

# Equation of state in neutron stars from a bottom-up holographic QCD model

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An ultimate purpose of QCD studies

To obtain the QCD phase diagram



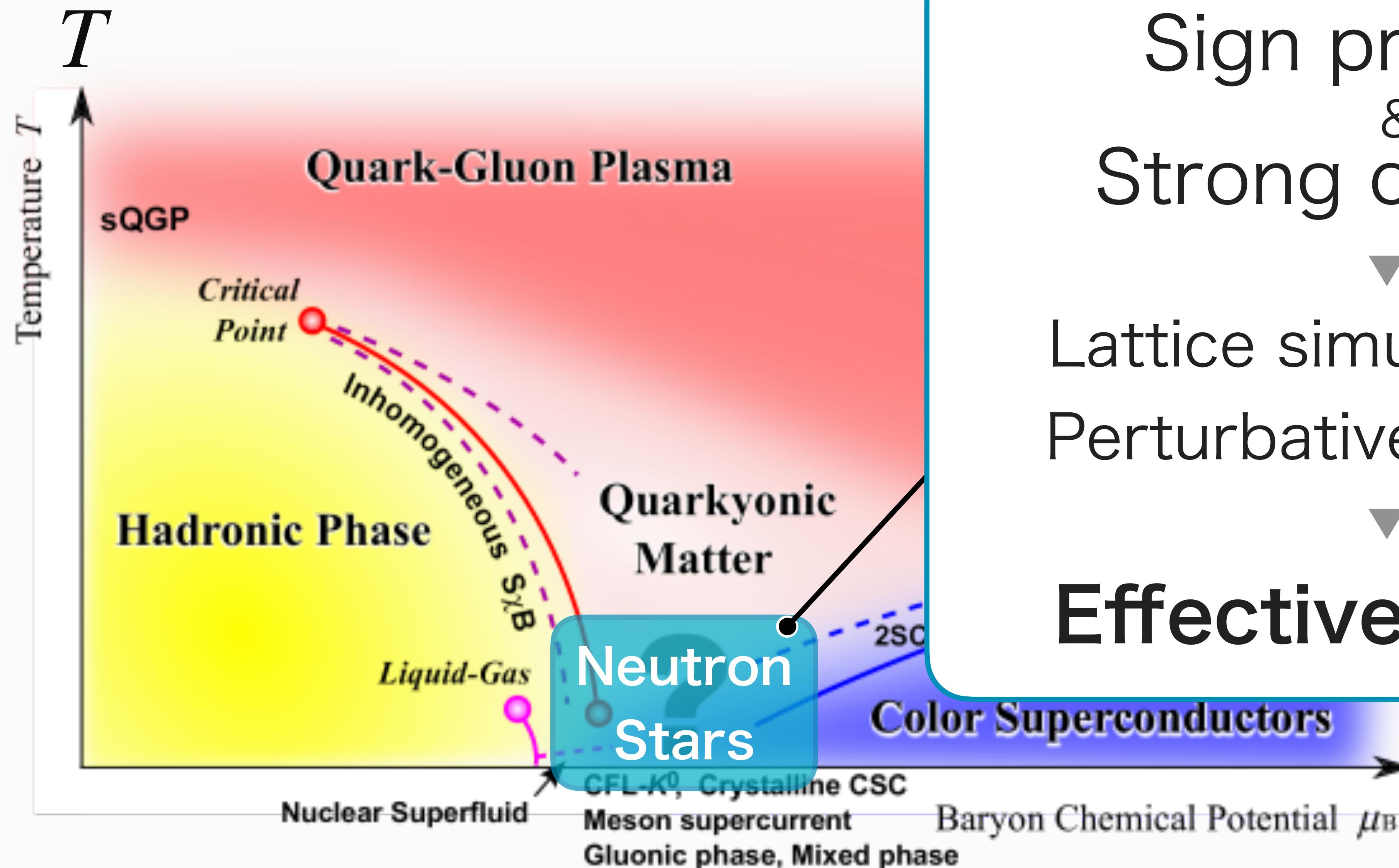
To challenge it

Focus on neutron stars

# QCD phase diagram

K. Fukushima and T. Hatsuda,  
Rep. Prog. Phys. **74** 014001 (2011)

Temperature



Sign problem  
&  
Strong coupling

▼  
Lattice simulation → ×

Perturbative QCD → ×

▼  
