Two-flavor color superconducting quark stars may not exist

Wen-Li Yuan (苑文莉)
Peking University

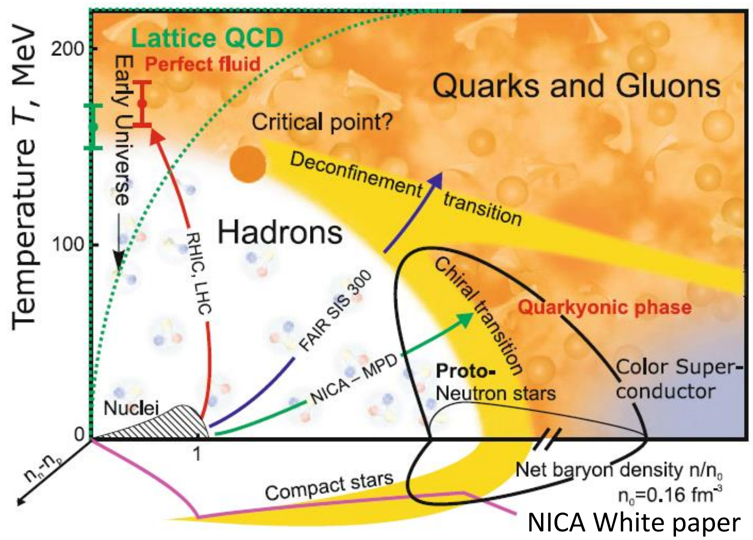
Compact Stars in the QCD phase diagram
October 7, 2024 —October 11, 2024

Outline

1. The states of dense quark matter and normal quark stars
2. Color superconducting quark matter and 2SC quark stars?
3. Summary and outlook

Wen-Li Yuan wlyuan@pku.edu.cn

What is the state of supra-nuclear matter?



- Hadronic phase:

$$\langle\bar{\psi}\psi\rangle \neq 0, \quad \langle\psi\psi\rangle = 0$$

- Quark-gluon plasma:

$$\langle\bar{\psi}\psi\rangle \approx 0, \langle\psi\psi\rangle = 0$$

- Two-flavor color superconductor (2SC):

$$\langle\bar{\psi}\psi\rangle \approx 0 \ \langle ud\rangle \neq 0$$

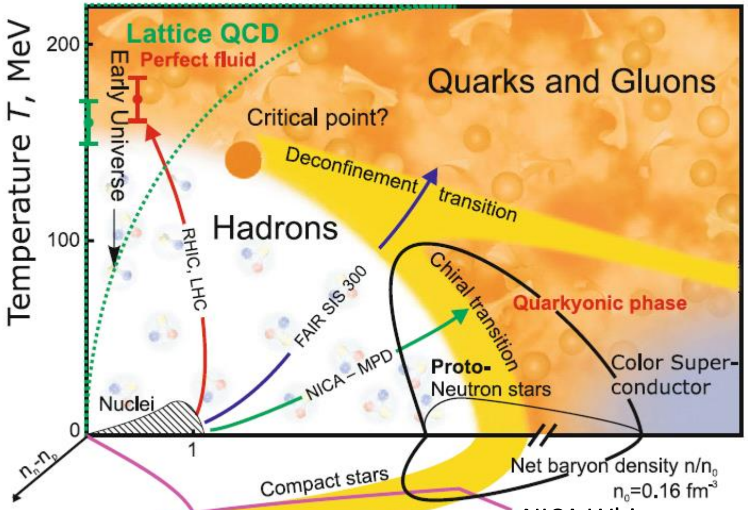
- Color-flavor locking (CFL):

$$\langle ud\rangle \approx \langle us\rangle = \langle ds\rangle \neq 0$$

- Terrestrial experiment: hard to reach
- **Compact stars**: natural laboratory

Maybe multi-messenger observations can tell us more about dense matter.

What is the state of supra-nuclear matter?



NICA White paper

- Hadronic phase:

$$\langle\bar{\psi}\psi\rangle \neq 0, \quad \langle\psi\psi\rangle = 0$$

- Quark-gluon plasma:

$$\langle\bar{\psi}\psi\rangle \approx 0, \langle\psi\psi\rangle = 0$$

- Two-flavor color superconductor (2SC):

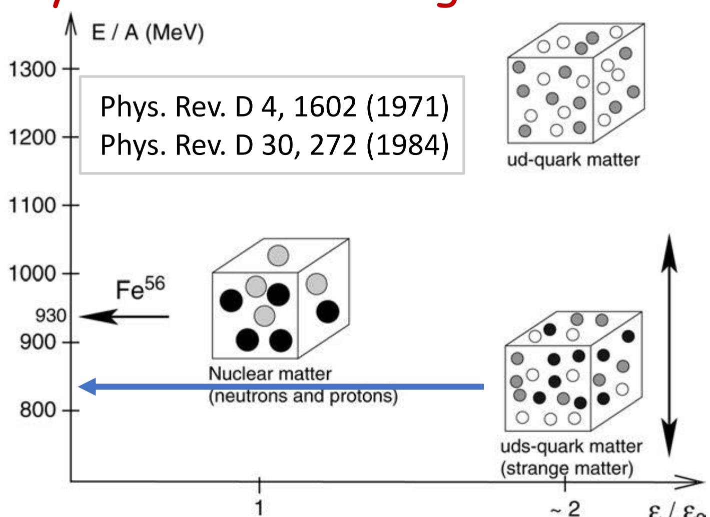
$$\langle\bar{\psi}\psi\rangle \approx 0 \ \langle ud\rangle \neq 0$$

- Color-flavor locking (CFL):

$$\langle ud\rangle \approx \langle us\rangle = \langle ds\rangle \neq 0$$

- Terrestrial experiment: hard to reach
- **Compact stars**: natural laboratory

Maybe multi-messenger observations can tell us more about dense matter.



