【 Compact Stars in the QCD Phase diagram (CSQCD)
October 7 (Mon.) – 11 (Fri.) , 2024, YITP, Kyoto 】

Properties of kaon condensation in hyperon-mixed matter with three-baryon repulsion

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1. Introduction

1-1 Multi-strangeness system and Kaon properties in medium

Various phases and phase equilibrium in High density QCD

Hadronic phase

Kaon condensation (KC) in hyperon-mixed matter = (Y+K) phase: Coexistent phase of KC with hyperons Y $(\Lambda, \Sigma^-, \Xi^-, \cdots)$

[T. Muto, T. Maruyama, and T. Tatsumi, Prog. Theor. Exp. Phys.2022, 093D03 (2022); Phys. Lett. B 820 (2021), 136587.]

- Chiral symmetry and its spontaneous breaking

Role of quark condensates on (Y+K) phase

Quark condensates in the nucleon:

$$\Sigma_{Kb} \equiv \frac{1}{2}(m_u + m_s)\langle b|(\bar{u}u + \bar{s}s)|b\rangle$$

Quark condensates $\langle \bar{q}q\rangle_{\mathrm{KC}}$ $(q = u, d, s)$ in the (Y+K) phase

(Y+K) phase

Driving force of KC
Softening of the EOS

Relation to Chiral restoration

Connection to quark matter

Plan of my talk

- Formulation of our interaction model for (Y+K) phase

- Chiral condensates in the nucleon
  → Estimation of KN (K-baryon) sigma term

- bulk properties of compact stars with the (Y+K) phase

  M-R relation → softening of the EOS

- Quark condensates in the (Y+K) phase
  → relevance to Chiral restoration

- Outlook : a possible pathway to quark matter

2. Formulation

2-1 our interaction model

[T. Muto, T. Maruyama, and T. Tatsumi,
Phys. Lett. B 820 (2021), 136587.
T. Muto, in preparation.]

K-Baryon and K-K interactions : effective chiral Lagrangian

Baryon-Baryon interaction

Minimal Relativistic Mean-Field theory

Meson-exchange (σ, ω, ρ …) → two-body force (without nonlinear self-interactions)

Slope : $L \equiv 3\rho_0 \left(\frac{\partial S}{\partial \rho_B}\right)_{\rho_B=\rho_0}$

$= (60 – 70)$ MeV

controls Stiffness of EOS from two-body B-B int.

+ Three-Baryon (many-body) forces

Universal Three-Baryon Repulsion (UTBR)

[S. Nishizaki, Y. Yamamoto and T. Takatsuka, Prog. Theor. Phys. 108 (2002) 703.]

Density-dependent effective two-baryon force

: String-Junction Model 2
[R. Tamagaki,
Prog. Theor. Phys. 119 (2008), 965.]

+ Three-Nucleon attraction (TNA) [c.f., I. E. Lagaris and V. R. Pandharipande,
Nucl. Phys. A359(1981),349.]

2-3 K-Baryon and K-K interactions

$SU(3)_L \times SU(3)_R$ chiral effective Lagrangian

[D. B. Kaplan and A. E. Nelson, Phys. Lett. B 175 (1986) 57.]

$$\mathcal{L}_{K,B} = \frac{1}{4} f^2\, \mathrm{Tr}(\partial^\mu U^\dagger \partial_\mu U) + \frac{1}{2} f^2 \Lambda_{\chi \mathrm{SB}} (\mathrm{Tr} M (U-1) + \mathrm{h.c.})$$

$$+ \ \mathrm{Tr}\overline{\Psi}(i\gamma^\mu \partial_\mu - M_B)\Psi$$

→ replaced by baryons' kinetic and mass terms in M-B Lagrangian

Octet Baryon Ψ

LO

$$+ \ \boxed{\mathrm{Tr}\overline{\Psi} i\gamma^\mu [V_\mu, \Psi]} + D\mathrm{Tr}\overline{\Psi}\gamma^\mu\gamma^5\{A_\mu, \Psi\} + F\mathrm{Tr}\overline{\Psi}\gamma^\mu\gamma^5[A_\mu, \Psi]$$

NLO

$$\boxed{+ \ a_1 \mathrm{Tr}\overline{\Psi}(\xi M^\dagger \xi + \mathrm{h.c.})\Psi + a_2 \mathrm{Tr}\overline{\Psi}\Psi(\xi M^\dagger \xi + \mathrm{h.c.}) + a_3(\mathrm{Tr} MU + \mathrm{h.c.})\mathrm{Tr}\overline{\Psi}\Psi}$$

$$\boxed{+ \ 2d_1 \mathrm{Tr}(A^\mu A_\mu)\mathrm{Tr}(\overline{\Psi}\Psi) + 4d_2 \mathrm{Tr}(\overline{\Psi} A^\mu A_\mu \Psi)},$$

