Possibility of phase transition on superfluid vortex under Higgs confinement crossover

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with Tomoya Hayata and Yoshimasa Hidaka

8/10 YITP Compact Starts in QCD Phase diagram

1. Introduction/Motivation
2. Analytical approach
3. Numerical approach
4. Conclusion

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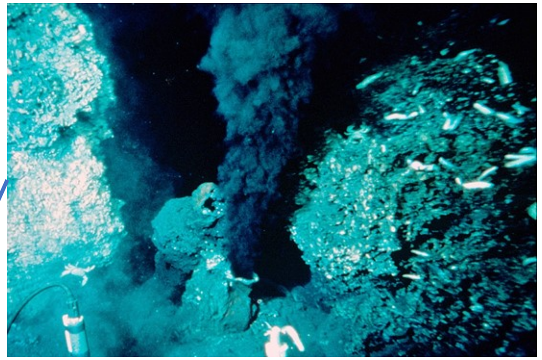
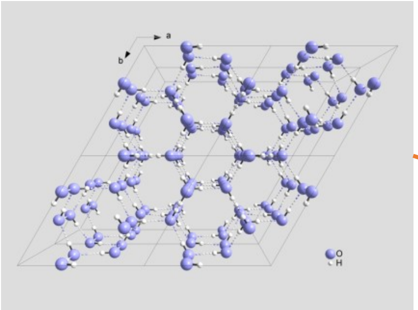
Symmetry and phase

