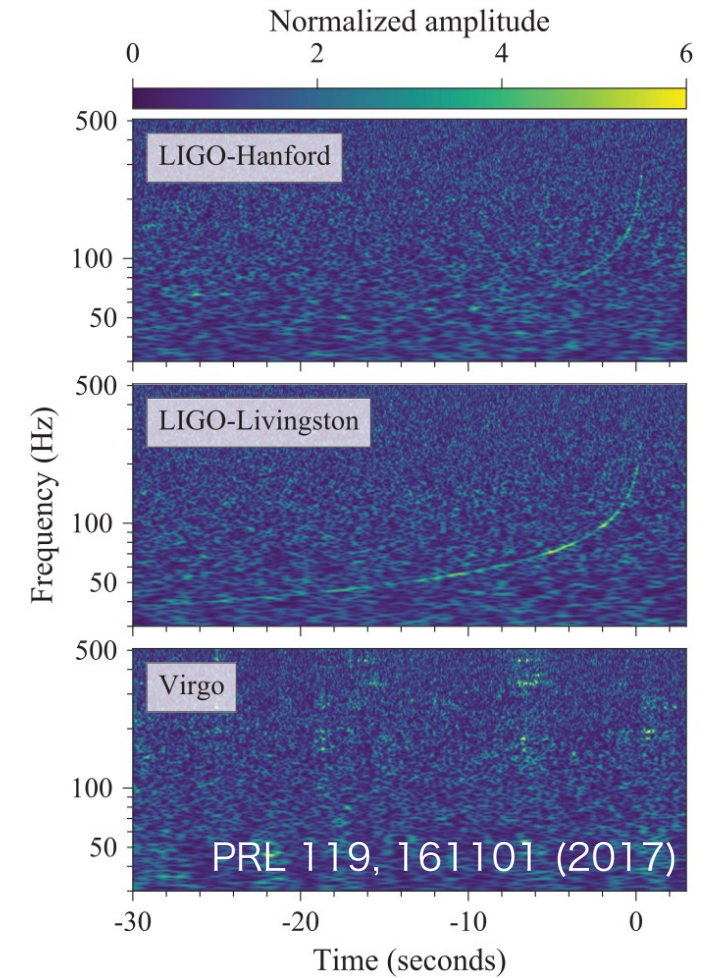
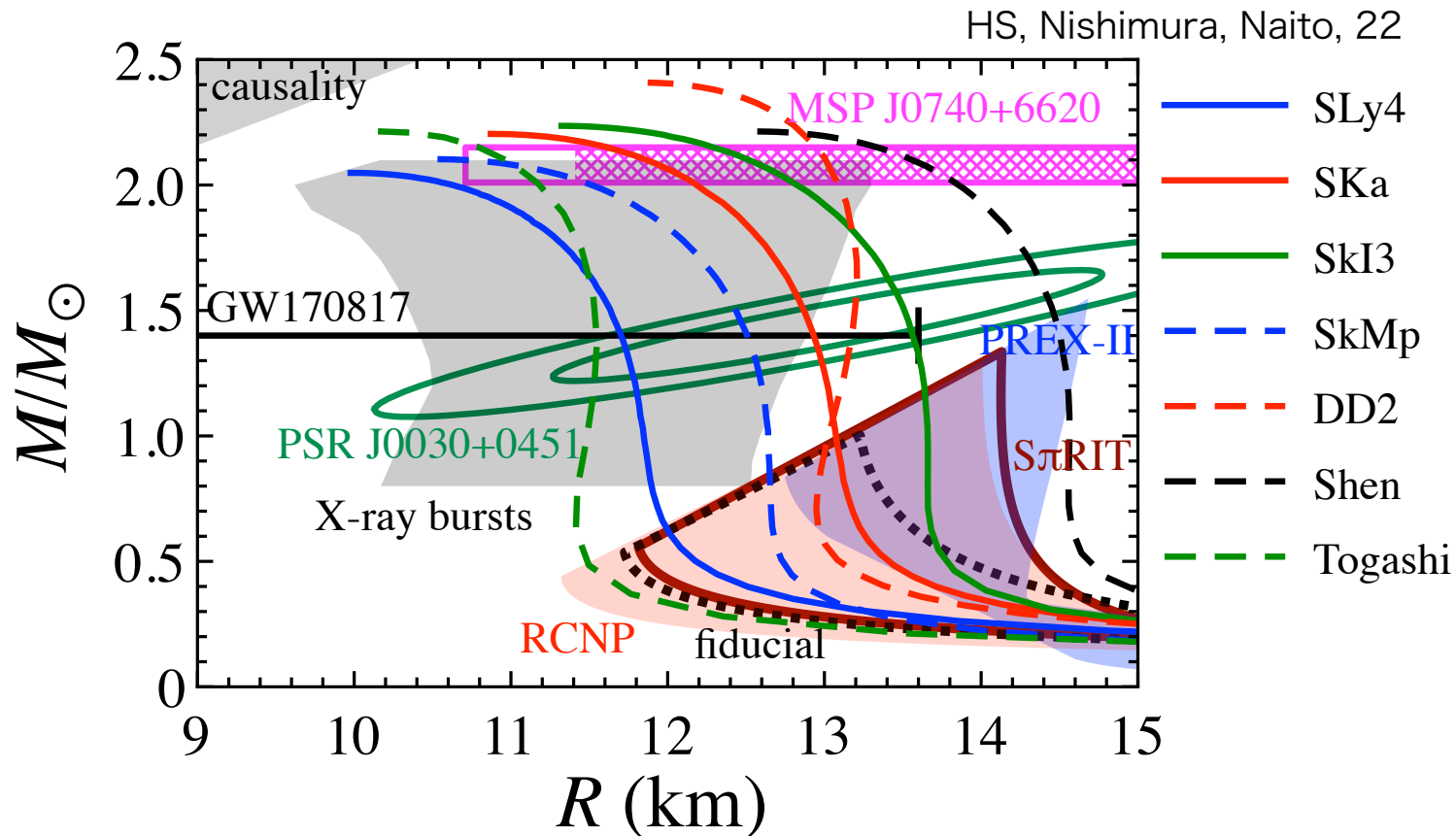


# Supernova gravitational waves and protoneutron stars

Hajime SOTANI (Kochi Univ.)

