Damping of density oscillations from bulk viscosity in quark matter

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Compact Stars in the QCD Phase Diagram
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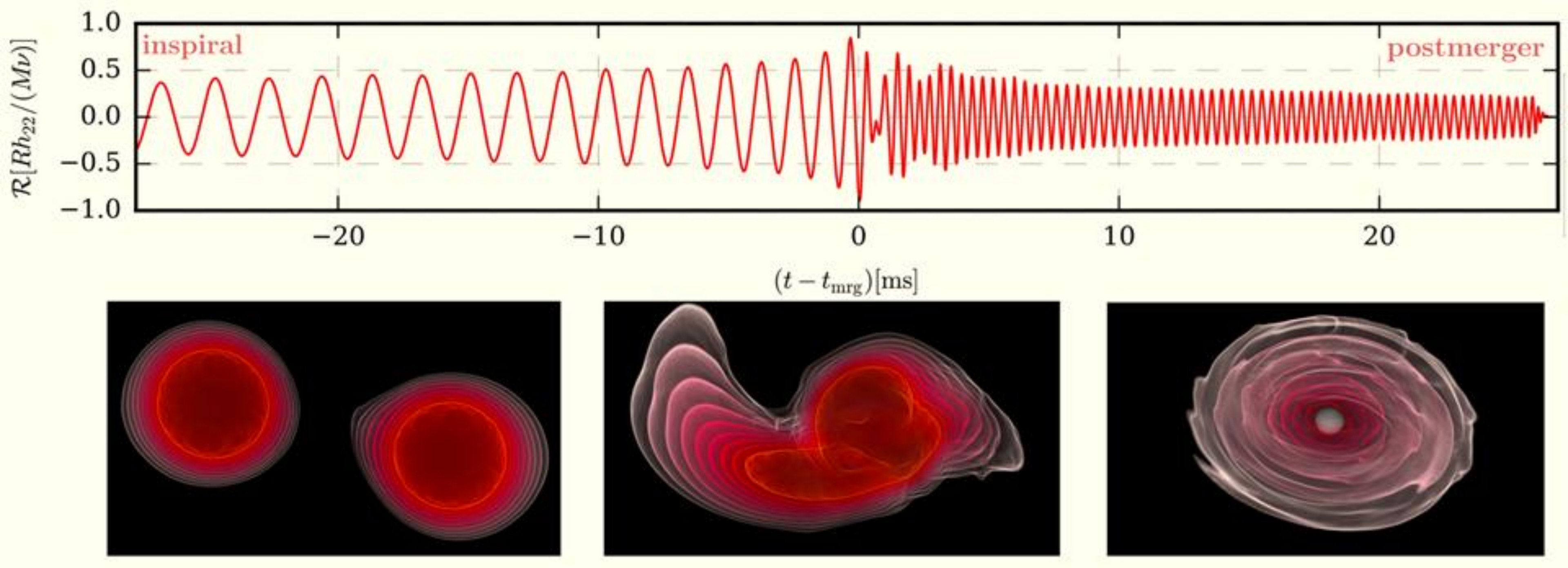
Cofinanciado por la Unión Europea
AGENCIA ESTATAL DE INVESTIGACIÓN

José Luis Hernández, C.M., Laura Tolós arXiv:2402.06595

Numerical simulations of binary mergers

Relativistic numerical simulations require macroscopic and microscopic physics

Dissipation from ζ may be relevant for mergers at $\omega/2\pi = 1$ kHz and for τ of a few ms[1]



Gravitational-wave profile and the matter evolution of a merger[2]

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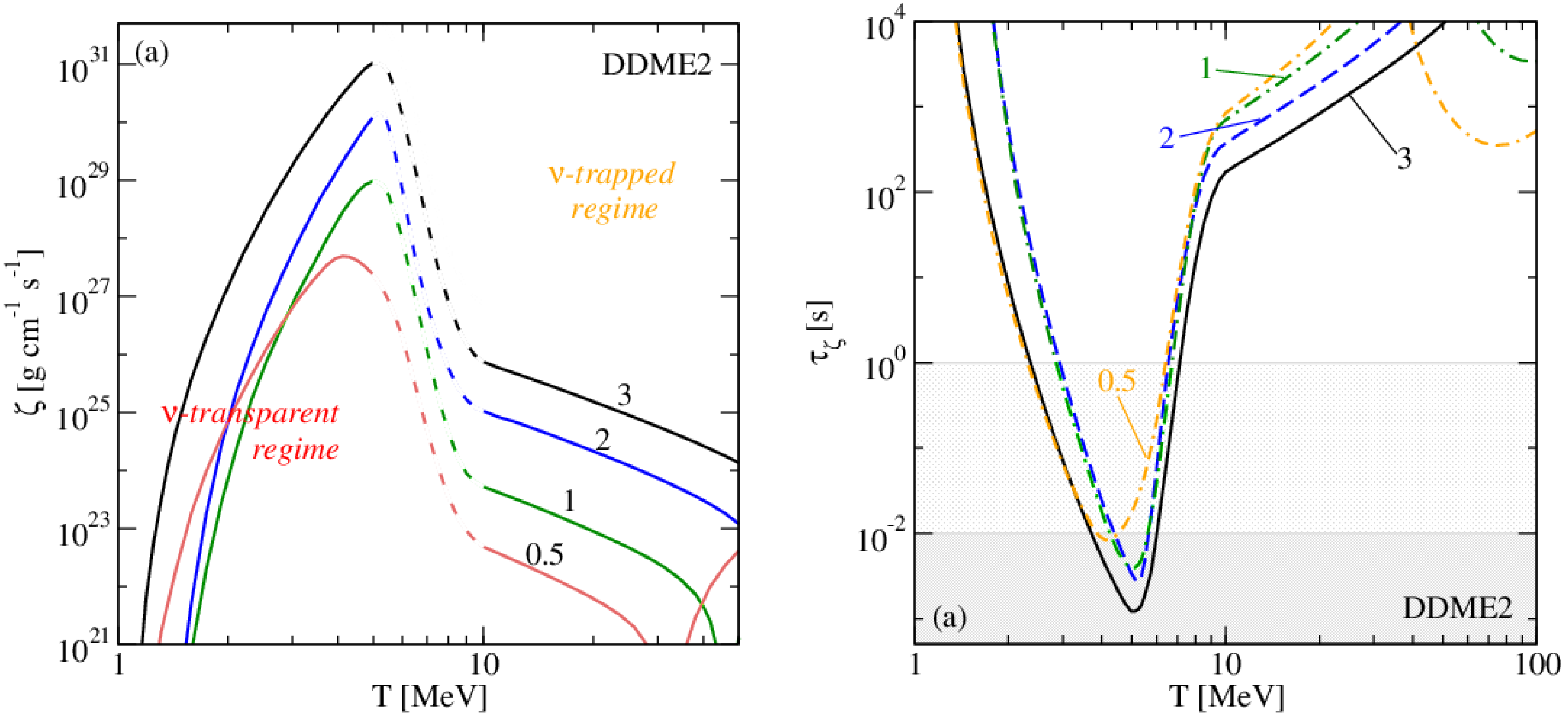
[1] Alford, Bovard, Hanauske, Rezzolla, Schwenzer, 2018, *arXiv:1707.09475*

