Parity violation of the weak interaction and supernovae

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Compact Stars in the QCD Phase diagram

Parity violation

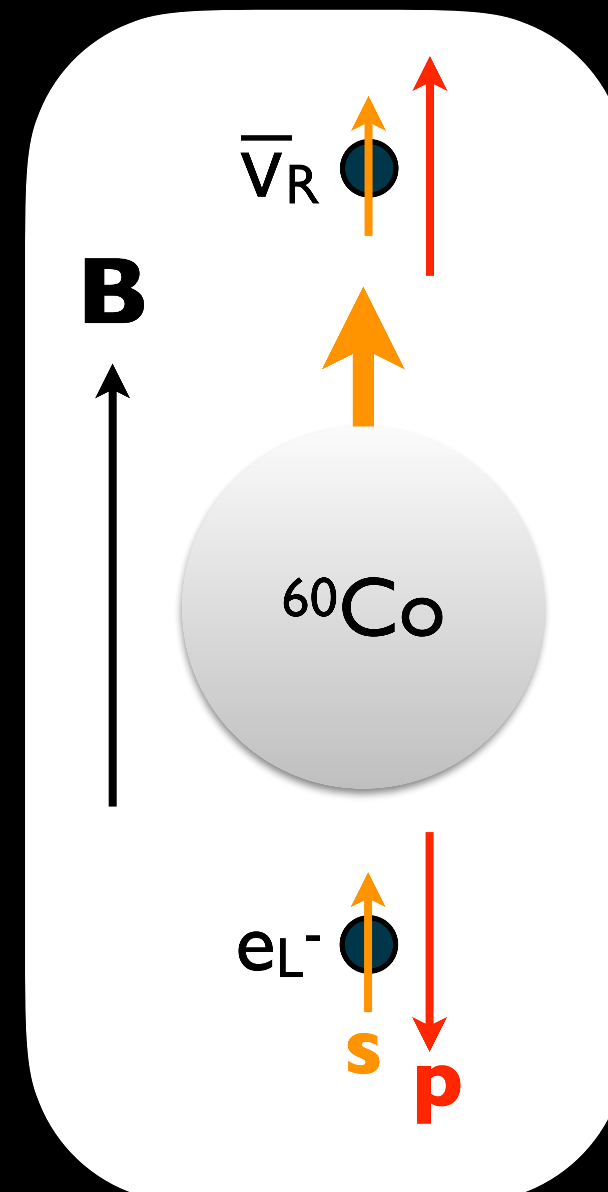
https://www.bnl.gov/newsroom/news.php?a=222034

Lee & Yang theory (1956)

 (1957)

https://en.wikipedia.org/wiki/Wu_experiment#/media/File:Chien-shiung_Wu_(1912-1997)_C.jpg

Wu experiment (1957)

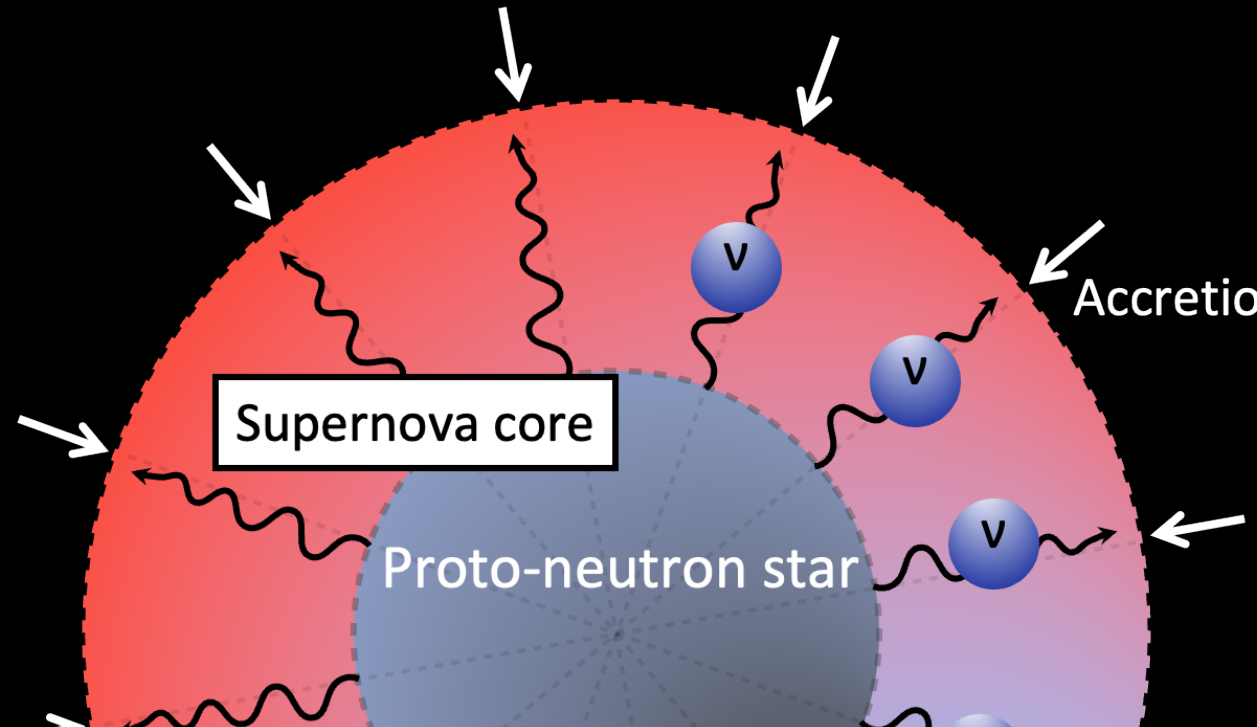


Weak interaction maximally (100%) violates parity

$\mathbf{p} \propto \mathbf{B}$

Main message

- Key to explosion and neutron star formation: neutrino transport
- Microscopic process: weak interaction
- Conventional neutrino transport theory ignores parity violation
- This qualitatively modifies the dynamical evolution of supernovae



$$\mathrm{n} + \nu_{\mathrm{e}}^{\mathrm{L}} \leftrightarrow \mathrm{p} + \mathrm{e}^{\mathrm{L}}$$

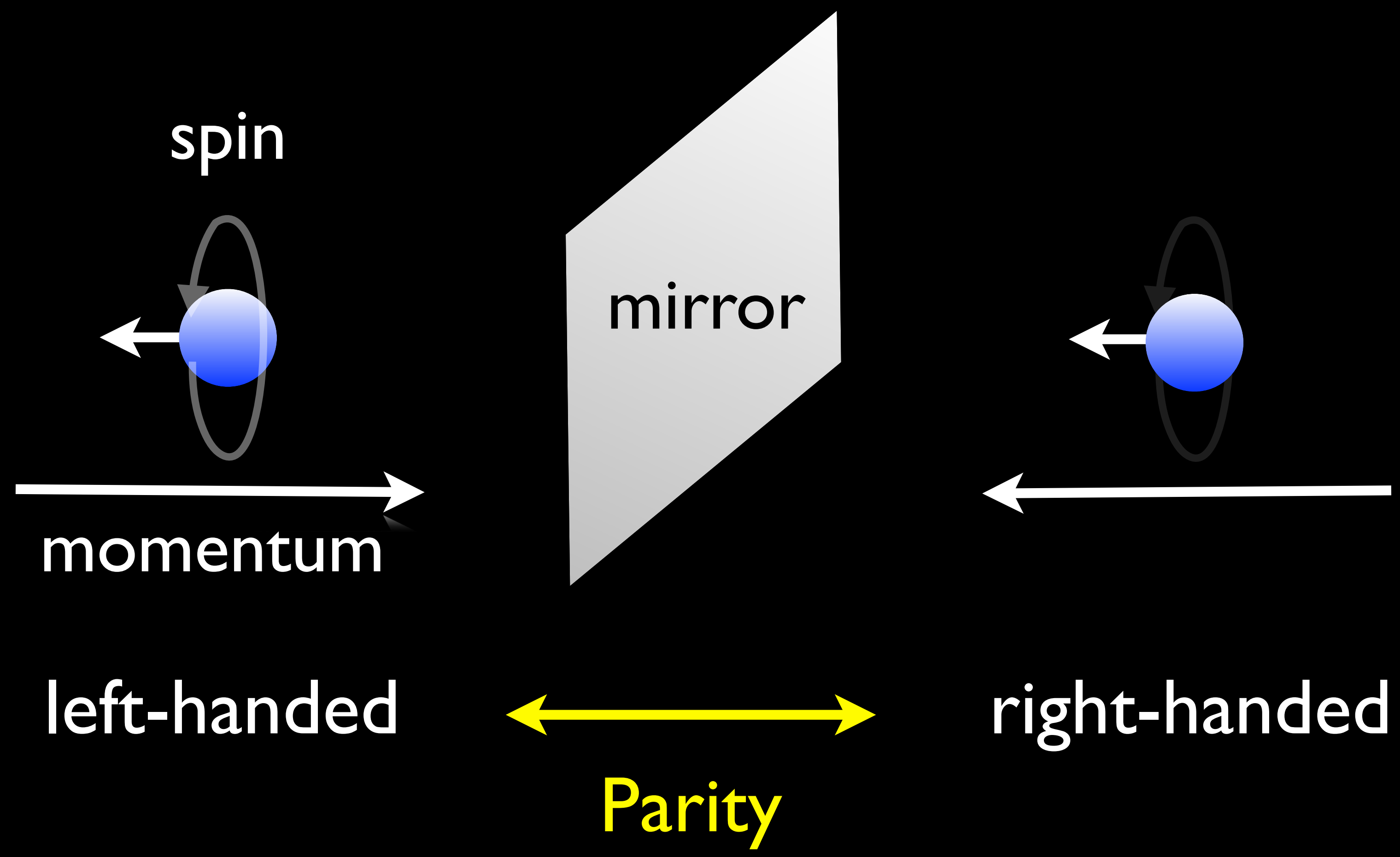
Contents

- Chiral effects & chiral matter
- Core-collapse supernovae and neutron stars
  - Chiral radiation transport theory
  - Chiral MHD simulations
  - Relevance to magnetars, explosion, pulsar kicks