+ Λ(1405) pole term

Nonlinear K⁻ field

$$\boxed{U = \exp(2i\Pi/f)} \qquad \Pi = \pi_a T_a = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & K^+ \\ 0 & 0 & 0 \\ K^- & 0 & 0 \end{pmatrix}$$

Kaon currents

$$V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \qquad A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \qquad \boxed{\xi \equiv U^{1/2}}$$

Classical Kaon field :

$$K^{\pm} = \frac{f}{\sqrt{2}}\theta \exp(\pm i \mu_K t)$$

θ : chiral angle

μ_{K} : kaon chemical potential

$f = \mathbf{93\ MeV}$: meson decay const.

Lagrangian density

Terms including a_1,a_2, a_3

$\mathrm{Tr}\bar{\Psi} i\gamma^{\mu}[V_{\mu}, \Psi]$

$$\mathcal{L}_{KB} = \frac{1}{2} f^2 \mu^2 \sin^2\theta - f^2 m_K^{*2}(1-\cos\theta) + 2X_0 \mu f^2 (1-\cos\theta)$$

S wave scalar int.

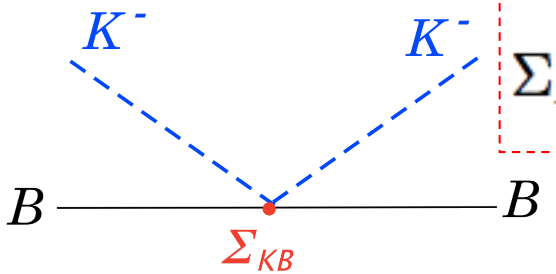
S wave vector int.

Effective kaon mass

$$m_K^{*2} \equiv m_K^2 - \frac{1}{f^2}\sum_i \rho_i^s \Sigma_{Ki}$$

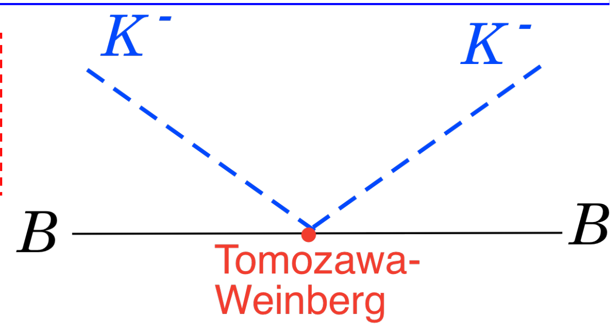
$$X_0 \equiv \frac{1}{2f^2}\left(\rho_p + \frac{1}{2}\rho_n - \frac{1}{2}\rho_{\Sigma^-} - \rho_{\Xi^-}\right)$$

$$\Sigma_{Kb} \equiv \frac{1}{2}(m_u + m_s)\langle b|(\bar{u}u + \bar{s}s)|b\rangle$$



Scalar int.

S wave KB interaction



vector int.

ambiguity of Σ_{KN}

$$\Sigma_{KN} = \frac{1}{2}(m_u + m_s)\langle N|\bar{u}u + \bar{s}s|N\rangle$$

$$\Sigma_{\pi N} = \hat{m}\langle N|(\bar{u}u + \bar{d}d)|N\rangle$$

with $\hat{m} \equiv (m_u + m_d)/2$

$$\Sigma_{KN} = \frac{m_u + m_s}{2\hat{m}}\left(\frac{\Sigma_{\pi N}}{1 + z_N} + \frac{\hat{m}}{m_s}\sigma_s\right)$$

$$\sigma_s \equiv m_s\langle N|\bar{s}s|N\rangle$$

$$z_N \equiv \langle N|\bar{d}d|N\rangle/\langle N|\bar{u}u|N\rangle$$

$\sigma_s \simeq 0$ (recent Lattice QCD results beyond chiral perturbation theory)

Chiral perturbation predicts

$$\sigma_3 \simeq \frac{1}{m_s/\hat{m} - 1}(M_\Xi - M_\Sigma) \simeq 5 \text{ MeV}$$

$$\begin{aligned}\sigma_0 &\equiv \hat{m}\langle N|(\bar{u}u + \bar{d}d - 2\bar{s}s)|N\rangle \\ &= \Sigma_{\pi N} - (2\hat{m}/m_s)\sigma_s \\ &\simeq \frac{3}{m_s/\hat{m} - 1}(M_\Xi - M_\Lambda) \simeq 25 \text{ MeV}\end{aligned}$$

Chiral perturbation →

$$\Sigma_{\pi N} = (40 - 60)\text{MeV}$$

Phenomenological analyses from πN scattering

Nonlinear effects on the $\bar{s}s$ condensates beyond chiral perturbation

[R. L. Jaffe and C. L. Korpa, Comm.Nucl.Part.Phys.17, 163 (1987).]