Bodmer-Witten hypothesis

A recent study: Phys. Rev. Lett. 120,222001 (2018)

Bob Holdom, Jing Ren, and Chen Zhang

Consider the flavor feedback dependence of the quark gas on the QCD vacuum.

Strange/Nonstrange quark stars could also exist.

The Strangeon matter is absolutely stable.

Renxin Xu Astrophys. J. 596:L59–L62 (2003)

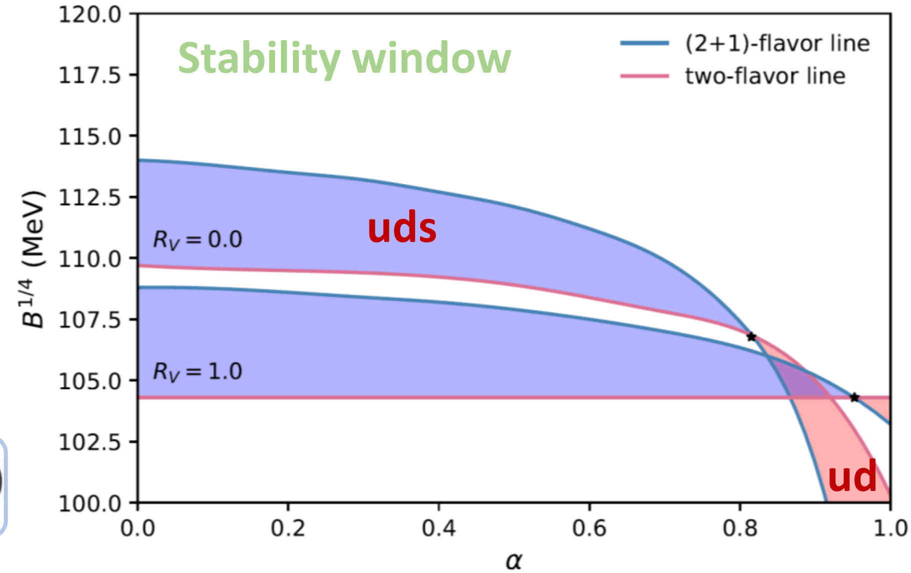
Interacting us and uds quark matter and quark stars

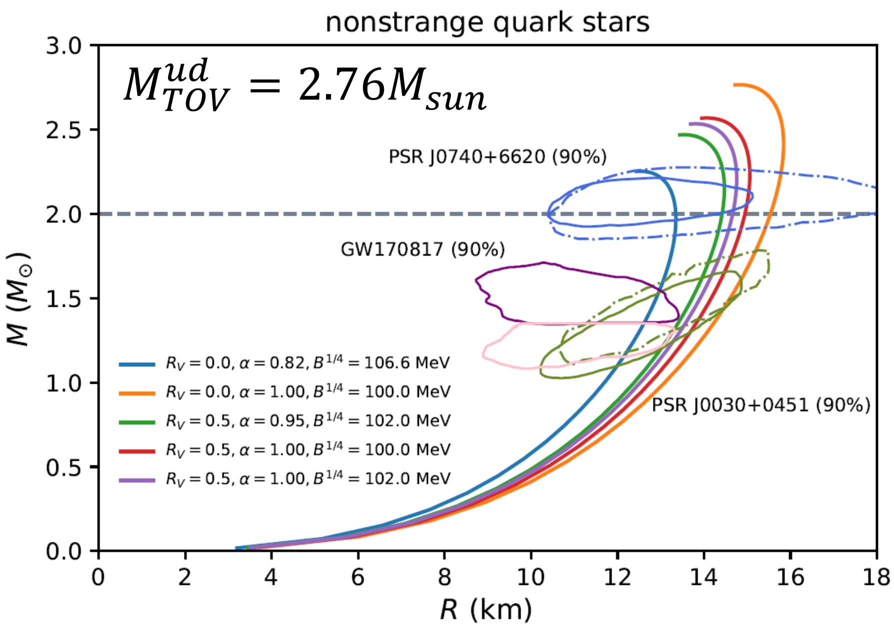
Modified NJL model:

