when chiral symmetry is restored

Interpolation
(crossover phase)
Masuda et al. (2011)
G. Baym et al. (2017)

Nambu-Jona-Lasinio (NJL)-type model to reproduce quark matter in the high density

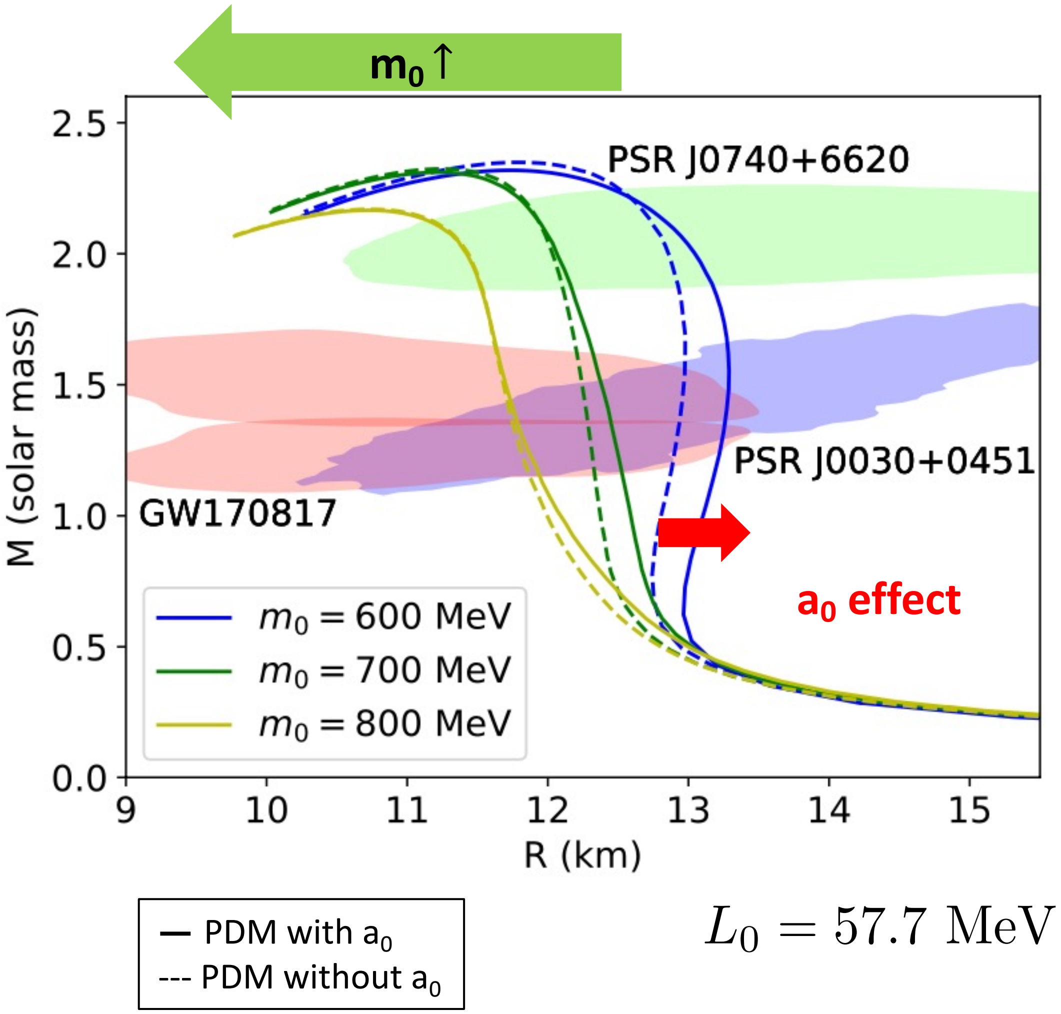
Neutron star M-R relation

- We compute the M-R relation by solving the TOV equation
- We find that a_0(980) **increase the radius of intermediate mass NS**

$$540 \text{ MeV} \lesssim m_0 \lesssim 870 \text{ MeV (without } a_0)$$

a_0(980)

$$580 \text{ MeV} \lesssim m_0 \lesssim 860 \text{ MeV}$$



$$L_0 = 57.7 \text{ MeV}$$

$$\Delta R \lesssim 1 \text{ km for } 0.5 \lesssim M/M_{\odot} \lesssim 2$$

Summary & future work

- **We find that the existence of $a_0(980)$ stiffens the matter:**
  - **increase the symmetry energy at $n_B > n_0$**
  - **increase the radius of intermediate mass NS**