Translational symmetry

indistinguishable

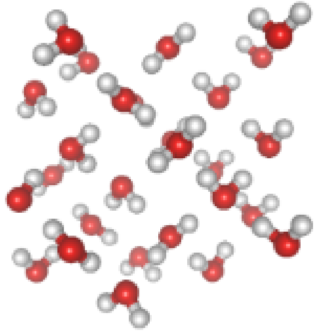


Hydrothermal vent

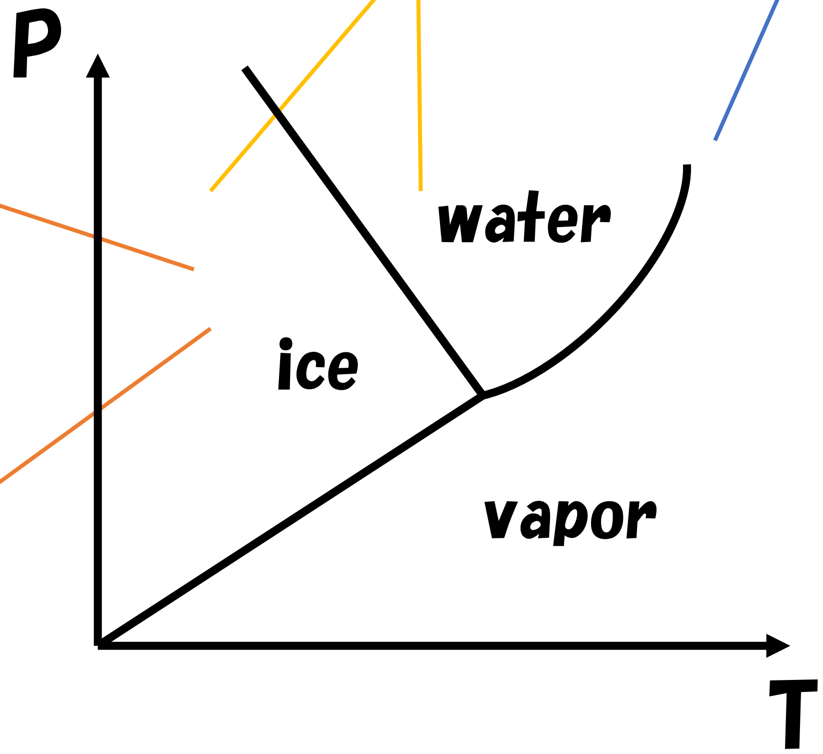


Ih phase
hexagonal $\mathbb{Z}_6$

Discrete symmetry



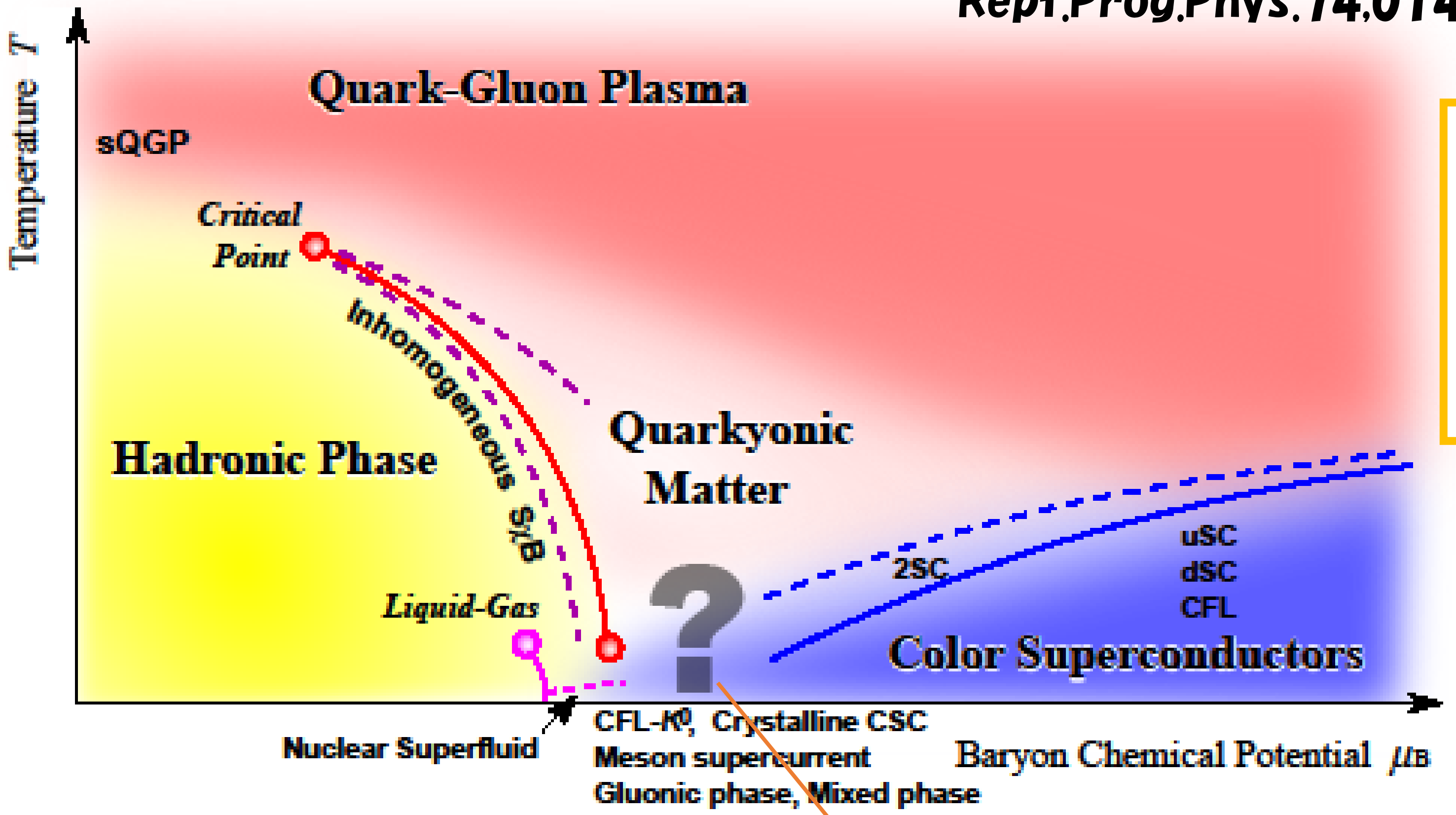
Ic phase cubic $\mathbb{Z}_4$



Water phase diagram

QCD phase diagram

K. Fukushima and T.Hatsuda,
Rept.Prog.Phys.74,014001(2011)



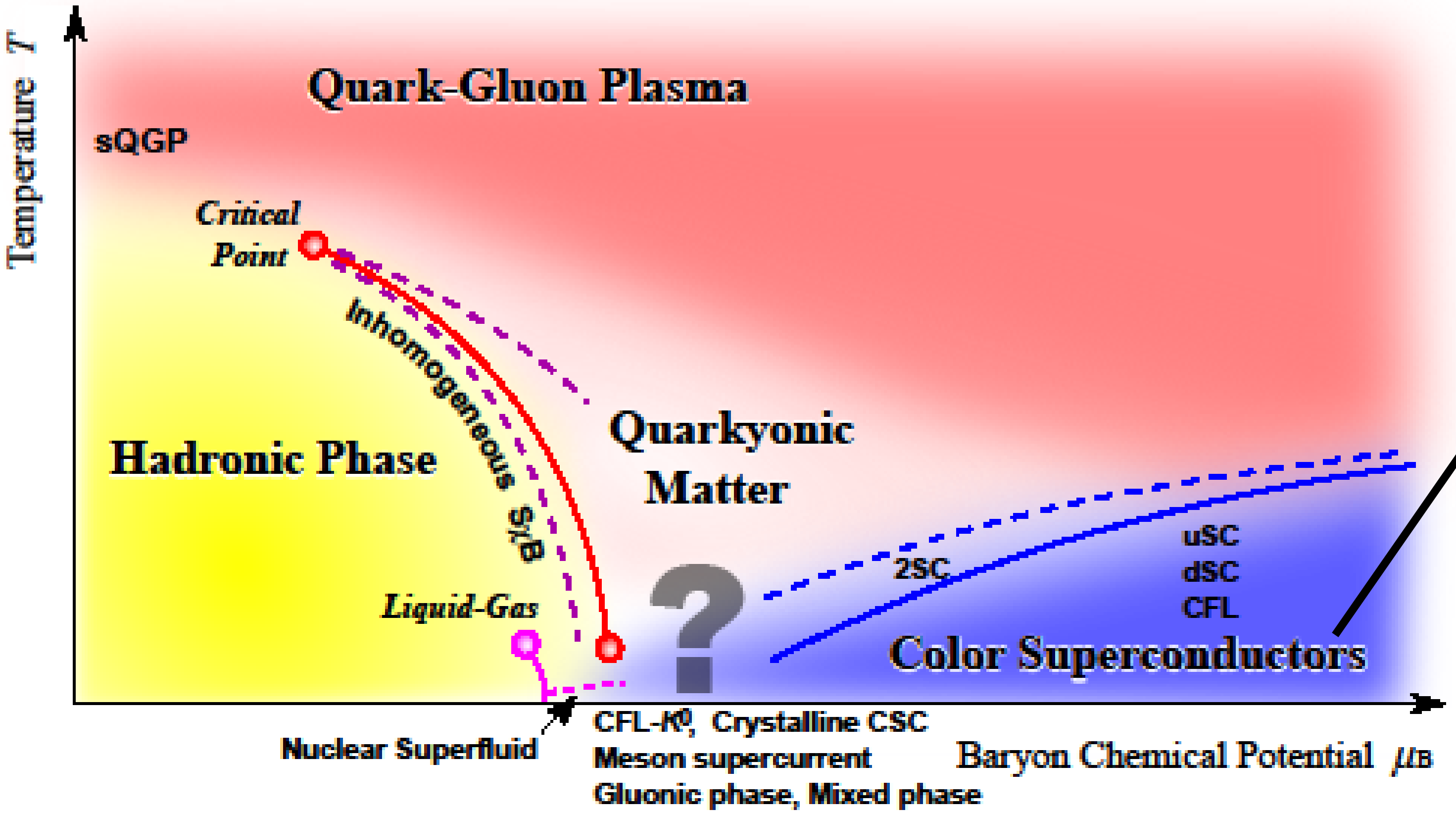
Global symmetry

$$G = SU(3)_L \times SU(3)_R \times U(1)_Q$$

$SU(3)$…**flavor (u,d,s)**

What happens between hadronic phase and color superconductor?
Can we distinguish the boundary?

K. Fukushima and T.Hatsuda,
Rept.Prog.Phys.74,014001(2011)



In high density…
Color Flavor Locking (CFL) phase
diquark condensate

$$(\Phi_L)_a^i = \epsilon^{ijk}\epsilon_{abc}\langle (q_L)_j^b (Cq_L)_k^c \rangle$$
$$(\Phi_R)_a^i = \epsilon^{ijk}\epsilon_{abc}\langle (q_R)_j^b (Cq_R)_k^c \rangle$$
$$\Phi \equiv \Phi_L = -\Phi_R = \begin{pmatrix} \Delta_{CFL} & 0 & 0 \\ 0 & \Delta_{CFL} & 0 \\ 0 & 0 & \Delta_{CFL} \end{pmatrix}$$

