Constraining Coupling Parameters of NJL Color Superconductivity for Compact Stars

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Compact Stars in the QCD phase diagram (CSQCD)

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Strong-interaction matter under extreme conditions

Alexander von HUMBOLDT STIFTUNG

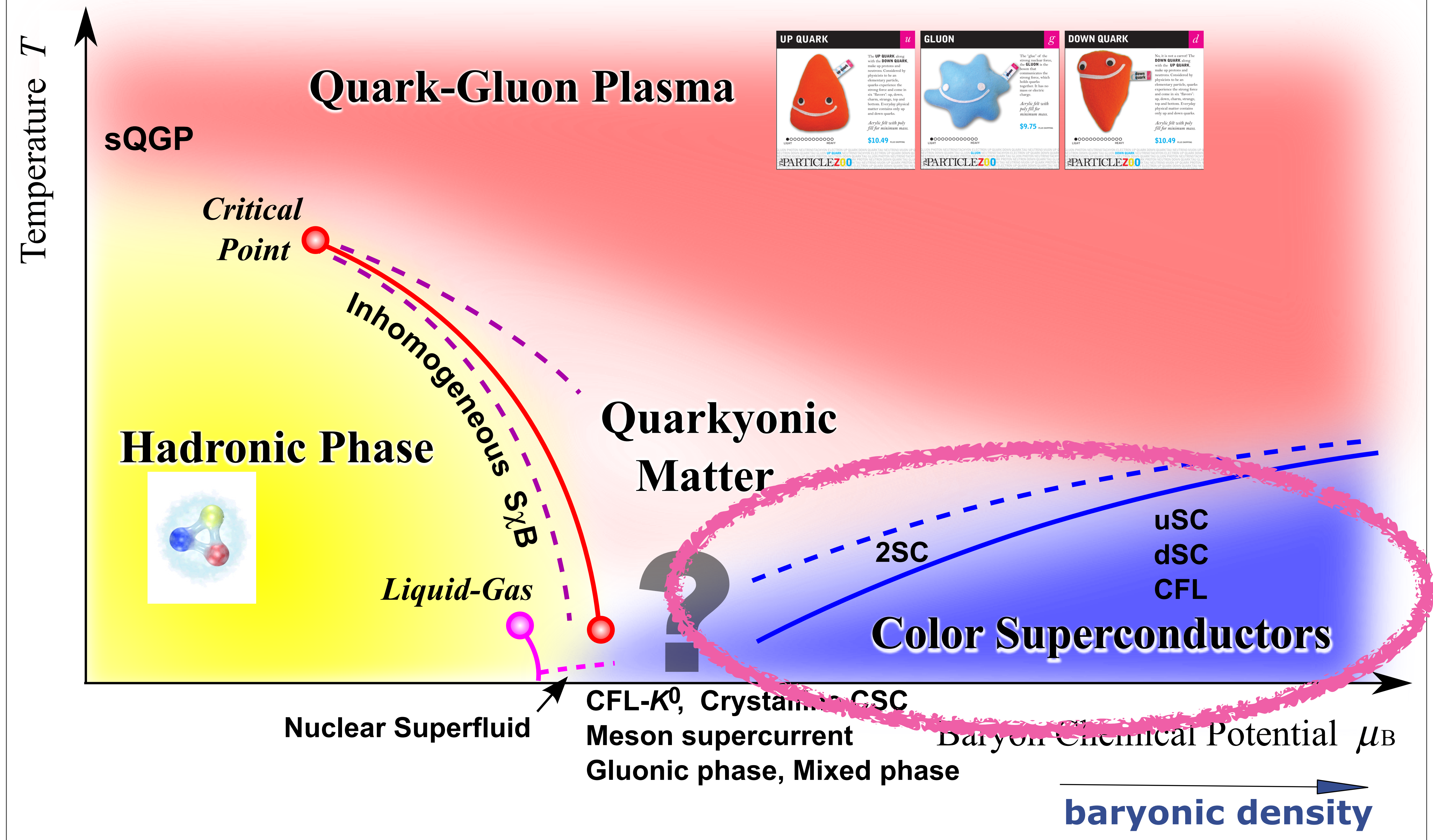
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- **NJL Model**
- Equation of State
- Constraining the Model Parameters
- EoS within the Allowed Parameter range
- Conclusion

Introduction

- CSC phases in the context of NS and quark stars have been studied in extensive detail, including the 2SC and CFL phases.
- The NJL model and its extension to CSC is a popular effective model for investigating dense QM.
- However, the reliability of its results is challenged by cutoff artifacts which emerge if T/μ are of the order of the cutoff energy scales.
- In this work, we explore the astrophysical implications of the **renormalization group-consistent (RG-consistent) NJL model**. **(arXiv:2408.06704.)**
- We extend the model with a repulsive vector interaction and vary the value of the diquark and vector couplings.

Phase diagram of QCD



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NJL Model

- We model quark matter within an NJL model with a diquark interaction, allowing for the formation of CSC condensates.
- Following the recent advancement for a RG-consistent description of the NJL model.
- Removes artifacts of the conventional regularization.
- Provides a consistent investigation of the phase structure at high chemical potentials.

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_{\bar{q}q} + \mathscr{L}_{qq} + \mathscr{L}_V + \mathscr{L}_{\mathrm{L}}.$$

$$\mathscr{L}_0 = \bar{\psi}(i\partial + \gamma^0\hat{\mu} - \hat{m})\psi$$

kinetic Lagrangian for the quark fields

$$\mathscr{L}_{\bar{q}q} = G_S \sum_{a=0}^{8} \left[(\bar{\psi}\tau_a\psi)^2 + (\bar{\psi}i\gamma_5\tau_a\psi)^2\right]$$
$$-K[\det_f(\bar{\psi}(1+\gamma_5)\psi) + \det_f(\bar{\psi}(1-\gamma_5)\psi)]$$

$U(3)_L \times U(3)_R$- symmetric scalar and pseudoscalar four-point interactions of the NJL model with the NJL coupling constant G_S

$$\mathscr{L}_{qq} = G_D \sum_{\gamma,c} (\bar{\psi}^a_\alpha i\gamma_5 \epsilon^{\alpha\beta\gamma}\epsilon_{abc}(\psi_C)^b_\beta)((\bar{\psi}_C)^r_\rho i\gamma_5\epsilon^{\rho\sigma\gamma}\epsilon_{rsc}\psi^s_\sigma)$$

quark pairing in the spin zero color-falvor anti-triplet channel in the Hartree approximation

$$\mathscr{L}_V = -G_V\left(\bar{\psi}\gamma^0\psi\right)^2,$$

Repulsive four-point vector interaction with vector coupling G_V

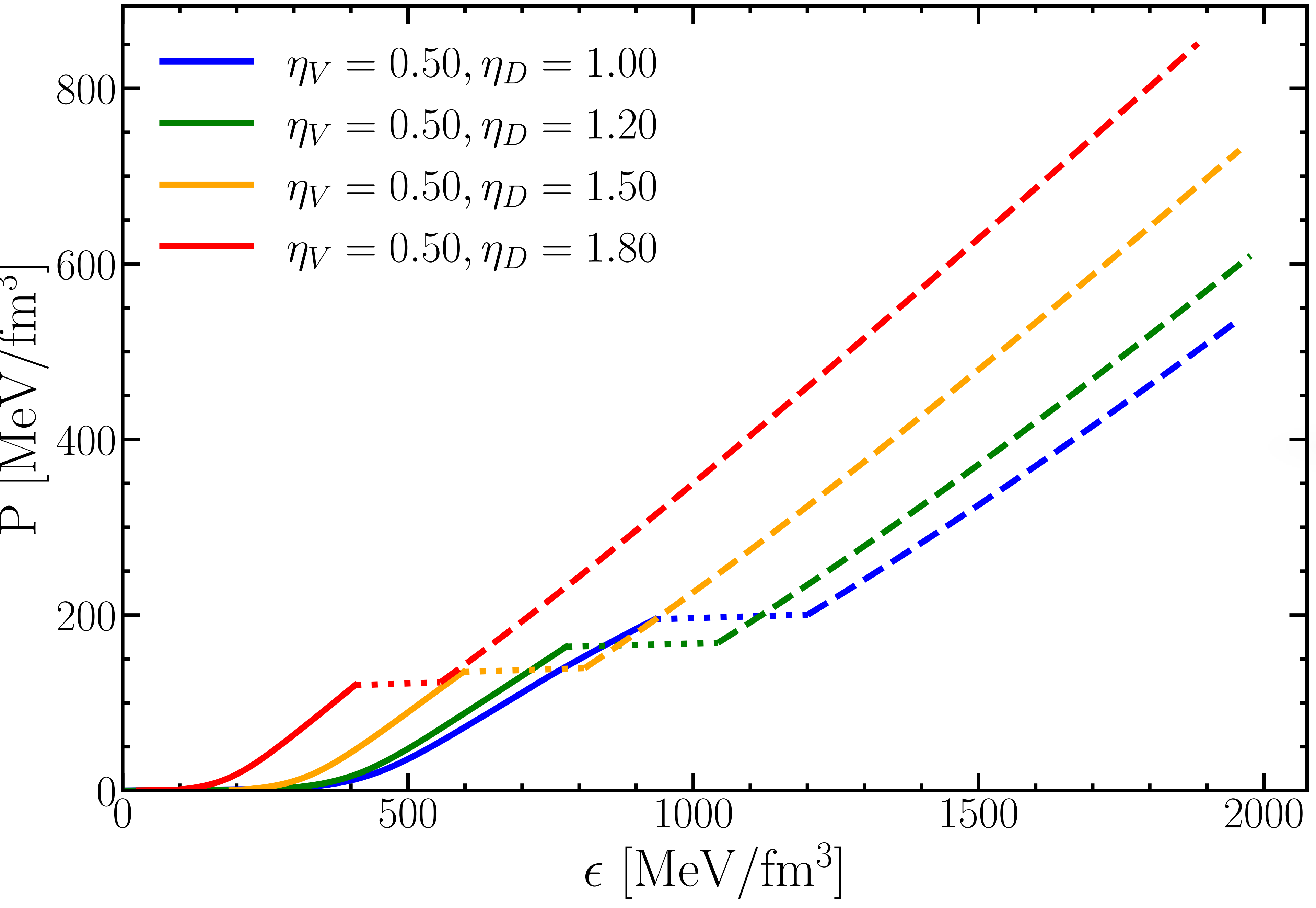
$$\mathscr{L}_L = \sum_{L=e,\mu} \bar{\psi}_L(i\partial - m_L)\psi_L$$

