Limiting phase transition scenarios in NSs: challenges and opportunities

Sophia Han, T.D. Lee Fellow

T.D. Lee Institute / Shanghai Jiao Tong U.

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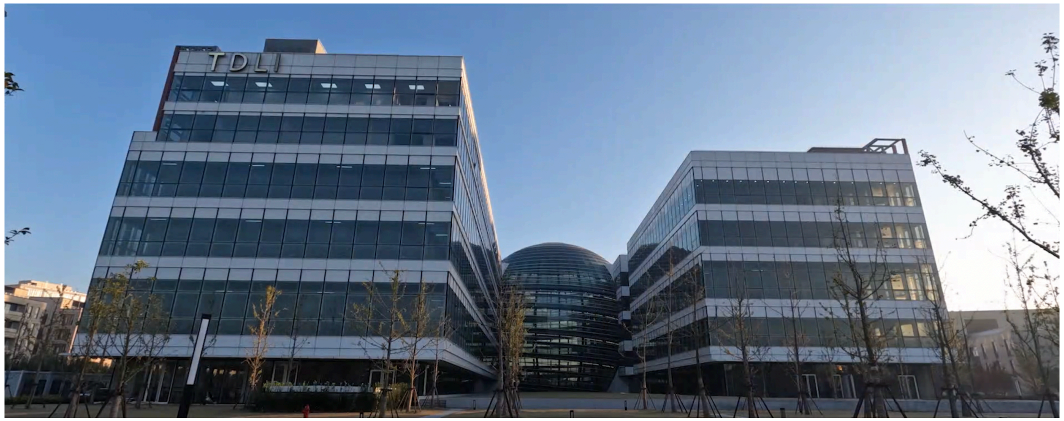

Compact Stars in the QCD Phase diagram (CSQCD 2024) @YITP

Oct. 7-11, 2024

Limiting phase transition scenarios in NSs: challenges and opportunities

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In collaboration with

- Mark Alford (Wash U), Christian Drischler (Ohio U), Katerina Chatziioannou (Caltech), C. Constantinou (INFN/ECT), Reed Essick (CITA), Prashanth Jaikumar (CSU Long Beach), S. Lalit (FRIB/MSU), Philippe Landry (CITA), Jim Lattimer (Stony Brook U), Isaac Legred (Caltech), M. A. A. Mamun (Ohio U), Madappa Prakash (Ohio U), Sanjay Reddy (INT, U Washington), Kai Schwenzer (Istanbul U), Andrew Steiner (UTK/ORNL), Tianqi Zhao (N3AS, UC Berkeley/Ohio U)

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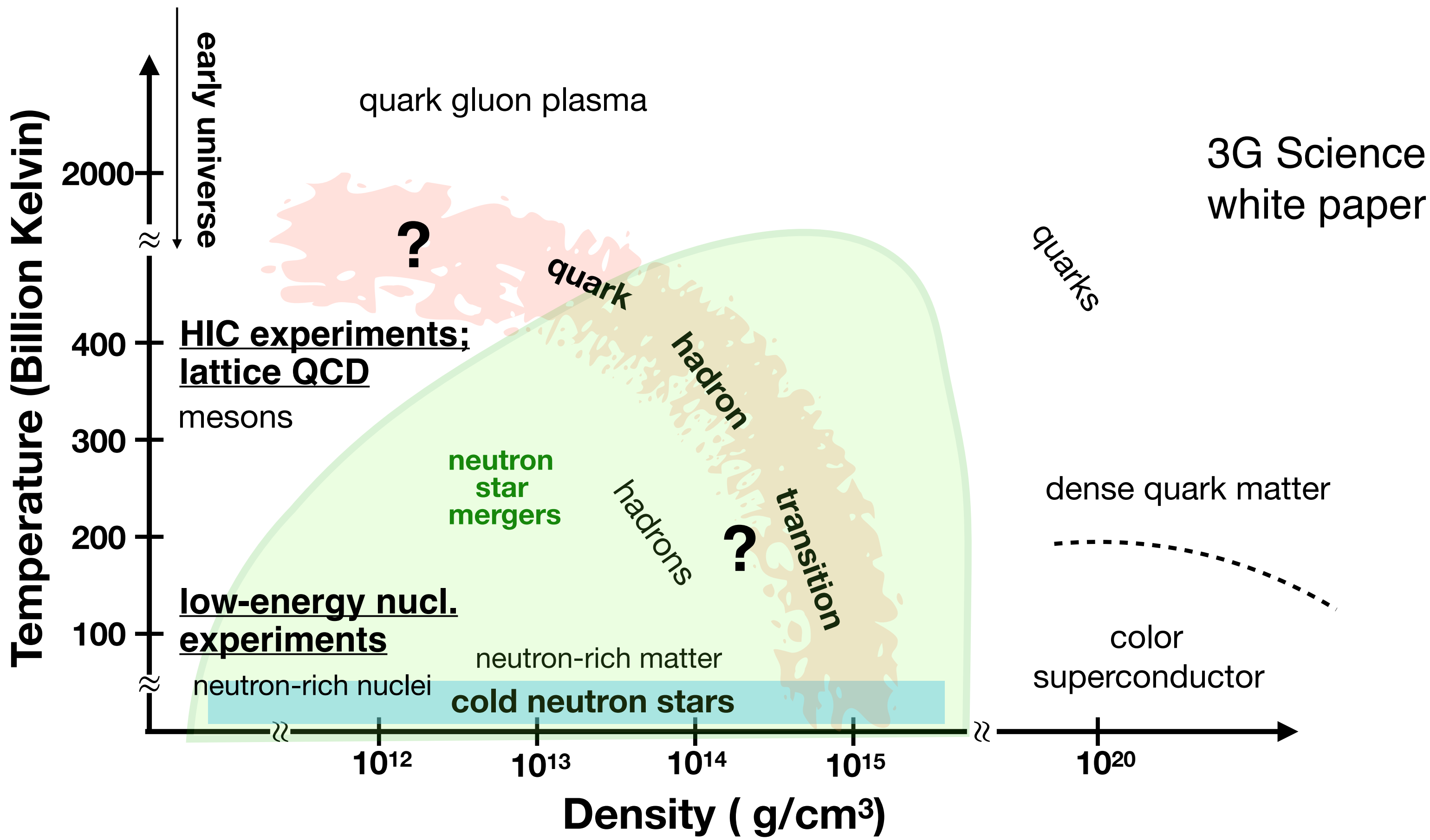
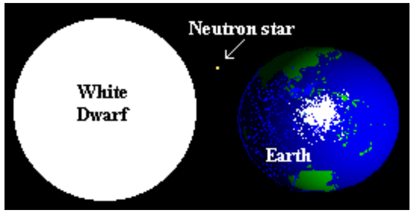
Laboratory for theoretical physics



- no terrestrial experiments can probe such high densities
- reliable first-principle calculations break down at the strongly-interacting regime

- can't calculate properties of cold dense matter; must **observe**!



NS mass-radius diagram

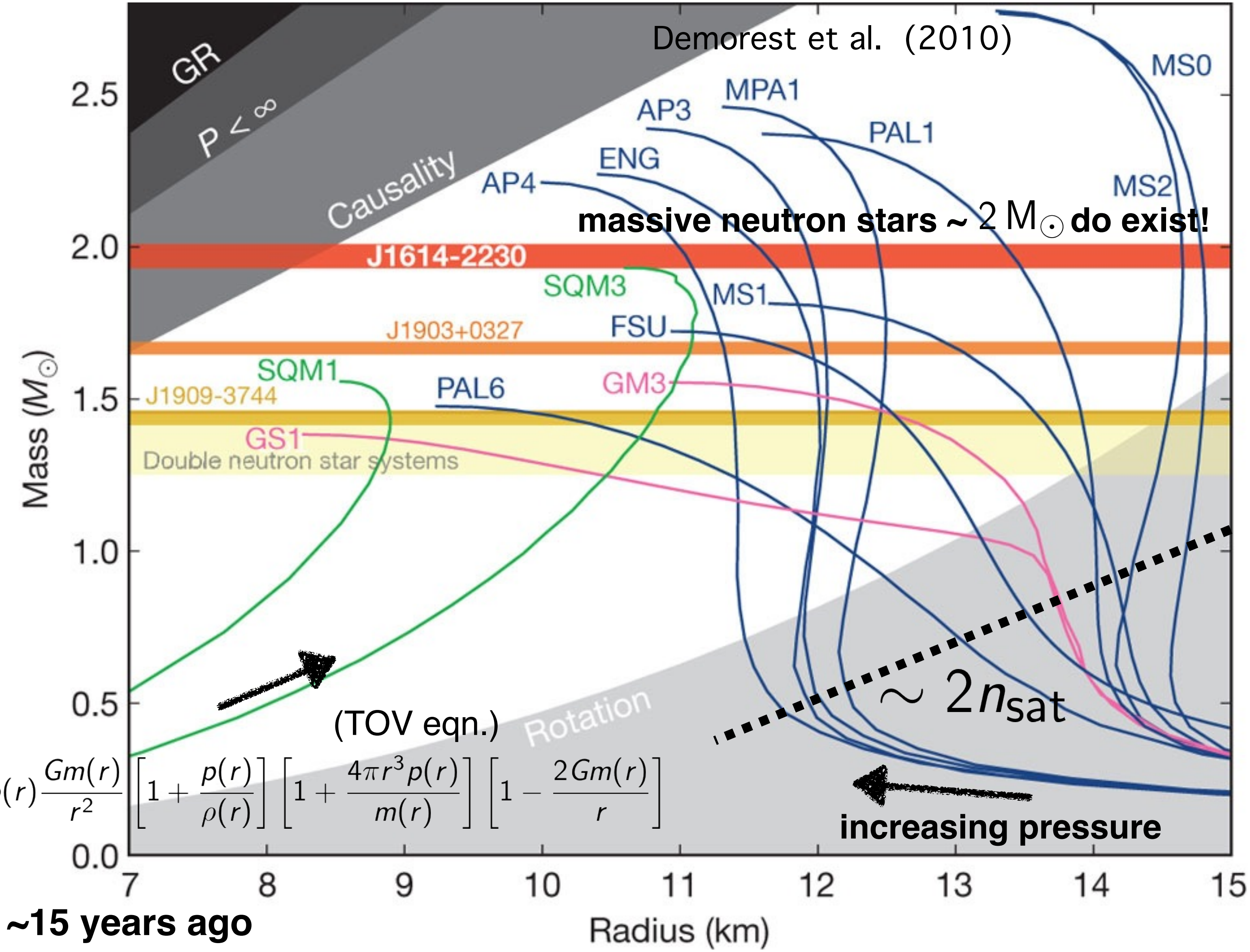
micro theo.

$p(\varepsilon)$

GR

$M(R)$

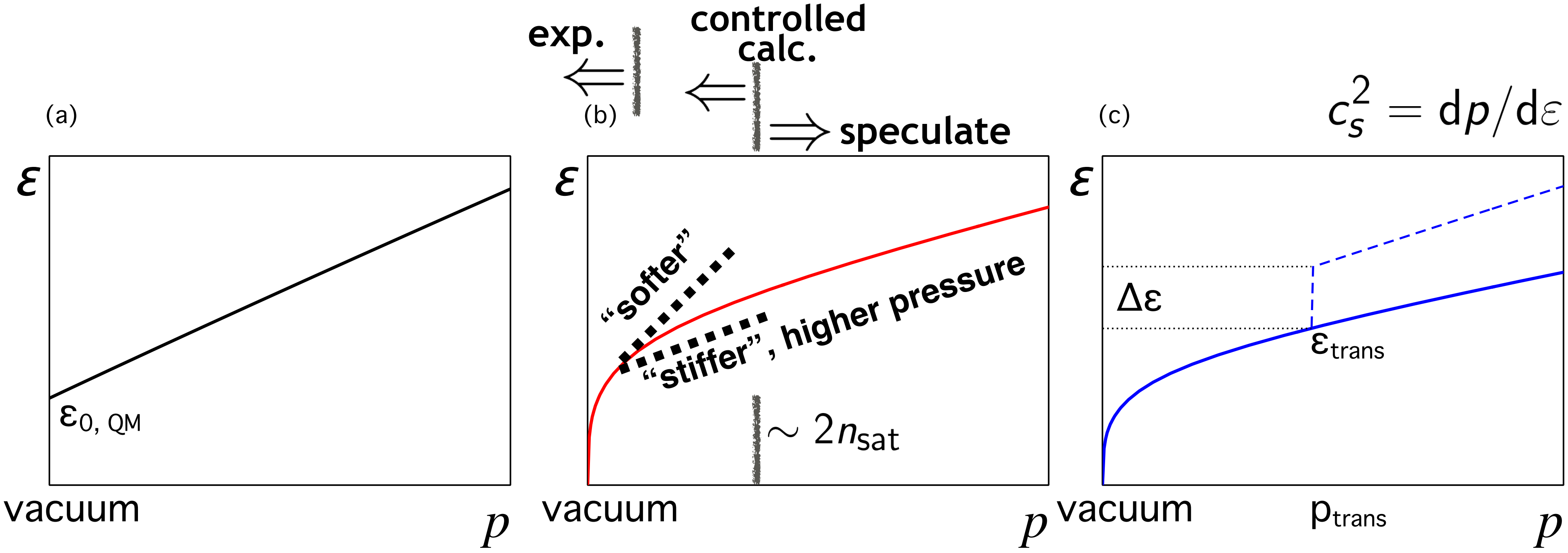
astro obs.



$$\frac{dp}{dr} = -\rho(r)\frac{Gm(r)}{r^2}\left[1+\frac{p(r)}{\rho(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{m(r)}\right]\left[1-\frac{2Gm(r)}{r}\right]$$

~15 years ago

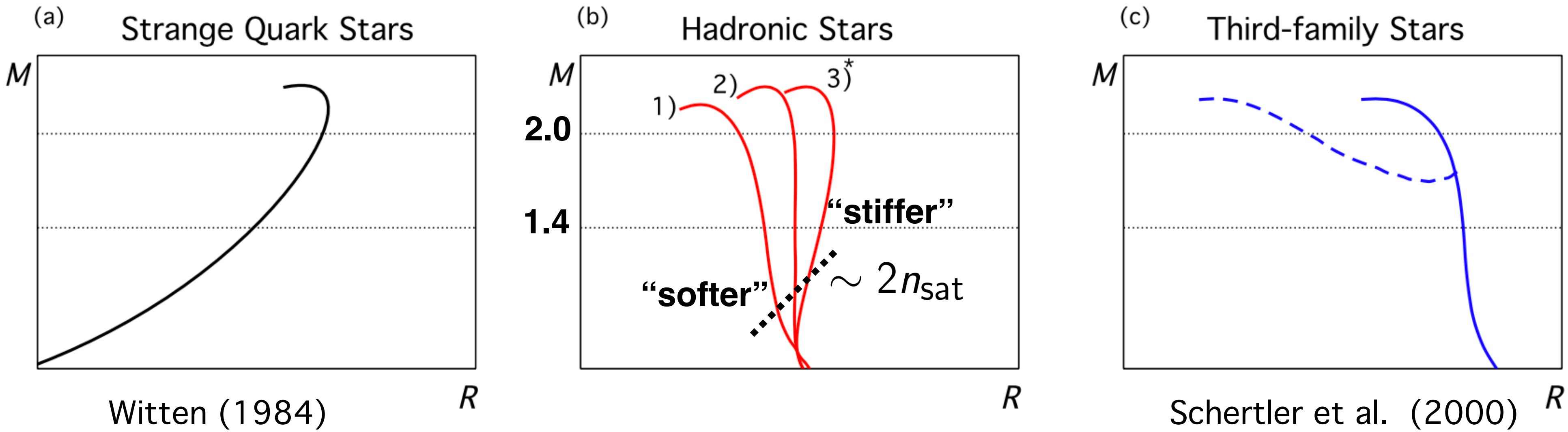
Schematic EoSs from theory



- self-bound stars with a bare surface e.g. strange matter hypothesis
- continuous (and mostly smooth) profile for normal hadronic EoSs; *also possible with weak/mild phase transition or crossover
- substantial softening e.g. discontinuity in the energy density induced by a strong sharp phase transition