[2] Dietrich, Hinderer and Samajdar, 2020, *arXiv:2004.02527*

Bulk viscous damping of density oscillations

Bulk viscosity and oscillation damping timescale in $npe\nu_e$ matter[1]



Bulk viscosity and damping timescale as a function of temperature for various densities at $\omega/2\pi = 1$ kHz

A complementary study assuming quark matter was required

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[1] Alford, Harutyunyan and Sedrakian , 2020, *arXiv:2006.07975*

Density oscillations in quark matter

The baryon number density

$$n_B = \frac{1}{3}(n_u + n_d + n_s)$$

For an oscillation proportional to $e^{i\omega t}$

$$n_B = n_{B,0} + \delta n_{B,0}$$

denotes β equilibrium

Conservation of baryon number

$$\partial_\mu(n_B u^\mu) = \theta n_{B,0} + \frac{\partial}{\partial t}\delta n_{B,0} = 0$$

with $\theta = \partial_\mu \delta u^\mu$

$$\delta n_{B,0} = -\frac{\theta}{i\omega} n_{B,0}$$

n_u, n_d, n_s and n_e are modified by processes that change particle species

Weak-interaction processes

Weak interaction happen in the time scale of density oscillations

ν-transparent regime, low T

Nonleptonic processes:

$u + s \longleftrightarrow u + d$

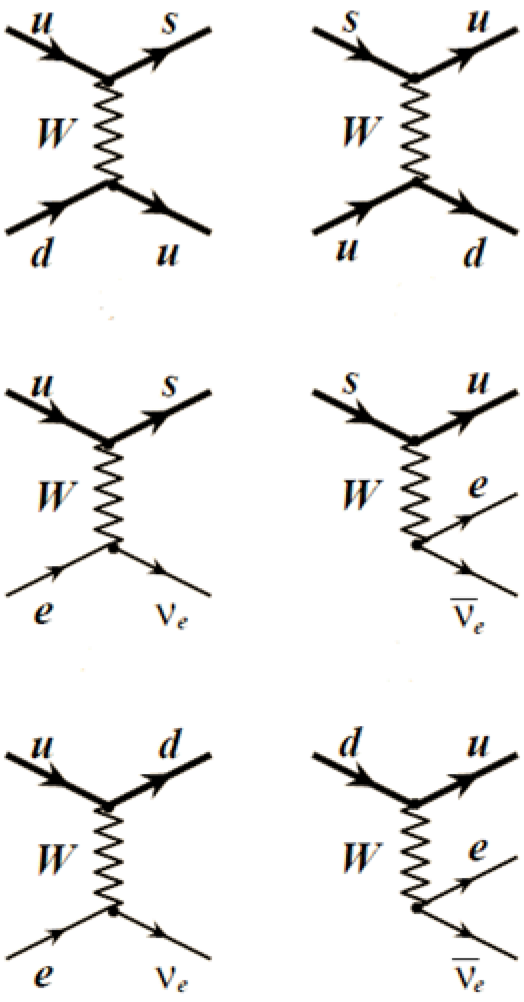
Urca-type processes:

$u + e^- \longrightarrow d + \nu_e$

$d \longrightarrow u + e^- + \bar{\nu}_e$

$u + e^- \longrightarrow s + \nu_e$

$s \longrightarrow u + e^- + \bar{\nu}_e$



Weak processes at tree level[1]

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[1] Sa'd, Shovkovy, Rischke, 2007, *arXiv:astro-ph/0703016*

Weak transition rates

In β equilibrium

$$\Gamma_{u+s\rightarrow d+u} = \Gamma_{d+u\rightarrow u+s}$$

and

$$\mu_d = \mu_s$$
$$\mu_s = \mu_u + \mu_e$$

Density oscillations drive the system out of β equilibrium

$$\Gamma_{d+u\rightarrow u+s} = \Gamma_{s+u\rightarrow d+u}(\mu_1 \rightarrow -\mu_1)^1$$

with chemical imbalances

$$\mu_1 = \mu_s - \mu_d$$
$$\mu_2 = \mu_s - \mu_e - \mu_u$$
$$\mu_3 = \mu_d - \mu_e - \mu_u$$

At linear order

$$\mu_1\lambda_1 = \Gamma_{s+u\rightarrow d+u} - \Gamma_{d+u\rightarrow s+u}$$
$$\mu_2\lambda_2 = \Gamma_{s\rightarrow u+e+\bar{\nu}_e} - \Gamma_{u+e\rightarrow s+\nu_e}$$
$$\mu_3\lambda_3 = \Gamma_{d\rightarrow u+e+\bar{\nu}_e} - \Gamma_{u+e\rightarrow d+\nu_e}$$

[1] Wang and Lu, 1984, *Phys. Lett. B 148, 211*

Particle number density oscillations

Out of β equilibrium, no conservation of particle number density

$$\partial_\mu(n_s u^\mu) = \theta n_{s,0} + \frac{\partial}{\partial t}\delta n_s = -\lambda_1\mu_1 - \lambda_2\mu_2$$
$$\partial_\mu(n_e u^\mu) = \theta n_{e,0} + \frac{\partial}{\partial t}\delta n_e = \lambda_2\mu_2 - \lambda_3\mu_3$$

The equilibration rates[1,2,3,4]

$$\lambda_1 = \frac{64}{5\pi^3} G_F^2 \sin^2\Theta_C \cos^2\Theta_C\, \mu_d^5 T^2$$
$$\lambda_2 = \frac{17}{40\pi} G_F^2 \sin^2\Theta_C\, \mu_s m_s^2 T^4$$
$$\lambda_3 = \frac{17}{15\pi^2} G_F^2 \cos^2\Theta_C\, \alpha_s \mu_d \mu_u \mu_e T^4$$

$$\mu_{u,d} = \left(1 + \frac{2}{3\pi}\alpha_s\right) p_F$$

[1] Wang and Lu, 1984, *Phys. Lett. B 148, 1, 2, 3*

[2] Iwamoto, 1980, *Phys. Rev. Lett. 44, 1637*

[3] Iwamoto, 1982, *Annals of Physics 141, 1*

[4] Sawyer, 1989, *Phys. Lett. B 233, 3, 4*

Out-of-equilibrium pressure

Deviations of the particle number density

$$\delta n_i = \delta n_{i,0} + \delta n_i'$$

denotes
out of β equilibrium

modify the total pressure

$$P(t) = P_0(t) + \delta p'$$

Gibbs-Duhem relation

$$\delta p' = s\delta T + \sum_i n_i \delta\mu_i' \qquad i = e, u, d, s$$

The nonequilibrium pressure $\delta p' = \Pi$

$$\Pi = \sum_i c_i \delta n_i'$$

with $c_i = n_{i,0} A_{ii}$ and $A_{ij} = \partial\mu_i / \partial n_j$

Bulk viscosity in strange quark matter

At first-order relativistic hydrodynamics

$$\zeta = -\frac{\mathrm{Re}[\Pi]}{\theta}$$

The weak-interaction-driven bulk viscosity in strange quark matter

$$\boxed{\zeta = \frac{\kappa_1 + \kappa_2 \omega^2}{\kappa_3 + \kappa_4 \omega^2 + \omega^4}}$$

