Impact of hot and cold dense matter on quasinormal oscillation modes in compact stars

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6. Summary

Unknowns in strong force physics...

- Neutron stars (NSs) can provide vital information about:
  - Equation of state (EoS) of dense bulk matter
  - Properties of neutron-rich nuclei above neutron drip density
- Hyperon, Δ-puzzles[1] due to observation of massive NSs ($\geq 2M_\odot$)
- Gravitational wave (GW) observations from binary NS merging demand dense matter to be softer at intermediate densities
- GWs associated with f-modes are most favoured to be observed first, by next generation telescopes

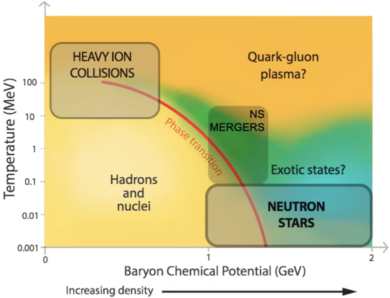
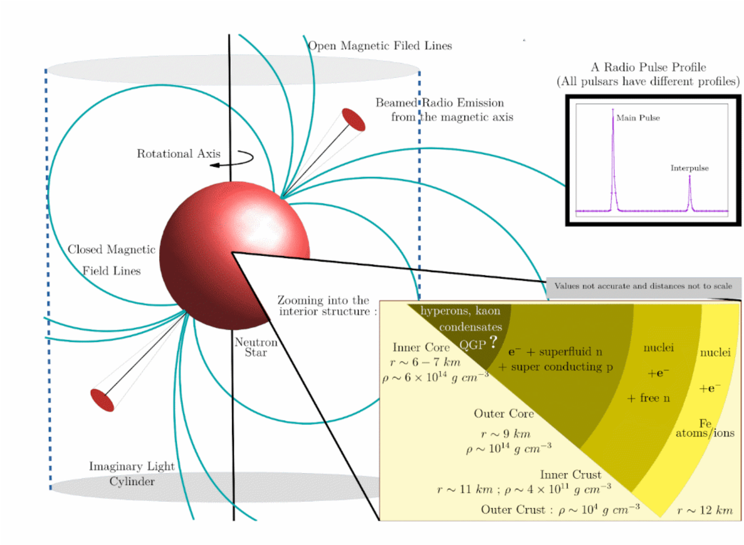


Figure: Exotic matter phase diagram
Image Courtesy: Anna Watts

[1] I. Bombaci, in *Proc. of the 12th Int. Conf. on Hyp. Nucl. and Strange Part. Phys.* (HYP2015, Jan. 2017) p.101 002. arXiv: 1601.05339 [nucl-th]

Anatomy of a neutron star



Neutron star mergers as astrophysical colliders

- $Q_{ij} = -\lambda \mathcal{E}_{ij} \Rightarrow \lambda = \frac{2}{3} k_2 R^5$
- Sources of GWs in isolated NSs:
  - Mountains on the surface
  - Strong magnetic fields
- Non-axisymmetric oscillations:
  - f - modes: Fundamental ($\sim$ kHz)
  - p - modes: Pressure ($\sim$ 4–7 kHz)
  - g - modes: Buoyancy ($\lesssim$ 100 Hz)
  - r - modes: Coriolis force
  - w - modes: space-time

GW170817:$\tilde{\Lambda} \sim 720$, GW190425:$\tilde{\Lambda} \sim 600$

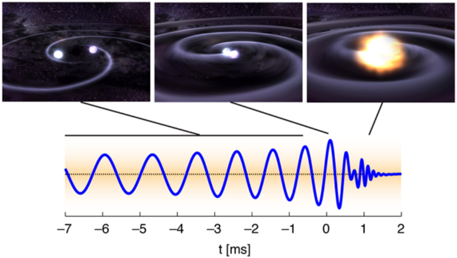


Figure: Gravitational wave signals from a NS merger in the time domain
Image Courtesy: LIGO

In presence of external or internal disturbances, compact objects may be perturbed, producing quasi-normal oscillation modes (QNMs)