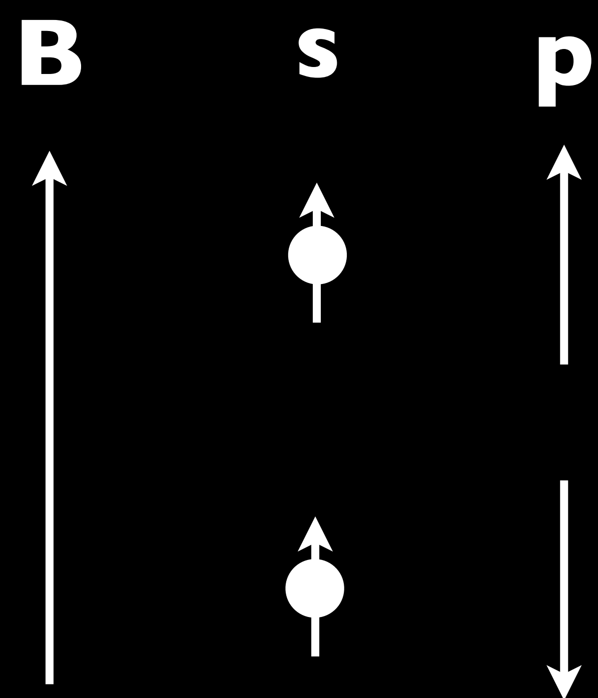
Units: $\hbar = c = k_{\rm B} = e = 1$

Kohei Kamada, NY, and Di-Lun Yang, "*Chiral Effects in Astrophysics and Cosmology,*"
Prog. Part. Nucl. Phys. (2023) 2207.09184 [astro-ph.CO]

Chirality/Helicity



Chiral magnetic effect



right-handed $\quad j_{\mathrm{R}} = \frac{\mu_{\mathrm{R}}}{4\pi^2} \boldsymbol{B}$

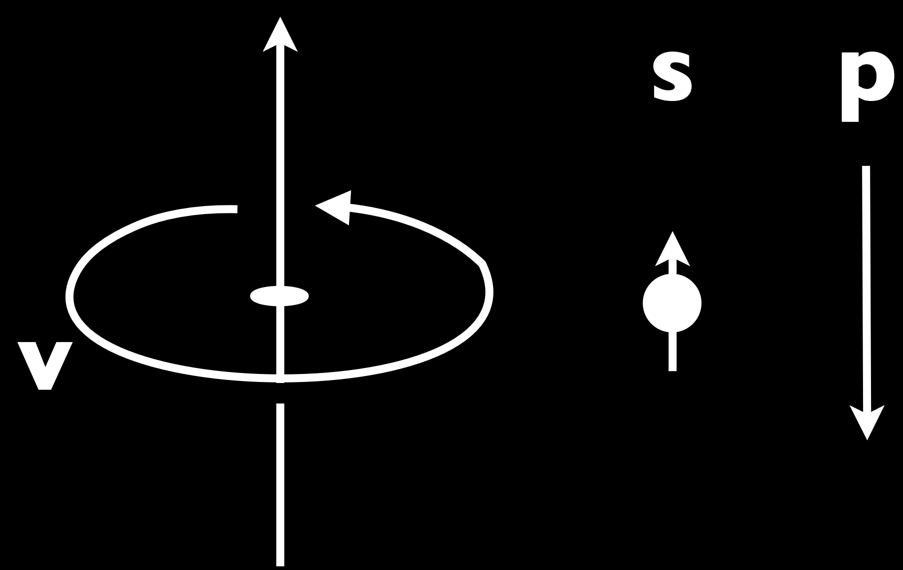
left-handed $\quad j_{\mathrm{L}} = -\frac{\mu_{\mathrm{L}}}{4\pi^2} \boldsymbol{B}$

$$\boldsymbol{j} = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{4\pi^2} \boldsymbol{B} \equiv \frac{\mu_5}{2\pi^2} \boldsymbol{B}$$

Vilenkin (1980); Nielsen, Ninomiya (1983); Fukushima, Kharzeev, Warringa (2008), …

Chiral vortical effect

$$\boldsymbol{\omega} \equiv \frac{1}{2}\boldsymbol{\nabla} \times \boldsymbol{v}$$



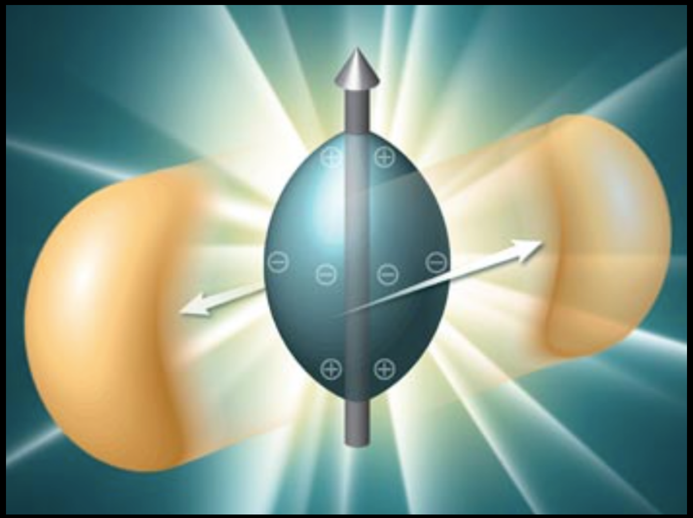
left-handed neutrino

$$\boldsymbol{j} = -\left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\boldsymbol{\omega}$$

Vilenkin (1979); Erdmenger et al. (2009); Banerjee et al. (2011);
Son, Surowka (2009); Landsteiner et al. (2011)

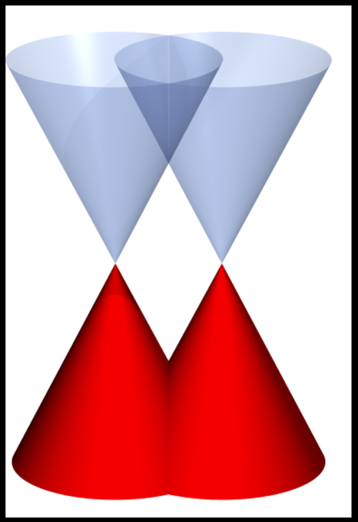
Chiral matter

- Electroweak plasma in early Universe — Joyce, Shaposhnikov (1997), …
- Quark-gluon plasma in heavy ion collision — Kharzeev, Mclerran, Warringa (2008), …
- Weyl semimetal — Nielsen, Ninomiya (1983), …
- Neutrino matter in supernovae — Yamamoto (2016), …



Quark-Gluon Plasma

http://www0.bnl.gov/rhic/news2/



Weyl semimetal

Supernovae

Core-collapse supernovae

Core-collapse supernova

- Explosion of giant stars at the end & transition to neutron stars
- Neutrinos carry away most of the gravitational energy
- Mechanism of the explosion and subsequent evolutions are unclear

Longstanding problem in astrophysics

Magnetars

- Neutrons stars with strong magnetic fields
- Surface magnetic field ~10^{15} G ("the strongest magnet")
- Origin of such a strong and stable magnetic field?

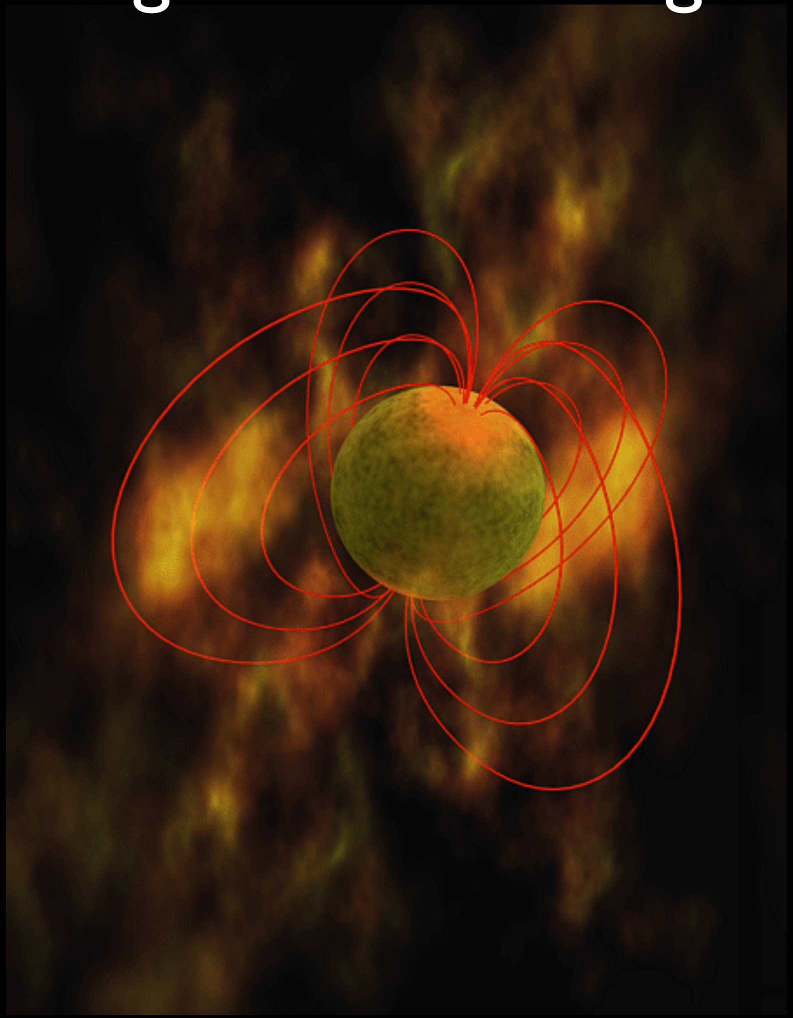
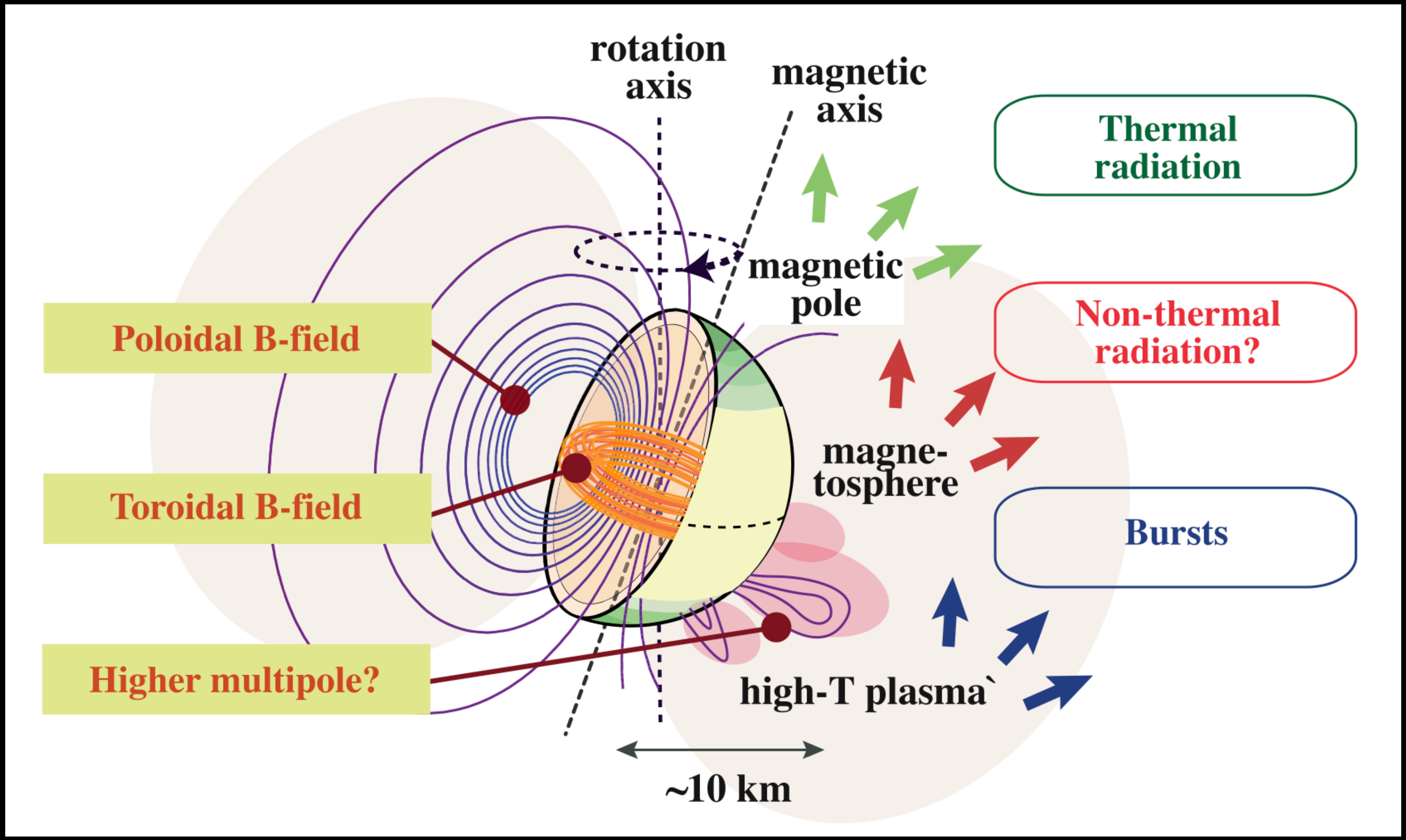