ω is the frequency of the perturbation

$$\kappa_j \left(\mu_{i,0}, A_{ii}, n_{B,0}, \lambda_1(T), \lambda_2(T), \lambda_3(T) \right)$$

with $i = e, u, d, s$ and $j = 1, 2, 3, 4$

EoS for strange quark matter

The bulk viscosity requires a model of strange quark matter

β-equilibrium conditions

$$\mu_{d,0} = \mu_{s,0}$$
$$\mu_{s,0} = \mu_{u,0} + \mu_{e,0}$$

Charge neutrality

$$n_e + \frac{1}{3} n_s + \frac{1}{3} n_d = \frac{2}{3} n_u$$

n_i can be obtained from the thermodynamic potential, Ω

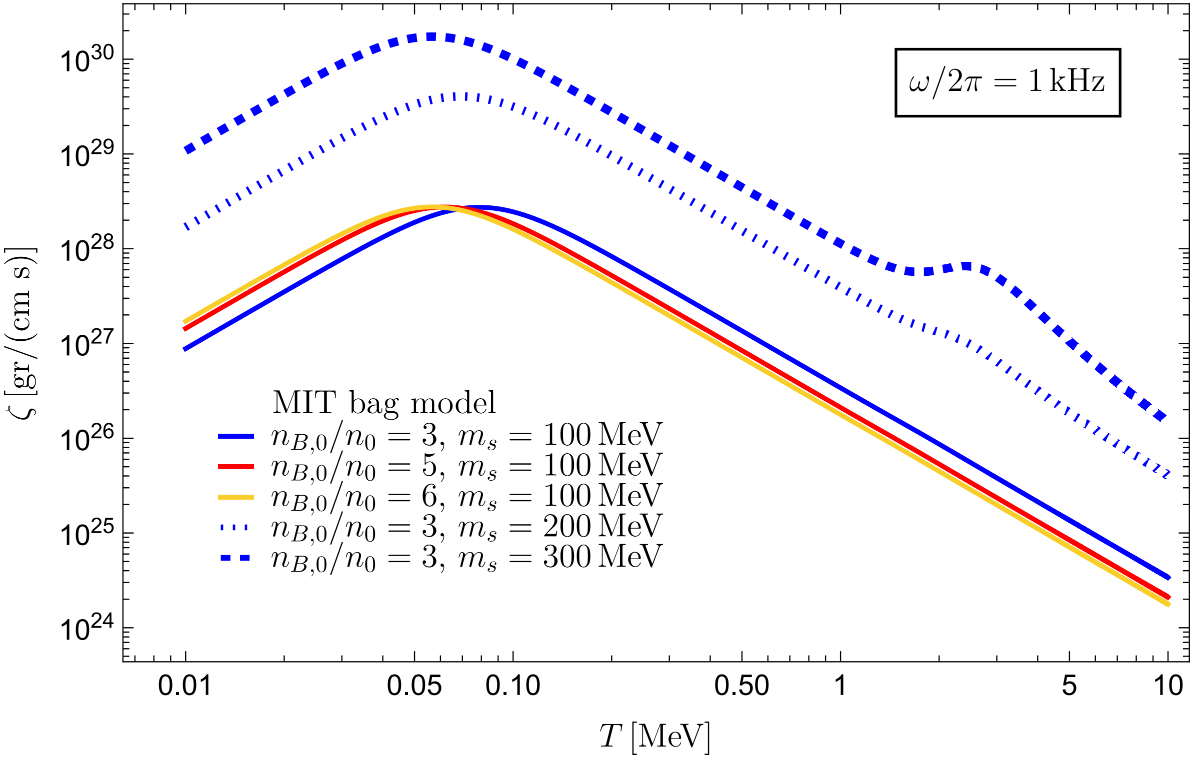
$$n_i = -\left(\frac{\partial \Omega}{\partial \mu_i}\right)_{T,V}$$

Bulk viscosity using the MIT bag model

$\zeta(T)$ varying $n_{B,0}$ and m_s

$\zeta_{\max}$ increases with $n_{B,0}$

Strong dependence on m_s



$\zeta_{\max} \in [10^{28}, 2\times10^{30}]$ gr/(cm · s) at $T \in [0.01, 0.1]$ MeV

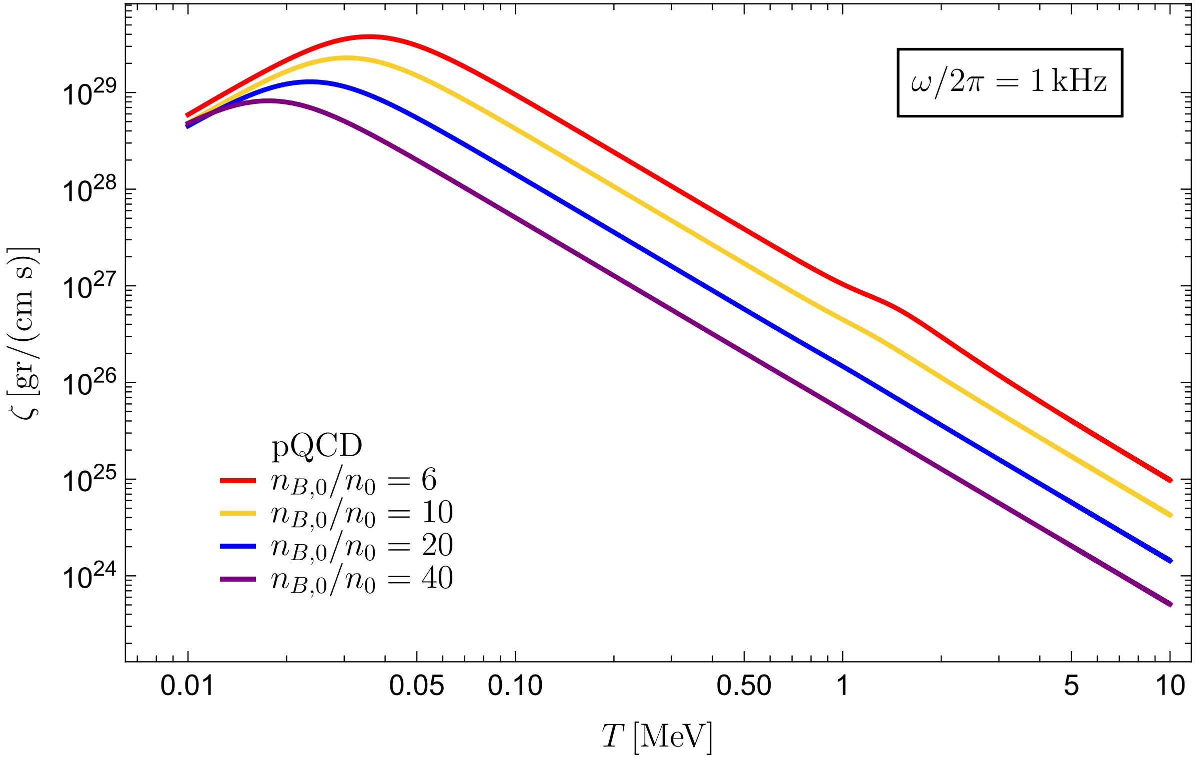
1 MeV$\sim 10^{10}$ K and $n_0 = 0.15\ \mathrm{fm}^{-3}$

