

# Probing the QCD phase structure with heavy-ion collisions

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*Compact Stars in the QCD Phase Diagram (CSQCD2024)*

*YITP, Kyoto, Japan*

**October 10, 2024**



## What we know

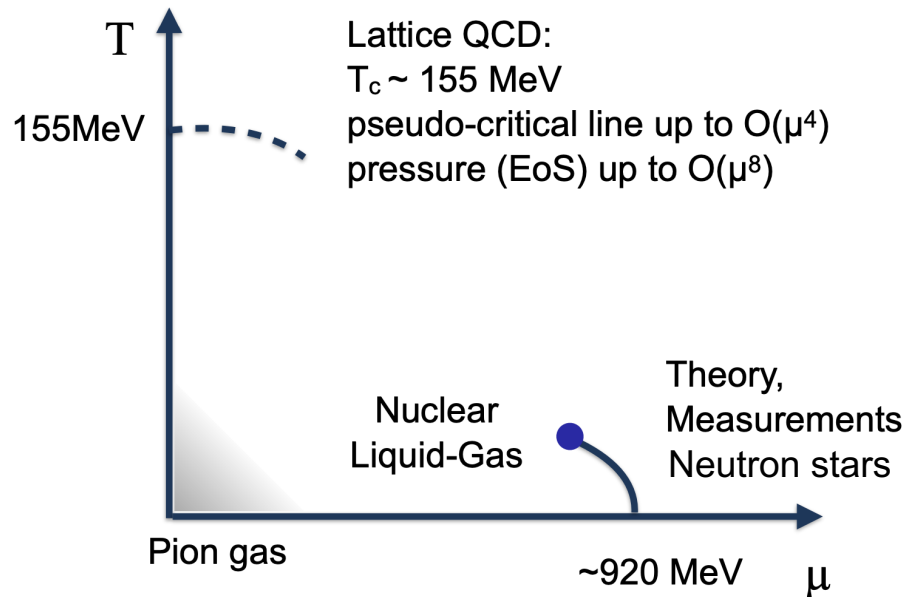


Figure courtesy of V. Koch

## What we hope to know

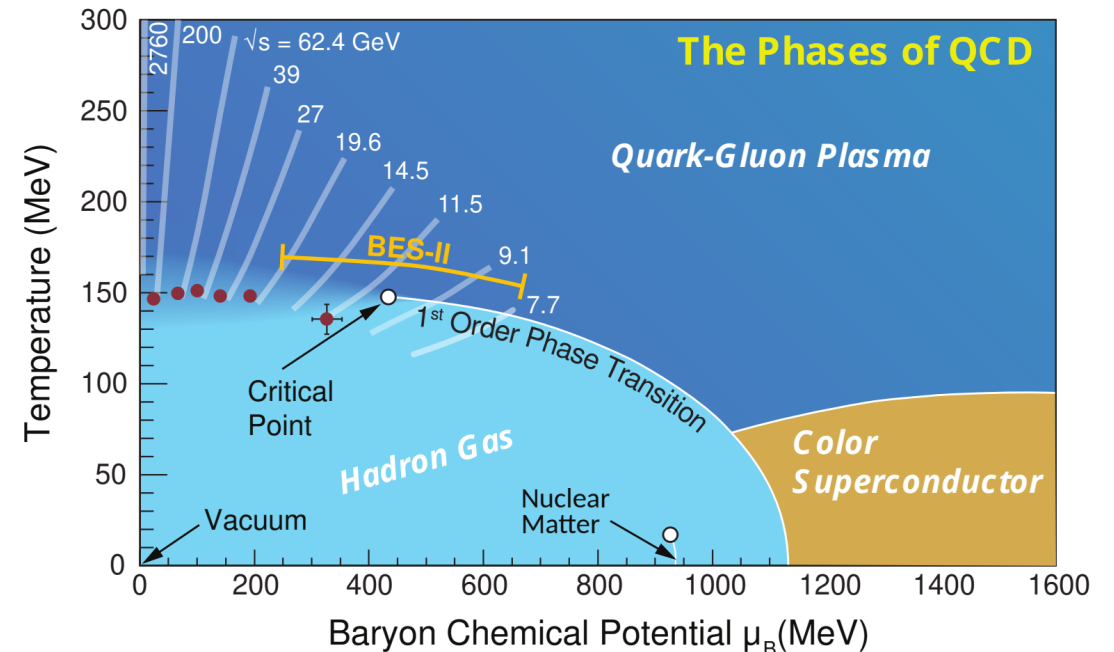


Figure from Bzdak et al., Phys. Rept. '20 & 2015 US Nuclear Long Range Plan

- Dilute hadron gas at low  $T$  &  $\mu_B$  due to confinement, quark-gluon plasma high  $T$  &  $\mu_B$
- Nuclear liquid-gas transition in cold and dense matter, lots of other phases conjectured
- Chiral crossover at  $\mu_B = 0$  which may turn into a *first-order phase transition* at finite  $\mu_B$

**Key question:** *Is there a QCD critical point and how to find it?*

# QCD critical point theory estimates: State-of-the-art

# Critical point predictions as of a few years ago

All over the place...

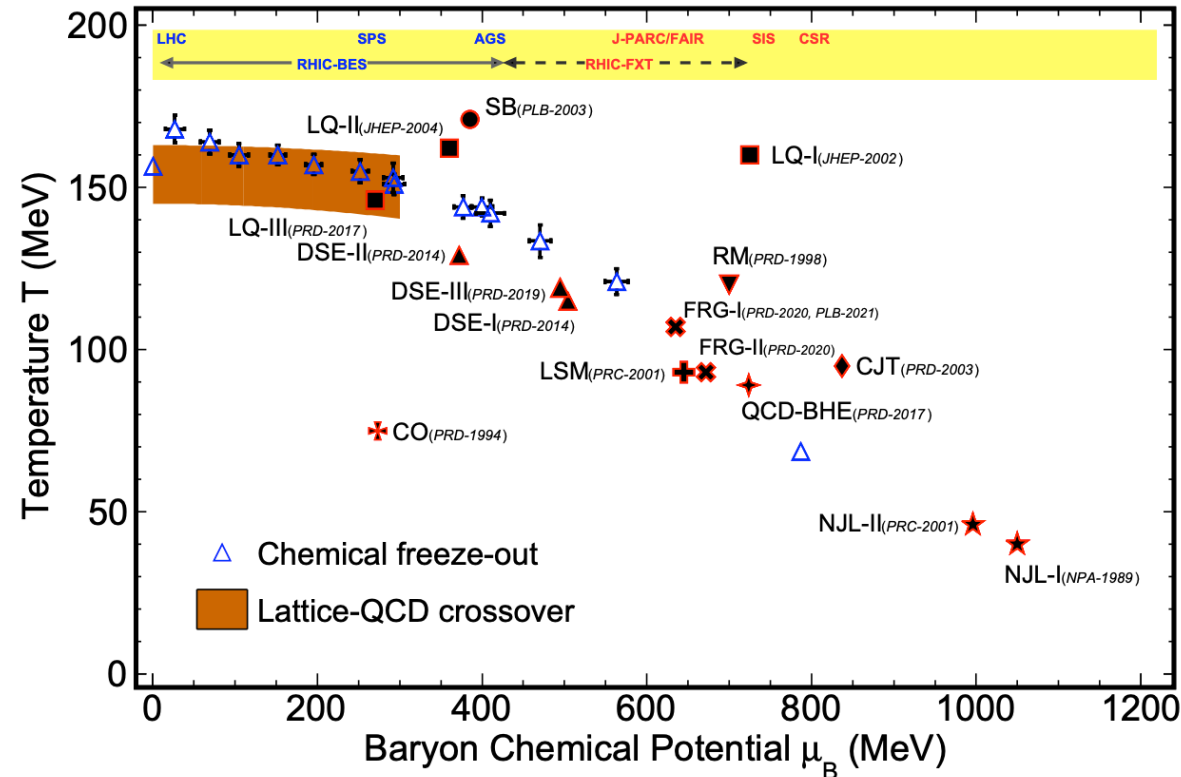


Figure adapted from A. Pandav, D. Mallick, B. Mohanty, Prog. Part. Nucl. Phys. 125 (2022)

Including the possibility that the QCD critical point does not exist at all

de Forcrand, Philipsen, JHEP 01, 077 (2007); VV, Steinheimer, Philipsen, Stoecker, PRD 97, 114030 (2018)



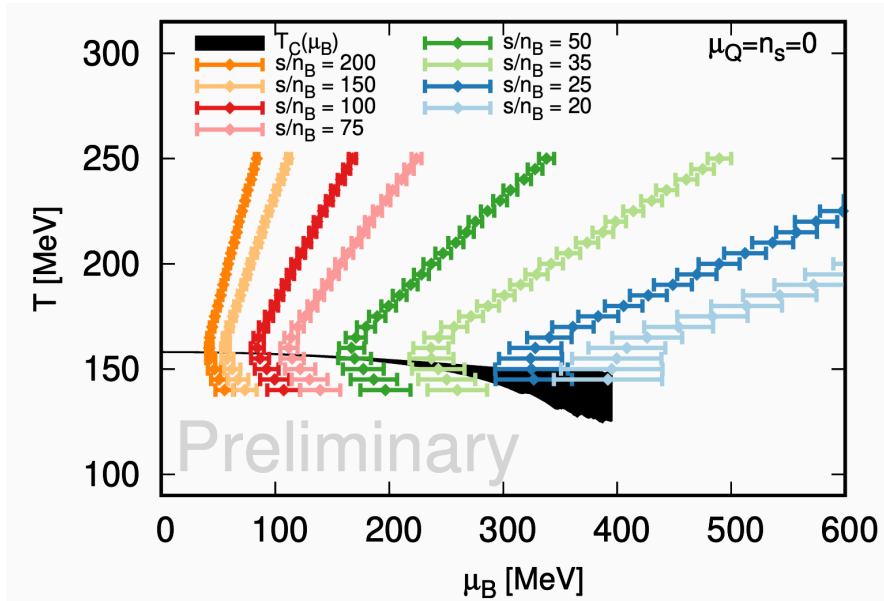
# Extrapolations from lattice QCD at $\mu_B = 0$

Ideally, find the critical point through first-principle **lattice QCD** simulations at finite  $\mu_B$

- Challenging (sign problem), but perhaps not impossible? [Borsanyi et al., Phys. Rev. D 107, 091503L (2023)]

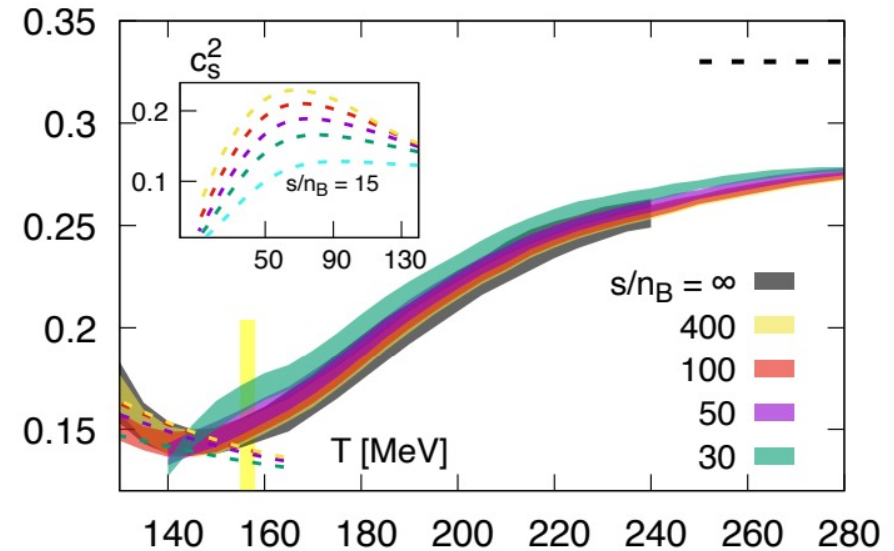
Taylor expansion + various resummations and extrapolation schemes from  $\mu_B = 0$

alternative expansion scheme



[Borsanyi et al. (WB), Phys. Rev. D 105, 114504 (2022)]

Padé approximants



[Bollweg et al. (HotQCD), Phys. Rev. D 108, 014510 (2023)]

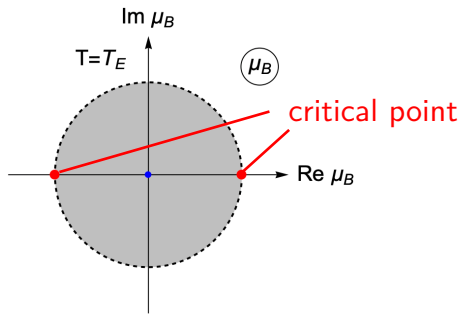
No indications for the strengthening of the chiral crossover or critical point signals

Disfavors QCD critical point at  $\frac{\mu_B}{T} < 3$

# Searching for singularities in the complex plane

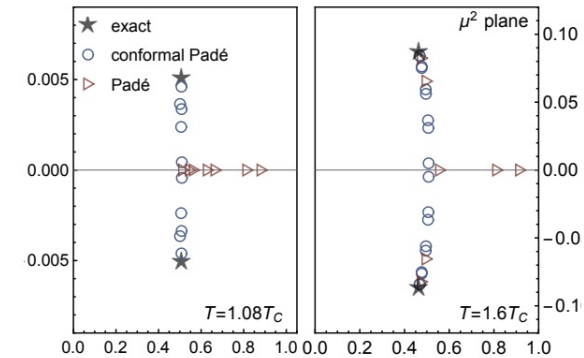
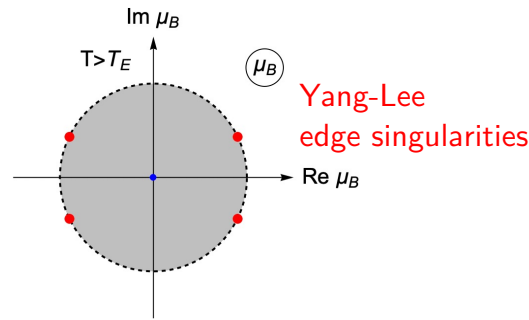
## Critical point:

- singularity in the partition function
- real  $\mu_B$  axis



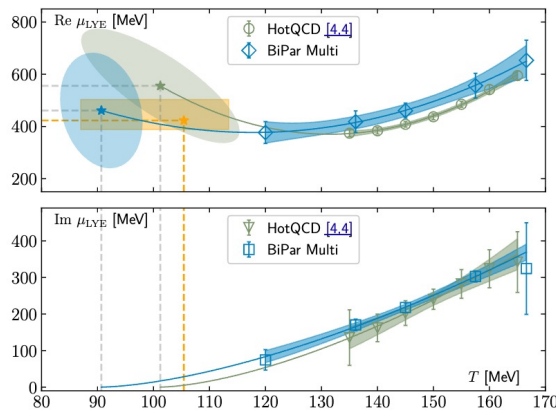
## Above the critical temperature:

Yang-Lee edge singularities in the complex plane



G. Basar, PRC 110, 015203 (2024)

**Strategy:** Extract YL edge singularity through (multi-point) Pade fits and see if it approaches the real axis as temperatures decreases



CP Z(2) scaling inspired fit:

$$\text{Im } \mu_{LY} = c(T - T_{CEP})^\Delta$$

$$\text{Re } \mu_{LY} = \mu_{CEP} + a(T - T_{CEP}) + b(T - T_{CEP})^2$$

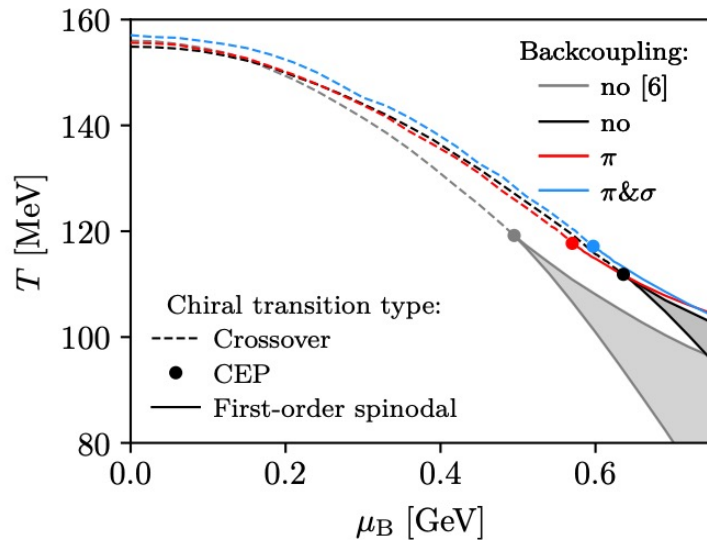
NB: many things have to go right, systematic error still very large (up to 100%)