Effective models

$\mu_B$  Baryon  
chemical  
potential

# Which model is better?

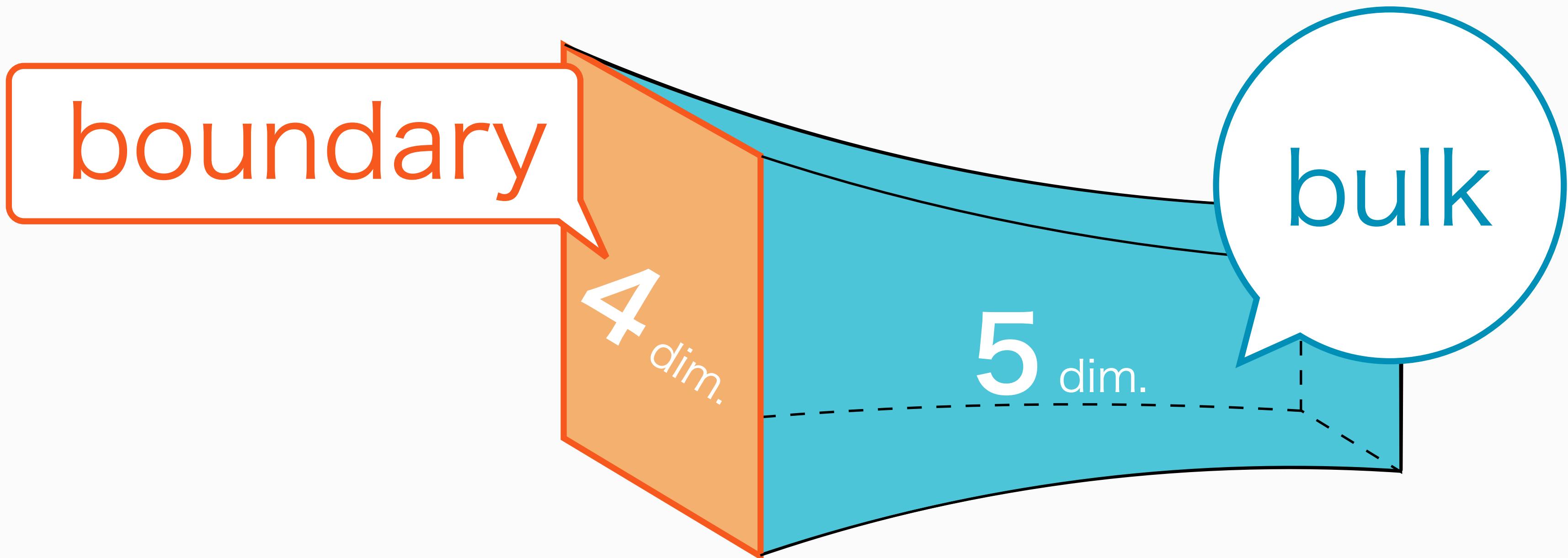
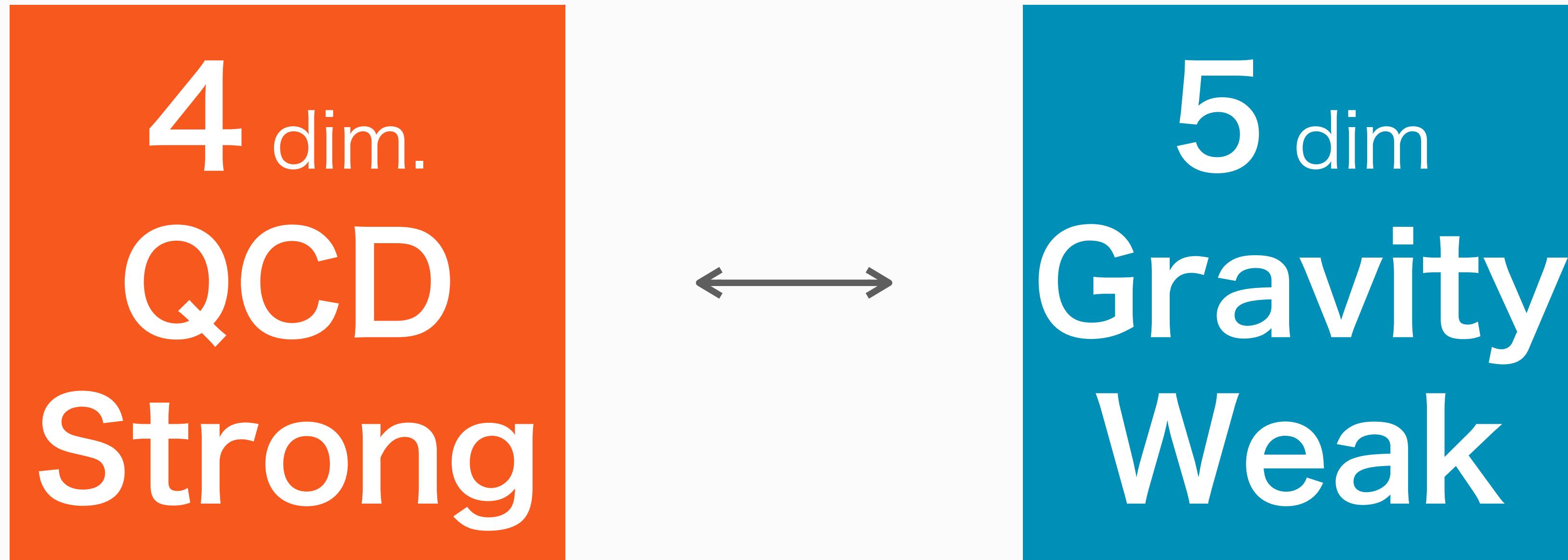
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## Holographic QCD

- 01 Finite density
- 02 Strong coupling
- 03 Chiral transition

# Holographic QCD

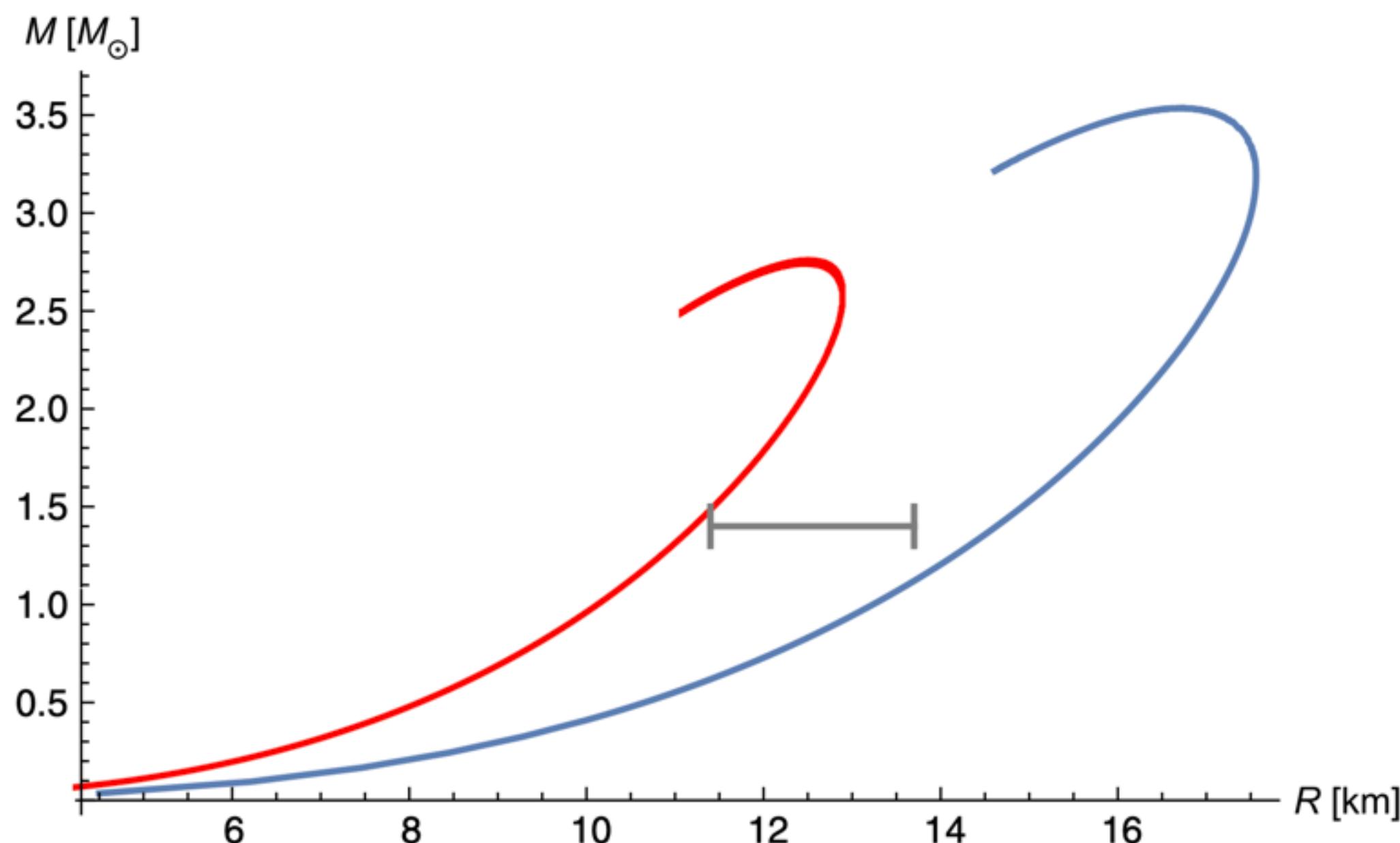
Note: Large  $N_c$  limit



# Previous study

## Hard-wall model

Lorenzo Bartolini, et al., Phys. Rev. D 105, 126014 (2022)