Bulk viscosity using perturbative QCD

$\zeta(T)$ varying $n_{B,0}$ fixing[1] $\bar{\Lambda} = 2\mu_s$

$\zeta_{\max}$ decreases with $n_{B,0}$

m_s, α_s decrease with $n_{B,0}$



$\zeta_{\max} \in [8 \times 10^{28}, 4 \times 10^{29}]$ gr/(cm · s) at $T \in [0.01, 0.1]$ MeV

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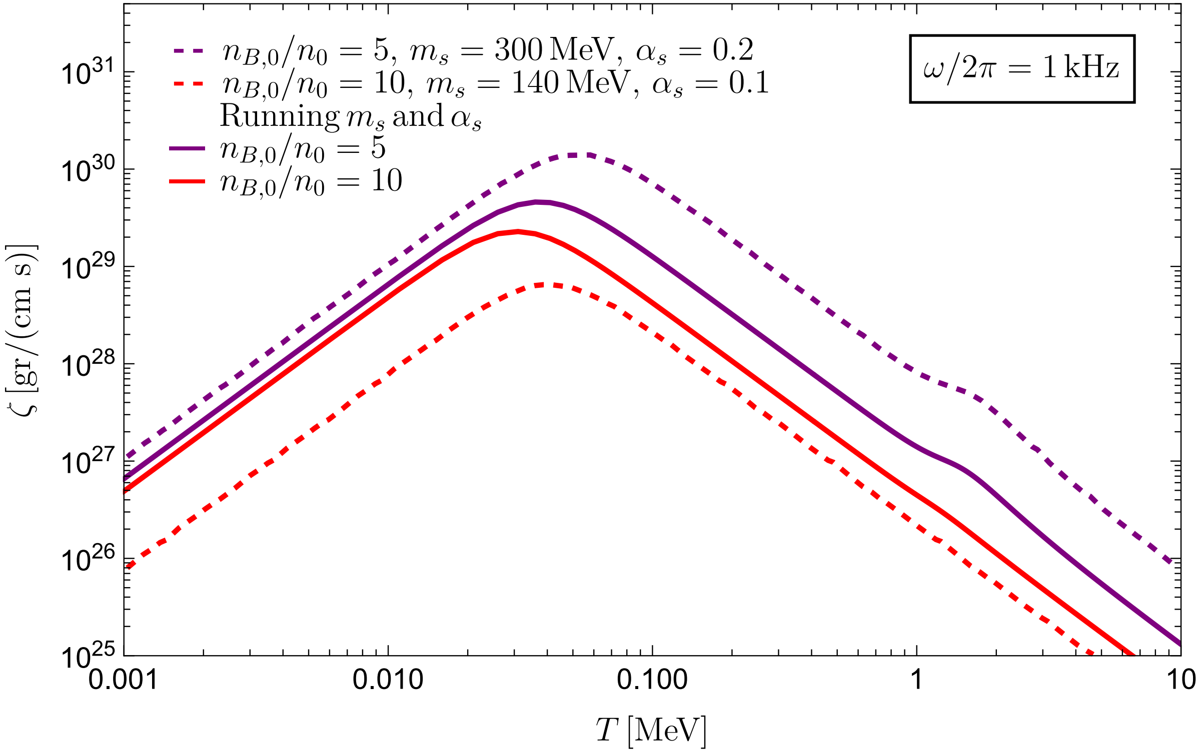
[1] Fraga, Pisarski and Schaffner-Bielich, 2001, *arXiv:hep-ph/0101143*

Previous studies in quark matter

Sawyer, 1989; Madsen, 1992; nonleptonic channel

Alford, Schmitt, 2006; Sa’d, Shovkovy, Rischke, 2007; all channels



Comparison of bulk viscosity using pQCD with fixed[1] and running m_s and α_s

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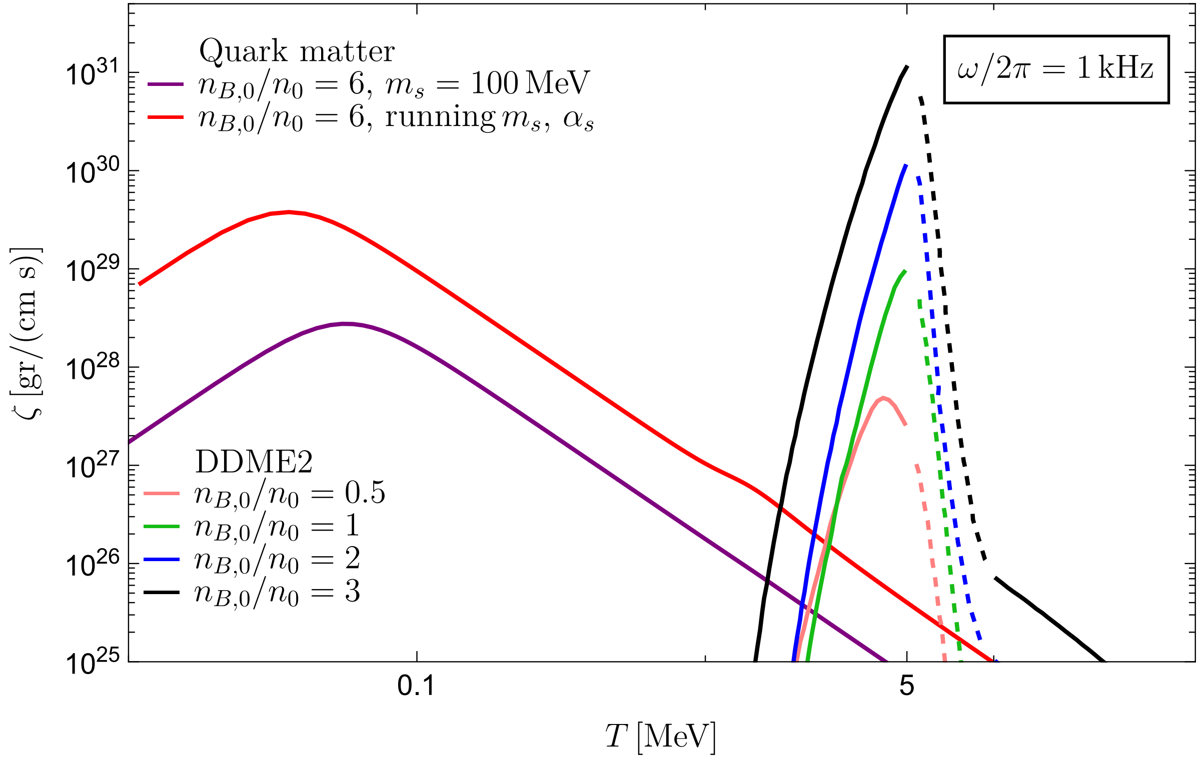
[1] Sa’d, Shovkovy, Rischke, 2007, *arXiv:astro-ph/0703016*