Categories of the *M-R* relation

SH & Prakash,
arXiv:2006.02207



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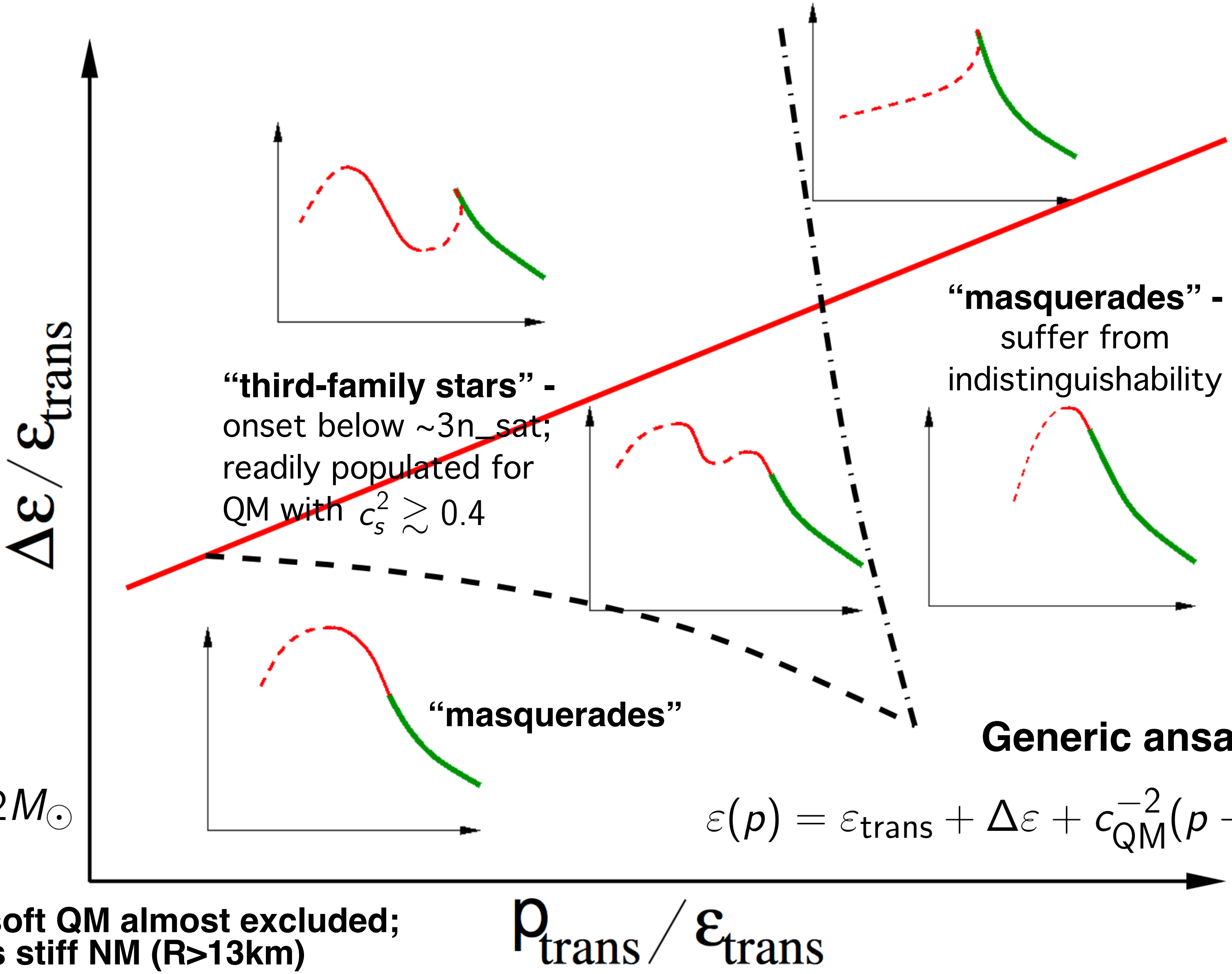
M-R morphology with 1st-OPTs

Alford, **SH** & Prakash
arXiv:1302.4732



Generic ansartz

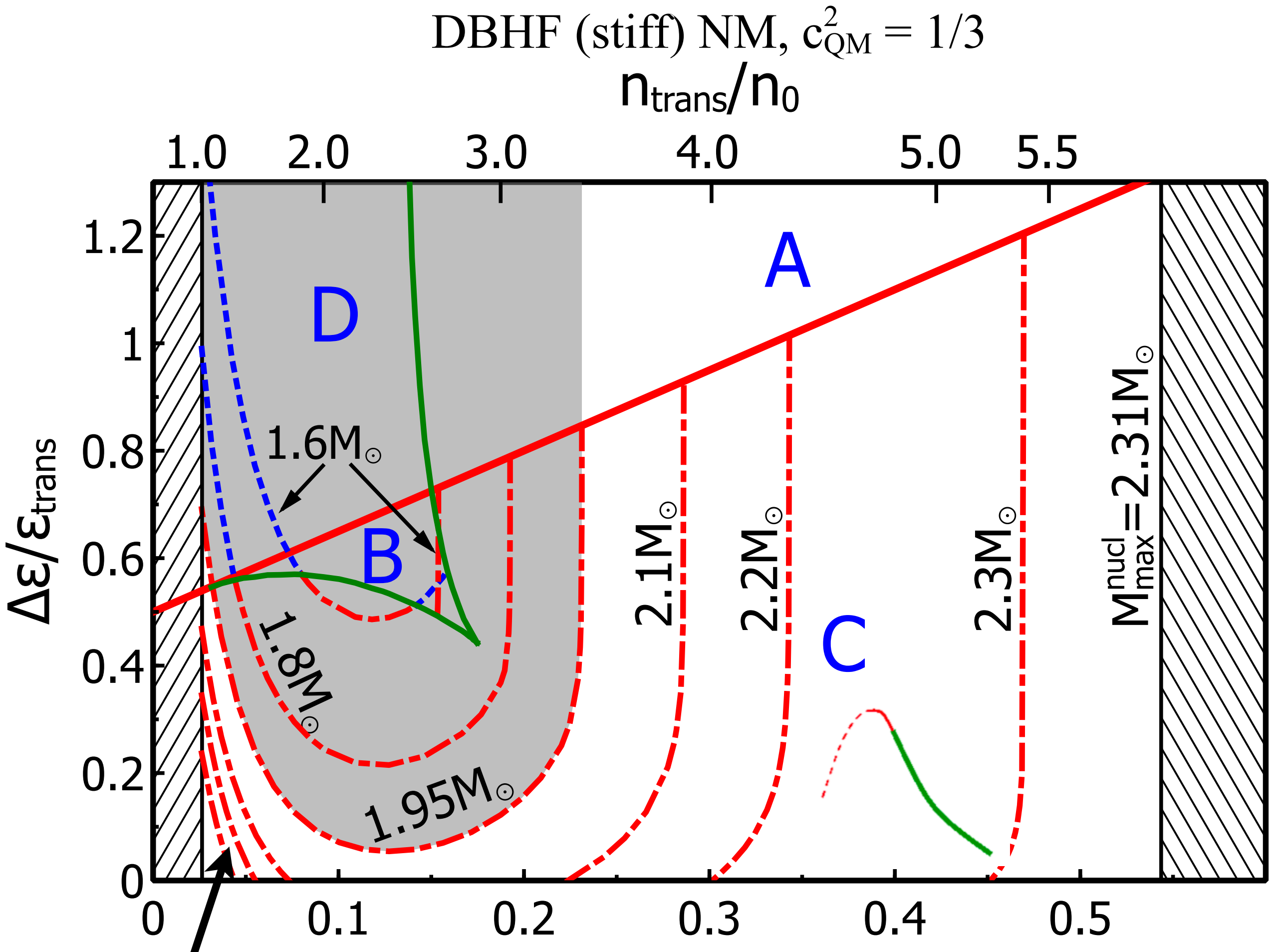
$$\varepsilon(p) = \varepsilon_{trans} + \Delta\varepsilon + c_{QM}^{-2}(p - p_{trans})$$

$M_{max} \geq 2M_{\odot}$
$\Rightarrow$

- **very soft QM almost excluded; favors stiff NM (R>13km)**

Alford & **SH**
arXiv:1508.01261

Constraints from heavy pulsars



still survives the conformal limit

- with weakly interacting quarks (**near conformal, pQCD-like** matter), very limited para. space to reach two solar masses
- **high transition density** scenario - resembles no PT; short extension
- **low transition density** scenario - no twin stars!

Generic ansartz

$$\varepsilon(p) = \varepsilon_{trans} + \Delta\varepsilon + c_{QM}^{-2}(p - p_{trans})$$

Constraints from heavy pulsars

Alford & **SH**
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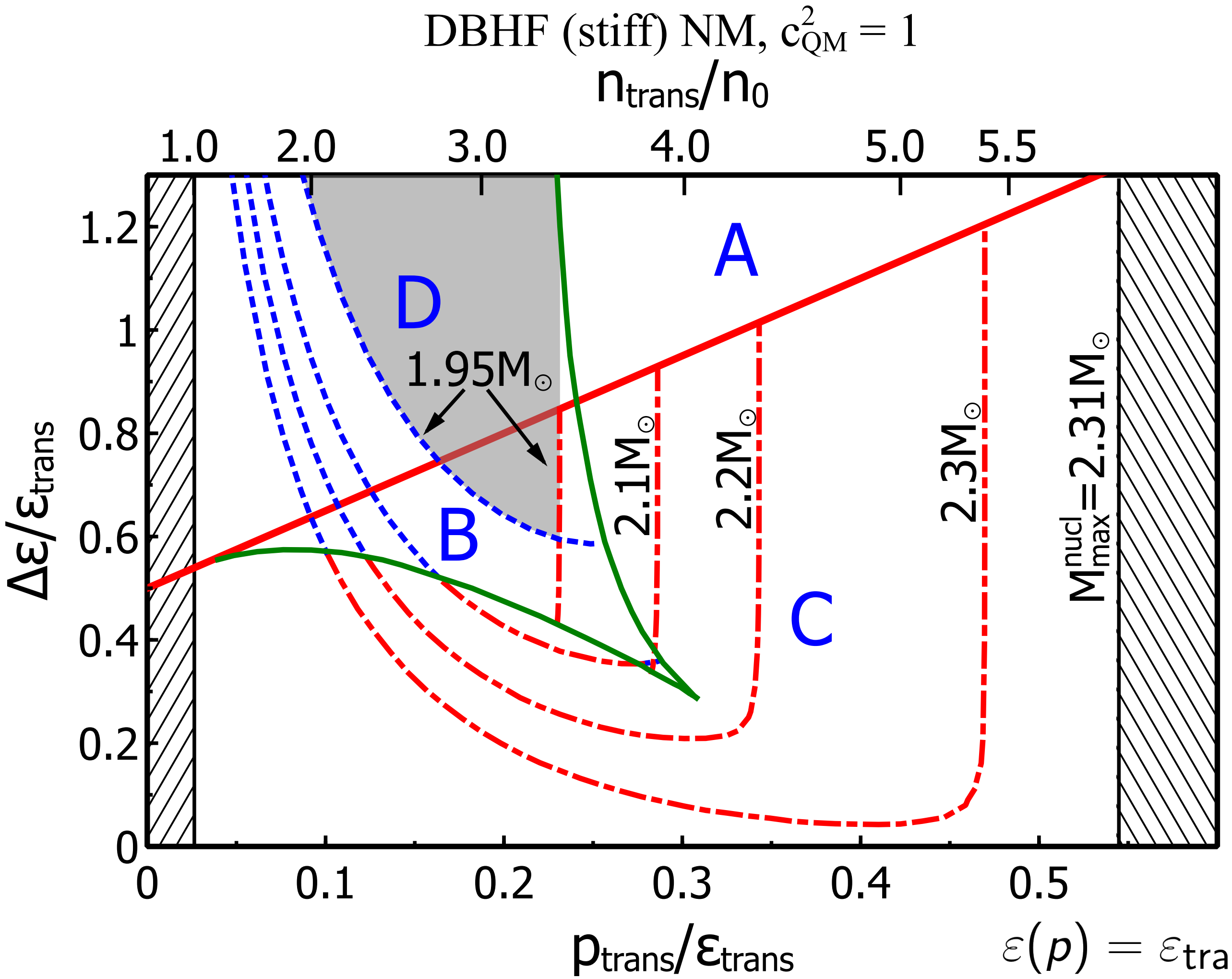


- with **maximally stiff** QM, a much broader range of transition densities is allowed
- distinct feature of the **disconnected** branch
- observability via e.g. future measurements of inspiral GWs from a **population** of events

Generic ansartz

$$\varepsilon(p) = \varepsilon_{trans} + \Delta\varepsilon + c_{QM}^{-2}(p - p_{trans})$$

Paths towards high-density regime

maximally distinguishable

purely hadronic/nucleonic matter (standard)

strong PT

sharp boundary - Maxwell

weak PT

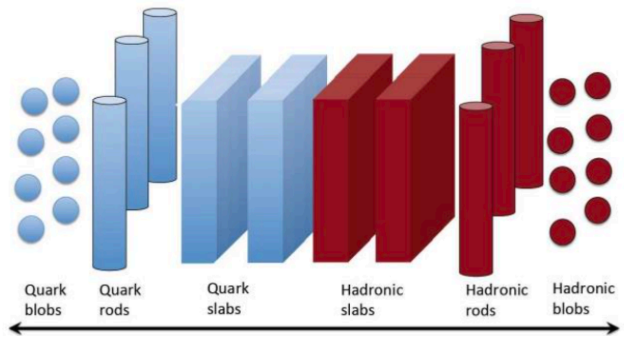
known up to ~saturation

1st-order PT? -> quarks

mixed phase - Gibbs (geometrically separated)

smooth crossover (quarkyonic)?

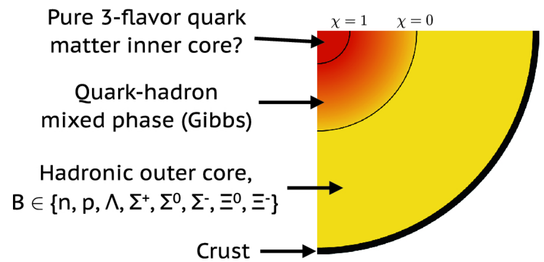
“golden window” in the vicinity of ~2*nsat; hints from exp.?



- **<u>masquerade</u>** <u>problem</u> - EoSs with or without PTs may hardly be distinguishable via observations that constrain *M-R* only

- crossover models motivated by e.g. lattice calc.