## Problems

- Definition of  $\mu_B$
- Selection of IR b.c.
- Selection of variables
- Renormalization

⋮

▼  
Revisit the model

# Method

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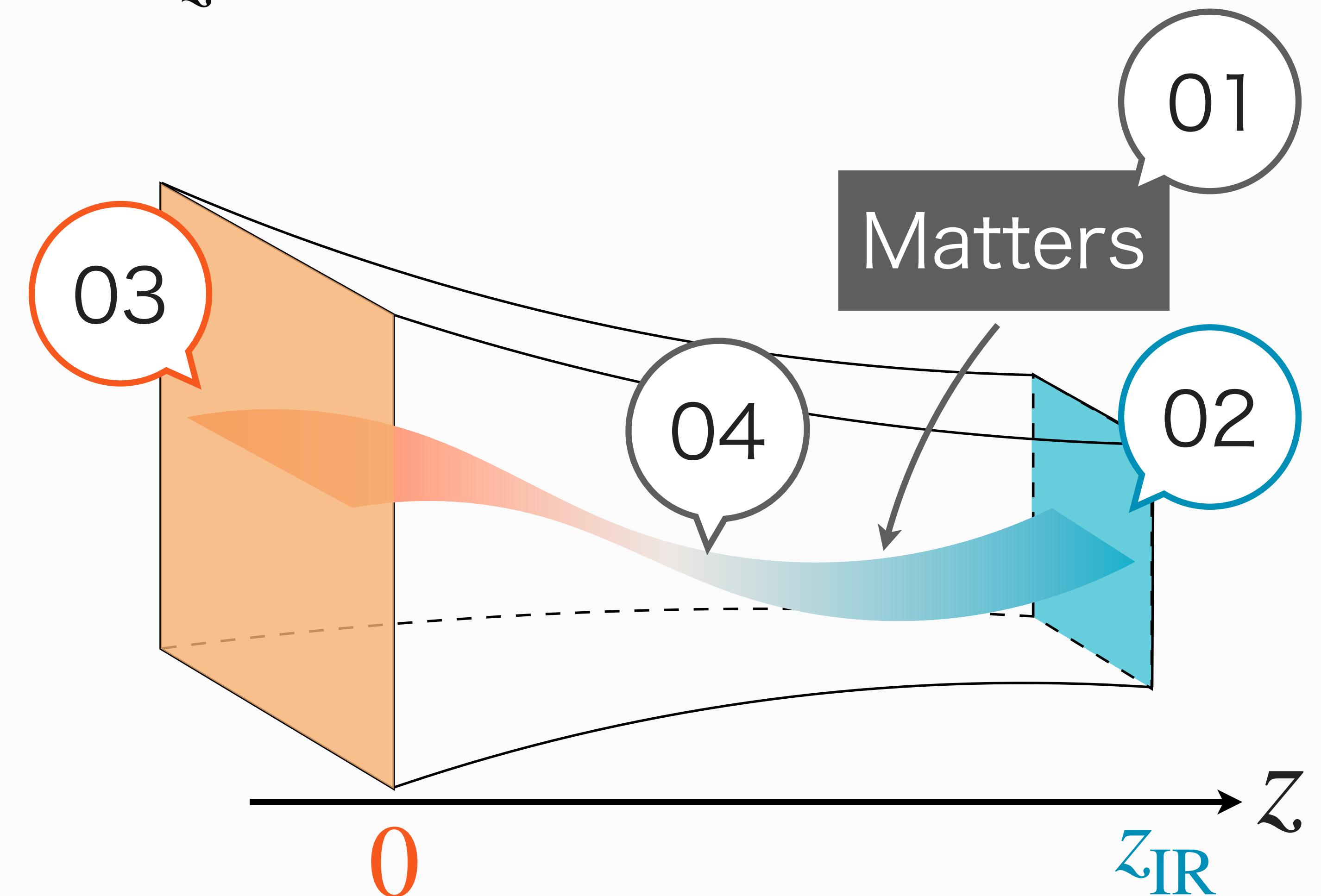
# Hard-wall model

J. Erlich, et al., PRL 95, 261602 (2005)

Cut-off AdS  
(Confined phase)

$$ds^2 = \frac{L^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right), \quad 0 \leq z \leq z_{\text{IR}}$$

- 01 Add matters
- 02 IR b.c.
- 03 UV b.c.
- 04 Solving EoM



# 01. Action of matters

Bi-fundamental  
Scalar fields

$U(2)$  Flavor gauge fields  
(Left&Right)

$$S = S_g + S_{\text{CS}} + S_\Phi + S_{\text{IR}} + S_c$$

Chern-Simons term

$$S_g = -\frac{N_c}{12\pi^2} \int d^4x dz \sqrt{-g} \left[ \frac{1}{2} \text{Tr}(L_{MN} L^{MN}) + \frac{1}{4} \hat{L}_{MN} \hat{L}^{MN} + \{R \leftrightarrow L\} \right],$$

$$S_{\text{CS}} = \frac{N_c}{16\pi^2} \int d^4x dz \epsilon_{MNPQR} \left[ \frac{1}{4} \hat{L}_M \left( \text{Tr}[L_{NP} L_{QR}] + \frac{1}{6} \hat{L}_{NP} \hat{L}_{QR} \right) - \{R \leftrightarrow L\} \right],$$

$$S_\Phi = \frac{N_c}{12\pi^2} \int d^4x dz \sqrt{-g} \left\{ \text{Tr} [(D_M \Phi)^\dagger D^M \Phi] + 3 \text{Tr} [\Phi^\dagger \Phi] \right\},$$

$$S_{\text{IR}} = - \int_{z=z_{\text{IR}}} d^4x m_b^2 \text{tr}[\Phi^\dagger \Phi].$$

$$\mathcal{L}_z = \mathcal{R}_z = 0 \quad (\text{gauge fixing})$$

$L_M, R_M : SU(2)$  gauge field  
 $\hat{L}_M, \hat{R}_M : U(1)$  gauge field  
 $\Phi$  : scalar field  
 $M, N, \dots = 0, 1, 2, 3, z$

$L_{MN} = \partial_M L_N - \partial_N L_M - i[L_M, L_N]$ ,  
 $L_{MN}^a = \partial_M L_N^a - \partial_N L_M^a + f^{abc} L_M^b L_N^c$ ,  
 $D_M \Phi = \partial_M \Phi - i \mathcal{L}_M \Phi + i \Phi \mathcal{R}_M$ ,  
 $\mathcal{L}_M = L_M^a \frac{\tau^a}{2} + \hat{L}_M \frac{I_2}{2}$   
 $(\tau^a : \text{Pauli matrix}, a = 1, 2, 3)$ ,  
 $N_c = 3, L = 1$ .