$\langle b|\bar{q}q|b\rangle = \partial M_b/\partial m_q$ (Feynman-Hellmann theorem)

$$
\begin{aligned}
M_p &= \bar{M}_B - 2(a_1 m_u + a_2 m_s) \ , \\
M_n &= \bar{M}_B - 2(a_1 m_d + a_2 m_s) \ , \\
M_\Lambda &= \bar{M}_B - 1/3 \cdot (a_1 + a_2)(m_u + m_d + 4m_s) \ , \\
M_{\Sigma^-} &= \bar{M}_B - 2(a_1 m_d + a_2 m_u) \ , \\
M_{\Xi^-} &= \bar{M}_B - 2(a_1 m_s + a_2 m_u)
\end{aligned}
$$

$$\bar{M}_B = M_B - 2a_3(m_u + m_d + m_s) + \underline{\Delta M(m_s)}$$

Contribution to baryon rest mass from nonlinear term with respect to m_s

$$
\begin{aligned}
\langle p|\bar{u}u|p\rangle &= \langle n|\bar{d}d|n\rangle = -2(a_1 + a_3) \ , \\
\langle p|\bar{d}d|p\rangle &= \langle n|\bar{u}u|n\rangle = -2a_3 \ , \\
\langle p|\bar{s}s|p\rangle &= \langle n|\bar{s}s|n\rangle = -2(a_2 + a_3) + \Delta
\end{aligned}
$$

nonlinear effect: $\Delta \equiv d\Delta M(m_s)/dm_s$

$$\langle\Lambda|\bar{u}u|\Lambda\rangle = \langle\Lambda|\bar{d}d|\Lambda\rangle = -\frac{1}{3}(a_1 + a_2) - 2a_3 \ ,$$

$$\boxed{\langle\Lambda|\bar{s}s|\Lambda\rangle = -\frac{4}{3}(a_1 + a_2) - 2a_3 + \Delta} \ ,$$

$$\langle\Sigma^-|\bar{u}u|\Sigma^-\rangle = -2(a_2 + a_3) \ , \langle\Sigma^-|\bar{d}d|\Sigma^-\rangle = -2(a_1 + a_3) \ ;$$

$$\boxed{\langle\Sigma^-|\bar{s}s|\Sigma^-\rangle = -2a_3 + \Delta} \ ,$$

$$\langle\Xi^-|\bar{u}u|\Xi^-\rangle = -2(a_2 + a_3) \ , \langle\Xi^-|\bar{d}d|\Xi^-\rangle = -2a_3 \ ,$$

$$\boxed{\langle\Xi^-|\bar{s}s|\Xi^-\rangle = -2(a_1 + a_3) + \Delta} \ .$$

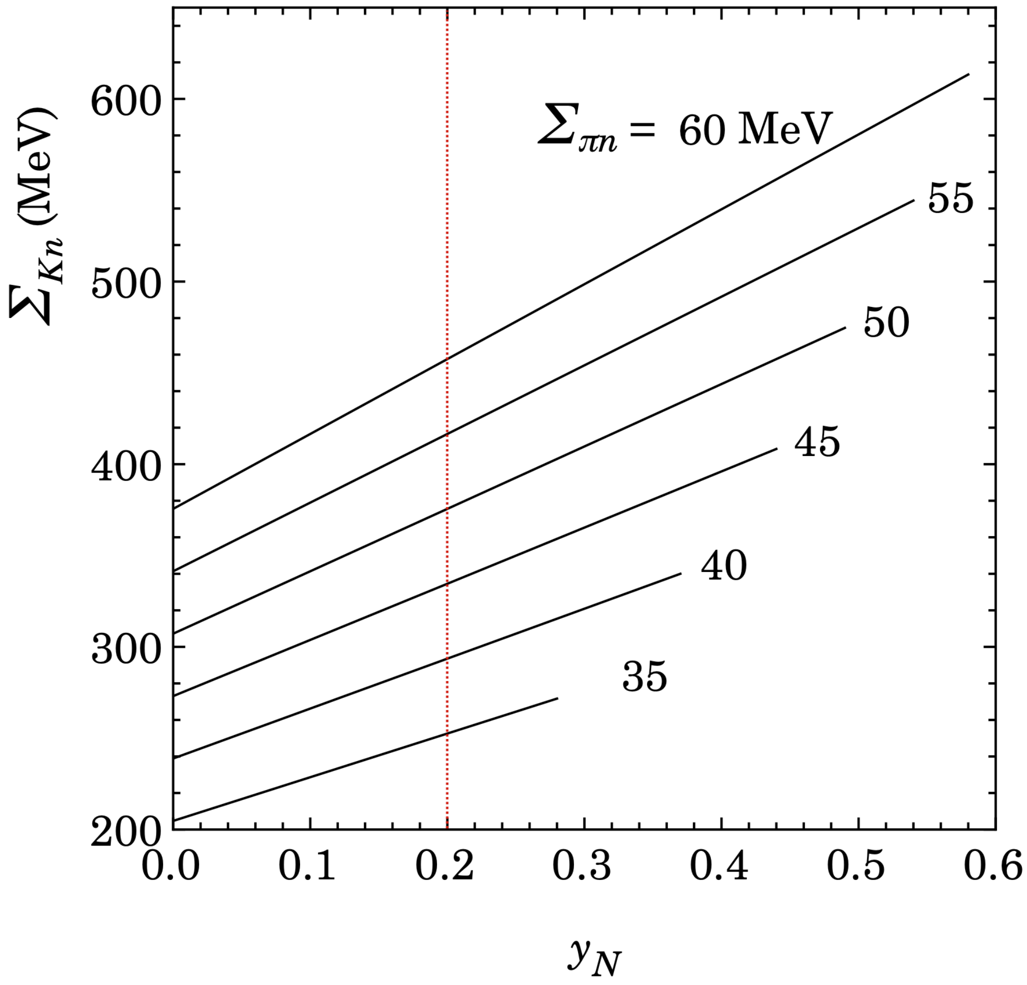
$$\Sigma_{Kn} = -(a_2 + 2\widetilde{a}_3)(m_u + m_s) = \Sigma_{K\Sigma^-} \ ,$$