- 1st-OPT: mixed phase (Gibbs) favored if the hadron/quark surface tension is small



Treating quarks within neutron stars

HM → QM	First-order Transition (Maxwell) [a]	Crossover Transition
stiff to soft	✗ vMIT: cannot support $M_{\rm max} \geq 2\,{\rm M}_\odot$ ✗ vNJL: $M_{\rm trans} \gtrsim 1.7\,{\rm M}_\odot$, $\tilde\Lambda(\mathcal{M} = 1.186\,{\rm M}_\odot) > 720$	✗ unphysical decreasing function of $P(n_{\rm B})$
soft to stiff	✗ no intersection for $P(\mu)$ [b]	$c_s^2 \gtrsim 0.7$ ✓interpolation: $M_{\rm trans} \lesssim 1.0\,{\rm M}_\odot$, $R_{1.4} < 13\,{\rm km}$ ✓quarkyonic: $M_{\rm max} \geq 2\,{\rm M}_\odot$; $R_{1.4}$ and $M_{\rm trans}$ vary
soft to soft	✗ cannot support $M_{\rm max} \geq 2\,{\rm M}_\odot$	✗ cannot support $M_{\rm max} \geq 2\,{\rm M}_\odot$
stiff to stiff	✓vMIT: $M_{\rm max} \geq 2\,{\rm M}_\odot$; $R_{1.4}$ and $M_{\rm trans}$ vary $c_s^2 \gtrsim 0.4$ ✗ vNJL: onset for quarks too high; immediately destabilize	✗ $\tilde\Lambda(\mathcal{M} = 1.186\,{\rm M}_\odot) > 720$, $R_{1.4} > 13\,{\rm km}$

[a] See text for details if the Gibbs construction is applied. Gibbs construction satisfies many observational constraints such as $R_{1.4}$ and $M_{\rm max}$ due to the earlier onset of quarks. However, distinguishability from purely-hadronic stars is lost.

[b] Limited by the specific quark models applied here; in a generic parametrization (e.g. CSS) the soft HM → stiff QM is possible.

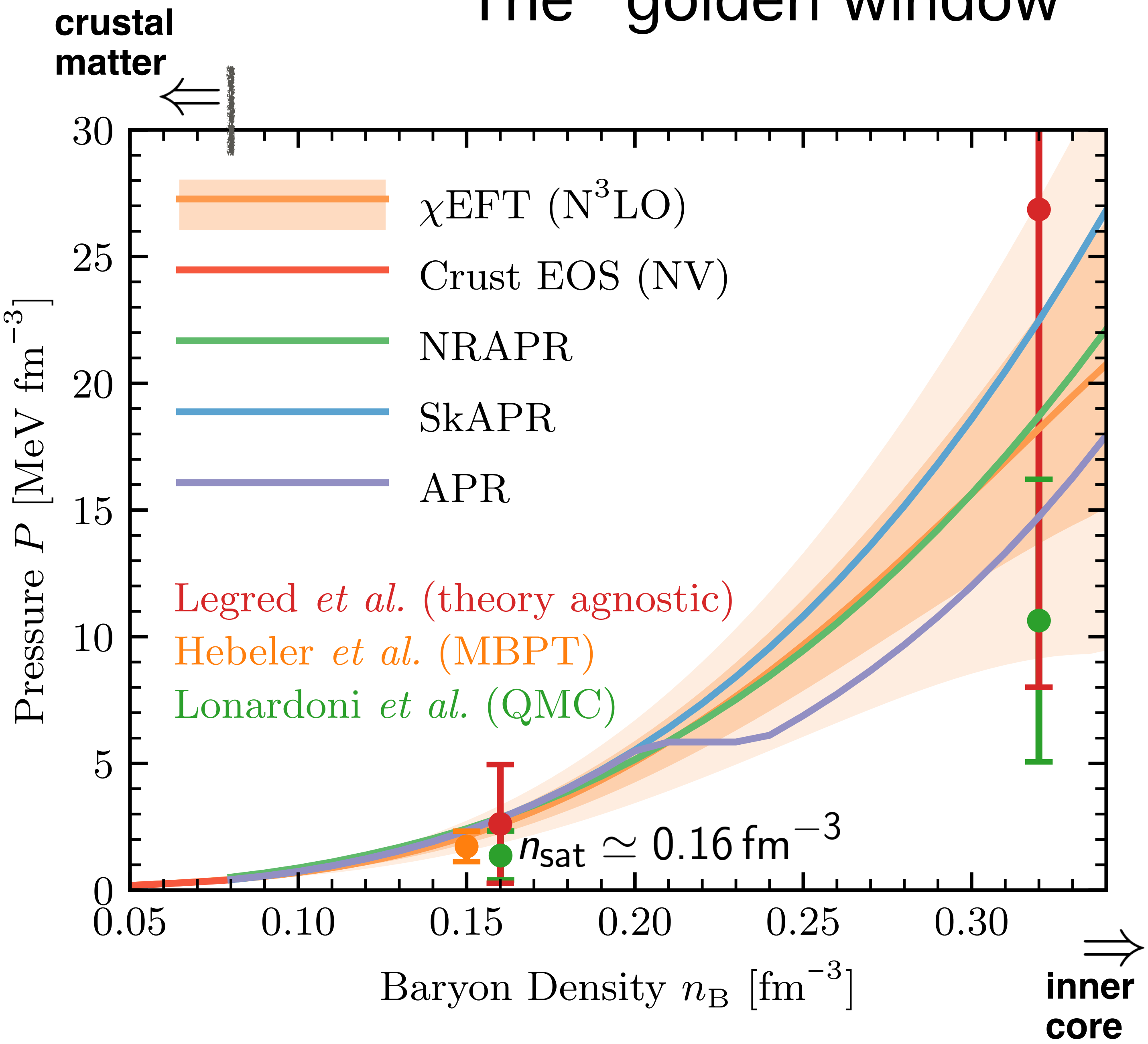
TABLE VI. Summary of the minimum density and minimum neutron star mass when quarks start to appear in various treatments of phase transitions explored in our work; see also the indicated figures for detailed information.

Treatment	$n_{\rm trans}/n_0$	$M_{\rm trans}$	Figure reference
Maxwell	1.77	0.97 $\rm M_\odot$	Fig. 2(b) and 2(c)
Gibbs	$\lesssim$1.5	$\lesssim$0.6 $\rm M_\odot$	Fig. 2(e) and 2(f)
Interpolation	2.0, 1.5	0.81, 0.48 $\rm M_\odot$	Fig. 5(b) and 5(c)
Quarkyonic	2.31	0.97, 1.21 $\rm M_\odot$	Fig. 6(b) and 6(c)

SH, Al Mamun, Lalit, Constantinou & Prakash, **arXiv:1906.04095**

The “golden window”

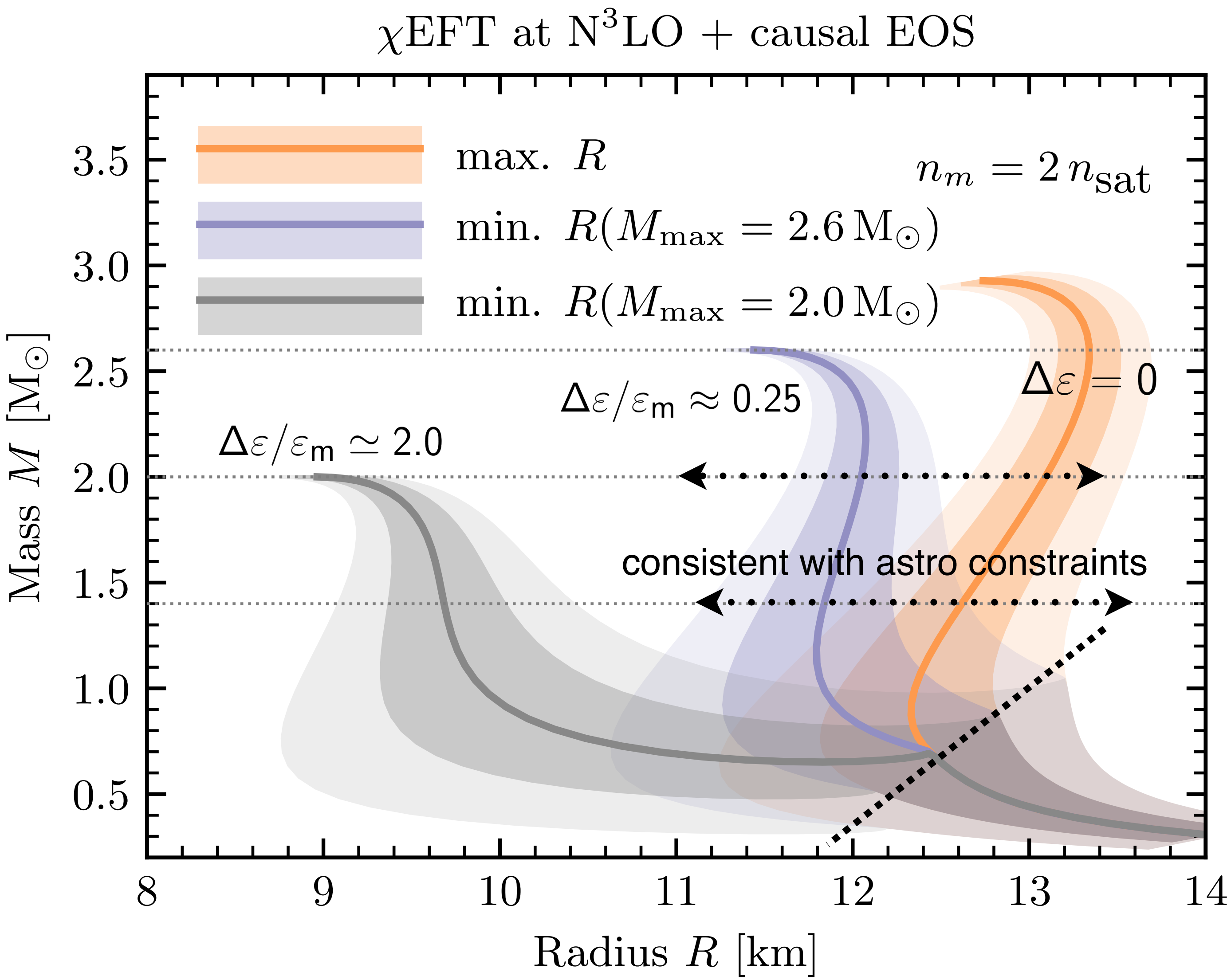
Drischler, **SH** & Reddy
arXiv:2110.14896



- pressure at low densities **[outer core]** controls typical NS radii: stiff or soft?
- reliably **quantified** uncertainties from chiEFT for beta-equilibrated NSM
- less than **~5%** deviation from PNM pressures
- to **extrapolate** or match at higher densities in the **inner core**

Bounds from causality

Drischler, **SH**, Lattimer, Prakash, Reddy and Zhao, **arXiv:2009.06441**

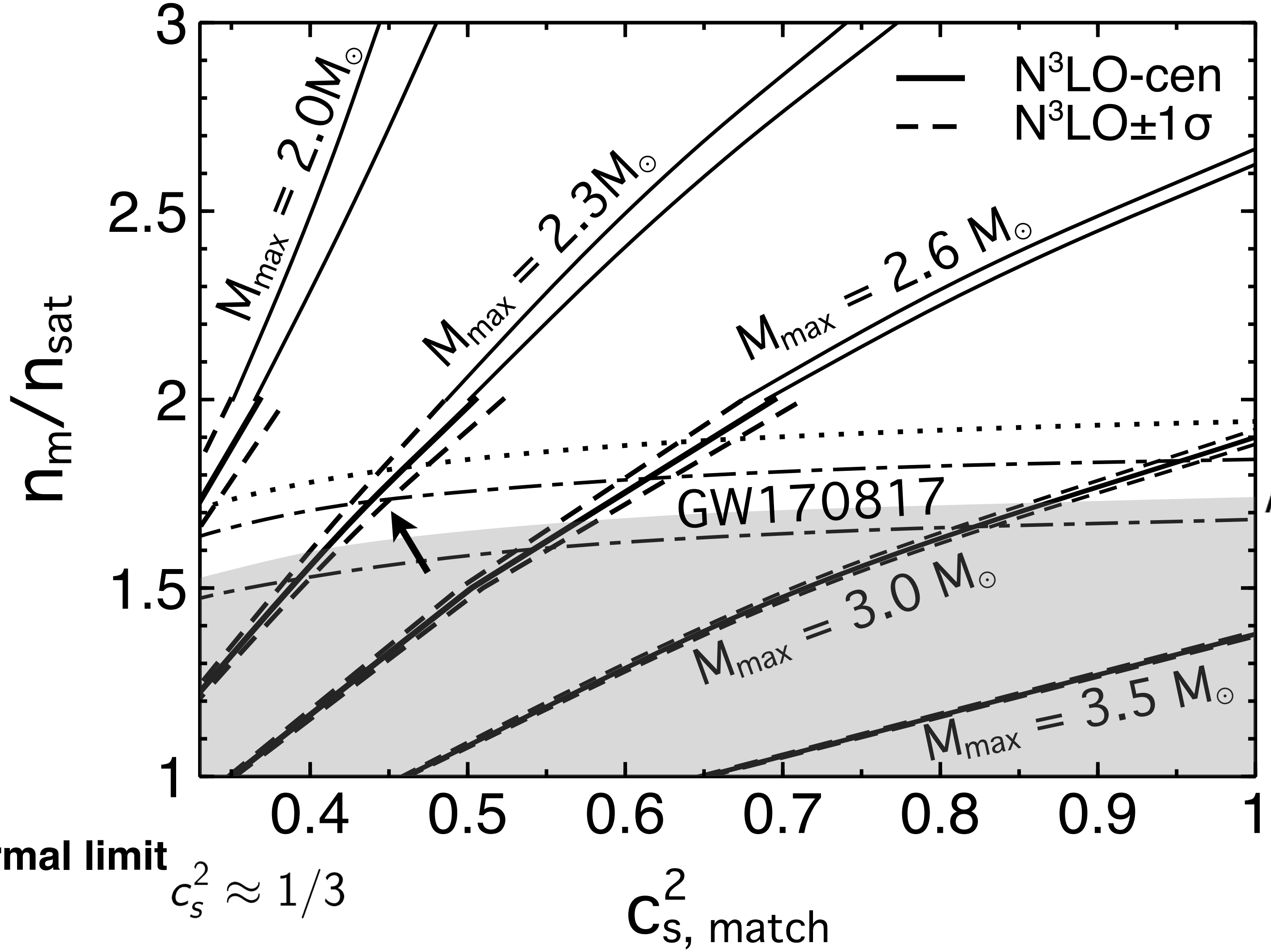


- pressure at low densities **[outer core]** controls typical NS radii: stiff or soft?
- reliably **quantified** uncertainties from chiEFT for beta-equilibrated NSM
- absolute **causal** limits imposed at high densities
- confronted with data: **interplay** between M_max and NS radii