Extrapolated CP estimate:  
 $T \sim 90-110$  MeV,  $\mu_B \sim 400-600$  MeV

## Dyson-Schwinger equations

*truncation errors*

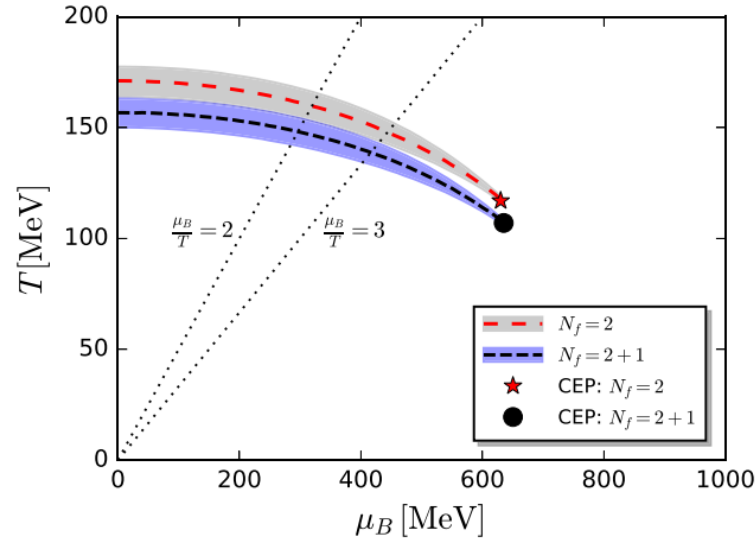


Gunkel, Fischer, PRD 104, 054202 (2021)

$T \sim 120$  MeV  $\mu_B \sim 600$  MeV

## Functional renormalization group

*truncation errors*

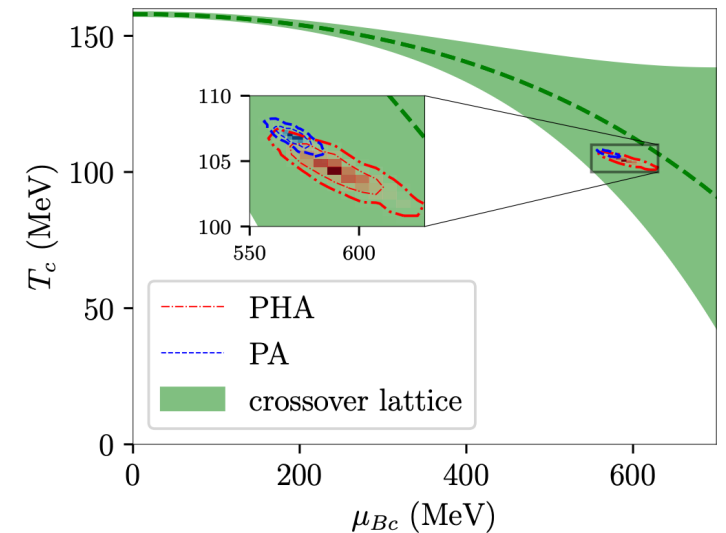


Fu, Pawłowski, Rennecke, PRD 101, 053032 (2020)

$T \sim 100$  MeV  $\mu_B \sim 600 - 650$  MeV

## Black-hole engineering

*strongly-coupled only ( $\eta/s=1/4\pi$ )*



Hippert et al., arXiv:2309.00579

$T \sim 105$  MeV  $\mu_B \sim 580$  MeV

- All in excellent agreement with lattice QCD at  $\mu_B = 0$  and predict QCD critical point in a similar ballpark of  $\mu_B/T \sim 5-6$
- Comparable to where onset of quarkyonic matter might take place [Bluhm, Fujimoto, McLerran, Nahrgang, arXiv:2409.12088](#)
- If true, reachable in heavy-ion collisions at  $\sqrt{s_{NN}} \sim 3 - 5$  GeV **RHIC-FXT, CBM-FAIR, J-PARC...**

# Search for critical point with heavy-ion collisions

## Control parameters

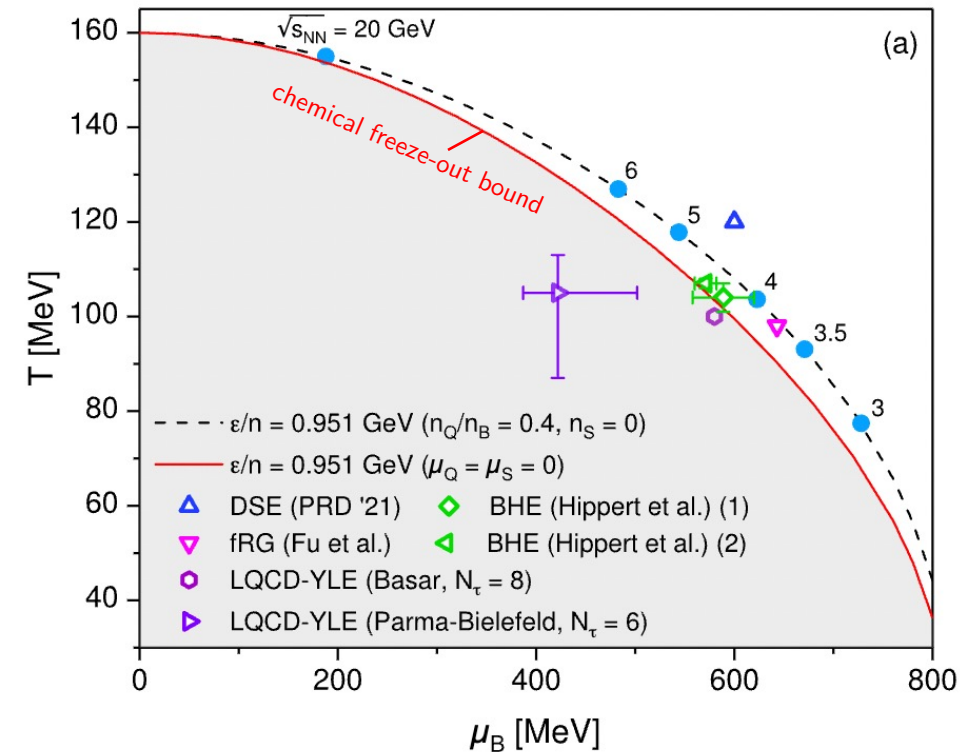
- Collision energy  $\sqrt{s_{NN}} = 2.4 - 5020$  GeV
  - Scan the QCD phase diagram
- Size of the collision region
  - Expect stronger signal in larger systems

## Measurements

- Final hadron abundances and momentum distributions **event-by-event**

## Chemical freeze-out curve and CP

- Sets the **lower bound** on the temperature of the CP
- **Caveats:** strangeness neutrality ( $\mu_S \neq 0$ ), uncertainty in the freeze-out curve



A. Lyenko, Poberezhnyuk, Gorenstein, VV, arXiv:2408.06473

# Critical point, cumulants, and heavy-ion collisions

# Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln \langle e^{tN} \rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

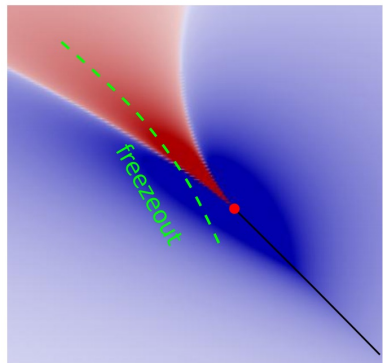
$$\kappa_n \propto \frac{\partial^n (\ln Z^{\text{gce}})}{\partial \mu^n}$$

Grand partition function

$$\ln Z^{\text{gce}}(T, V, \mu) = \ln \left[ \sum_N e^{\mu N/T} Z^{\text{ce}}(T, V, N) \right]$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- (QCD) critical point: large correlation length and fluctuations



M. Stephanov, PRL '09, '11  
Energy scans at RHIC (STAR)  
and CERN-SPS (NA61/SHINE)

$$\kappa_2 \sim \xi^2, \quad \kappa_3 \sim \xi^{4.5}, \quad \kappa_4 \sim \xi^7$$

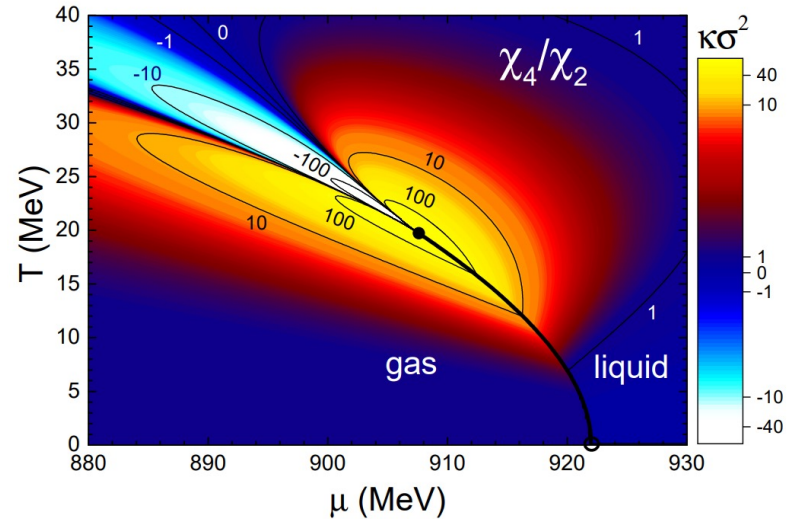
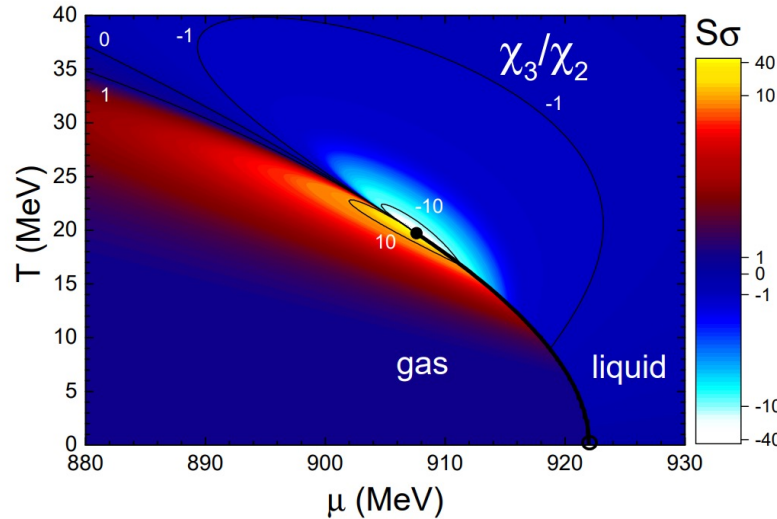
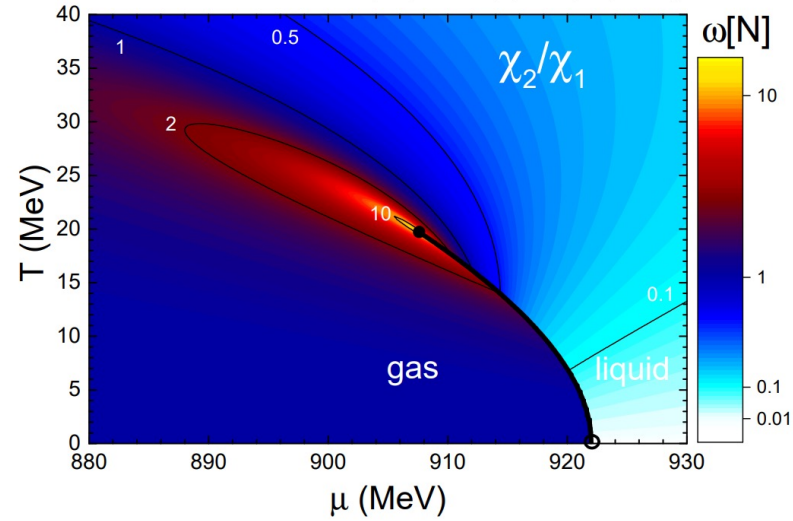
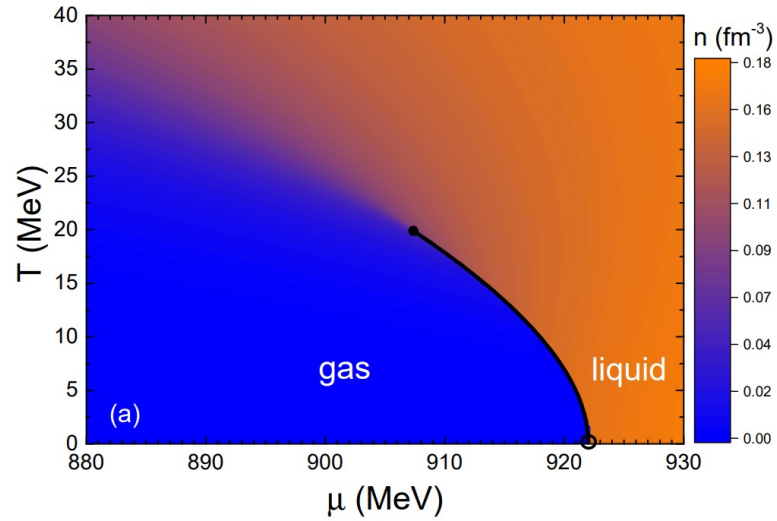
$$\xi \rightarrow \infty$$

Looking for enhanced fluctuations  
and non-monotonicities

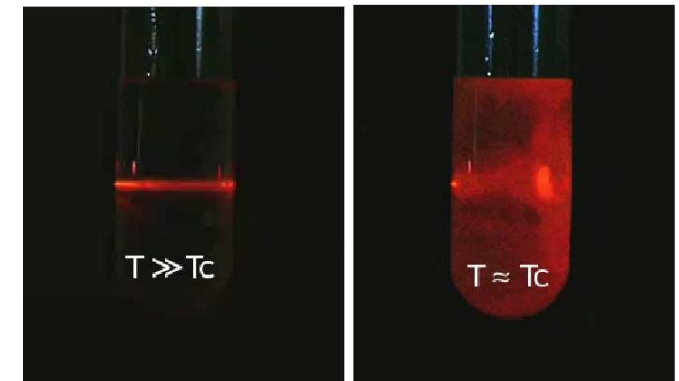
Other uses of cumulants:

- QCD degrees of freedom  
[Jeon, Koch, PRL 85, 2076 \(2000\)](#)  
[Asakawa, Heinz, Muller, PRL 85, 2072 \(2000\)](#)
- Extracting the speed of sound  
[A. Sorensen et al., PRL 127, 042303 \(2021\)](#)
- Conservation volume  $V_C$   
[VV, Donigus, Stoecker, PRC 100, 054906 \(2019\)](#)

# Example: (Nuclear) Liquid-gas transition



## Critical opalescence



$$\langle N^2 \rangle - \langle N \rangle^2 \sim \langle N \rangle \sim 10^{23}$$

in equilibrium



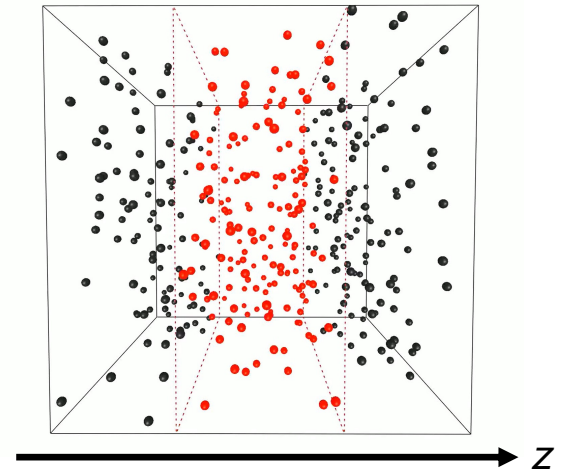
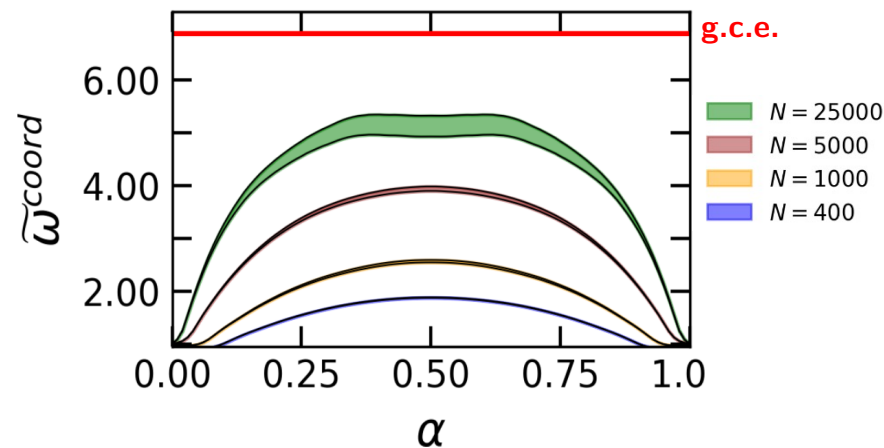
# Example: Critical fluctuations in a microscopic simulation

