Superfluid Band Calculations for Neutron Star Inner Crust

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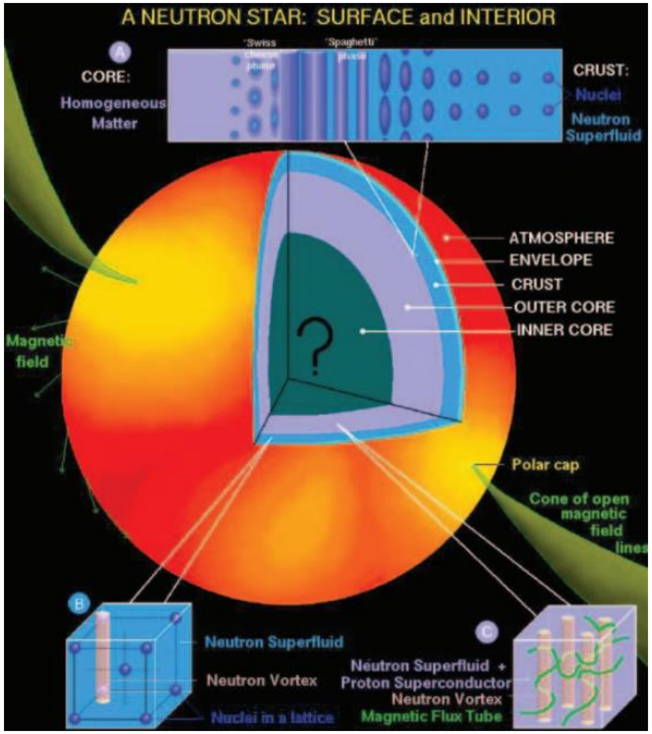
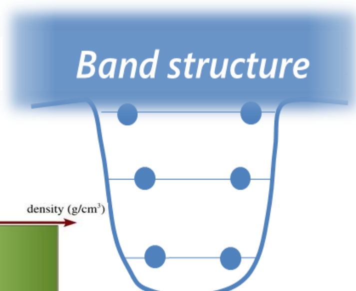
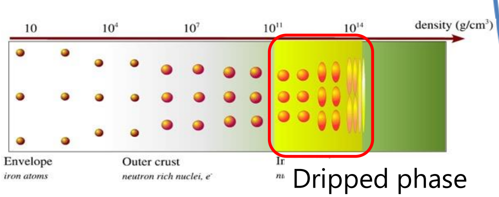
Neutron Star Crust



Band structure?



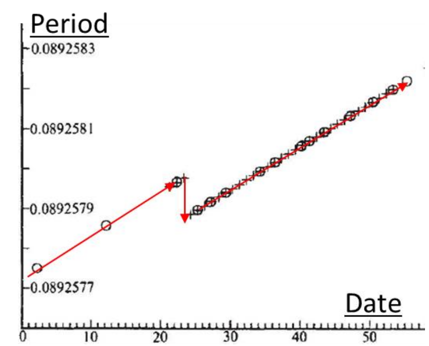


Core region

- Uniform matter
- M-R relation
- Hyperons? Quarks?
- QCD phase diagram

Crust region

- Infinite systems of nuclei
- Source of astro-phenomena
  - -- Pulsar glitches
  - -- quasi-periodic oscillations
  - -- Rapid nucleosynthesis
  - -- Supernovae
- Issues
  - -- Very neutron-rich
  - -- Neutron superfluidity
  - -- Band structure effects
  - -- Temperature, magnetar



Nature, 302, 319 (1983)

Nuclear Band Theory

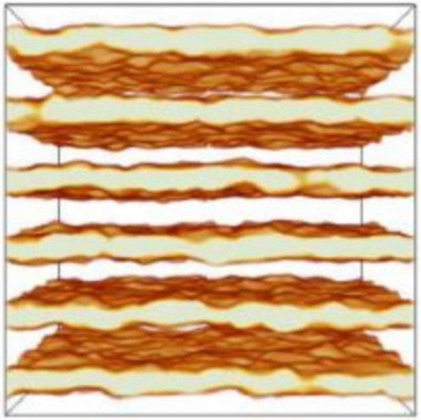
Nuclear HFB theory

$$\begin{pmatrix} \hat{h}_q - \lambda & \Delta \\ \Delta^* & -\hat{h}_q^* + \lambda \end{pmatrix} \begin{pmatrix} u_\mu^{(q)} \\ v_\mu^{(q)} \end{pmatrix} = E_\mu^{(q)} \begin{pmatrix} u_\mu^{(q)} \\ v_\mu^{(q)} \end{pmatrix}$$