Supporting massive NSs

Drischler, **SH**, Lattimer, Prakash, Reddy and Zhao, **arXiv:2009.06441**

- sound speed in the **core** and **where** rapid stiffening in the EoS begins



$\tilde{\Lambda}_{1.186} \leq 720$

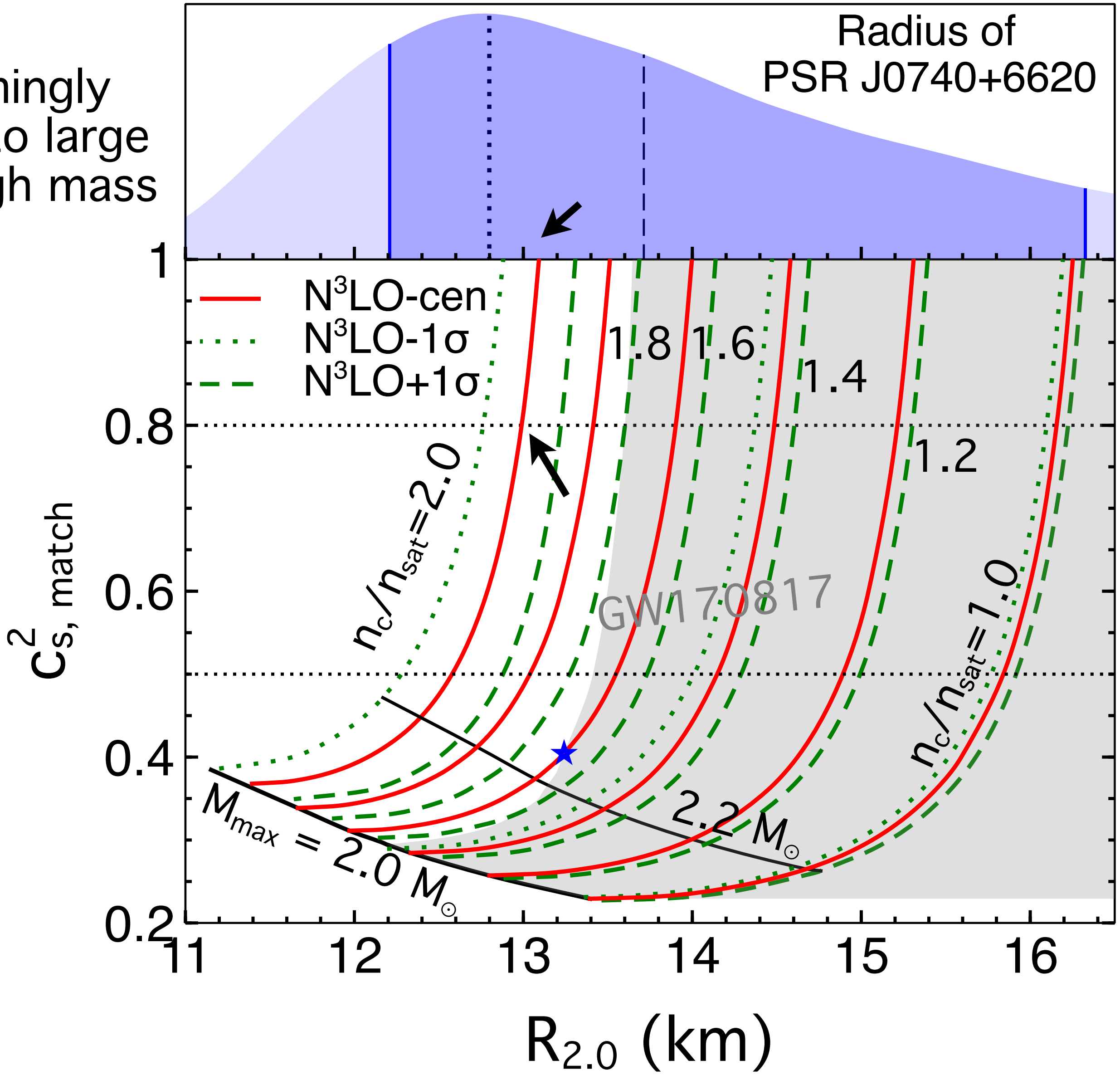
conformal limit $c_s^2 \approx 1/3$

causal limit

Large massive NSs (?)

Drischler, **SH** & Reddy
arXiv:2110.14896

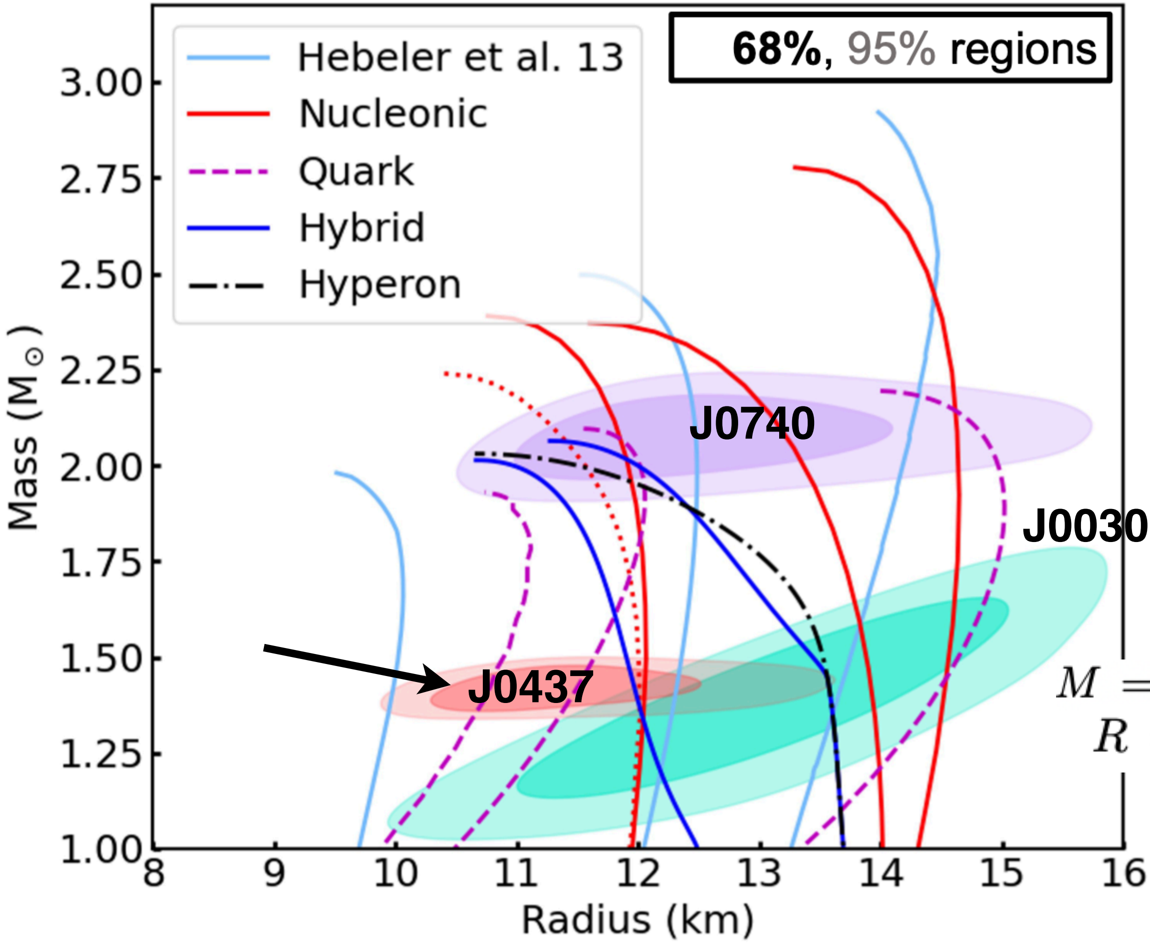
- overwhelmingly sensitive to large radii at high mass



A NICER VIEW OF PSR J0437−4715 (new!)

nearest and brightest millisecond pulsar

Choudhury et al. arXiv:2407.06789
Rutherford et al. arXiv:2407.06790



$$M = 1.418 \pm 0.037\ M_\odot$$
$$R = 11.36^{+0.95}_{-0.63}\ \text{km}$$

mass, distance, inclination all well known from pulsar timing

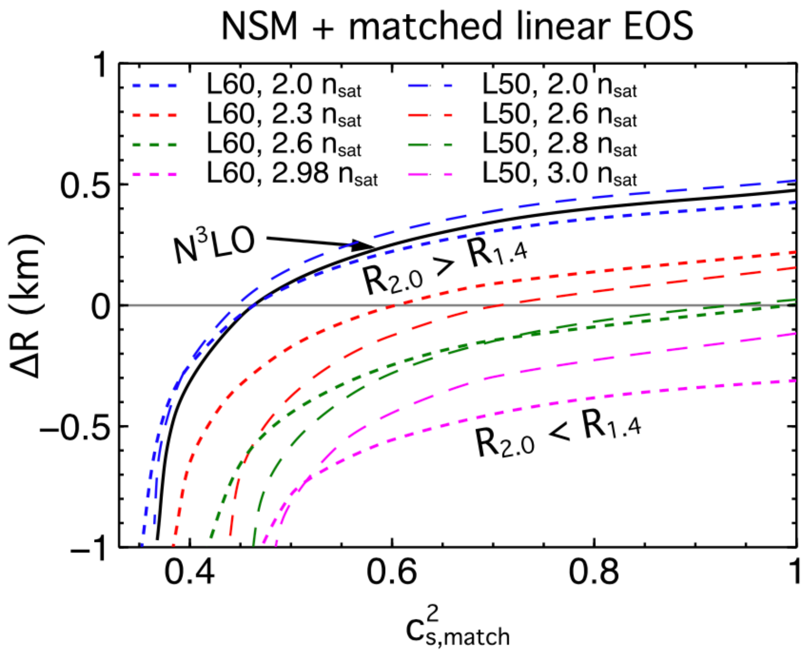
Courtesy: Anna Watts

Relating $R_{1.4}$ and $R_{2.0}$

Legred et al. (including **SH**), **arXiv:2106.05313**

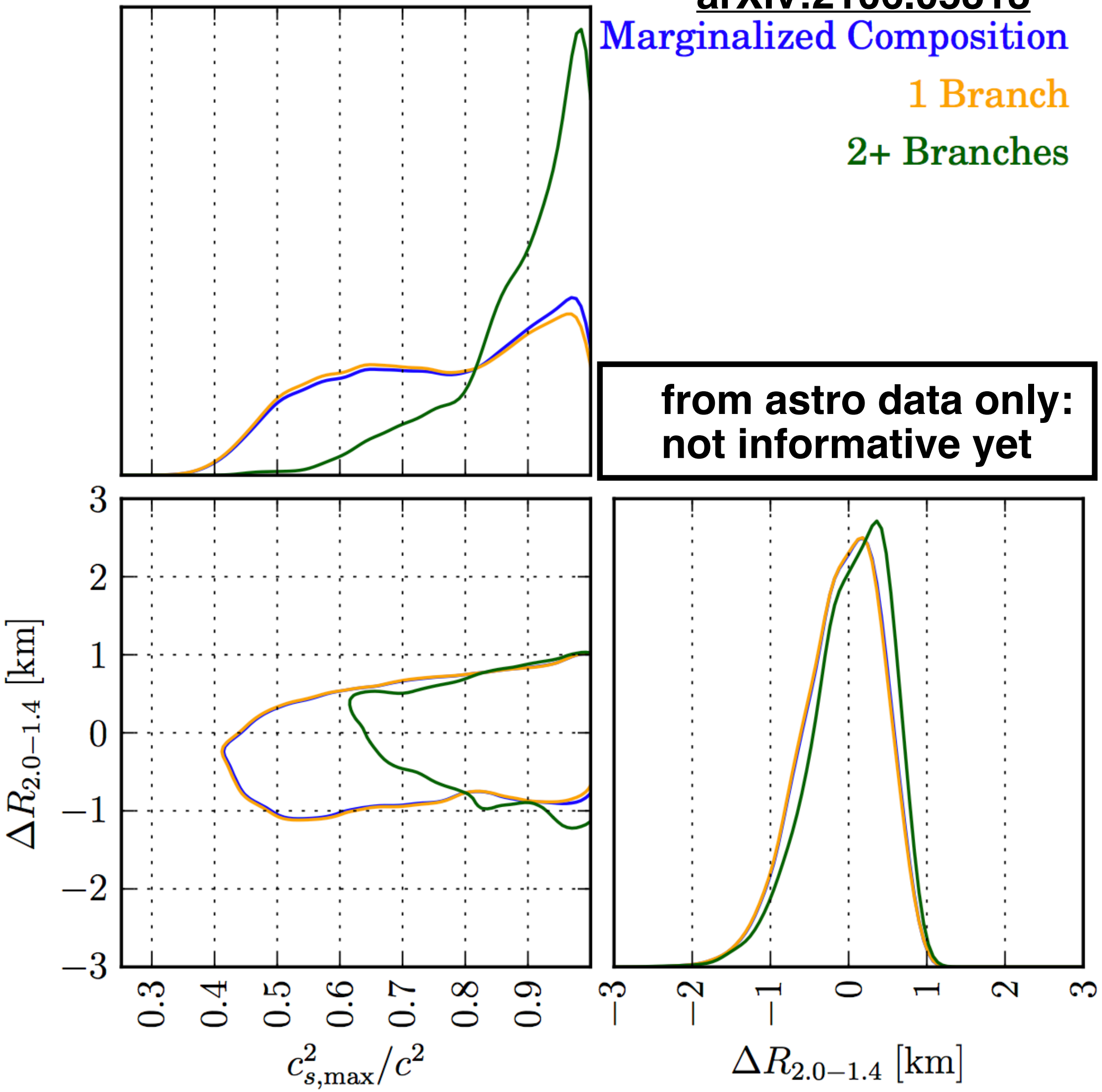
- Is the neutron star radius different at high masses than low masses?