Indices are color (r,g,b) and flavor (u,d,s) because of the Cooper pairing

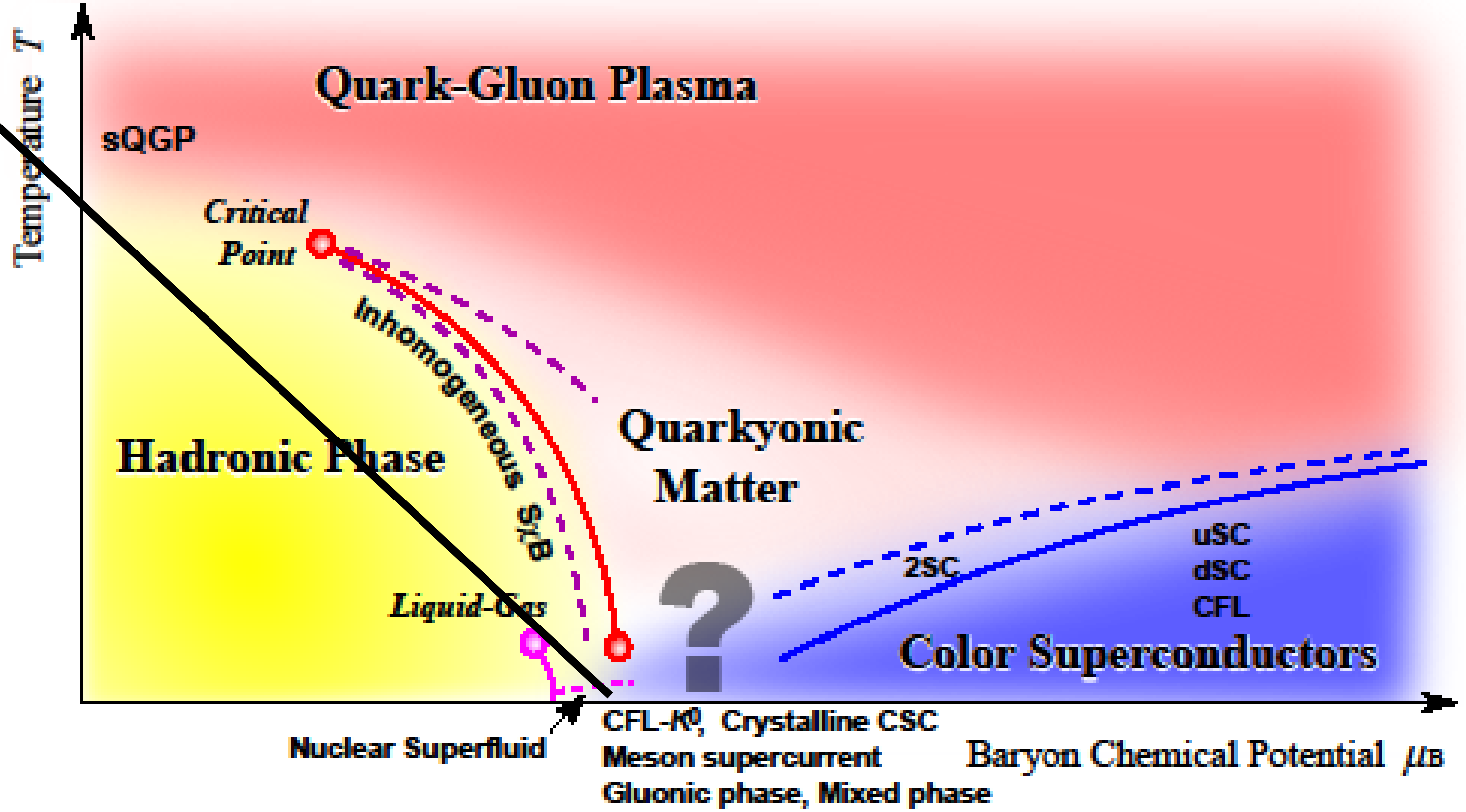
Global symmetry breaking pattern

$$G = SU(3)_L \times SU(3)_R \times U(1)_Q \rightarrow H = SU(3)_V$$

In low density…Hadronic phase
2 Λ (uds)= dibaryon condensate

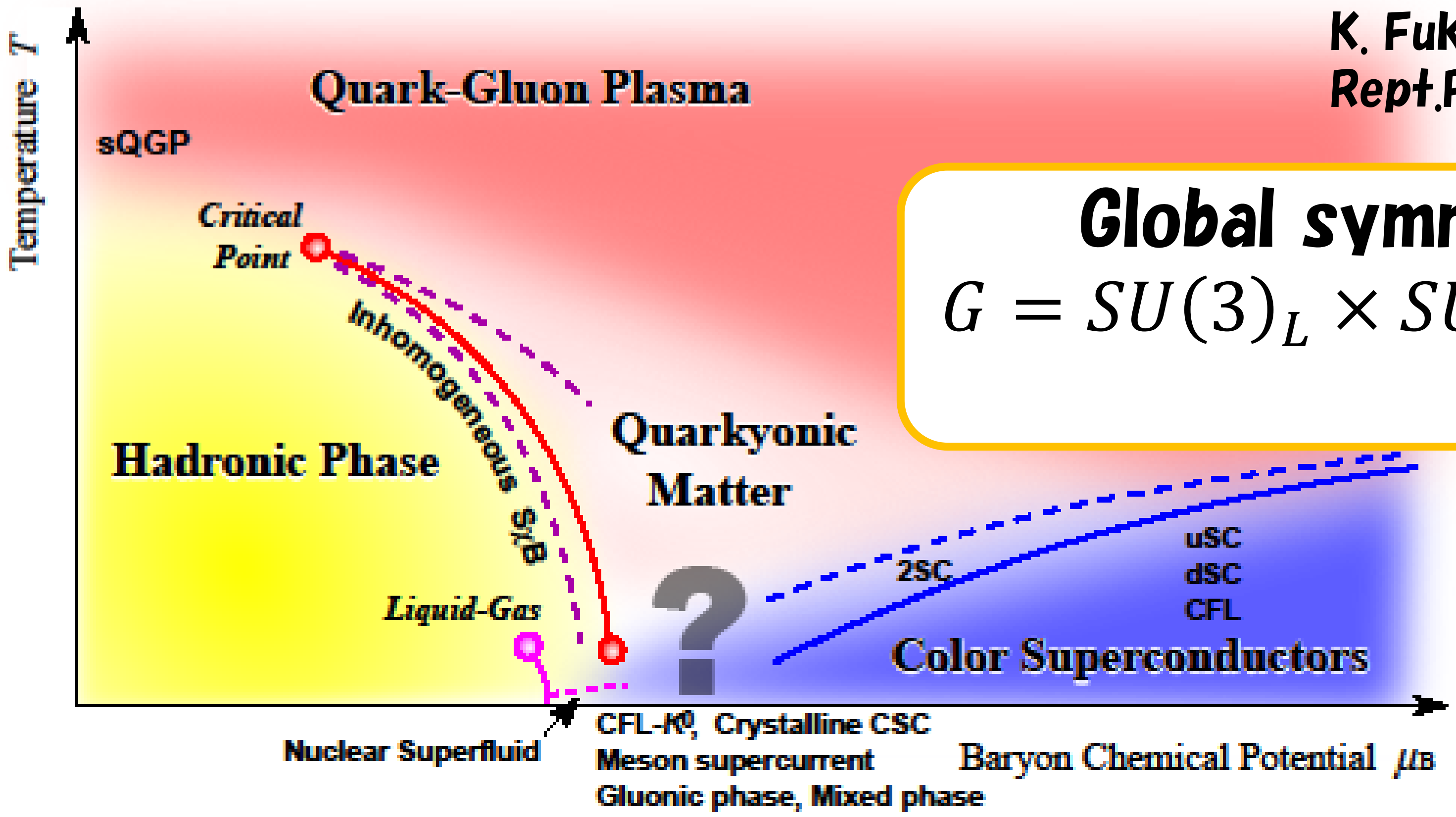
K. Fukushima and T.Hatsuda, Rept.Prog.Phys.74,014001(2011)

$$\Lambda = \epsilon^{abc}\epsilon_{ijk}q_a^i q_b^j q_c^k$$
$$\langle\Lambda\Lambda\rangle \neq 0$$



Global symmetry breaking pattern

$$G = SU(3)_L \times SU(3)_R \times U(1)_Q \rightarrow H = SU(3)_V$$



K. Fukushima and T.Hatsuda, Rept.Prog.Phys.74,014001(2011)

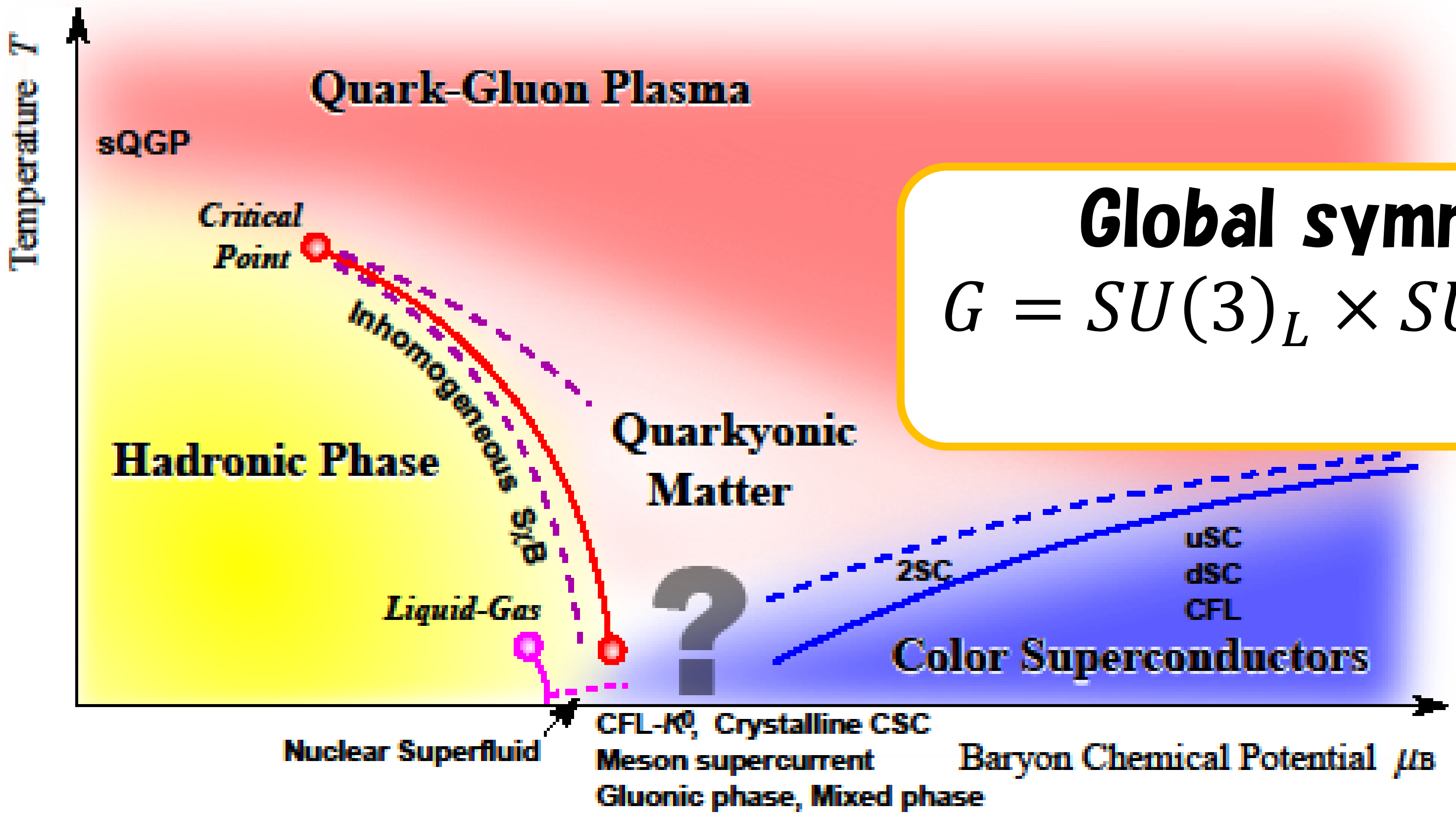
Global symmetry breaking pattern

$$G = SU(3)_L \times SU(3)_R \times U(1)_Q \rightarrow H = SU(3)_V$$

Same!

Possibility 1…These two phase cannot be distinguished.

Schafer and Wilczek,PRL82,3956(1999)
Hatsuda,Tachibana,Yamamoto,Baym,PRL97,122001(2006)
Alford,Baym,Fukushima,Hatsuda,Tachibana,Phys.Rev.D99,036004 (2019)
Chatterjee, Nitta,Yasui Phys.Rev.D99,034001(2019)
Hirono,Tanizaki,PRL122,212001(2019)
Hayashi PRL132,221901(2024)



K. Fukushima and T.Hatsuda, Rept.Prog.Phys.74,014001(2011)

Global symmetry breaking pattern

$$G = SU(3)_L \times SU(3)_R \times U(1)_Q \to H = SU(3)_V$$

Same!