Illustration from Wikipedia

Poloidal and toroidal fields

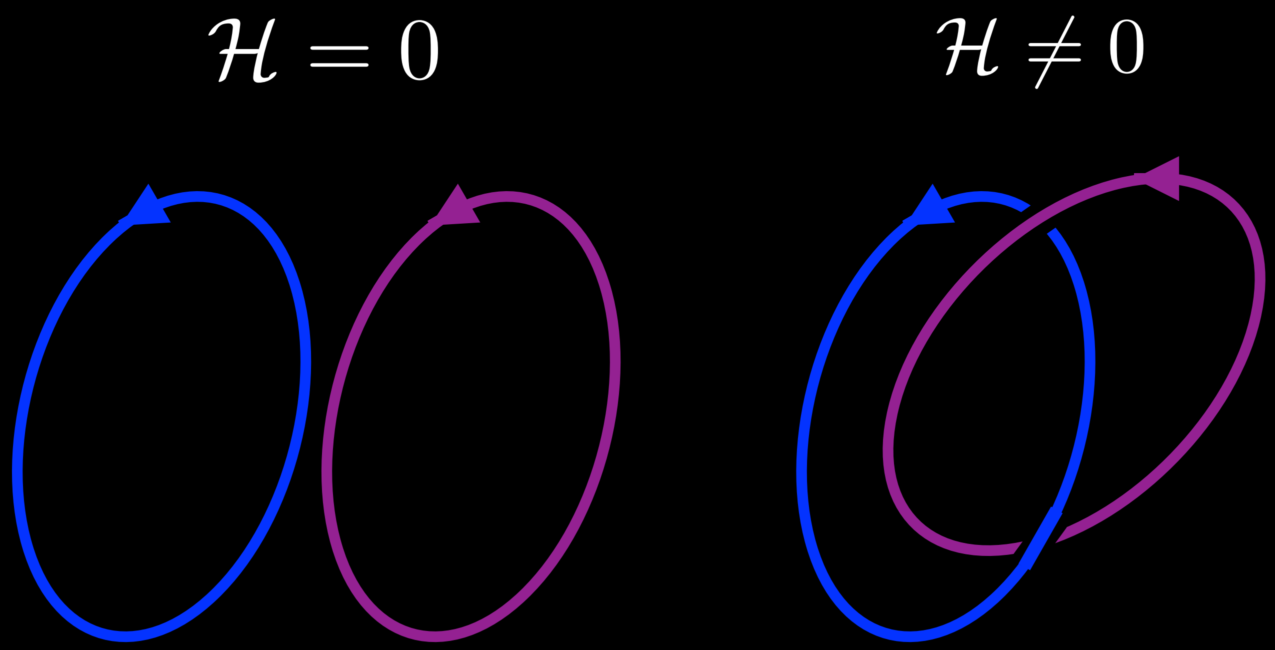
- Purely poloidal or toroidal magnetic fields are unstable.
- Both components are necessary for the stability (linked structure)
- Possible evidence for the toroidal field e.g., Makishima, Enoto, et al. (2014)



Enoto, Kisaka, Shibata (2019)

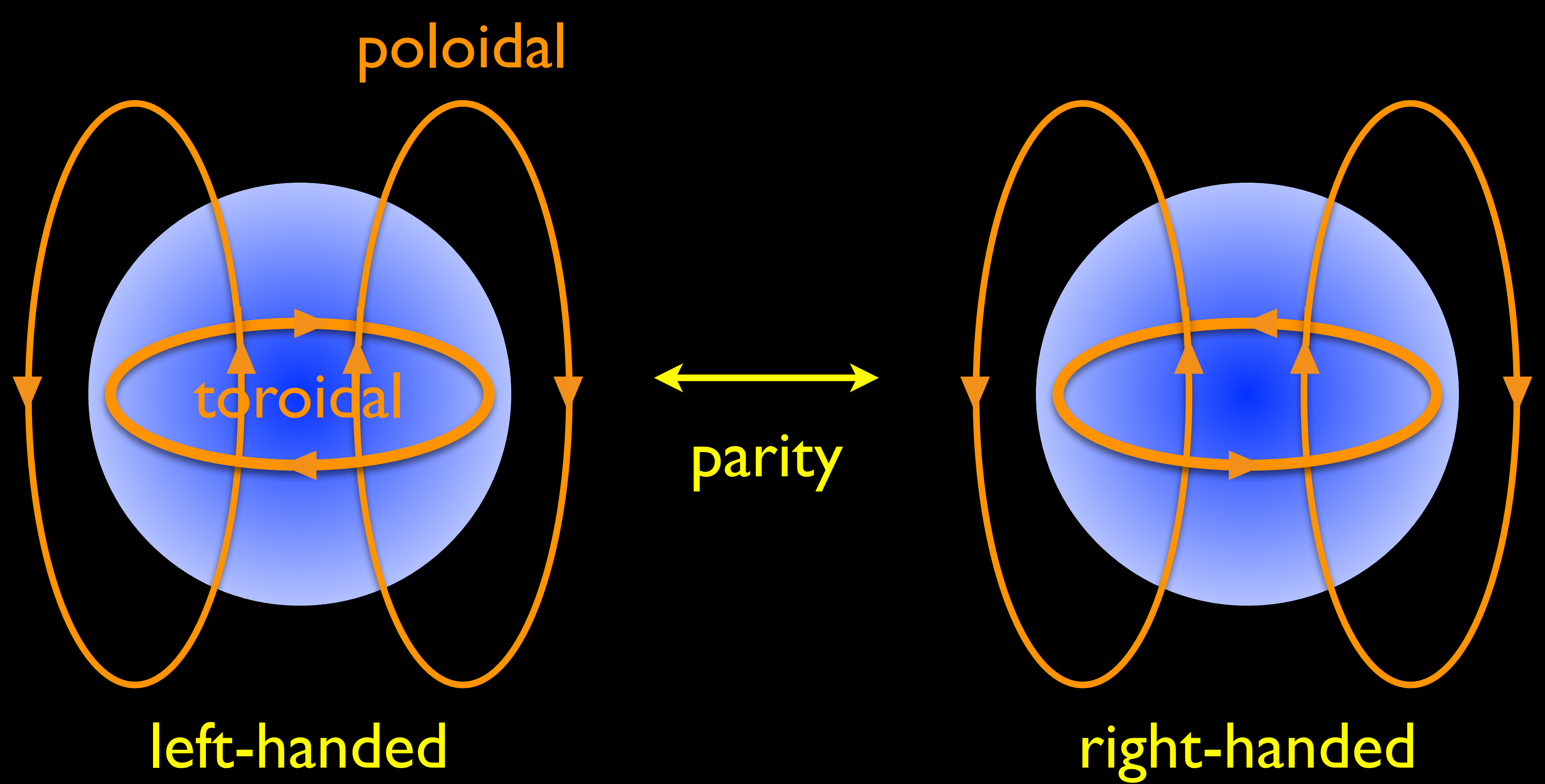
Magnetic helicity

- $\mathcal{H} = \int \mathrm{d}^3\boldsymbol{x}\boldsymbol{A}\cdot\boldsymbol{B}$: linking of magnetic fluxes (topological stability)



Magnetic helicity

- $\mathcal{H} = \int \mathrm{d}^3\boldsymbol{x}\,\boldsymbol{A}\cdot\boldsymbol{B}$: linking of magnetic fluxes (topological stability)
- Typically assumed as initial conditions, but its origin is unclear (how is parity-*odd* $\mathscr{H}$ generated from parity-*even* MHD?)