$$\Sigma_{K\Lambda} = -\left(\frac{5}{6}a_1 + \frac{5}{6}a_2 + 2\widetilde{a}_3\right)(m_u + m_s) \ ,$$

$$\Sigma_{Kp} = -(a_1 + a_2 + 2\widetilde{a}_3)(m_u + m_s) = \Sigma_{K\Xi^-}$$

with $\widetilde{a}_3 \equiv a_3 - \Delta/4$

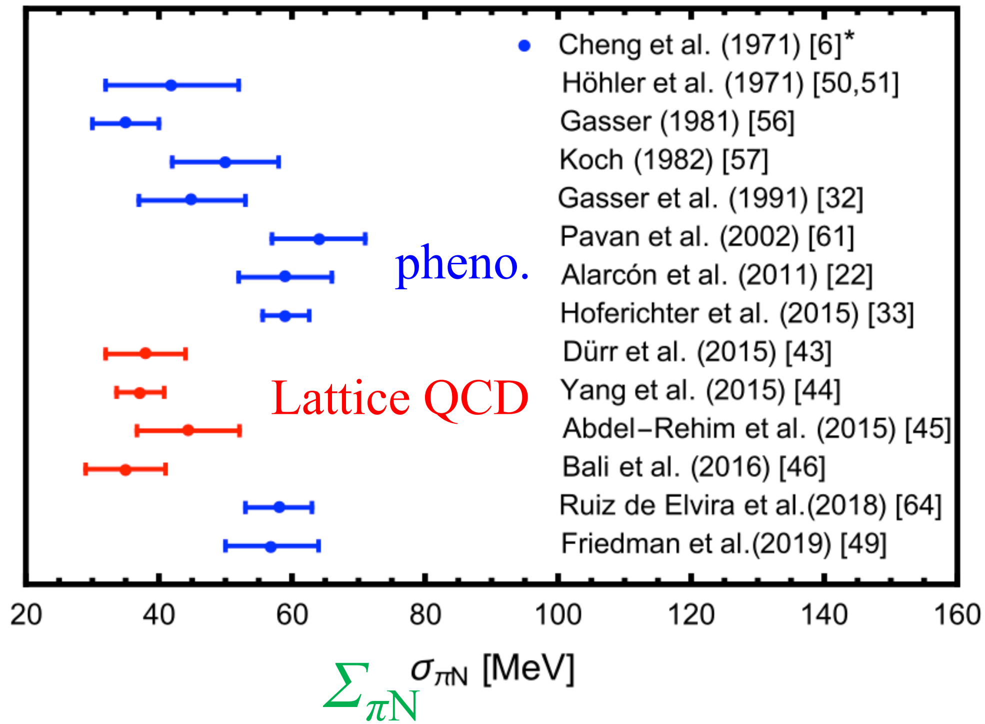
Allowable value of Σ_{KN}



$$y_N = \frac{2(a_2 + a_3) - \Delta}{a_1 + 2a_3} \qquad (N = p, n)$$

$\Sigma_{\pi N} = 40 - 60$ MeV

[J. M. Alarcon, Eur. Phys. J. Spec.Top. (2021) 230:1609-1622]