V. Kuznietsov et al., Phys. Rev. C 105, 044903 (2022)

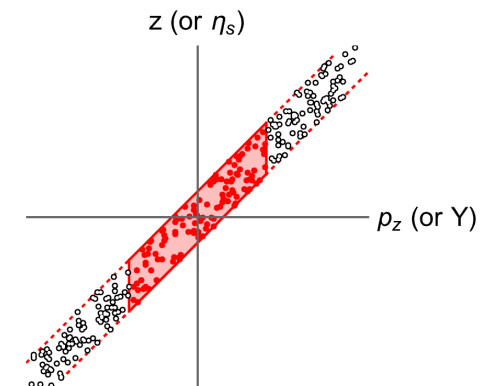
Classical molecular dynamics simulations of the **Lennard-Jones fluid** near Z(2) critical point ( $T \approx 1.06T_c$ ,  $n \approx n_c$ ) of the liquid-gas transition

Scaled variance in coordinate space acceptance  $|z| < z^{max}$

$$\tilde{\omega}^{coord} = \frac{1}{1 - \alpha} \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}$$



**Heavy-ion collisions:**  
flow correlates  $p_z$  and  $z$  cuts



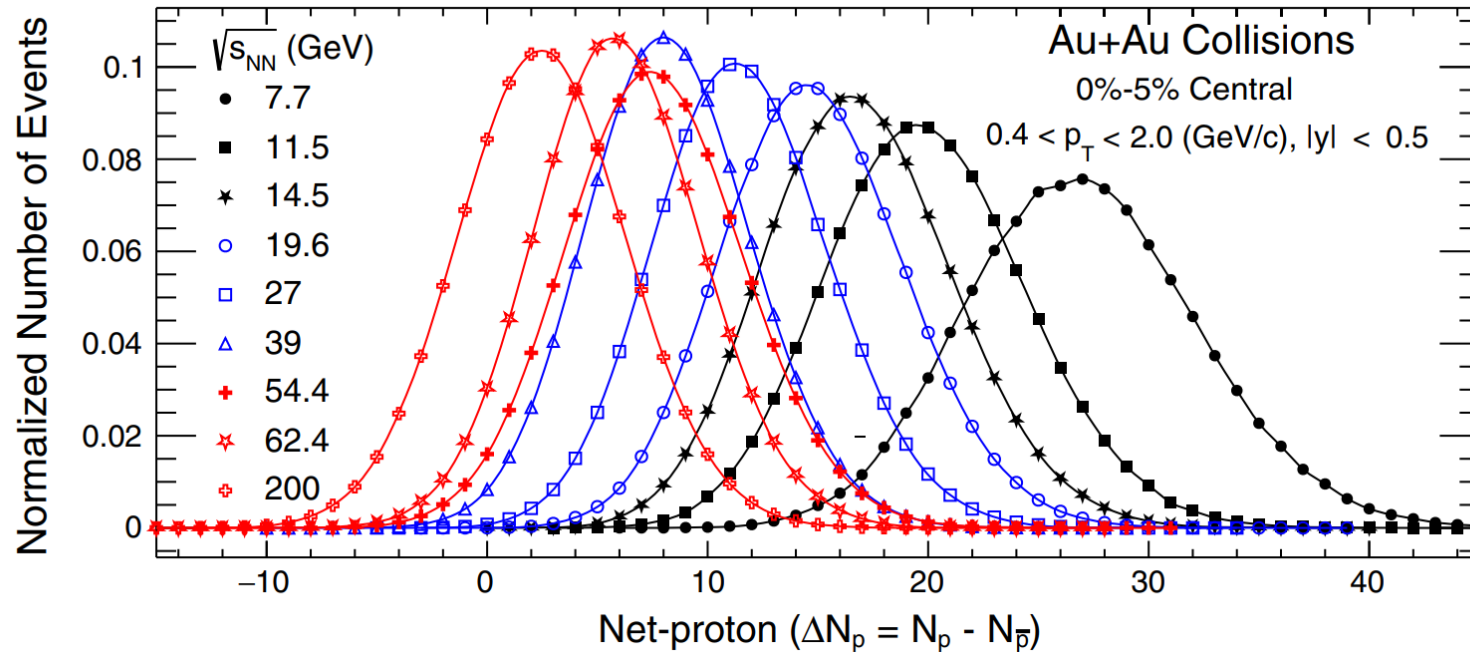
- Large fluctuations survive despite strong finite-size effects
- Need coordinate space cuts (collective flow helps)
- Here no finite-time effects



# Measuring cumulants in heavy-ion collisions

Count the number of events with given number of e.g. (net) protons  $P(\Delta N_p) \sim \frac{N_{\text{events}}(\Delta N_p)}{N_{\text{events}}^{\text{total}}}$

STAR Collaboration, Phys. Rev. Lett. 126, 092301 (2021)



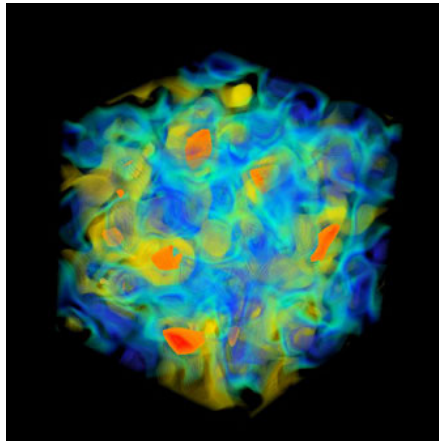
Cumulants are extensive,  $\kappa_n \sim V$ , use ratios to cancel out the volume

$$\frac{\kappa_2}{\langle N \rangle}, \quad \frac{\kappa_3}{\kappa_2}, \quad \frac{\kappa_4}{\kappa_2}$$

Look for subtle critical point signals

# Theory vs experiment: Challenges for fluctuations

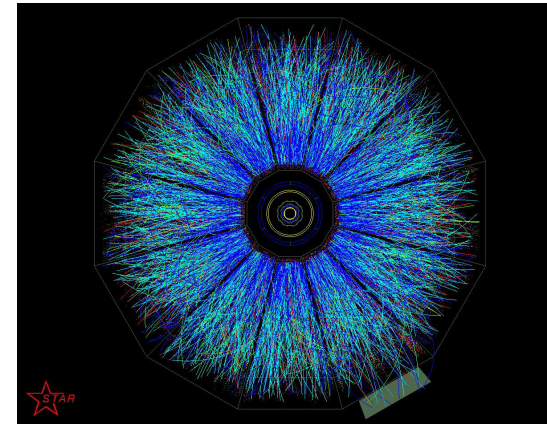
## Theory



© Lattice QCD@BNL

- Coordinate space
- In contact with the heat bath
- Conserved charges
- Uniform
- Fixed volume

## Experiment



STAR event display

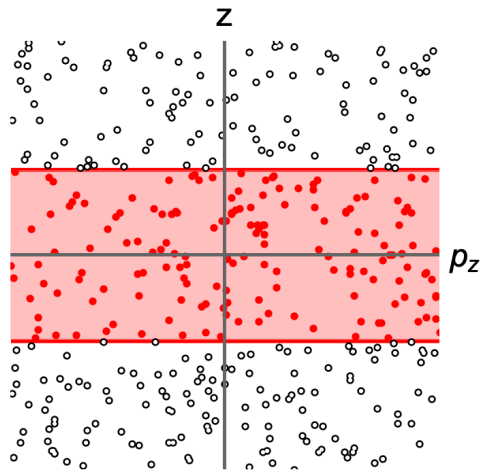
- Momentum space
- Expanding in vacuum
- Non-conserved particle numbers
- Inhomogenous
- Fluctuating volume

***Need dynamical description***

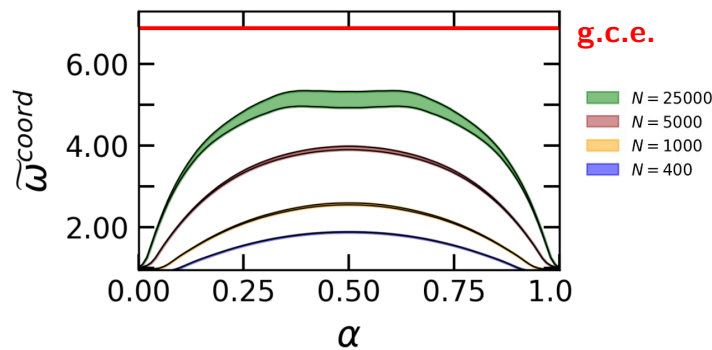
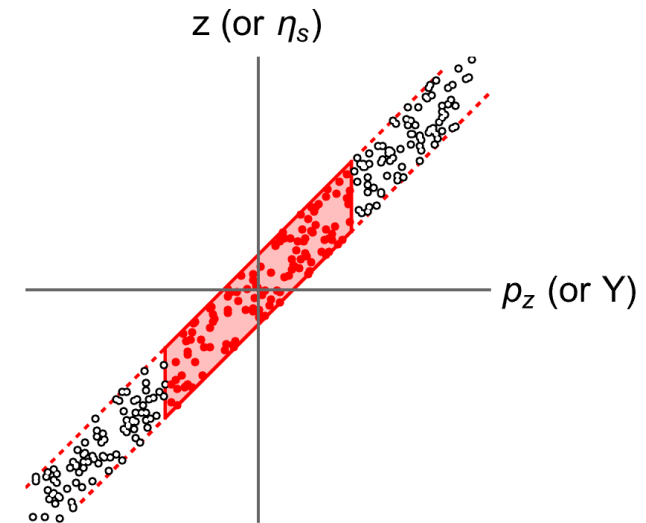
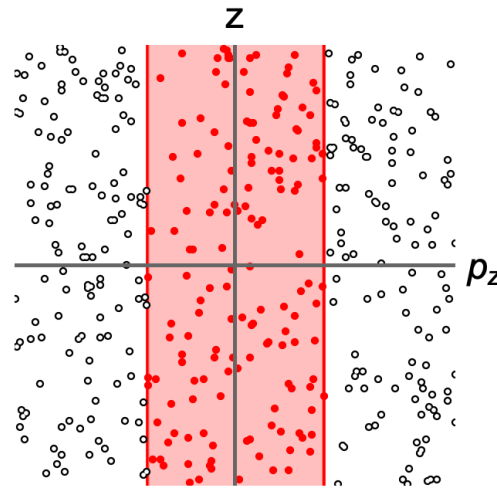
**Box setup:** Coordinates and momenta are uncorrelated

**HICs:** Flow (e.g. Bjorken)

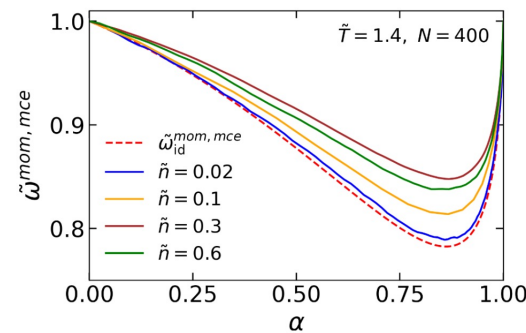
Coordinate space cut



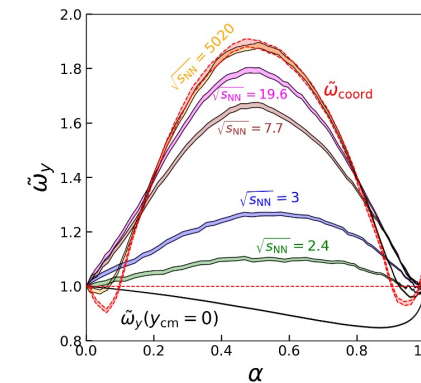
Momentum space cut



Large fluctuations



Nothing left



momentum cut  $\sim$  coordinate cut + smearing

Utilizing the canonical partition function in thermodynamic limit compute **n-point density correlators**

$$\mathcal{C}_1(\mathbf{r}_1) = \rho(\mathbf{r}_1)$$

$$\mathcal{C}_2(\mathbf{r}_1, \mathbf{r}_2) = \chi_2 \delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{\chi_2}{V}$$

**local correlation**      **balancing contribution**  
(e.g. baryon conservation)

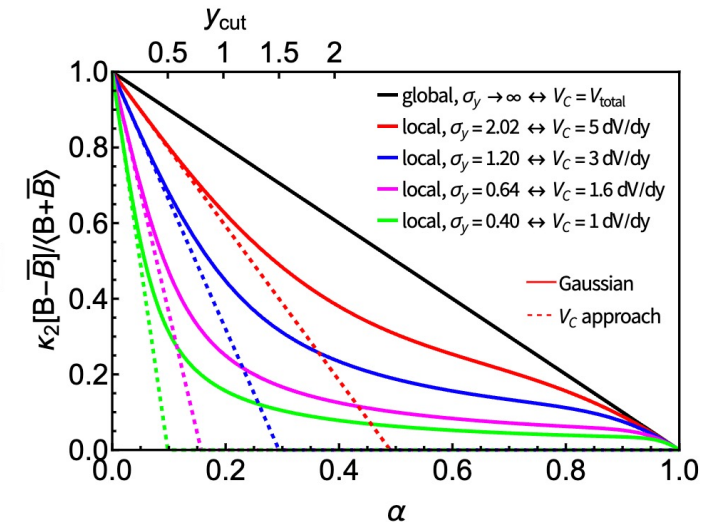
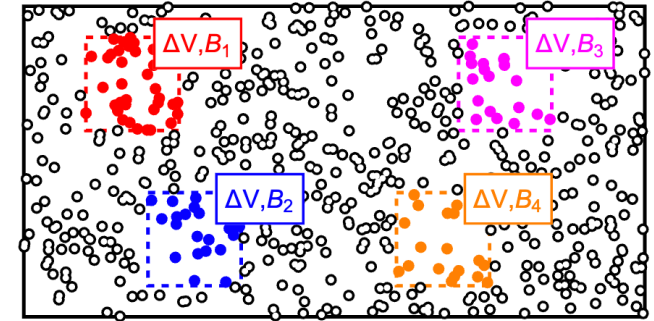
$$\mathcal{C}_3(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \chi_3 \delta_{1,2,3} - \frac{\chi_3}{V} [\delta_{1,2} + \delta_{1,3} + \delta_{2,3}] + 2 \frac{\chi_3}{V^2} \quad \delta_{1,\dots,n} = \prod_{i=2}^n \delta(\mathbf{r}_1 - \mathbf{r}_i)$$

**local correlation**      **balancing contributions**

$$\mathcal{C}_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = \chi_4 \delta_{1,2,3,4} - \frac{\chi_4}{V} [\delta_{1,2,3} + \delta_{1,2,4} + \delta_{1,3,4} + \delta_{2,3,4}] - \frac{(\chi_3)^2}{\chi_2 V} [\delta_{1,2} \delta_{3,4} + \delta_{1,3} \delta_{2,4} + \delta_{1,4} \delta_{2,3}]$$

$$+ \frac{1}{V^2} \left[ \chi_4 + \frac{(\chi_3)^2}{\chi_2} \right] [\delta_{1,2} + \delta_{1,3} + \delta_{1,4} + \delta_{2,3} + \delta_{2,4} + \delta_{3,4}] - \frac{3}{V^3} \left[ \chi_4 + \frac{(\chi_3)^2}{\chi_2} \right]$$

**balancing contributions**



Integrating the correlator reproduces known cumulant inside a subsystem

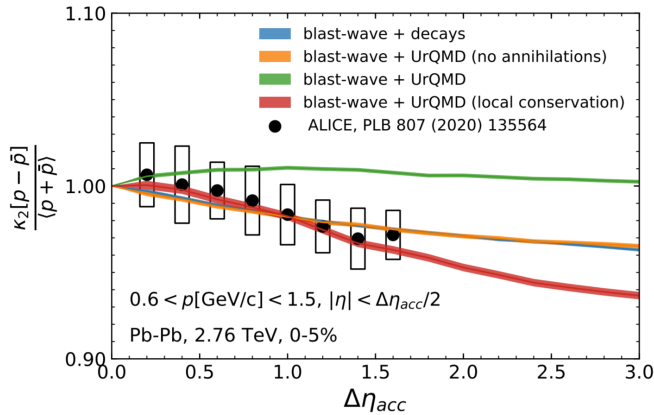
$$\kappa_n[B_{V_s}] = \int_{\mathbf{r}_1 \in V_s} d\mathbf{r}_1 \dots \int_{\mathbf{r}_n \in V_s} d\mathbf{r}_n \mathcal{C}_n(\{\mathbf{r}_i\})$$