Effective theory for supernovae

Kinetic theory

- statistically describes time evolutions of nonequilibrium systems
  - systems in global equilibrium → thermodynamics
  - systems in local equilibrium → hydrodynamics
  - systems out of equilibrium → kinetic theory

collision

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{x}} = C[f] \qquad f = f(t, \boldsymbol{x}, \boldsymbol{p})$$

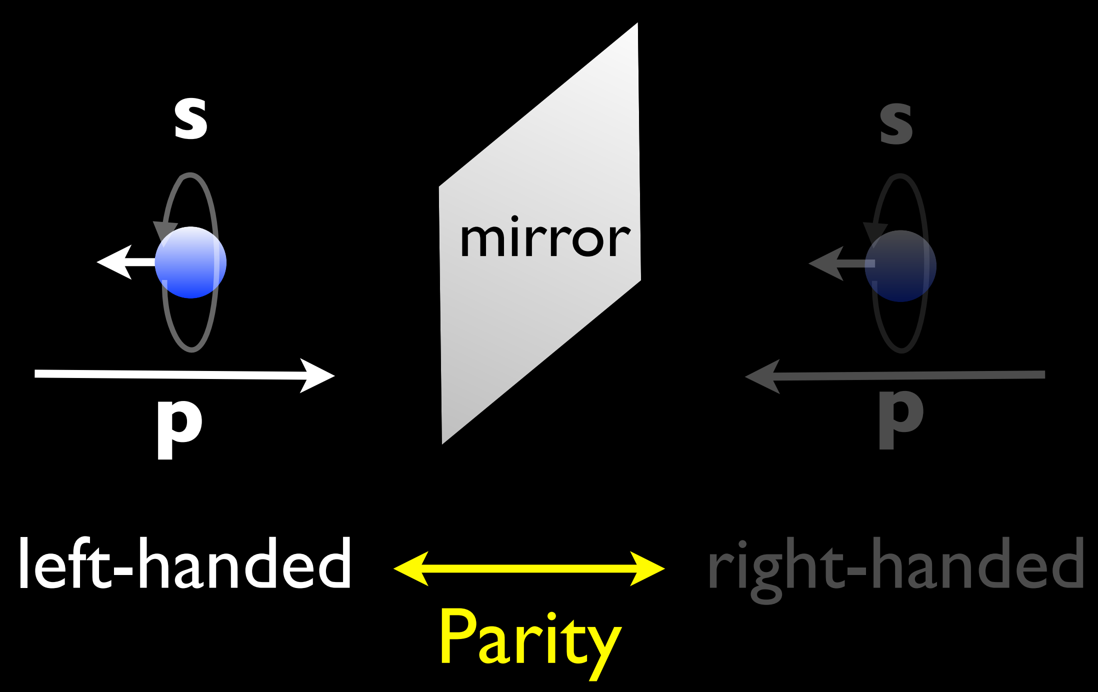
- Effective theories based on systematic expansions & symmetries

  e.g., superfluids differ from normal fluids by U(1) symmetry breaking

Problem w/ conventional theory

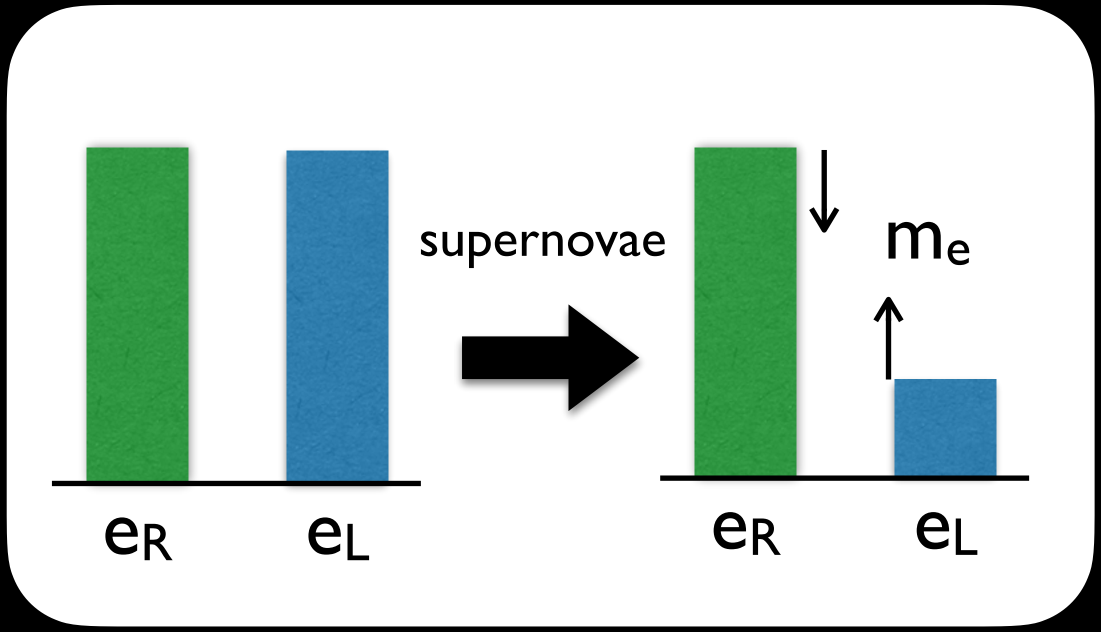
Conventional ν kinetic theory violates the basic principle of EFT:

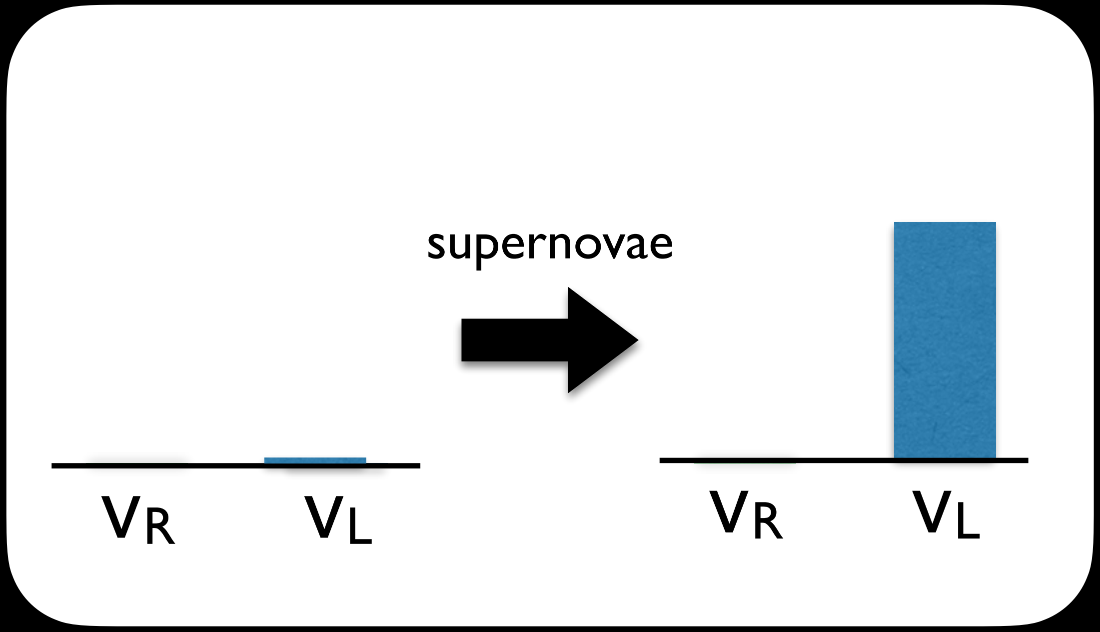
100 % parity violation by left-handedness



Supernova = Giant Parity Breaker

$$p + e^{L} \to n + \nu_e{}^{L}$$

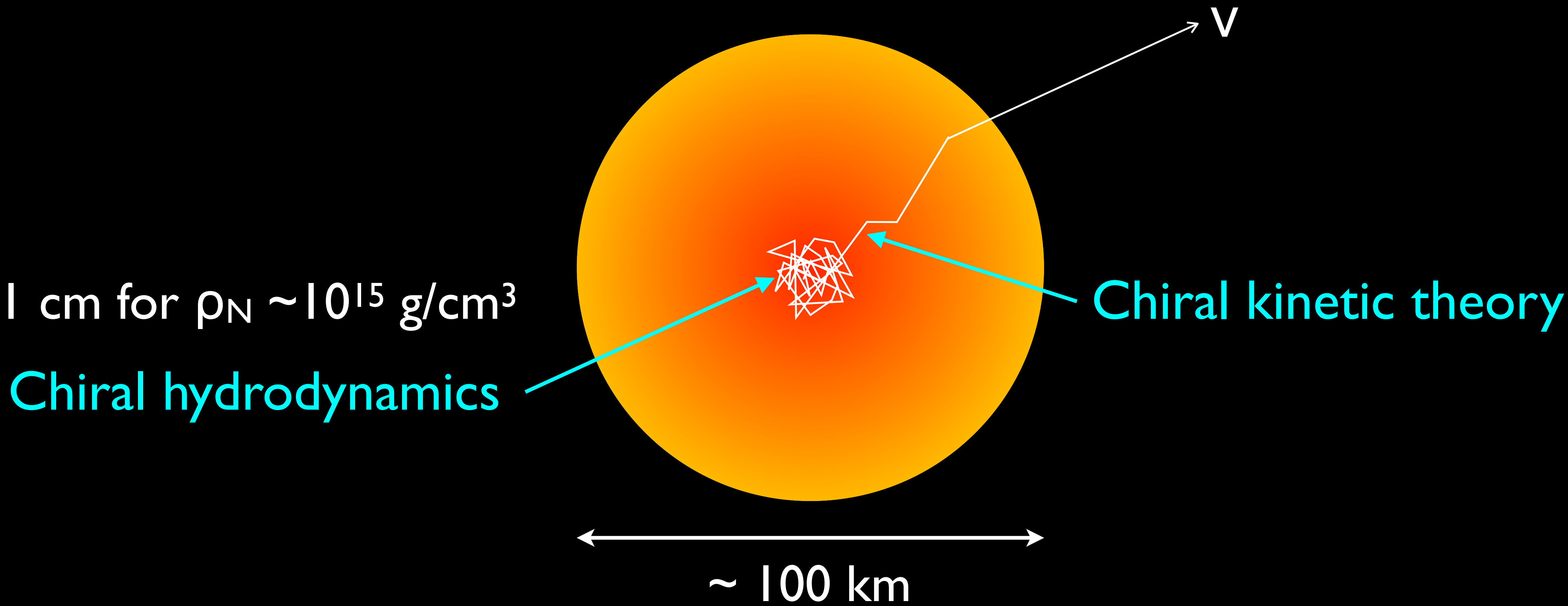




Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

Neutrino radiation transfer

$l_\nu \sim 1$ cm for $\rho_N \sim 10^{15}$ g/cm^3



$$\nabla_\alpha T^{\alpha\beta}_{\rm mat} = -\nabla_\alpha T^{\alpha\beta}_{\nu}$$