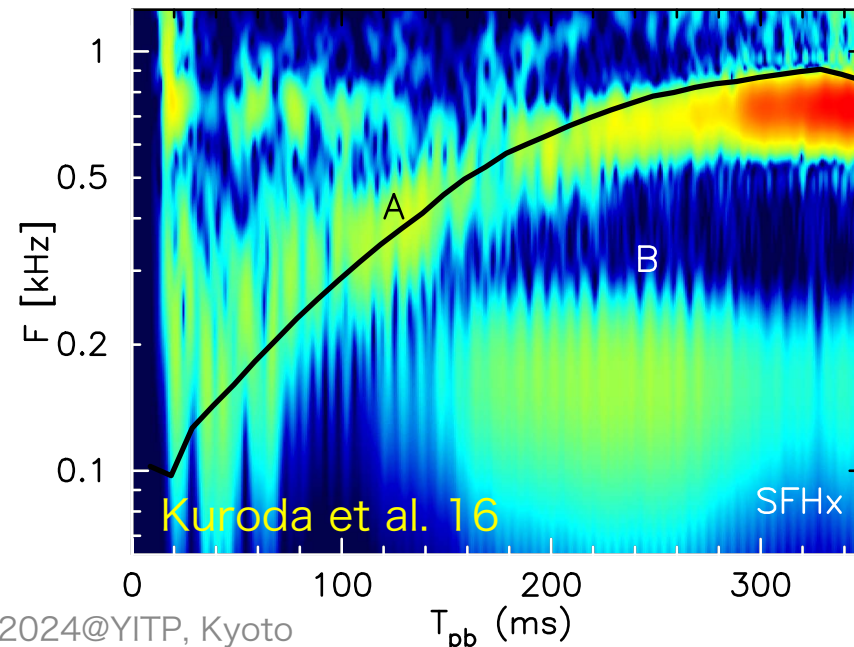
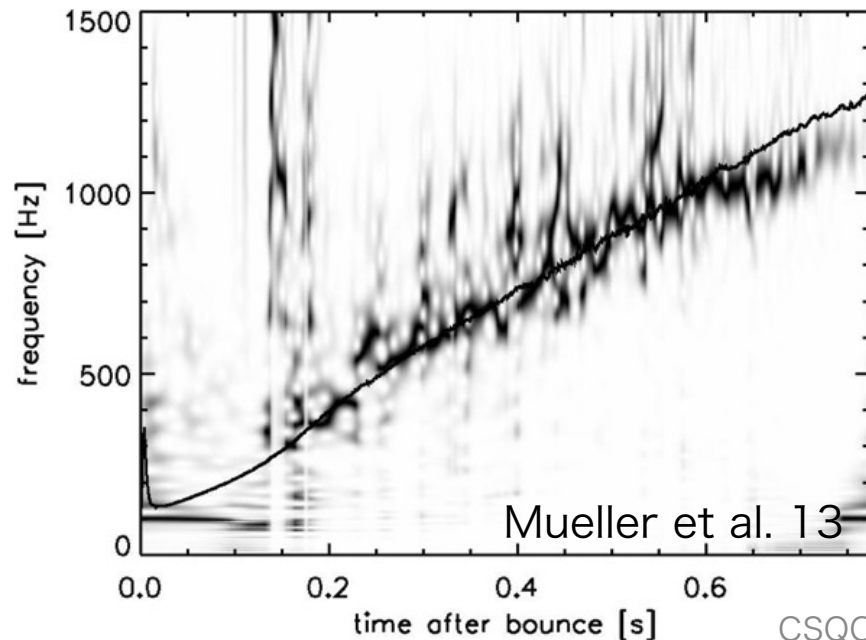
# Dawn of GW astronomy

- GWs from compact binary mergers have been detected.
  - GWs become a new tool for extracting astronomical information.



# Next candidate of GW sources

- core-collapse supernovae
  - compared to the binary merger, the system is almost spherically symmetric
    - less energy of gravitational waves
  - many numerical simulations show the existence of GW signals
  - SN GWs depend on the SN models, such as progenitor mass and EOS
    - how to extract the astronomical information from the GW observations?
    - what is the origin of the SN GWs?
- we adopt [asteroseismology](#) on the PNSs



# How to determine the eigenfrequencies

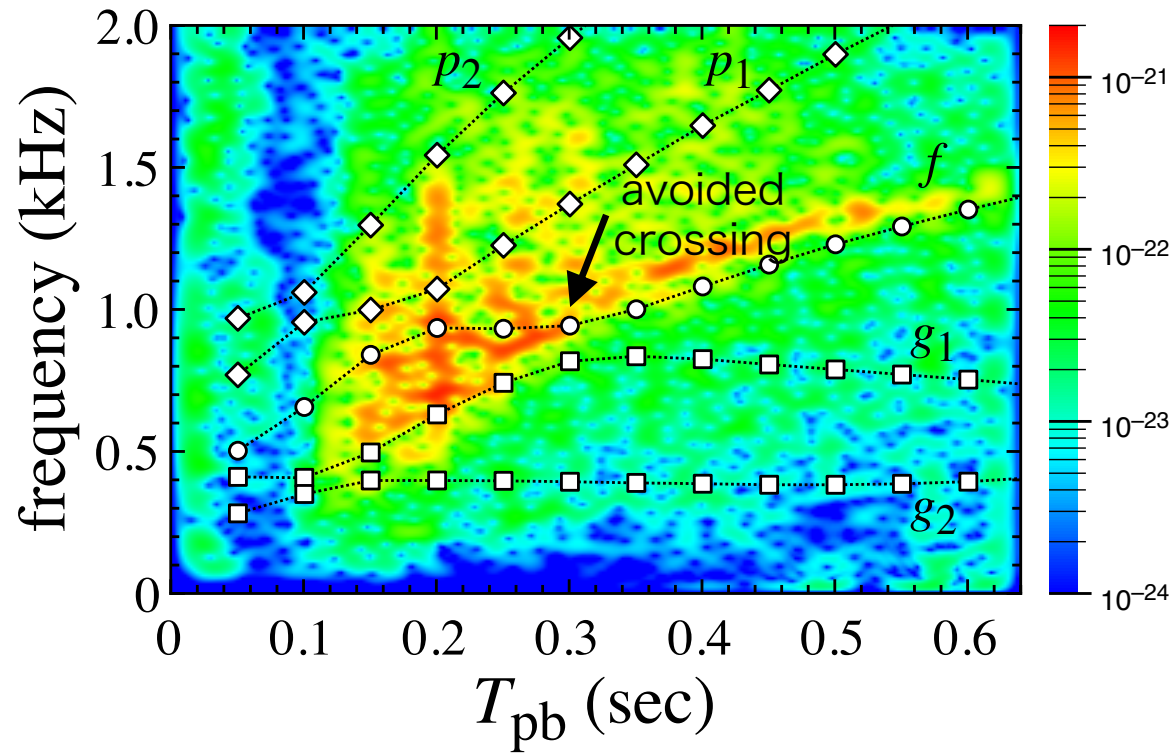
- Variables can be expanded as  $f = f^{(B)} + \delta f$   
 $\delta f$  is decomposed as  $\delta f(t, r, \theta, \phi) = \delta f(r) e^{i\omega t} Y_{\ell m}(\theta, \phi)$
- Perturbation eqs. can be derived by linearizing the energy-momentum conservation law
- Imposing appropriate boundary conditions,  
the problem to solve becomes an eigenvalue problem with respect to the eigenvalue of  $\omega$ .
- Various eigenmodes can be excited, depending on the input physics, e.g.,
  - fundamental (f-) mode
  - pressure (p-) mode
  - gravity (g-) mode
  - gravitational wave (w-) mode
  - shear (s-) mode
  - interface (i-) mode
  - ...