Bulk viscosity in nuclear matter

Bulk viscosity in quark matter differs from the one in hadronic matter



Comparison of the bulk viscosity in quark matter with the one predicted using the DDME2 model[1]

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[1] Alford, Harutyunyan and Sedrakian , 2020, *arXiv:2006.07975*

Damping of baryon density oscillations

Dissipation of energy due to volume expansion and compression

Energy density stored in an oscillation, ϵ

$$\epsilon = \frac{1}{2}\frac{d^2\varepsilon}{dn_B^2}\left(\delta n_{B,0}\right)^2$$

Energy dissipation time, $\dot{\epsilon}$

$$\dot{\epsilon} = \frac{\omega^2\zeta}{2}\left(\frac{\delta n_{B,0}}{n_{B,0}}\right)^2$$

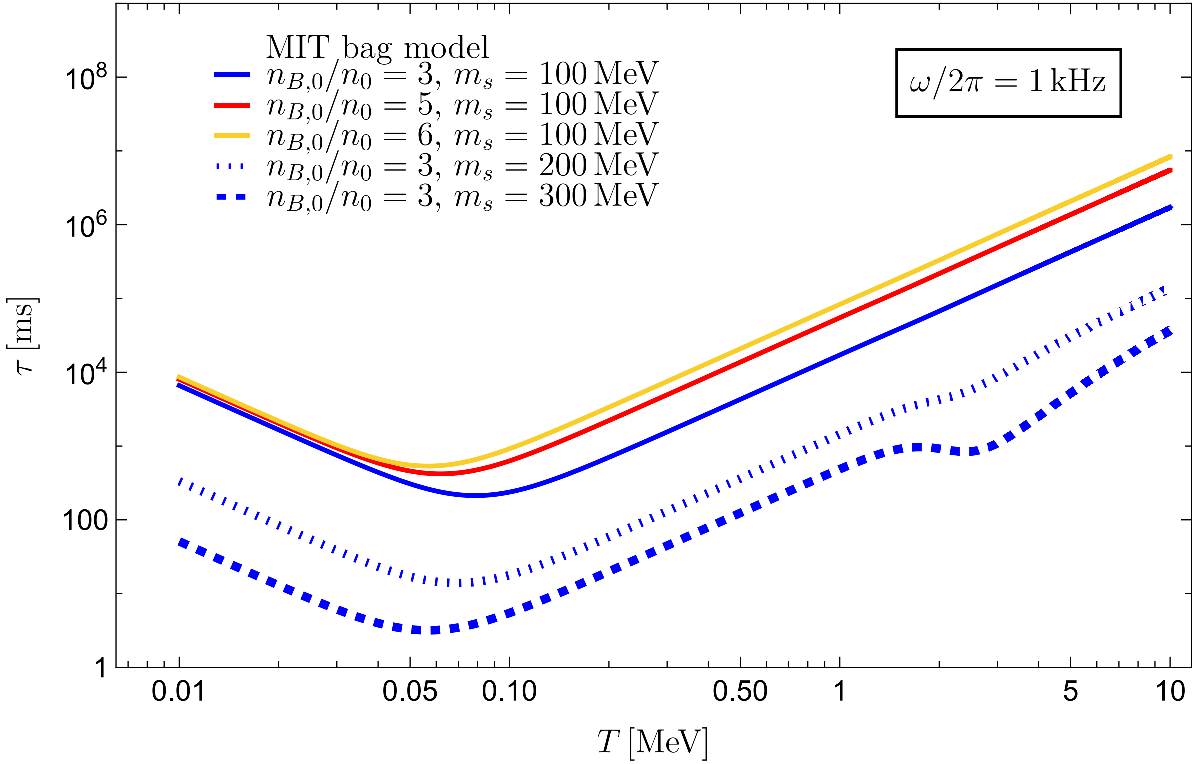
The damping time of a baryon number density oscillation,

$$\tau \equiv \epsilon/\dot{\epsilon}$$

$$\boxed{\tau = \frac{n_{B,0}^2}{\omega^2\zeta}\ \frac{d^2\varepsilon}{dn_B^2}}$$

Damping times using the MIT bag model

ζ determines temperature dependence of τ



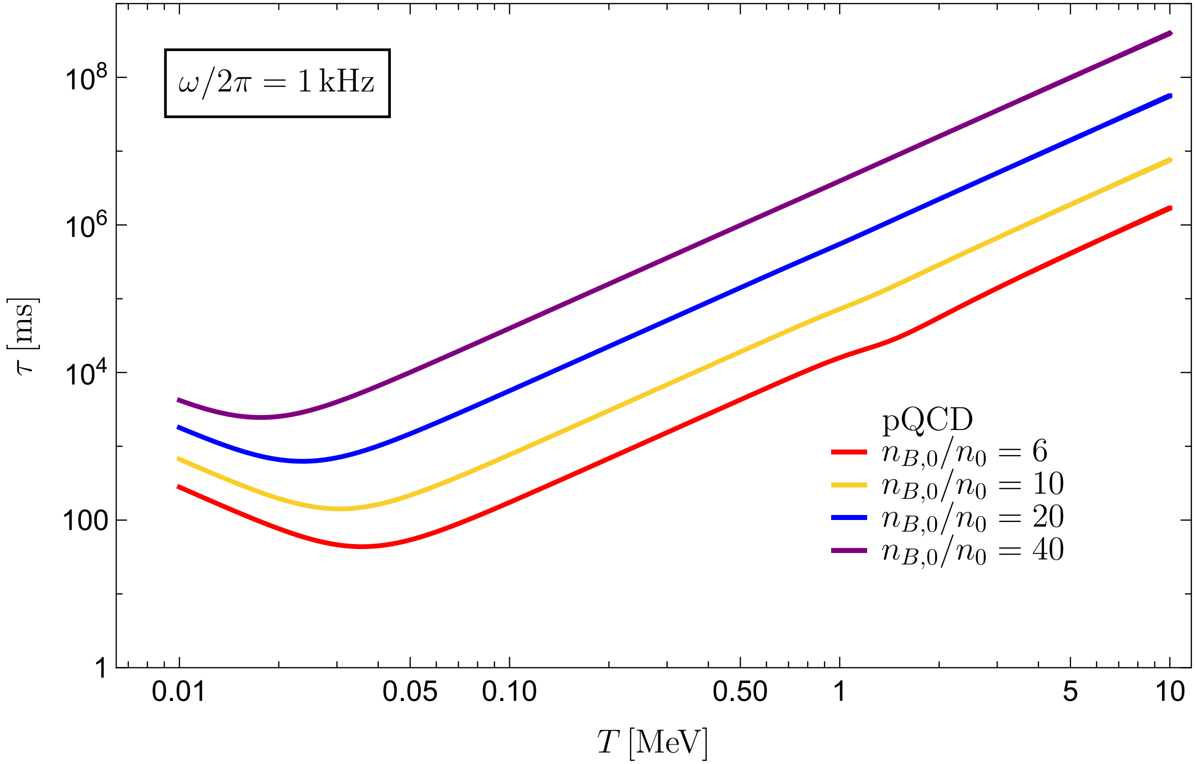
Damping times of density oscillations using the MIT bag model