- **We constrains the chiral invariant mass m_0 in a_0 model to**

580 – 860 MeV

- **Further experiment data on $S(n_B > n_0)$ may help us to constrain the chiral invariant mass**

- **We are now studying the effect of $a_0(980)$ to finite nuclei with isospin asymmetry**

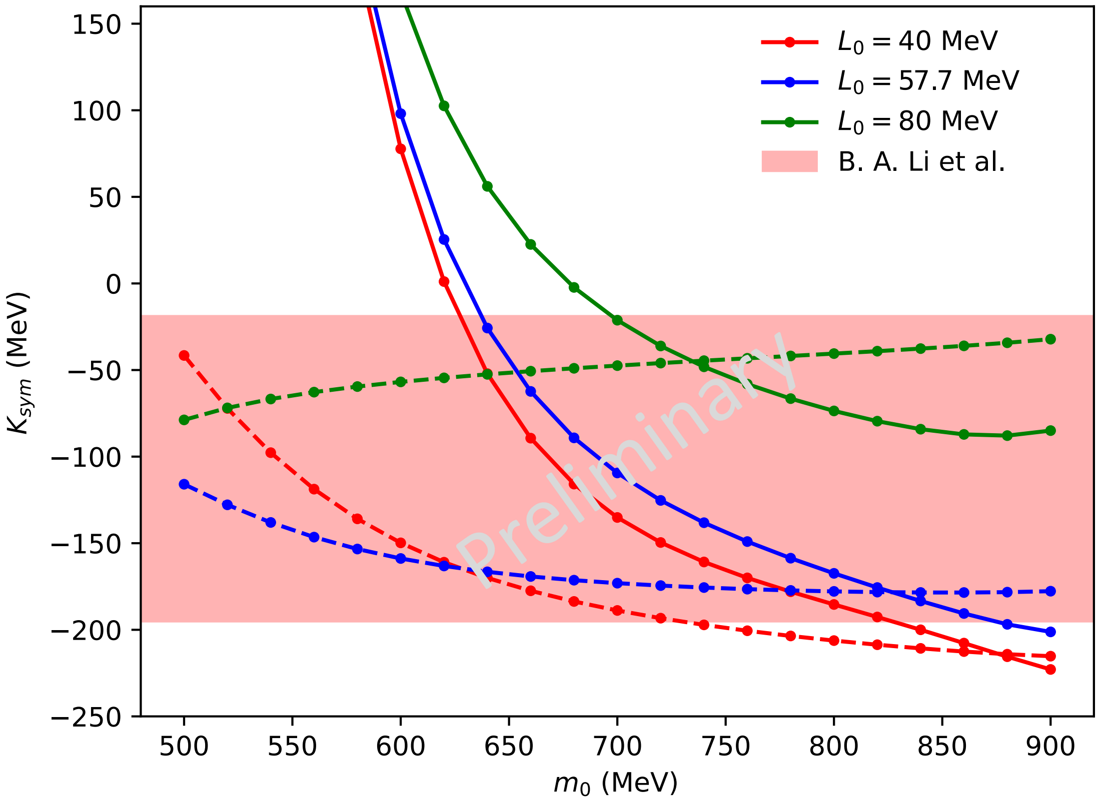
Thank you!