# PNS asteroseismology

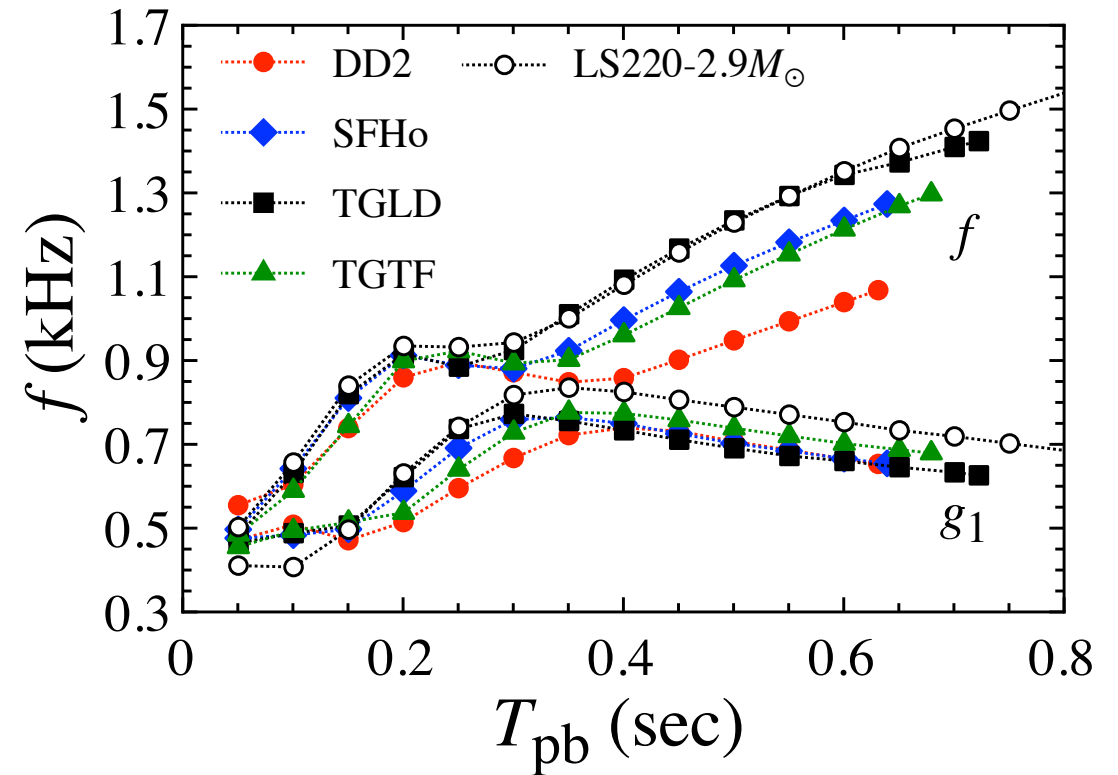
- First, one has to prepare the background PNS model
  - unlike a cold NS, PNS structure depends on not only **the pressure and density profiles** but also **the entropy and electron fraction profiles**.
  - these profiles can be determined via a numerical simulation of core-collapse supernovae.
  - Using the numerical data obtained from the simulation, **the spherically symmetric PNS model** is constructed by averaging the quantities in the angular direction.
- On such a PNS model, we determine the eigenfrequencies by solving an eigenvalue problem.

# Comparison with GW signals

- GW signals correspond to  $g_1$ -mode in an early phase and  $f$ -mode after avoided crossing.
  - time evolution of frequencies strongly depends on supernova models