Stress tensor for matter (e, N)
(Hydrodynamics)

Stress tensor for ν
(Chiral kinetic theory)

From micro to macro

Micro Standard Model of Particle Physics

↓ ← Systematic low-energy expansion

Nonequilibrium kinetic theory for ν

↓

Macro Hydrodynamic evolution of core-collapse supernovae

Conventional derivation

for neutral massless scalar field (spin 0)

- Green's function: $S^{<}(x,y) = \langle \phi^{\dagger}(y)\phi(x)\rangle, \quad S^{>}(x,y) = \langle \phi(x)\phi^{\dagger}(y)\rangle$
- Equation of motion: $\square_x S^{<}(x,y) = 0$
- Wigner function: $S^{<}(q,X) = \int_s \mathrm{e}^{-\mathrm{i}q\cdot s} S^{<}\left(X + \frac{s}{2}, X - \frac{s}{2}\right) \sim f(q,X)$
- Derivative expansion: $\partial_X \ll q \longrightarrow q\cdot\partial_X f(q,X) = 0$

collisionless Boltzmann equation

Chiral radiation transport theory for neutrinos

From QFT to chiral kinetic theory

see, e.g., a review by Hidaka, Pu, Wang, Yang, PPNP (2022)

- Wigner function: $S^{<}(q,x) = \int_y \mathrm{e}^{-\mathrm{i}q\cdot y} \langle \psi^\dagger(x+y/2)\psi(x-y/2)\rangle \equiv \sigma^\mu \mathcal{L}^{<}_{\mu}$
- Equations of motion:

$$\mathcal{D}_\mu \mathcal{L}^{<\mu} = 0, \qquad \cdots(1)$$

$$q_\mu \mathcal{L}^{<\mu} = 0, \qquad \cdots(2)$$

$$\mathcal{D}_\mu \mathcal{L}^{<}_{\nu} - \mathcal{D}_\nu \mathcal{L}^{<}_{\mu} = -2\epsilon_{\mu\nu\rho\sigma} q^\rho \mathcal{L}^{<\sigma} \qquad \cdots(3)$$

where $\mathcal{D}_\mu \mathcal{L}^{<}_{\nu} \equiv \partial_\mu \mathcal{L}^{<}_{\nu} - \Sigma^{<}_{\mu} \mathcal{L}^{>}_{\nu} + \Sigma^{>}_{\mu} \mathcal{L}^{<}_{\nu}$

- Solution of (2), (3): $\mathcal{L}^{<\mu} = 2\pi\delta(q^2)\big(q^\mu - S^{\mu\nu}\mathcal{D}_\nu\big) f^{<}$

frame vector

where $S^{\mu\nu} = \dfrac{\epsilon^{\mu\nu\alpha\beta} q_\alpha n_\beta}{2q\cdot n}$

- Inserting it into (1) → transport equation with collisions

$$J^\mu = 2\int_q \mathcal{L}^{<\mu}, \quad T^{\mu\nu} = \int_q (\mathcal{L}^{<\mu} q^\nu + \mathcal{L}^{<\nu} q^\mu)$$

Chiral radiation transport theory

Yamamoto, Yang, Astrophysical Journal (2020)

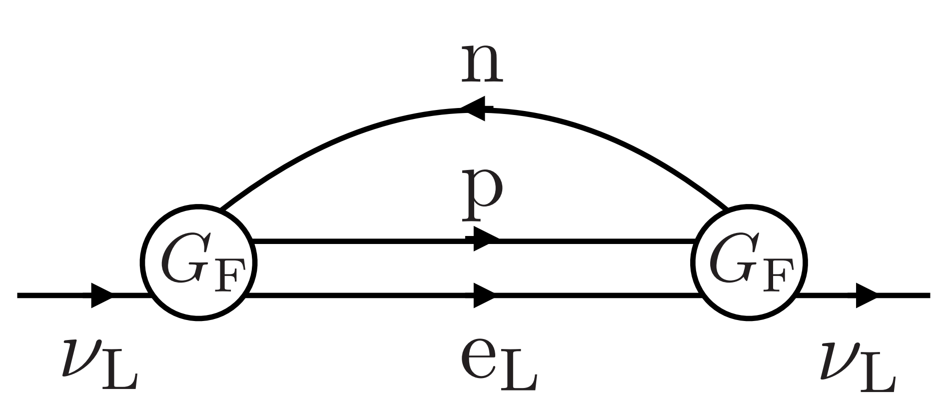
General Relativity + Standard Model + Nonequilibrium Field Theory

emission absorption

$$\left[q^\mu D_\mu - (D_\mu S^{\mu\nu})\partial_\nu + S^{\mu\nu} q^\rho R^\lambda_{\rho\mu\nu} \partial_{q\lambda}\right] f = (1-f)\Gamma^< - f\Gamma^>$$

$$D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu} q^\nu \partial_{q\lambda}, \quad \Gamma^{\lessgtr} = (q^\nu - D_\mu S^{\mu\nu})\,\Sigma^{\lessgtr}_\nu, \quad S^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta} q_\alpha n_\beta}{2q\cdot n}$$

Example of the neutrino self-energy



Chiral radiation transport theory

Yamamoto, Yang, Astrophysical Journal (2020)

A practical version (with $n^\mu = (1,\mathbf{0})$ ignoring curvature)

emission absorption

$$q^\mu D_\mu f = (1-f)\Gamma^{<} - f\Gamma^{>}, \quad \Gamma^{\lessgtr} \approx \Gamma^{(0)\lessgtr} + \Gamma^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma^{(B)\lessgtr}(q\cdot B)$$

$$\omega^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta, \quad B^\mu \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$$

$$\Gamma^{(0)>} \approx \frac{G_{\mathrm{F}}^2}{\pi}(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2)E_\nu^3(1-f^{(\mathrm{e})})\left(1-\frac{3E_\nu}{M_{\mathrm{N}}}\right)\frac{n_{\mathrm{p}}-n_{\mathrm{n}}}{1-\mathrm{e}^{\beta(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})}}$$

$$\Gamma^{(B)>} \approx \frac{G_{\mathrm{F}}^2}{2\pi M_{\mathrm{N}}}(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2)E_\nu(1-f^{(\mathrm{e})})\left(1-\frac{8E_\nu}{3M_{\mathrm{N}}}\right)\frac{n_{\mathrm{p}}-n_{\mathrm{n}}}{1-\mathrm{e}^{\beta(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})}}$$

$$\Gamma^{(\omega)>} \approx \frac{G_{\mathrm{F}}^2}{2\pi}(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2)E_\nu(1-f^{(\mathrm{e})})\left(2+\beta E_\nu f^{(\mathrm{e})}\right)\frac{n_{\mathrm{p}}-n_{\mathrm{n}}}{1-\mathrm{e}^{\beta(\mu_{\mathrm{n}}-\mu_{\mathrm{p}})}}$$

$\Gamma^{(0)}$ was computed in Reddy, Prakash, Lattimer, PRD (1998)

Chiral radiation transport theory

Yamamoto, Yang, Astrophysical Journal (2020)

A practical version (with $n^\mu = (1,\mathbf{0})$ ignoring curvature)

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Neutrino current and energy-momentum tensor

$$J^\mu = \int_{\boldsymbol{q}} \frac{1}{|\boldsymbol{q}|}(q^\mu - S^{\mu\nu}D_\nu)f, \quad T^{\mu\nu} = \int_{\boldsymbol{q}} \frac{1}{|\boldsymbol{q}|}\left[q^\mu q^\nu - \frac{1}{2}\left(q^\mu S^{\nu\rho} + q^\nu S^{\mu\rho}\right)D_\rho\right] f$$

$$D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu}q^\nu\partial_{q\lambda}, \quad S^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}$$

& corresponding corrections for electrons interacting w/ neutrinos

Nonequilibrium chiral effects by $\boldsymbol{B}$

Yamamoto, Yang, PRD (2021), PRL (2023)

- Neutrino current near equilibrium:

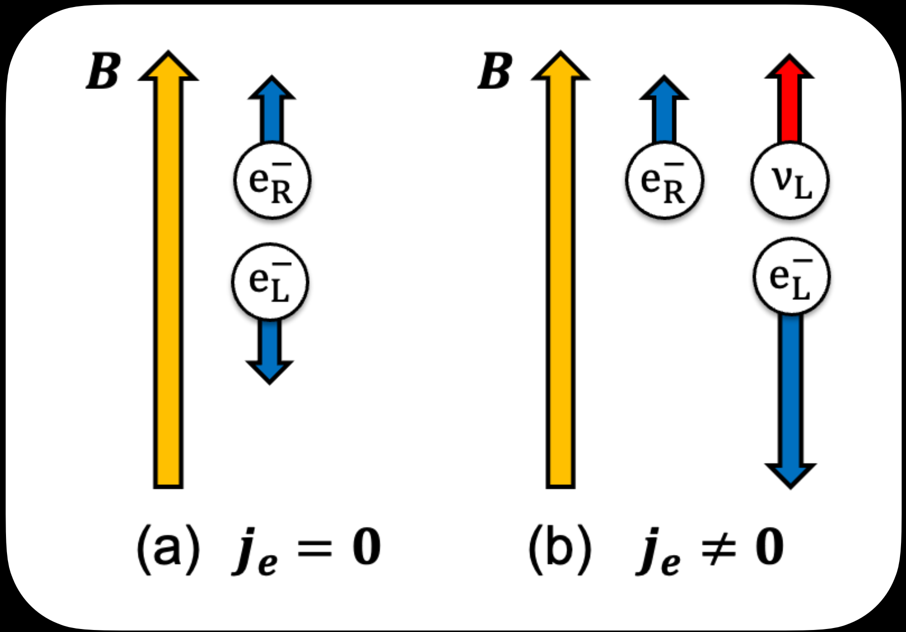
$$T_\nu^{i0} \approx \mu_\nu j_\nu^i \approx -\frac{1}{72\pi M_{\rm N} G_{\rm F}^2 (g_{\rm V}^2 + 3g_{\rm A}^2)} \frac{{\rm e}^{2\beta(\mu_{\rm n}-\mu_{\rm p})}}{n_{\rm n} - n_{\rm p}} (\boldsymbol{\nabla}\cdot\boldsymbol{v})\mu_\nu B^i$$

