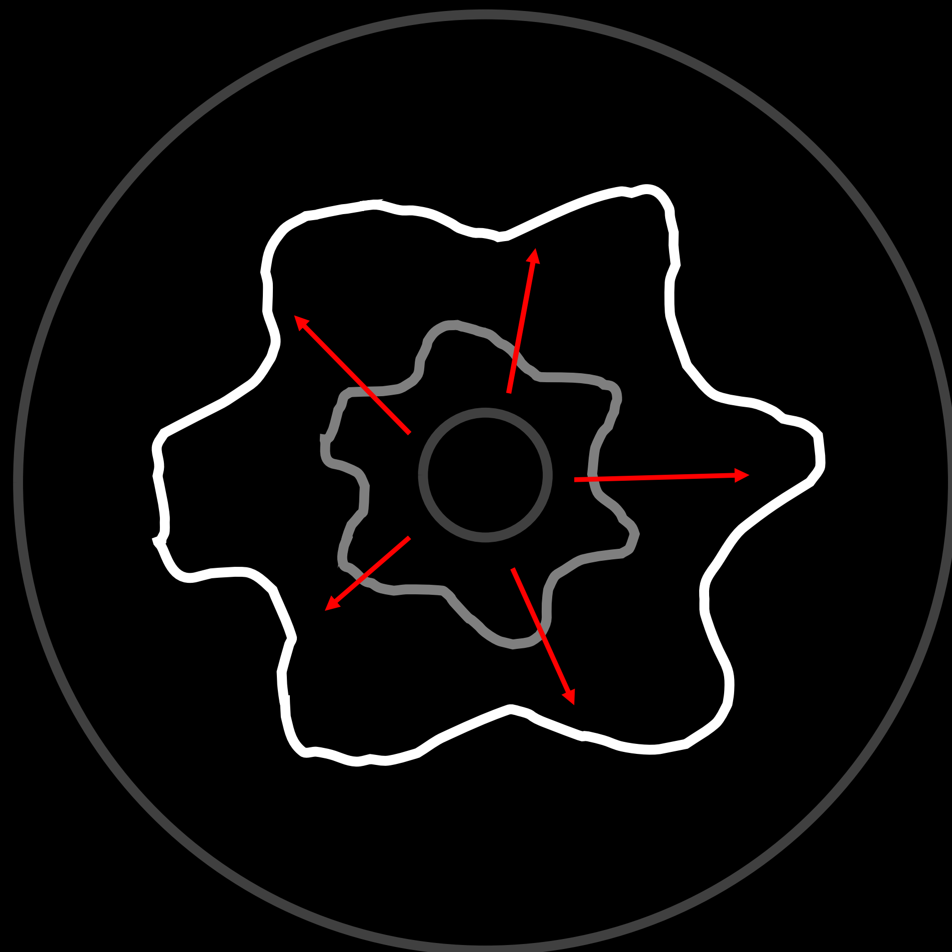
Gravitational waves from the hadronic-quark matter interface (HQMI) in hybrid stars?



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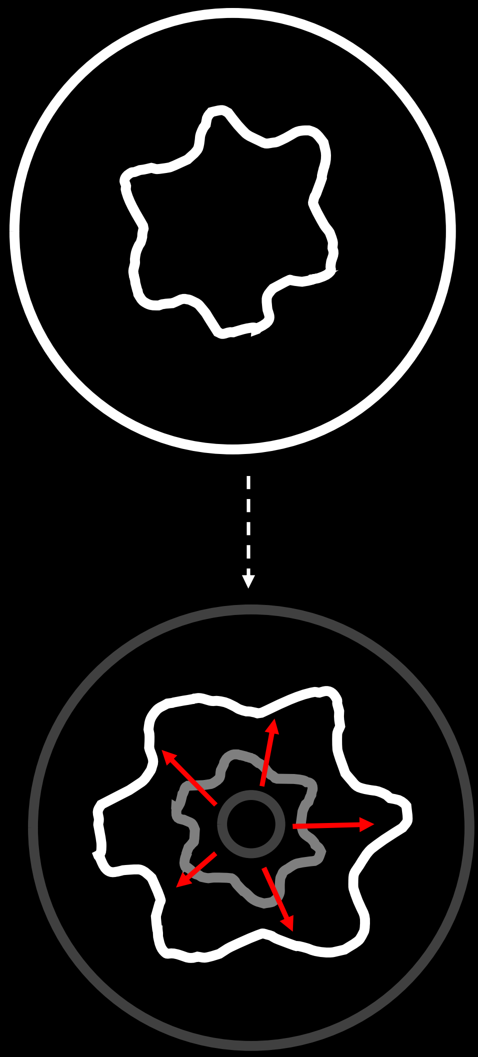
Project goal