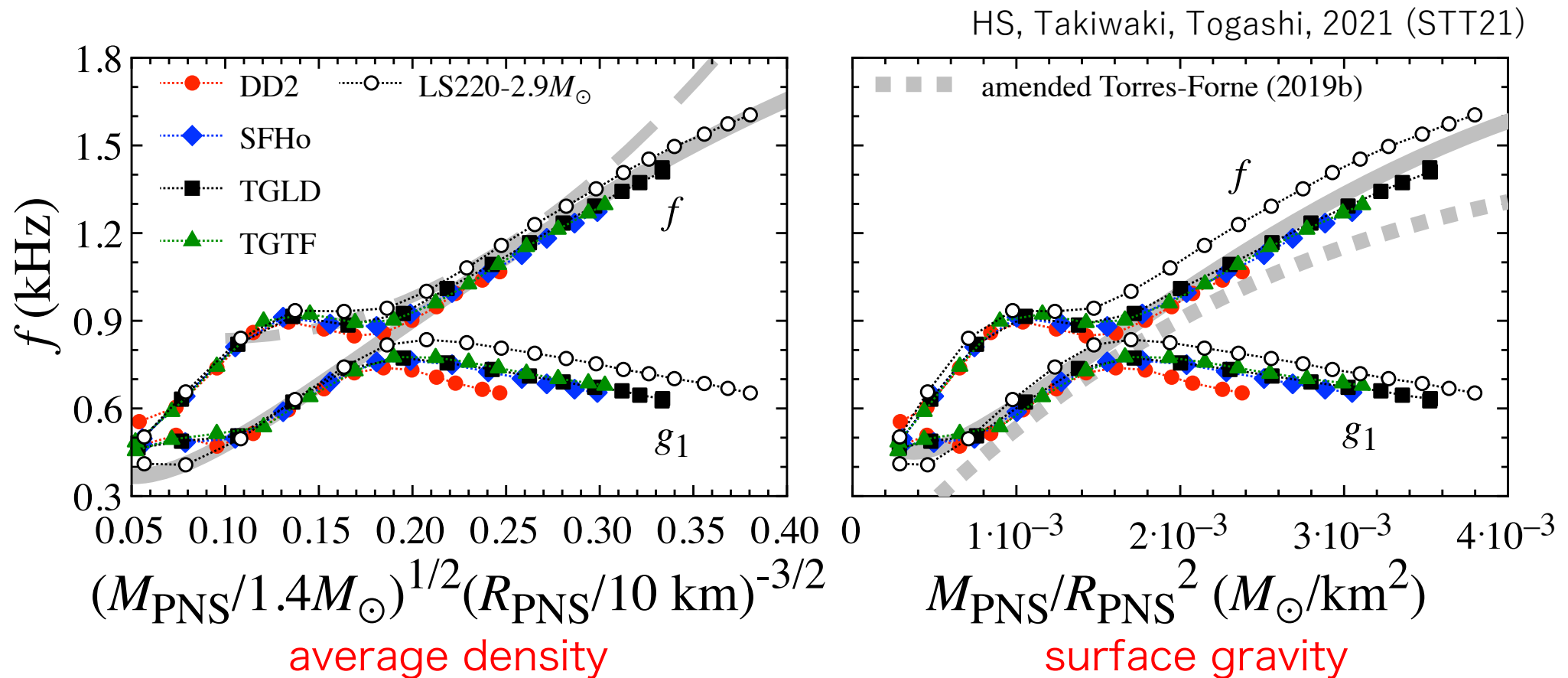
HS, Takiwaki, MNRAS 2020



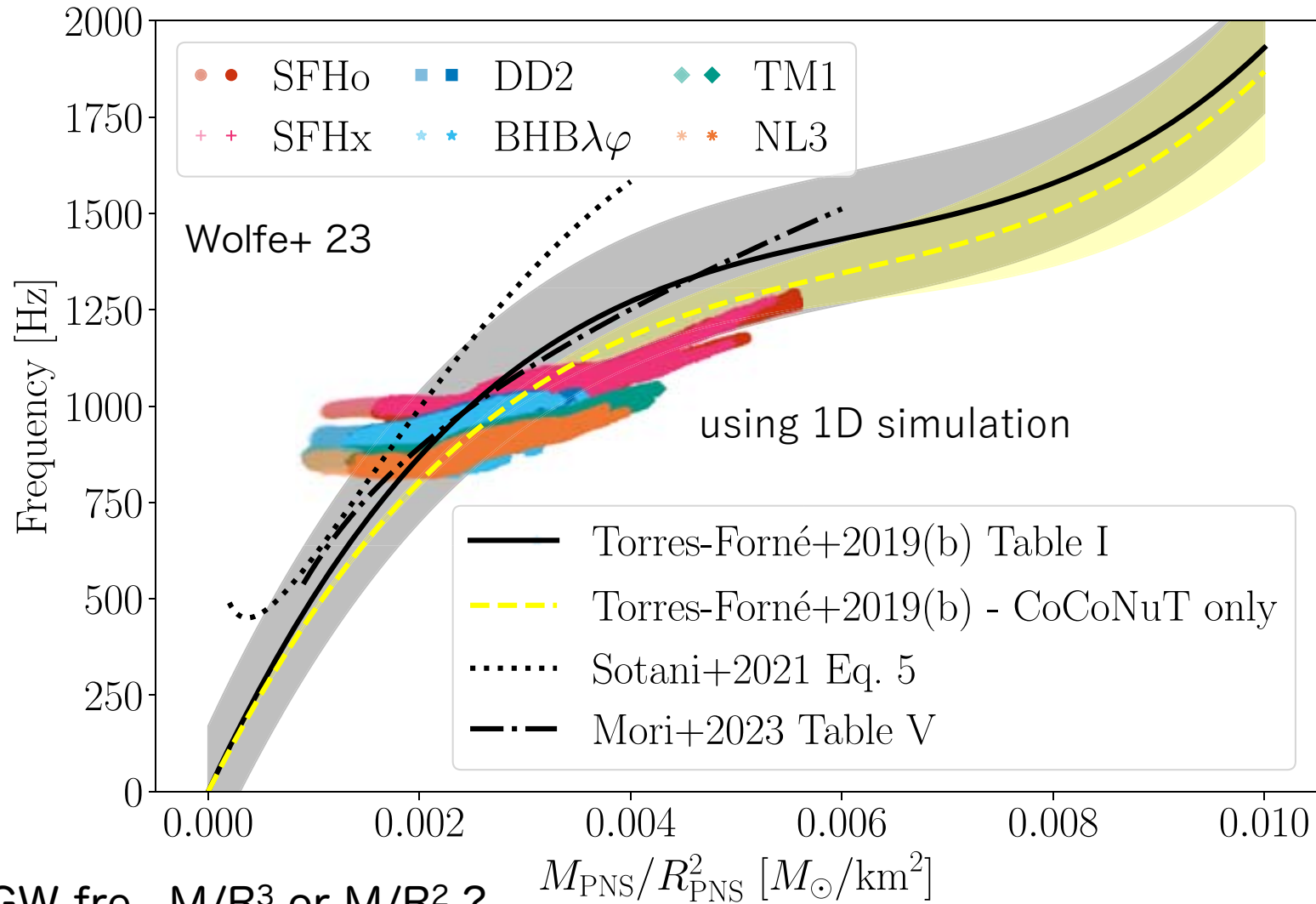
HS, Takiwaki, Togashi, 2021

# Universal relations

- The  $g_1$ - and  $f$ -mode frequencies can be well expressed as a PNS properties
  - average density seems to be more suitable than surface gravity