τ_{min} up to a few hundred ms at $T \in [0.01, 0.1]$ MeV

$n_{B,0}/n_0$	T_{m} [MeV]	$\zeta_{\max}$ [gr/(cm·s)]	$\tau_{\min}$ [ms]	m_s [MeV]
3	7.9×10^{-2}	2.73×10^{27}	213.24	100
3	6.9×10^{-2}	4.05×10^{29}	13.93	200
3	5.7×10^{-2}	1.73×10^{30}	3.19	300
5	6.2×10^{-2}	2.75×10^{28}	420.14	100
6	5.7×10^{-2}	2.76×10^{28}	535.37	100

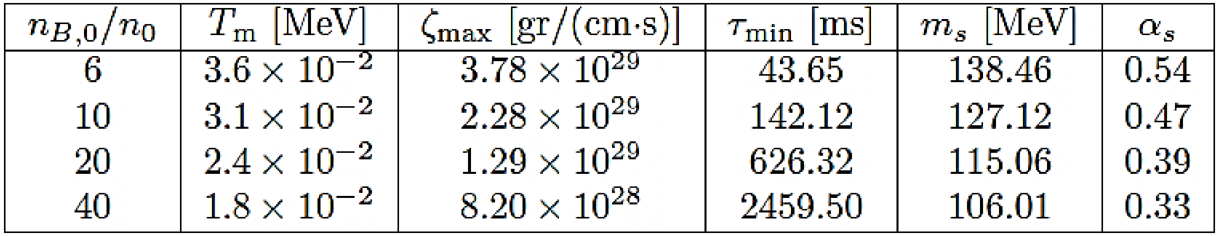
Damping times using perturbative QCD

Lower times as the baryon number density decreases



Damping times of density oscillations using the pQCD up to $\mathcal{O}(\alpha_s)$

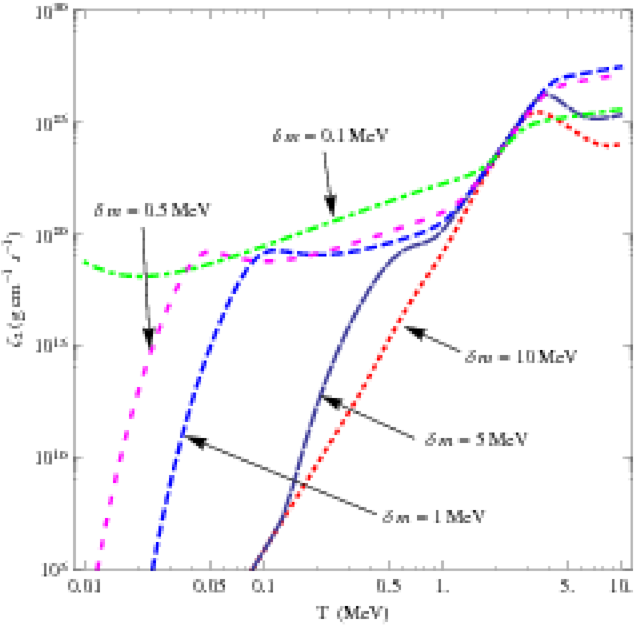
τ_{min} up to a few hundred ms at $T \in [0.01, 0.1]$ MeV



$n_{B,0}/n_0$	T_m [MeV]	ζ_max [gr/(cm·s)]	τ_min [ms]	m_s [MeV]	α_s
6	3.6×10^{-2}	3.78×10^{29}	43.65	138.46	0.54
10	3.1×10^{-2}	2.28×10^{29}	142.12	127.12	0.47
20	2.4×10^{-2}	1.29×10^{29}	626.32	115.06	0.39
40	1.8×10^{-2}	8.20×10^{28}	2459.50	106.01	0.33

CFL quark matter

- It is a superfluid; three possible bulk viscosity coefficients

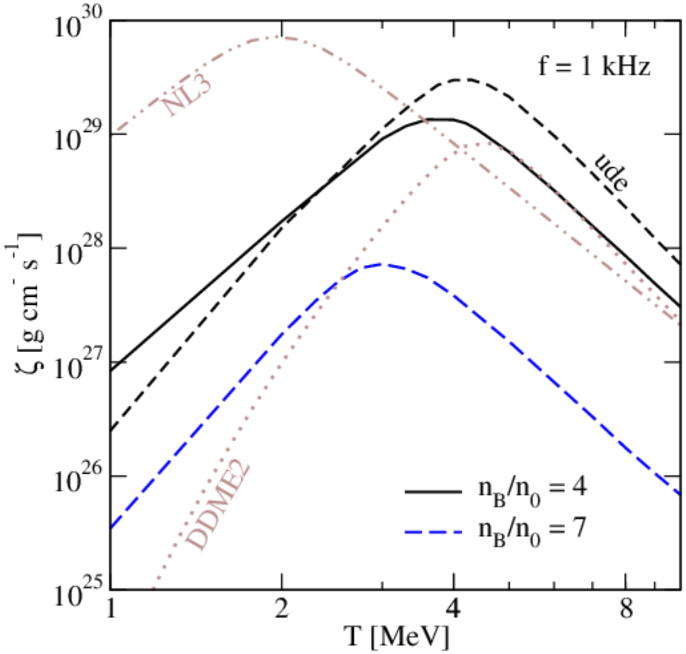


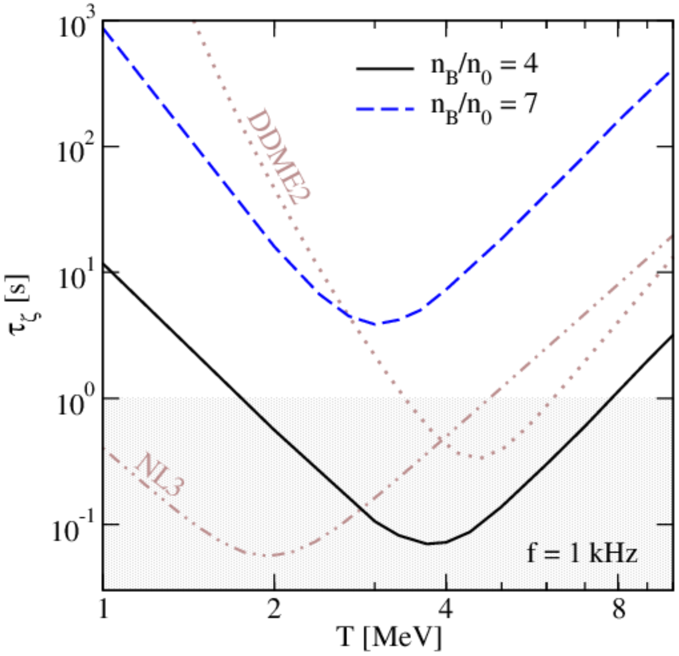
$$\phi + \phi \longrightarrow \phi + \phi + \phi$$

$$K^0 \longrightarrow \phi + \phi$$

Bierkandt and CM, 2011; Mannarelli and CM ; Alford, Braby, Reddy and Schafer

2SC quark matter





Alford, Harutyunyan,
Sedrakian, 24

Conclusions

ζ and τ were addressed in strange quark matter using the MIT bag model and pQCD up to $\mathcal{O}(\alpha_s)$