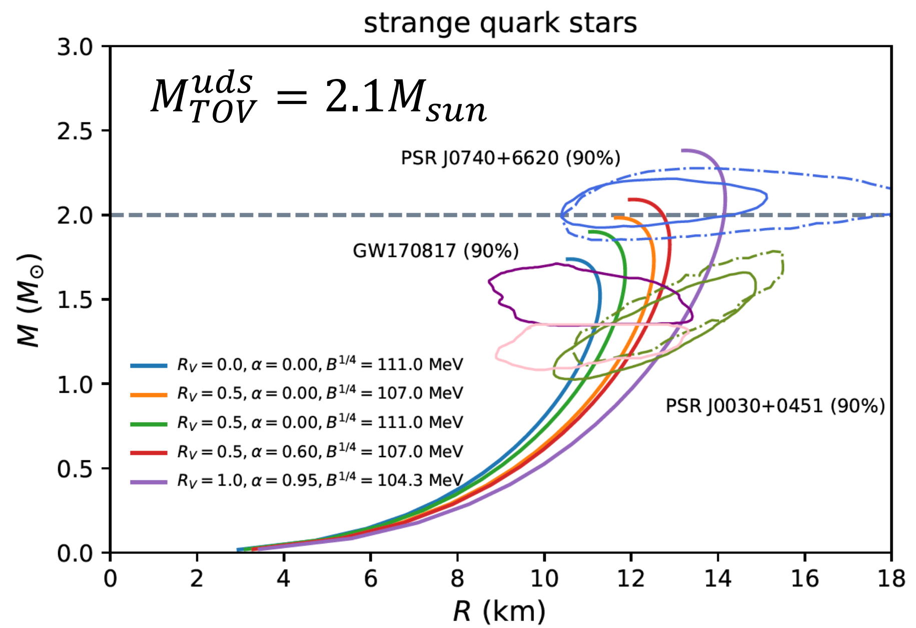
Consider the effect of a rearrangement of fermion field operators.

$$\mathcal{L}_{\rm eff}^{2f} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m + \mu\gamma^{0})\psi + (1-\alpha)\mathcal{L}_{\rm int}^{2f} + \alpha\mathcal{F}(\mathcal{L}_{\rm int}^{2f})$$

$$\mathcal{L}_{\rm eff}^{3f} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m + \mu\gamma^{0})\psi + (1-\alpha)\mathcal{L}_{\rm int}^{3f} + \alpha\mathcal{F}(\mathcal{L}_{\rm int}^{3f})$$







W. L. Yuan, A. Li, Z. Q. Miao, B. J. Zuo, and Z. Bai, Phys.Rev.D 105 (2022)

Outline

1. The states of dense quark matter and normal quark stars

2. Color superconducting quark matter and 2SC quark stars?

3. Summary and outlook

Wen-Li Yuan wlyuan@pku.edu.cn

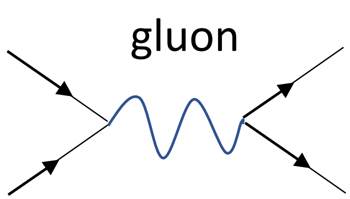
Color superconducting phases

The birth of CSC

1977-1984

$\Delta \sim 1$ MeV

The interaction between quarks at high density is dominated by one-gluon-exchange interaction, which is attractive in the color-antitriplet channel.



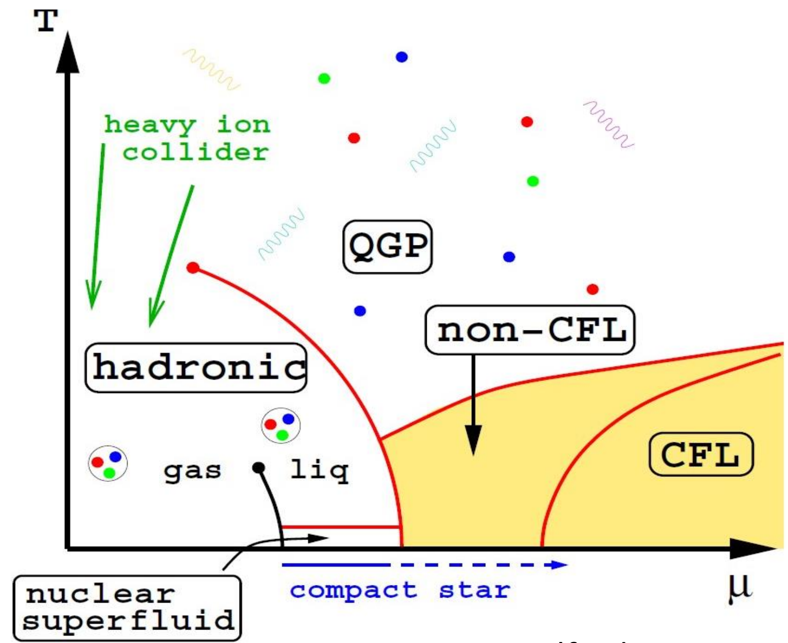
$< qq >$

Barrois, Nucl. Phys. B 129, 390 (1977);
Bailin, Love, Phys. Rep. 107, 325 (1984).

1988

$\Delta \sim 100$ MeV

Rapp, Schaefer, Shuryak, Velkovsky,
Phys. Rev. Lett. 81, 53 (1998);
Alford, Rajagopal, Wilczek, Phys. Lett. B 422, 247 (1998)



Alford 1999

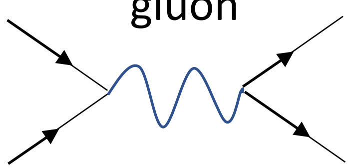
Color superconducting phases

The birth of CSC

1977-1984

$\Delta \sim 1$ MeV

The interaction between quarks at high density is dominated by one-gluon-exchange interaction, which is attractive in the color-antitriplet channel.