# Comparison of universal relations



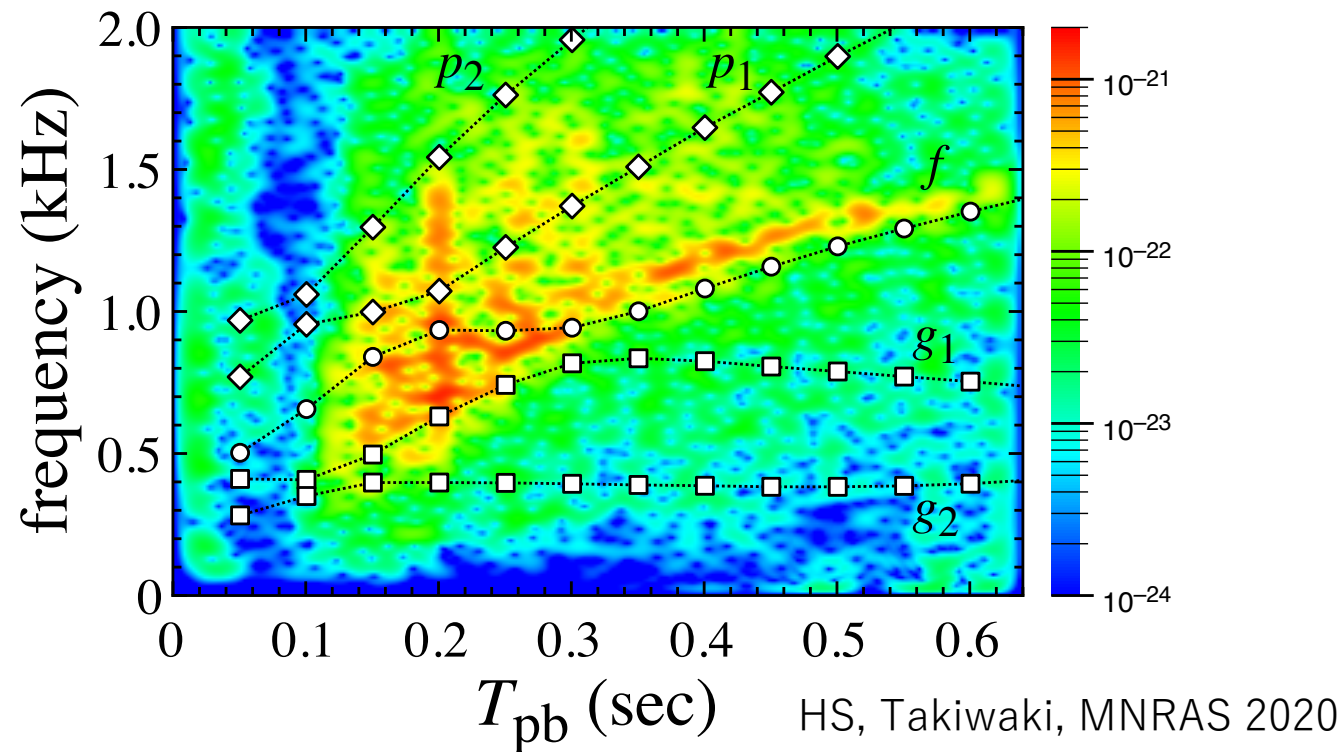
to express the GW fre.,  $M/R^3$  or  $M/R^2$  ?

$$M_{\text{PNS}}/R_{\text{PNS}}^2 [M_{\odot}/\text{km}^2]$$



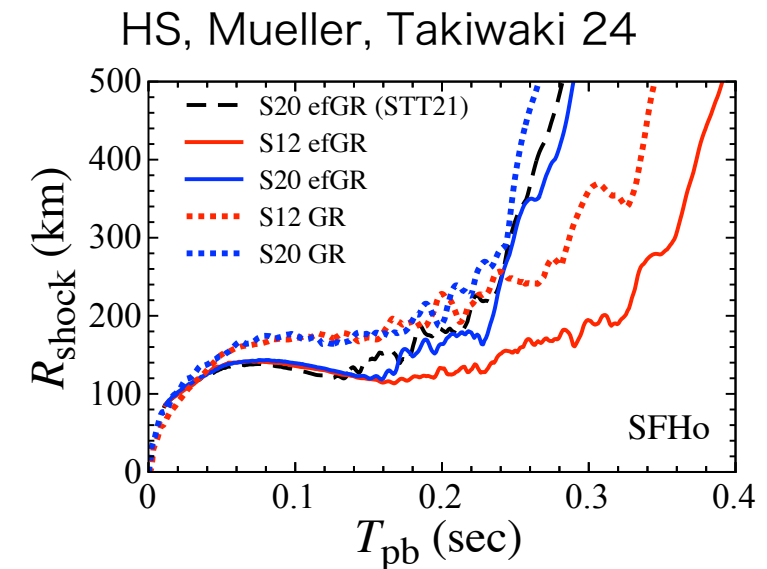
# Open issues

- Which is better to express the supernova gravitational waves,  $M/R^3$  or  $M/R^2$ ?
  - numerical scheme (different interpolation)
  - effective GR vs GR
- The gravitational wave spectra in the simulation deviate from the PNS oscillation frequency.



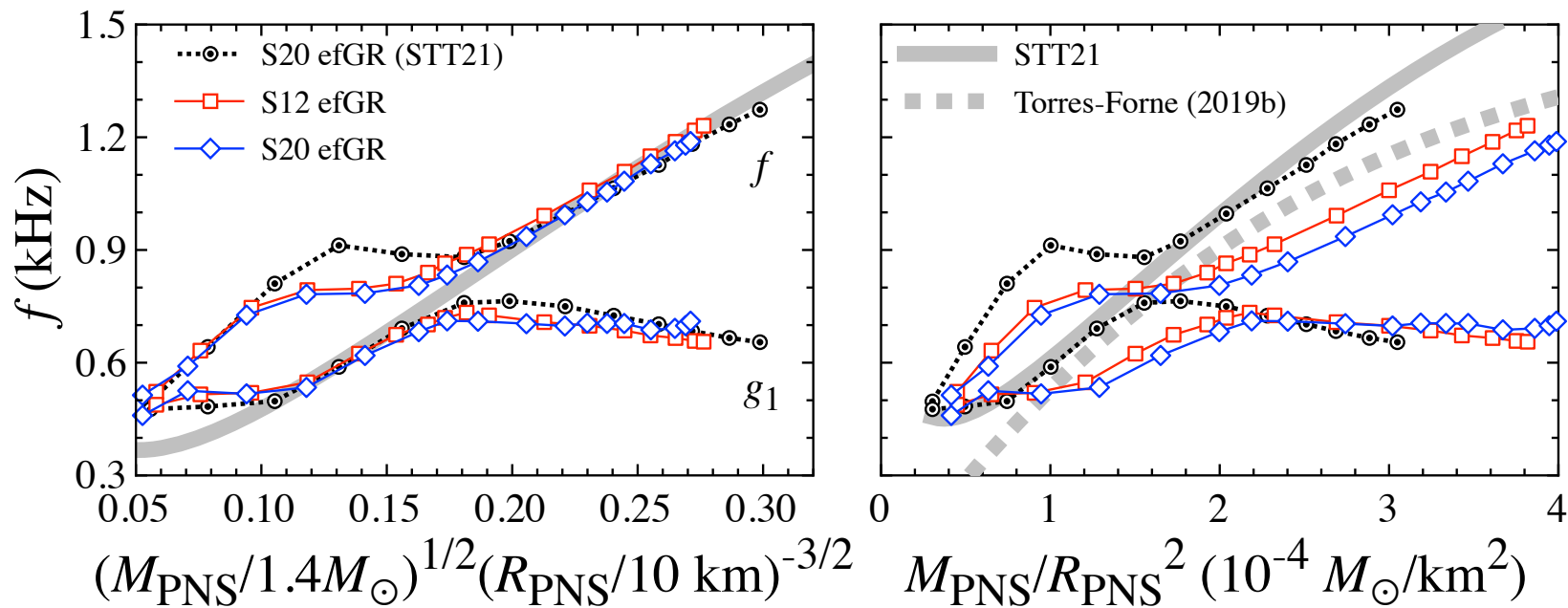
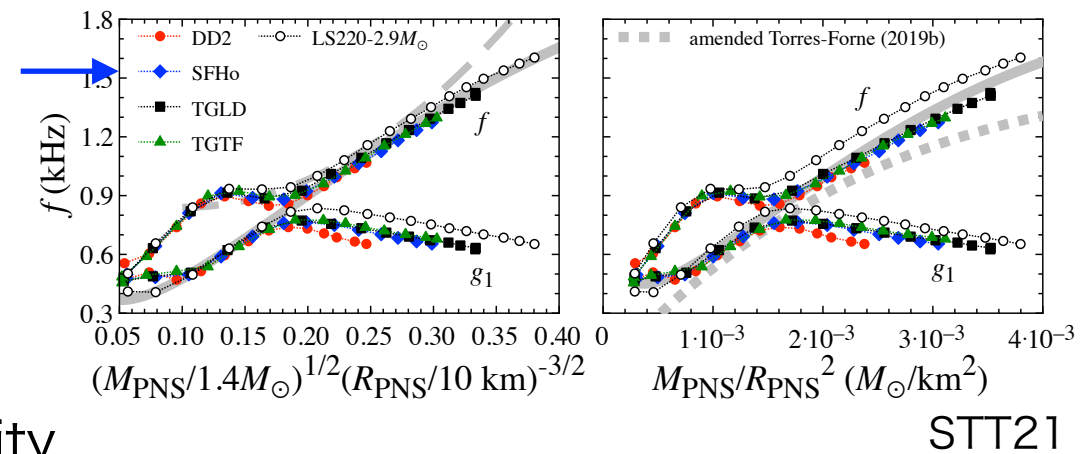
# Average density or Surface gravity