Motivation & Objectives

- **Motivation:**
  - Due to the extreme sensitivity to the composition interior to these compact objects, the QNMs can provide great insight in constraining the EoS of dense matter
  - GWs associated with $f-$modes are most favoured to be observed first, by next generation telescopes such as the Cosmic Explorer, Einstein Telescope and the Advanced LIGO detector
  - Proto-neutron stars (PNSs) are also potential sources of GWs where temperatures may reach up to $\sim$50 MeV, and due to high densities one may expect the nucleation of heavier exotic particles in NSs. EoSs must reflect these conditions
- **Objectives:**
  - To investigate correlations between the frequencies of $f-$ and $p-$oscillation modes of cold NSs with the properties of nuclear matter and NSs
  - To investigate the effects of finite temperature and exotic particle degrees of freedom on the $f-$ and $p-$modes

Model for EoS

- We consider the Density-Dependent Relativistic Mean Field (DDRMF) model with matter composition as the full baryon octet b, the quartet of Δ-resonances d, (anti)kaons and leptons ($l = e^-, \mu^-$). Strong interactions between baryons are mediated by σ, ω, ρ, σ^*, ϕ-mesons
- The baryonic matter part of the Lagrangian density[1],

$$
\begin{aligned}
\mathcal{L}_B = & \sum_b \bar{\psi}_b (i\gamma_\mu D^\mu - m_b + g_{\sigma b}\sigma + g_{\sigma^* b}\sigma^* - g_{\omega b}\gamma_\mu\omega^\mu - g_{\phi b}\gamma_\mu\phi^\mu \\
& - g_{\rho b}\gamma_\mu \boldsymbol{\tau}_b \cdot \boldsymbol{\rho}^\mu)\psi_b + \sum_d (\psi_b \longrightarrow \psi_d^\nu) + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) \\
& + \frac{1}{2}(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\
& - \frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2}m_\phi^2\phi_\mu\phi^\mu - \frac{1}{4}\boldsymbol{\rho}_{\mu\nu}\cdot\boldsymbol{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\cdot\boldsymbol{\rho}^\mu \\
& - \left(\frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4\right)
\end{aligned}
\tag{1}
$$

[1] L. Tolos, M. Centelles, and A. Ramos, *Publications of the Astronomical Society of Australia*, vol. 34, e065, e065, Dec. 2017. DOI: 10.1017/pasa.2017.60. arXiv: 1708.08681 [astro-ph.HE].

Equation of State

- The matter pressure can be evaluated from the thermodynamic (Gibbs-Duhem) relation as

$$p_m = \sum_b \mu_b n_b + \sum_d \mu_d n_d + \sum_l \mu_l n_l - \varepsilon_m, \tag{2}$$

- Self-energy re-arrangement term is introduced in the chemical potential expression to maintain thermodynamic consistency[1,2]

$$\begin{aligned} \Sigma^r = \sum_b \Bigg[& \frac{\partial g_{\omega b}}{\partial n}\omega_0 n_b - \frac{\partial g_{\sigma b}}{\partial n}\sigma n_b^s + \frac{\partial g_{\rho b}}{\partial n}\rho_{03}\boldsymbol{\tau}_{b3} n_b - \frac{\partial g_{\sigma^* b}}{\partial n}\sigma^* n_b^s \\ & + \frac{\partial g_{\phi b}}{\partial n}\phi_0 n_b \Bigg] + \sum_d \left[\frac{\partial g_{\omega d}}{\partial n}\omega_0 n_d - \frac{\partial g_{\sigma d}}{\partial n}\sigma n_d^s + \frac{\partial g_{\rho d}}{\partial n}\rho_{03}\boldsymbol{\tau}_{d3} n_d \right] \end{aligned} \tag{3}$$

[1] C. Fuchs, H. Lenske, and H. H. Wolter, *Phys. Rev. C*, vol. 52, no. 6, pp. 3043–3060, Dec. 1995. DOI: 10.1103/PhysRevC.52.3043. arXiv: nucl-th/9507044 [nucl-th].