Appendix

Symmetry incompressibility K_{sym}

Recent value of K_{sym} = −107±88 MeV

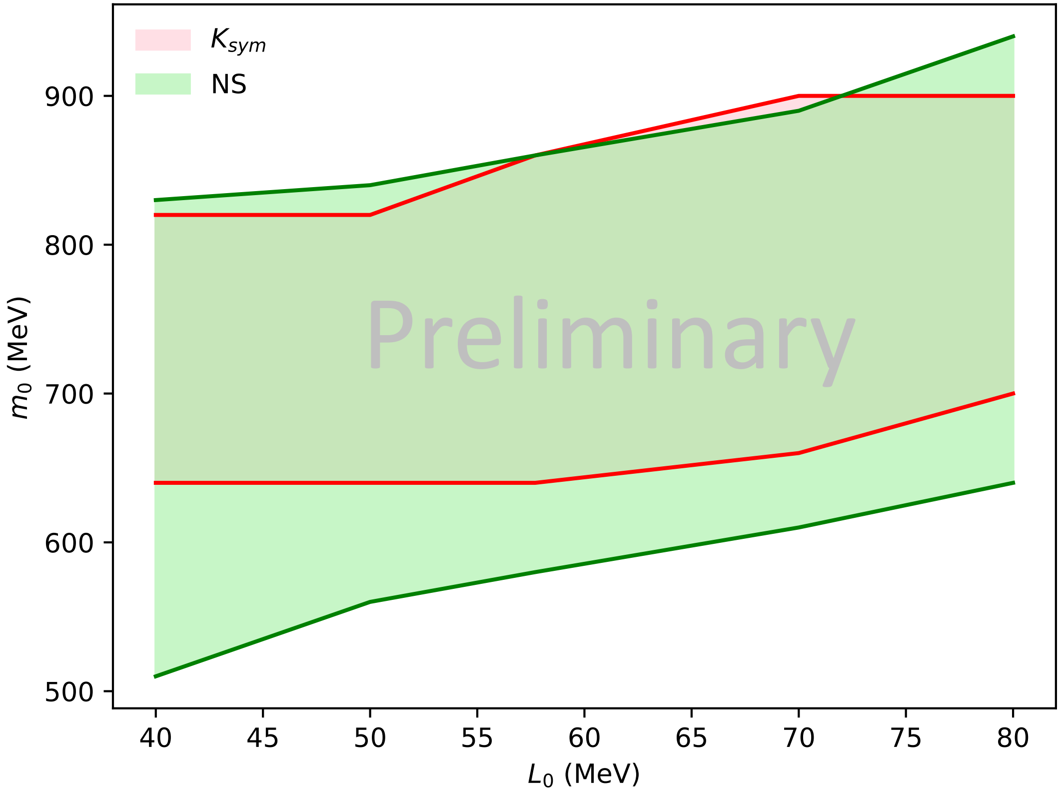
B. A. Li et al. Universe, 2021, 7(6).



Solid line: with a0 meson

Dash line: without a0 meson

Constraint to m_0 in the a0-PDM model



The constraint from NS (high density) and K_{sym} (low density) basically agree with each other

Parity Doublet Model (PDM)

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The mesonic Lagrangian is based on the extended linear sigma model

$$
\begin{aligned}
\mathcal{L}^{L\sigma M}(M) = &\frac{1}{4}\mathrm{tr}\left[\partial_\mu M \partial^\mu M^\dagger\right] \\
&+ \frac{\bar{\mu}^2}{4}\mathrm{tr}[M^\dagger M] \\
&- \frac{\lambda_{41}}{8}\mathrm{tr}[(M^\dagger M)^2] + \frac{\lambda_{42}}{16}\{\mathrm{tr}[M^\dagger M]\}^2 \\
&+ \frac{\lambda_{61}}{12}\mathrm{tr}[(M^\dagger M)^3] + \frac{\lambda_{62}}{24}\mathrm{tr}[(M^\dagger M)^2]\mathrm{tr}[M^\dagger M] + \frac{\lambda_{63}}{48}\{\mathrm{tr}[M^\dagger M]\}^3 \\
&+ \frac{m_\pi^2 f_\pi}{4}\mathrm{tr}[M + M^\dagger] \\
&+ \frac{K}{8}\{\det M + \det M^\dagger\}
\end{aligned}
$$

Parity Doublet Model (PDM)