- I want to model the gravitational waves (GWs) from the HQMI in hybrid stars using full general relativity (GR) in three dimensions
- My first step is to perturb the HQMI and obtain the GWs
- The next step is to find the GWs from a HQMI that is both perturbed and moving
    - Interface could be slow or fast moving



GWs from oscillation modes

$$h_{ij}^{TT} = \frac{1}{r} \sum_{l=2}^{\infty} \sum_{m} \ddot{I}_{lm} T_{ij}^{E2,2m}$$

$$I_{2m} = \frac{16\pi\sqrt{3}}{15} \int \rho Y_{2m}^{*} r^{2}\, dV$$

$$\rho(\vec{r}, t) = \rho_s(r, t) + \Re \sum_{lm} \rho_{lm}(r) e^{i(\omega_{lm} t + \Phi_{lm})} Y_{lm}(\theta, \phi)$$

GWs from the Weyl scalar

$$\mathrm{Re}[\Psi_4] \propto \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left(\frac{d}{dt}\right)^{l+2} I^{lm}$$

$$\Psi_4 = C_{\alpha\beta\gamma\delta} \mathrm{n}^{\alpha} \bar{\mathrm{m}}^{\beta} \mathrm{n}^{\gamma} \bar{\mathrm{m}}^{\delta}$$

GW Source	Strain	Frequency (Hz)	Mass ($M_{\odot}$)	Characteristic size (km)	Distance to source (Mpc)
Supernova	10^{-21}	-	1.4	-	10
NS-NS inspiral	10^{-21}	100	2.8	90	15
MBH-MBH inspiral	10^{-16}	10^{-4}	10^{7}	10	1000
HQMI	**~10^{-22} (?)**	**<1.25E3 (?)**	**0-1.4 (?)**	**0-10 (?)**	**?**

NS = neutron star

MBH = massive black hole

Table adapted from Table 1 of J. B. Camp and N. J. Cornish, Annu. Rev. Nucl. Part. Sci. **54**, 525 (2004).

Equations from K. Thorne, Rev. Mod. Phys. **52**, 299 (1980).

Oscillation modes

- Oscillations in a perturbed star can generate quadrupole moments -> GW emission

- Relativistic star oscillation modes are characterized by their restoring force (e.g., Fundamental (f), gravity (g), pressure (p))[1]
    - The g-mode is excited by oscillations in a stratified fluid (e.g., the HQMI)[2]

- How do these modes couple to spacetime?

	Frequency	**GW damping timescale**
F-mode	1.5-2.5 kHz	Fraction of a second
G-mode	$< \nu_{fmode}$	> Seconds?

[1]M. G. Orsaria *et al.*, J. Phys. G: Nucl. Part. Phys. **46**, 073002 (2019).

[2]C. J. Kreuger and K. D. Kokkotas, Phys. Rev. Lett. **125**, 111106 (2020).