# Ansatz

## Homogeneous Ansatz

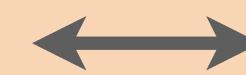
“Mean-field approximation”

$$\Phi = \omega_0(z) \frac{I_2}{2}$$



Current quark mass  
Chiral condensate

$$\mathcal{L}_0 = -\mathcal{R}_0 = \hat{a}_0(z) \frac{I_2}{2}$$



Baryon chemical potential  
Baryon number density

$$\mathcal{L}_i = -\mathcal{R}_i = -H(z) \frac{\tau^i}{2}$$



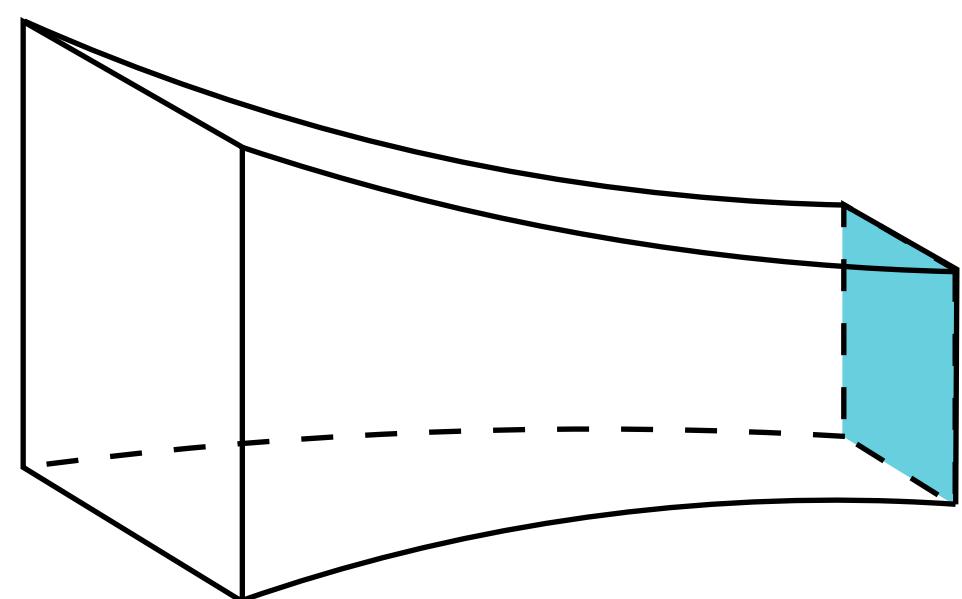
Axial vector potential  
Axial vector meson condensate

# 02. IR b.c.

Note:  $z_{\text{IR}} = 1$

## Mesonic IR b.c. ( $z = z_{\text{IR}}$ )

$$\mu_B < \mu_c$$



b.c.

Neumann

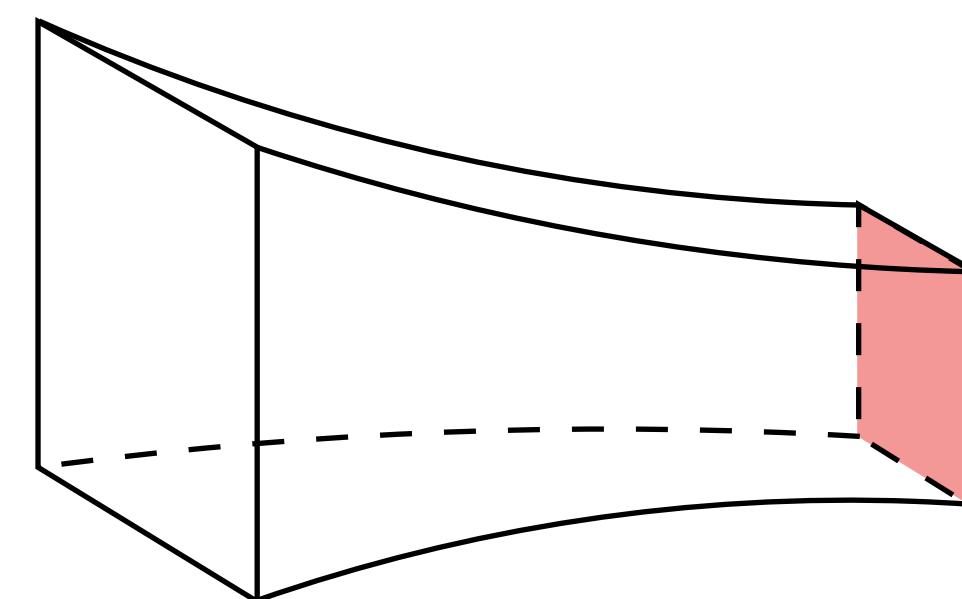
$$\partial_z \omega_0(z_{\text{IR}}) = -\frac{12\pi^2}{N_c} \left( 3kH^2\omega_0 + m_b^2\omega_0 + \frac{\lambda}{4}\omega_0^3 \right),$$

$$\partial_z \hat{a}_0(z_{\text{IR}}) = 0,$$

$$\partial_z H(z_{\text{IR}}) = 0.$$

## Baryonic IR b.c. ( $z = z_{\text{IR}}$ )

$$\mu_B \geq \mu_c$$



b.c.

Neumann  
+  
Dirichlet

$$\partial_z \omega_0(z_{\text{IR}}) = -\frac{12\pi^2}{N_c} \left( 3kH^2\omega_0 + m_b^2\omega_0 + \frac{\lambda}{4}\omega_0^3 \right),$$

$$\hat{a}_0(z_{\text{IR}}) = A = 4,$$

$$H(z_{\text{IR}}) = B.$$

# 03. UV b.c.

UV b.c. ( $z = 0$ )