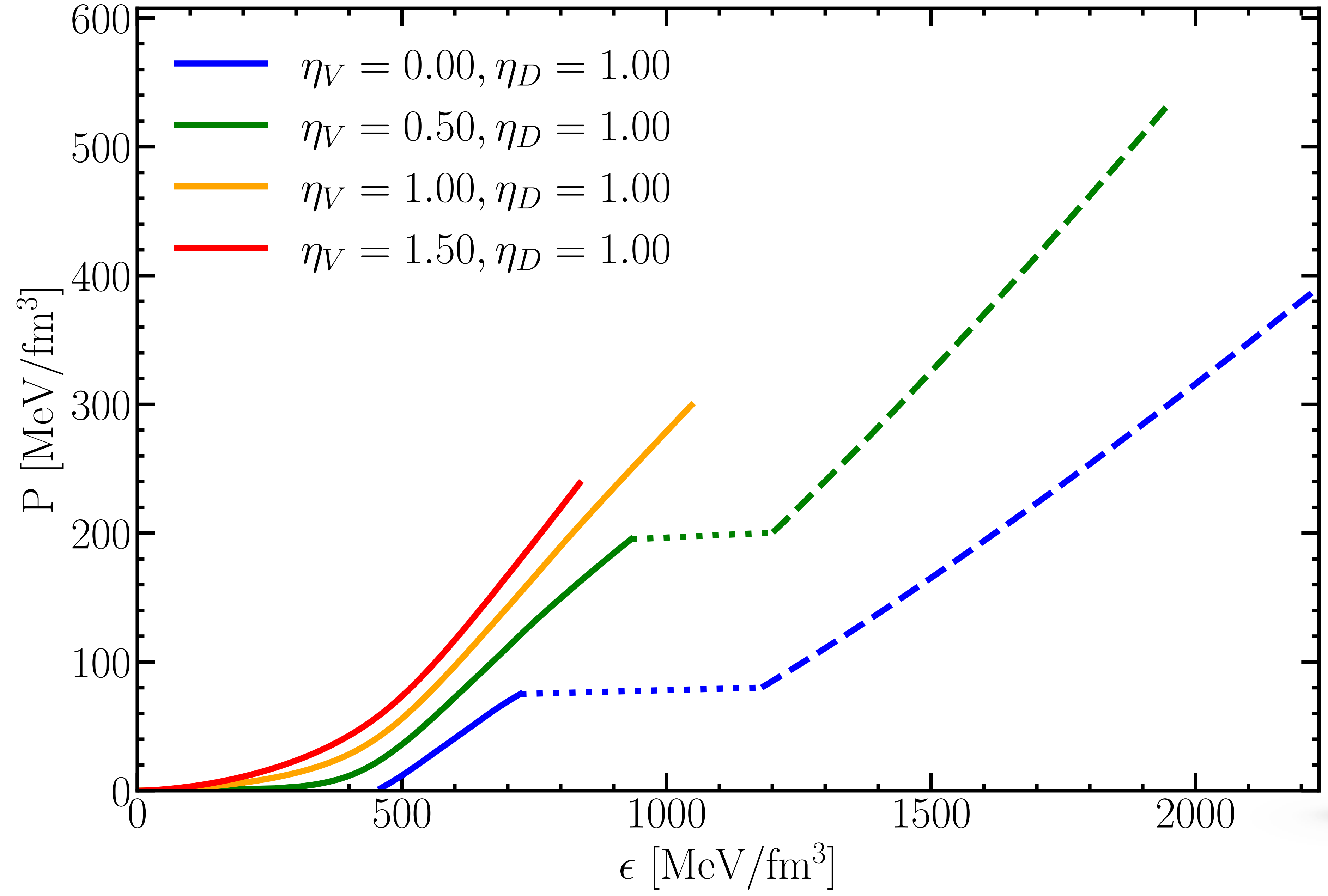
kinetic terms for electrons and muons

The mean-field effective potential per volume in the RG-consistent regularization

$$\begin{aligned}\Omega_{\mathrm{eff}}(\boldsymbol{\mu}, T, \boldsymbol{\chi}, \tilde{\mu}) = \mathcal{V}(\boldsymbol{\chi}, \tilde{\mu}) - \frac{1}{2\pi^2}\Bigg(&\int_0^\Lambda dp\, p^2 \mathcal{A}(\boldsymbol{\mu}, T, \boldsymbol{\chi}, \tilde{\mu}) \\ &- \int_{\Lambda'}^{\Lambda} dp\, p^2 \mathcal{A}_{\mathrm{vac}}(\boldsymbol{\chi}) \\ &- \int_{\Lambda'}^{\Lambda} dp\, p^2 \sum \frac{1}{2}\mu^2_{\alpha a,\beta b} \\ &\times \left(\frac{\partial^2}{\partial\mu^2_{\alpha a,\beta b}}\mathcal{A}(\boldsymbol{\mu}, 0, \boldsymbol{\chi})\right)\Bigg|_{\boldsymbol{\mu}=\tilde{\mu}=\mathbf{M}=0;\Delta_{\alpha a,\beta b}\neq 0}\Bigg) \\ &+ \Omega_L(\mu_e, T).\end{aligned}$$