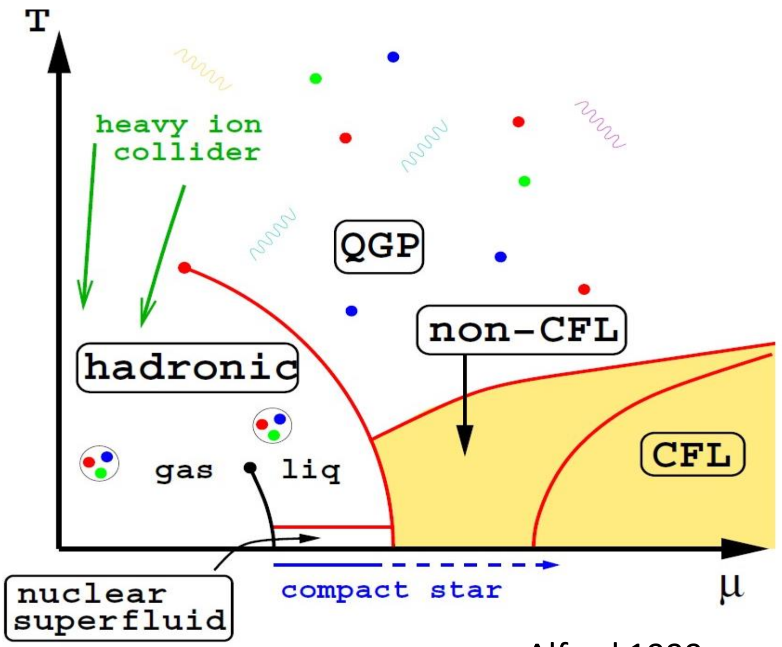
$< qq >$

Barrois, Nucl. Phys. B 129, 390 (1977);
Bailin, Love, Phys. Rep. 107, 325 (1984).

1988

$\Delta \sim 100$ MeV

Rapp, Schaefer, Shuryak, Velkovsky, Phys. Rev. Lett. 81, 53 (1998);
Alford, Rajagopal, Wilczek, Phys. Lett. B 422, 247 (1998)



Alford 1999

$< q_{ia}^{\alpha} q_{jb}^{\beta} >$ Quark Cooper pair

color $\alpha,\ \beta =$r, g, b

flavor $i,\ j$ =u, d, s

spin $a,\ b =$ ↑ ↓

Color superconducting phases

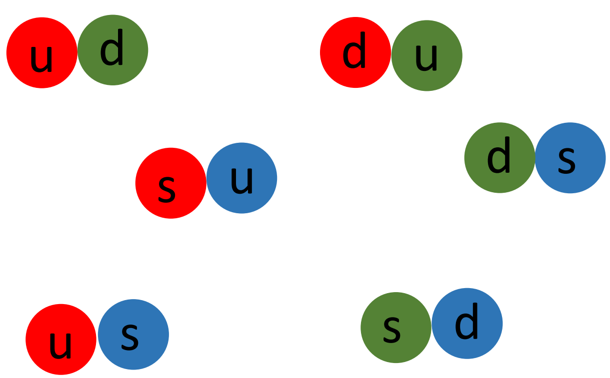
Two flavor CSC (2SC) : spin-0

Intermediate density region, 2SC is suggested to exist.

s quarks do not participate in pairing.

$< ud >\neq 0$ $< du >\neq 0$

Color-flavor-locked (CFL) phase: spin-0



Color superconducting phases

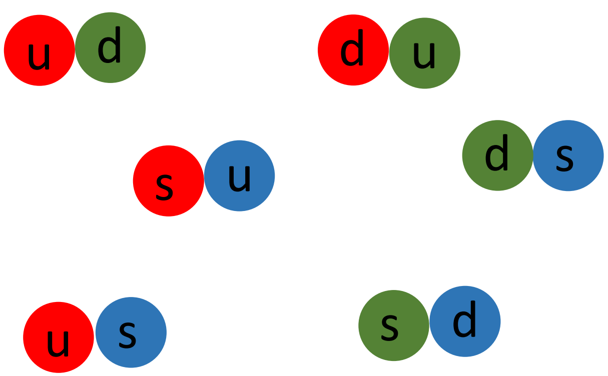
Two flavor CSC (2SC) : spin-0

Intermediate density region, 2SC is suggested to exist.

s quarks do not participate in pairing.