with nuclear theory inputs: smaller R2.0 are more common

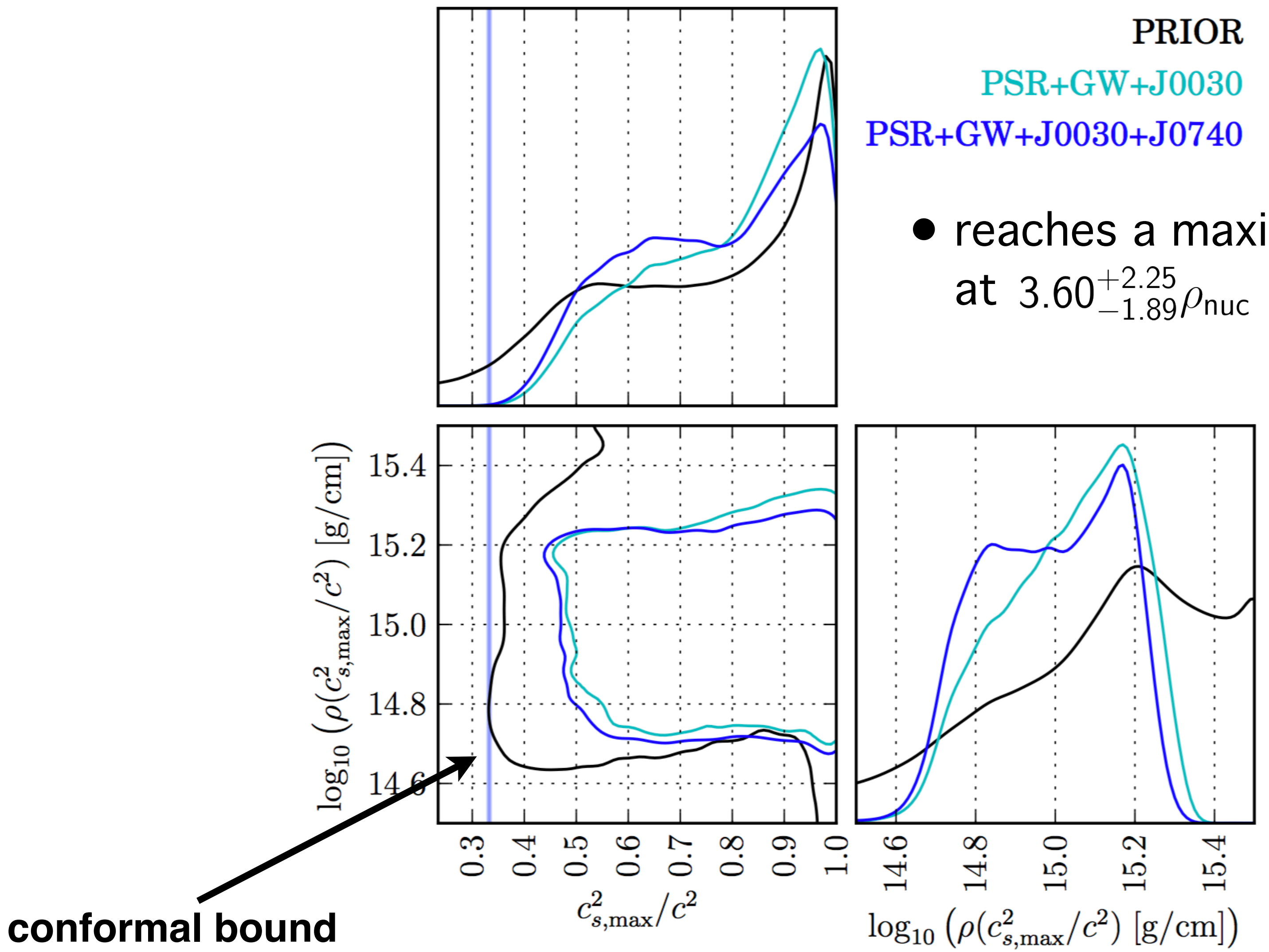


Drischler, **SH**, Lattimer, Prakash, Reddy and Zhao, **arXiv:2009.06441**

from astro data only: not informative yet

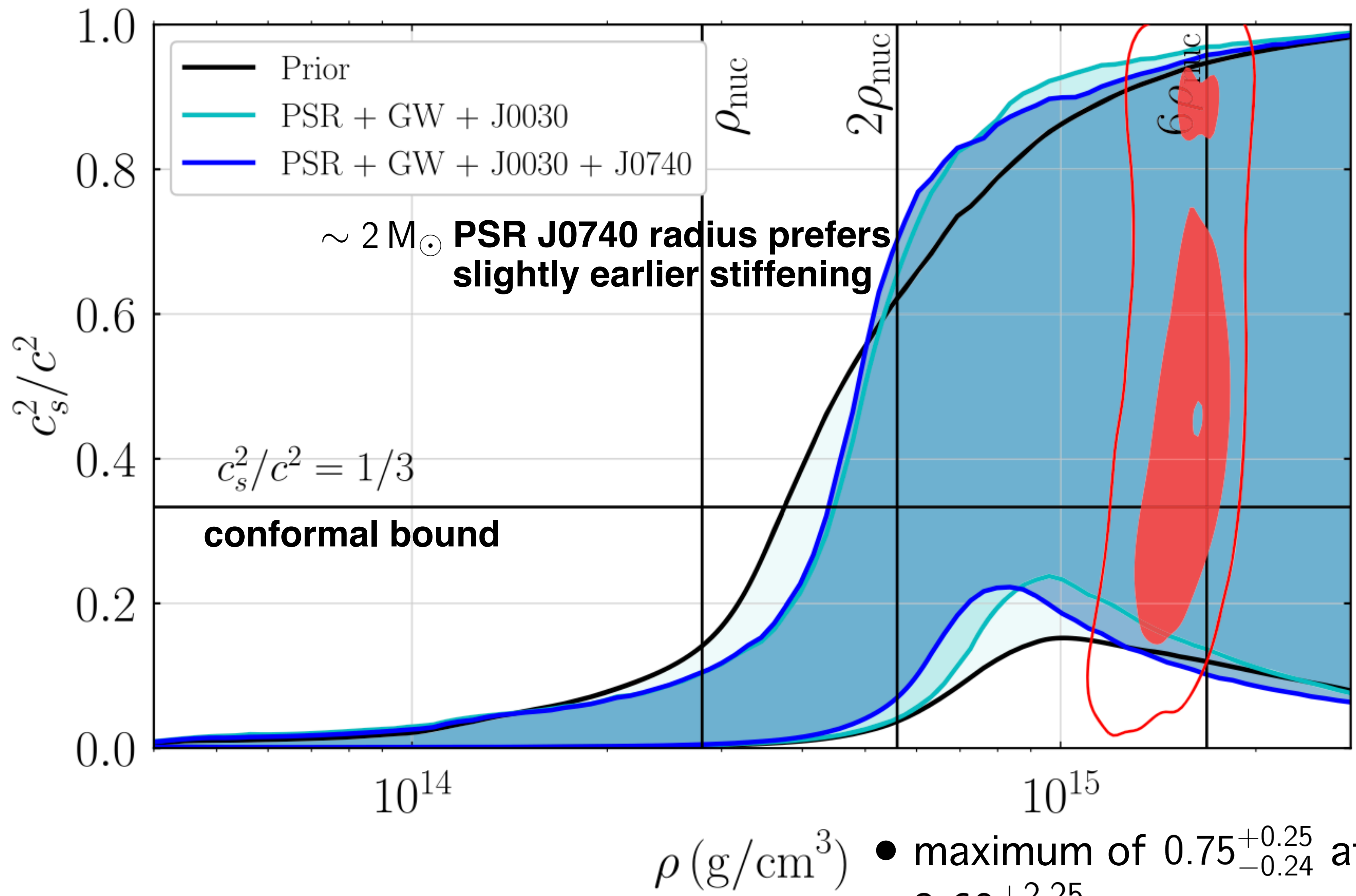


Inferring the peak sound velocity



- reaches a maximum of $0.75^{+0.25}_{-0.24}$ at $3.60^{+2.25}_{-1.89}\rho_{\rm nuc}$

Inferring the peak sound velocity



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Sound speed in the core

$$c_s^2(r) \equiv dp(r)/d\varepsilon(r)$$

how fast pressure rises with energy density

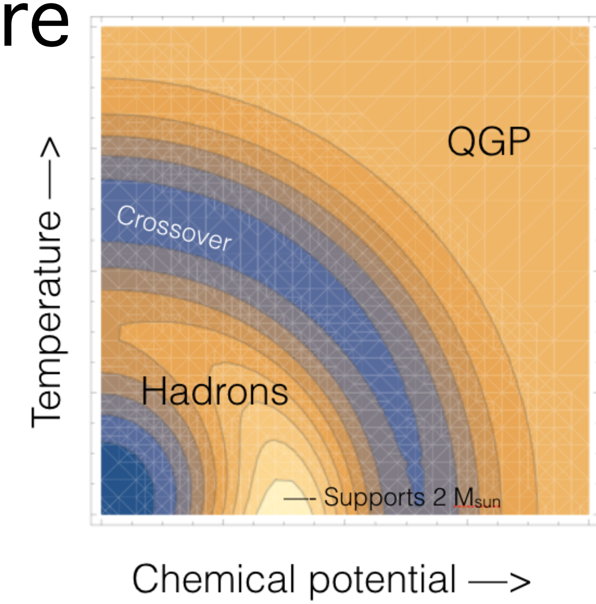
Possible behavior in neutron star interiors

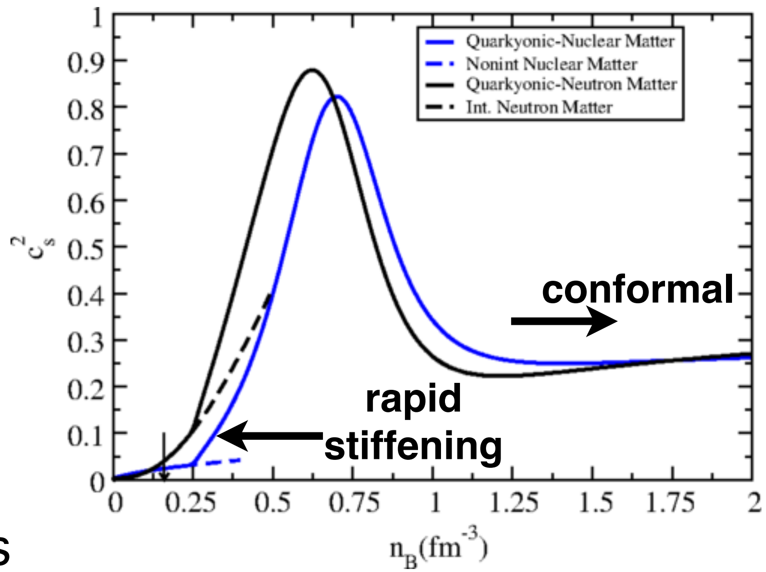
- minimal scenario of normal nuclear matter: (smoothly) continuous function of pressure
- first-order phase transition scenario: finite energy density discontinuity induces sudden softening near the phase boundary
- crossover/quarkyonic: local **peak** structure

Limits

- asymptotically high density: ~1/3
- ~4-8 times saturation: supports massive NSs
- high-T: matches lattice calc./heavy-ion data

Phases of Dense Matter
(INT Program INT-16-2b)





McLerran & Reddy,
PRL 122, 122701 (2019)

Paths towards high-density regime

maximally distinguishable

known up to ~saturation

→ **purely hadronic/nucleonic matter (standard)**

→ **1st-order PT? -> quarks**

- → **sharp boundary - Maxwell**
  - → **strong PT**
  - → **weak PT**
- → **mixed phase - Gibbs (geometrically separated)**

→ **smooth crossover (quarkyonic)?**

“golden window” in the vicinity of ~2*nsat; hints from exp.?

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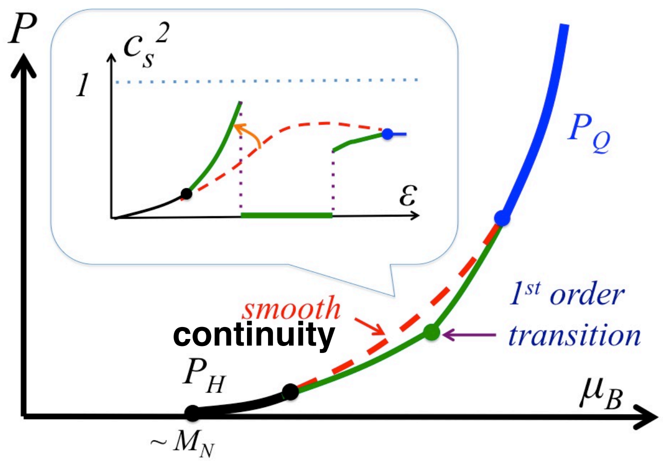
- 1st-OPT: mixed phase (Gibbs) favored if the hadron/quark surface tension is small

Baym et al. Rept. Prog. Phys. 81, 056902 (2018)



BNS/NSBH mass distribution

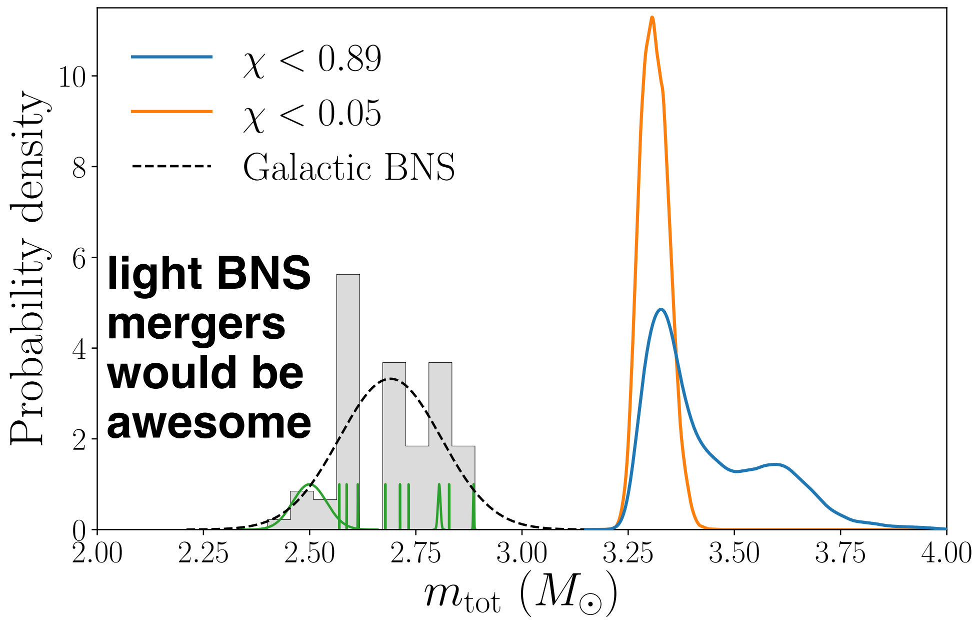
arXiv:2006.12611

GW190425

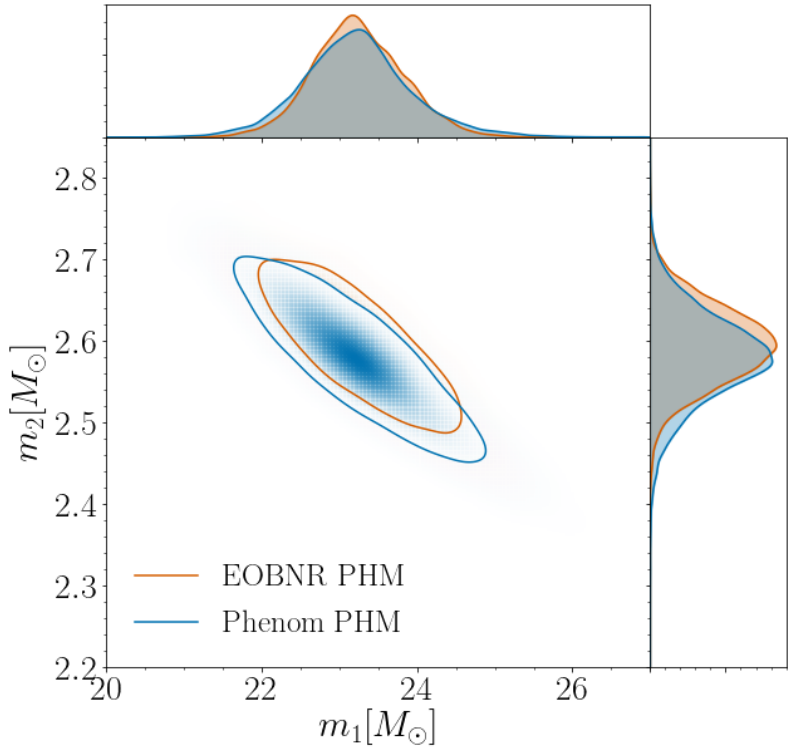
hunting for surprises..