$$\partial_z \omega_0(0) = m = 3 \text{ MeV}$$

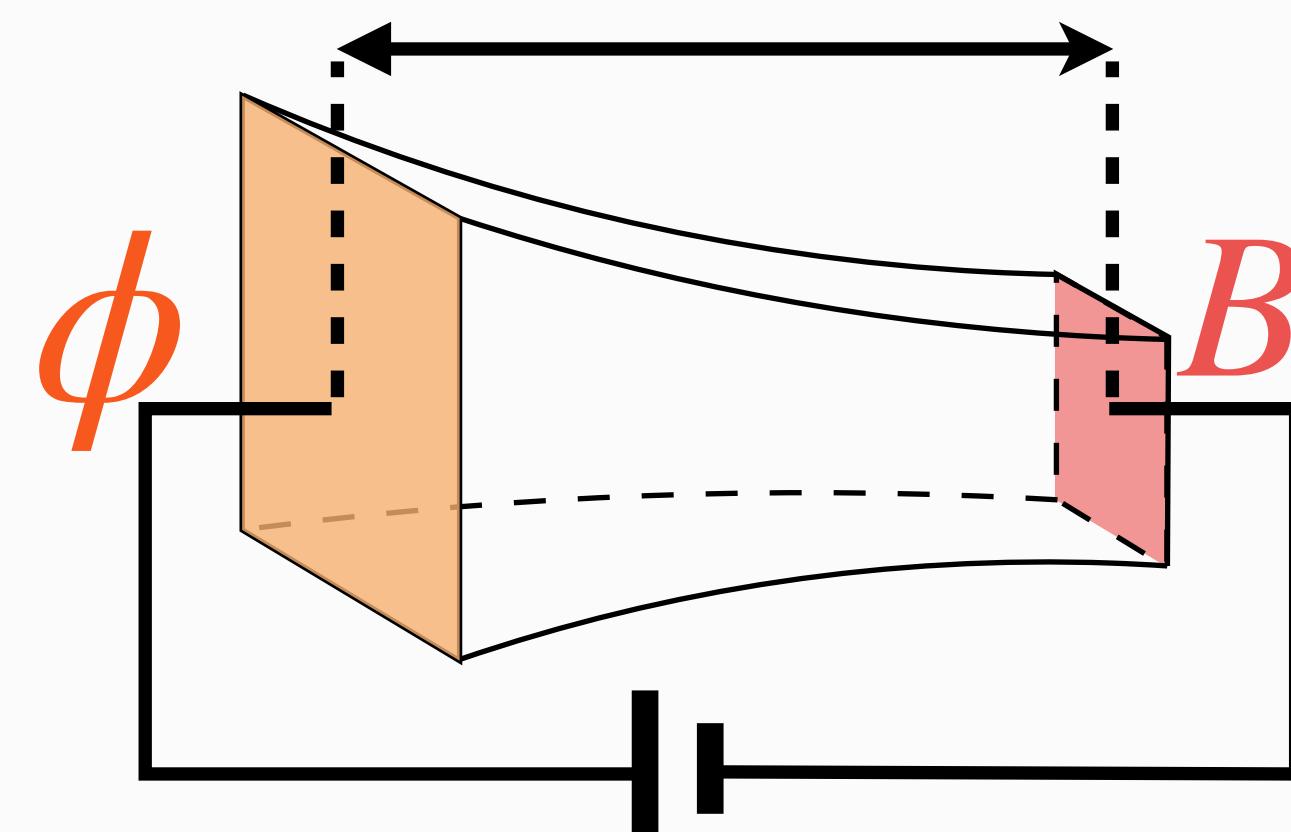
$\longleftrightarrow$  Current quark mass

$$\hat{a}_0(0) = \mu$$

$\longleftrightarrow$  Baryon chemical potential

$$H(0) = \phi = B \in \{0.2, 0.4, 0.6\}$$

$\longleftrightarrow$  Axial vector potential



Make “no difference of potential”