<u>Possibility 2…There is quantum phase transition</u>

Cherman, Sen, Yaffe, PRD 100, 034015 (2019)
Cherman, Jacobson, Sen, Yaffe Phys.Rev.D 102 (2020) 10, 105021
Cherman, Jacobson, Sen, Yaffe JHEP06 (2024)200

Today…

No phase transition on bulk,
but phase transition on vortex in
$U(1) \times U(1)$ model in 3+1 dim

Action

$U_\mu(x)$ ··· $U(1)$ **valued link variable**

$\phi_1(x) = e^{i\theta_1(x)}, \phi_2(x) = e^{i\theta_2(x)}$ ··· **charged scalar field**

$$S = \frac{\beta_g}{2}\sum_x U_p(x) - \frac{\beta_H}{2}\sum_{x,\mu}\phi_1^*(x)U_\mu(x)\phi_1(x+\hat{\mu}) - \frac{\beta_H}{2}\sum_{x,\mu}\phi_2^*(x)U_\mu(x)\phi_2(x+\hat{\mu}) + c.c$$

or

$$S = -\frac{\beta_g}{2}\sum_x U_p(x) - \beta_H\sum_{x,\mu}(\cos\left(\Delta_\mu\theta_1 + A_\mu(x)\right) + \cos\left(\Delta_\mu\theta_2 + A_\mu(x)\right))$$

Action

$U_\mu(x)\cdots U(1)$ **valued link variable**

$\phi_1(x) = e^{i\theta_1(x)}, \phi_2(x) = e^{i\theta_2(x)}\cdots$ **charged scalar field**

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Symmetry

$U(1)$ **gauge** : $\phi_1(x) \to e^{i\lambda(x)}\phi_1,\ \phi_2 \to e^{i\lambda(x)}\phi_2$

$U(1)$ **global** : $\phi_1(x) \to e^{i\alpha}\phi_1,\ \phi_2 \to e^{-i\alpha}\phi_2$

Z_2 **Flavor** : $\phi_1 \leftrightarrow \phi_2$

Charge conjugation : $\phi_1 \to \phi_1^*,\ \phi_2 \to \phi_2^*,\ U_\mu \to U_\mu^*$

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Action

$U_\mu(x)$…$U(1)$ valued link variable

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$$S = \frac{\beta_g}{2}\sum_x U_p(x) - \frac{\beta_H}{2}\sum_{x,\mu} \phi_1^*(x)U_\mu(x)\phi_1(x+\hat{\mu}) - \frac{\beta_H}{2}\sum_{x,\mu} \phi_2^*(x)U_\mu(x)\phi_2(x+\hat{\mu}) + c.c$$

Put a domain wall that exchange ϕ_1 and ϕ_2,
and study the expectation value by path integral.
Or evaluate $Z' = e^{S'}$ action after exchanging ϕ_1 and ϕ_2 to compare the original $Z = e^S$.

In bulk structure (no vortex)

$$S = \frac{\beta_g}{2}\sum_x U_p(x) - \frac{\beta_H}{2}\sum_{x,\mu}\phi_1^*(x)U_\mu(x)\phi_1(x+\hat{\mu}) - \frac{\beta_H}{2}\sum_{x,\mu}\phi_2^*(x)U_\mu(x)\phi_2(x+\hat{\mu}) + c.c$$

$\phi_1 = \phi_2 = 1$ **or** $\theta_1 = \theta_2 = 0$ **minimize the action.**

$\phi_1 \leftrightarrow \phi_2$ **does not change anything.**

$$\frac{Z'}{Z} = 1$$

On the vortex

On the vortex

Configuration is specified by winding number (n_1, n_2)

On the vortex

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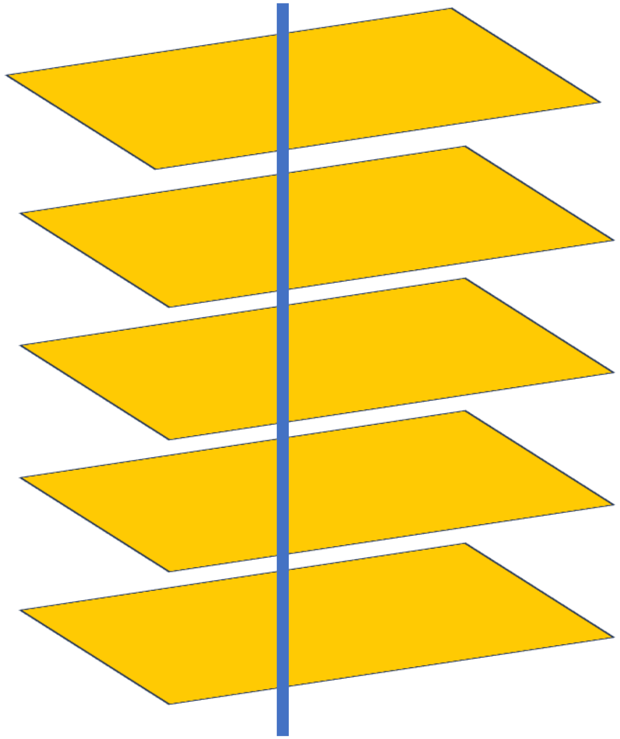
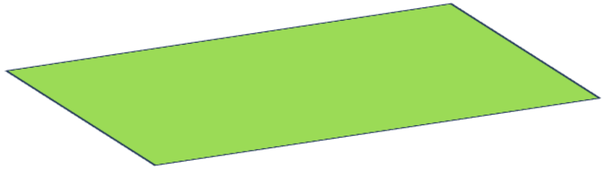
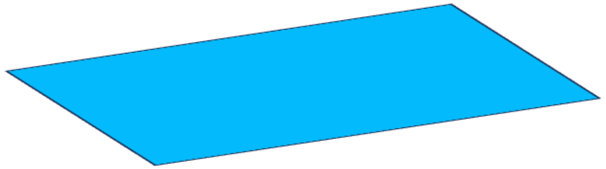
On each z**, winding number is** 1; **either** $(1{,}0)$ **or** $(0{,}1)$







Strong coupling $\beta_g = 0$

Integrate the gauge field

$$Z = \int DU_\mu D\phi_1 D\phi_2 \exp\left[2\beta_H \sum_{x,\mu} \cos\left(\frac{\Delta_\mu\theta_1(x) - \Delta_\mu\theta_2(x)}{2}\right) \cos\left(\frac{\Delta_\mu\theta_1(x) + \Delta_\mu\theta_2(x)}{2} + A_\mu\right)\right]$$

$$= \int D\phi_1 D\phi_2 \prod_{x,\mu} I_0\left[2\beta_H \cos\left(\frac{\Delta_\mu\theta_1(x) - \Delta_\mu\theta_2(x)}{2}\right)\right]$$

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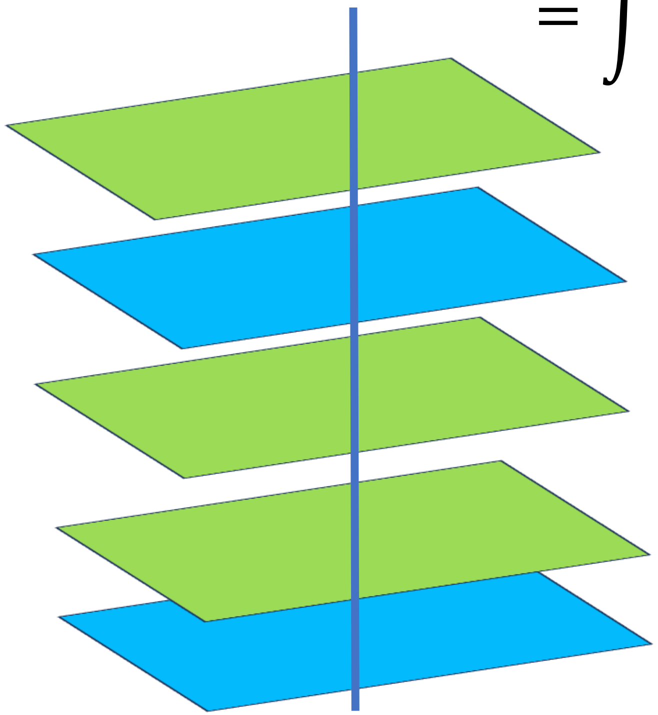
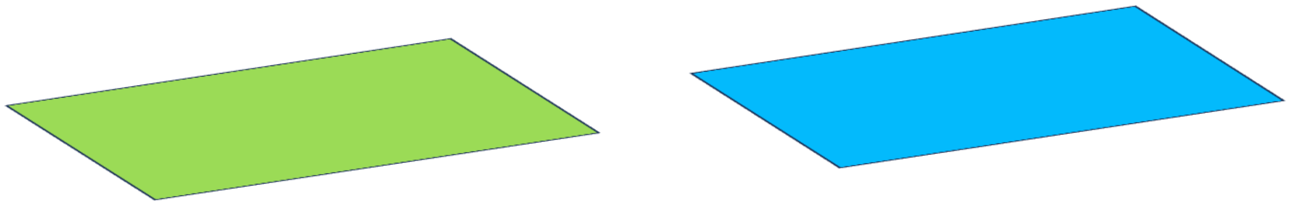