- total mass ~3.4 solar masses
- signal too weak to provide further EoS constraints R<16 km

arXiv:2001.01761



see events of GWTC-2: arXiv:2010.14527

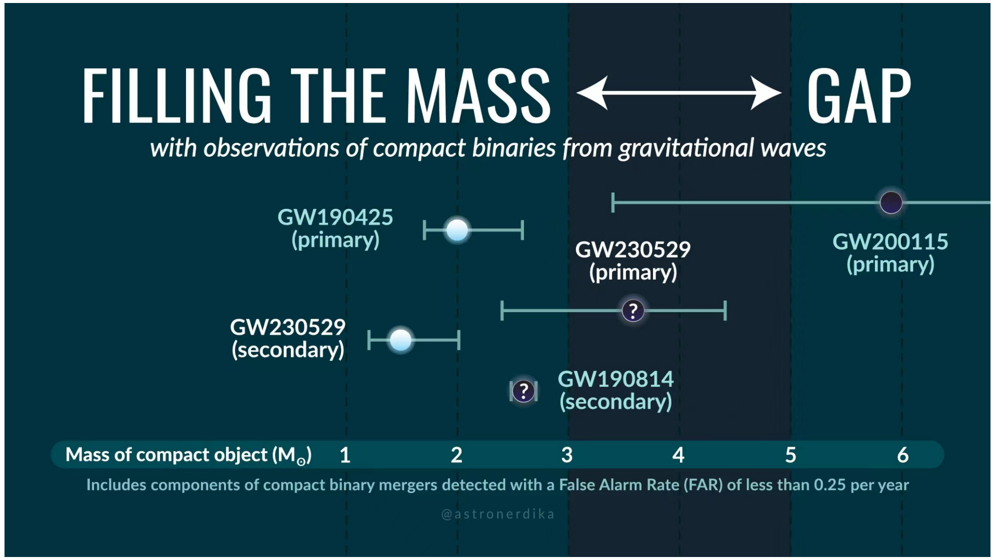


GW190814

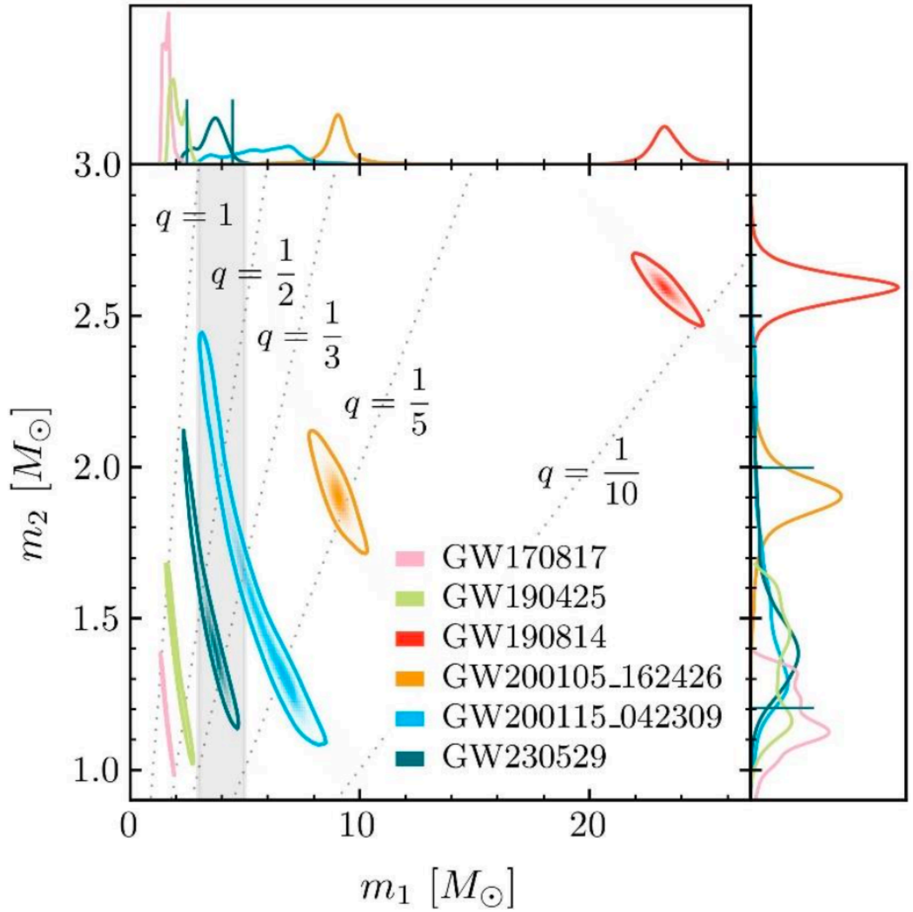
more mass-gap objects?

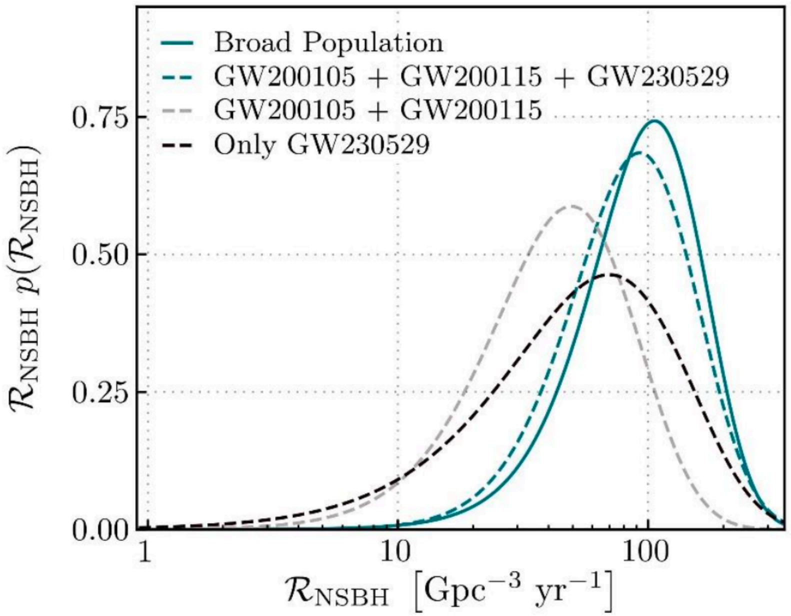
- component of ambiguous nature
- most **<u>asymmetric</u>** system observed

GW230529 (new!)



LVK collaboration
arXiv:2404.04248



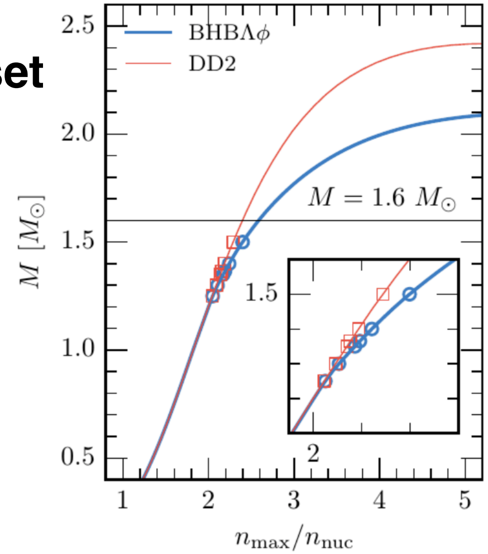


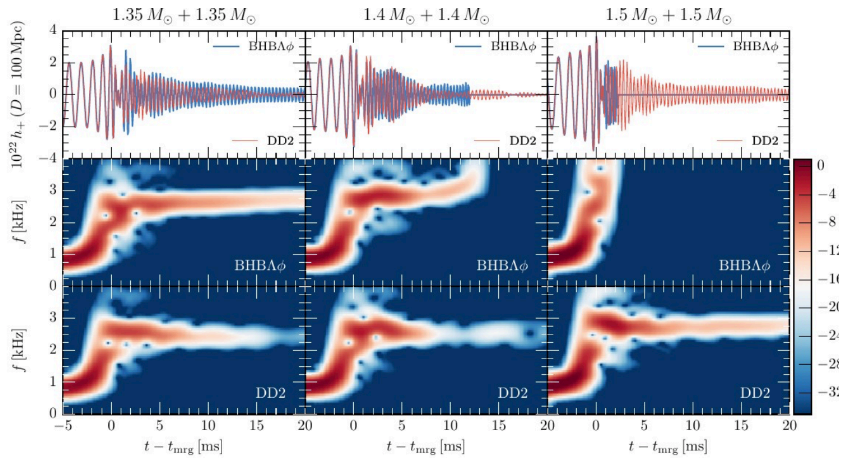
- ~1.4 (NS) + ~3.6 (BH) solar masses
- most **symmetric** NSBH event so far

e.g. softening effects on post-merger GWs

hyperon onset

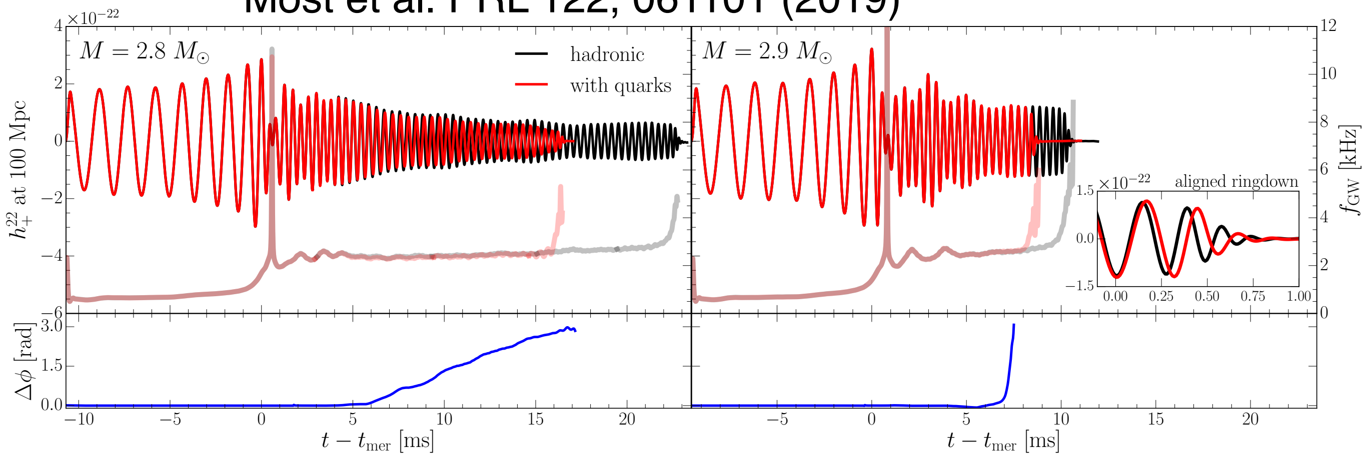
- more compact remnant (higher central density)
- earlier collapse; higher frequency





Radice et al. ApJL 842, L10 (2017)

Most et al. PRL 122, 061101 (2019)

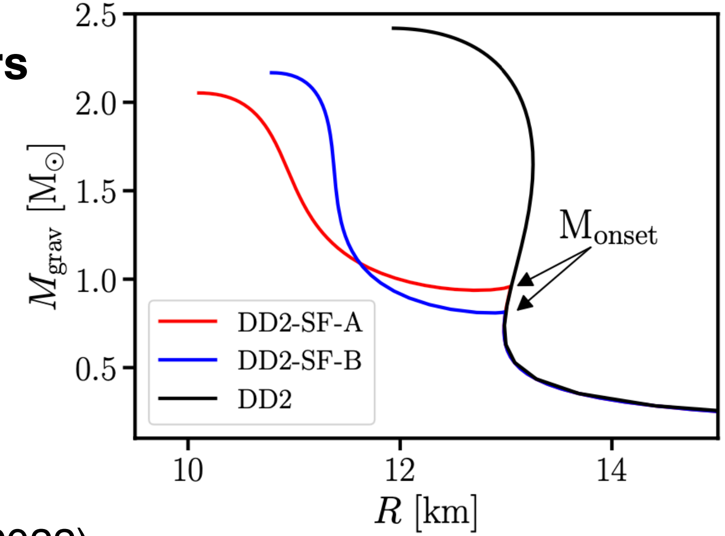


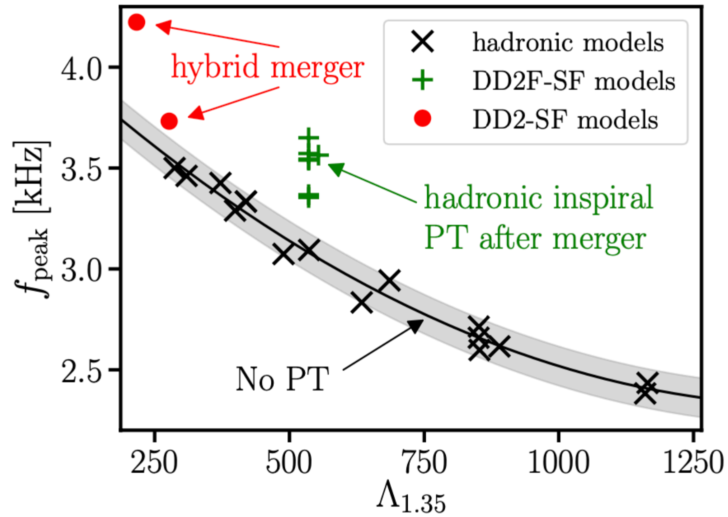
1st-OPT to soft quark matter after merger

e.g. softening effects on post-merger GWs

third-family stars

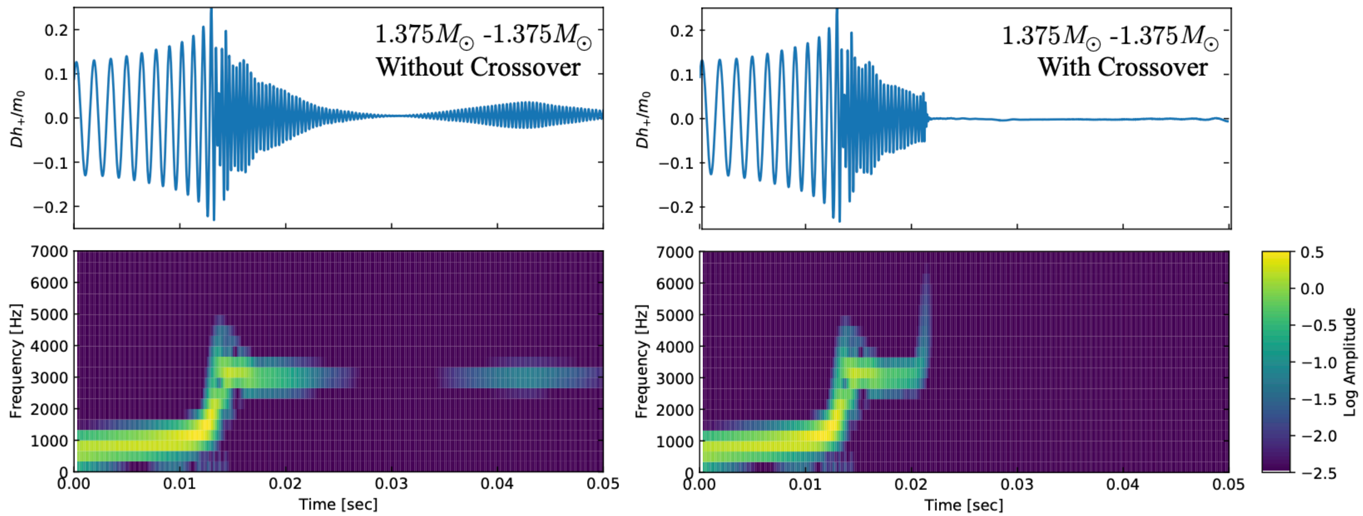
- ***stiff*** EoS at low density -DD2
- strong **1st-OPT** to ***stiff*** quark matter **before merger**





Bauswein & Blacker (2020)

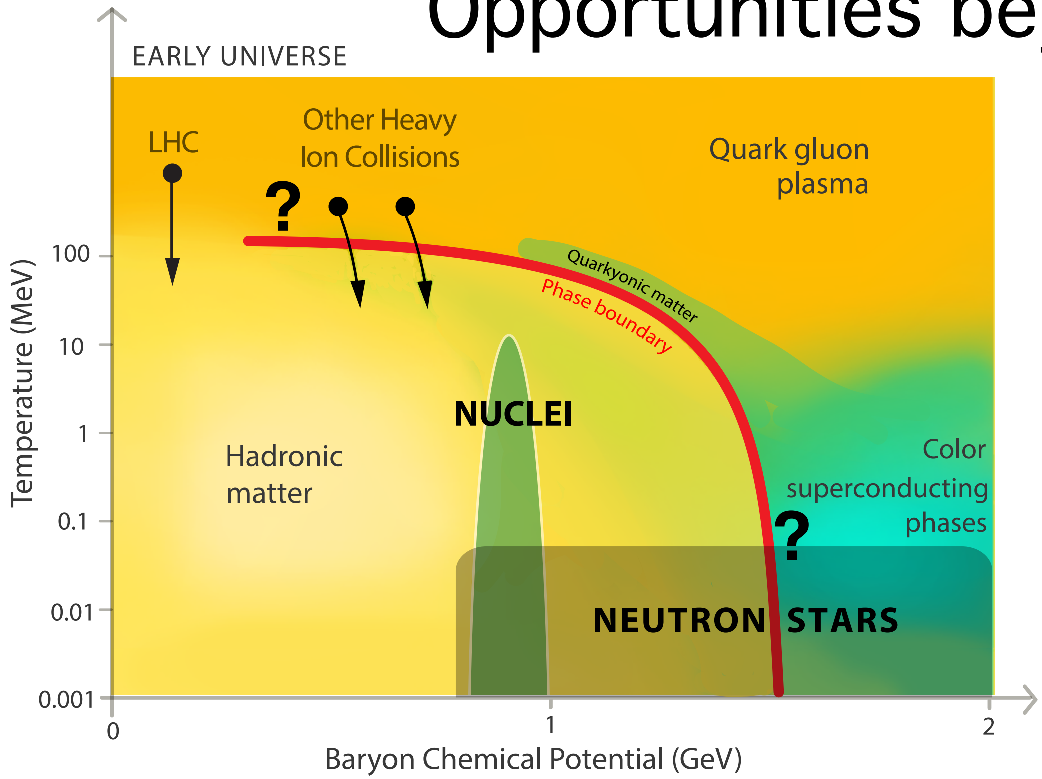
Fujimoto et al. (2022)