$$\hat{\phi} \propto \phi - B = 0$$

# Parameters

01

AdS radius  
&  
place of the hard-wall

Fit from M-R plot

$$L = z_{\text{IR}} = (800 \text{ MeV})^{-1}$$

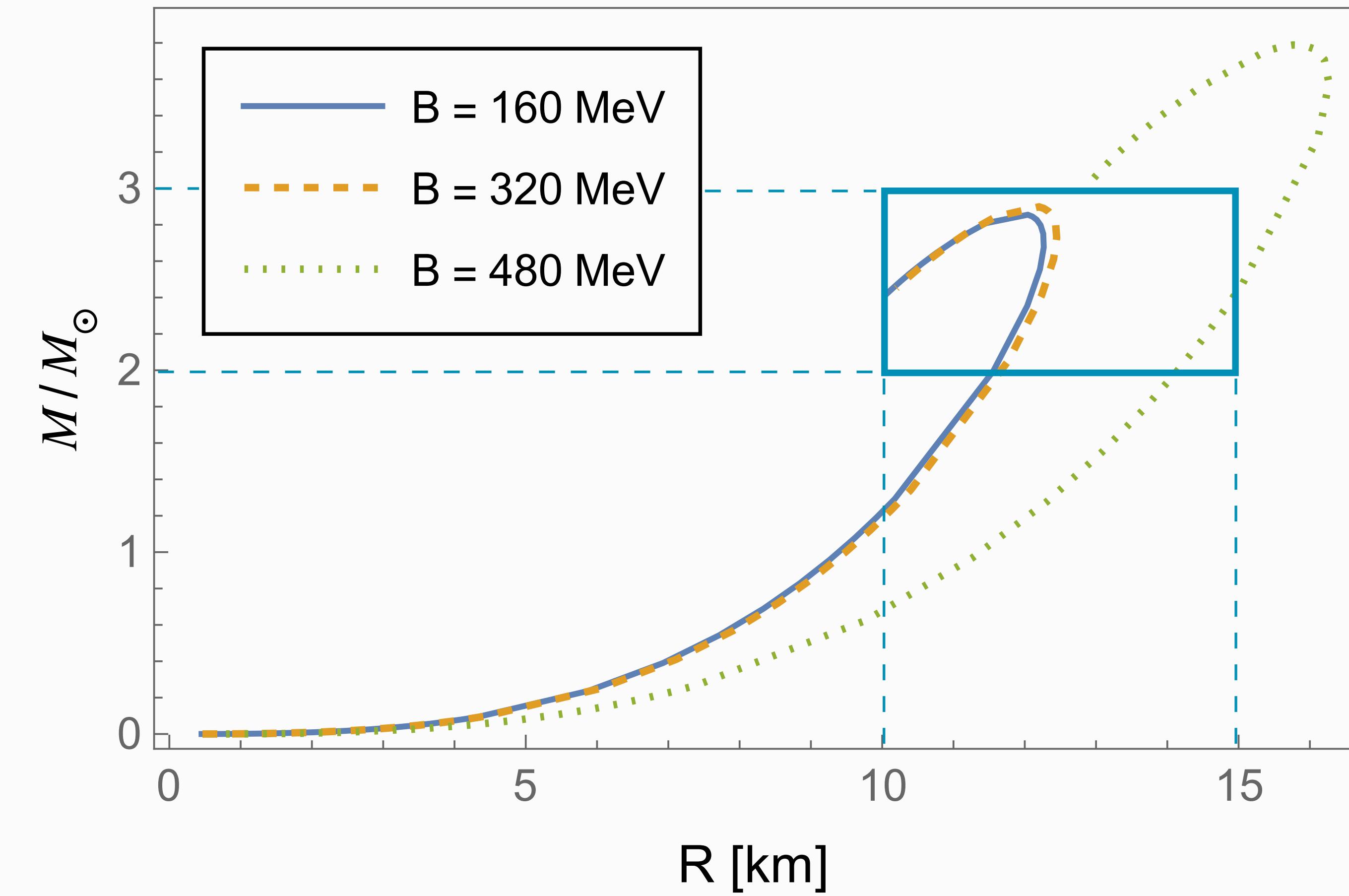
02

Chiral condensate  
in the mesonic phase

Lattice result

$$\xi_0 = (251 \text{ MeV})^3$$

H. Fukaya, et al., PRL 98, 172001 (2007)



**Self-binding(?)**, stiff matter  
(Quark star like)

# Results

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# Grand potential density

Two transitions

- Chirality