# Fluctuations at the LHC

# Proton cumulants at high energy

Second-order cumulants such as  $\kappa_2[p - \bar{p}]/\langle p + \bar{p} \rangle$ :

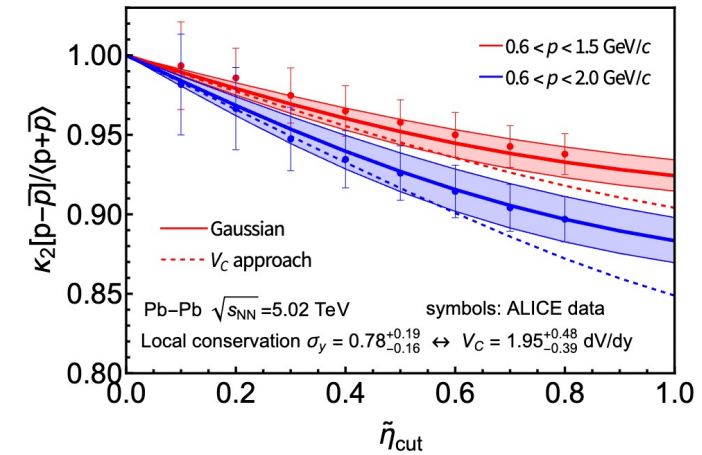
Pb-Pb 2.76 TeV



O. Savchuk et al., PLB 827, 136983 (2022)

- Largely understood as driven by baryon conservation
- baryon annihilation(↗) vs local conservation(↘)
  - Additional measurement of  $\kappa_2[p + \bar{p}]$  can resolve it
- For some quantities like net-charge (or net-pion/net-kaon) fluctuations, resonance decays are important

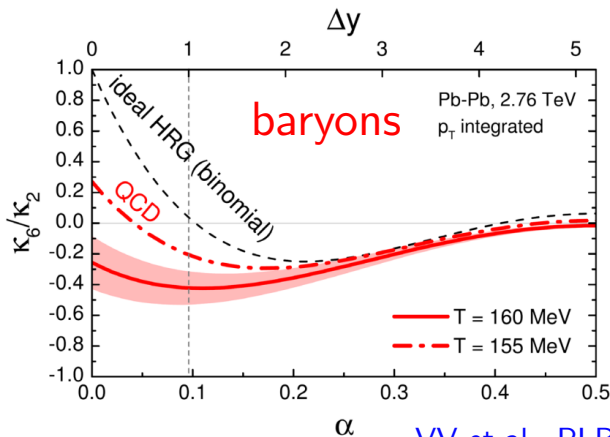
Pb-Pb 5.02 TeV



VV, arXiv:2409.01397

High-order cumulants: probe remnants of **chiral criticality**

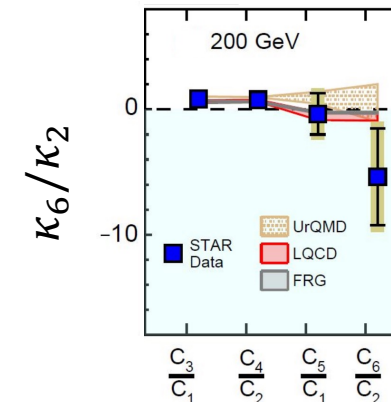
Friman et al., EPJC 71, 1694 (2011)



VV et al., PLB 811, 135868 (2020)

- negative  $\kappa_6$  of **baryons**

RHIC 200 GeV: hints of negative  $\kappa_6 < 0$  (**protons**)



- are **baryons** even more negative?

STAR Collaboration, PRL 130, 082301 (2023)



Introduce Gaussian (space-time) rapidity correlation into baryon-conservation balancing term

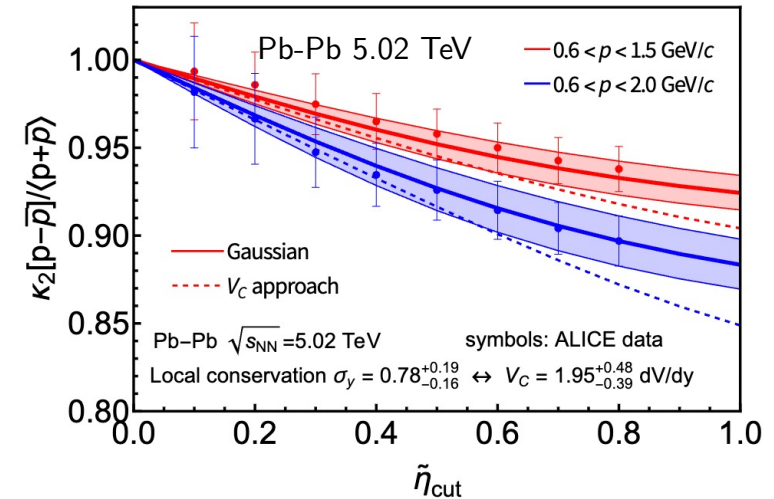
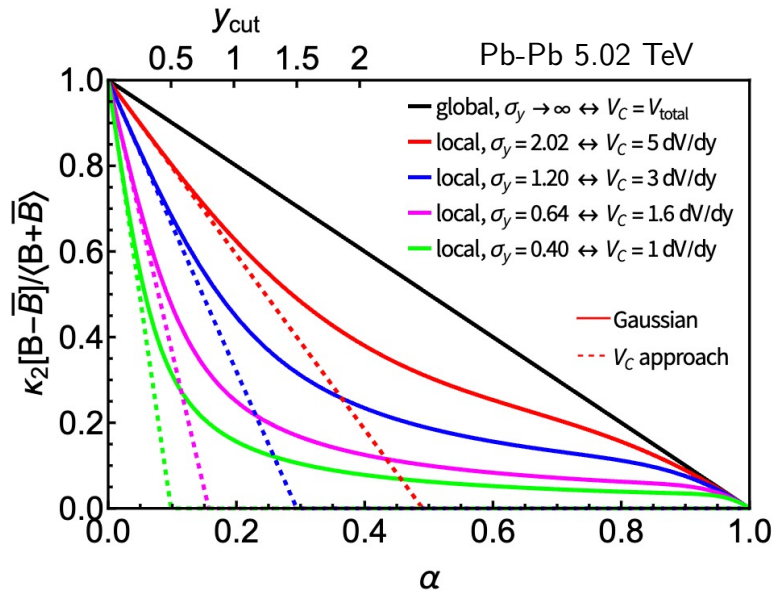
global conservation

$$C_2^B(\eta_1, \eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[ \delta(\eta_1 - \eta_2) - \frac{1}{2\eta_{\max}} \right]$$



+ local conservation

$$C_2^B(\eta_1, \eta_2) = \langle n_B + n_{\bar{B}} \rangle \left[ \delta(\eta_1 - \eta_2) - \frac{\tilde{A} e^{-\frac{(\eta_1 - \eta_2)^2}{2\sigma_\eta^2}}}{2\eta_{\max}} \right]$$



- Linear regime at small  $a$  establishes connection to the  $V_C$  approach ( $V_C = kdV/dy$ ,  $k \approx \sqrt{2\pi}\sigma_\eta$ )
- $V_C$  approach has limitations, likely provides upper bound on the conservation volume
- Evidence for local (not just global) baryon conservation for 5 TeV data (in contrast to 2.76 TeV data)

# Fluctuations and beam energy scan



## 1. Dynamical model calculations of critical fluctuations [X. An et al., Nucl. Phys. A 1017, 122343 (2022)]

- Fluctuating hydrodynamics (hydro+) and (non-equilibrium) evolution of fluctuations
- Equation of state with a tunable critical point [P. Parotto et al, PRC 101, 034901 (2020); J. Karthein et al., EPJ Plus 136, 621 (2021)]
- Generalized Cooper-Frye particlization [M. Pradeep, et al., PRD 106, 036017 (2022); PRL 130, 162301 (2023)]

Alternatives at high  $\mu_B$ : hadronic transport/molecular dynamics with a critical point

[A. Sorensen, V. Koch, PRC 104, 034904 (2021); V. Kuznetsov et al., PRC 105, 044903 (2022)]

## 2. Deviations from precision calculations of non-critical fluctuations

- Non-critical baseline is not flat [Braun-Munzinger et al., NPA 1008, 122141 (2021)]
- Include essential non-critical contributions to (net-)proton number cumulants
- Exact **baryon conservation** + **hadronic interactions** (hard core repulsion)
- Based on realistic hydrodynamic simulations tuned to bulk data

[VV, C. Shen, V. Koch, Phys. Rev. C 105, 014904 (2022)]

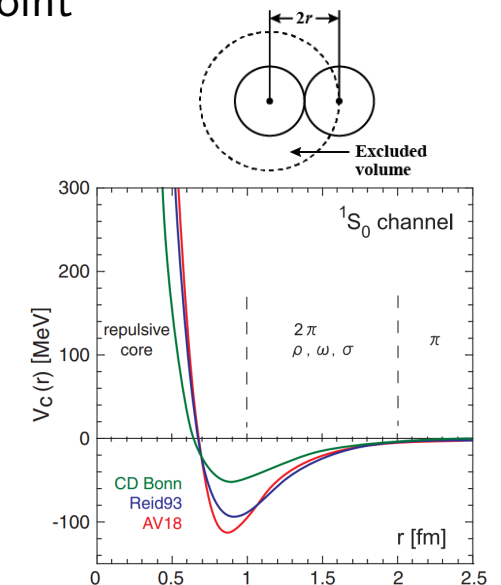
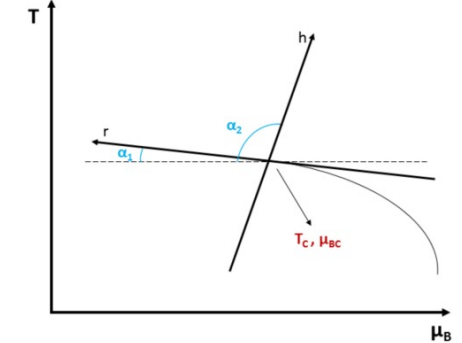
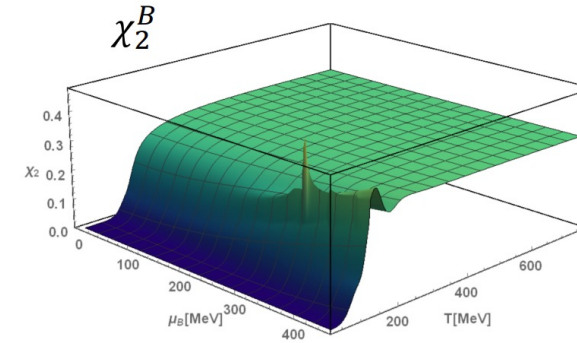


Figure from Ishii et al., PRL '07

# Equation of state with a tunable critical point

**BEST equation of state:** [P. Parotto et al, PRC 101, 034901 \(2020\)](#)

- 3D-Ising CP mapped onto the QCD
- Tunable CP location along the pseudocritical line
- Matched to lattice data at  $\mu_B = 0$

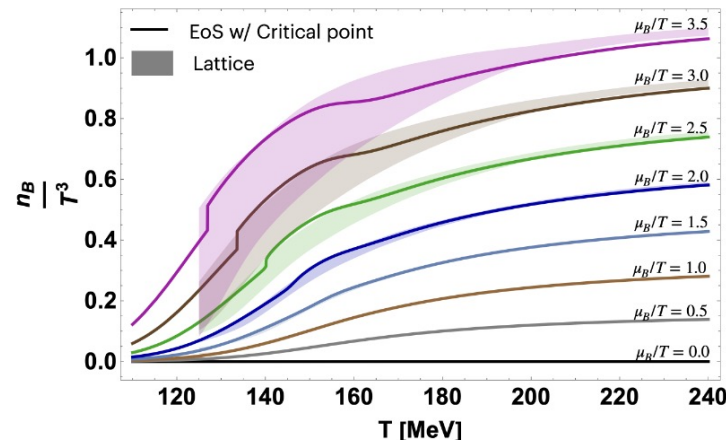


**New development:** [M. Kahangirwe et al, PRD 109, 094046 \(2024\)](#)

Match to **alternative expansion scheme** from lattice QCD instead of Taylor expansion, extending the range to whole BES range

$$p(T, \mu_B) = p^{\text{non-Ising}}(T, \mu_B) + p^{\text{Ising}}(T, \mu_B)$$

**regular**
**critical**



*Alternative ways to embed the critical point:*

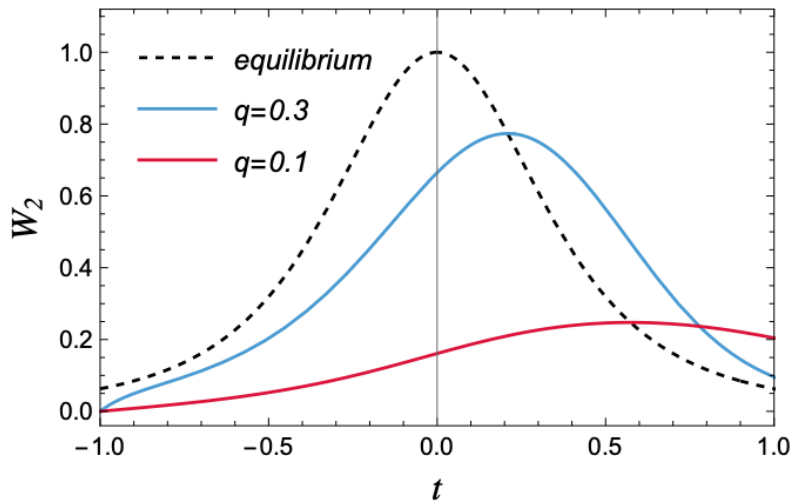
[\[J. Kapusta, T. Welle, C. Plumberg, PRC 106, 014909 \(2022\); PRC 106, 044901 \(2022\)\]](#)

*Equilibrium expectations for fluctuations:*

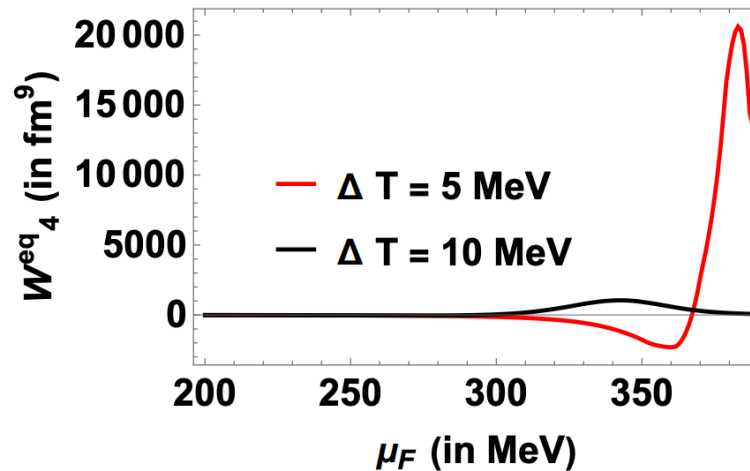
[\[J.M. Karthein et al., 2402.18738; SQM2024\]](#)

# Non-equilibrium evolution and critical slowing down

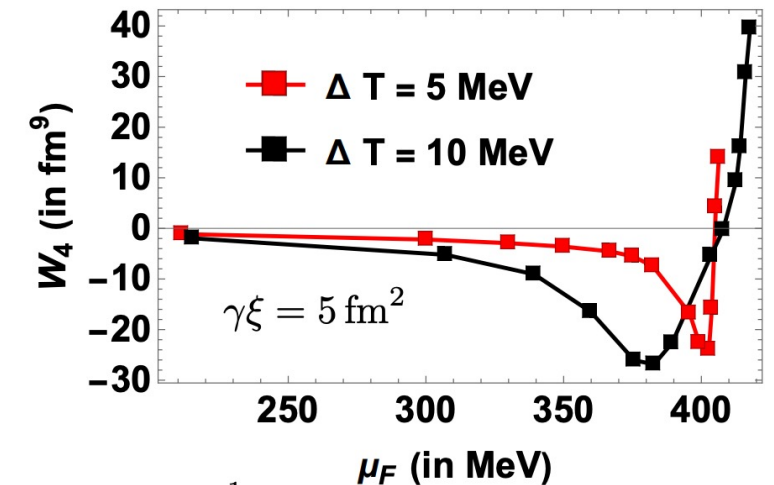
- Non-equilibrium evolution of (non-)Gaussian fluctuations
  - Strong suppression of critical point signals due to critical slowing down and (local) conservation