Cowling approximation

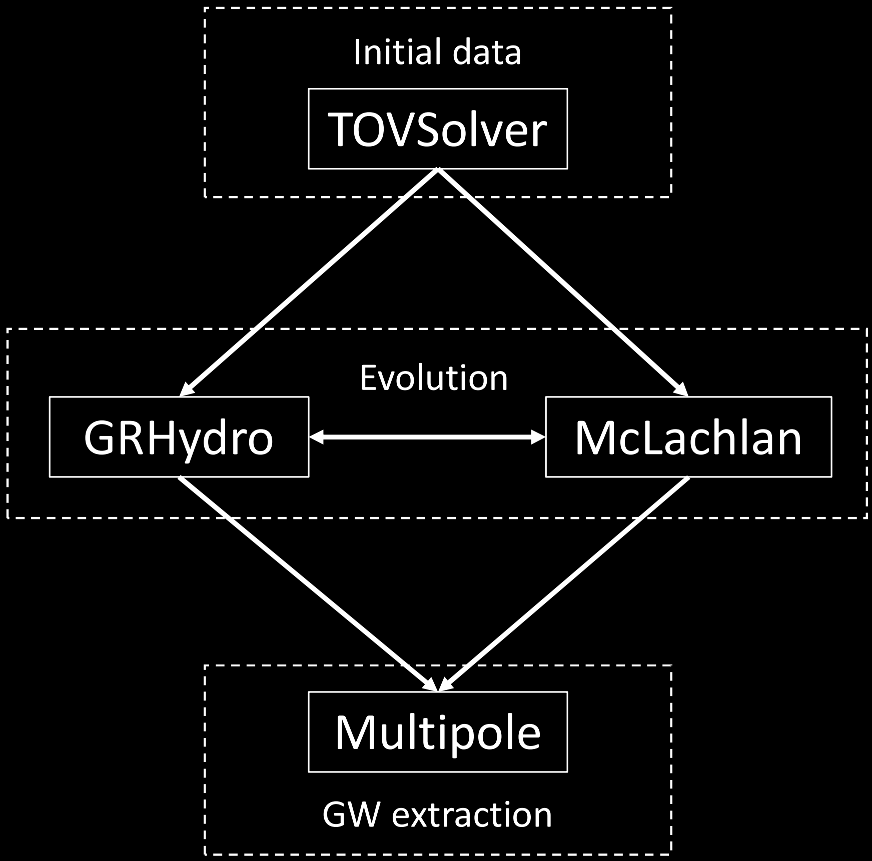
- Most studies of stellar oscillations use the Cowling approximation
  - Fluid oscillations are de-coupled from metric perturbations[3, 4]

- I intend to study HQMI oscillations in full GR[5]
  - To explore spacetime-matter coupling and how it affects hybrid star oscillation modes

[3]P. Jaikumar, A. Semposki, M. Prakash, and C. Constantinou, Phys. Rev. D. **103**, 123009 (2021).
[4]H. Sotani and T. Takiwaki, Phys. Rev. D. **102**, 063025 (2020).
[5]T. Zhao and J. M. Lattimer, Phys. Rev. D. **106**, 123002 (2022).

The Einstein Toolkit

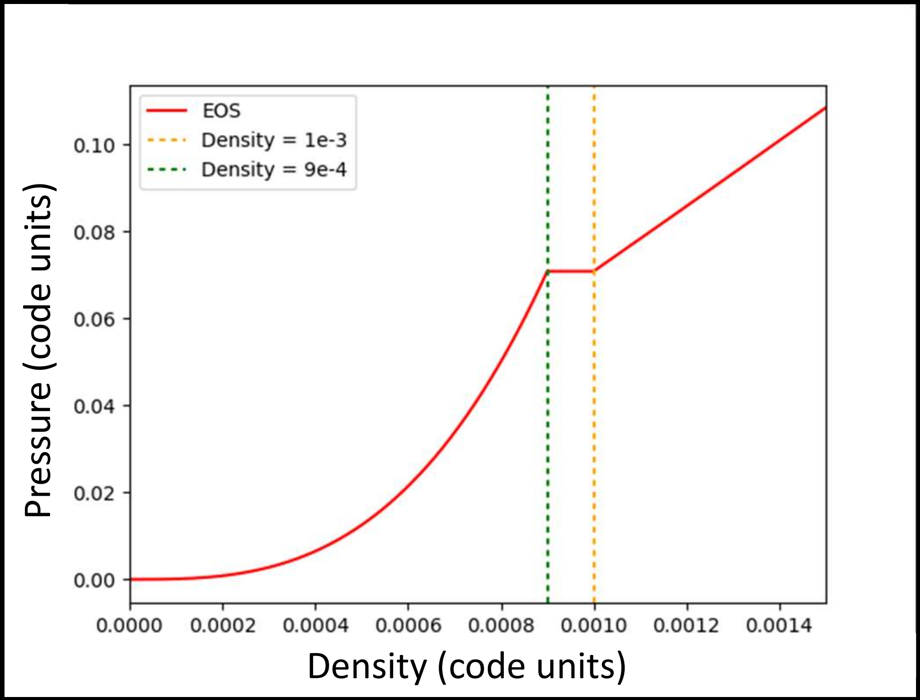
- The Einstein Toolkit (ET) is a suite of numerical relativity codes[6]
  - This code lets me simulate astrophysical systems in 3D and full GR
- The ET is made up of code modules that talk to each other
  - TOVSolver – sets up a stable NS
  - GRHydro – matter evolution
  - McLachlan – spacetime evolution
  - Multipole – wave extraction



[6]The Einstein Toolkit, doi:10.5281/zenodo.12588764 (key: EinsteinToolkit:2024_05) (2024).