$$\Delta_\mu\theta_1 - \Delta_\mu\theta_2 = 0$$

Both 1 and 2 vortex can be put on each z



Weak coupling $\beta_g \gg 1$

Gauge field is in pure gauge $U_\mu(x) = e^{i\Delta_\mu \alpha(x)}$

$$Z = \int DU_\mu D\phi_1 D\phi_2 \exp\left[\beta_g P + \beta_H \sum_{x,\mu} (\cos\left(\Delta_\mu \theta_1 + \Delta_\mu \alpha(x)\right) + \cos\left(\Delta_\mu \theta_2 + \Delta_\mu \alpha(x)\right))\right]$$

$P \cdots$ **# of Plaquets**

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P… # **of Plaquets**

When $\Delta_\mu\theta_1 = \Delta_\mu\theta_2 = 0$, **they are in the same form of kinetic term**

→maximize the action.

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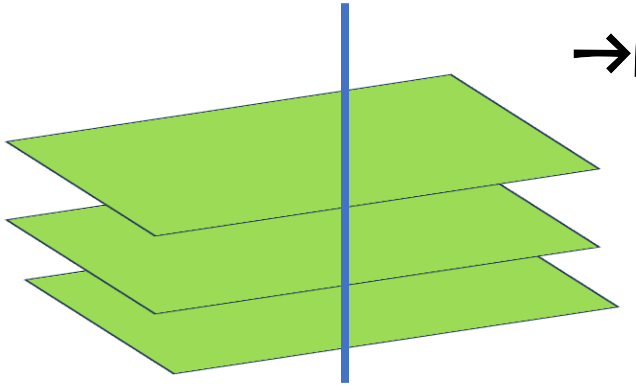
P··· **# of Plaquets**

When $\Delta_\mu\theta_1 = \Delta_\mu\theta_2 = 0$, they are in the same form of kinetic term
→maximize the action.

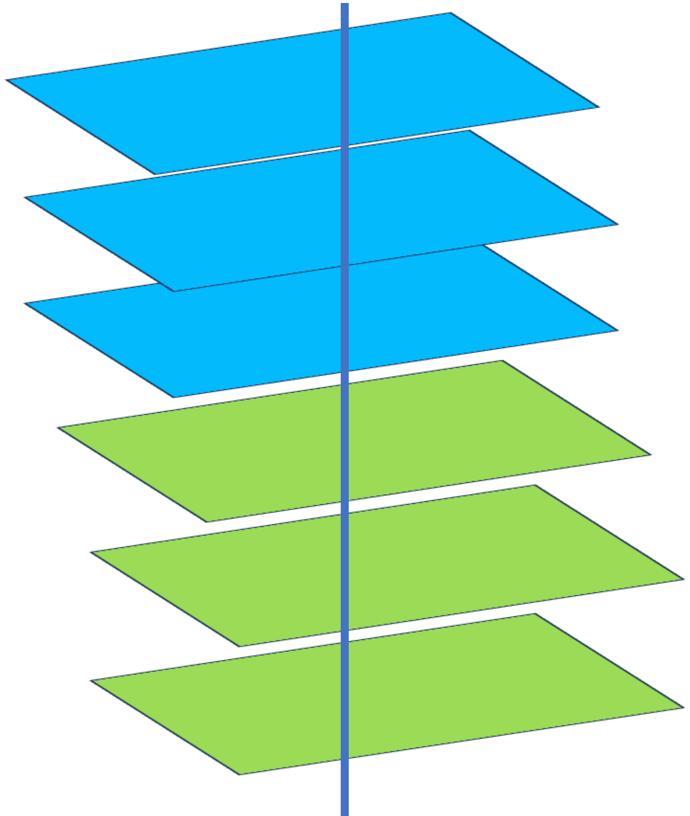


→Either 1 or 2 vortex is put on each z

$$Z = \int DU_\mu D\phi_1 D\phi_2 \exp\left[\beta_g P + \beta_H \sum_{x,\mu} \left(\cos\left(\Delta_\mu \theta_1 + \Delta_\mu \alpha(x)\right) + \cos\left(\Delta_\mu \theta_2 + \Delta_\mu \alpha(x)\right)\right)\right]$$

Put a domain wall at $z = \frac{1}{2}$

tend to align for neighboring z
→domain wall arises.



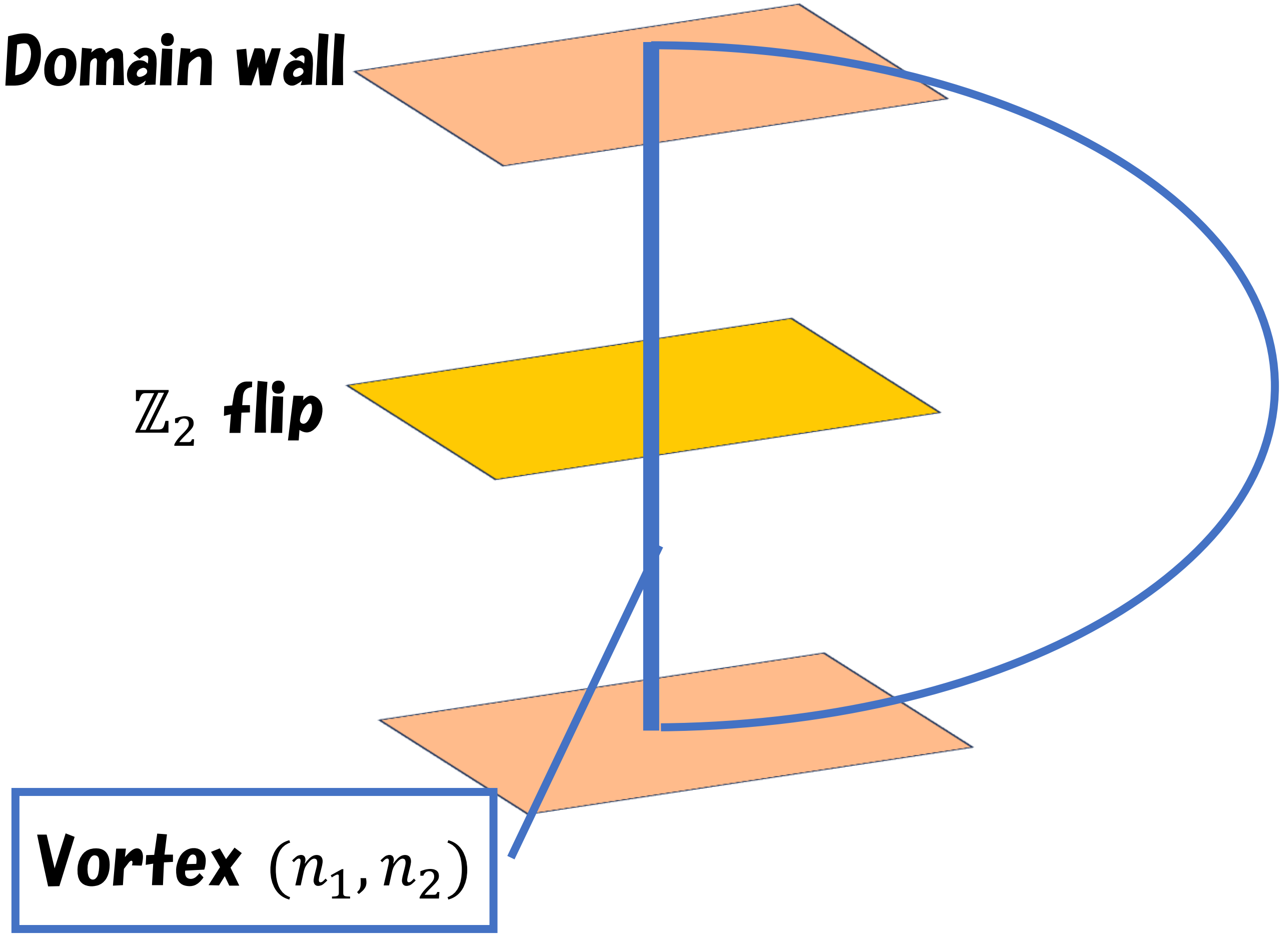
The evaluation for domain wall

$$Z' \supset \exp[2\beta_H(\cos\theta_1(x_\perp) + \cos\theta_2(x_\perp))] = \exp[4\beta_H \cos\theta_1(x_\perp)]$$

$\frac{Z'}{Z} = \exp[-2CT\beta_H(1 - \cos\theta(x_\perp))] \to 0$ **as** $C, T \to \infty$

C…**area of each z slice**
T…**time duration**

Result $\frac{Z'}{Z}$



Bulk $\cdots 1$ for strong/weak coupling

Vortex strong coupling $\cdots 1$

Vortex weak coupling $\cdots 0$

SSB of Z_{2F} is happens on the vortex when the coupling is weak.