- Electric current due to the scattering w/ general nonequilibrium ν:

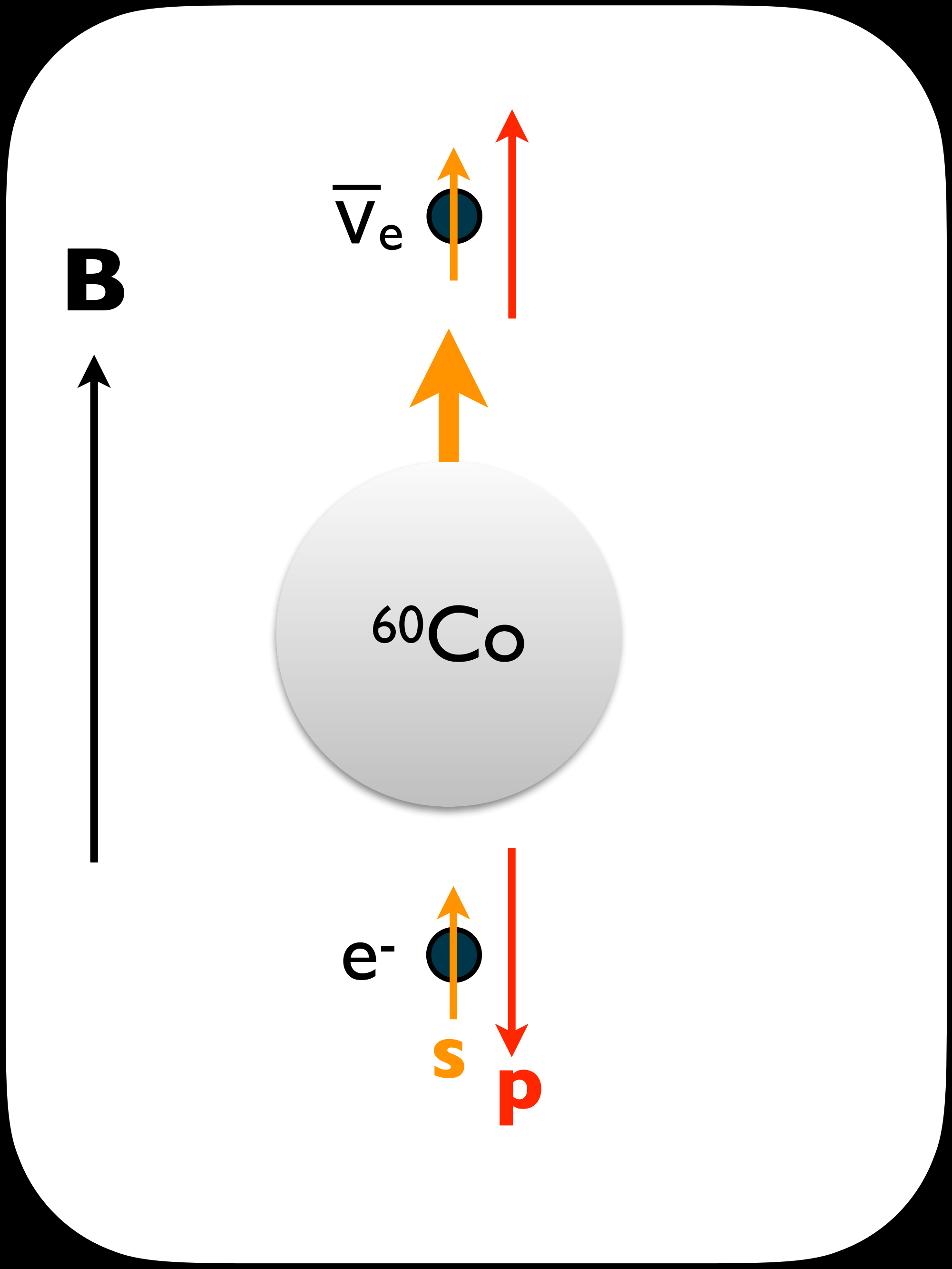
$\boldsymbol{j}_{\rm e} = \xi_B \boldsymbol{B}$ effective CME (even without μ_5)

$$\dot{\xi}_B = \frac{1}{4\pi^3}(g_{\rm V}^2 + 3g_{\rm A}^2) G_{\rm F}^2 (n_{\rm p} - n_{\rm n}) \int_0^\infty p^2 {\rm d}p \left[\frac{\bar{f}_{\rm e}(1-f_\nu)}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}} + \frac{(1-\bar{f}_{\rm e})f_\nu}{1-{\rm e}^{\beta(\mu_{\rm p}-\mu_{\rm n})}} \right]$$

$|\xi_B| \sim 0.1\text{-}1\,{\rm MeV}$ for the gain region



Wu experiment



$J_{\mathrm{e},\nu} \propto B$: nonequilibrium many-body manifestation of the chiral effect

Other chiral effects

- Neutrino chiral vortical effect (CVE):

$$j_\nu^i = -\frac{1}{2\pi^2}\omega^i \int_0^\infty \mathrm{d}p\; p f_\nu + (\mathrm{antiparticle's})\,, \quad T_\nu^{i0} = -\frac{1}{2\pi^2}\omega^i \int_0^\infty \mathrm{d}p\; p^2 f_\nu + (\mathrm{antiparticle's})$$

- Neutrino spin Hall effect:

$$\boldsymbol{j}_\nu = \int_{\boldsymbol{p}} \frac{1}{2|\boldsymbol{p}|^3}(\boldsymbol{p} \times \boldsymbol{\nabla} V) f_\nu + (\mathrm{antiparticle's})$$

Yamamoto, Yang, PRD (2024)

$$V = \frac{G_\mathrm{F}}{\sqrt{2}}\left[(1+4\sin^2\theta_\mathrm{W})n_\mathrm{e} - n_\mathrm{n} + (1-4\sin^2\theta_\mathrm{W})n_\mathrm{p}\right]$$

Potential *V* also leads to matter effect on neutrino oscillations (MSW effect)

Phenomenological applications

Chiral MHD equations

Masada, Kotake, Takiwaki, Yamamoto, PRD (2018); Matsumoto, Yamamoto, Yang, PRD (2022)

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

$$\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \boldsymbol{v}) = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \text{(dissipation)}$$

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \boldsymbol{\nabla} \times \underline{(\xi_B \boldsymbol{B})}$$

$$\partial_t \mathcal{H}(\xi_B) = \frac{\eta}{2\pi^2} (\boldsymbol{\nabla} \times \boldsymbol{B} - \xi_B \boldsymbol{B}) \cdot \boldsymbol{B}$$

see also Rogachevskii et al. (2017), Brandenburg et al. (2017), Schober et al. (2018)