We use

$\Sigma_{Kn} = (300 - 400)$ MeV

3. Equation of state for the (Y+K) phase

3-1 Chiral Lagrangian for K-B dynamics + Minimal RMF (MRMF) + UTBR+TNA

Energy expressions for kaon condensation in Y-mixed matter

$$\mathcal{E} = \boxed{\frac{1}{2}(\mu_K f \sin\theta)^2 + f^2 m_K^2 (1-\cos\theta)} + \boxed{\frac{\mu_e^4}{4\pi^2} \quad + \text{ muons}}$$

Kaon condensates (KC)

free leptons

$$\boxed{\begin{aligned} &+ \sum_{b=p,n,\Lambda,\Sigma^-,\Xi^-} \frac{2}{(2\pi)^3}\int_{|\mathbf{p}|\le p_F(b)} d^3|\mathbf{p}|(|\mathbf{p}|^2 + \widetilde{M}_b^{*2})^{1/2} \\ &+ \frac{1}{2}\left(m_\sigma^2\sigma^2 + m_{\sigma^*}^2\sigma^{*2}\right) + \frac{1}{2}\left(m_\omega^2\omega_0^2 + m_\rho^2 R_0^2 + m_\phi^2\phi_0^2\right)\end{aligned}}$$

(effective baryon mass)