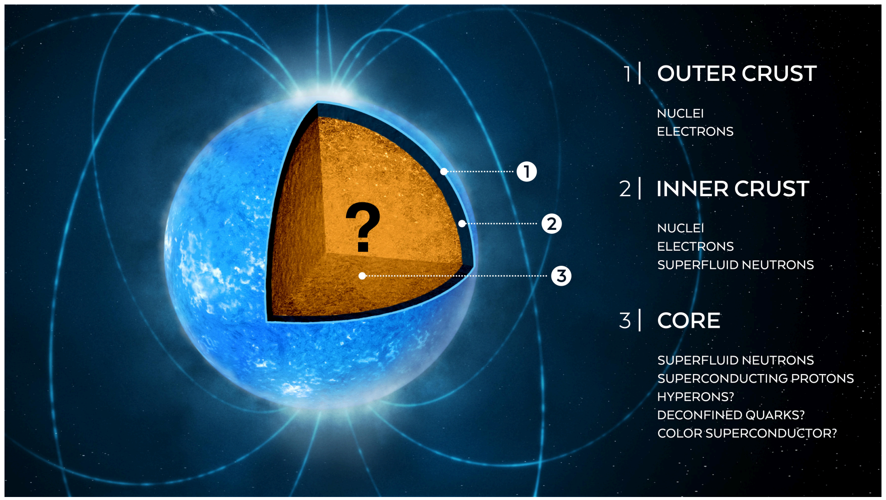


- ***soft*** EoS at low density ~N3LO chiEFT
- rapid ***stiffening*** within the crossover regime

crossover into *soft* quark matter after merger

Opportunities beyond the EoS

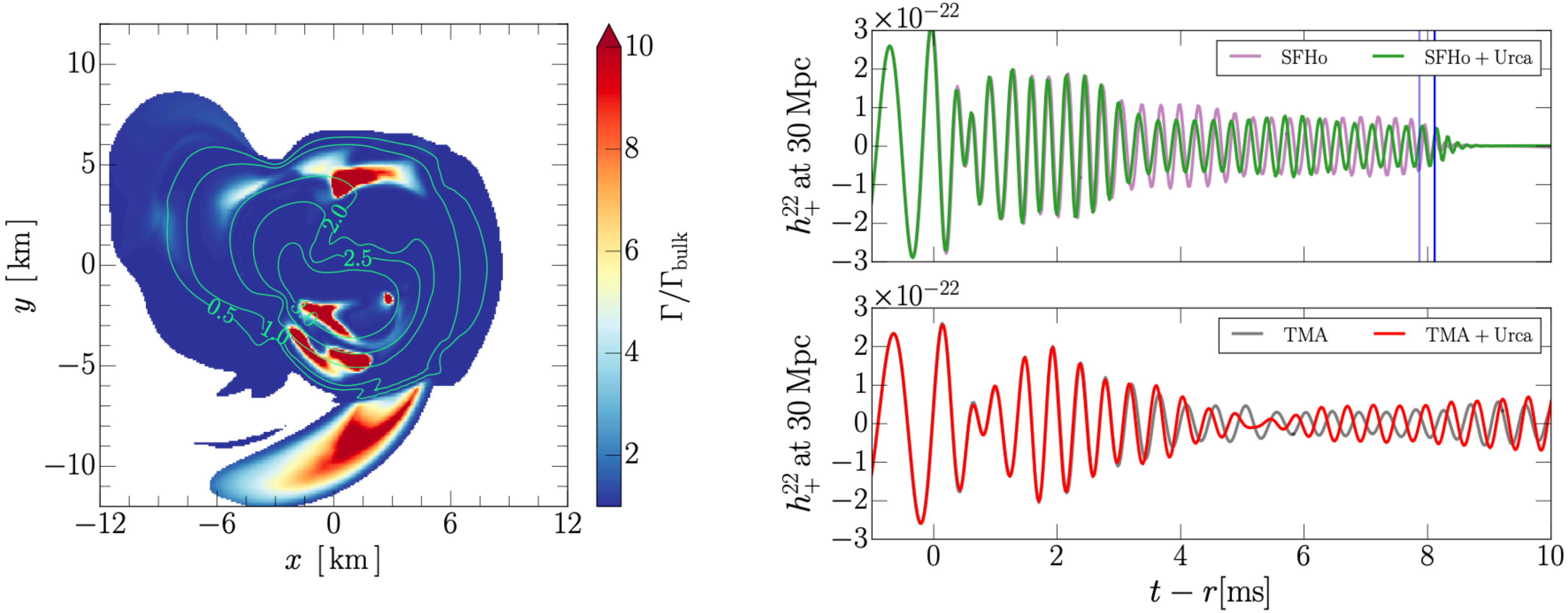




- thermal evolution - neutrino emissivity, stellar superfluids **[nuclear theory, transport prop.]**
- merger dynamics with astro/GW signals - out-of-equilibrium (visc.) physics; composition details **[simulations, nucleosynthesis]**
- next Galactic supernova? **[neutrino physics]**
- spin-down, glitches, asteroseismology, **[hydrodynamics, GR, nucl-th]**
- ...and more - add your own

Rev. Mod. Phys. 88, 021001 (2016)

Bulk viscous phase in merger



Most et al. arXiv:2207.00442
ApJL 967, L14 (2024)

- remnant evolution: impact of weak-interaction driven out-of-equilibrium dynamics; phase shift of the gravitational-wave spectrum

- dissipation via **nucleonic** Urca processes on a millisecond timescale

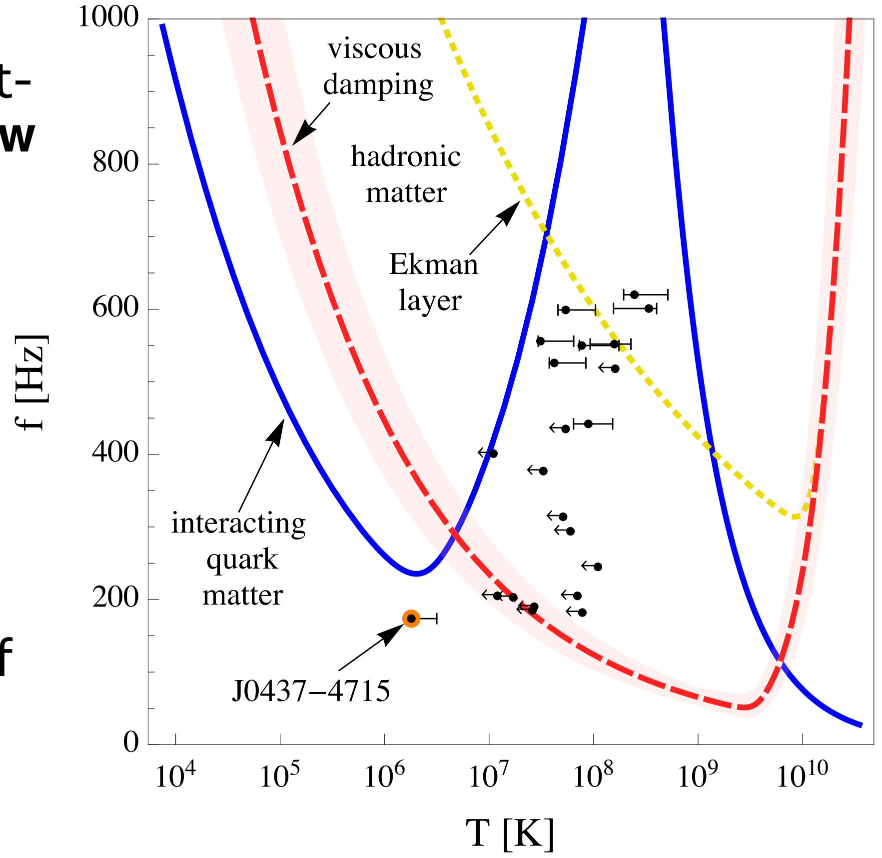
- different channels of chemical equilibration for **hyperons, quarks** etc. -> bulk viscosities with different dependencies on temperature and density

Spin evolution

- puzzles: long periods of young NSs; fast-rotating NSs in r-mode **instability window** of hadronic matter; glitches..

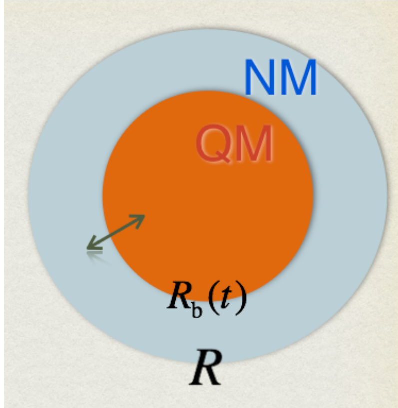
e.g. r-modes

- transport properties of dense matter: shear & bulk **viscosity**
- r-modes both heat and spin down NS: standard (minimal) model **inconsistent** with temperature and frequency data of LMXBs
- promising **saturation** mechanisms: superfluid mutual friction; phase-conversion at hadron/quark interface



Alford, **SH** & Schwenzer, arXiv:1904.05471
Haskell, Degenaar & Ho, arXiv:1201.2101

Spin evolution



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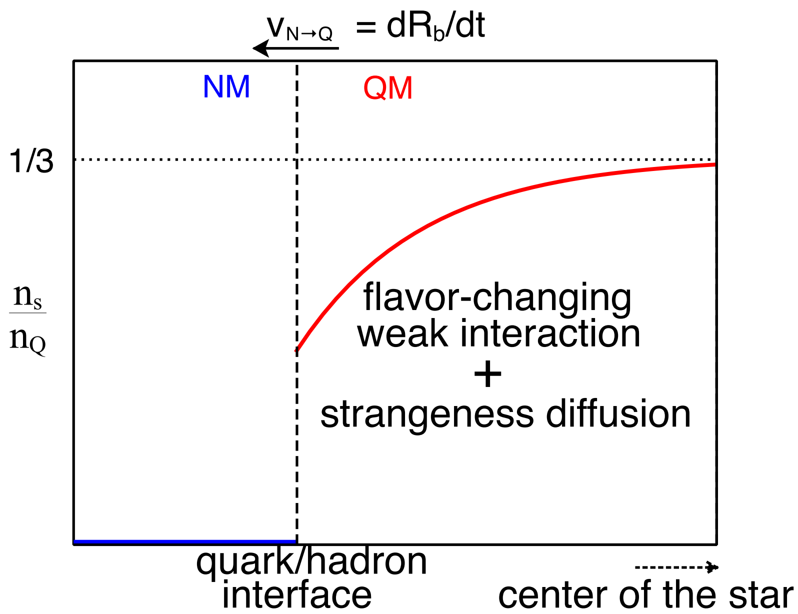
- promising **saturation** mechanisms: superfluid mutual friction; phase-conversion at hadron/quark interface

-steady-state transport
-no acceleration/deceleration effects; no turbulence
-no leptons; no superfluids

$$(udd) \leftrightarrow (uds)$$

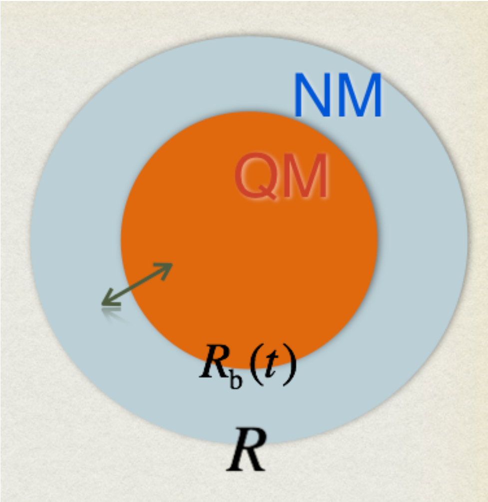


e.g. dissipation at an interface

- Ekman layer damping from shear rubbing of a fluid core along a solid crust

Phase-conversion dissipation (PCD)

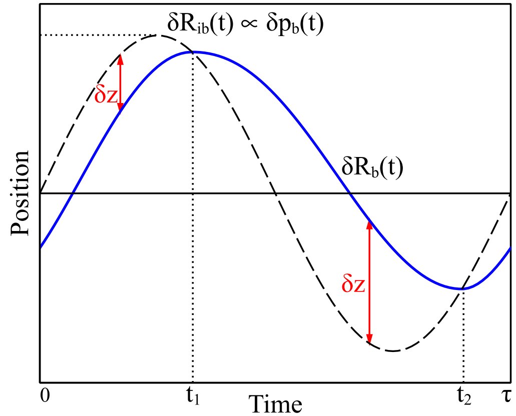
- between fluids in different phases with first-order transition separated by a **sharp** interface
- quark/hadron conversion



1) **flavor-changing** process $d \leftrightarrow s$ out of equilibrium due to global oscillations

2) instantaneous restoration $\Leftrightarrow$
phase boundary moves arbitrarily fast (no diss.)

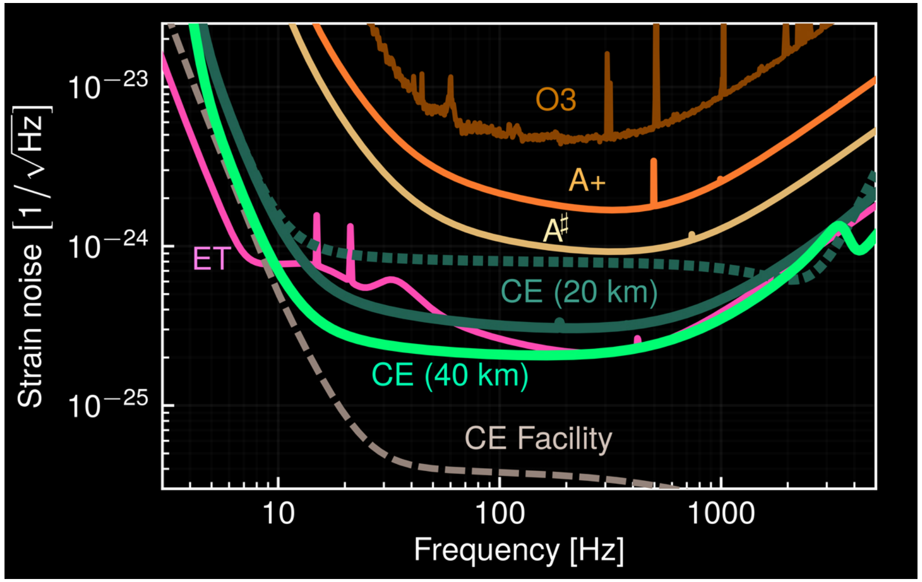
3) finite rate of weak interaction and flavor diffusion
$\longrightarrow$ a **phase lag** in system response
$\longrightarrow$ dissipates energy



Alford, **SH** & Schwenzer
arXiv:1404.5279

Stellar oscillations

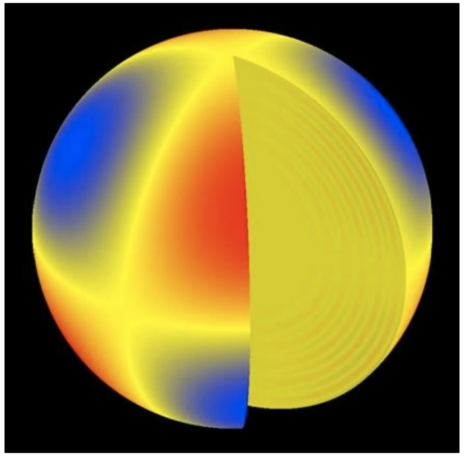
©LIGO-Virgo-KAGRA

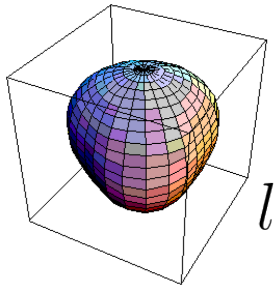


stable vibration modes (“ringing”)

- f-mode (fundamental mode) scales with average density
- p-mode (pressure mode) probes the sound speed
- ***g*-mode (gravity mode)** sensitive to **composition**/thermal gradients
- w-mode, s-mode, i-mode/***r*-mode**..