Equation of State

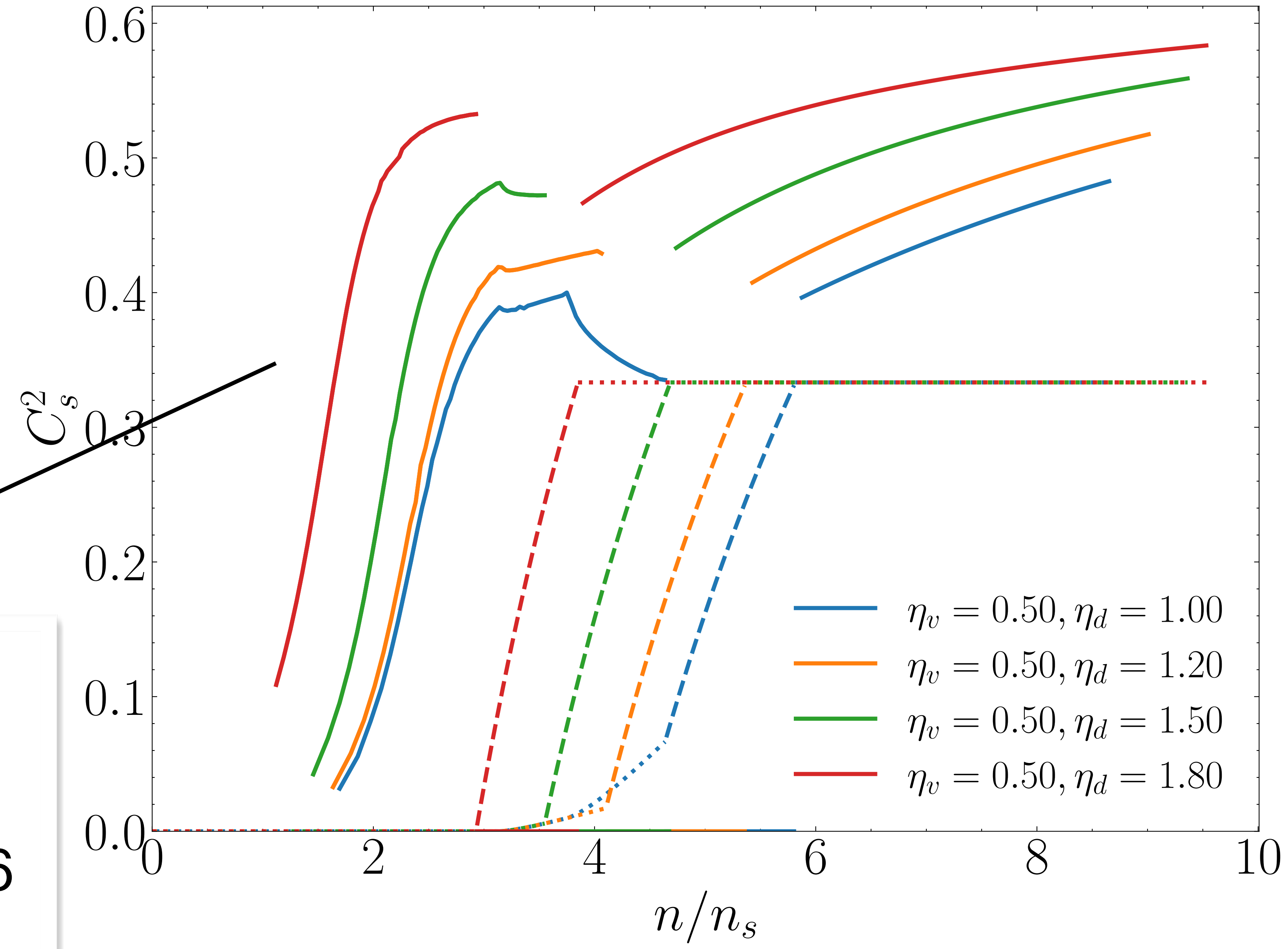




SoS and Ys

with increasing η_D

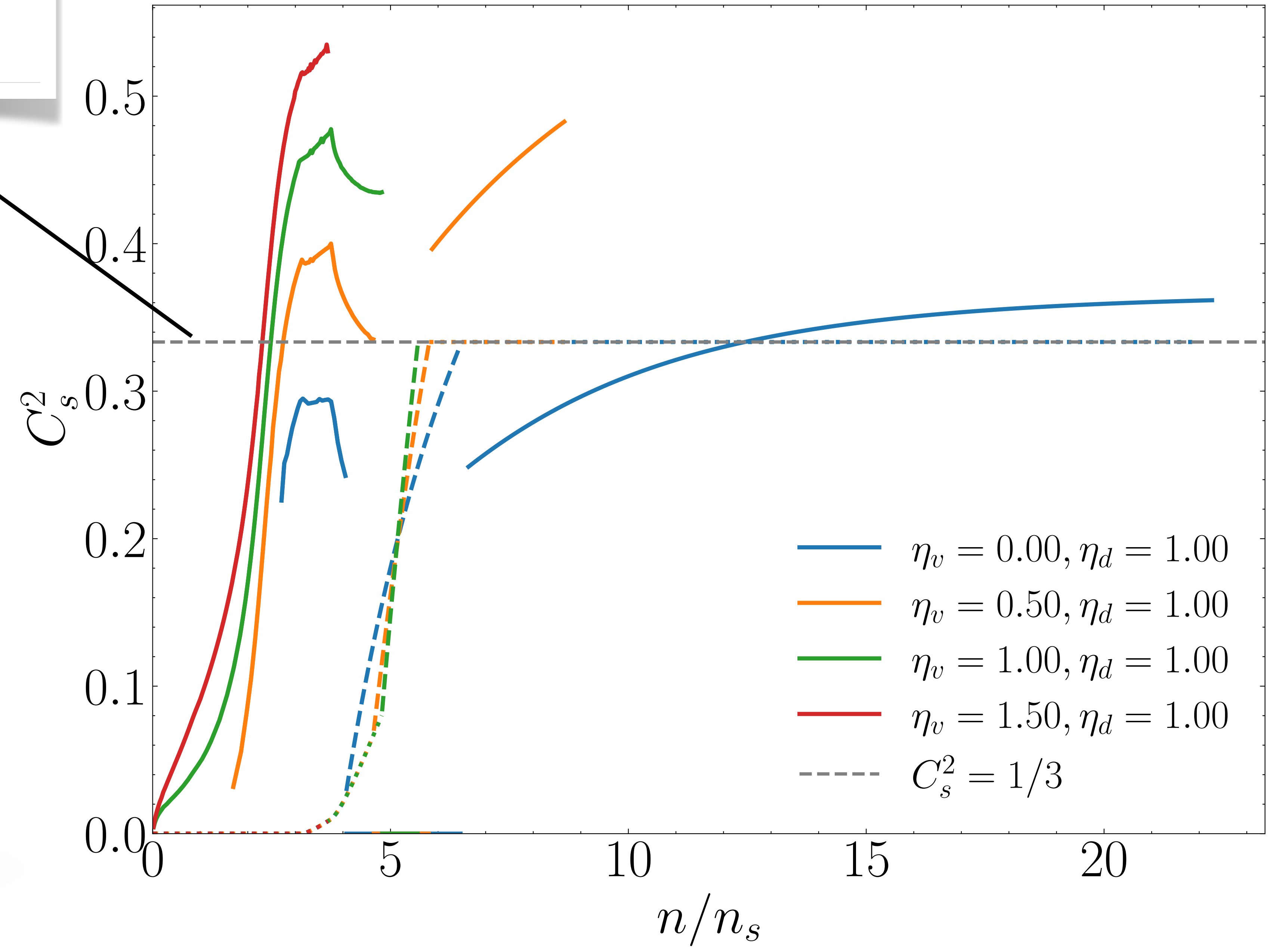
- PT shifts to low denisty
- Density jump decreases



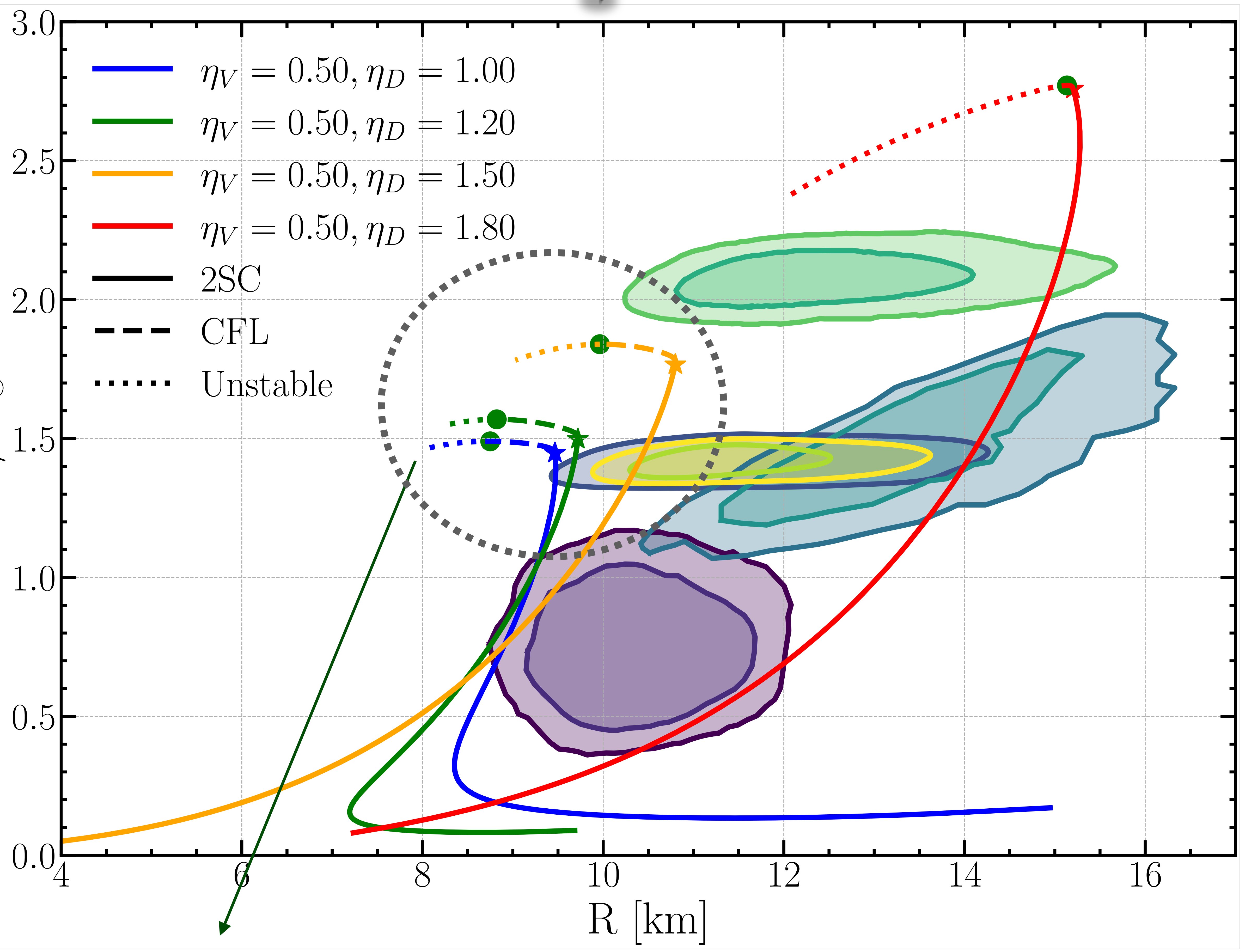
- SoS approaches conformal limit for $\eta_V = 0$
- For $\eta_V \neq 0$, still less than 0.6
- Characteristic features with PT = discontinuity