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Action

$U_\mu(x)\cdots U(1)$ **valued link variable**

$\phi_1(x) = e^{i\theta_1(x)}, \phi_2(x) = e^{i\theta_2(x)}\cdots$**charged scalar field**

$$S = -\beta_g \sum_{x,\mu<\nu} \cos F_{\mu\nu} - \beta_H \sum_{x,\mu,a} \cos\big(\theta_a(x+\hat{\mu}) - \theta_a(x) + A_\mu(x) + (-1)^a a_\mu(x)\big)$$

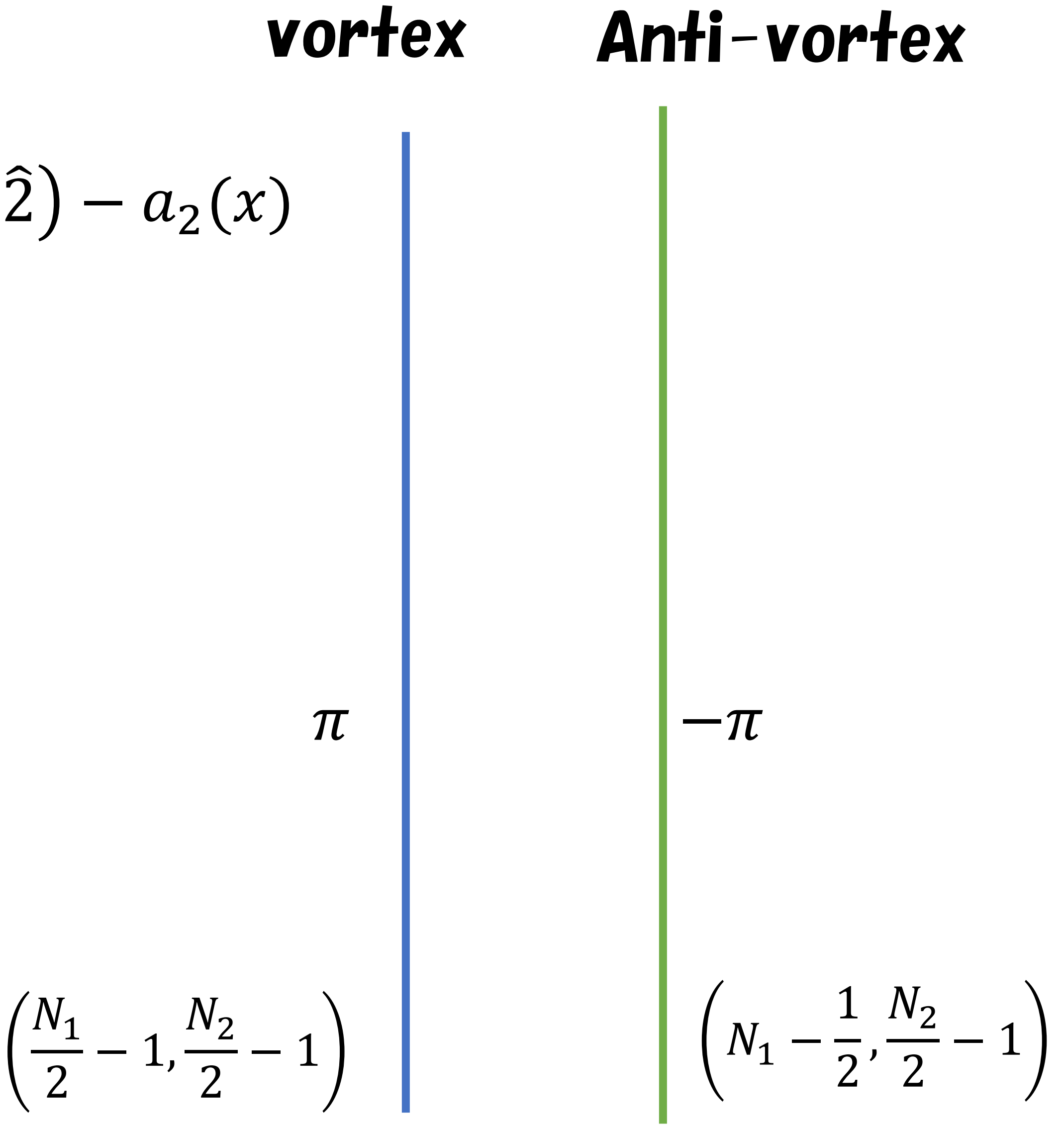
$$F_{\mu\nu} = A_\mu(x) + A_\nu(x+\hat{\mu}) - A_\mu(x+\hat{\nu}) - A_\nu(x)\cdots$$ **Plaquett**

$$a_\mu(x) = \begin{cases} \pi & \textbf{If } \mu = 2,\ x_1 > \frac{N_1}{2} - 1,\ x_2 > \frac{N_2}{2} - 1 \\ 0 & \textbf{otherwise} \end{cases}$$

$$a_\mu(x) = \begin{cases} \pi & \textbf{If } \mu = 2,\ x_1 > \frac{N_x}{2} - 1,\ x_2 = \frac{N_y}{2} - 1 \\ 0 & \textbf{otherwise} \end{cases}$$

Non-zero flux… $f_{12} = a_1(x) + a_2(x + \hat{1}) - a_1(x + \hat{2}) - a_2(x)$

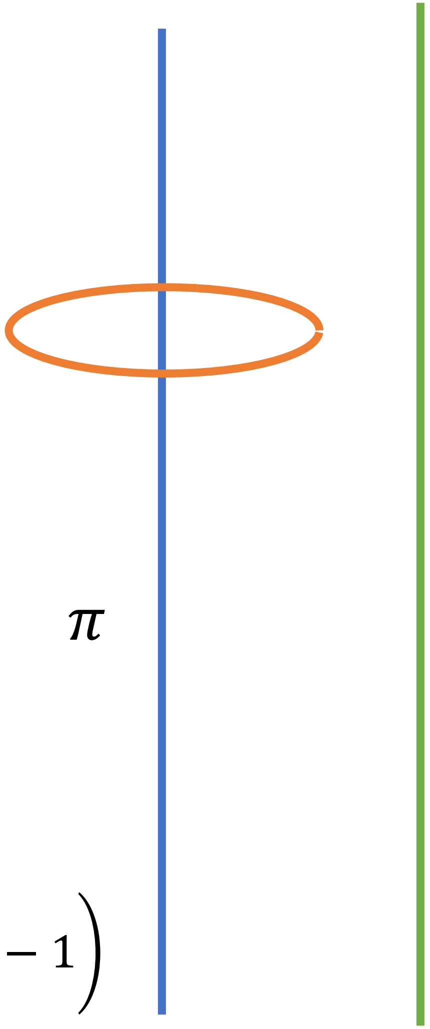
$$f_{12} = \begin{cases} \pi & \textbf{at } \left(\frac{N_1}{2} - 1, \frac{N_2}{2} - 1\right) \\ -\pi & \textbf{at } \left(N_1 - 1, \frac{N_2}{2} - 1\right) \end{cases}$$



Correlation function

Wilson loop $W(x_3, x_4) = e^{\oint A_\mu} = e^{iF_{12}\left(\frac{N_1}{2}-1,\frac{N_2}{2}-1,x_3,x_4\right)}$