Stable hybrid star using the ET

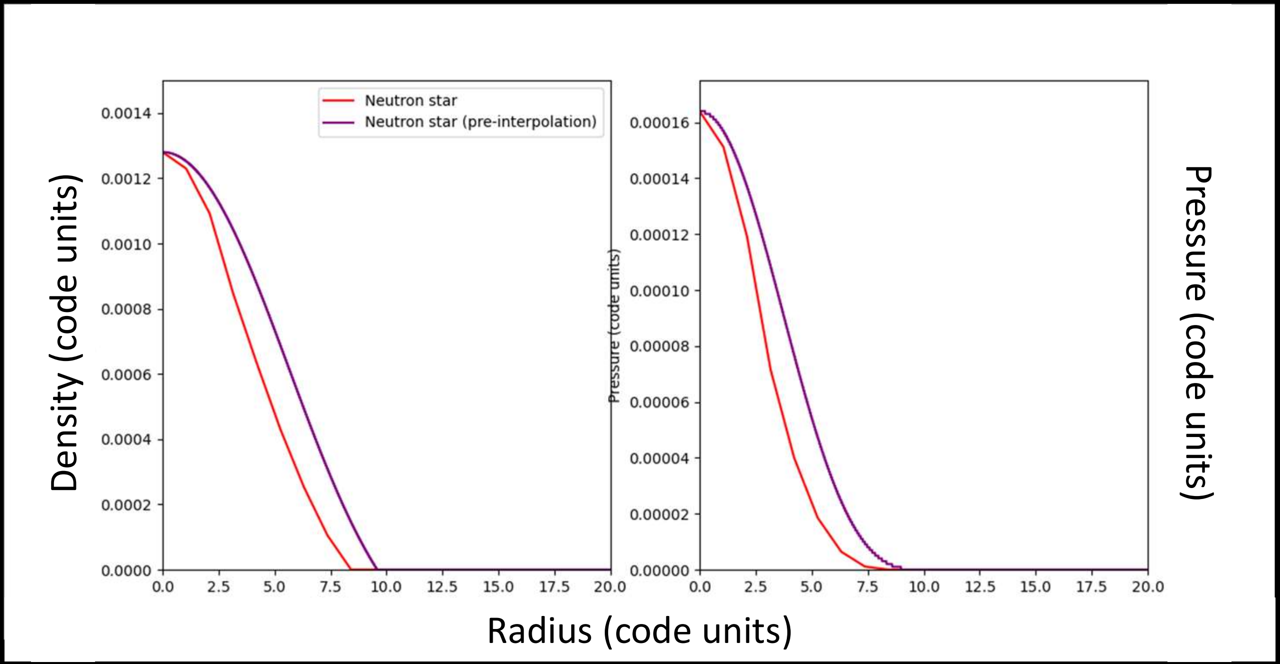
- I am creating a stable hybrid star in the ET
  - This involves adding a custom equation of state (EOS) to the code
  - I am currently using two polytropic EOS for the hadronic and quark matter
- I am currently following the prescription of Pereira et al. (2018)[7]
  - Pressure balance across the HQMI