with increasing η_V

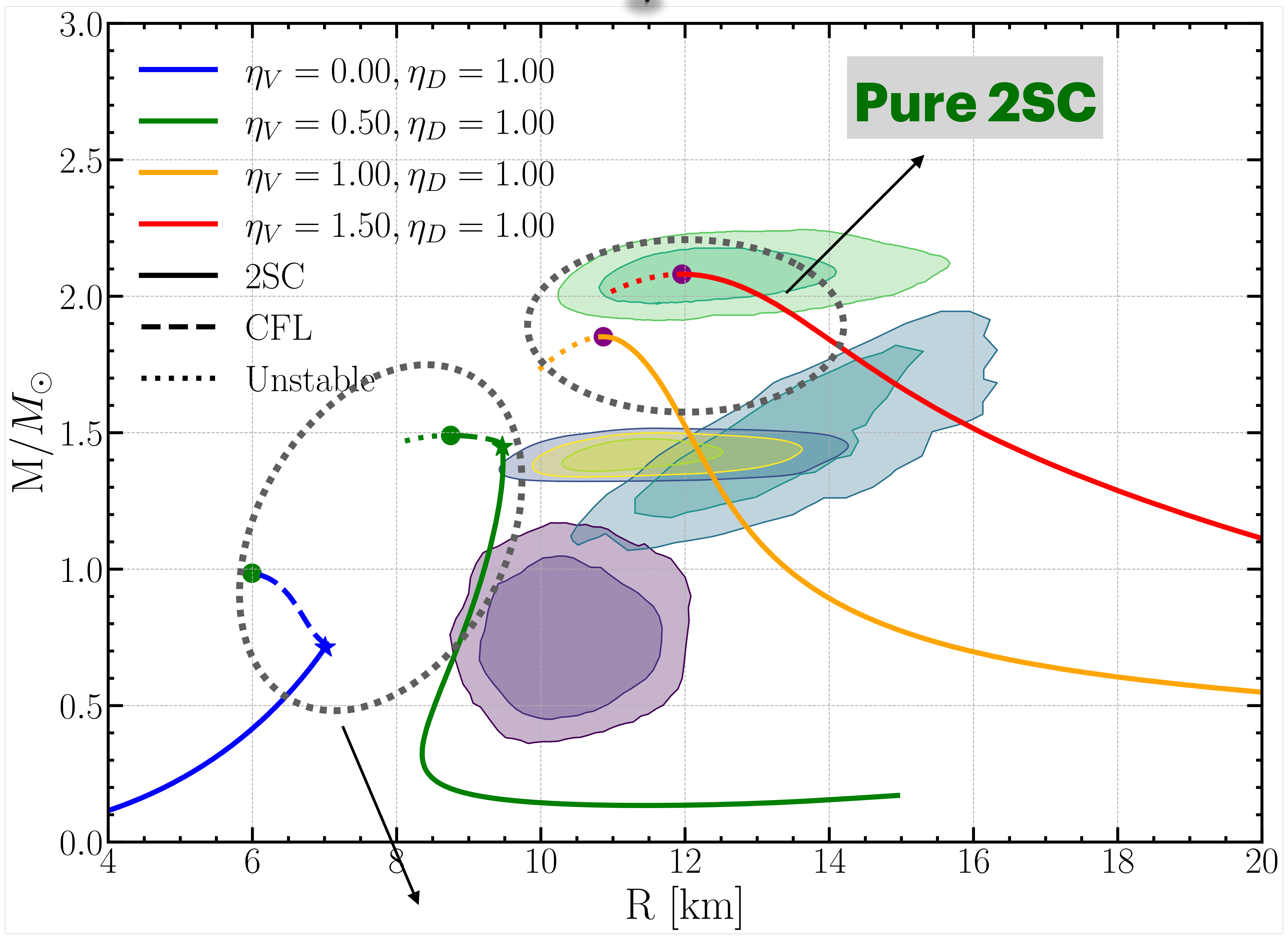
- Stiff EoS
- Less CFL to 2SC phase only



Mass-Radius Plots



More CFL



2SC to CFL

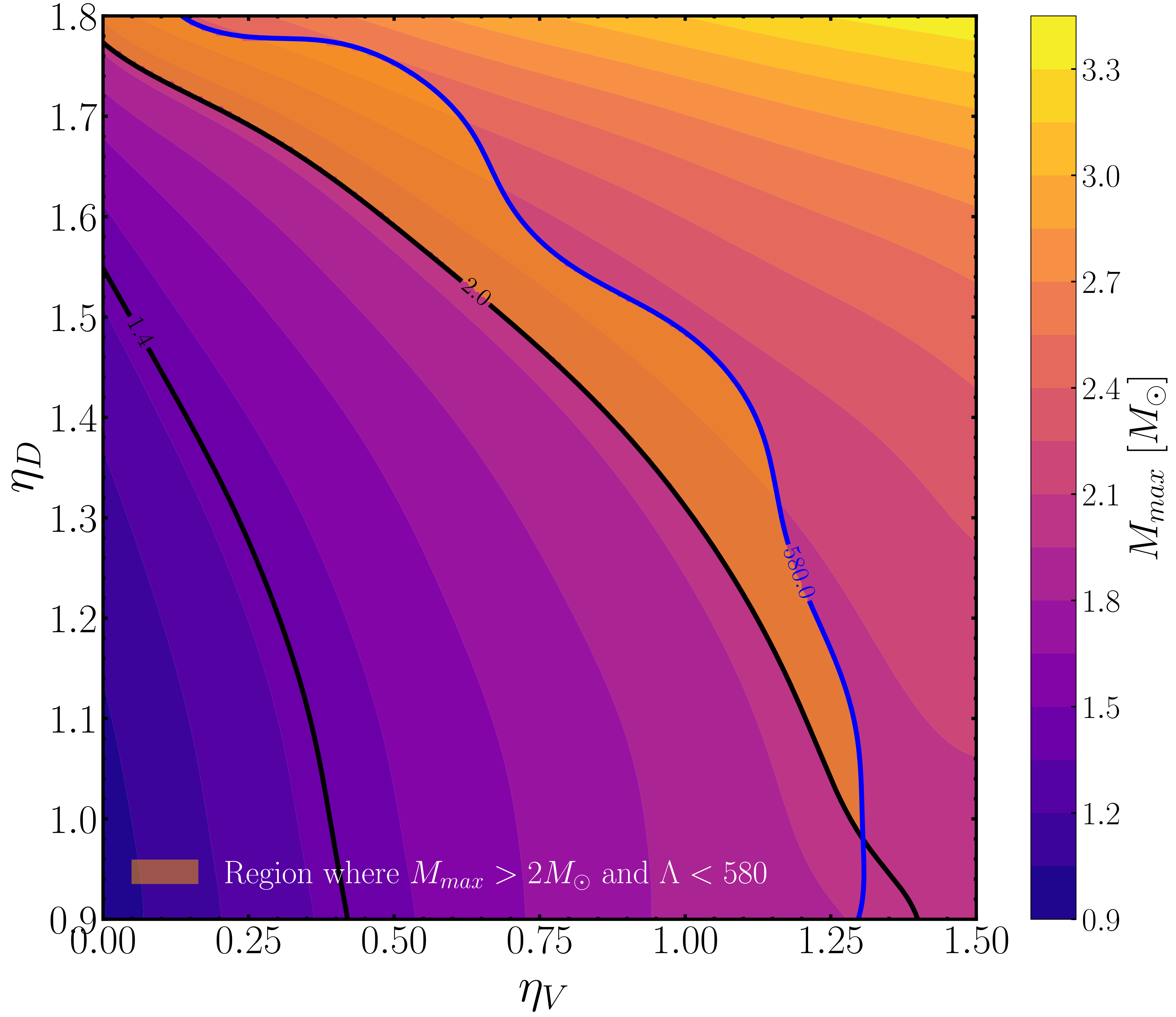
Low η_V

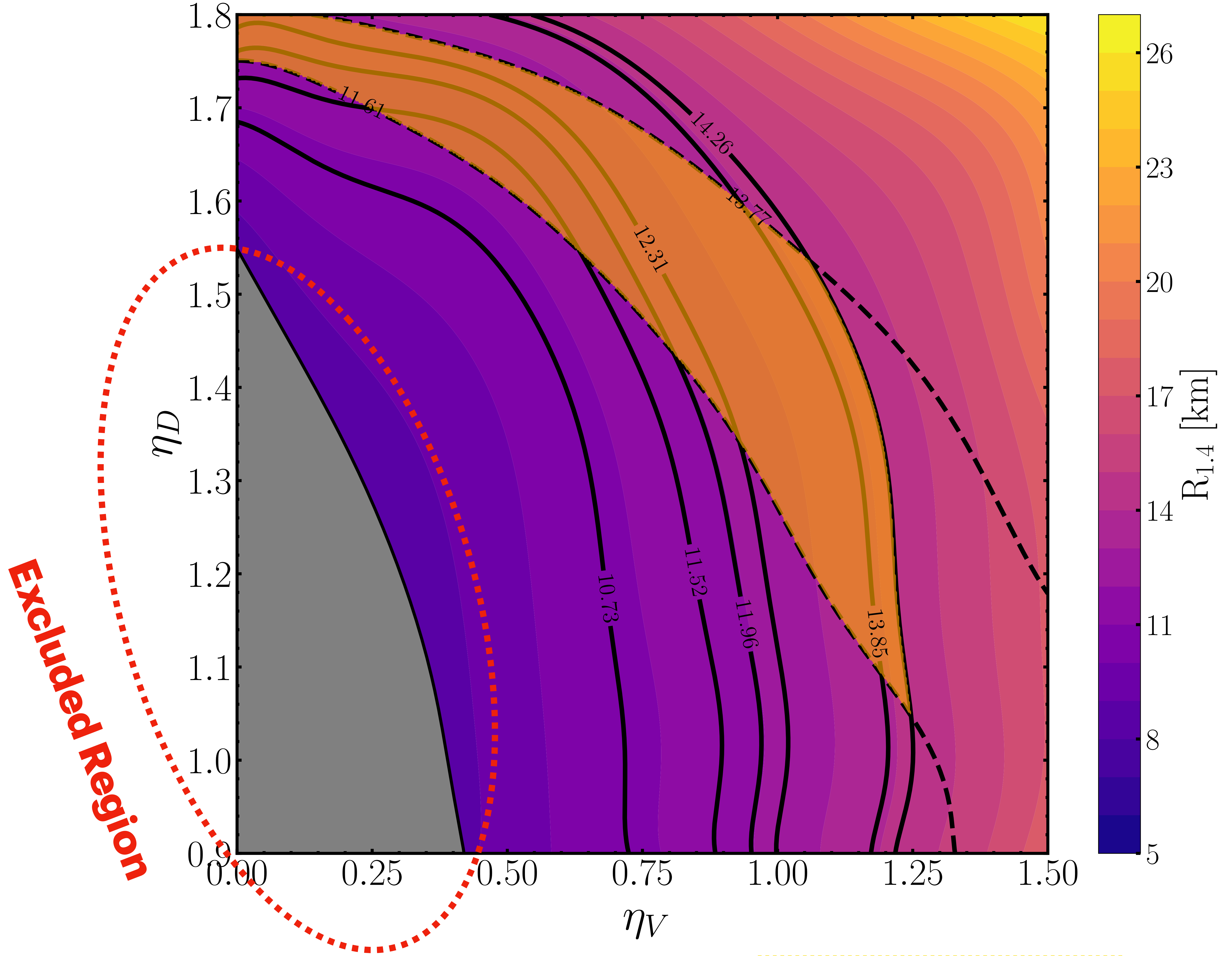
Self-Bound (low η_V)