- Supernova models
  - SFHo EOS
  - S12 & S20 (Woosley & Heger 07)
- We see how PNS frequencies (corresponding to GW signals in the simulations) depend on
  - **interpolations in the simulations** (2nd or 5th-order) in Newtonian
  - **treatment of gravity in the simulations** (Newtonian or GR)
- Newtonian simulation with approximate potential (**effective GR**)
  - 2nd-order interpolation with Harten-Lax-van Leer contact (STT21)
  - 5th-order interpolation with Harten-Lax-van Leer discontinuities
- Relativistic simulations (GR)
  - conformal flatness approximation



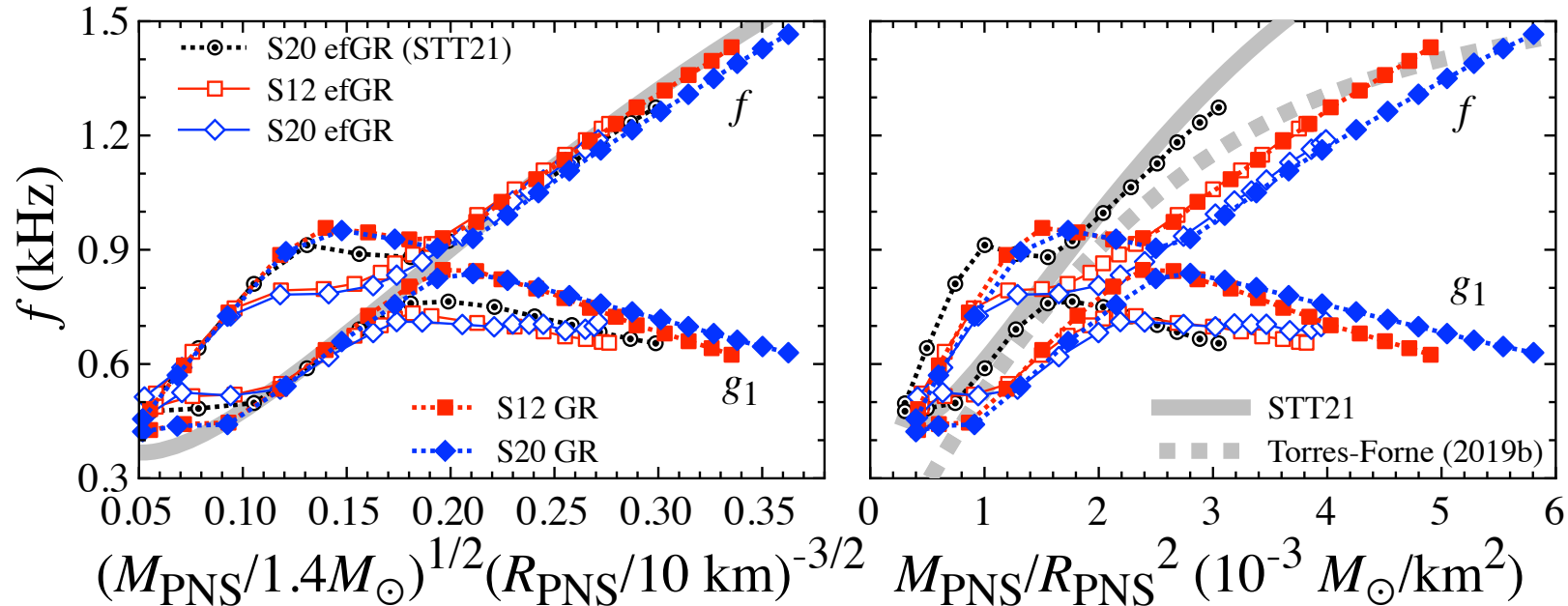
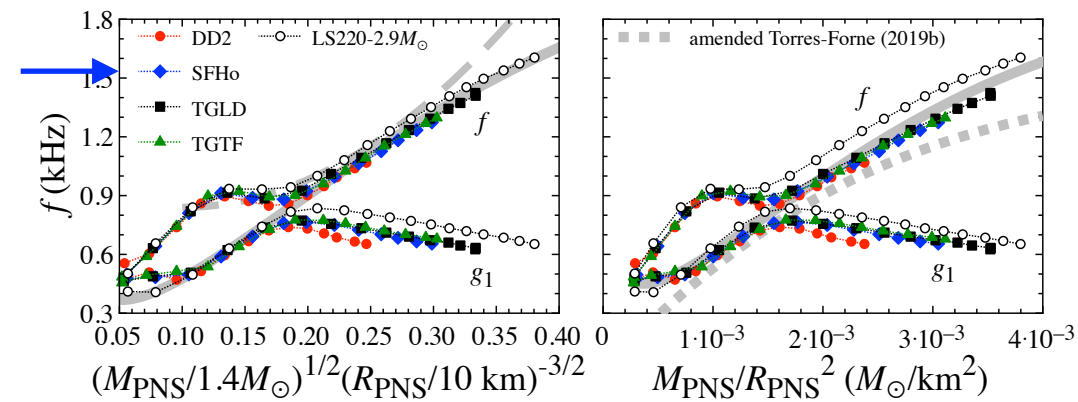
# universal relations in effective GR