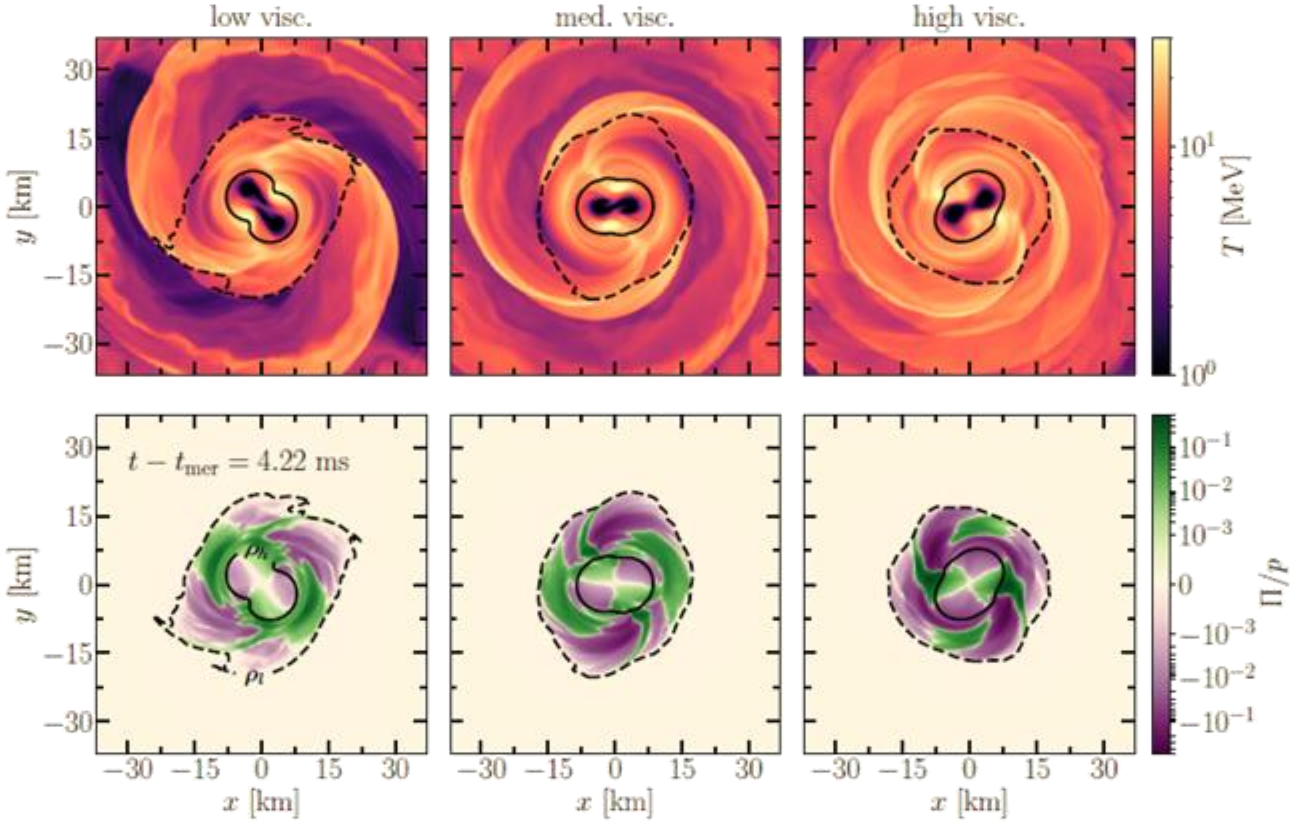
arXiv:2402.06595

- Other quark matter phases are being considered: different behavior of the damping times

- It would be interesting to carry out numerical simulations of the NS mergers incorporating viscosity of different matter phases

Bulk-viscous damping might be relevant in the post-merger phase

Inclusion of bulk-viscous dissipation in numerical simulations of binary mergers has been recently achieved[1]



Cross-sections of the temperature and the ratio of the bulk-viscous pressure over the EOS pressure[1]

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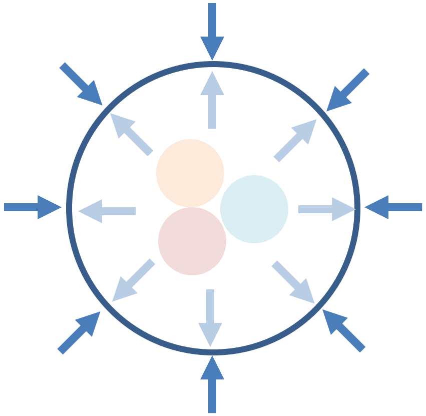
[1] Chabanov M. and Rezzolla L., 2023, *arXiv:2311.13027* and *arXiv:2307.10464*

BACKUP SLIDES

MIT bag model

Quarks inside the bag
reproduce asymptotic freedom and confinement[1]



Outward pressure from confined quarks
is balanced by
the inward vacuum pressure B

- $m_s \neq 0$ and $m_u = m_d = 0$
- $\Omega(m_s, \mu_e, \mu_u, \mu_d, \mu_s, B)$
- Stability windows[2] at $B^{1/4} \in [148 - 159]$ MeV

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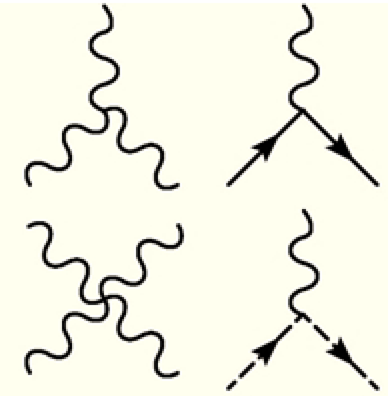
[1] Chodos, Jaffe, Johnson, Thorn, and Weisskopf, 1974, *Phys. Rev. D 9, 3471*

[2] Lopes, Biesdorf, Menezes, 2021, *arXiv*: 2005.13136

Perturbative QCD up to $\mathcal{O}(\alpha_s)$

QCD is studied assuming small $\alpha_s = g^2/4\pi$ at ultrahigh density[1,2]



Interaction among quarks and gluons
described by
gauge, spinor and ghost fields

Studies of the EoS[3] in N3LO suggest pQCD is feasible
for $n_B \sim 40 n_0$ with $n_0 = 0.15\ \text{fm}^{-3}$