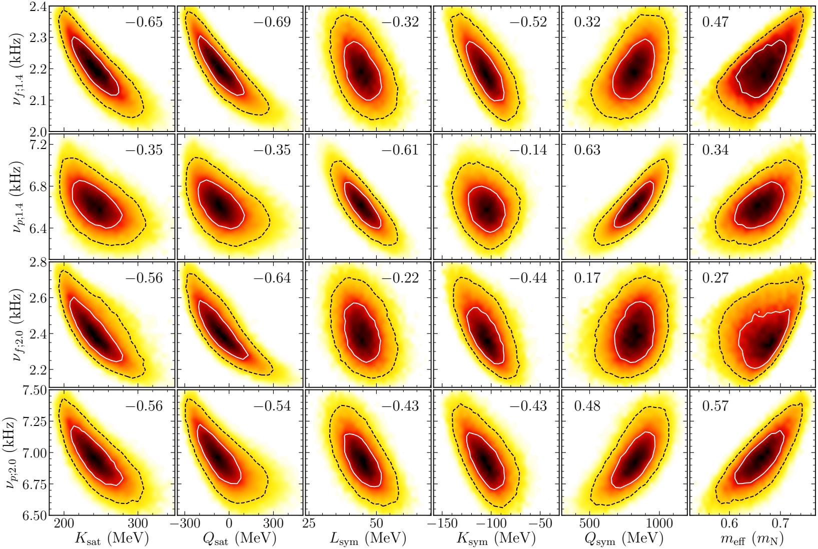
[2] H. Lenske and C. Fuchs, *Physics Letters B*, vol. 345, no. 4, pp. 355–360, Feb. 1995. DOI: 10.1016/0370-2693(94)01664-X.

Equation of state models

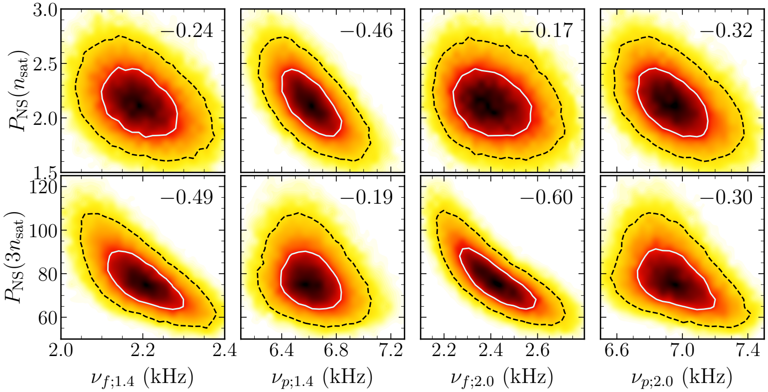
TABLE I. List of EOS models used in this work. For each model we provide information on: considered degrees of freedom; maximum mass of cold β-equilibrated NS ($M_G^{\max}$); radius of canonical 1.4 $M_\odot$ NS ($R_{1.4}$); radius of a 2.072 $M_\odot$ NS ($R_{2.072}$); limits of combined tidal deformability $\tilde{\Lambda} = 16\left[(M_1+12M_2)\,M_1^4\Lambda_1 + (M_2+12M_1)\,M_2^4\Lambda_2\right]/13\,(M_1+M_2)^5$ corresponding to the GW170817 event with an estimated total mass $M_T = 2.73^{+0.04}_{-0.01}$ $M_\odot$ and a mass ratio range $0.73 \le q = M_2/M_1 \le 1$. As in Ref. [21, 22], values outside the ranges 11.80 km $\le R_{1.4} \le$ 13.10 km [23]; 11.41 km $\le R_{2.072} \le$ 13.69 km [24]; $110 \le \tilde{\Lambda} \le 800$ [25] are marked in bold. n.a. (not available) means that quantities could not be calculated or the calculation is not meaningful (data extracted from CompOSE fall in the first category while those corresponding to DDB* in the second); "—" means that the quantities do not exist. Other notations are: q stands for quarks; Λ denotes the Λ-hyperon; Δ is the $\Delta(1232)$ resonance; Y generically denotes the Λ, $\Sigma^{-,0,+}$ and $\Xi^{-,0}$ hyperons; K respectively stands for kaons. For DDB* median values and 68% confidence intervals are provided.

Model	d.o.f	$M_G^{\max}$ ($M_\odot$)	R_{14} (km)	$R_{2.072}$ (km)	$\tilde{\Lambda}$ q=0.73	$\tilde{\Lambda}$ q=1
DDB*	N	$2.15^{+0.14}_{-0.1}$	$12.6^{+0.34}_{-0.36}$	n.a.	607^{+118}_{-105}	587^{+112}_{-100}
HS(DD2)	N	2.42	**13.2**	13.1	799	758
BHB(DD2$\Lambda\phi$)	N, Λ	2.10	**13.2**	12.2	790	757
OMHN(DD2Y)	N, Y	2.03	**13.2**	—	787	756
R(DD2YDelta)(1.2;1.1;1.0)	N, Y, Δ	2.05	12.3	—	470	434
MBB(DD2K)	N, K^-	2.19	**13.2**	13.0	n.a.	n.a.
BBKF(DD2F-SF)1.2	N, q	2.15	12.2	11.4	501	473
BBKF(DD2-SF)1.8	N, q	2.06	**11.0**	—	218	180