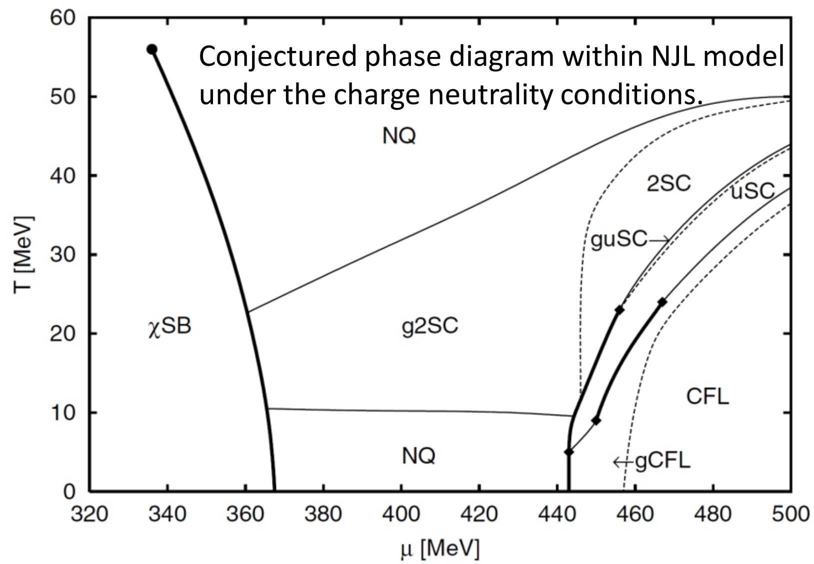
$< ud > \neq 0$ $< du > \neq 0$

Color-flavor-locked (CFL) phase: spin-0



Where can we find CSC?

CSC phase may exist inside compact stars, which has an effect on ***cooling***、***r-mode instability***、***pulsar glitch***…



Rüster et al., Phys. Rev. D **72**, 034004 (2005)

The existing research: gap size and free energy

Our work： **first study to examine the absolute stability of the 2SC quark matter (NJL)**

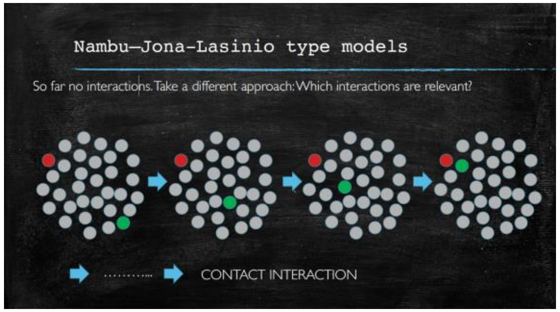
Nambu-Jona-Lasinio (NJL) model

“for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature”
—— the Nobel Prize (2008)

Yoichiro Nambu

Giovanni Jona-Lasinio

- An important and valid effective quark theory (a suitable approximation to QCD in the low-energy and long-wavelength limit)
- Reproduce the basic symmetries of QCD
- Dynamical chiral symmetry breaking (DCSB)
- Confinement



Modified Nambu-Jona-Lasinio (NJL) model

The original two-flavor NJL model:

$$\mathcal{L} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma^0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

- ✓ The vector interaction is important for the study of compact stars.
- ✓ We aim to explore the stability of two-flavor color superconducting quark matter.

$$M = m_0 - 2G_S\sigma$$

$$\sigma = \langle \bar{q}q \rangle$$

The modified two-flavor NJL model:

$$\mathcal{L} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0 + \mu\gamma^0)q + G_S\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right]$$

$$-G_V(\bar{\psi}\gamma^{\mu}\psi)^2 + G_D\left[(\bar{q}i\gamma_5\tau_2\lambda_A q_c)(\bar{q}_c i\gamma_5\tau_2\lambda_A q)\right]$$

$$\Delta = -2G_D\langle \bar{q}_c i\gamma_5\tau_2\lambda_2 q \rangle$$

Parameter fixing in two-flavor NJL model:

Λ, G_S: determined by fitting experimental data on the pion decay constant and pion mass

G_V and G_D: treated as free parameter for our purpose of exploring whether a parameter space exists for absolutely stable 2SC quark matter.

2SC quark matter under charge neutrality

Beta-equilibrium and charge neutrality :

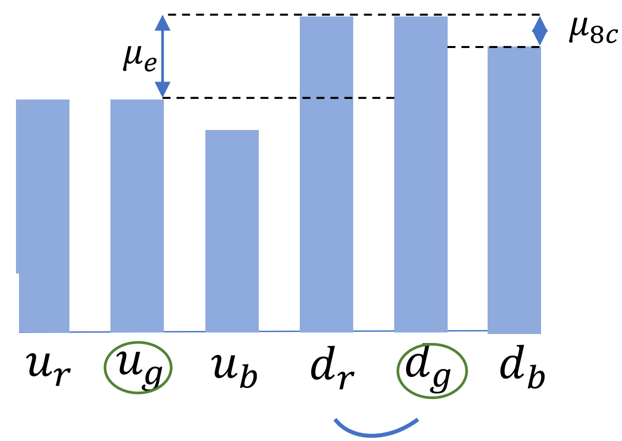
$$\mu_{ur} = \mu_{ug} = \frac{1}{3}\mu_B - \frac{2}{3}\mu_e + \frac{1}{3}\mu_{8c}$$

$$\mu_{dr} = \mu_{dg} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_e + \frac{1}{3}\mu_{8c}$$

$$\mu_{ub} = \frac{1}{3}\mu_B - \frac{2}{3}\mu_e - \frac{2}{3}\mu_{8c}$$

$$\mu_{db} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_e - \frac{2}{3}\mu_{8c}$$