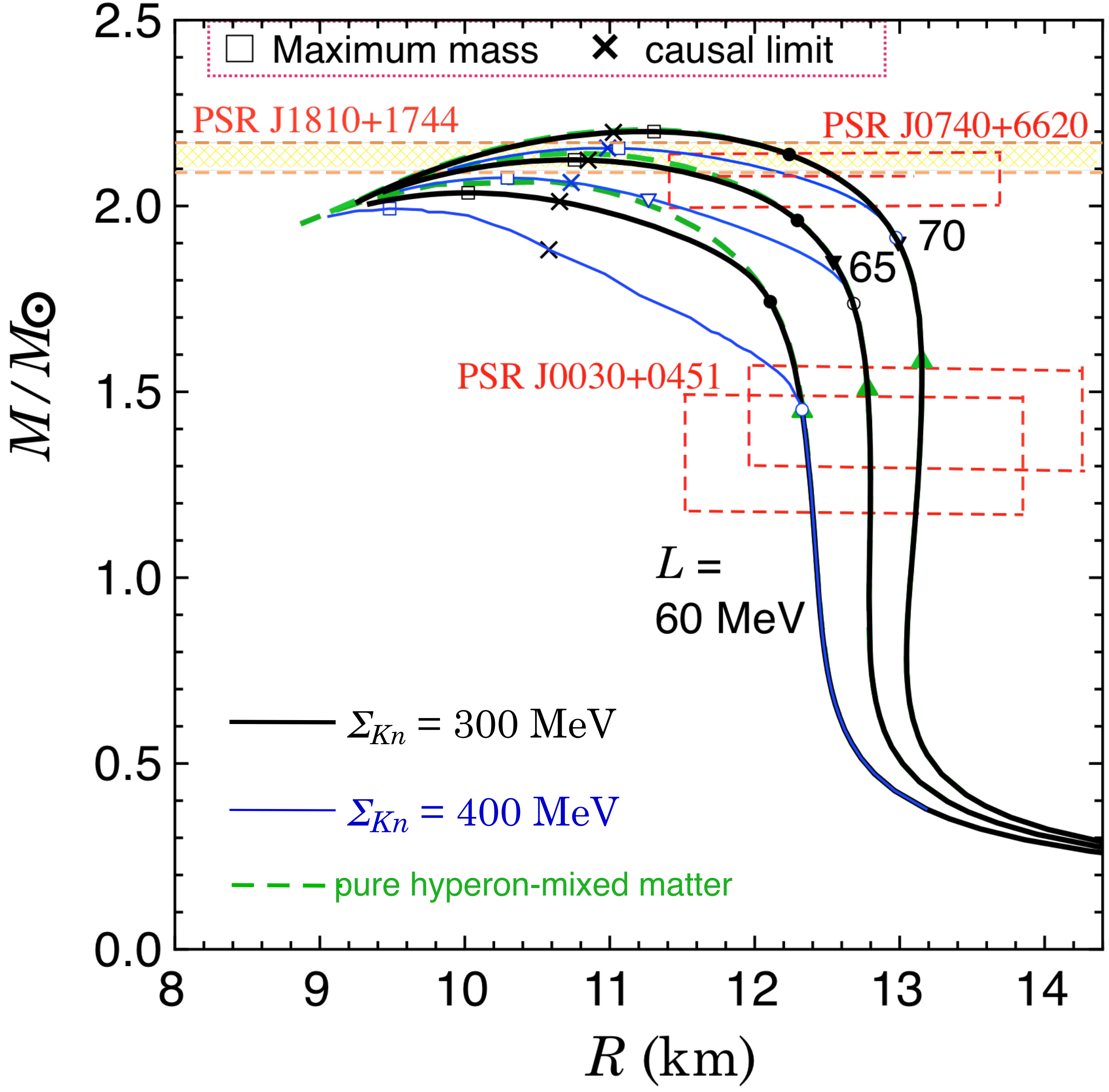
Two-body

UTBR

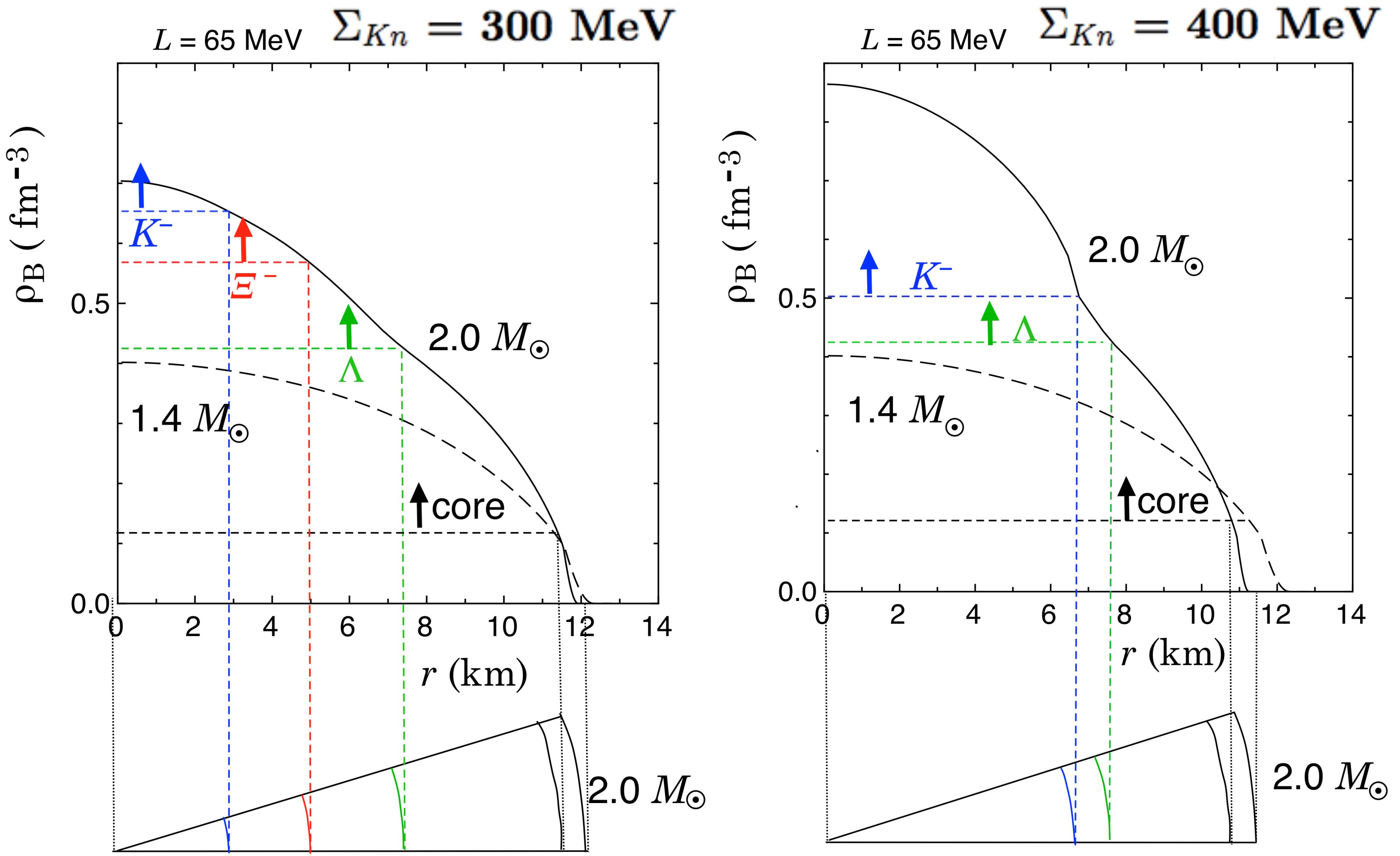
TNA

$$+ \boxed{\frac{\pi^{3/2}}{2} V_r C \rho_{\mathrm{B}}^3 \left(1 + c_r \frac{\rho_{\mathrm{B}}}{\rho_0}\right) + \gamma_a \rho_{\mathrm{B}}^3 e^{-\eta_a \rho_{\mathrm{B}}}\{3 - 2(1-2x_p)^2\}}$$