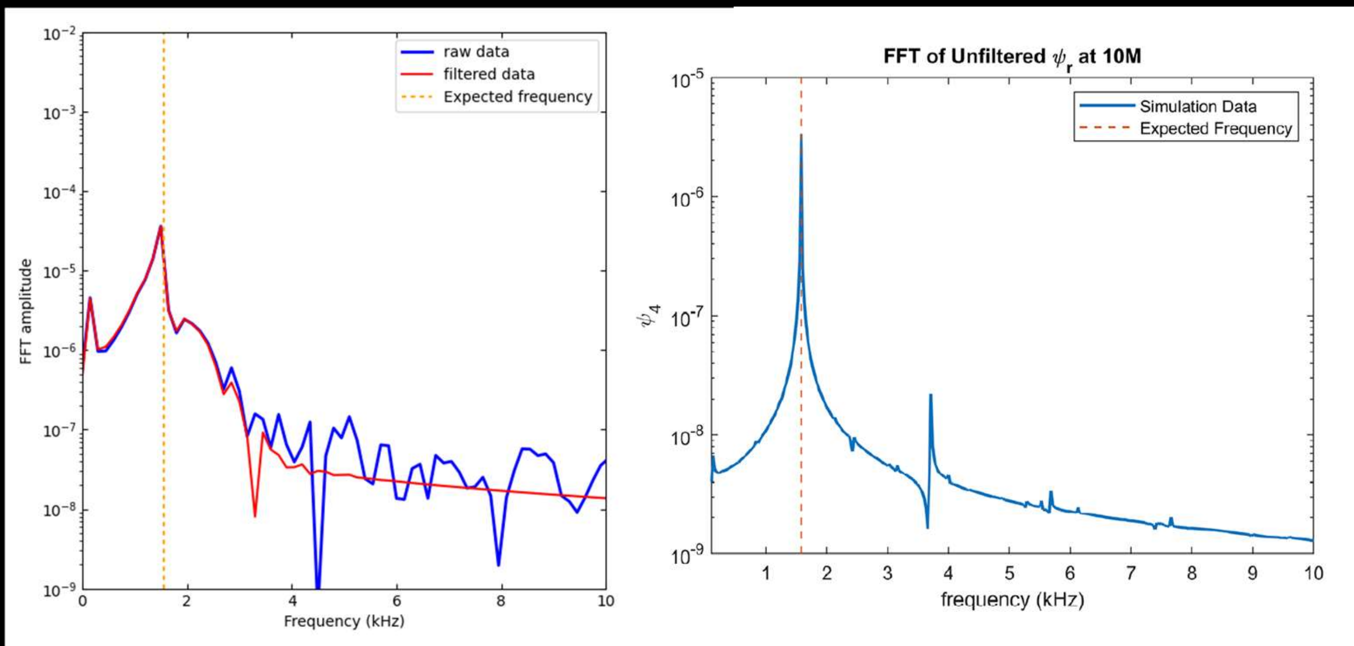
[7]J. P. Pereira, C. V. Flores, and G. Lugones, Astrophys. J. **860**, 12(2018).

1D – 3D interpolation

- To set up an initial NS, the ET solves the Tolman-Oppenheimer-Volkoff (TOV) equations in 1D
  - Interpolates to 3D afterward
  - Changes from Schwarzschild to isotropic coordinates
- This process changes the density/radius and pressure/radius curves
  - This might wash out the HQMI