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}}\mu_8 \delta_{ij} (T_8)_{\alpha\beta}$$



mean-filed approximation

The thermodynamical potential:

$$\Omega_q = \frac{(m_0 - M)^2}{4G_S} - \frac{(\mu - \tilde{\mu})^2}{4G_V} + \frac{\Delta^2}{4G_D}$$
$$- 2\int \frac{d^3p}{(2\pi)^3} \{2E_p + 2E_\Delta^+ + 2E_\Delta^-$$
$$+ T\ln\left[1 + \exp\left(-\beta E_{ub}^+\right)\right] + T\ln\left[1 + \exp\left(-\beta E_{ub}^-\right)\right]$$
$$+ T\ln\left[1 + \exp\left(-\beta E_{db}^+\right)\right] + T\ln\left[1 + \exp\left(-\beta E_{db}^-\right)\right]$$
$$+ 2T\ln\left[1 + \exp\left(-\beta E_{\Delta+}^+\right)\right] + 2T\ln\left[1 + \exp\left(-\beta E_{\Delta-}^+\right)\right]$$
$$+ 2T\ln\left[1 + \exp\left(-\beta E_{\Delta+}^-\right)\right] + 2T\ln\left[1 + \exp\left(-\beta E_{\Delta-}^-\right)\right]\}$$

Wen-Li Yuan wlyuan@pku.edu.cn

2SC quark matter under charge neutrality

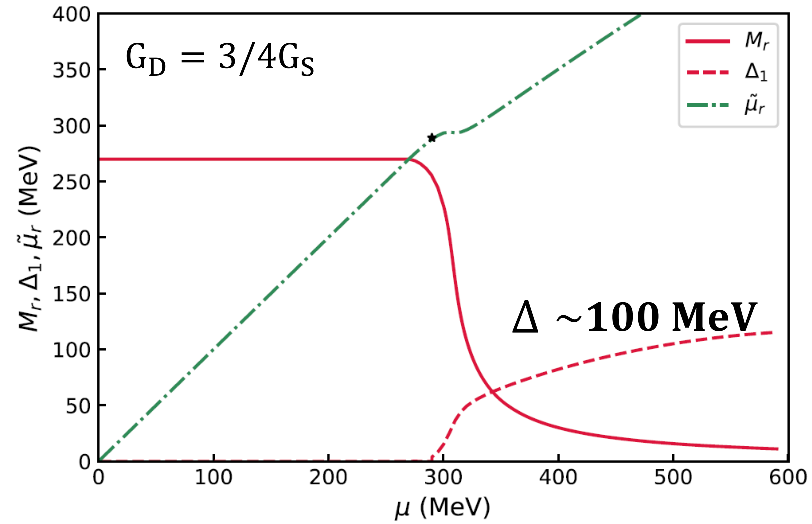
The dynamical mass gap equation:

$$M = m_0 - 2G_S\sigma, \quad \sigma = \langle \bar{q}q \rangle$$

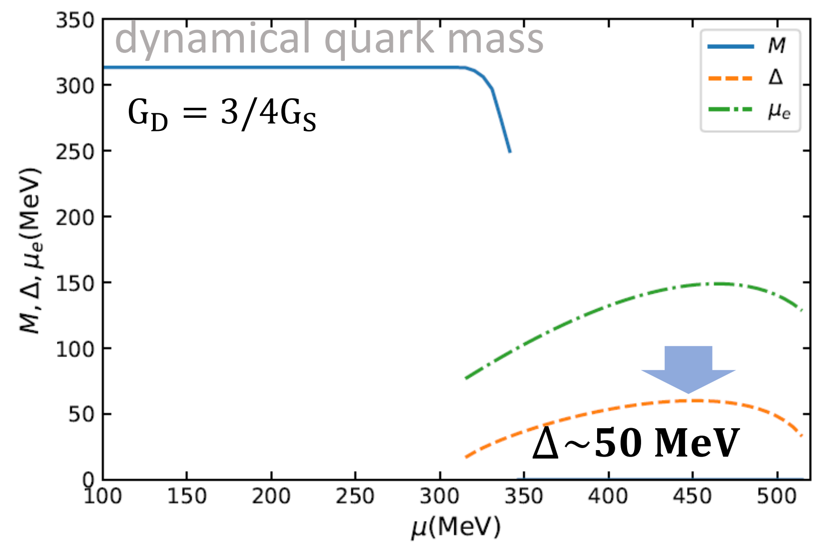
$$M = m_0 + 4G_s M \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E_p}\Big\{\Big[1 - f\big(E_{ub}^{+}\big) - f\big(E_{ub}^{-}\big)\Big] + \Big[1 - f\big(E_{db}^{+}\big) - f\big(E_{db}^{-}\big)\Big] + 2\frac{E_p^+}{E_\Delta^+}\Big[1 - f\big(E_{\Delta^+}^{+}\big) - f\big(E_{\Delta^-}^{+}\big)\Big] + 2\frac{E_p^-}{E_\Delta^-}\Big[1 - f\big(E_{\Delta^+}^{-}\big) - f\big(E_{\Delta^-}^{-}\big)\Big]\Big\}$$