Correlations between QNMs of cold NSs, parameters of nuclear matter and NS EoS



Correlations among QNMs in 1.4 $M_\odot$ and 2 $M_\odot$ NSs and the pressure of NS matter at $n_{\rm sat}$ ($3n_{\rm sat}$) for NS EoS models



Correlations between ν_f & ν_p and NS properties

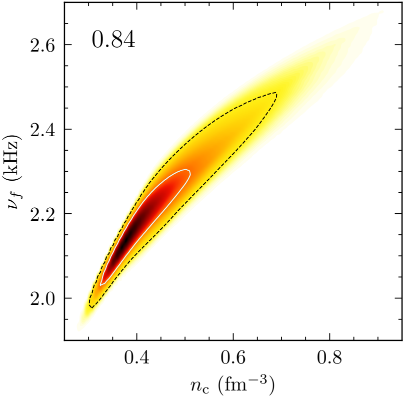


Figure: Correlations between $f-$mode frequencies and central densities

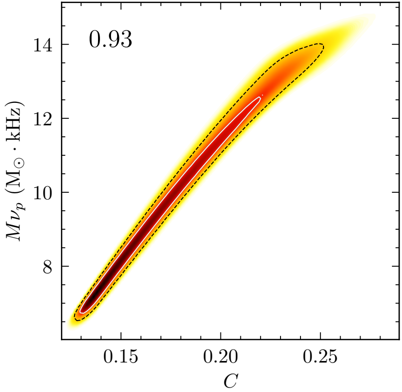


Figure: Scaling of $M\nu_p$ as a function of compactness

Results corresponding to the DDB$*$ family of models and the mass range $1 \leq M/M_\odot \leq 2$. The light cyan solid and black dashed contours demonstrate 50% and 90% confidence regions, respectively. The numbers in each panel represent Kendall rank correlation coefficients.

Thermal EoSs

- HS(DD2) $\longrightarrow$ Nucleonic
- BHB(DD2Lphi) $\longrightarrow$ $\Lambda\Lambda$ interactions mediated by ϕ meson
- OMHN(DD2Y) $\longrightarrow$ YY interactions mediated by the scalar σ^* and hidden vector ϕ mesons
- R(DD2YD)(1.2,1.1) $\longrightarrow$ $\Delta(1232)$ resonances
- MBB(DD2K) $\longrightarrow$ Antikaon condensates ($U_{\bar{K}}^{(N)} = -120$ MeV)
- BBKF(DD2F-SF) $\longrightarrow$ Hadron-quark phase transition

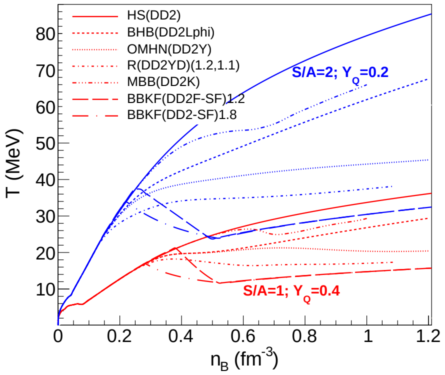


Figure: Temperature as a function of baryonic number density

EoSs extracted from publicly available online CompOSE database

NS structure solutions

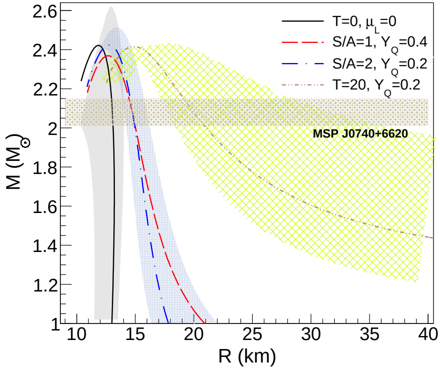


Figure: Mass-radius relations for non-rotating spherically-symmetric stars (nucleonic)