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The mean field Lagrangian is then given by

$$\mathcal{L}^{L\sigma M} = \frac{\bar{\mu}_\sigma^2}{2}\sigma^2 + \frac{\bar{\mu}_a^2}{2}a^2 - \frac{\lambda_4}{4}(\sigma^4 + a^4) - \frac{\gamma_4}{2}\sigma^2 a^2 + \frac{\lambda_6}{6}(\sigma^6 + 15\sigma^2 a^4 + 15\sigma^4 a^2 + a^6) - \lambda_6'(\sigma^2 a^4 + \sigma^4 a^2) + m_\pi^2 f_\pi \sigma$$

$$\bar{\mu}_\sigma^2 \equiv \bar{\mu}^2 + \frac{1}{2}K \,,$$

$$\bar{\mu}_a^2 \equiv \bar{\mu}^2 - \frac{1}{2}K = \bar{\mu}_\sigma^2 - K \,,$$

$$\lambda_4 \equiv \lambda_{41} - \lambda_{42} \,,$$

$$\gamma_4 \equiv 3\lambda_{41} - \lambda_{42} \,,$$

$$\lambda_6 \equiv \lambda_{61} + \lambda_{62} + \lambda_{63} \,,$$

$$\lambda_6' \equiv \frac{4}{3}\lambda_{62} + 2\lambda_{63}$$

Hidden Local Symmetry (HLS)

The vector meson is included basing on Hidden Local Symmetry (HLS) to account for the repulsive interaction in the matter:

$$
\begin{aligned}
\mathcal{L}^{HLS} = & \, a_{VNN}\left[\bar{N}_{1l}\gamma^{\mu}\xi_L^{\dagger}\hat{\alpha}_{\parallel\mu}\xi_L N_{1l} + \bar{N}_{1r}\gamma^{\mu}\xi_R^{\dagger}\hat{\alpha}_{\parallel\mu}\xi_R N_{1r}\right] \\
& + a_{VNN}\left[\bar{N}_{2l}\gamma^{\mu}\xi_R^{\dagger}\hat{\alpha}_{\parallel\mu}\xi_R N_{2l} + \bar{N}_{2r}\gamma^{\mu}\xi_L^{\dagger}\hat{\alpha}_{\parallel\mu}\xi_L N_{2r}\right] \\
& + a_{0NN}\sum_{i=1,2}\left[\bar{N}_{il}\gamma^{\mu}\mathrm{tr}[\hat{\alpha}_{\parallel\mu}]N_{il} + \bar{N}_{ir}\gamma^{\mu}\mathrm{tr}[\hat{\alpha}_{\parallel\mu}]N_{ir}\right] \\
& + \frac{m_{\rho}{}^2}{g_{\rho}{}^2}\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel\mu}] + \left(\frac{m_{\omega}{}^2}{8g_{\omega}{}^2} - \frac{m_{\rho}{}^2}{2g_{\rho}{}^2}\right)\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}]\mathrm{tr}[\hat{\alpha}_{\parallel\mu}] - \frac{1}{8g_{\omega}{}^2}\mathrm{tr}[\omega^{\mu\nu}\omega_{\mu\nu}] - \frac{1}{2g_{\rho}{}^2}\mathrm{tr}[\rho^{\mu\nu}\rho_{\mu\nu}] \\
& + \lambda_{\omega\rho}(a_{VNN} + a_{0NN})^2 a_{VNN}^2\left[\frac{1}{2}\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}\hat{\alpha}_{\parallel\mu}]\mathrm{tr}[\hat{\alpha}_{\parallel}^{\nu}]\mathrm{tr}[\hat{\alpha}_{\parallel\nu}] - \frac{1}{4}\left\{\mathrm{tr}[\hat{\alpha}_{\parallel}^{\mu}]\mathrm{tr}[\hat{\alpha}_{\parallel\mu}]\right\}^2\right],
\end{aligned}
$$

$$M = \xi_L^{\dagger} S \xi_R$$

$$\xi_R = \xi_L^{\dagger} = \exp\left(iP/f_{\pi}\right)$$

$$D^{\mu}\xi_R = \partial^{\mu}\xi_R - ig_{\rho}\rho^{\mu}\xi_R - ig_{\omega}\omega^{\mu}\xi_R + i\xi_R\mathcal{R}^{\mu} + i\xi_R\mathcal{A}^{\mu}$$

$$D^{\mu}\xi_L = \partial^{\mu}\xi_L - ig_{\rho}\rho^{\mu}\xi_L - ig_{\omega}\omega^{\mu}\xi_L + i\xi_L\mathcal{L}^{\mu} - i\xi_L\mathcal{A}^{\mu}$$

$$\hat{\alpha}_{\perp}^{\mu} \equiv \frac{1}{2i}[D^{\mu}\xi_R\xi_R^{\dagger} - D^{\mu}\xi_L\xi_L^{\dagger}]$$

$$\hat{\alpha}_{\parallel}^{\mu} \equiv \frac{1}{2i}[D^{\mu}\xi_R\xi_R^{\dagger} + D^{\mu}\xi_L\xi_L^{\dagger}]$$