The diquark condensate gap equation:

$$\Delta = 4G_D\Delta \int \frac{d^3\mathbf{p}}{(2\pi)^3}\Big\{\frac{2}{E_\Delta^-}\Big[1 - f\big(E_{\Delta^+}^{-}\big) - f\big(E_{\Delta^-}^{-}\big)\Big] + \frac{2}{E_\Delta^+}\Big[1 - f\big(E_{\Delta^+}^{+}\big) - f\big(E_{\Delta^-}^{+}\big)\Big]\Big\}$$



without charge neutrality



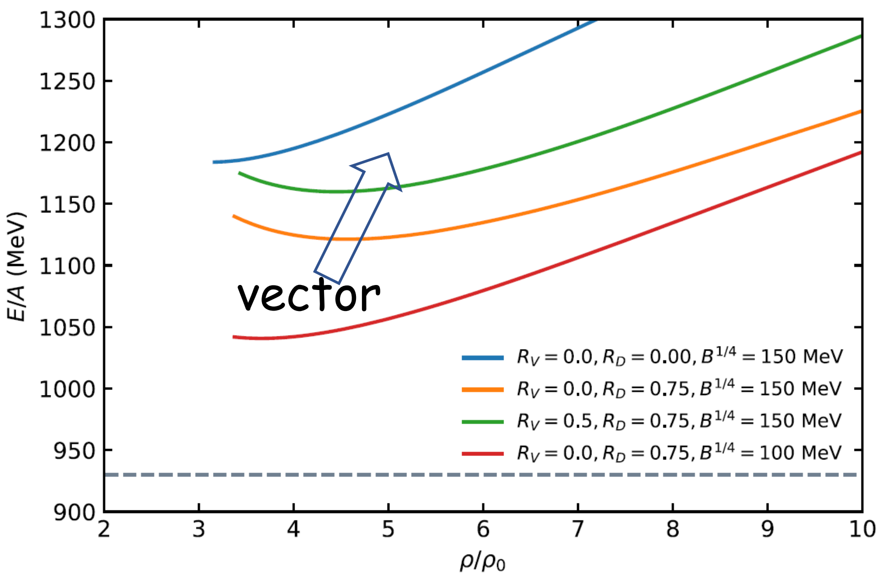
with charge neutrality

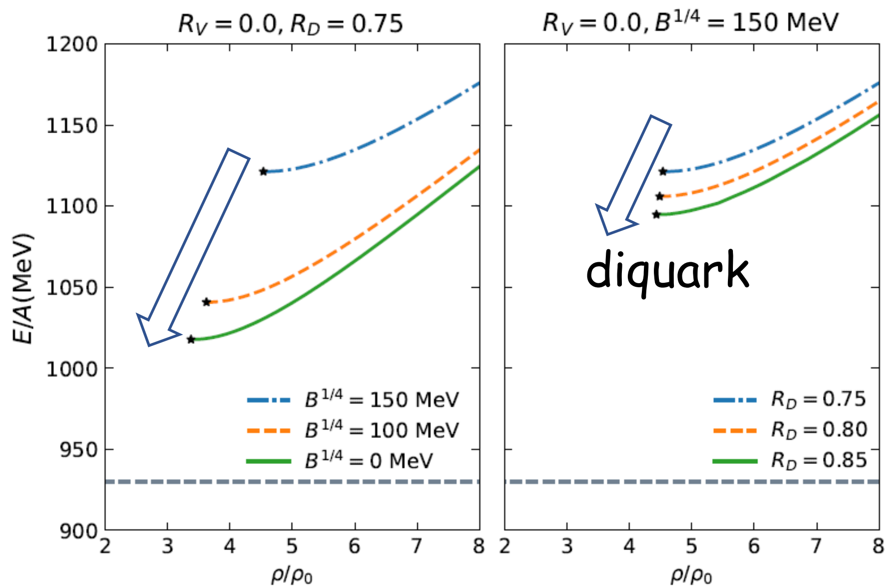
Stability for self-bound 2SC quark matter

Stability condition for self-bound 2SC quark matter:

- 2SC quark matter is more stable than Fe nuclei.

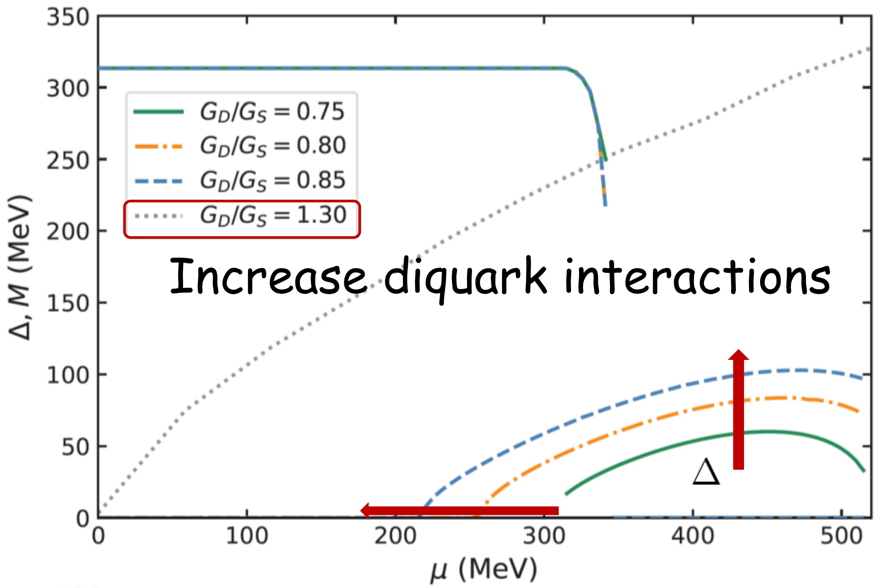
$$E/A(P=0) < (56m_N - 56\times 8.8\ \text{MeV})/56 = 930\ \text{MeV}$$