Chiral plasma instability

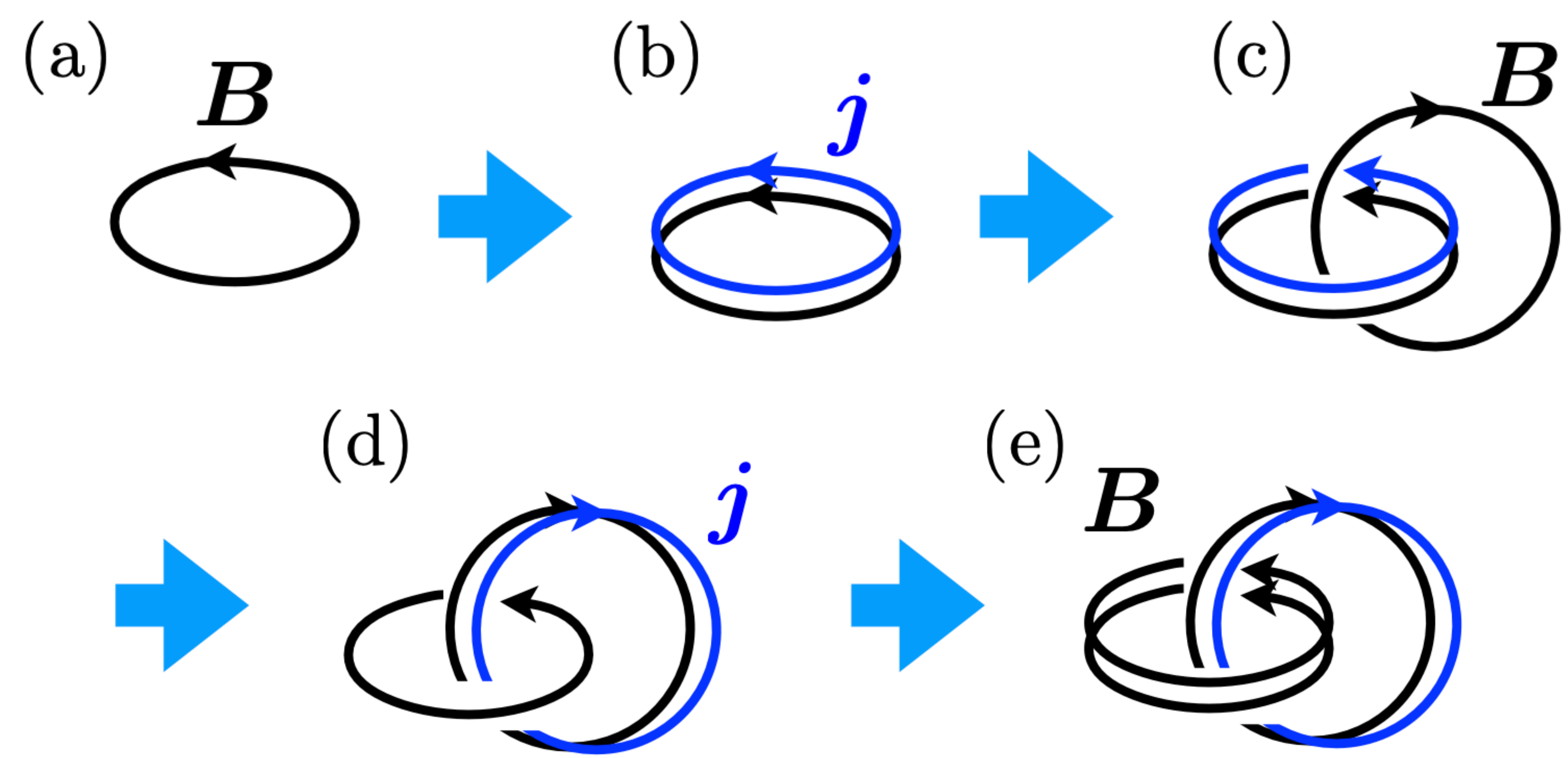


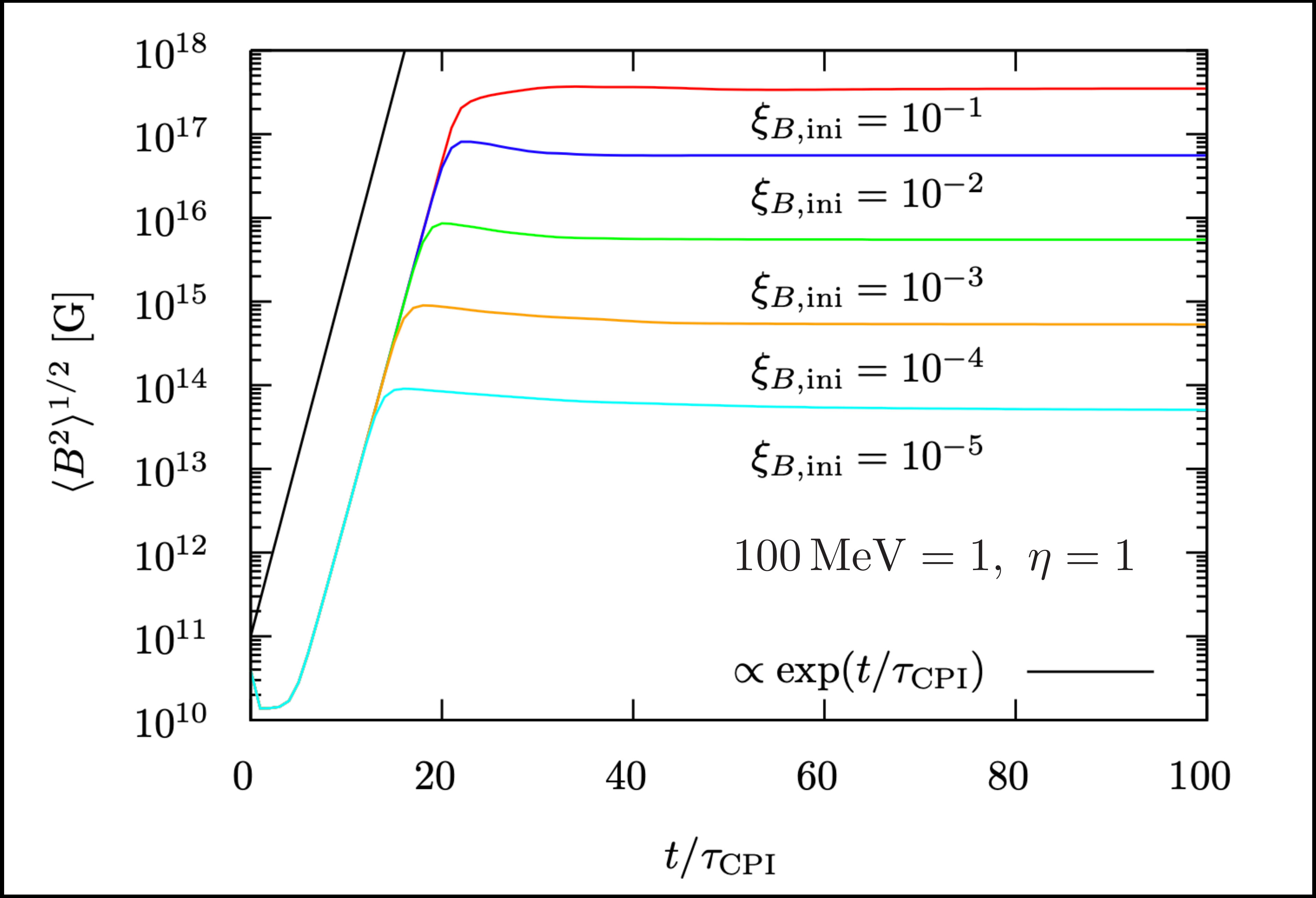
figure taken from arXiv:2305.01234

Positive feedback → Strong magnetic field with magnetic helicity

Akamatsu, Yamamoto (2013), Ohnishi, Yamamoto (2014), …

Generation of ***B*** by CPI

Matsumoto, Yamamoto, Yang, PRD (2022); Matsumoto, Takiwaki, Yamamoto, in prep.



A possible new mechanism for magnetars

Sources for CME and CPI

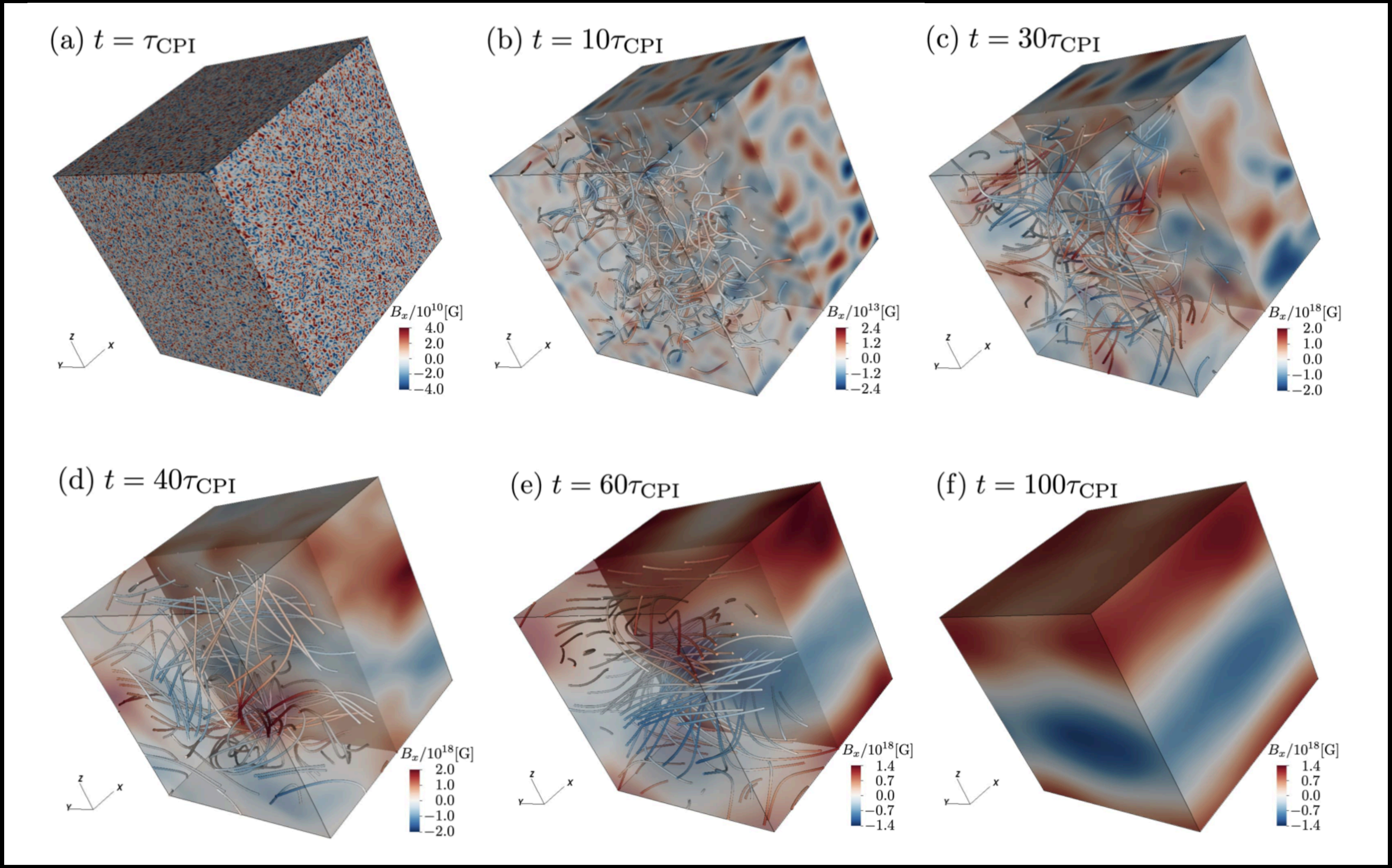
$$\boldsymbol{j}_{\rm e} = [\#\mu_5 + \#\boldsymbol{v}\cdot\boldsymbol{\omega} + \xi_B(f_\nu) + \cdots]\boldsymbol{B}$$

- μ_5 due to $\rm p + e^L \to n + \nu_e^L$ (SN core) Ohnishi, Yamamoto (2014)
  → suppressed by chirality flipping due to $m_{\rm e}$ Grabowska, Kaplan, Reddy (2015)
  → at higher *T*, CPI induced B~10^{14} G Sigl, Leite (2016), …
- Fluid kinetic helicity due to the chiral vortical effect (SN core)
  Yamamoto, PRD (2016)
- $\xi_B(f_\nu)$ due to the scattering w/ nonequilibrium ν → $10^{15\text{-}16}$ G
  Matsumoto, Yamamoto, Yang, PRD (2022); Yamamoto, Yang, PRL (2023)
- μ_5 due to $\rm p + e^L \to n + \nu_e^L$ (NS crust) → $10^{15\text{-}16}$ G for ~100 yrs
  Dehman, Pons, 2408.05281

Time evolution of *B*

$\xi_{B,\mathrm{ini}} = 10^{-1}$

Matsumoto, Yamamoto, Yang, PRD (2022)