- the sequence of the GW signals from the  $g_1$ - to  $f$ -modes is well expressed with the empirical relation as a function of the root square of the average density, even if the interpolation in the simulations is different.
- Meanwhile, those frequencies seem to depend on the interpolation in the simulations if one considers the function of the surface gravity.



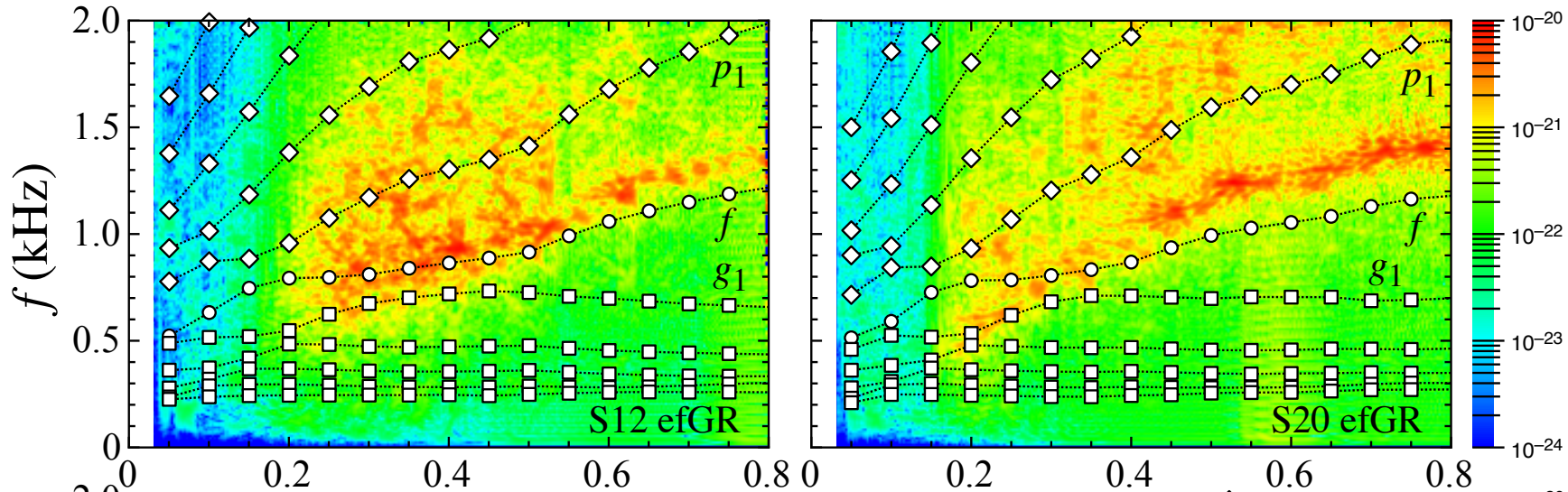
# universal relations in GR

- The sequence from the  $g_1$ - to  $f$ -mode frequencies are well expressed with the protoneutron star average density, even for the case of GR simulations.
- The dependence of the  $g_1$ - and  $f$ -modes on the surface gravity with GR simulation are more or less similar to those with effective GR simulations newly done in this study, which deviate from the empirical relation derived in STT21