Gravitational Mass – radius R relations

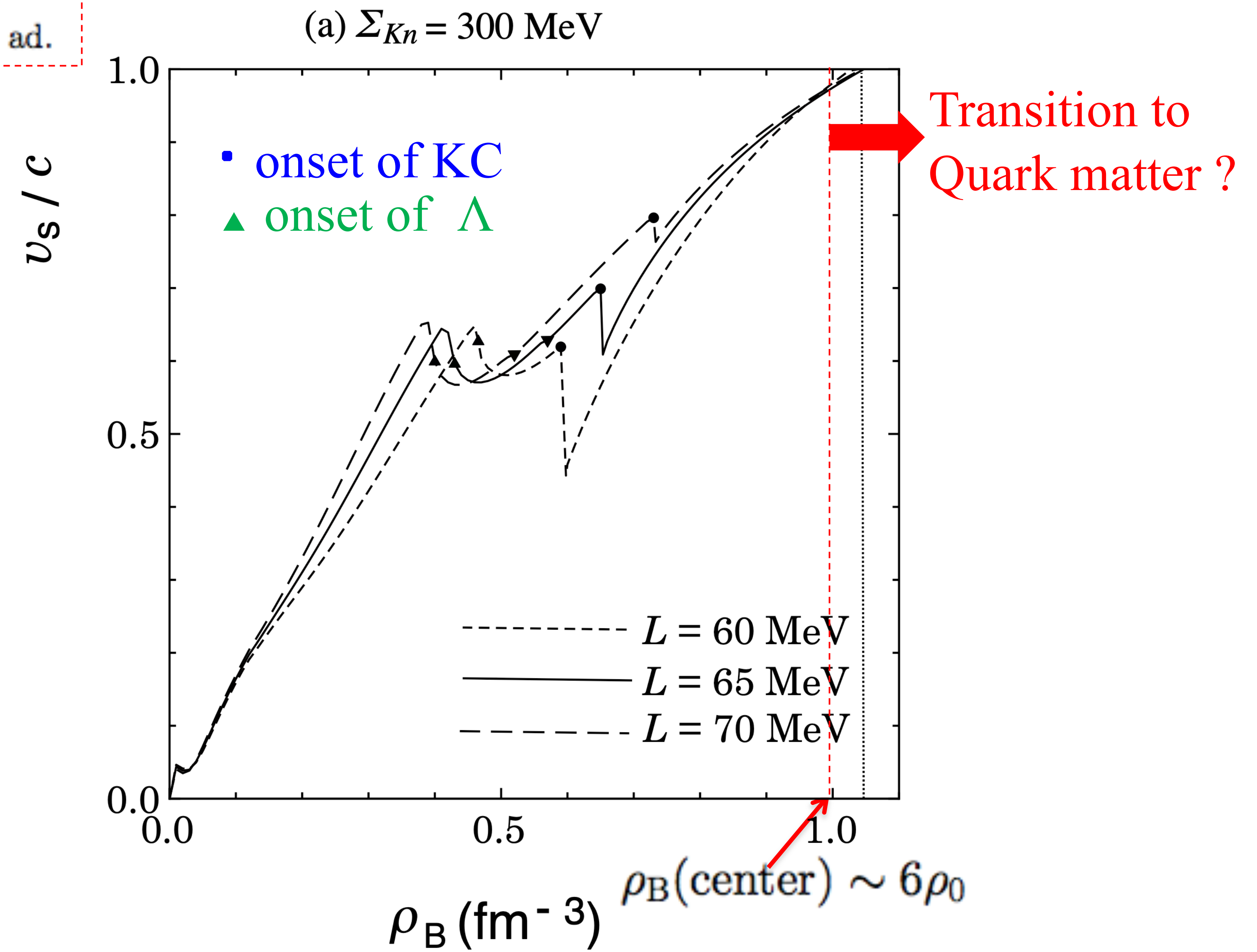


Density distributions --- L = 65 MeV ---

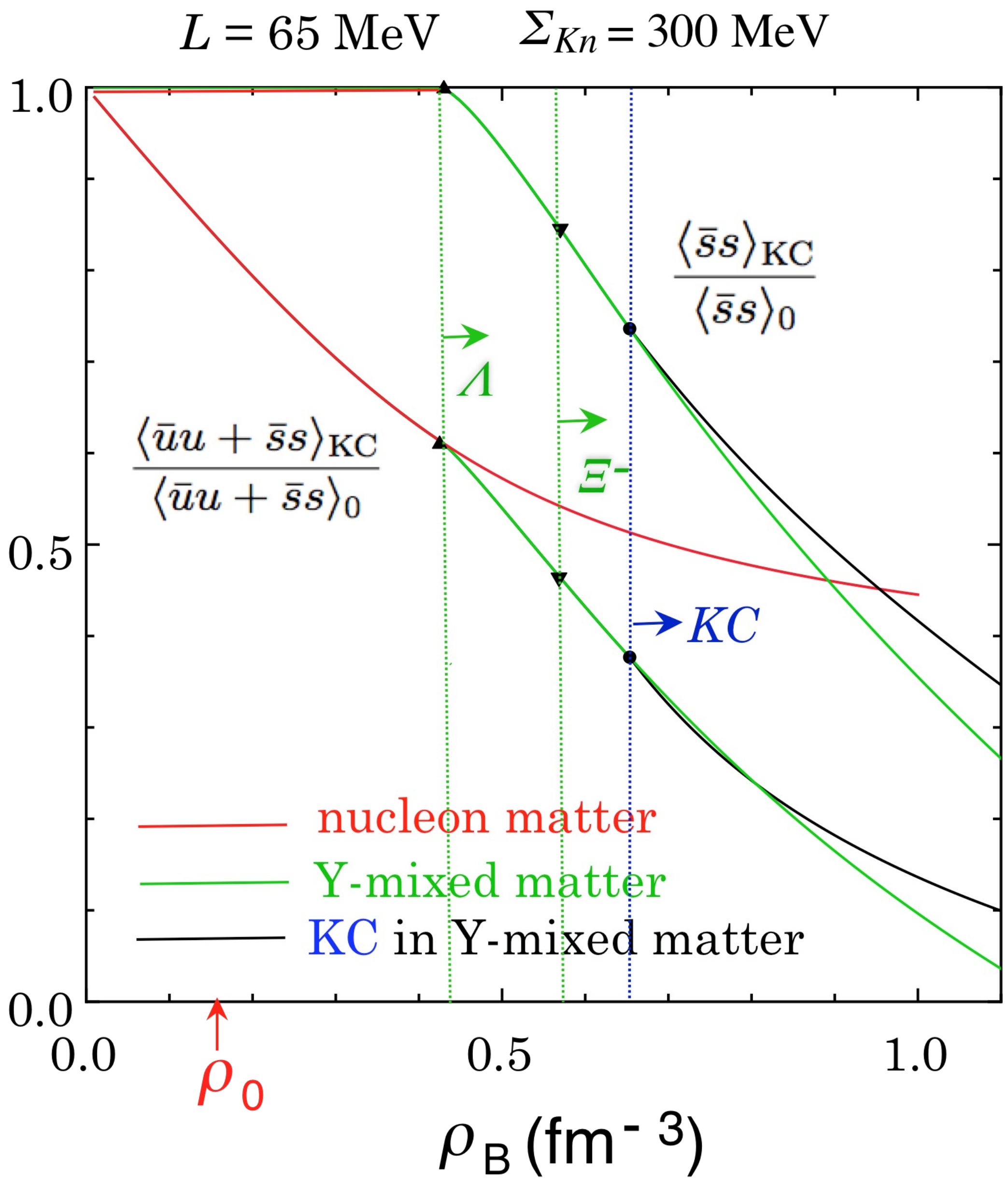


Sound velocity (v_s) – baryon density ρ_B

$$v_s = \left(\frac{dP}{d\mathcal{E}}\right)^{1/2}_{\mathrm{ad.}}$$



5.Quark condensates in the (Y+K) phase and Chiral restoration



Hellman-Feynman theorem

$$\langle \bar{q}q \rangle_{\rm KC} = d\langle {\rm KC}|\mathcal{H}_{QCD}|{\rm KC}\rangle / dm_q$$

$$\frac{\langle \bar{u}u + \bar{s}s \rangle_{\rm KC}}{\langle \bar{u}u + \bar{s}s \rangle_0} = \left(\frac{m_K^{*2}}{m_K^2}\right) \cos\theta$$

$$m_K^{*2} = m_K^2 - \sum_b \frac{\rho_b^S}{f^2}\Sigma_{Kb}$$

Y-mixing and KC assist restoration of chiral symmetry

6 Outlook

Possible Hadron–Quark crossover and Role of Meson Condensation

hadron matter ⟶ quark matter

H-Q crossover

[K.Masuda, T. Hatsuda, T.Takatsuka,
Astrophys. J. 764,12 (2013); PTEP 2016, 021D01(2016).]

[G. Baym, T. Hatsuda, T. Kojo, P.D.Powell, Y.Song, T.Takatsuka,
Rept. Prog. Phys. 81, 056902 (2018).]

Meson condensation

Role of MC as a pathway ?

in the context of chiral restoration in dense matter