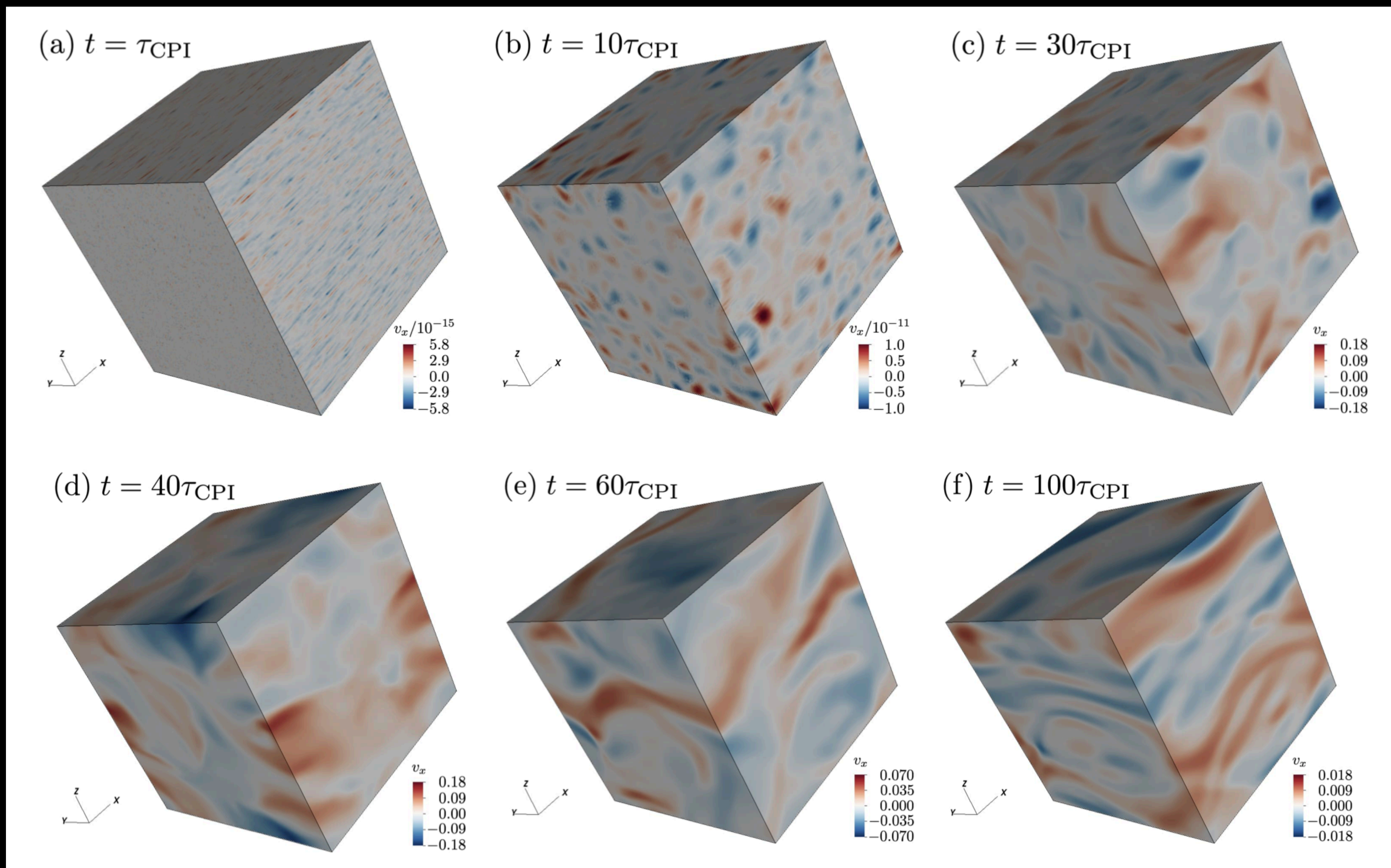


see also Brandenburg et al. (2017); Masada et al. (2018)

Time evolution of *v*

$\xi_{B,\mathrm{ini}} = 10^{-1}$

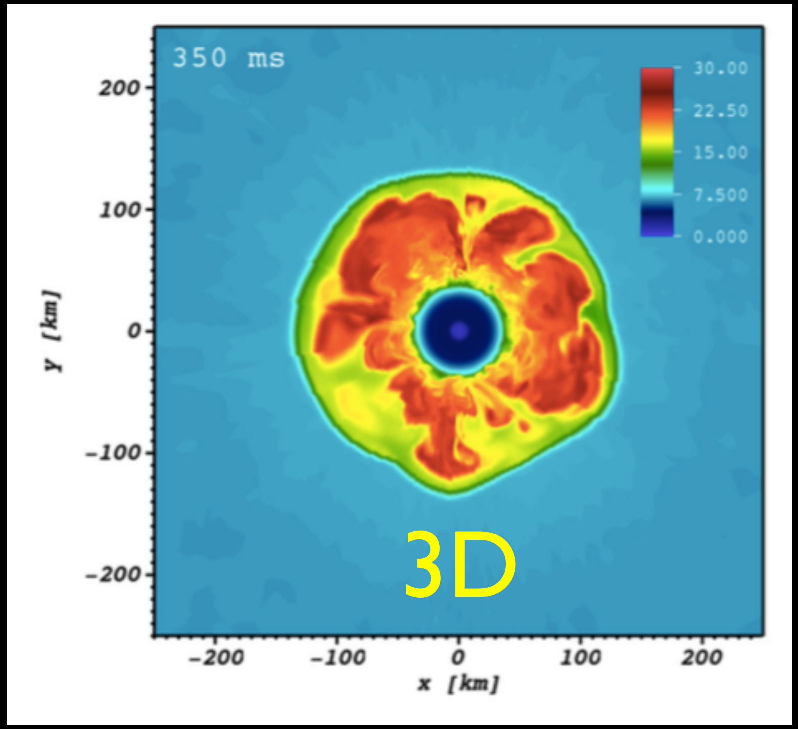
Matsumoto, Yamamoto, Yang, PRD (2022)



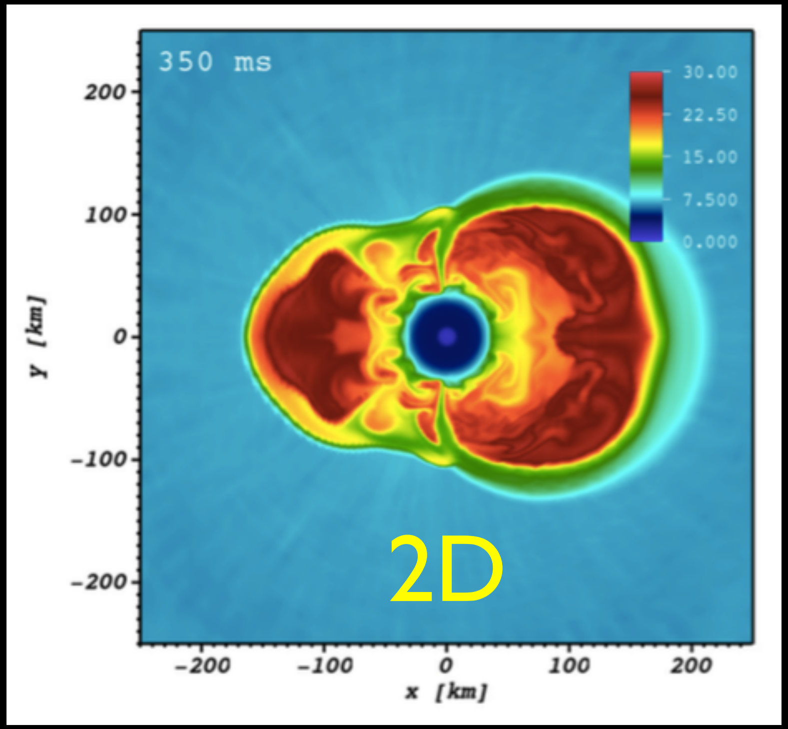
Chiral effects lead to inverse cascade, which may affect explosion dynamics

Turbulent cascade & explodablity

Direct cascade (3D w/o chirality):
energy



Inverse cascade (2D):
energy & enstrophy



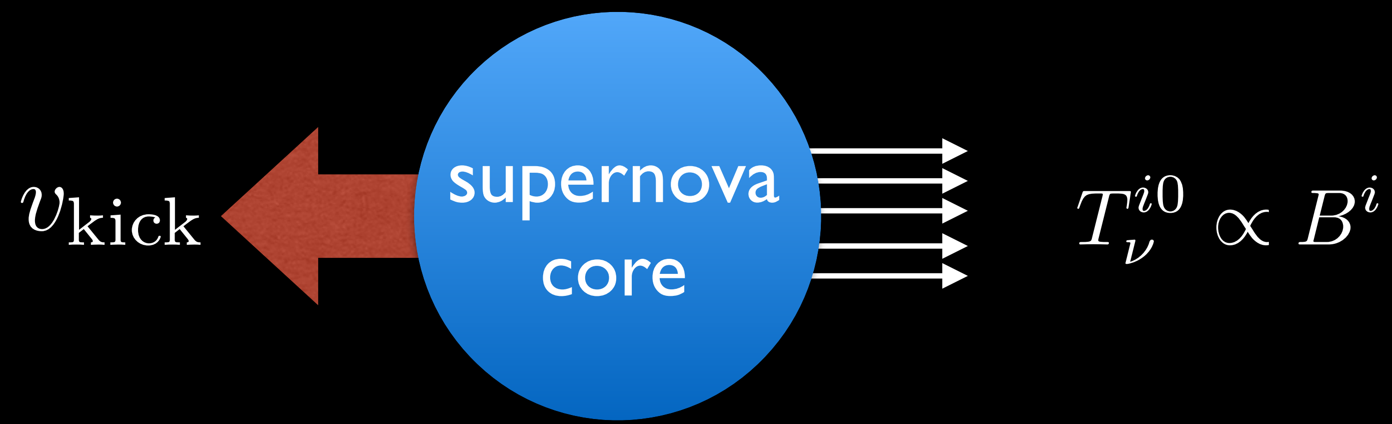
Hanke (2014)

Inverse cascade for 3D matter w/ chiral effects: energy & helicity

How does it affect the global evolution of supernovae?

Pulsar kicks

- Typical NS velocity ~ a few 100-1000 km/s ← anisotropy of explosion?



- Phenomenology: $v_{\rm kick} \sim 100 \left(\frac{B}{10^{15}\ {\rm G}} \right)\ {\rm km/s}$ Chugai (1984), Vilenkin (1995), …
- ν near equilibrium: $T_\nu^{i0} \propto B^i$ analytically computable Yamamoto, Yang, PRD (2021)
- Interplay of chiral effects & momentum anisotropy Fukushima, Yu, 2401.04568

→ Global simulations with chiral radiation hydro is necessary

Summary & Outlook

- Conventional supernova theory ignores P violation of the weak int.
- Chiral effects drastically modify the hydrodynamic behaviors: chiral plasma instability and inverse cascade
- Relevant to magnetars, explosion dynamics, and pulsar kicks
- Other chiral effects (e.g., chiral vortical effect, spin Hall effect)?
- Other quantum effects (neutrino collective oscillations)? e.g., Nagakura
- Global simulations of chiral radiation hydro would be important