Dynamical circulation $\Gamma(x_3, x_4) = \frac{1}{\pi}\log W(x_3, x_4)$



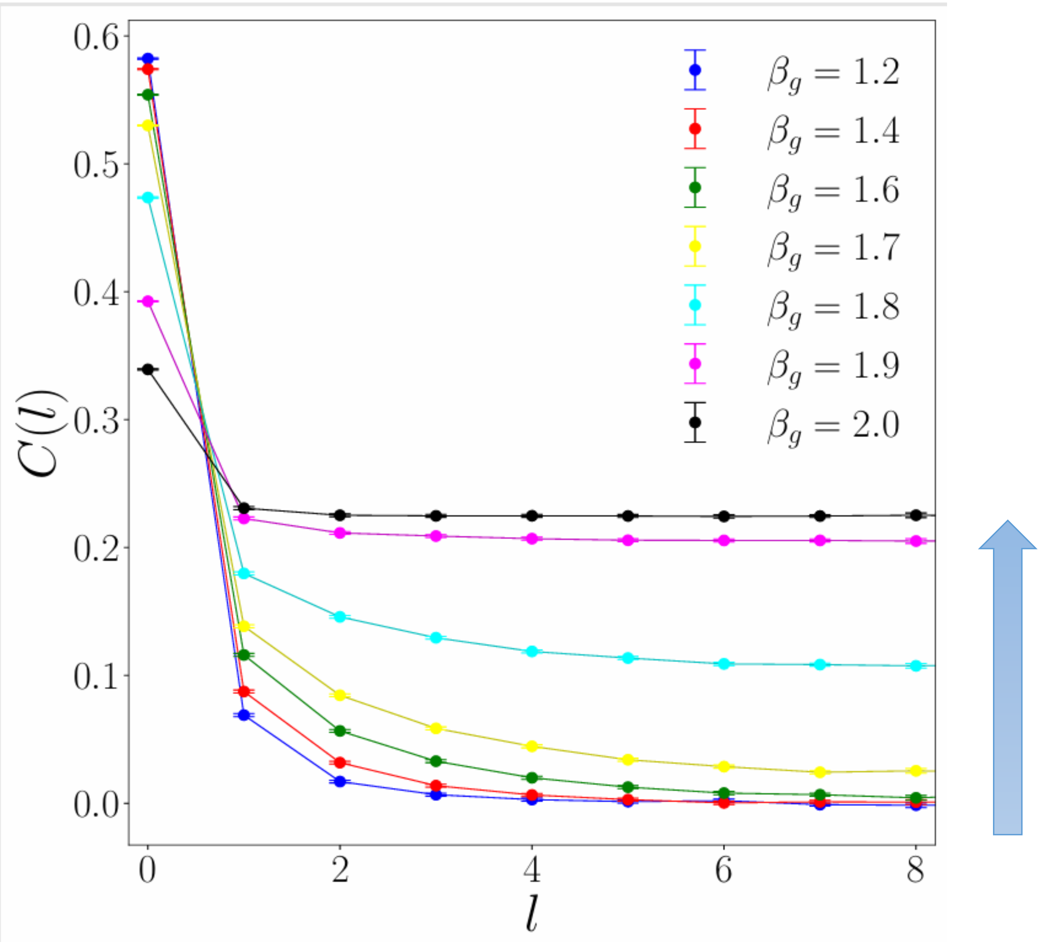
Correlation function

$$C(l) = \frac{1}{N_3N_4}\sum_{x_3,x_4}\langle\Gamma(x_3+l,x_4)\Gamma(x_3,x_4)\rangle$$

Dynamical circulation $\Gamma(x_3, x_4) = \frac{1}{\pi} \log W(x_3, x_4)$

Correlation function

$$C(l) = \frac{1}{N_3 N_4} \sum_{x_3, x_4} \langle \Gamma(x_3 + l, x_4) \Gamma(x_3, x_4) \rangle$$

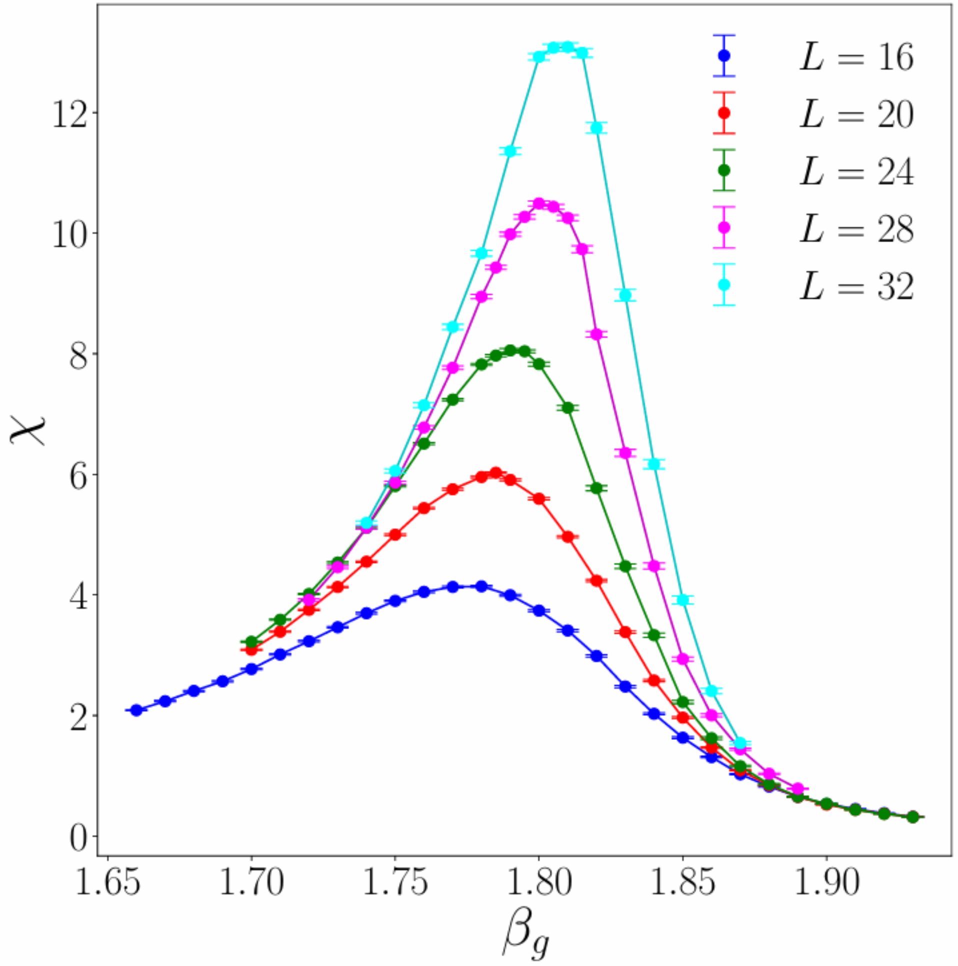


Correlation remains as coupling becomes weak.

Susceptibility…

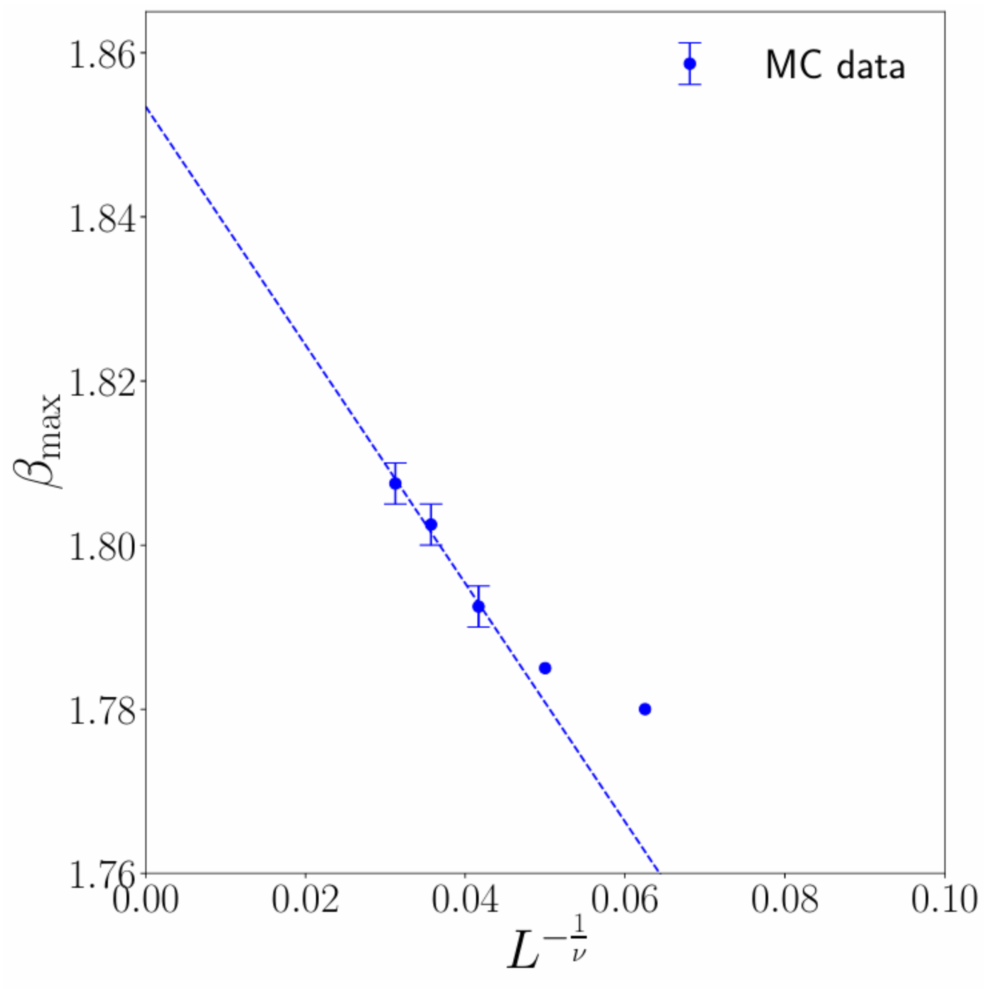
$$m = \frac{1}{N_3 N_4} \sum_{x_3, x_4} \Gamma(x_3, x_4)$$

$$\chi = \langle m^2 \rangle - \langle m \rangle^2$$



showing the same phase transition as quantum Ising model.