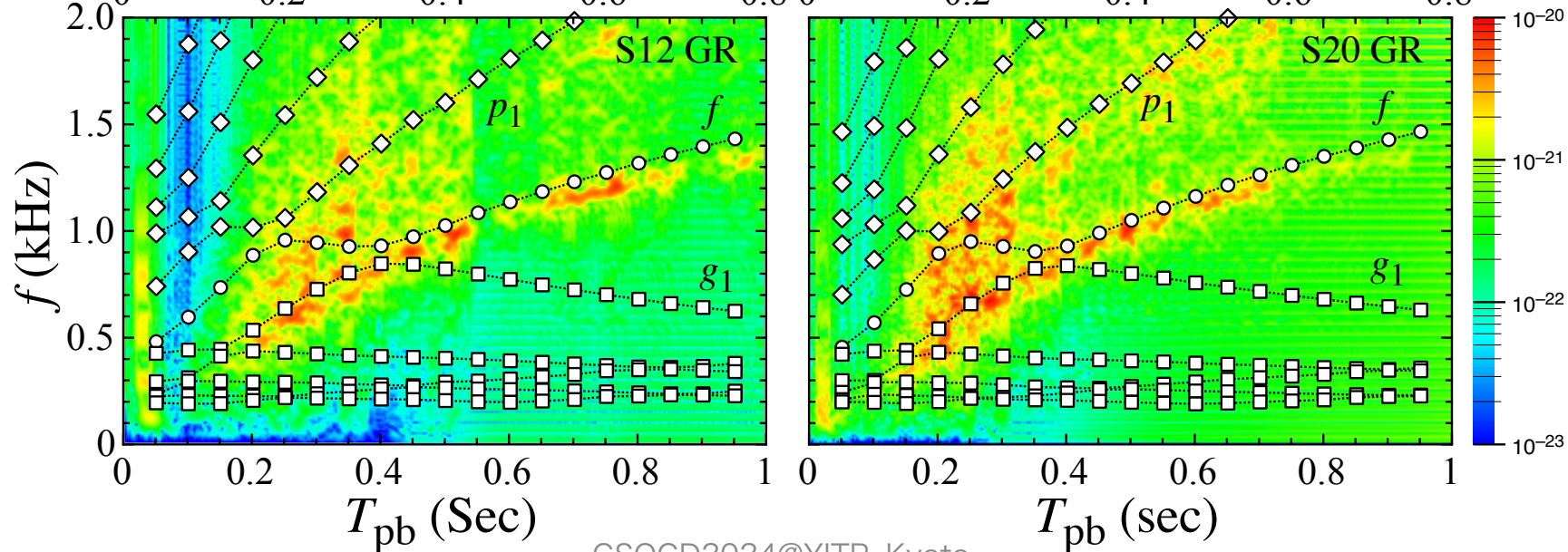


# GW spectrum & PNS frequencies

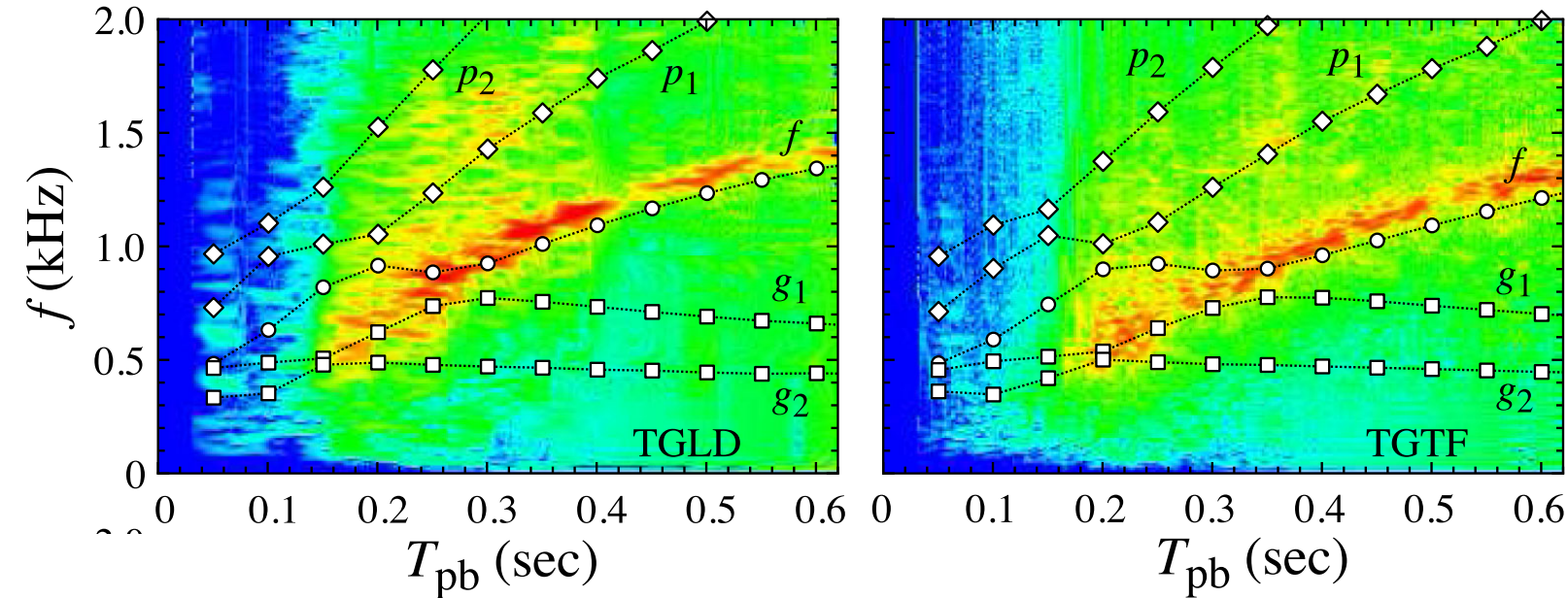
effective GR



GR

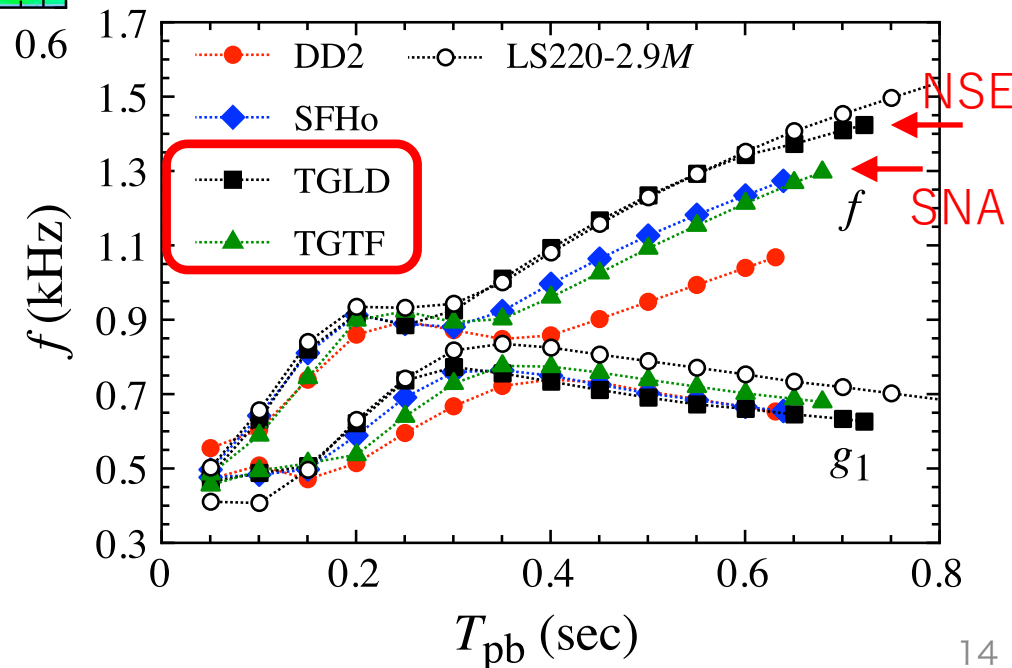


# One comment on the EOS dependence



effective GW with  $20M_{\odot}$  progenitor  
 HS, Takiwaki, Togashi, 2021

- Supernova GWs depend on the nuclear matter EOS.
- Also, they depend on the treatment of nuclei in a low-density region, i.e.,
  - single nucleus approx. (SNA)
  - nuclear statistical equilibrium (NSE)



# Conclusion

- To see how the PNS frequencies (corresponding to the GW signals in the simulations) depend on the difference of the interpolation and/or the treatment of gravity in the simulation, we newly did the simulations with effective GR and GR
  - we did a linear analysis, using the resultant simulation data
- The sequence of GW signals is well expressed as a function of PNS average density, instead of the PNS surface gravity, independently of the supernova models and numerical scheme.
- With the simulation data done in the GR framework, GW spectrum can be well identified with the PNS oscillation frequencies.
- One can extract the PNS average density via the observations of GWs from SN.