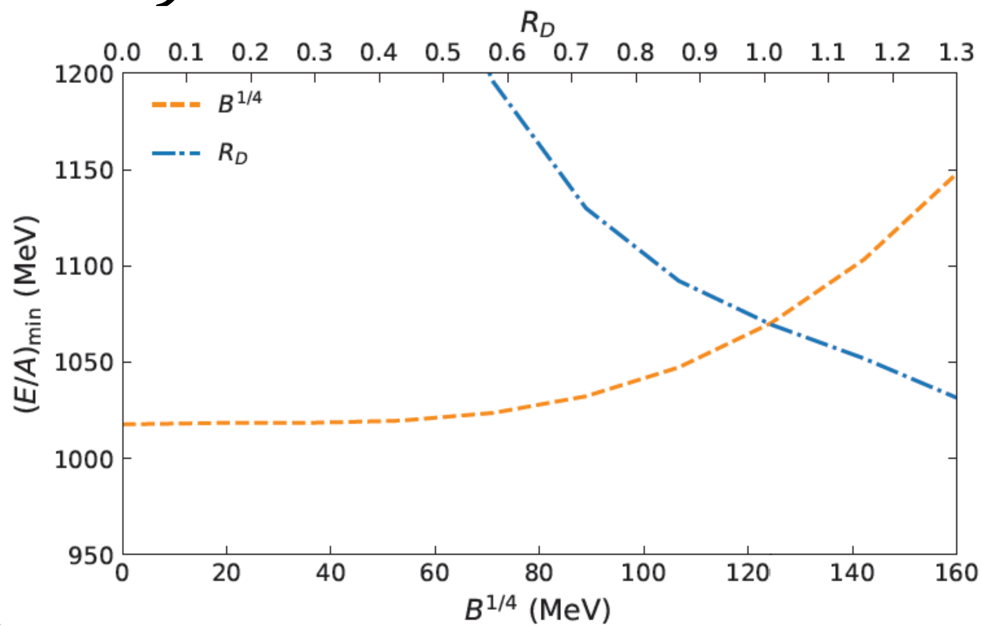


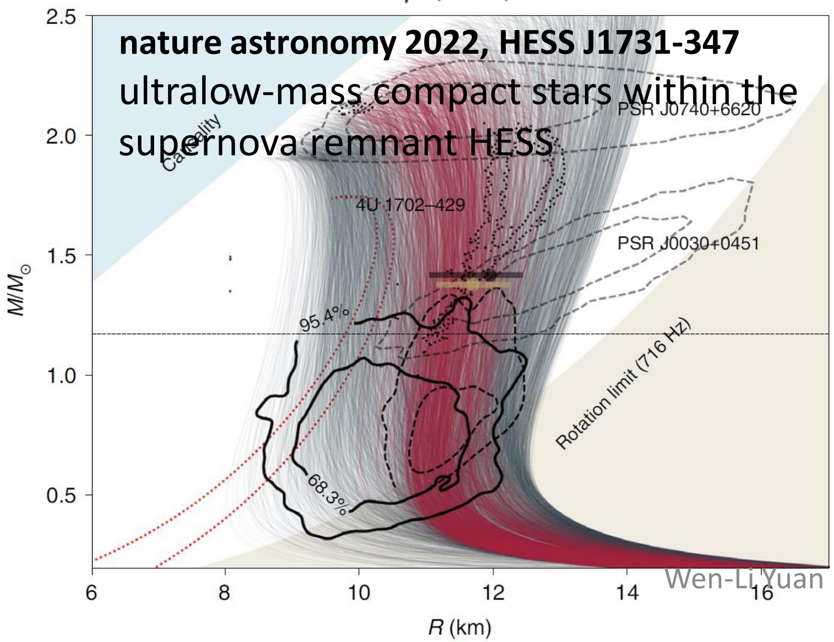


W. L. Yuan, J. Y. Chao, and A. Li, Phys.Rev.D 108 (2023); W. L. Yuan, and A. Li, Astrophys.J. 966 (2024)

No parameter space for absolutely stable 2SC quark matter under charge neutrality







Two-flavor color superconducting quark stars may not exist

$$M = 0.77^{+0.20}_{-0.17}\, M_\odot$$

Summary and outlook

- We find that there is an ample parameter space in the NJL-type model calculations for stable quark matter. **Theoretically supports quark stars as viable alternative physical model for neutron stars**.

- Both **nonstrange and strange quark stars** can, in general, reconcile with the available mass and radius **constraints from observational data.**

- Within the modified NJL model, we investigate the stability of beta-stable two-flavor color superconducting (2SC) phase of quark matter, but find **no physically-allowed parameter space for the existence of 2SC quark stars**.

Thank you!

Wen-Li Yuan wlyuan@pku.edu.cn