Renormalization Group Consistent Treatment of NJL Color-Superconductivity

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TECHNISCHE UNIVERSITÄT DARMSTADT

CRC-TR 211
Strong-interaction matter under extreme conditions

DFG

Renormalization-group consistent treatment of color superconductivity in the NJL model

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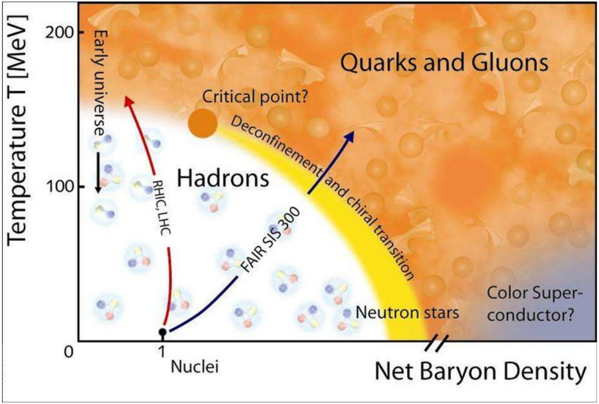
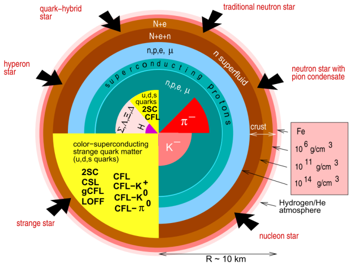
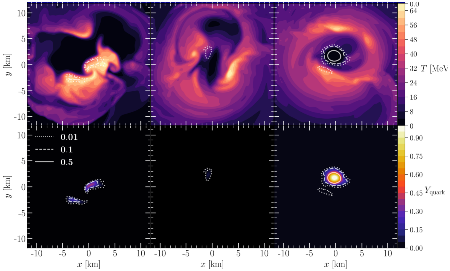
(Dated: August 14, 2024)



- Phase structure at large (but not asymptotically large) density and moderate T is relevant for neutron stars and neutron star mergers



[Weber (1999)]



[Tootle et al. (2022)]

- Merger simulations: densities produced in merger remnant might be sufficient to produce quark matter
- Expected implications: Modified post-merger frequency spectrum [Elias R. Most et al. (2019)] [Bauswein et al. (2019)]

Motivation: signatures of color superconducting phases in neutron star cores or merger remnants?

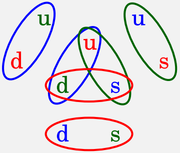
Method: Use effective models for studying color superconductivity

- This talk: Treatment of unphysical regularization artefacts in NJL color superconductivity (cut-off artefacts)
- Ishfaq's talk → Investigations of some astrophysical aspects of this model

Color Superconductivity

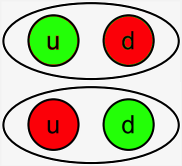
- Cooper theorem: Fermi surface unstable against finite attractive interaction of particles
- Strong interactions: Attractive Diquark interaction in scalar color-, flavor antitriplet channel $\rightarrow$ diquark condensate: $< q_i \mathcal{O}_{ij} q_j >$
- Pairing of particular color-flavor combinations

Color-flavor-locking (CFL)



Large $\mu \gg M_s$
3 finite gap parameters
$\Delta_{ud}, \Delta_{us}, \Delta_{ds}$

2SC



Intermediate $\mu \lesssim M_s$
1 finite gap parameter
Δ_{ud}

Model

Nambu Jona-Lasinio (NJL)-type model

$$\mathcal{L} =$$

$$\bar{\psi}(i\partial\!\!\!/ - m)\psi \qquad \text{kinetic term}$$

$$+ G \sum \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2 \right] \qquad \text{scalar NJL interaction}$$

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$+ G \sum \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right]$	scalar NJL interaction
$- K \left[\mathrm{det_f}(\bar{\psi}(\mathbb{1} + \gamma_5)\psi) + \mathrm{det_f}(\bar{\psi}(\mathbb{1} - \gamma_5)\psi)\right]$	't Hooft (KMT) interaction