[X. An et al., PRL 127, 072301 (2021)]



[M. Pradeep et al., Phys. Rev. D 106 (2022) 036017; QM2024]



- Generalized Cooper-Frye particlization: maximum entropy freeze-out of fluctuations
 

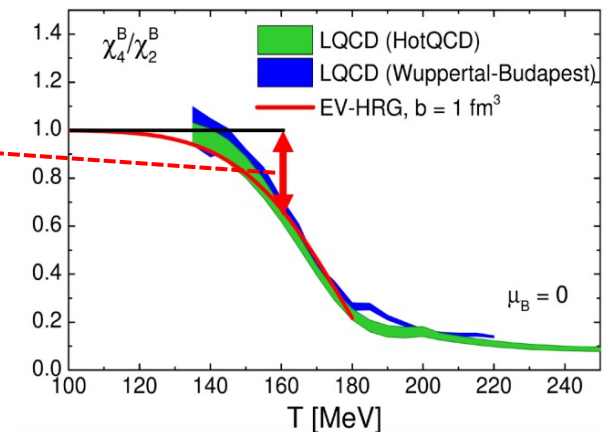
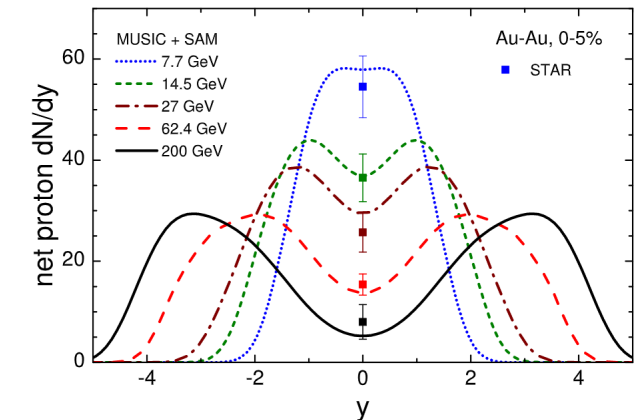
[M. Pradeep, M. Stephanov, PRL 130, 162301 (2023)]
- Diffusion and cross-correlations of multiple conserved charges and energy-momentum, balancing conservation laws
 

[O. Savchuk, S. Pratt, PRC 109, 024910 (2024)]

# Calculation of non-critical contributions at RHIC-BES

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)

- (3+1)-D viscous hydrodynamics evolution (MUSIC-3.0)
  - Collision geometry-based 3D initial state [Shen, Alzhrani, PRC 102, 014909 (2020)]
  - Crossover equation of state based on lattice QCD [Monnai, Schenke, Shen, Phys. Rev. C 100, 024907 (2019)]
  - Cooper-Frye particlization at  $\epsilon_{sw} = 0.26 \text{ GeV}/\text{fm}^3$
- Non-critical contributions are computed at particlization
  - QCD-like baryon number distribution ( $\chi_n^B$ ) via **excluded volume**  $b = 1 \text{ fm}^3$  [VV, V. Koch, Phys. Rev. C 103, 044903 (2021)]
  - Exact global baryon conservation\* (and other charges)
    - Subensemble acceptance method 2.0 (analytic) [VV, Phys. Rev. C 105, 014903 (2022)]
    - or FIST sampler (Monte Carlo) [VV, Phys. Rev. C 106, 064906 (2022)]  
<https://github.com/vlvovch/fist-sampler>



- **Absent:** critical point, local conservation, initial-state/volume fluctuations, hadronic phase

\*If baryon conservation is the only effect (no other correlations), non-critical baseline can be computed without hydro

Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)

# Calculating cumulants from MUSIC hydro

Cooper-Frye formula:

$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x) p^\mu f_j[u^\mu(x)p_\mu; T(x), \mu_j(x)]$$

Calculation of the cumulants incorporates **balancing contributions from baryon conservation\***

$$C_1^B(x_1) = \chi_1^B(x_1),$$

$$C_2^B(x_1, x_2) = \underbrace{\chi_2^B(x_1) \delta(x_1 - x_2)}_{\text{local correlation}} - \underbrace{\frac{\chi_2^B(x_1) \chi_2^B(x_2)}{\int_{\sigma(x)} d\sigma_\mu(x) u^\mu(x) \chi_2^B(x)}}_{\text{balancing contribution (baryon conservation)}},$$

...

global baryon conservation:

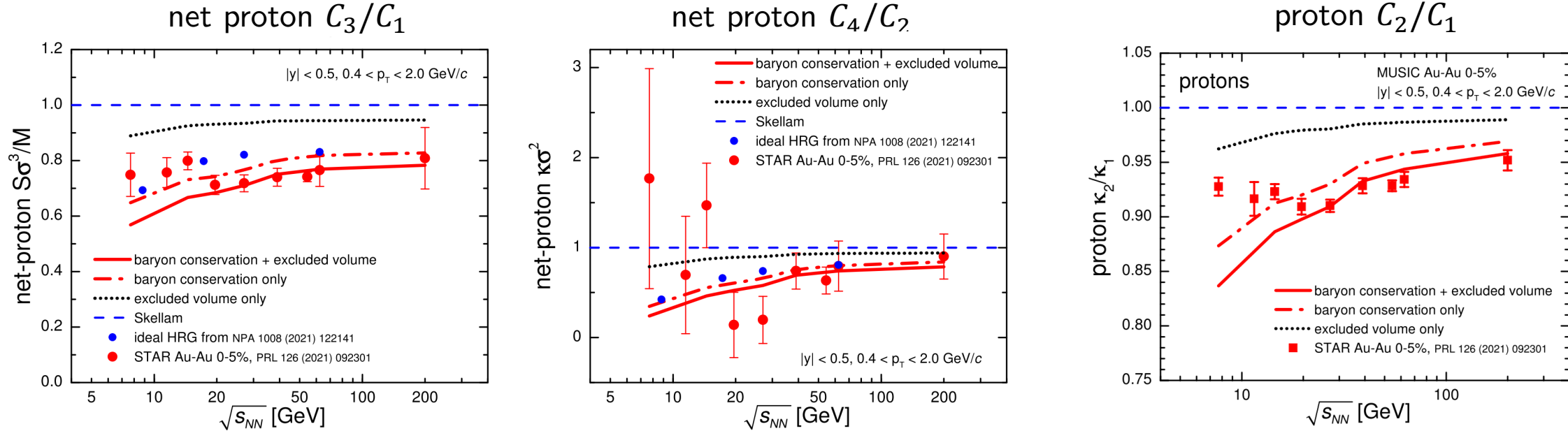
$$\int d\sigma_\mu(x_i) u^\mu(x_i) C_n^B(x_1, \dots, x_n) = 0 \quad \text{for } n > 1$$

Generalized Cooper-Frye:

$$\kappa_n^B = \prod_{i=1}^n \int_{x_i \in \sigma(x)} d\sigma_\mu(x_i) \int_{|y_i| < 0.5, 0.4 < p_T < 2} \frac{d^3p_i}{\omega_{p_i}} p_i^\mu \exp\left[-\frac{p_i^\mu u_\mu(x_i)}{T(x_i)}\right] C_n^B(x_1, \dots, x_n)$$

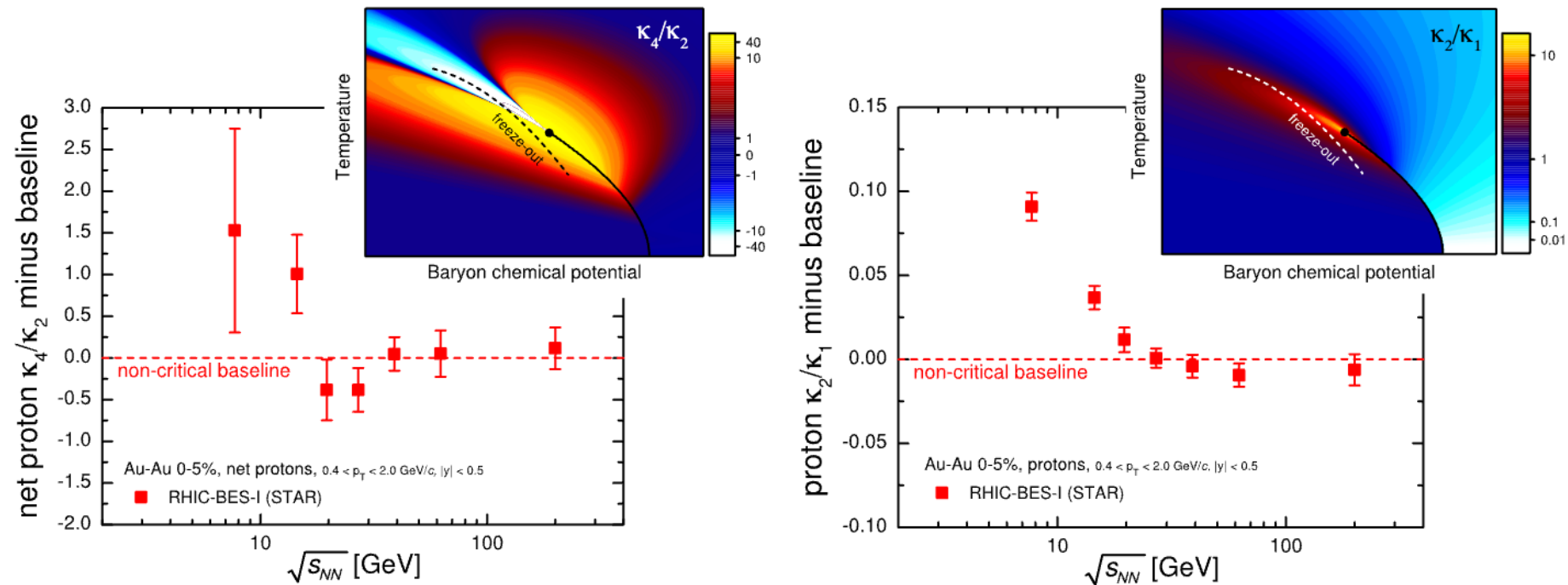
# RHIC-BES-I: Net proton cumulant ratios (MUSIC)

VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 (2022)



- Data at  $\sqrt{s_{NN}} \geq 20$  GeV consistent with non-critical physics (BQS conservation and repulsion)
- Effect from baryon conservation is stronger than repulsion but both are required at  $\sqrt{s_{NN}} \geq 20$  GeV
- Deviations from baseline at lower energies?

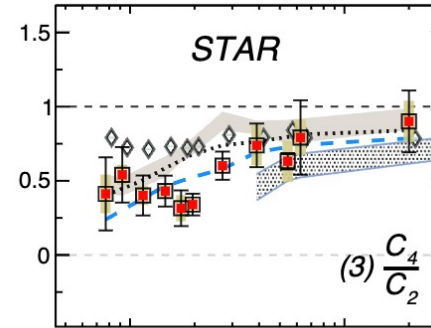
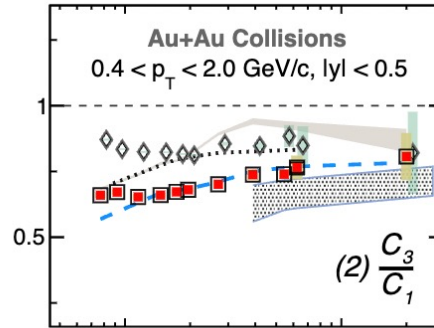
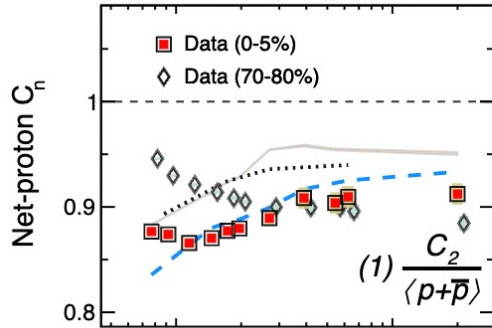
## Subtracting the hydrodynamic non-critical baseline



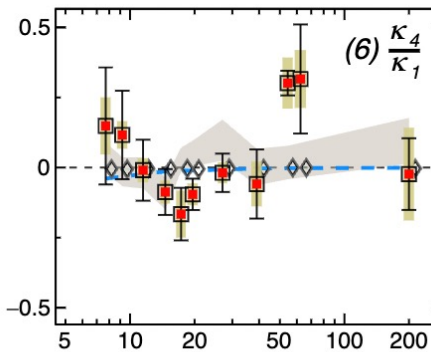
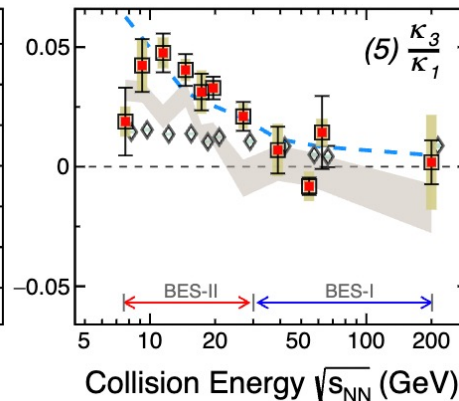
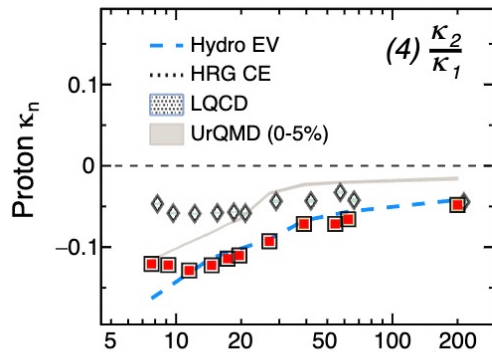
# Proton cumulants from RHIC-BES-II



## Net-proton cumulant ratios

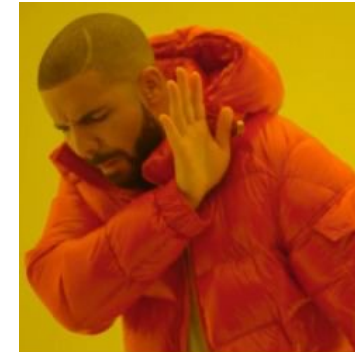


## Proton/antiproton factorial cumulant ratios



- No smoking gun signature for CP in ordinary cumulants
- More structure seen in factorial cumulants

### Conclusion 1:



Ordinary  
cumulants



Factorial  
cumulants

Hydro EV: [VV, V. Koch, C. Shen, Phys. Rev. C 105, 014904 \(2022\)](#)

# Factorial cumulants $\hat{C}_n$ vs ordinary cumulants $C_n$

**Factorial cumulants:** ~irreducible n-particle correlations

$$\hat{C}_n \sim \langle N(N-1)(N-2)\dots \rangle_c$$

$$\hat{C}_1 = C_1$$

$$\hat{C}_2 = C_2 - C_1$$

$$\hat{C}_3 = C_3 - 3C_2 + 2C_1$$

$$\hat{C}_4 = C_4 - 6C_3 + 11C_2 - 6C_1$$

**Ordinary cumulants:** mix corrs. of different orders

$$C_n \sim \langle \delta N^n \rangle_c$$

$$C_1 = \hat{C}_1$$

$$C_2 = \hat{C}_2 + \hat{C}_1$$

$$C_3 = \hat{C}_3 + 3\hat{C}_2 + \hat{C}_1$$

$$C_4 = \hat{C}_4 + 6\hat{C}_3 + 7\hat{C}_2 + \hat{C}_1$$

[Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017); Kitazawa, Luo, PRC 96, 024910 (2017); C. Pruneau, PRC 100, 034905 (2019)]