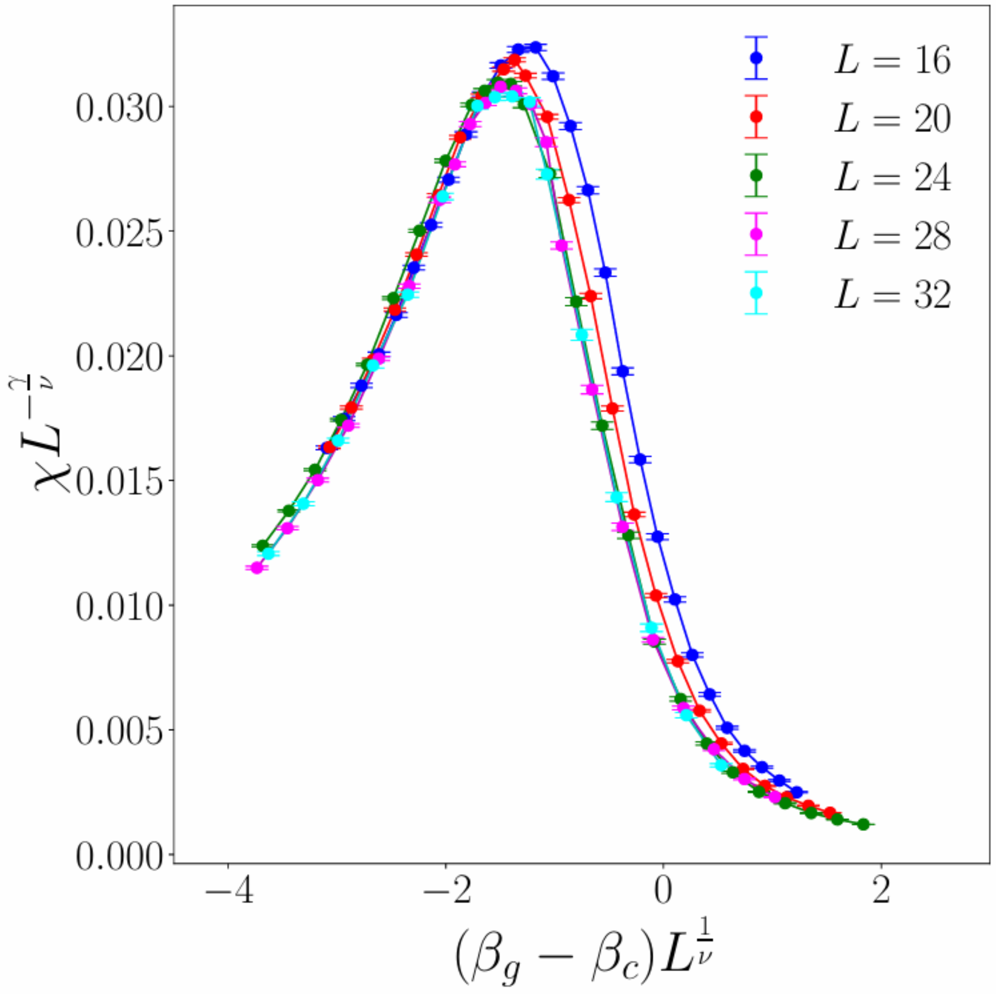
Fitting β_g vs $L^{-\frac{1}{\nu}}$



$\nu = 1$ **for quantum Ising universality class**

$$\beta_c = 1.853 \pm 0.006$$

Renormalized dependence



$$\chi L^{-\frac{\gamma}{\nu}} \text{ vs } (\beta_g - \beta_c)^{\frac{1}{\nu}}$$

$$\gamma = \frac{7}{4}$$

⇨ **Volume independence**

Quantum Ising universality class

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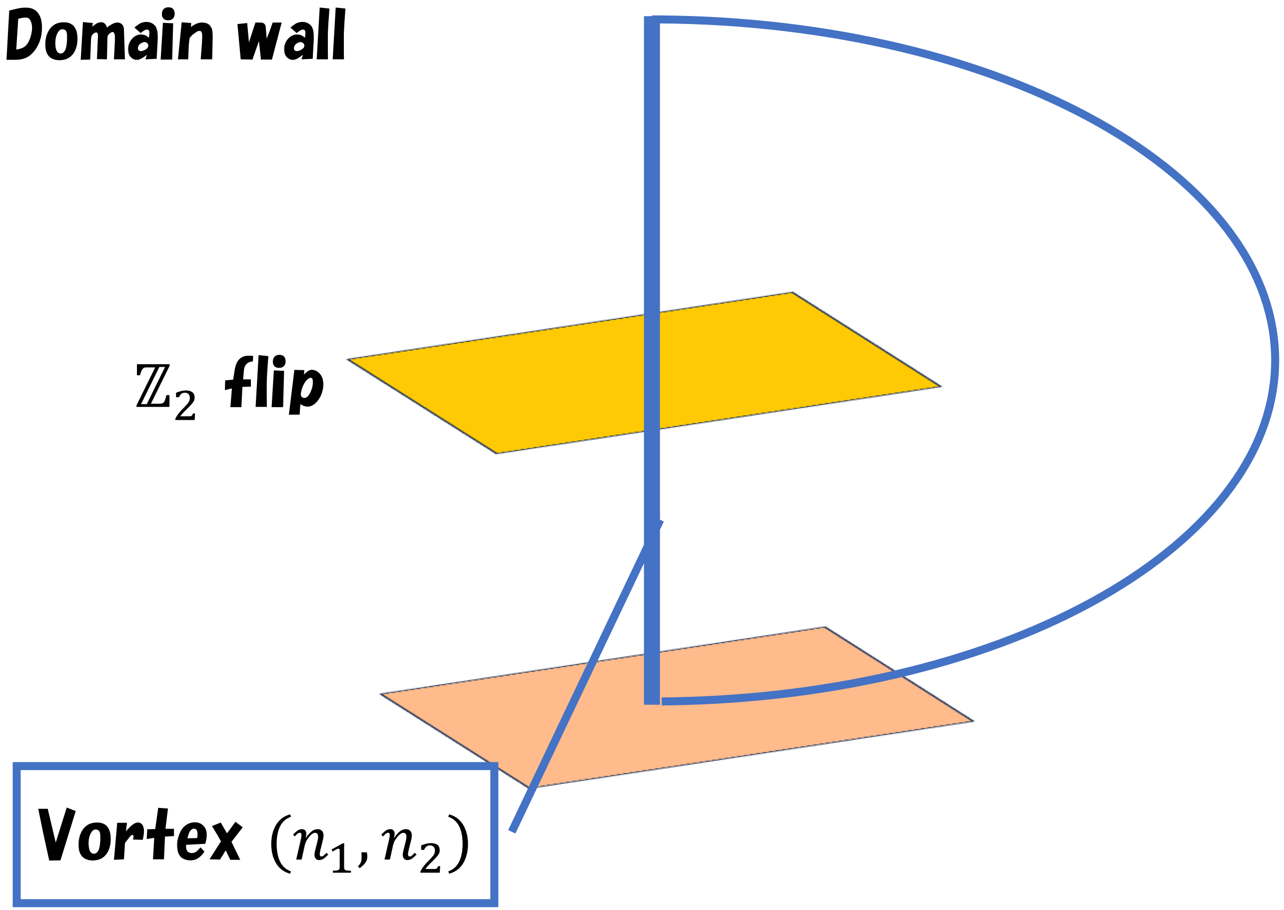
In $U(1) \times U(1)$ model, we saw that as we decrease the coupling, bulk does not change, but vortex goes through a phase transition.

Analytical way



Numerical way