High η_V

Gravitationally bound

Constraining the Model Parameters





Mass and Tidal Def. constraints

$2.0M_\odot$ $\Lambda = 190^{+390}_{-120}$

(PRL 121 161101 (2018))

NICER constraints

$R_{2.0} = 12.39^{+1.30}_{-0.98}\ km$ $R_{2.0} = 13.7^{+2.6}_{-1.5}\ km$

$R_{1.4} = 11.36^{+0.95}_{-0.63}\ km$

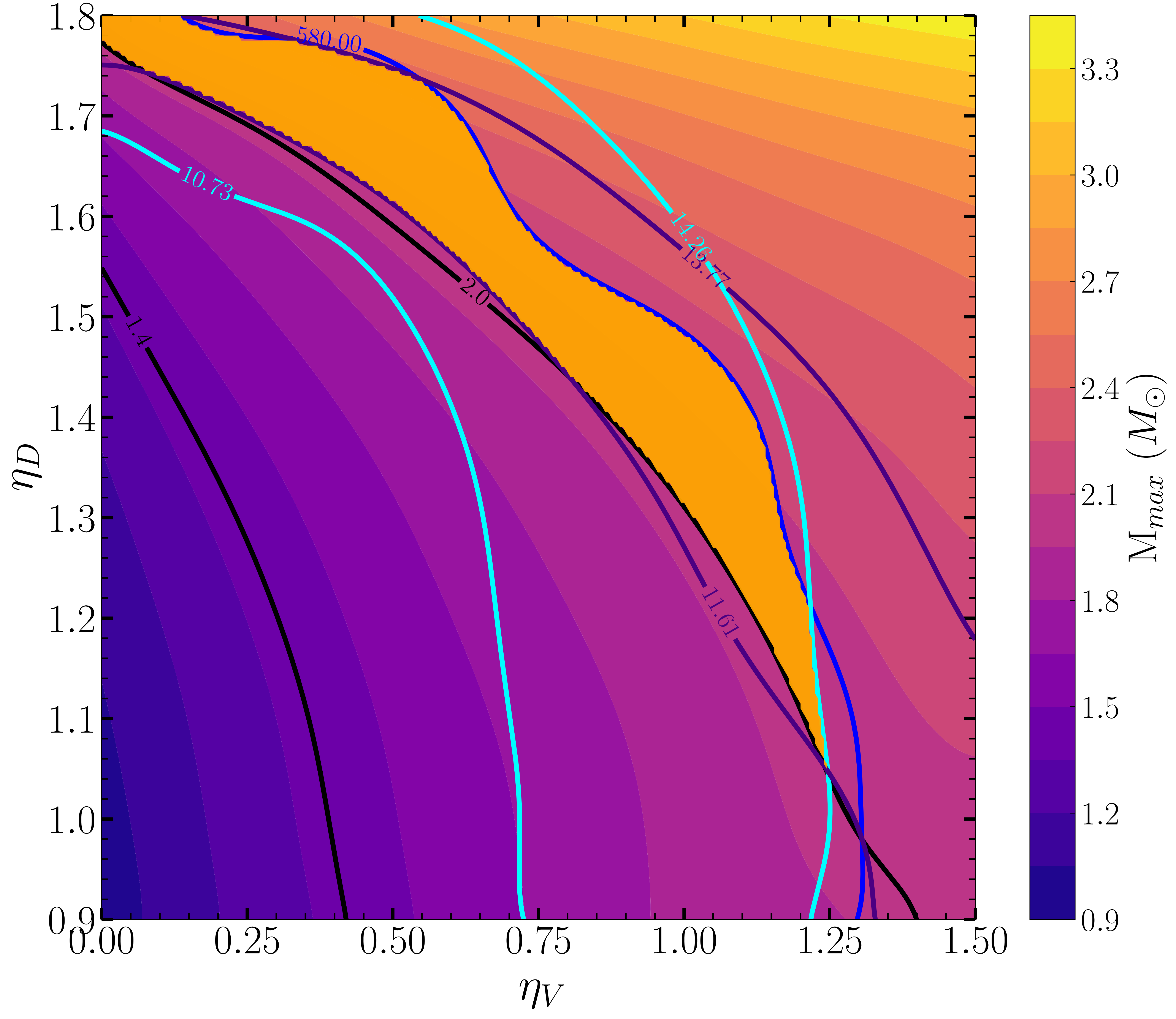
(Riley 2021) (Miller 2021) APJL. 971, L20 (2024)