## Factorial cumulants and different effects

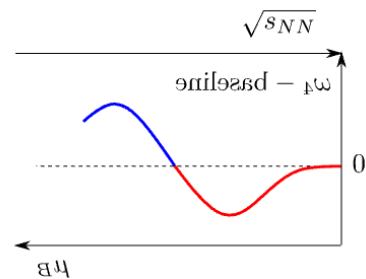
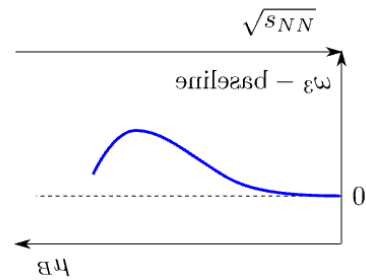
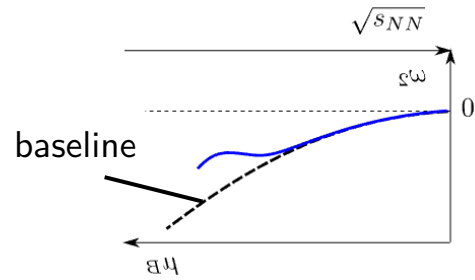
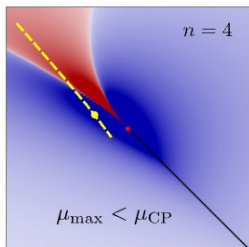
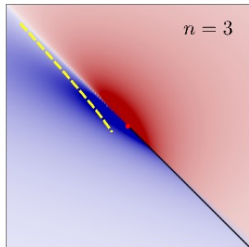
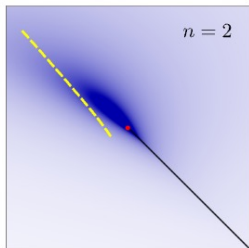
- Baryon conservation  $\hat{C}_n^{\text{cons}} \propto (\hat{C}_1)^n / \langle N_{\text{tot}} \rangle^{n-1}$  *small*  
[Bzdak, Koch, Skokov, EPJC '17]
- Excluded volume  $\hat{C}_n^{\text{EV}} \propto b^n$  *small*  
[VV et al, PLB '17]
- Volume fluctuations  $\hat{C}_n^{\text{CF}} \sim (\hat{C}_1)^{n\kappa_n} [V]$  *depends on volume cumulants*  
[Holzman et al., arXiv:2403.03598]
- **Critical point**  $\hat{C}_2^{\text{CP}} \sim \xi^2$ ,  $\hat{C}_3^{\text{CP}} \sim \xi^{4.5}$ ,  $\hat{C}_4^{\text{CP}} \sim \xi^7$  *large*  
[Ling, Stephanov, PRC '16]
- proton vs baryon  $\hat{C}_n^B \sim 2^n \times \hat{C}_n^p$  **same sign!**  
[Kitazawa, Asakawa, PRC '12]

# Factorial cumulants from RHIC-BES-II

From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1$$

(universal EOS) critical  $\chi_n$ :



Bzdak et al review 1906.00936

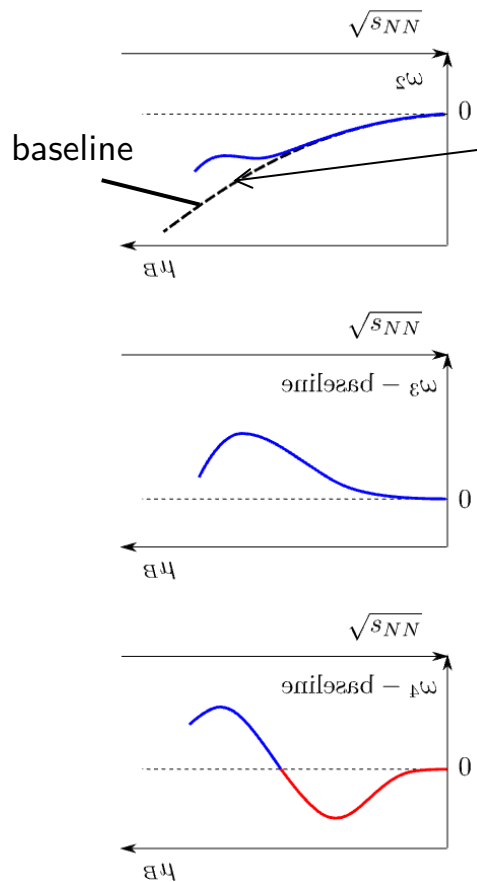
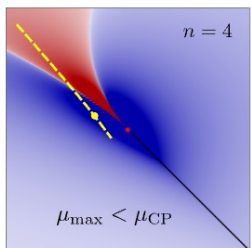
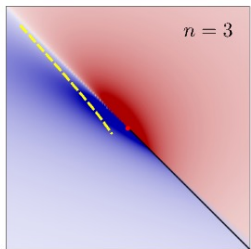
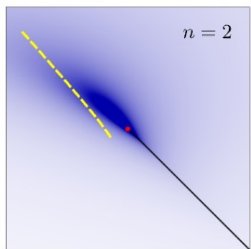
Expected signatures: **bump** in  $\omega_2$  and  $\omega_3$ , **dip** then **bump** in  $\omega_4$  for CP at  $\mu_B > 420$  MeV

# Factorial cumulants from RHIC-BES-II

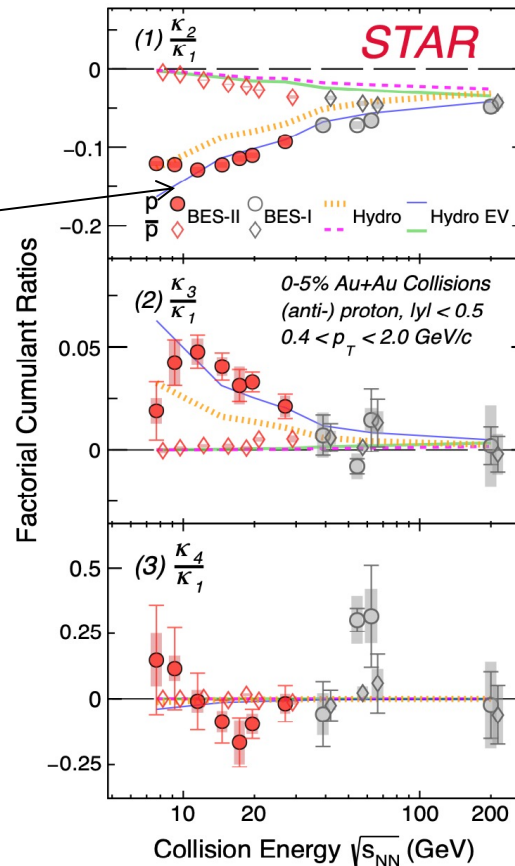
From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1^n$$

(universal EOS) critical  $\chi_n$ :



STAR data:



A. Pandav, CPOD2024

baseline (hydro):

VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

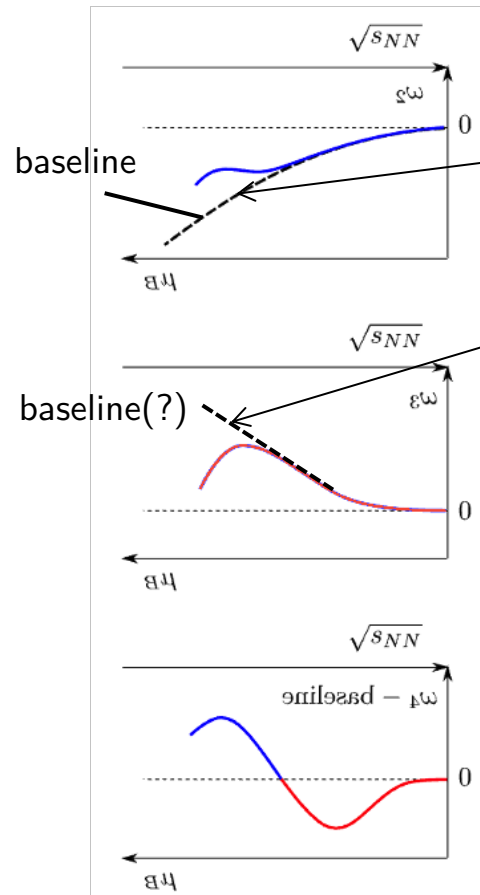
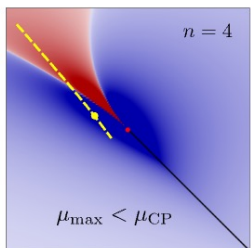
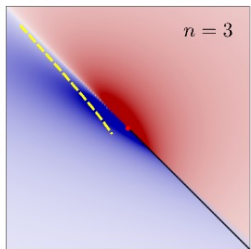
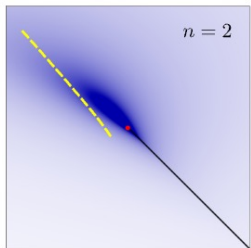
Expected signatures: **bump** in  $\omega_2$  and  $\omega_3$ , **dip** then **bump** in  $\omega_4$  for CP at  $\mu_B > 420$  MeV

# Factorial cumulants from RHIC-BES-II

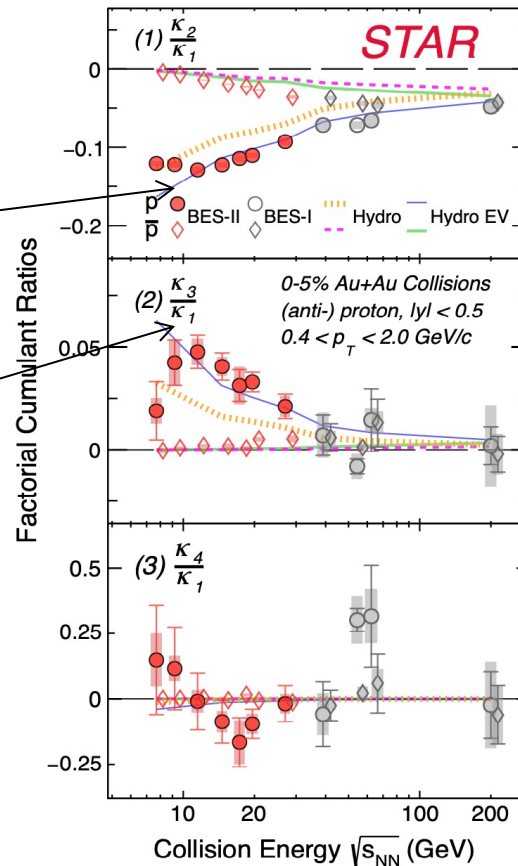
From M. Stephanov (SQM2024):

$$\omega_n = \hat{C}_n / \hat{C}_1^n$$

(universal EOS) critical  $\chi_n$ :



STAR data:



A. Pandav, CPOD2024

baseline (hydro):

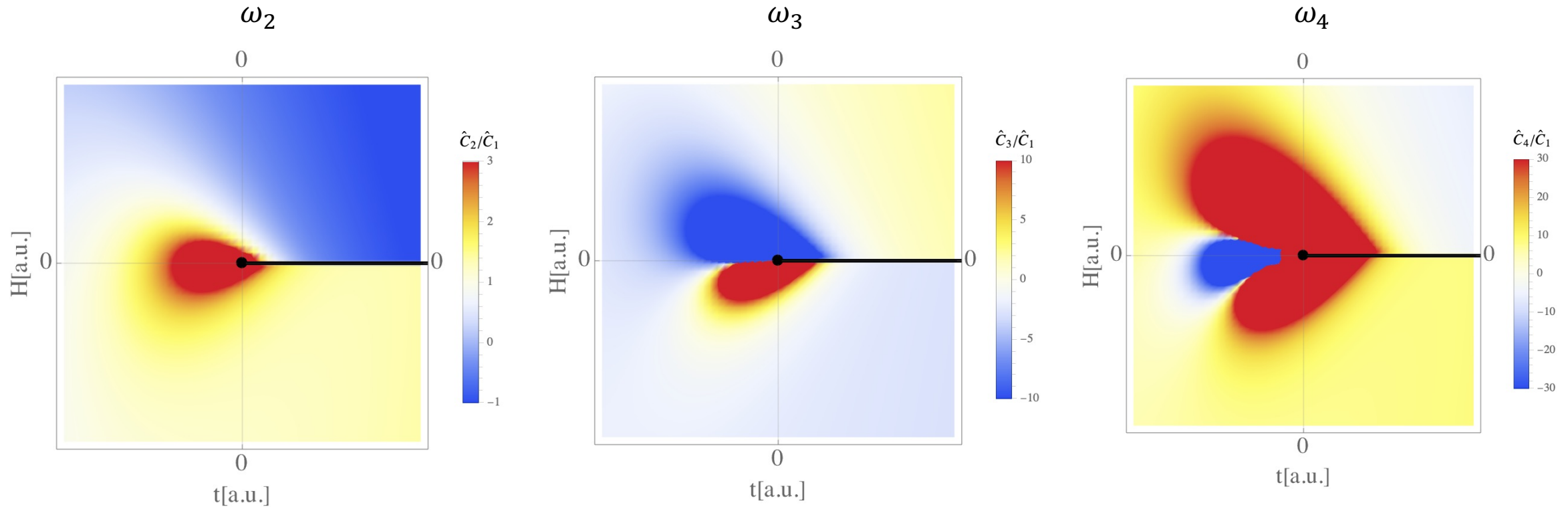
VV, V. Koch, C. Shen, PRC 105, 014904 (2022)

- describes right side of the peak in  $\hat{C}_3$
- signal relative to baseline:
  - positive  $\hat{C}_2 > 0$
  - negative  $\hat{C}_3 < 0$

**Conclusion 2:**

Controlling the non-critical baseline is essential

## Factorial cumulants in 3D-Ising model



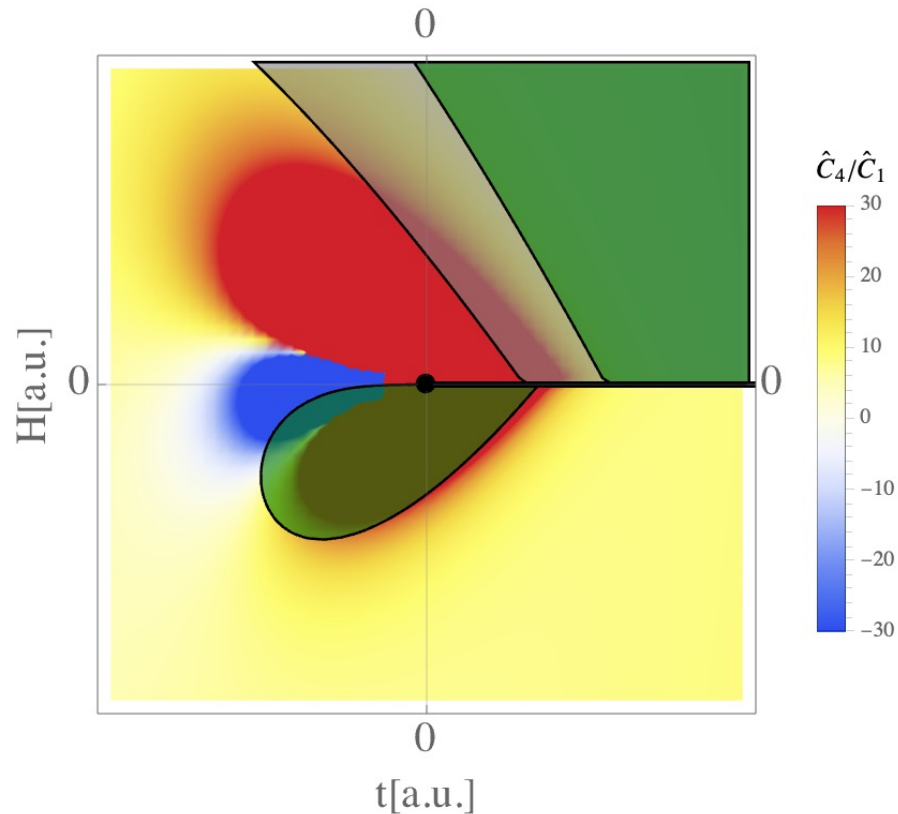
Adapted from Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

$$\omega_n = \hat{C}_n/\hat{C}_1$$



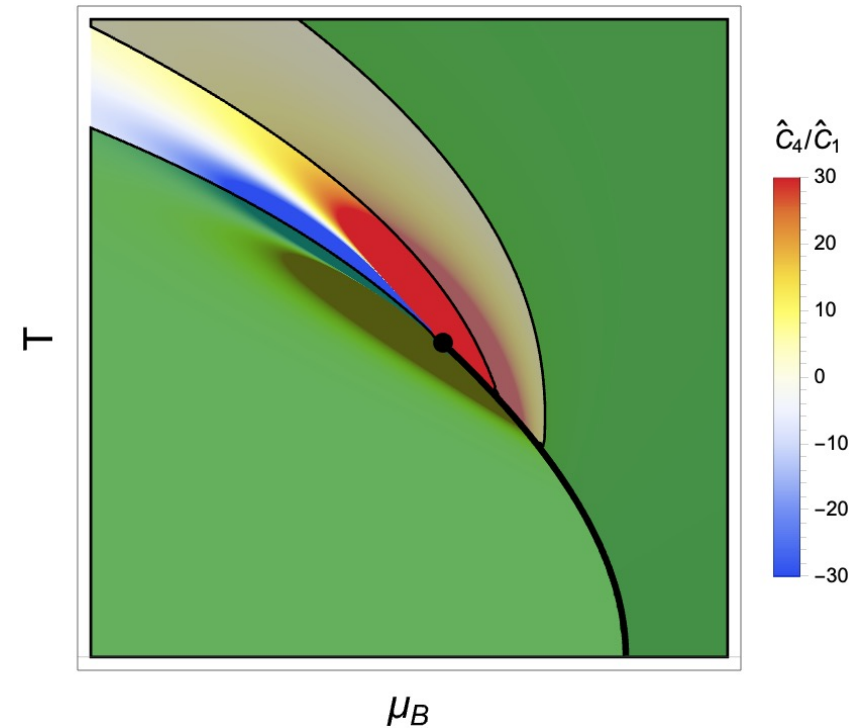
# Factorial cumulants from RHIC-BES-II and CP