f-modes from the ET

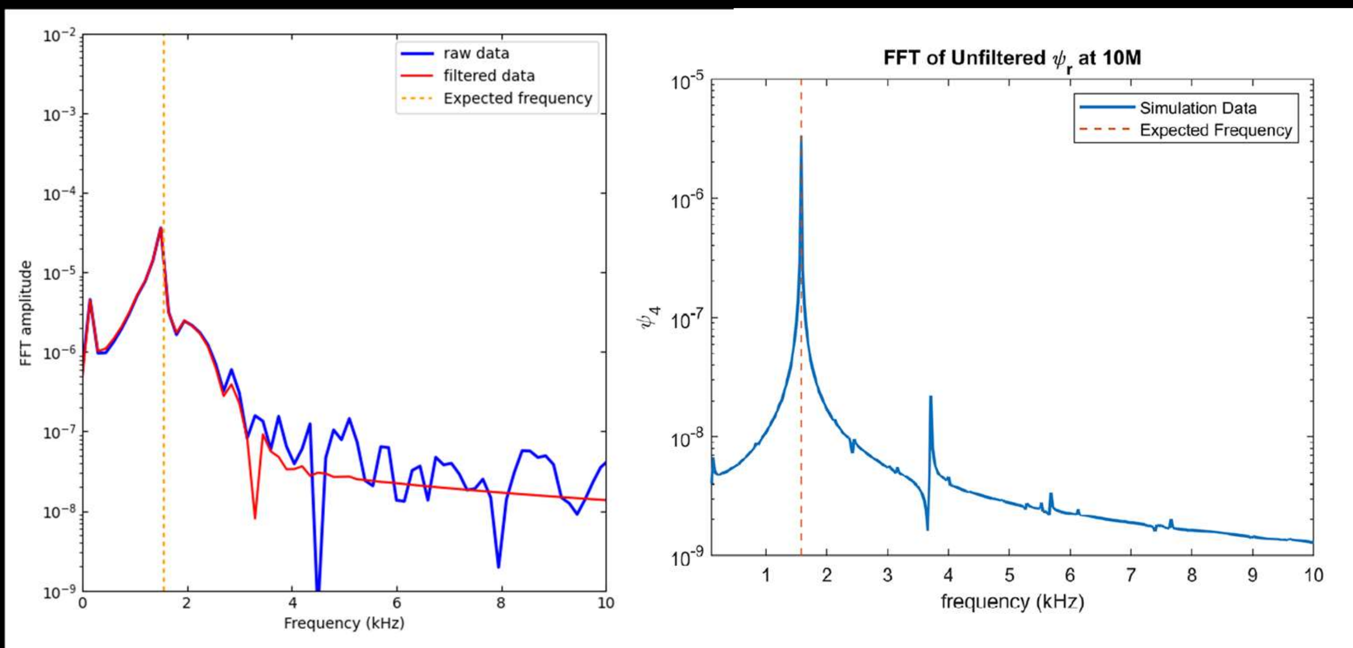


FFT of the Ψ_4 signal. Left panel is my reproduction, right panel is the original data from Rosofsky et al. (2019)

- Rosofsky *et al.* (2019)[8] showed that you could extract NS f-mode frequencies using the ET
- Also found the GW signal and damping times

[8]S. G. Rosofsky *et al.*, Phys. Rev. D. **99**, 084024 (2019).

g-modes from the ET



FFT of the Ψ_4 signal. Left panel is my reproduction, right panel is the original data from Rosofsky et al. (2019)

- I am looking for GWs from g-mode oscillations at the HQMI[3]
  - Perturbed HQMI
  - Perturbed + moving HQMI
- The speed of the HQMI depends on the rate of burning
  - Slow burning = slow HQMI and vice versa

[3]P. Jaikumar, A. Semposki, M. Prakash, and C. Constantinou, Phys. Rev. D. **103**, 123009 (2021).

Summary

- I want to find potential GWs from the HQMI
  - I will do so in full GR
- The plan is to adapt the ET to the HQMI system
  - The ET can evolve systems with matter coupled to spacetime
  - I am currently setting up a stable hybrid star in the ET as an initial state
- The aim is to simulate g-mode oscillations at the HQMI with the ET and look for GWs
  - Perturbed HQMI -> moving + perturbed HQMI

Gravitational waves: radiation field

- In globally vacuum spacetime we can choose a gauge where $h_{\alpha\beta}$ is purely spatial ($h_{tt} = h_{ti} = 0$) and traceless ($h = h_i^i = 0$)
  - This implies the metric perturbation is transverse ($\partial_i h_{ij} = 0$)

- This is known as transverse traceless (TT) gauge
  - The TT part of the perturbation completely describes GW radiation, even when a source is present[7]

[7]K. Thorne, Rev. Mod. Phys. **52**, 299 (1980).

Gravitational waves: multipole expansion

- The GW radiation field can be expanded in terms of tensor spherical harmonics[7]

$$h_{ij}^{TT} = \frac{1}{r}\sum_{l=2}^{\infty}\sum_{m} \ddot{I}_{lm} T_{ij}^{E2,2m}$$

[7]K. Thorne, Rev. Mod. Phys. **52**, 299 (1980).

Gravitational waves: oscillation modes

- The density of the system can be decomposed into an unperturbed part + a perturbation

$$\rho(\vec{r},t) = \rho_s(r,t) + \Re \sum_{lm} \rho_{lm}(r) e^{i(\omega_{lm} t + \Phi_{lm})} Y_{lm}(\theta,\phi)$$

The TOV equation

- The Tolman-Oppenheimer-Volkoff (TOV) equation relates the total mass m, density ρ, and pressure P as functions of distance from the stellar core r

$$\frac{dP}{dr} = -\frac{1}{r^2}(\rho + P)(m + 4\pi r^3 P)\left(1 - \frac{2m}{r}\right)^{-1}$$

- It is derived from the Einstein equations and is necessary to construct a relativistic stellar model[8]

[8]K. S. Thorne and A. Campollataro, Astrophys. J. **149**, 591 (1967).