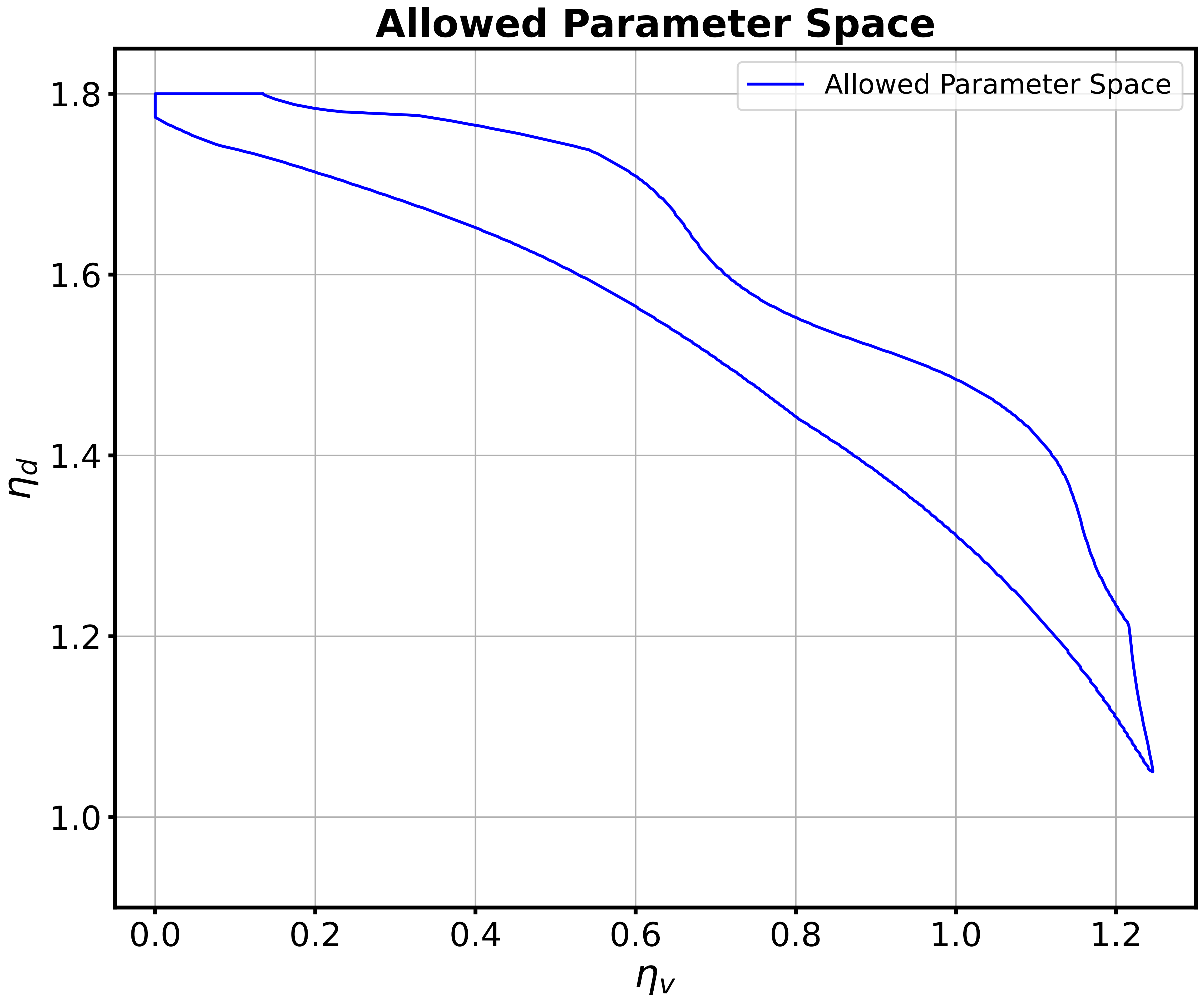
$R = 12.49^{+1.28}_{-0.88}\ km$

$R = 12.76^{+1.49}_{-1.02}\ km$

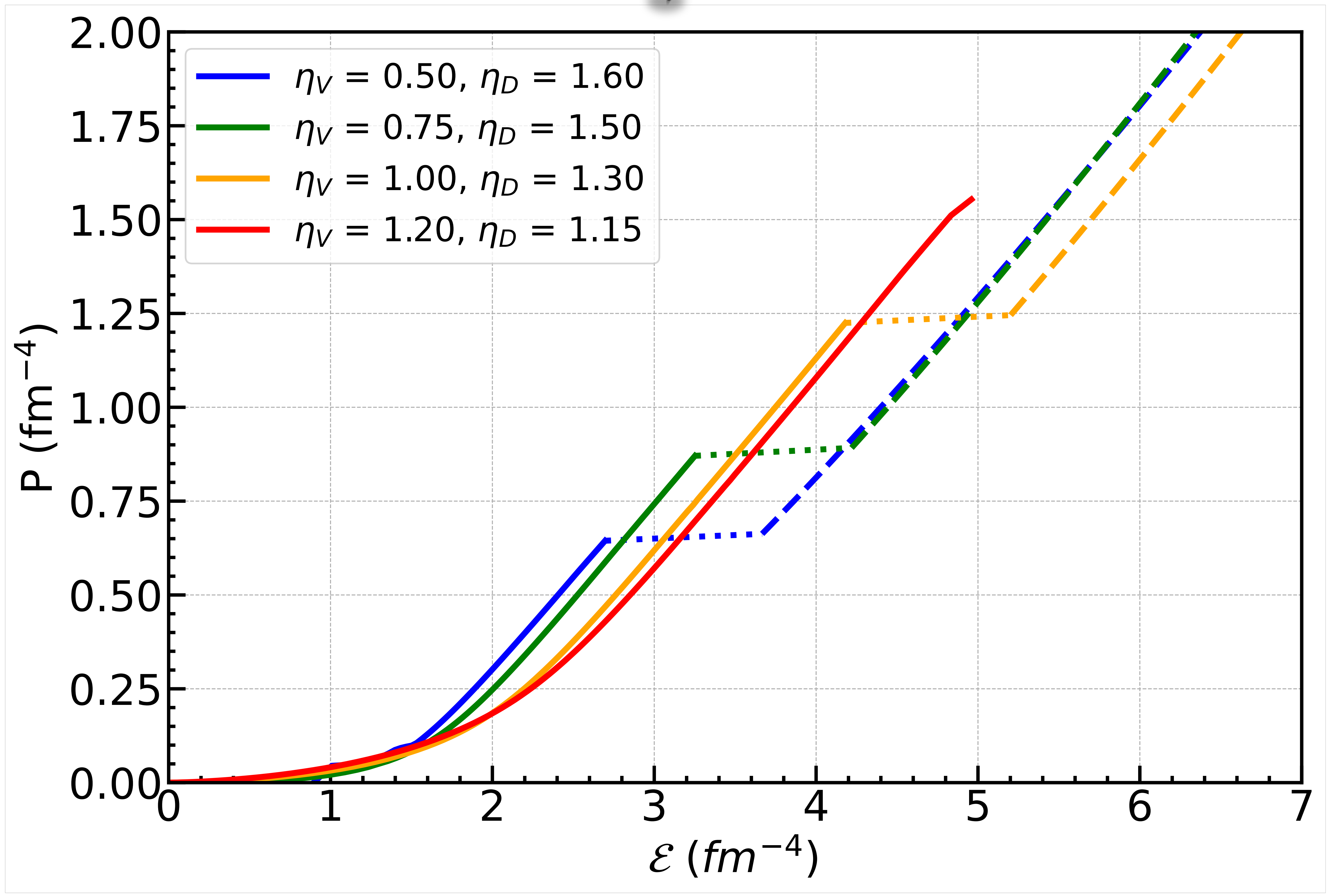
arXiv:2406.14466, arXiv:2406.14467

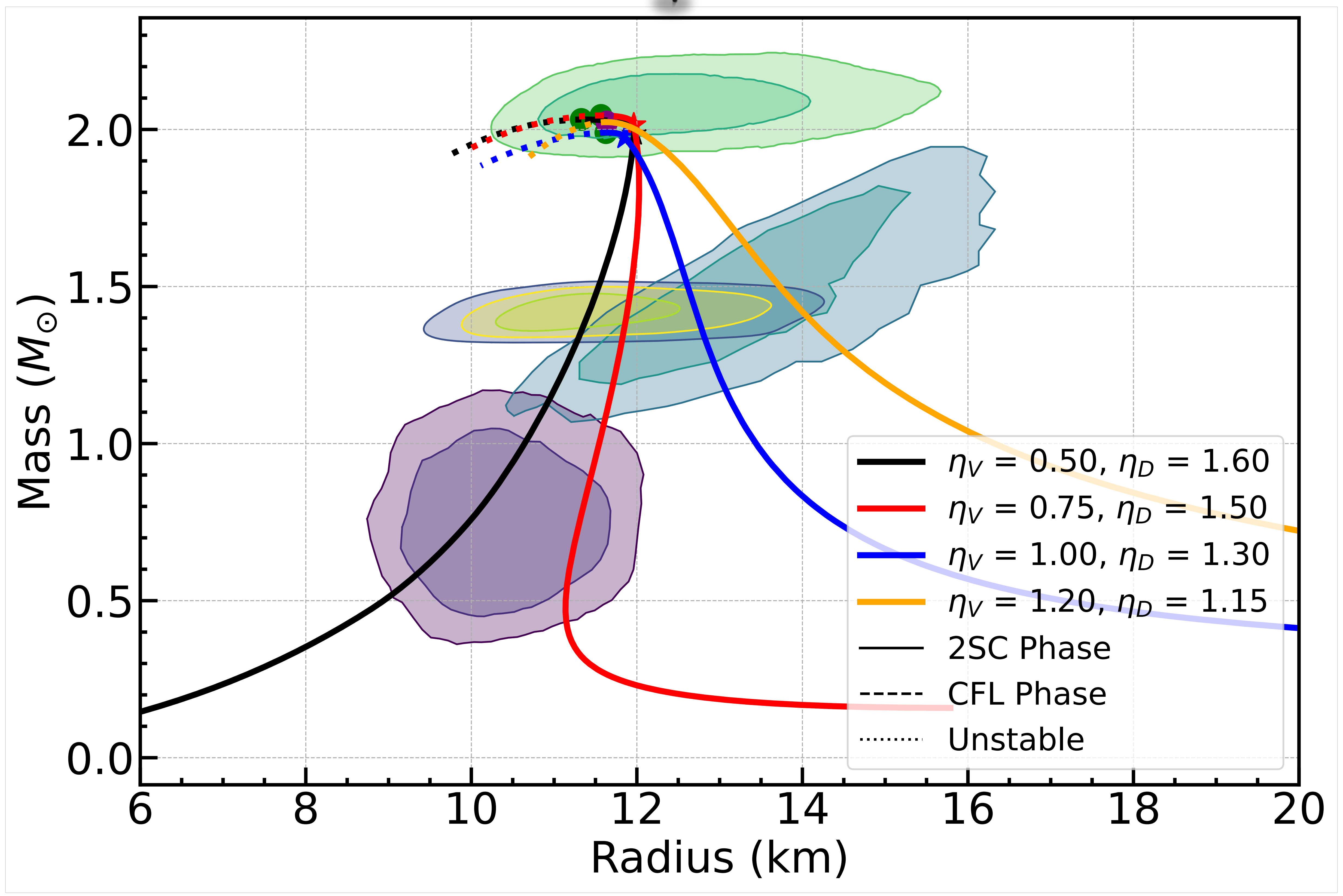
All astrophysical constraints





EoS within the Allowed Parameter range

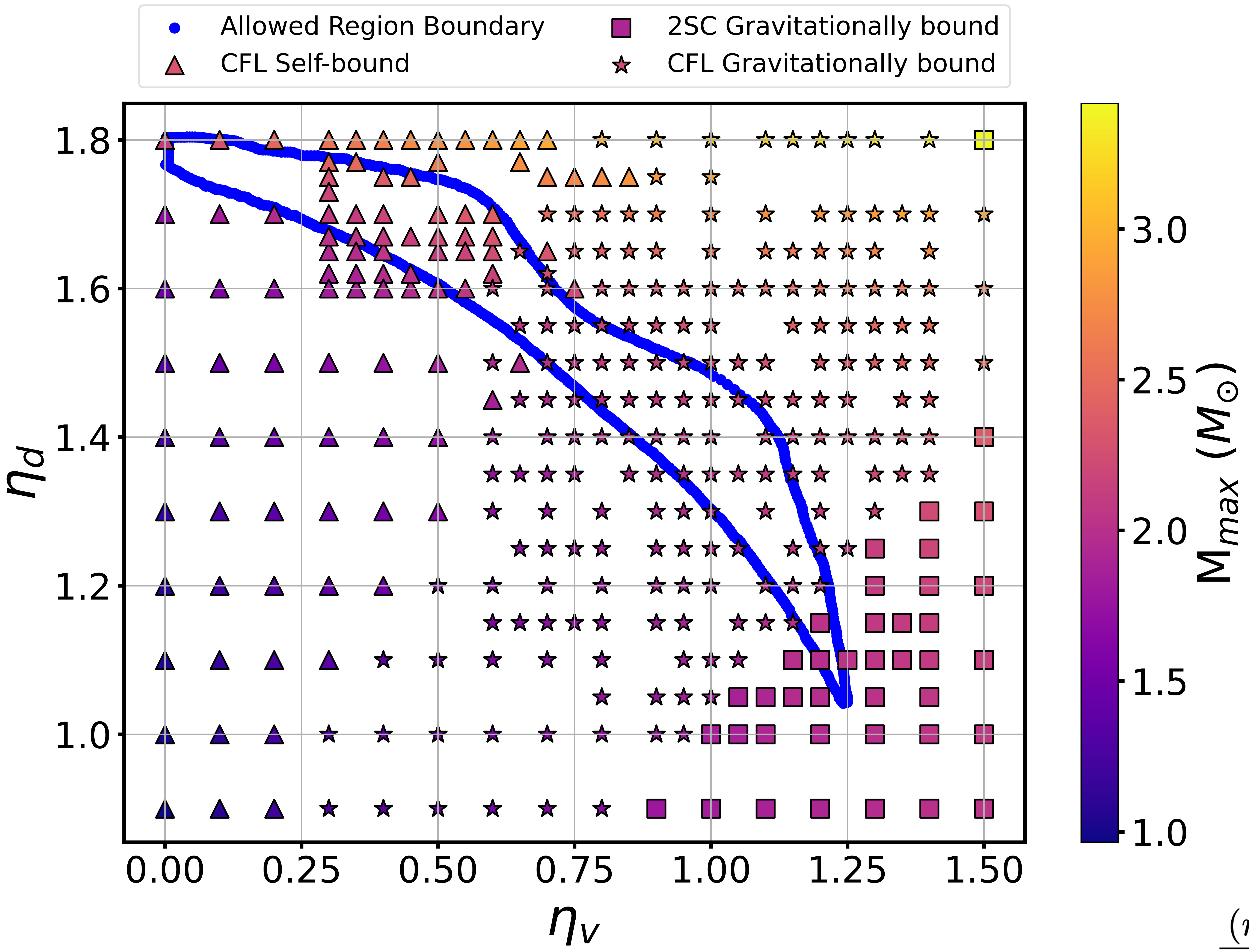