## Exclusion plots



Shaded regions exclude  $\hat{C}_2 < 0$  &  $\hat{C}_3 > 0$

How it may look like in  $T - \mu_B$  plane



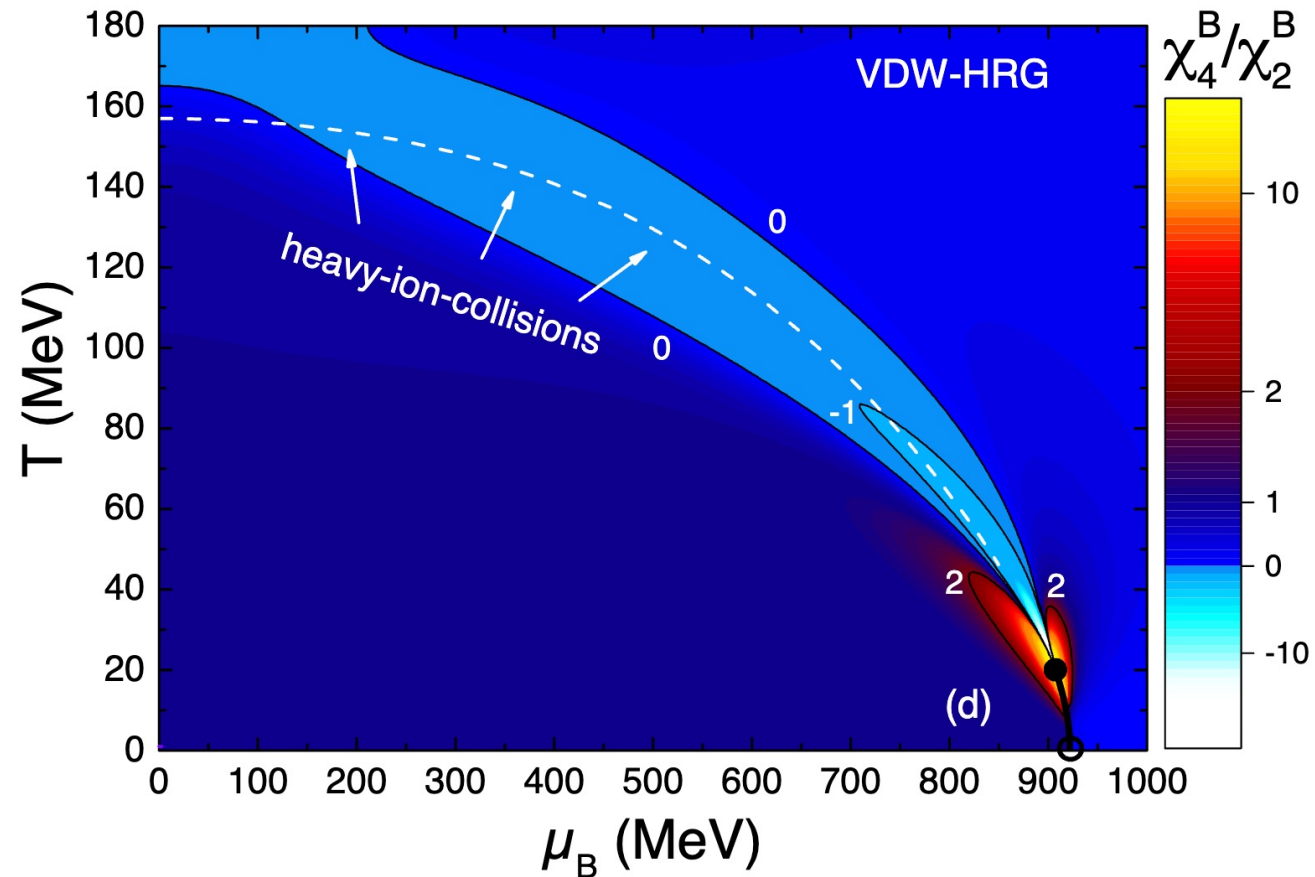
Based on QvdW model of nuclear matter

VV, Anchishkin, Gorenstein, Poberezhnyuk, PRC 92, 054901 (2015)

Freeze-out of fluctuations on the QGP side of the crossover?

# Nuclear liquid-gas transition

HRG with attractive and repulsive interactions among baryons

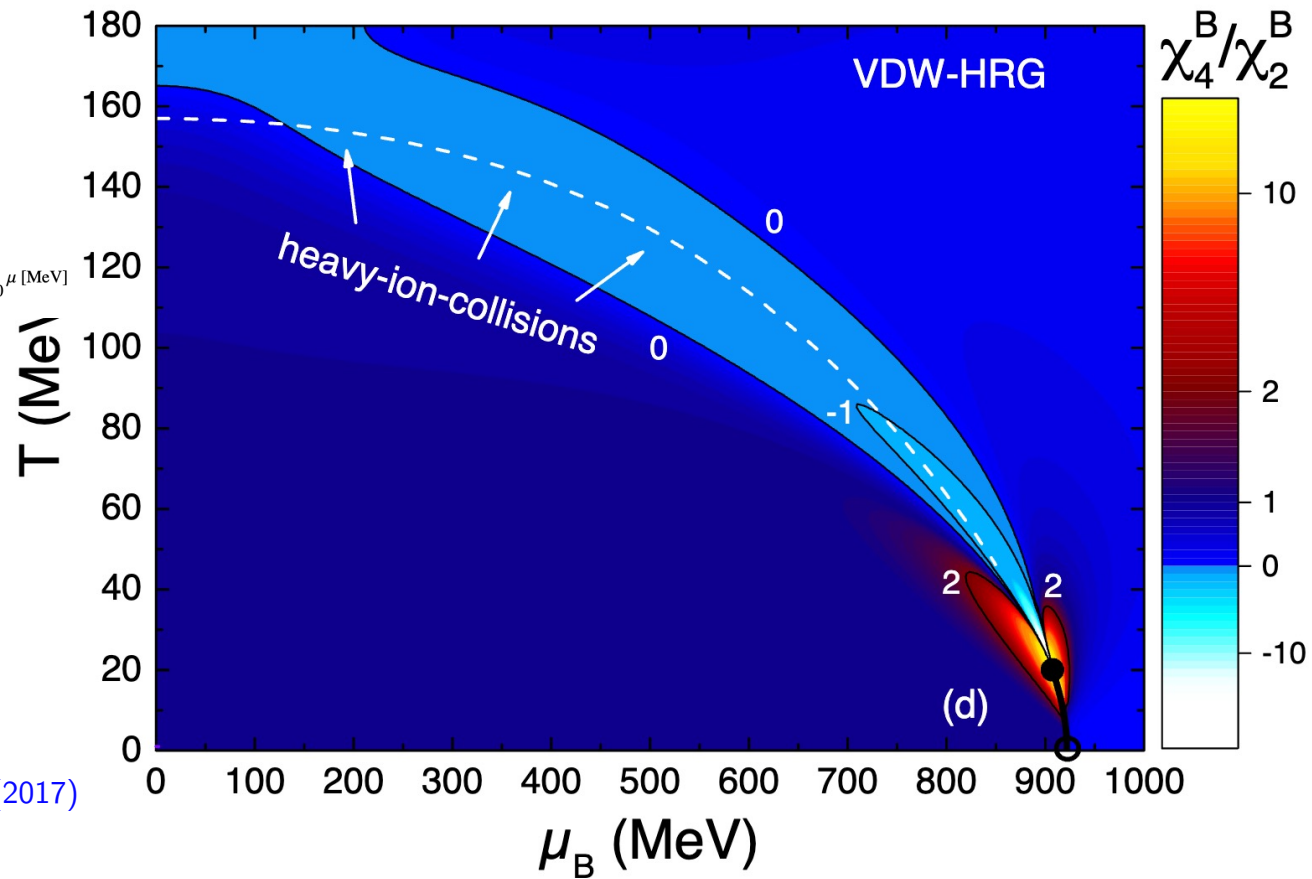


VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)

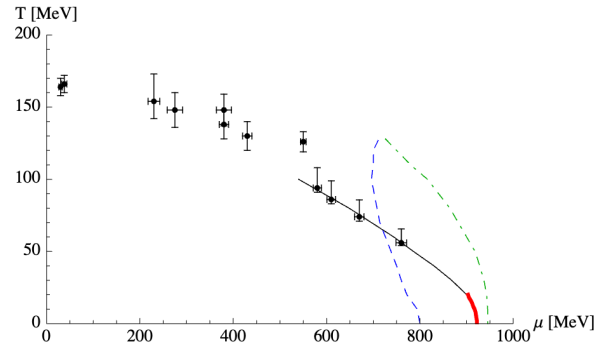


# Nuclear liquid-gas transition

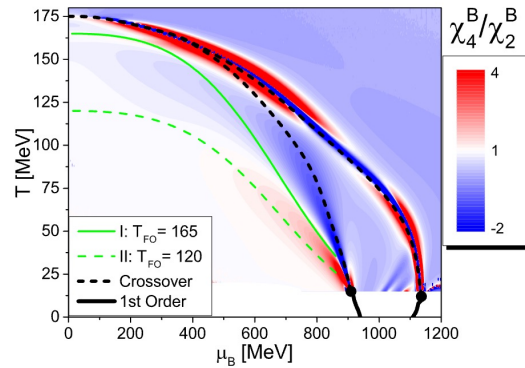
HRG with attractive and repulsive interactions among baryons



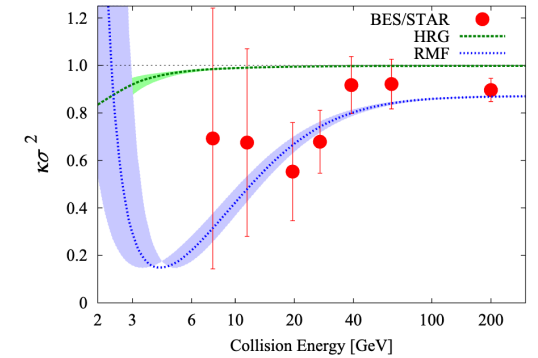
VV, Gorenstein, Stoecker, Phys. Rev. Lett. 118, 182301 (2017)



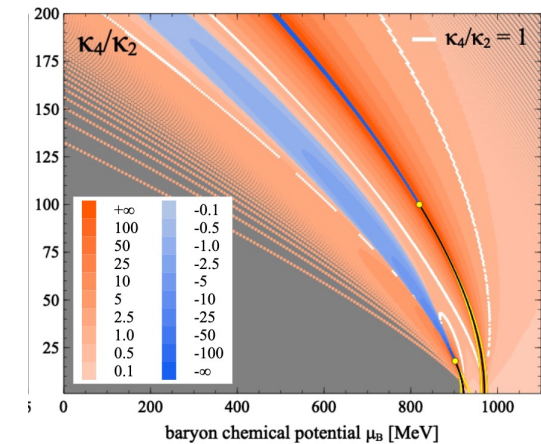
Floerchinger, Wetterich, NPA (2012)



Mukherjee, Steinheimer, Schramm, PRC (2017)



Fukushima, PRC (2014)



Sorensen, Koch, PRC (2020)

# Factorial cumulants and long-range correlations

# Acceptance dependence and long-range correlations

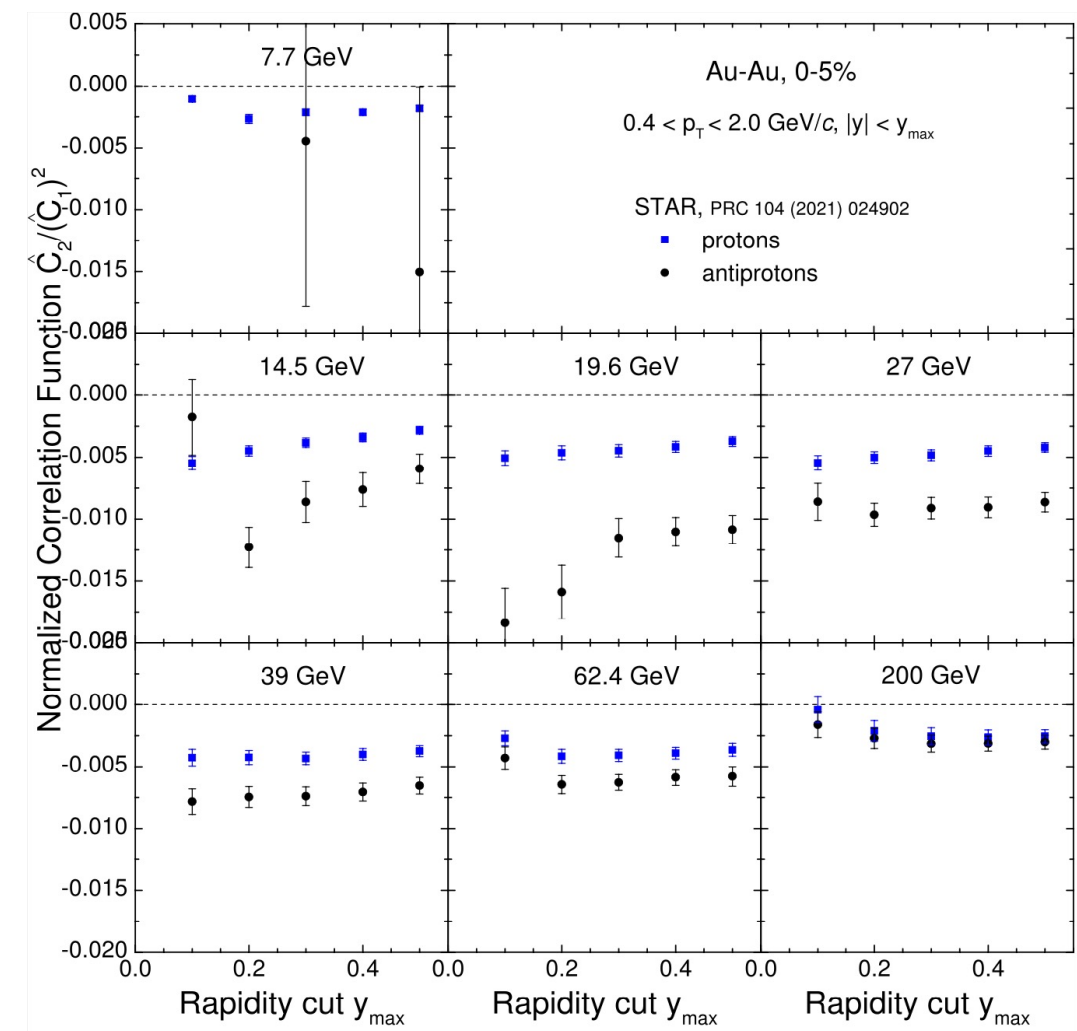
Long-range correlations:  $\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$  at given  $\sqrt{s_{NN}}$

- Global (not local) baryon conservation  
[Bzdak, Koch, Skokov, EPJC 77, 288 (2017)]
- + volume fluctuations  
[Holzmann, Koch, Rostamov, Stroth, arXiv:2403.03598]
- + (uniform) efficiency  
[Pruneau, Gavin, Voloshin, PRC 66, 044904 (2002)]

In particular  $\frac{\hat{C}_2^p}{(\hat{C}_1^p)^2} \approx \frac{\hat{C}_2^{\bar{p}}}{(\hat{C}_1^{\bar{p}})^2} = \text{const.}$

- **Significant difference between  $p$  and  $\bar{p}$  in BES-I**
- With BES-II one can test the scaling with greater precision and extended coverage in rapidity
  - No need for volume fluctuations corrections (CBWC)