Band Theory

$$\psi_{\mu\boldsymbol{k}}(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\tilde{\psi}_{\mu\boldsymbol{k}}(\boldsymbol{r})$$

1D periodical phase



Band HFB equation

$$\begin{pmatrix} \hat{h}_q + \hat{h}_{q,\boldsymbol{k}} - \lambda & \Delta \\ \Delta^* & -\hat{h}_q^* - \hat{h}_{q,-\boldsymbol{k}}^* + \lambda \end{pmatrix} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix} = E_{\mu\boldsymbol{k}}^{(q)} \begin{pmatrix} \tilde{u}_{\mu\boldsymbol{k}}^{(q)} \\ \tilde{v}_{\mu\boldsymbol{k}}^{(q)} \end{pmatrix}$$

There are so many orbitals in the computational space…

For 1D case, $N_z \times N_{k_\parallel} \times N_{k_z} \times (n,p) \times (u,v)$

$$\sim 60 \times 150 \times 80 \times 2 \times 2 = 28800000$$

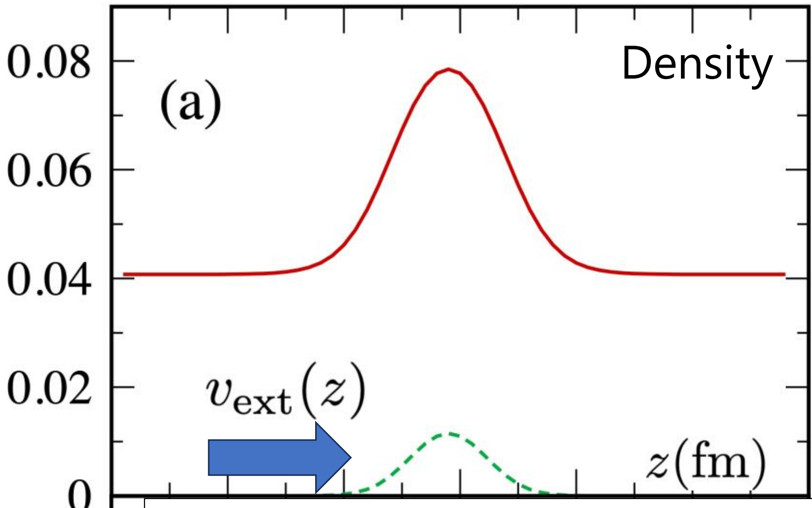
!!!????

SUPERCOMPUTING in CPU Parallelization !!!

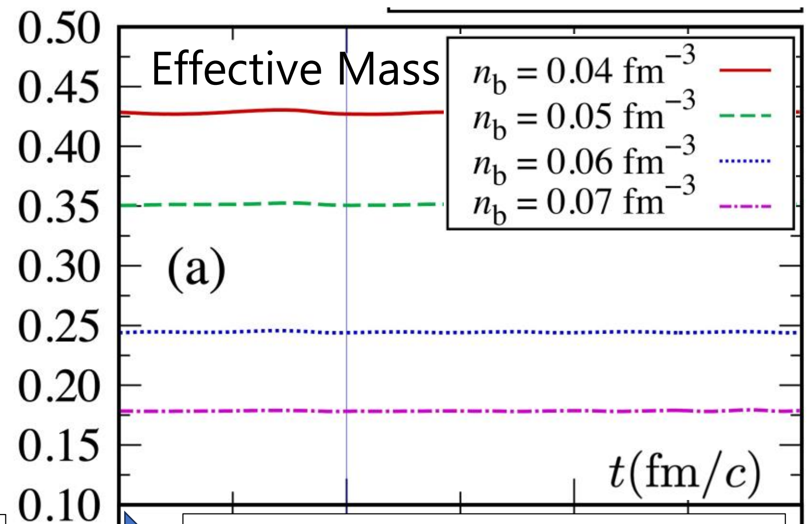
Anti-Entrainment

Time-dependent form

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\tilde{u}^{(q)}_{\mu\boldsymbol{k}}\\ \tilde{v}^{(q)}_{\mu\boldsymbol{k}}\end{pmatrix}=\begin{pmatrix}\hat{h}_q(t)+\hat{h}_{q,\boldsymbol{k}}(t) & \Delta(t)\\ \Delta^*(t) & -\hat{h}^*_q(t)-\hat{h}^*_{q,-\boldsymbol{k}}(t)\end{pmatrix}\begin{pmatrix}\tilde{u}^{(q)}_{\mu\boldsymbol{k}}\\ \tilde{v}^{(q)}_{\mu\boldsymbol{k}}\end{pmatrix}$$



Impose ext. field and calc. response…



Results less eff. mass than bare!!