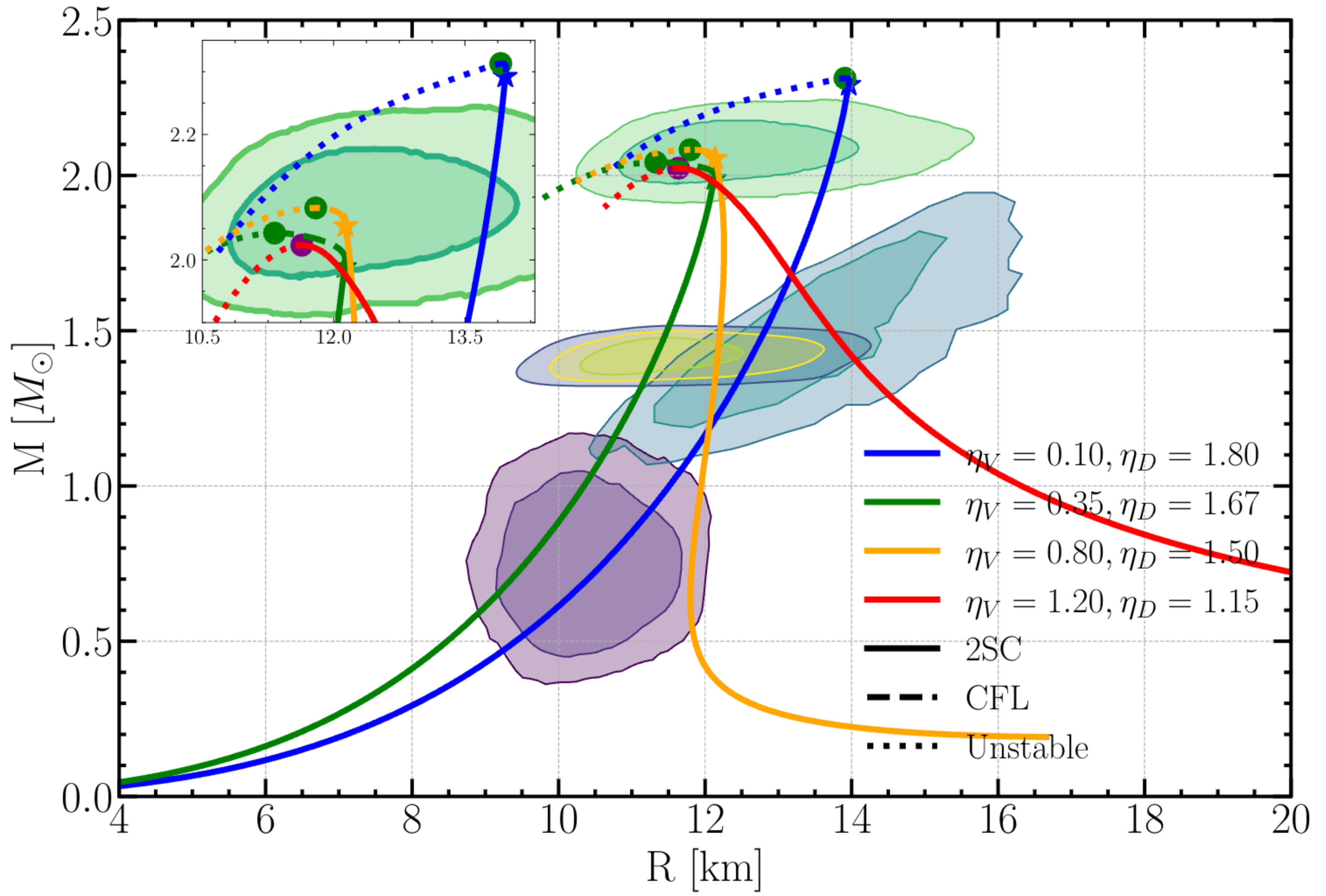
2SC to CFL Phase Transition at different density points.
Different amount of 2SC and CFL phases
Pure 2SC phase also.

Different (η_V, η_D)

≈ Similar Maximum mass.
Self-bound as well as Gravitationally bound.
CFL stars and pure 2SC stars.



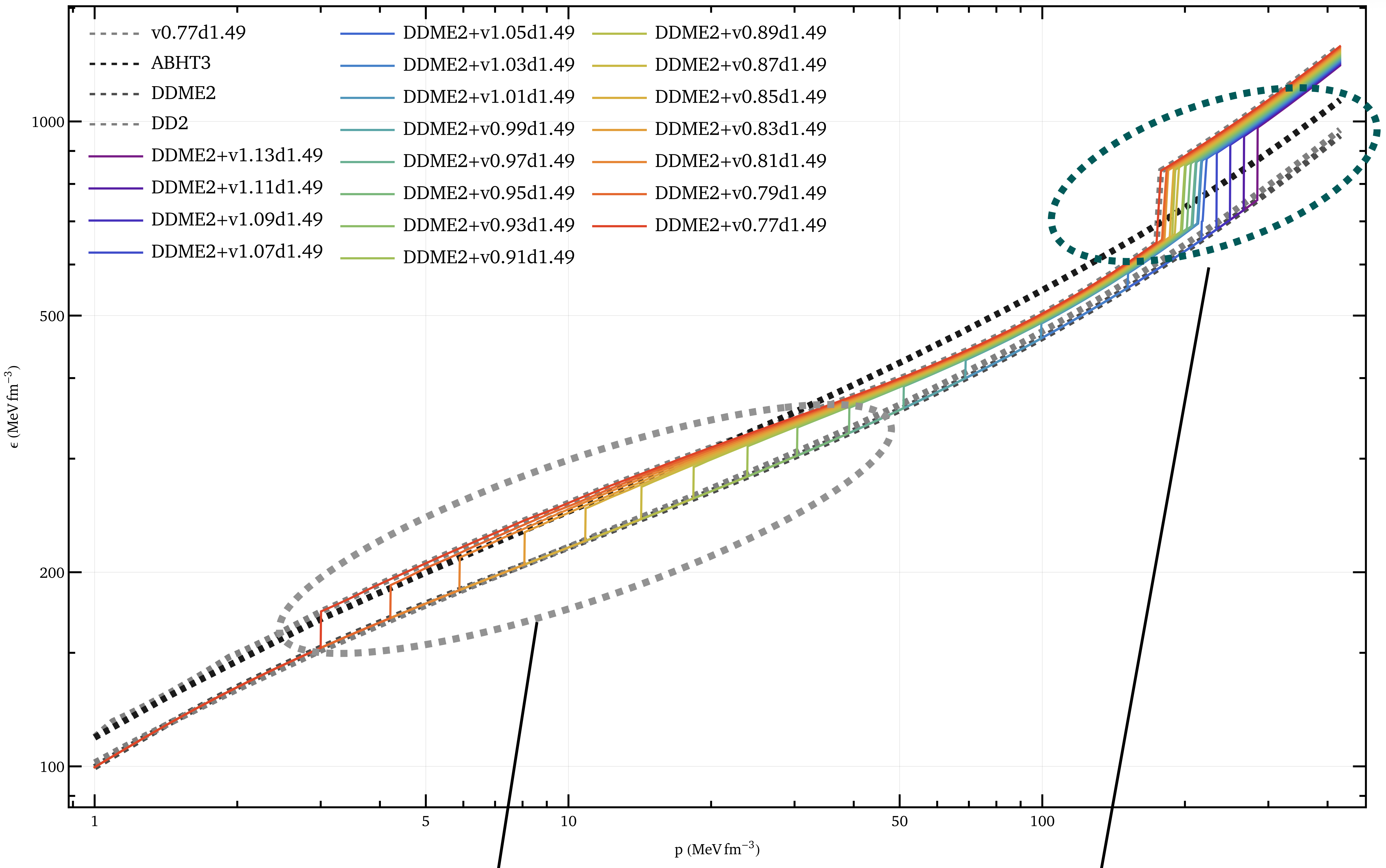
Some exemplary (η_V, η_D)