Model

Nambu Jona-Lasinio (NJL)-type model

$$
\begin{aligned}
\mathcal{L} = \; & \\
& \bar{\psi}(i\partial\!\!\!/ - m)\psi && \text{kinetic term} \\
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& - K \left[\det{}_\mathsf{f}(\bar{\psi}(\mathbb{1} + \gamma_5)\psi) + \det{}_\mathsf{f}(\bar{\psi}(\mathbb{1} - \gamma_5)\psi) \right] && \text{'t Hooft (KMT) interaction} \\
& + G\,\eta_D \sum (\bar{\psi}i\gamma_5\tau_A\lambda_{A'}\psi^c)(\bar{\psi}^c i\gamma_5\tau_A\lambda_{A'}\psi) && \text{diquark interaction}
\end{aligned}
$$

with charge conjugated spinor $\psi^c = C\bar{\psi}^T$

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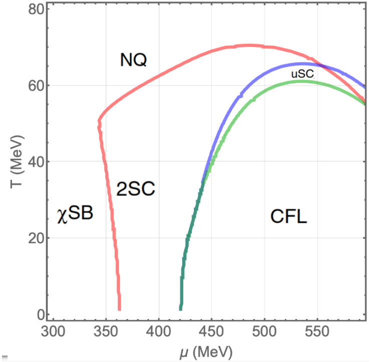
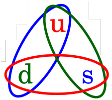
with charge conjugated spinor $\psi^c = C\bar{\psi}^T$

• Mean field approximation: Linearise theory around condensates then minimizing with respect to them

• Neutron star: Enforce charge and color-neutrality + β-equilibrium

- Regularization: sharp 3-momentum cutoff $\Lambda^{'}$
- $\Lambda^{'}, G, K, m$ fitted to vacuum meson spectrum→$\Lambda^{'} = 602MeV$

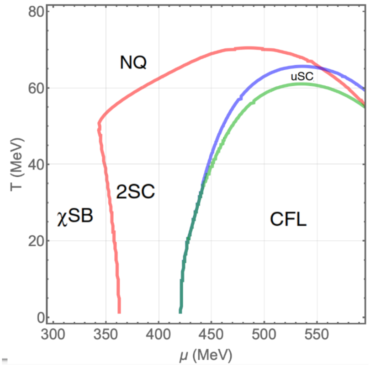


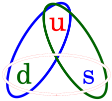


CFL pairing

- Cutoff artefacts
  - Gaps and phase boundary to normal phase decrease in value at $\mu \sim \Lambda'$
  - Appearance of uSC phase [Fukushima 2005]
- Previous explanations for uSC: T=0 arguments $\rightarrow$ Not relevant for $T \neq 0$

Phase Diagram of Neutral Quark Matter





uSC pairing

- Cutoff artefacts
  - Gaps and phase boundary to normal phase decrease in value at $\mu \sim \Lambda'$
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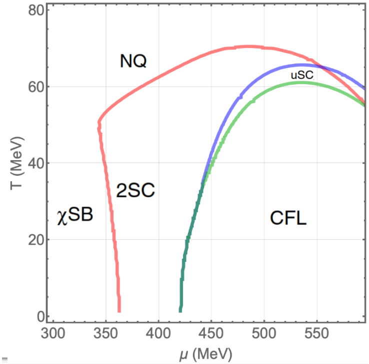
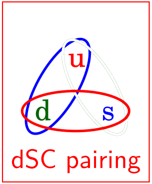
Phase Diagram of Neutral Quark Matter





?

- Cutoff artefacts
  - Gaps and phase boundary to normal phase decrease in value at $\mu \sim \Lambda'$
  - Appearance of uSC phase [Fukushima 2005]
- Previous explanations for uSC: T=0 arguments $\rightarrow$ Not relevant for $T \neq 0$
- Puzzle: Absence of expected dSC phase in CFL melting pattern [Iida et al 2004]

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Solution to Regularization artefacts:

- Use *renormalization group-consistent* regularization presented by Braun et al (2016)

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Renormalization group consistency
and low-energy effective theories

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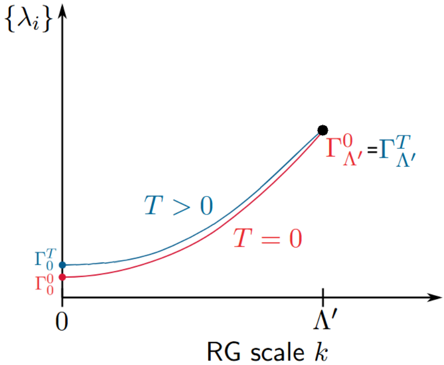
Philosophenweg 16, D-69120 Heidelberg, Germany