Takeaway!!

Band structure effects "enhance" the dynamics of neutrons, even with the superfluidity (*anti-entrainment* effect).

Finite-temperature, magnetic-field

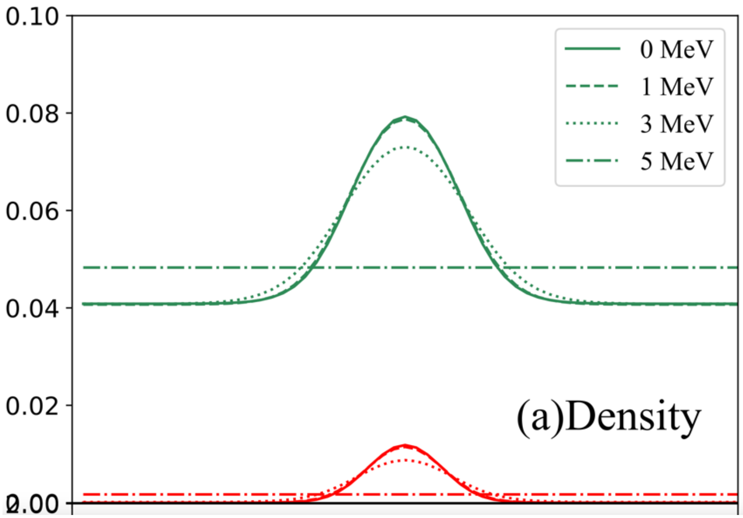
Finite temperature form

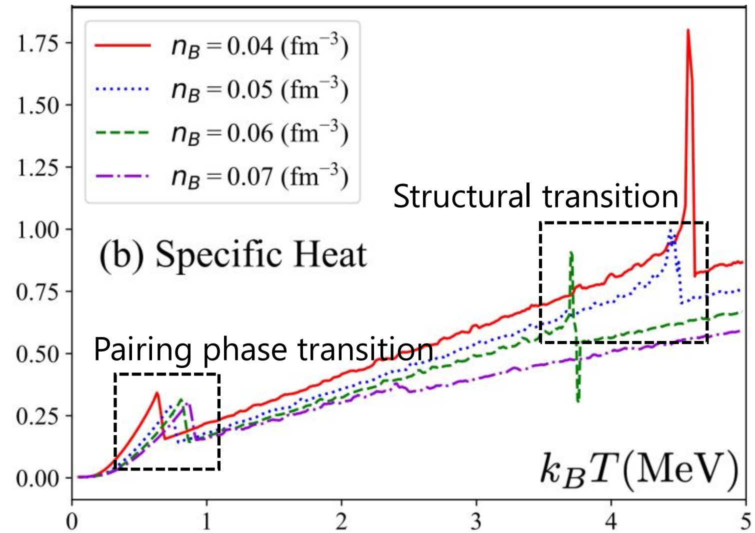
$$\rho_q(\boldsymbol{r}) = \sum_{\mu\boldsymbol{k}} \left[f_D(-E_\mu)|v_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 + f_D(E_\mu)|u_{\mu\boldsymbol{k}}(\boldsymbol{r})|^2 \right]$$

Finite magnetic-field form

$$\hat{h}_q = \hat{h}_q^{(0)} + \hat{h}_q^{(B)} \quad \hat{h}_q^{(B)} = -\left(l\delta_{q,p} + g_q \frac{\boldsymbol{\sigma}}{2} \right) \cdot \tilde{\boldsymbol{B}}_q$$

w/ g-factors $g_n = -3.826$ $g_p = 5.585$





Takeaway!!

Our framework is applicable to more realistic systems,
and can explore the phase structure of neutron star matter.