- $m_s \neq 0$ and $m_u = m_d = 0$
- $\Omega(m_s, \mu_e, \mu_u, \mu_d, \mu_s, \alpha_s, \bar{\Lambda})$
- running $\alpha_s(\bar{\Lambda})$ and $m_s(\alpha_s)$

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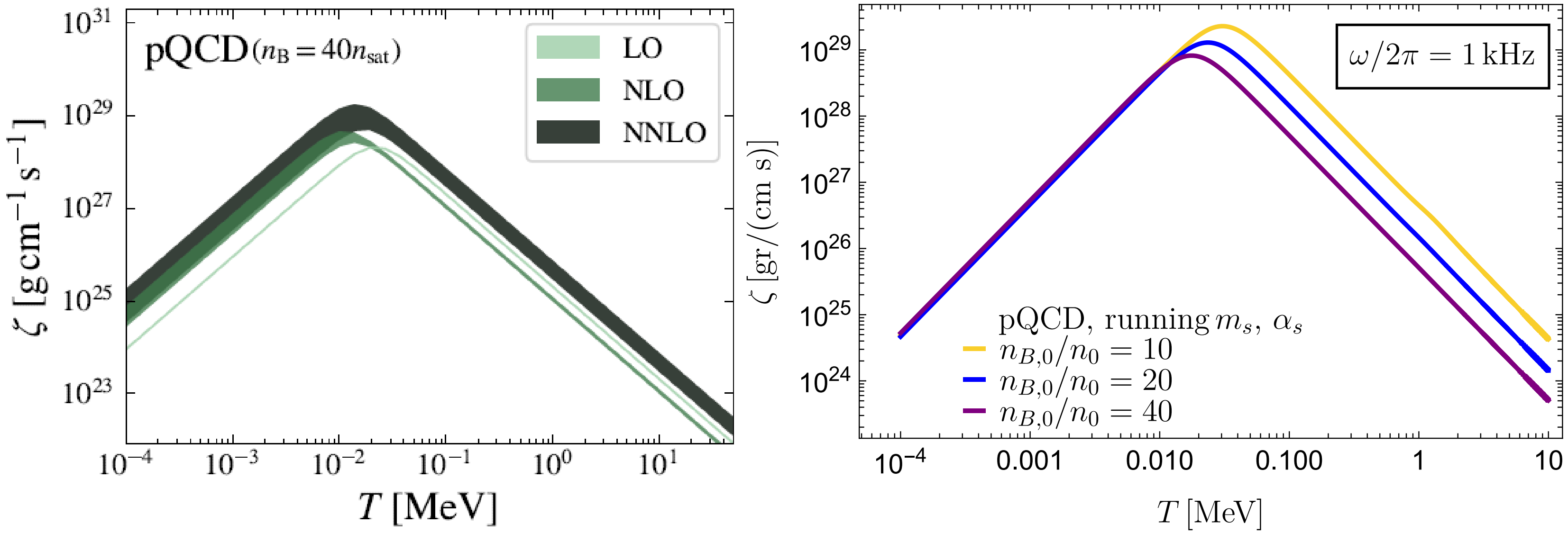
[1] Fraga and Romatschke, 2005 , *arXiv:hep-ph/0412298*
[2] Kurkela, Romatschke and Vuorinen, 2010, *arXiv:0912.1856*
[3] Gorda, Paatelainen, Säppi and Seppänen, 2023, *arXiv:2307.08734*

Recent studies in quark matter

Rojas, Gorda, et al., 2024; nonleptonic channel, NNLO pQCD and D3-D7 and V-QCD models[1]



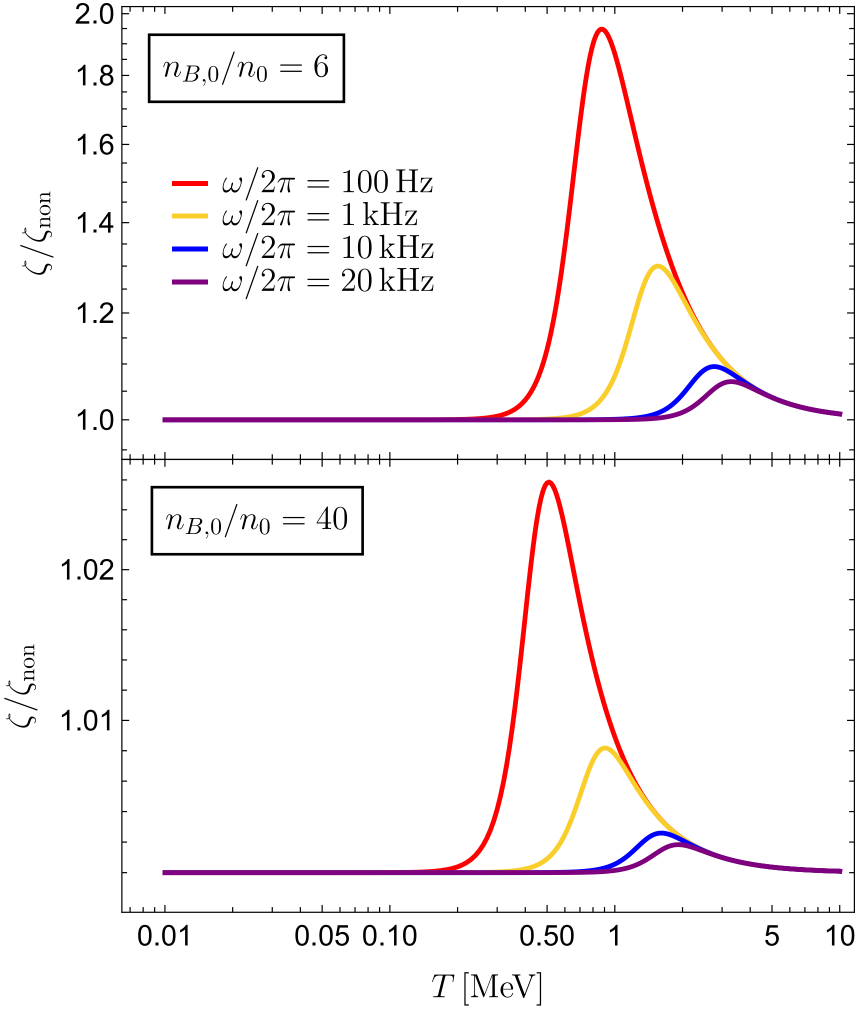
Comparison of bulk viscosity using perturbative QCD, Rojas, Gorda, et al., 2024 (left), this study (right)

This study; all channels, MIT bag model and pQCD with λ_3 and running $\alpha_s(\bar{\Lambda})$ $m_s(\alpha_s)$ up to $\mathcal{O}(\alpha_s)$

[1] Rojas, Gorda, Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen, 2024, *arXiv:2402.00621*

Relevance of Urca-type processes

Semileptonic processes play a relevant role at low temperatures[1,2]



Ratio of the full bulk viscosity with that arising only from the nonleptonic channel using pQCD

[1] Sa'd, Shovkovy, Rischke, 2007, *arXiv:astro-ph/0703016*

[2] Dong, Su, Wang, 2007, *arXiv:astro-ph/0702104*