Parity Doublet Model (PDM)

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Under mean field approximation, the vector meson Lagrangian is written in a more familiar form:

$$\mathcal{L}_V = -\, g_{\omega NN} \sum_{\alpha j} \bar{N}_{\alpha j} \gamma^0 \omega N_{\alpha j} - g_{\rho NN} \sum_{\alpha j} \bar{N}_{\alpha j} \gamma^0 \frac{\tau_3}{2} \rho N_{\alpha j} + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \rho^2 + \underline{\lambda_{\omega\rho} g_{\omega NN}^2 g_{\rho NN}^2 \omega^2 \rho^2}$$

$$g_{\omega NN} = \left(a_{VNN} + a_{0NN}\right) g_\omega$$

$$g_{\rho NN} = a_{VNN} g_\rho$$

PDM with mean field approximation

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In our works, we employ the mean field approximation:

$$\sigma(x) \to \sigma, \qquad \pi(x) \to 0, \qquad \eta(x) \to 0.$$

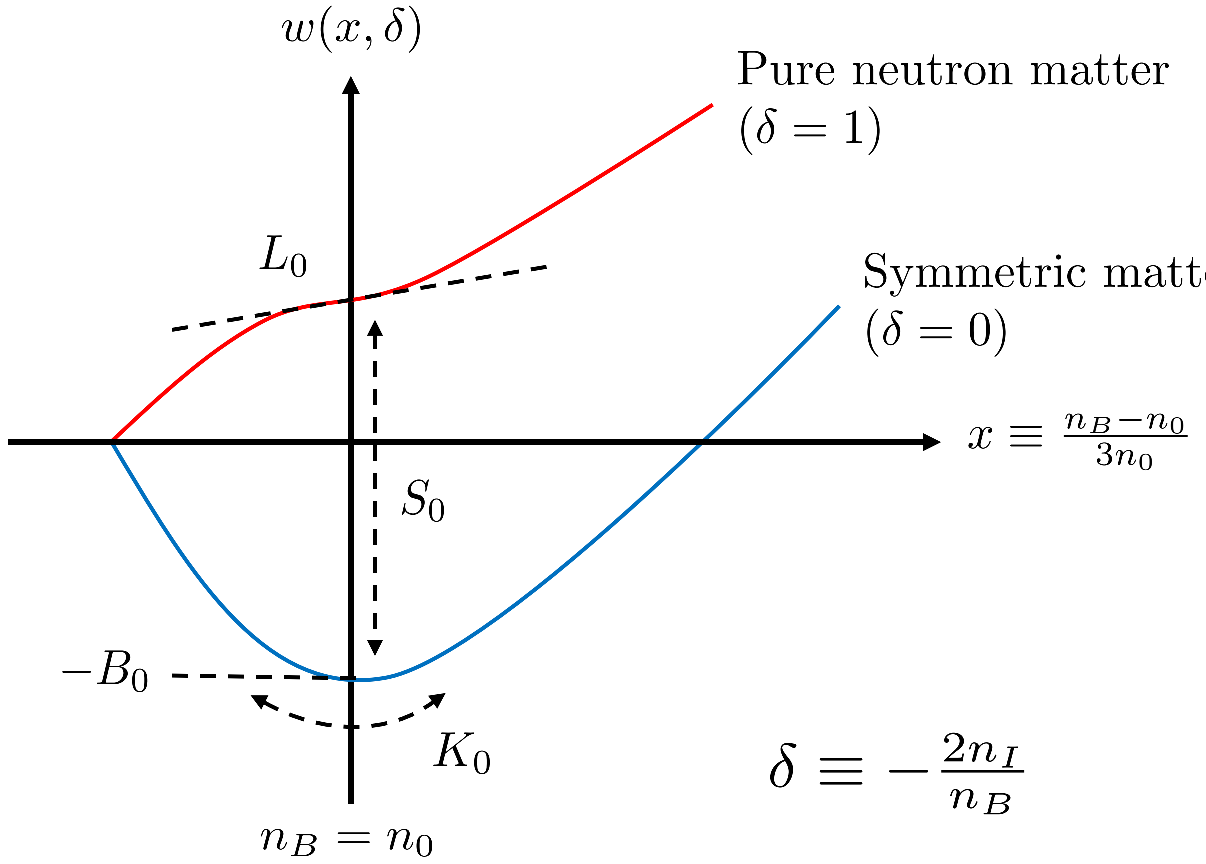
$$\vec{a_0}(x) \to a\delta_{i3}, \qquad a_0^{i=3} \equiv a.$$