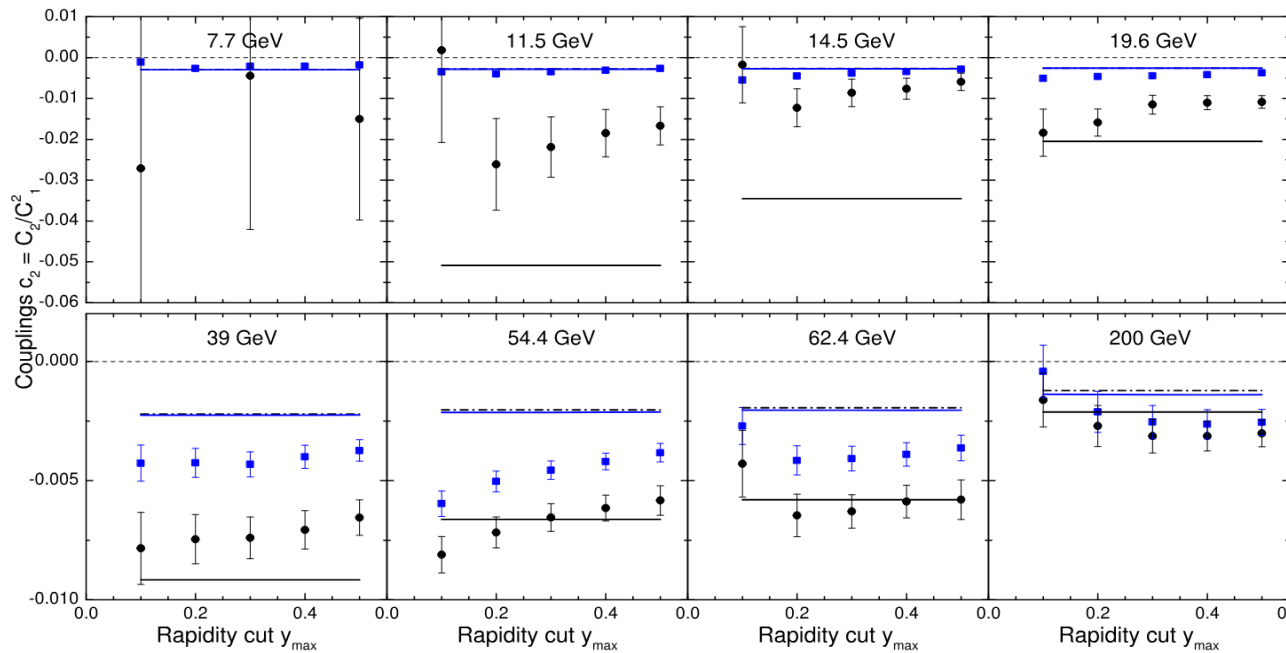
BES-I data



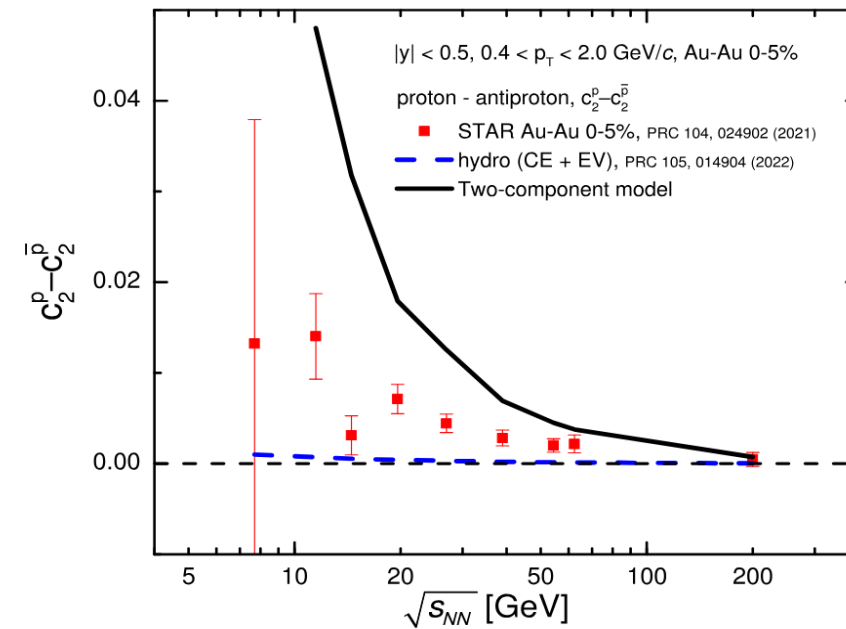
# Factorial cumulants and two-component model

**Two-component model:** produced ( $p\bar{p}$  pairs) and stopped protons come from two independent fireballs

Data lie in-between single and two-fireball models



Difference between  $p$  and  $\bar{p}$



## Opportunities for BES-II:

- Further test the splitting between protons and antiprotons in 2<sup>nd</sup> order cumulants

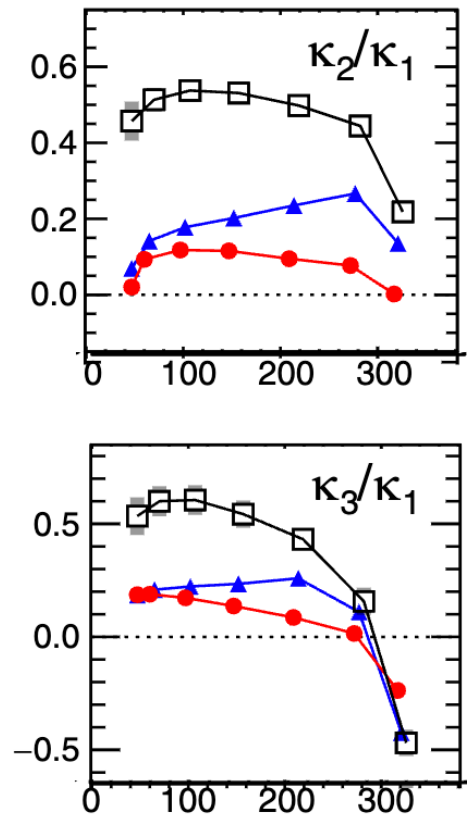
- Critical point signal expected to break the scaling  $\frac{\hat{C}_n}{(\hat{C}_1)^n} = \text{const.}$

Lower energies

# Lower energies $\sqrt{s_{NN}} \leq 7.7$ GeV

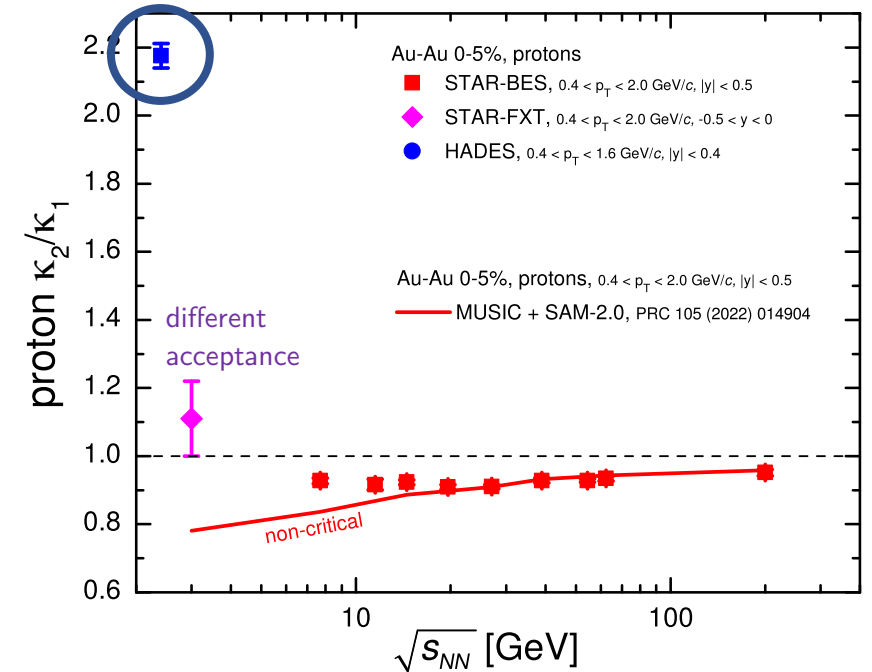
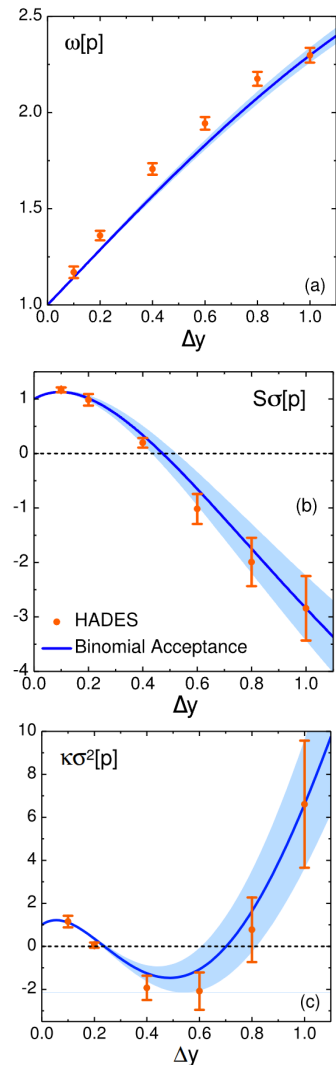
STAR 2209.11940

$$\sqrt{s} = 3 \text{ GeV}$$



HADES 2002.08701

$$\sqrt{s} = 2.4 \text{ GeV}$$



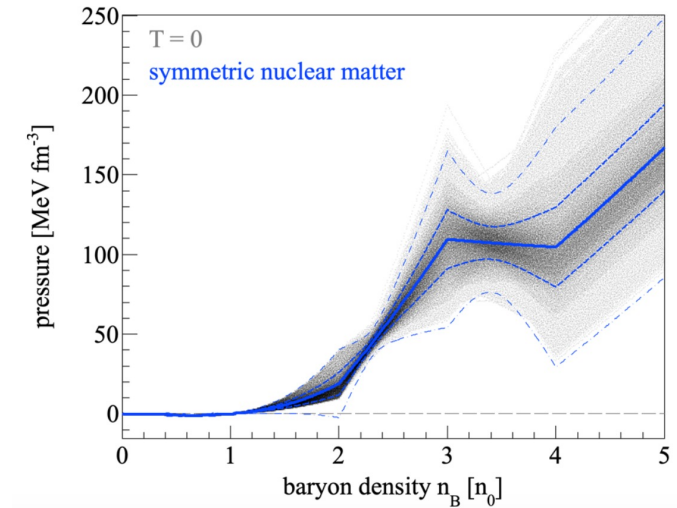
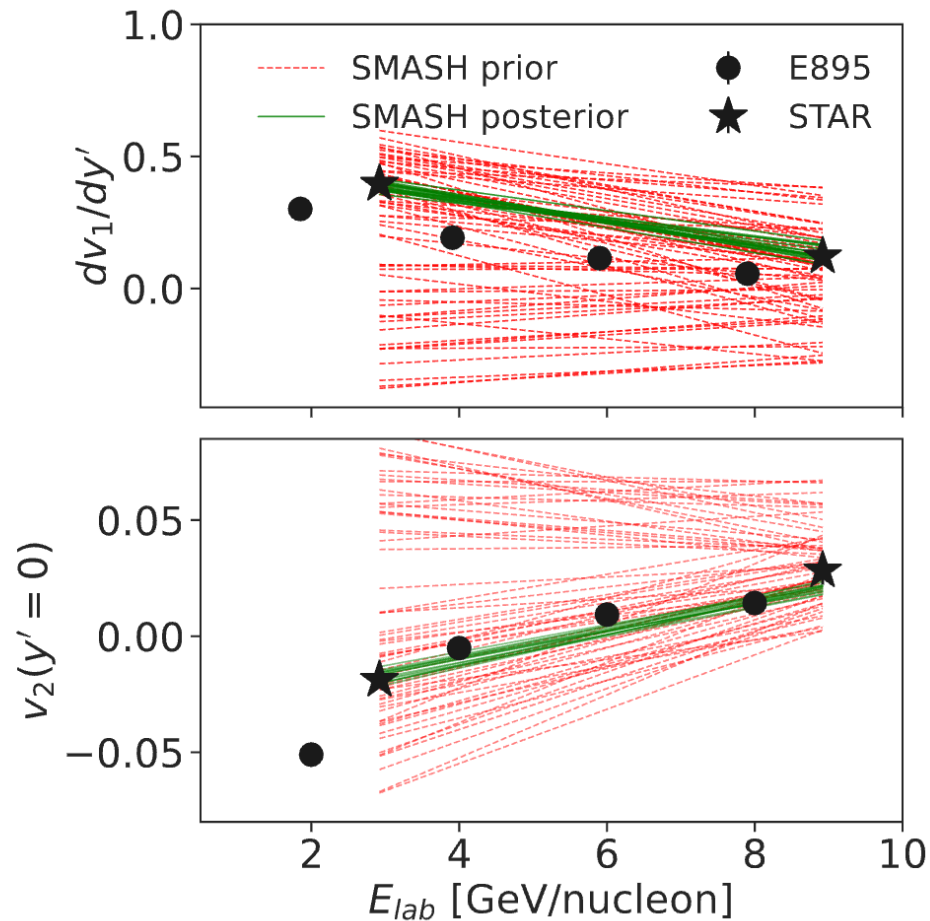
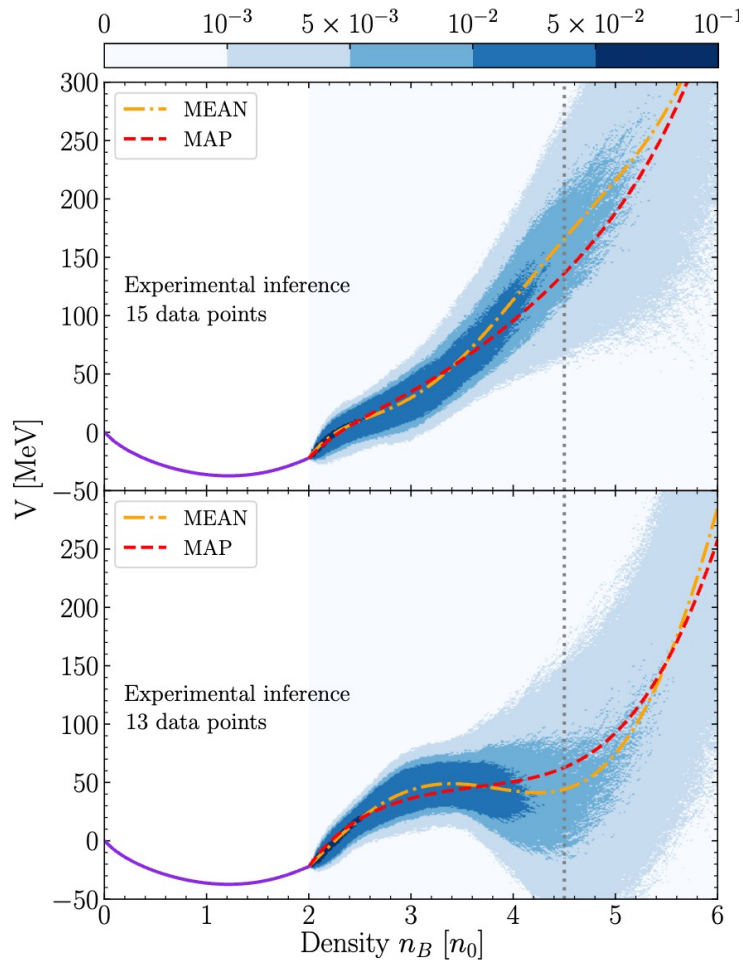
**Challenges:** volume fluctuations, fragments, equilibration

**Crucial to close the gap between 3 and 7.7 GeV energies**

**RHIC-FXT, CBM-FAIR, J-PARC...**

# Dense matter EoS from flow measurements

- Use hadronic transport (UrQMD and SMASH) with adjustable mean field to use a flexible EoS
- Extract the EoS from proton flow measurements



- QCD equation of state
  - Well-controlled at small baryon densities with lattice QCD where the transition is a chiral crossover
  - New developments point to the possible CP location at  $T \sim 90\text{-}120$  MeV and  $\mu_B \sim 400 - 600$  MeV
- Proton cumulants are uniquely sensitive to the the CP but challenging to model dynamically
  - factorial cumulants are especially advantageous to distinguish critical and non-critical features
- BES-II data
  - Consistent with predictions due to non-critical physics at  $\sqrt{s_{NN}} \geq 20$  GeV (as was BES-I data)
  - Shows (non-monotonic) structure in factorial cumulants
  - Positive  $\hat{C}_2$  and negative  $\hat{C}_3$  after subtracting non-critical baseline at  $\sqrt{s_{NN}} < 10$  GeV

## Outlook:

- Improved description of non-critical baselines and quantitative predictions of critical fluctuations
- Acceptance dependence of factorial cumulants, understanding antiprotons

**Thanks for your attention!**