- Baryon number density

All transitions are

**1** st transition

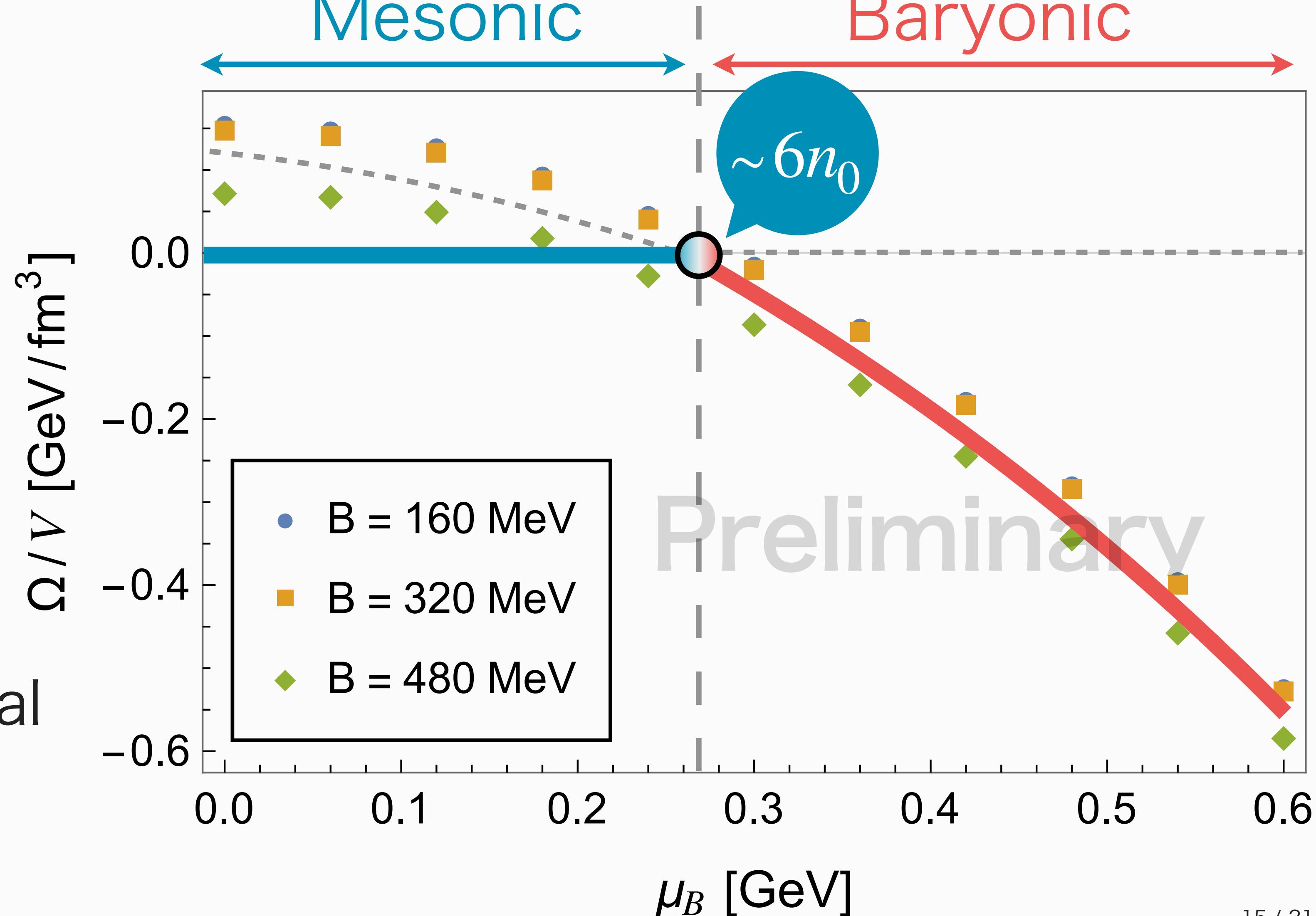
Critical chemical potential

$$\mu_B \sim 270 \text{ MeV}$$

Grand potential density

Mesonic

Baryonic



# Chiral condensate

Chiral condensate remains

$\sim 35\%$

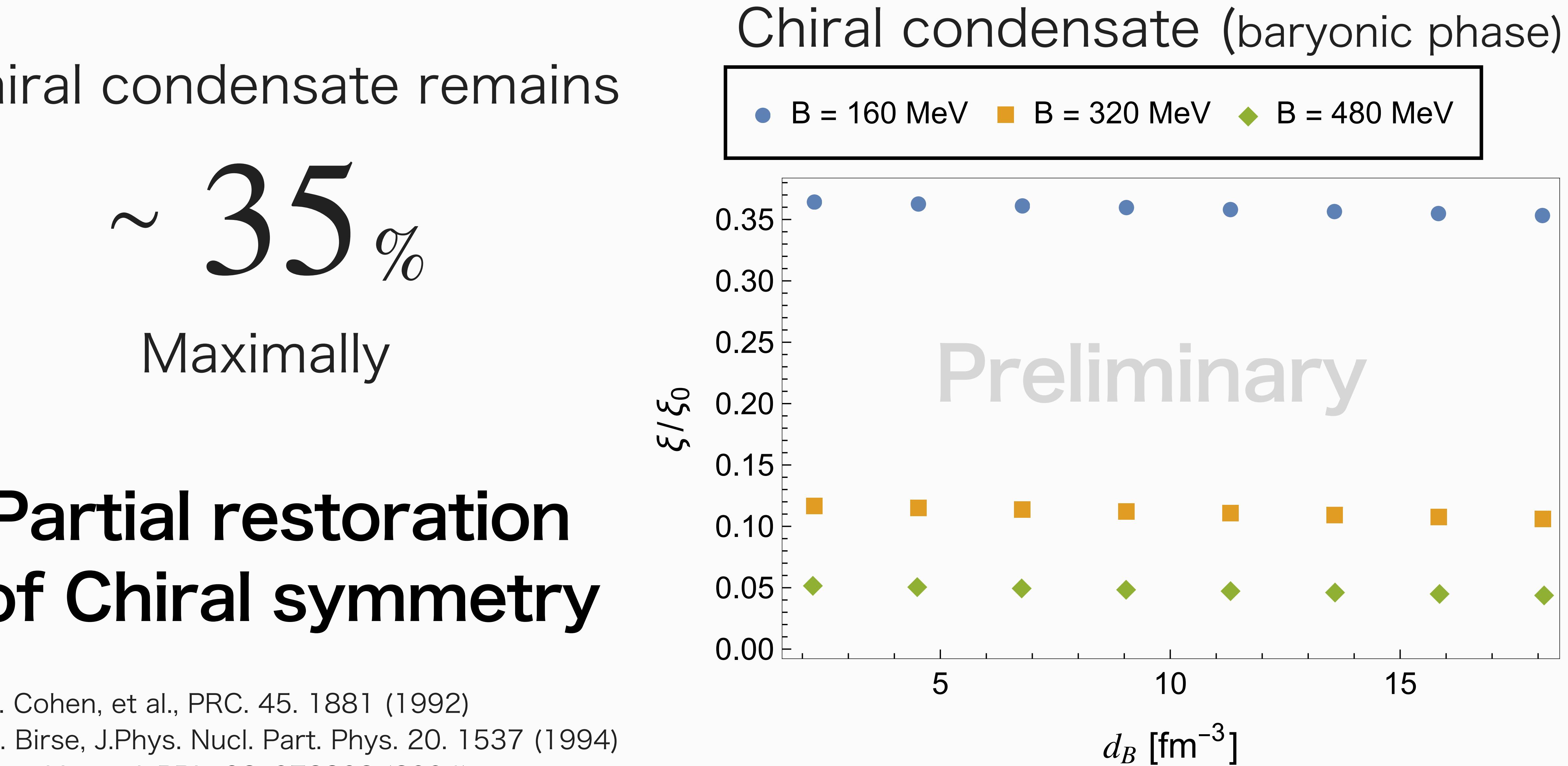
Maximally

Partial restoration  
of Chiral symmetry

T. D. Cohen, et al., PRC. 45. 1881 (1992)

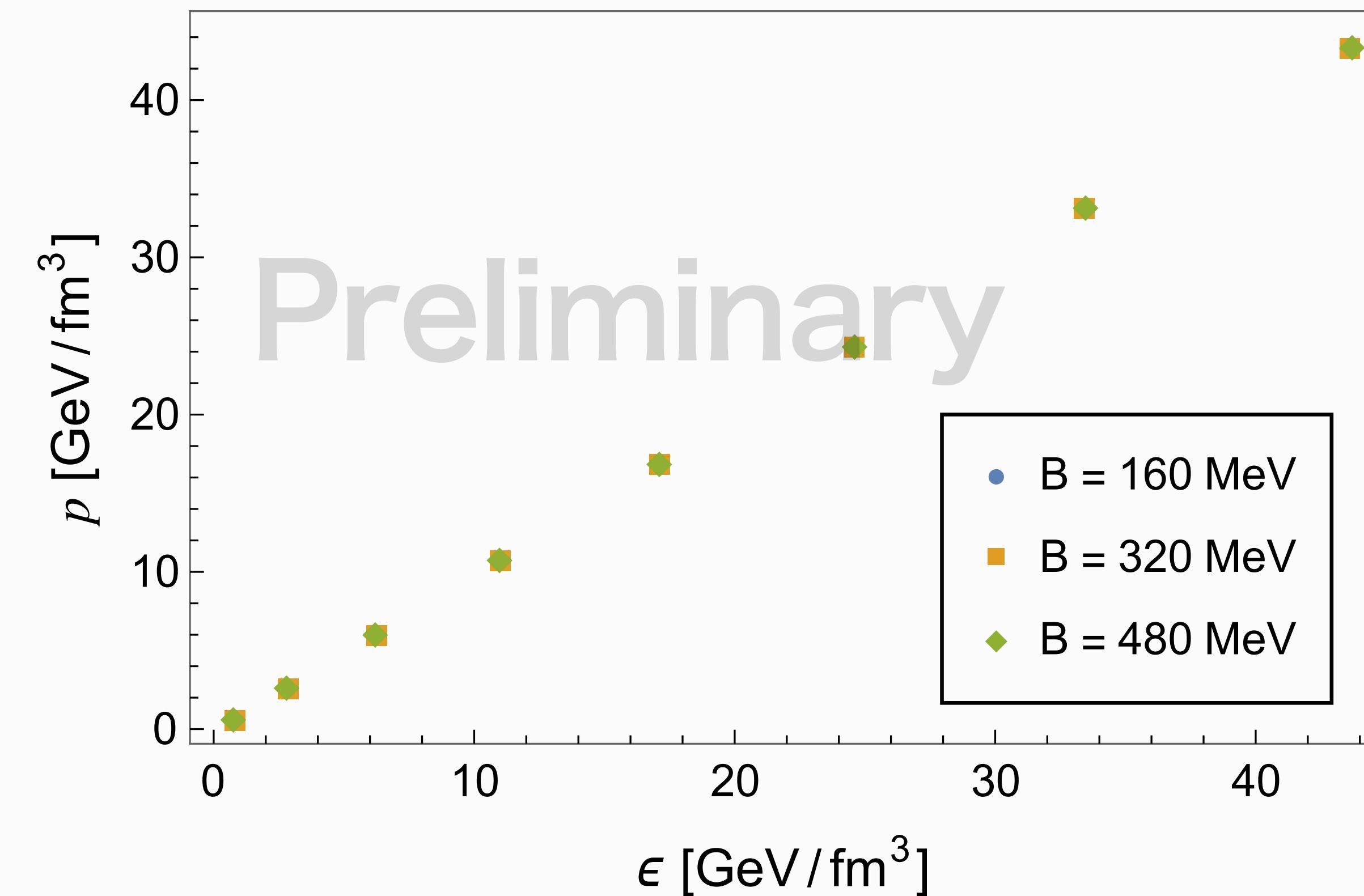
M.C. Birse, J.Phys. Nucl. Part. Phys. 20. 1537 (1994)