$$M = (\sigma + i\vec{\tau}\cdot\vec{\pi}) - (\vec{\tau}\cdot\vec{a}_0 + i\eta)$$

$$\Rightarrow \begin{pmatrix} \sigma - a & 0 \\ 0 & \sigma + a \end{pmatrix}$$

$$\omega_\mu(x) \to \omega\delta_{\mu 0}, \qquad \rho_\mu^i(x) \to \rho\delta_{\mu 0}\delta_{i3},$$

Properties of nuclear matter



$$\delta \equiv -\frac{2n_I}{n_B}$$

$n_0 = 0.16\ \mathrm{fm}^{-3}$ (Saturation density)
$K_0 = 215$ MeV (Incompressibility)
$B_0 = 16$ MeV (Binding energy)
$S_0 = 31$ MeV (Symmetry energy)
$L_0 = 57.7$ MeV (Slope parameter)

Determination of model parameters

- **The physical input we used are as follows:**

$$
\begin{aligned}
m_\pi &= 140 MeV \\
m_a &= 980 MeV \\
m_\eta &= 550 MeV \\
m_\omega &= 783 MeV \\
m_\rho &= 776 MeV \\
m_{N-} &= 1535 MeV \\
m_{N+} &= 939 MeV \\
m_e &= 0.511 MeV \\
m_\mu &= 105 MeV
\end{aligned}
$$

$$
\begin{aligned}
m_0 &= 500 - 900 MeV \\
L_0 &= 50 MeV \\
f_\pi &= 92.4 MeV \\
K_0 &= 240 MeV \\
S_0 &= 31 MeV
\end{aligned}
$$

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- **The coupling of a0 meson (g1,g2) is smaller for larger m0**

m0 (MeV)	500	600	700	800	900
g_1	9.02	8.48	7.81	6.99	5.96
g_2	15.47	14.93	14.26	13.44	12.41

$$m_{\pm j} = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2(\sigma - ja)^2 + 4m_0^2} + \pm(g_1 - g_2)(\sigma - ja)\right]$$

$$(p,n) : j = (+,-)$$

ρ meson coupling constant

With a0

	M0 (MeV)	500	600	700	800	900
L0 = 40 (MeV)	$g\rho$	19.4301	15.5224	13.8896	12.6416	11.3991
L0 = 50 (MeV)	$g\rho$	18.7510	15.0349	13.3523	12.0009	10.6865
L0 = 60 (MeV)	$g\rho$	18.1385	14.5907	12.8728	11.4486	10.0927
L0 = 70 (MeV)	$g\rho$	17.5823	14.1836	12.4416	10.9662	9.5880
L0 = 80 (MeV)	$g\rho$	17.0743	13.8088	12.0510	10.5401	9.1522

Without a0

	M0 (MeV)	500	600	700	800	900
L0 = 40 (MeV)	$g\rho$	12.4751	10.9932	10.7241	10.6424	10.6102
L0 = 50 (MeV)	$g\rho$	10.7175	10.0050	9.9065	9.893	9.9148
L0 = 60 (MeV)	$g\rho$	9.5406	9.2430	9.2512	9.2912	9.3404
L0 = 70 (MeV)	$g\rho$	8.6820	8.6321	8.7109	8.7836	8.8555
L0 = 80 (MeV)	$g\rho$	8.0201	8.1283	8.2554	8.3511	8.4391

Vector meson mixing interaction

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In our a_0 PDM without vector meson mixing interaction, we can compute the slope parameter L_0 :