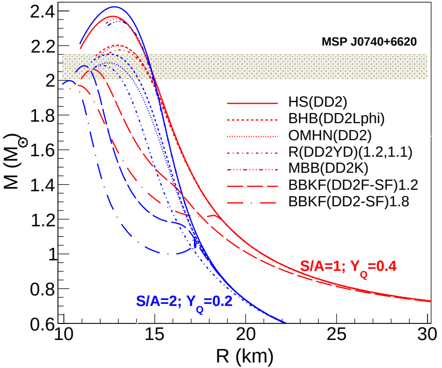


Figure: Similar to left panel, but for exotic dense matter

The adhoc Γ−law is as follows: $P_{\rm th} = \varepsilon_{\rm th}(\Gamma_{\rm th} - 1)$ with '$\Gamma_{\rm th}$' in the range [1.5 - 2.0]

Thermal and composition effects on oscillation modes

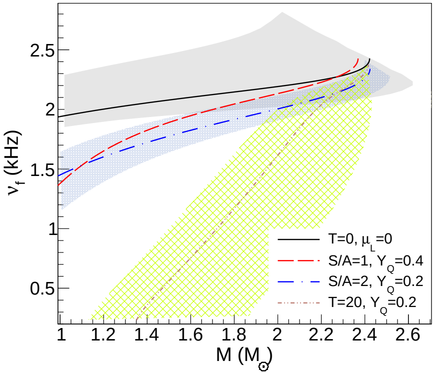


Figure: $f-$ mode frequencies as a function of gravitational mass

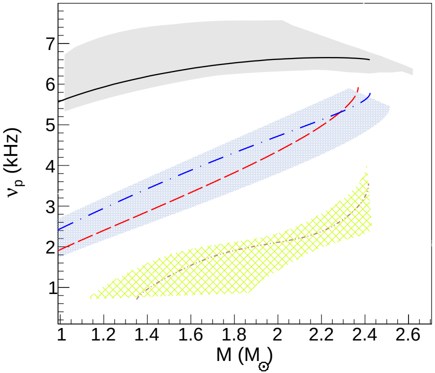


Figure: Similar to left panel, but for $p-$ mode frequencies

Considered nucleonic models: HS(DD2) (curves) and DDB$*$ (gray shaded regions)

Thermal and composition effects on oscillation modes (continued)

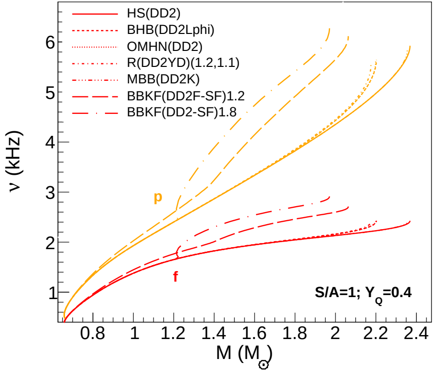


Figure: $f-$ and $p-$ mode frequencies as a function of M for ($S/A = 1$, $Y_Q = 0.4$)

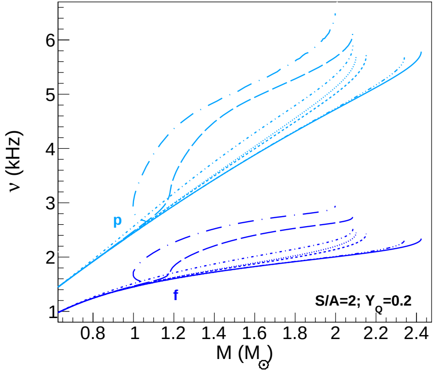


Figure: Similar to left panel, but for $S/A = 2$, $Y_Q = 0.2$

Summary & Future prospects

- It is found that the oscillation mode frequencies mostly correlates negatively with the nuclear matter paramaters. These can be explained considering the scaling of ν_f and ν_p with NS average density and compactness
- A correlation exists between ν_f and n_c suggesting that the $f-$mode probes the inner core of NSs
- Thermal effects result in a strong reduction of oscillation frequencies in nucleonic stars, they are more important in low mass stars and influence more the $p-$modes that the $f-$modes
- $f-$ and $p-$modes have different sensitivities to different density domains
- Hot stars with exotic degrees of freedom have larger values of ν_f and ν_p than their nucleonic counterparts. This may be attributed to higher compactness of exotica containing NSs

As a future objective, we plan to study the impact of other exotic particle spectrum with context to QNMs.

For detailed analysis, please refer to:

- Thapa, V. B., Beznogov, M., Raduta, A. R., & Thakur, P., *Physical Review D*, **107**, 103054 (2023). (***arXiv:*** 2302.11469)
- Beznogov, M. & Raduta, A. R., *Physical Review C*, **107**, 045803 (2023). (***arXiv:*** 2212.07168)

Thank You!