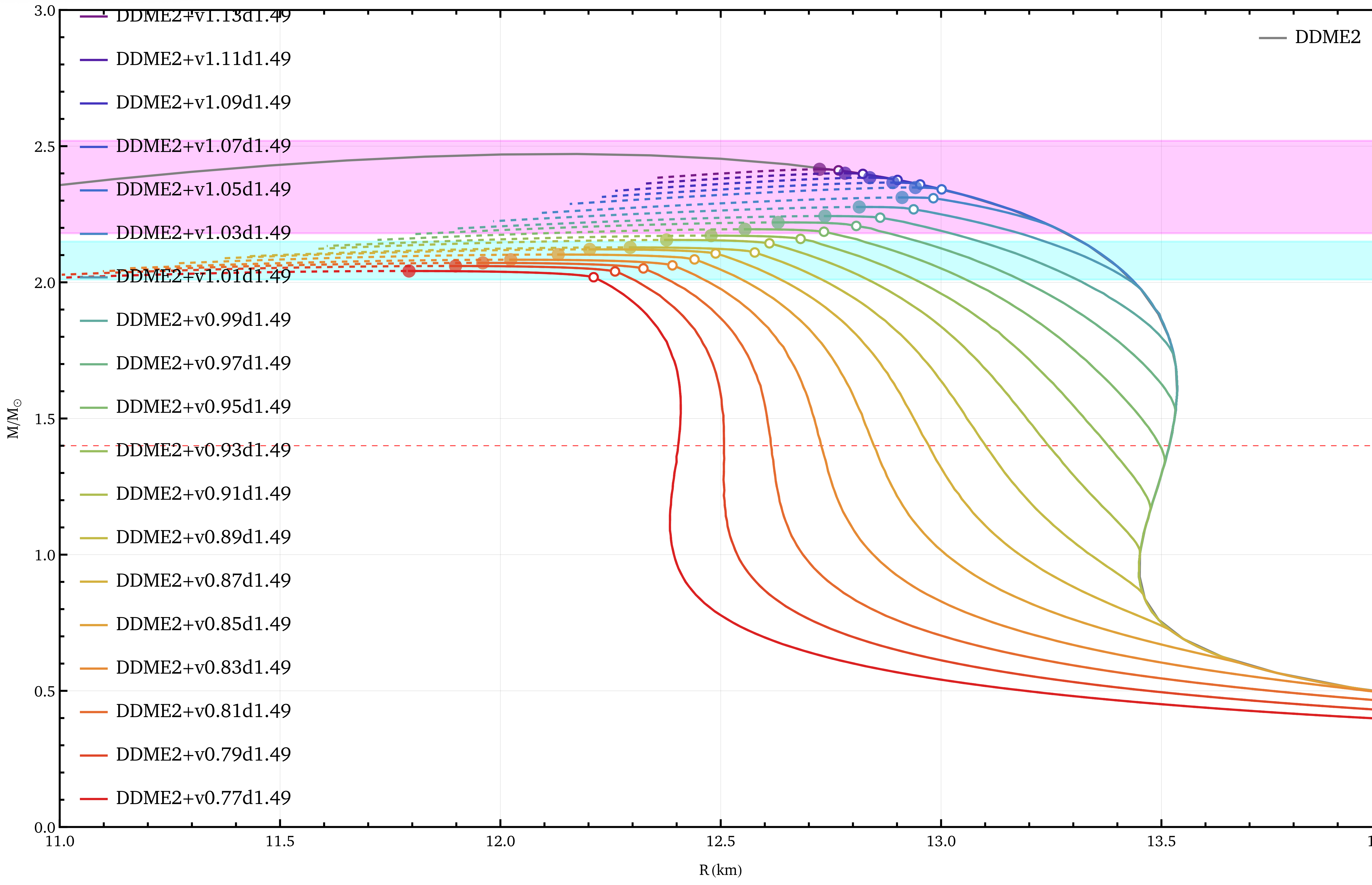


(η_V, η_D)	(0.10, 1.80)	(0.35, 1.67)	(0.80, 1.50)	(1.20, 1.15)
M_{max}	2.31	2.04	2.08	2.02
R_{max}	13.92	11.33	11.80	11.65
$R_{1.4}$	12.60	11.28	12.16	14.07
$R_{2.0}$	13.67	12.13	12.18	11.98
$\Lambda_{1.4}$	511	216	432	492
Phase	CFL (SB)	CFL (SB)	CFL (GB)	2SC (GB)

- Low η_V = CFL self-bound
- For a fixed η_V, the amount of CFL phase decreases with increasing η_D.
- For η_D = 1.80, a very small amount of stable CFL is present.
- Increasing η_V changes theMR profile from SB to GB stars because of the extended 2SC phase at low densities.

Hadron-Quark PT





Low density ⇕ Hadron to 2SC

High density ⇕ 2SC to CFL

- PT Before and after 1.4 $M_\odot$
- Large 2SC phase
- More CFL phase with small η_V

Summary

- We investigated a range of astrophysical properties of compact stars in the context of CSC phases using the NJL model with a RG-consistent treatment.
- Change in response to variations in the η_V and η_D parameters.
- Adjusting the η_V and η_D parameters significantly influence the stiffness of the EoS and, consequently, the MR of the resulting compact stars.
- Self-bound → Gravitationally bound stars
- Pure 2SC star configurations.
- CFL phase is generally stable (except at high η_V).
- This enable us to set constraints on the existence of CSC phases in neutron stars and emphasize the importance of incorporating CSC phases into neutron star models to meet astrophysical constraints

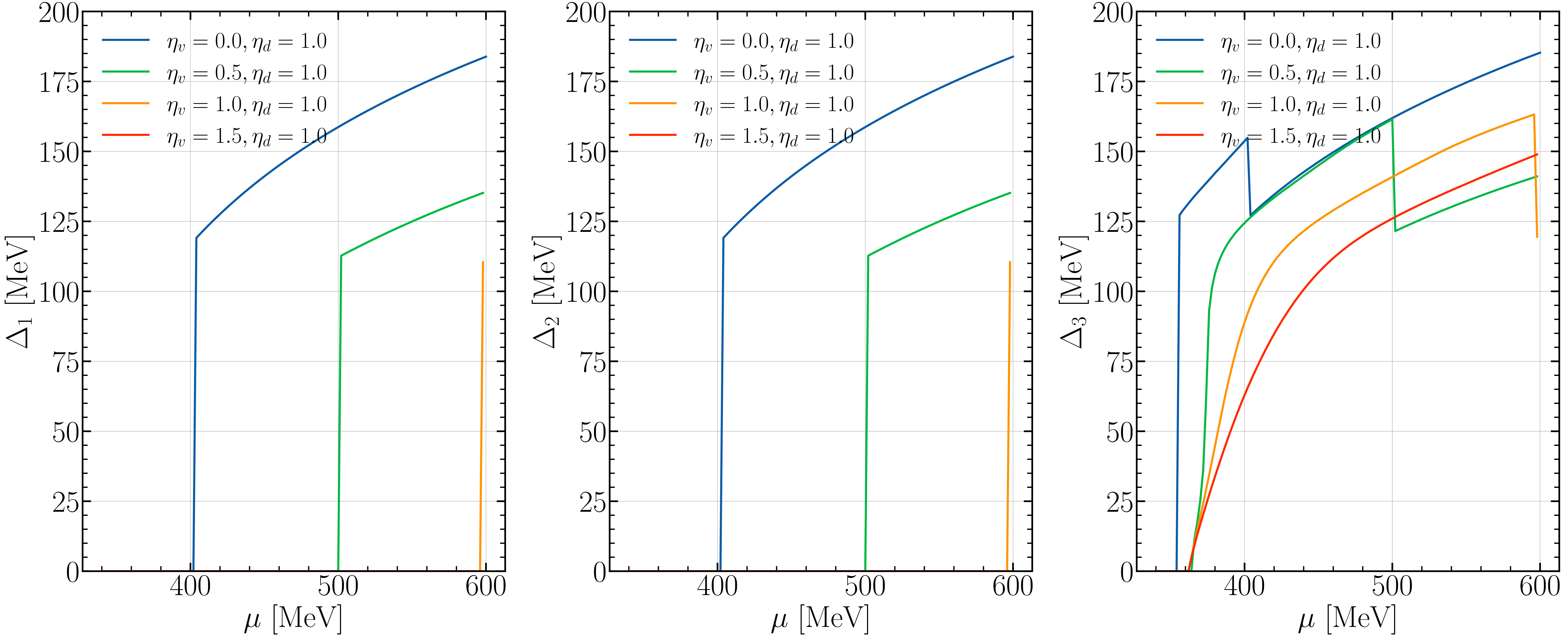
Ongoing work:

- Hybrid stars with CSC phases.
- Neutron star merger simulations.
- Empirical relations connecting f-mode frequencies with tidal deformation.

Behaviour of Diquark

At fixed η_V

Behaviour of Diquark



At fixed η_D

Seidov Limit

This limit serves as a criterion for gravitational stability during a sudden phase transition in a neutron star

$$\frac{\Delta\epsilon}{\epsilon} = \frac{3}{2} + \frac{1}{2}\left(\frac{P}{\epsilon}\right),$$

If $(\Delta\epsilon/\epsilon)$ exceeds this limit, the star cannot maintain hydrostatic equilibrium and becomes gravitationally unstable.