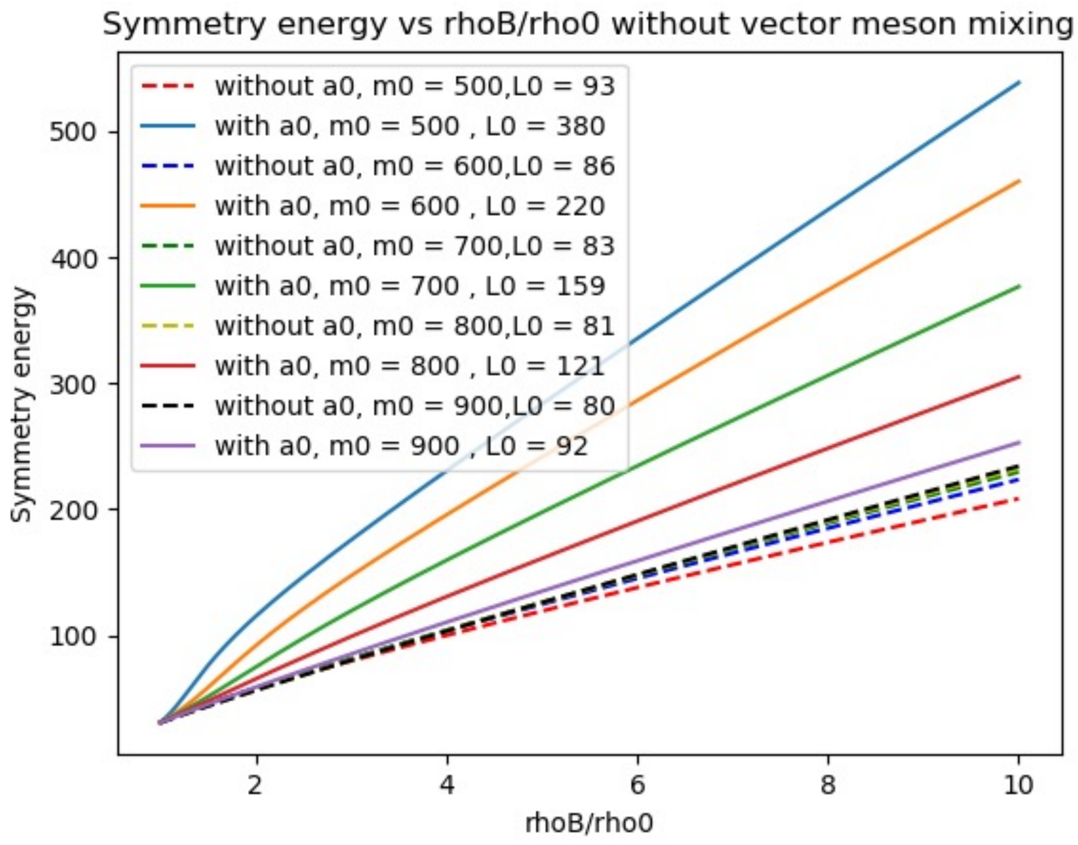
	m_0 [MeV]	600	700	800	900
$K_0 = 215\,\mathrm{MeV}$	$g_{\rho NN}$	12.52	11.20	9.94	8.90
	L_0 [MeV]	120.14	105.21	97.05	87.65
$K_0 = 240\,\mathrm{MeV}$	$g_{\rho NN}$	12.47	11.16	9.90	8.86
	L_0 [MeV]	126.58	108.78	98.67	87.75
$K_0 = 260\,\mathrm{MeV}$	$g_{\rho NN}$	12.43	11.13	9.86	8.83
	L_0 [MeV]	131.19	111.45	99.86	87.75

Recent accepted L0 = 57.7 ± 19 MeV

Li, B.A. et al., Universe 2021, 7

Reduce the stiffness of the matter with vector meson mixing interaction

My model without vector meson mixing



ρ meson coupling constant

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With a0

	M0 (MeV)	500	600	700	800	900
L0 = 60 (MeV)	gρ	18.138 5	14.590 7	12.872 8	11.448 6	10.092 7

Without a0

	M0 (MeV)	500	600	700	800	900
L0 = 60 (MeV)	gρ	9.5406	9.2430	9.2512	9.2912	9.3404