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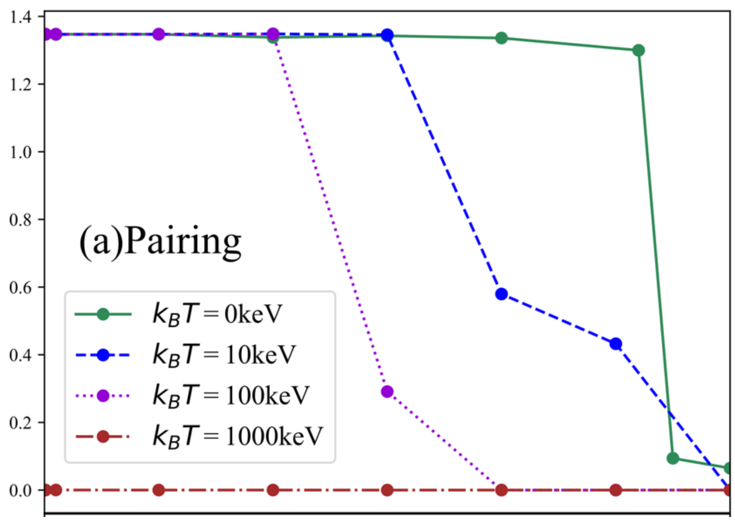

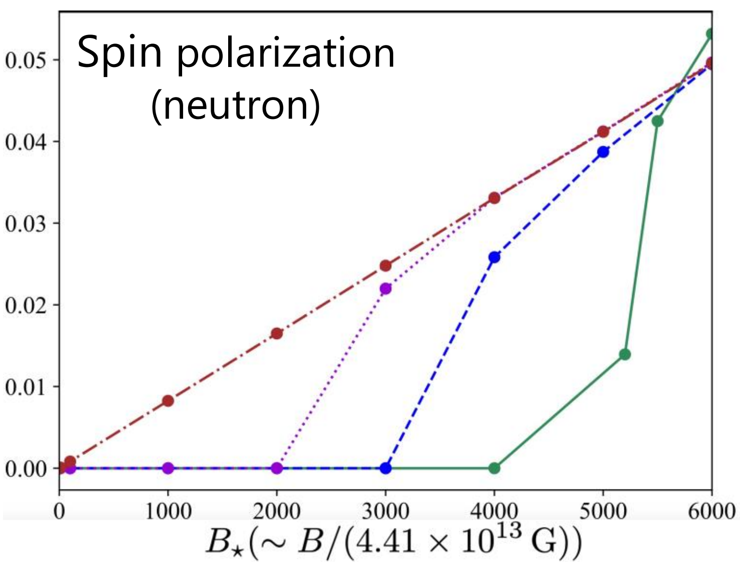

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Towards further extensions

2D or 3D extensions are computationally challenging…

→ *More efficient comp. method necessary!!*

Normal HF calculation

Hamiltonian

$$\hat{H}^{\mathrm{KS}}\psi_i = \varepsilon_i \psi_i$$

Diagonalization

eigenvectors

$$\{\varepsilon_i, \psi_i\}$$

Re-input

densities

$$\rho(\boldsymbol{r}) = \sum_i |\psi_i|^2$$

Shifted-COCG method

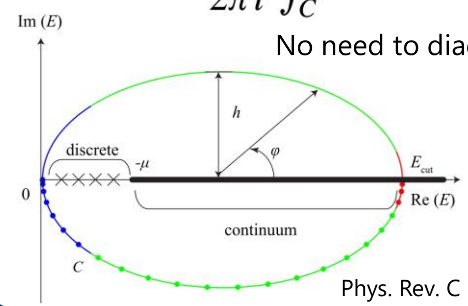
Green function

$$(z - H)G(z,\boldsymbol{r};\boldsymbol{r}') = \delta(\boldsymbol{r} - \boldsymbol{r}'),$$

Densities as contour integral

$$\rho(\boldsymbol{r}') = \frac{1}{2\pi i}\oint_C dz\, G(z,\boldsymbol{r};\boldsymbol{r}')|_{\boldsymbol{r}=\boldsymbol{r}'}$$

No need to diagonalization!



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Takeaway!!

Our project on fully comprehensive band calculations

for neutron star inner crust is still being on-going!!

Summary and Prospect

What we've done

- Superfluid band calculations for neutron star matter within the inner crust
- Integrate nuclear HFB theory and band theory on the same footing
- Extend framework into finite-temperature and magnetic-field systems

What's been found

- Neutron dynamics is *enhanced* in the inner crust (*anti-entrainment*)
- With high temperatures two phase transitions take place
- With magnetic fields spin-polarized phase appears

What we plan

- Extend the framework into the 2D and 3D systems
- Complete the *table* of the Equation of State, and the neutron effective mass

Thank you for your careful attention