K. Suzuki, et. al. PRL. 92. 072302 (2004)



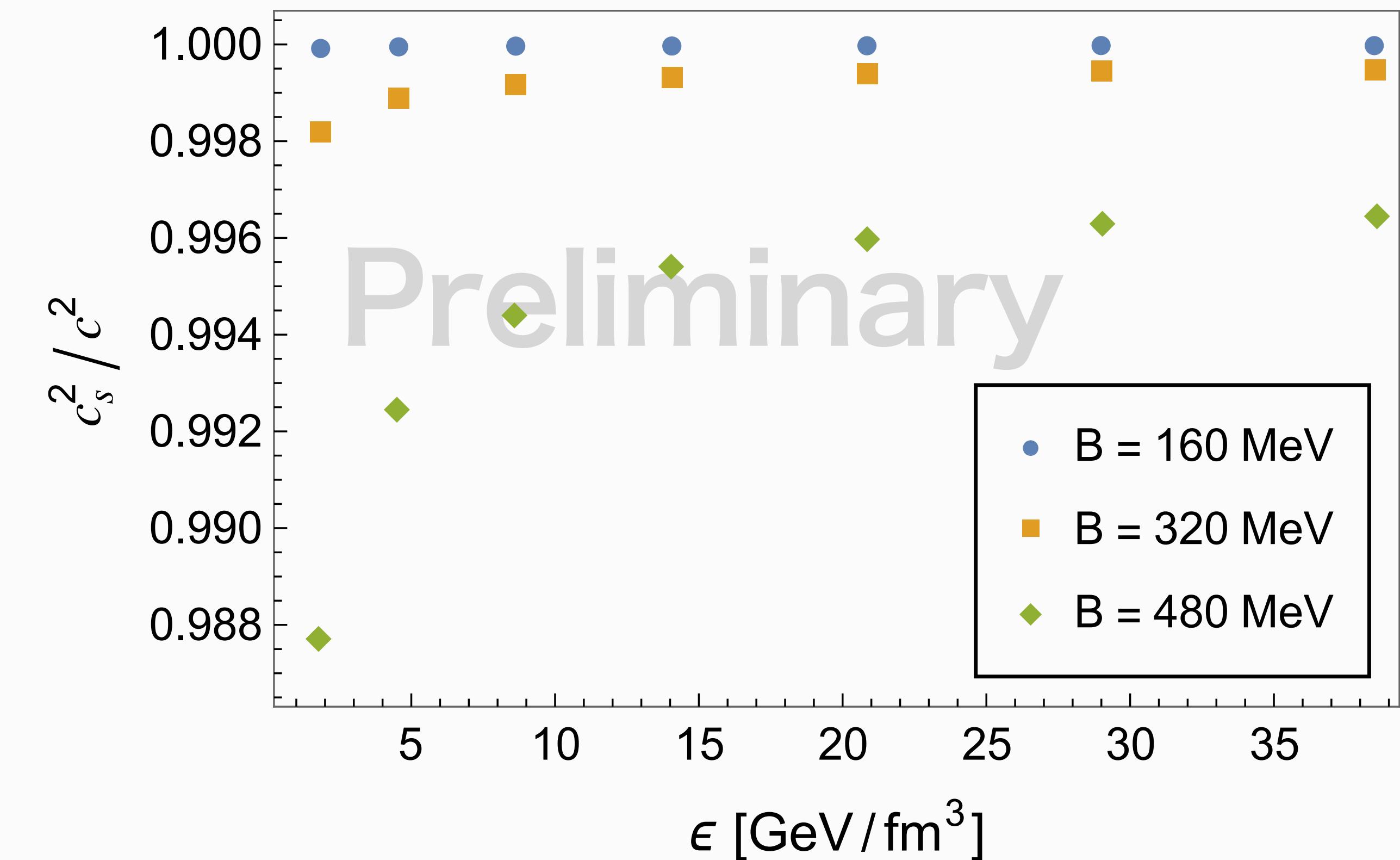
# EoS & Speed of sound

EoS (baryonic phase)



The EoS is  
Linear function  
with gradient  $\approx 1$

Speed of sound (baryonic phase)



Speed of sound  $\approx 1$

# Discussion

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# What is the matter?

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## Properties

- Six times  $n_0 \simeq 1.7 \text{ fm}^{-3}$
- Self-binding matter (?)
- Speed of sound  $\sim 1$
- Color confinement



**High density  
nuclear matter**

# What is the matter?

## Properties

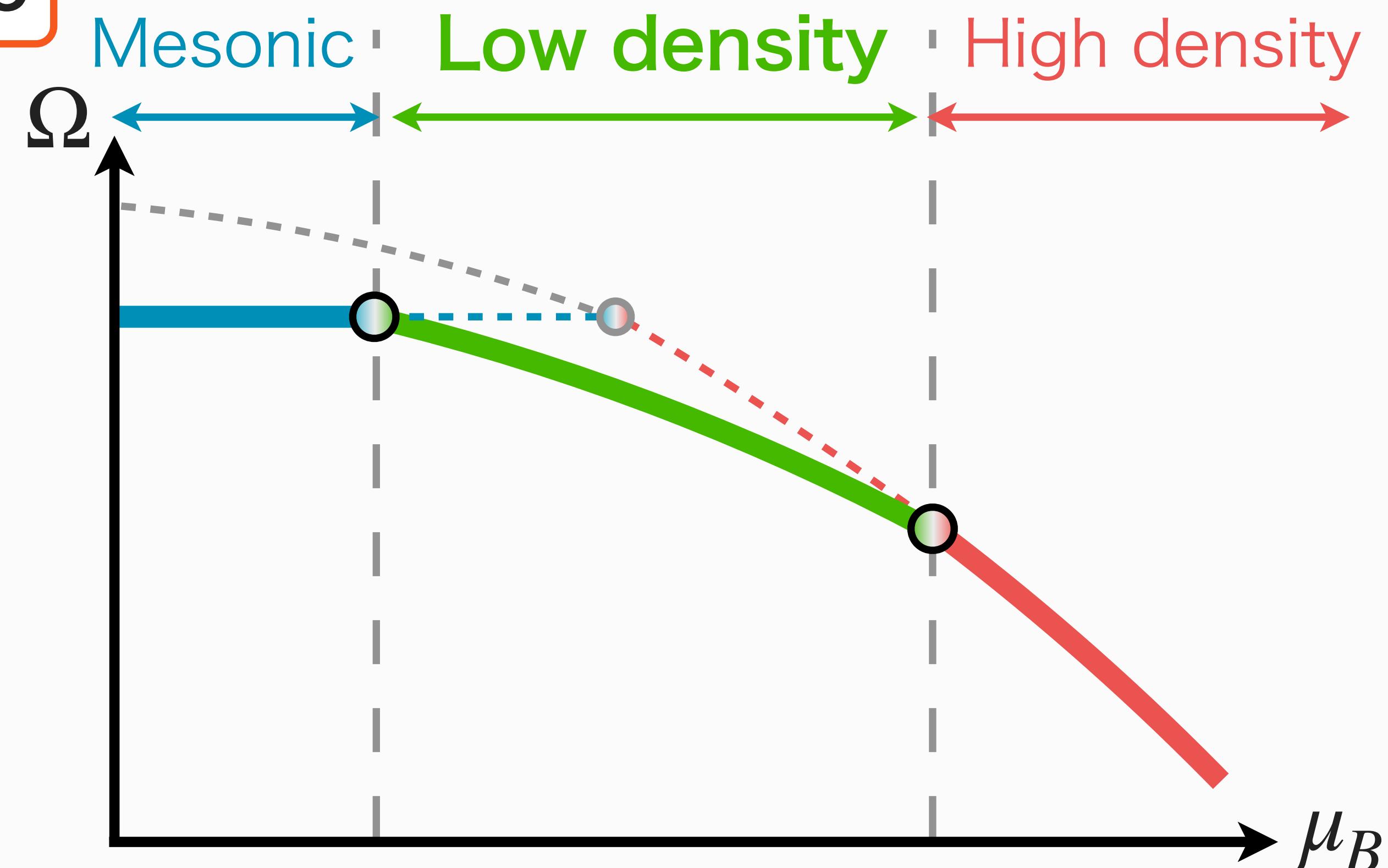
- Six times  $n_0 \simeq 1.7 \text{ fm}^{-3}$
- Self-binding matter (?)
- Speed of sound  $\sim 1$
- Color confinement

Unreliable

High density  
nuclear matter

The Ansatz is not suitable  
for low density region

## Need another Ansatz



# Abstract

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## Pourpose

- Studying the QCD EoS from holographic QCD

## Method

- Hard-wall model + **Switching IR b.c.**

## Result

- Baryonic matter appears with first transition
- **Partial restoration of chiral symmetry**

## Discussion

- The matter is a high density nuclear matter
- We need low density Ansatz

Fin.