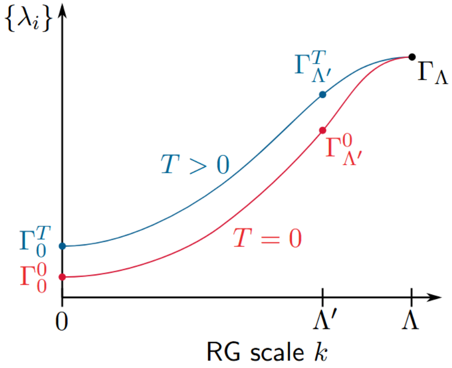
Renormalization Group-Consistency

Problem: Λ' isn't big enough with respect to all scales of the system when in medium: $\Lambda' \sim \mu, T, \Delta_i, M_j$.



Renormalization Group-Consistency

Assumption: Λ bigger than all scales of the system: $\Lambda \gg \mu, T, \Delta_i, M_j, \Lambda'$.
Then $\Gamma_\Lambda(\mu, T) \approx \Gamma_\Lambda(\mu = 0, T = 0)$ can be calculated from flow equation in vacuum



- Issue of divergences in medium $\rightarrow$ Medium renormalization
- $\lambda = \frac{\Lambda}{\Lambda'} = 10$ is enough to satisfy the RG consistency conditions for our region of interest

Results: Phase Diagram

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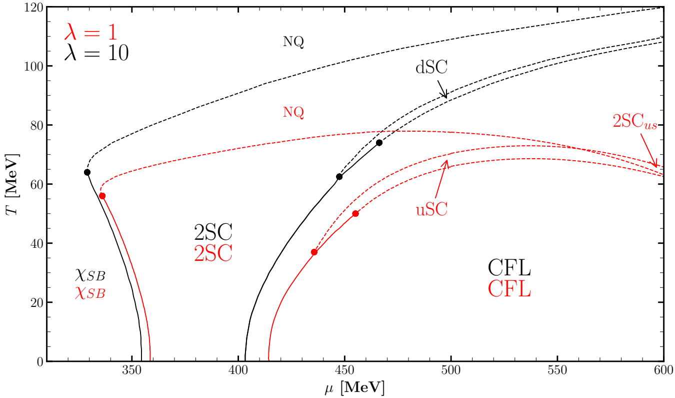
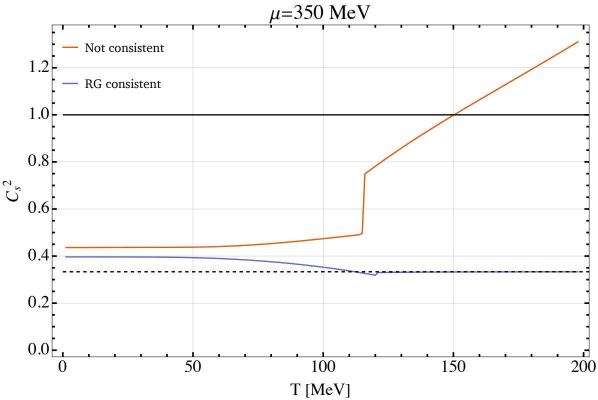


- Cut-off artefacts are removed
  - Expected increasing trend of phase boundaries
  - No uSC phase
- Expected dSC phase appears in CFL melting: Puzzle solved

A natural outcome of the RG consistent treatment: correct thermodynamic limits

- Correct thermodynamics are mostly important for the astrophysics related calculations
- Example: Above the T_c at high densities speed of sound should go to the chiral limit $C_s^2 = 1/3$



Summary and Outlook

Summary

- NJL color-superconductivity suffers from cut-off artefacts
- RG-consistent formulation systematically removes the cutoff artefacts and changes the phase diagram in terms of critical temperatures, diquark condensate values and phase transition points
- RG-consistent formulation for neutral CSC matter is in agreement with expected dSC phase in CFL melting patter and also it gives the correct thermodynamics for the model

Outlook

- Main interest: Study imprints of color superconductivity in neutron star mergers
- Constraining free parameters of the model for astrophysical calculations → Next talk by Ishfaq

Thank you for listening!