©NASA/Kepler





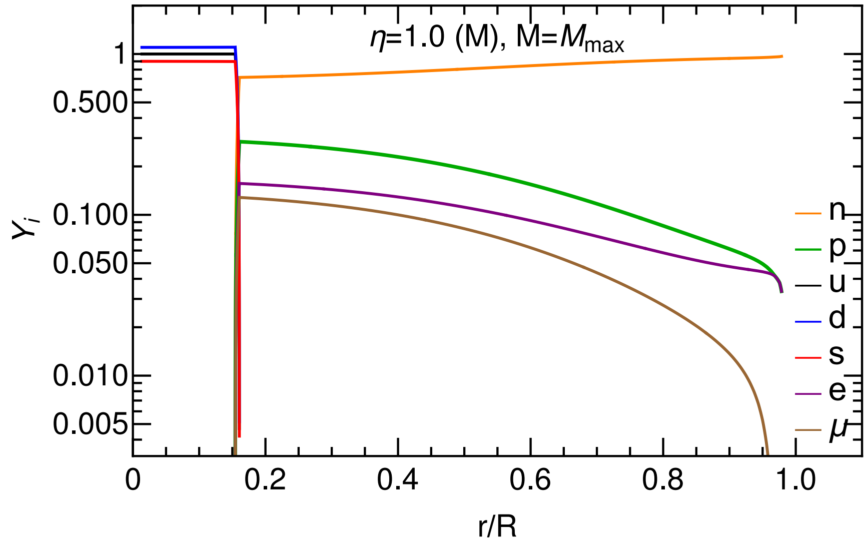
$l = 3,\ m = 3$

promising sources for XG detectors

small amplitude oscillations -> weak (continuous) emission of GWs

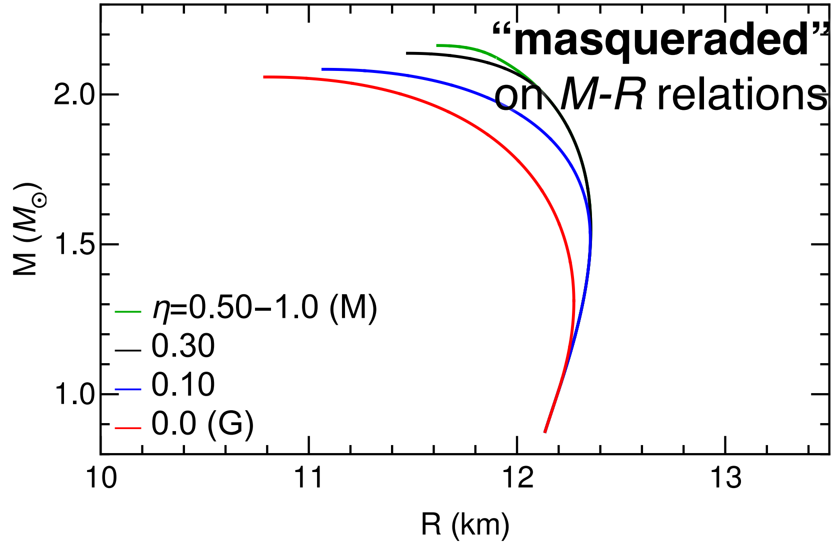
Global g-mode frequency with PTs

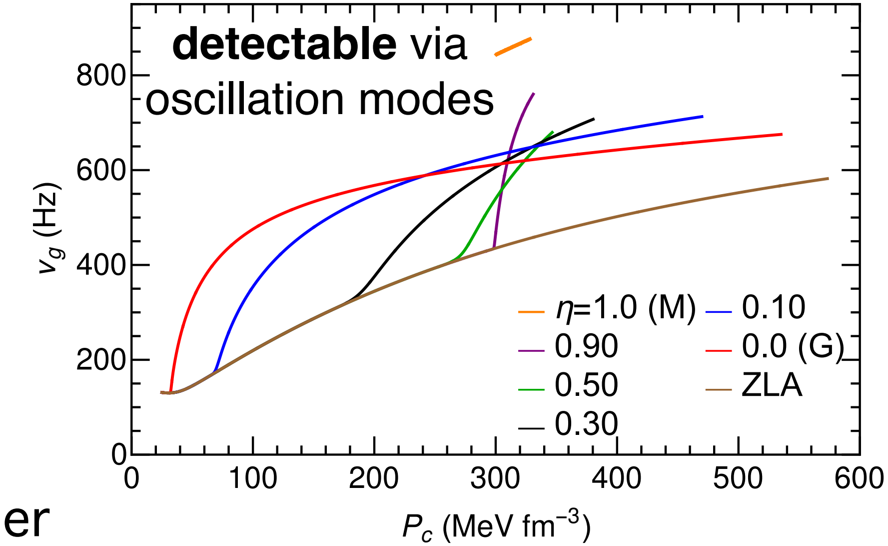
- hybrid system under local vs. global charge neutrality in Maxwell (M) vs. Gibbs (G) construction for a 1st-OPT



- "discontinuity" g-mode observed when there exists a sharp boundary

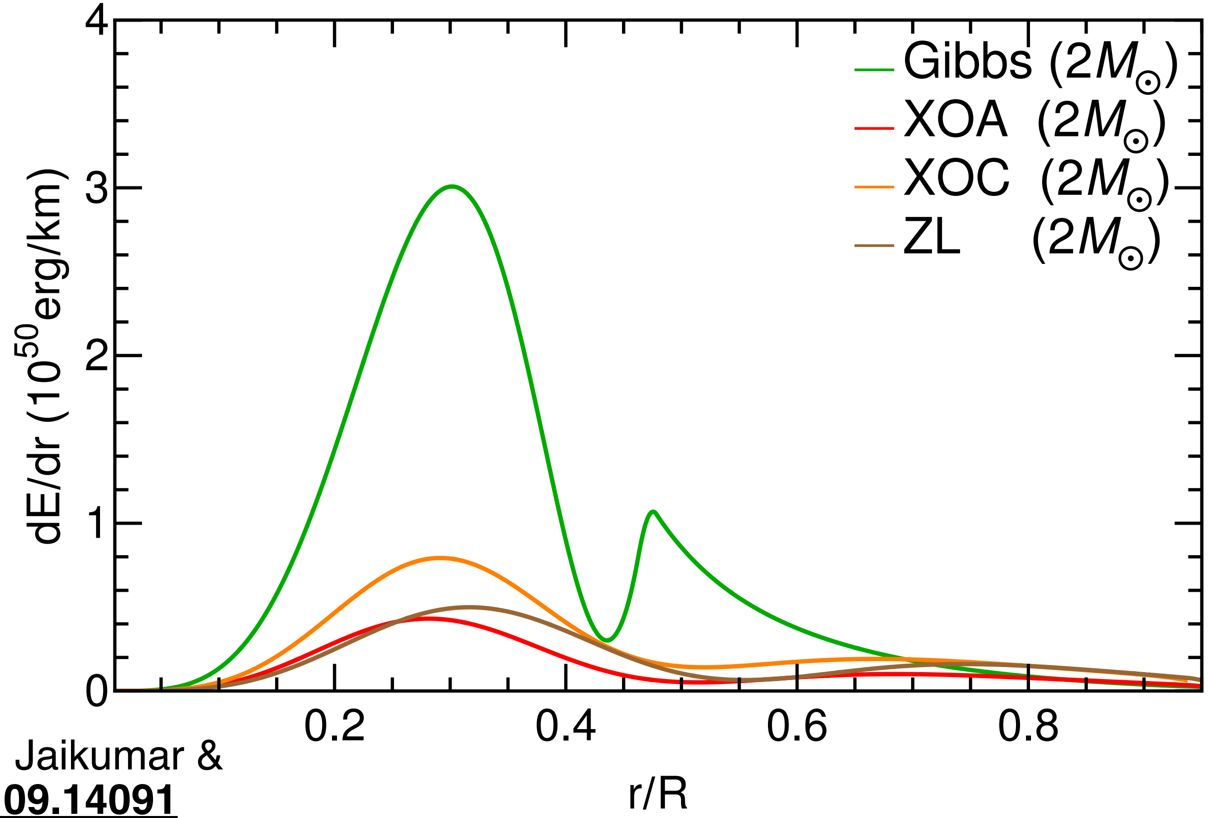
- distinct signature of exotic phases: higher frequency implies larger fraction of quarks





Constantinou, Zhao, **SH** & Prakash, **arXiv:2302.04289**

Mode energies and tidal forcing



Constantinou, **SH**, Jaikumar & Prakash, **arXiv:2109.14091**

$$\frac{dE}{dr} = \frac{\omega^2 r^2}{2}(\varepsilon + p)e^{(\lambda-\nu)/2}\left[\xi_r^2 e^\lambda + l(l+1)\xi_h^2\right]$$

$$U = r^2 e^{\lambda/2}\xi_r, \text{ radial}$$

$$V = \omega^2 r \xi_h, \text{ tangential}$$

- energy per unit radial distance contained in the oscillatory motion

Crossover vs. Gibbs

- Kapusta-Welle approach: switching function of baryon chemical potential (arXiv:2103.16633)

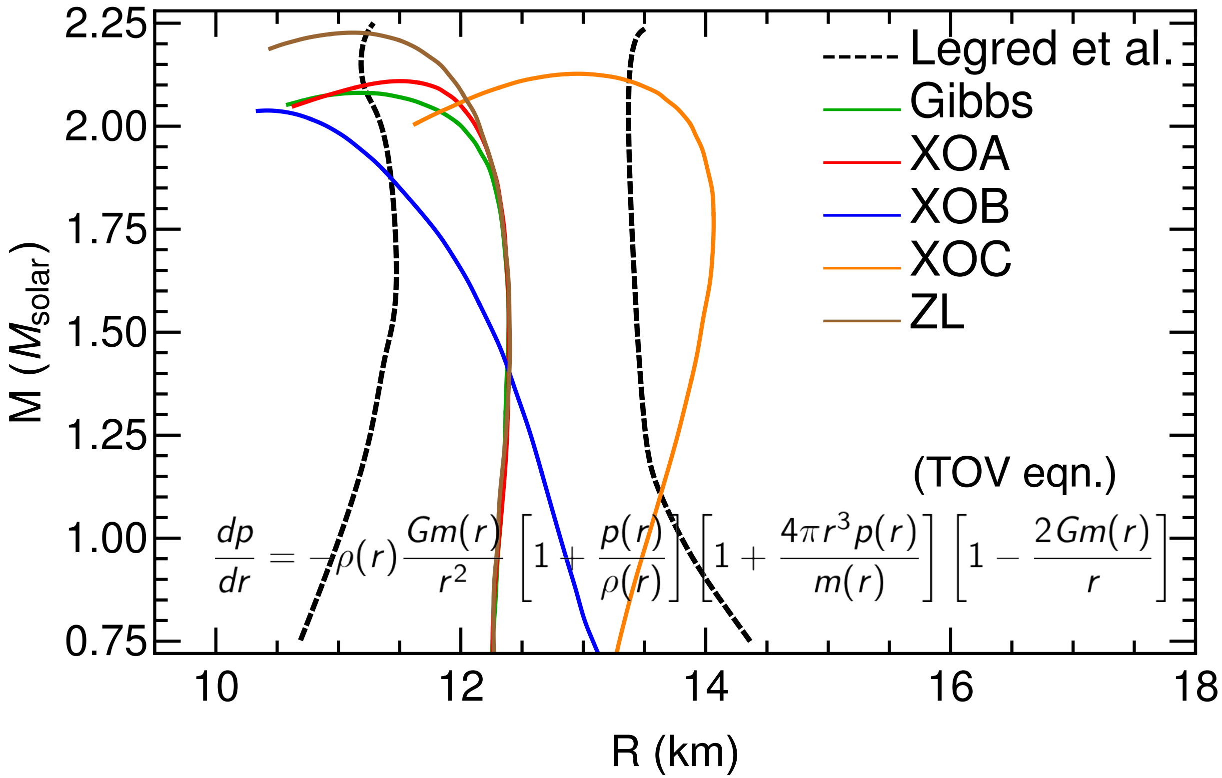
$$P_B = (1 - S)P_H + SP_Q$$

$$S = \exp\left[-(\mu_0/\mu)^4\right]$$

$$\mu_0 \sim 2.0\,\mathrm{GeV}$$

$$\mu = \frac{n_n\mu_n + n_p\mu_p}{n_n + n_p}$$

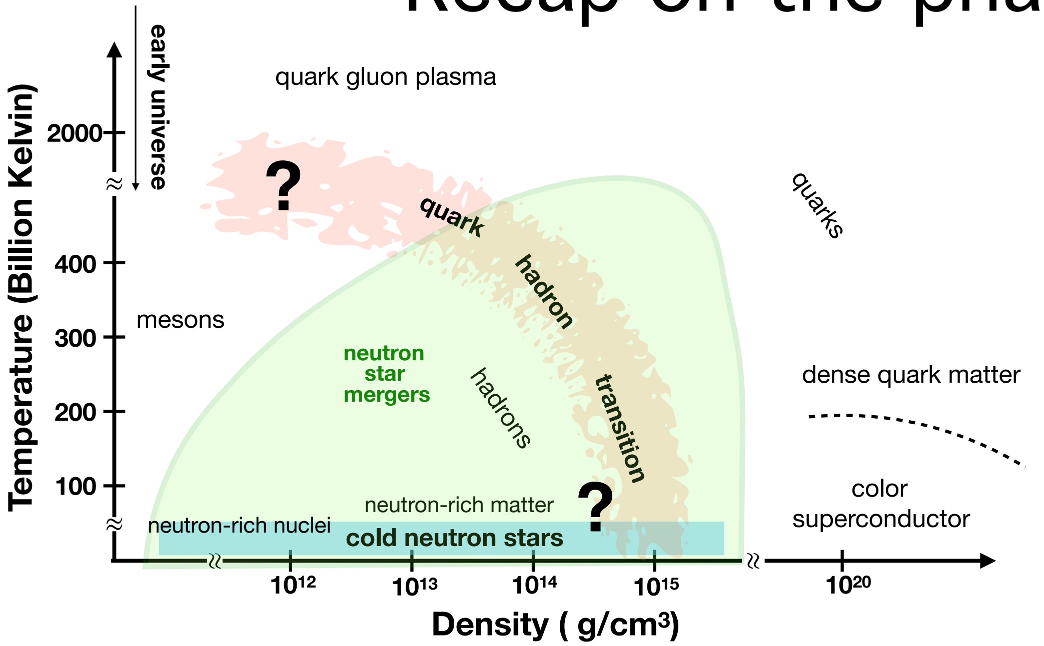
$$n_i = \left.\frac{\partial P}{\partial \mu_i}\right|_{\mu_j}$$

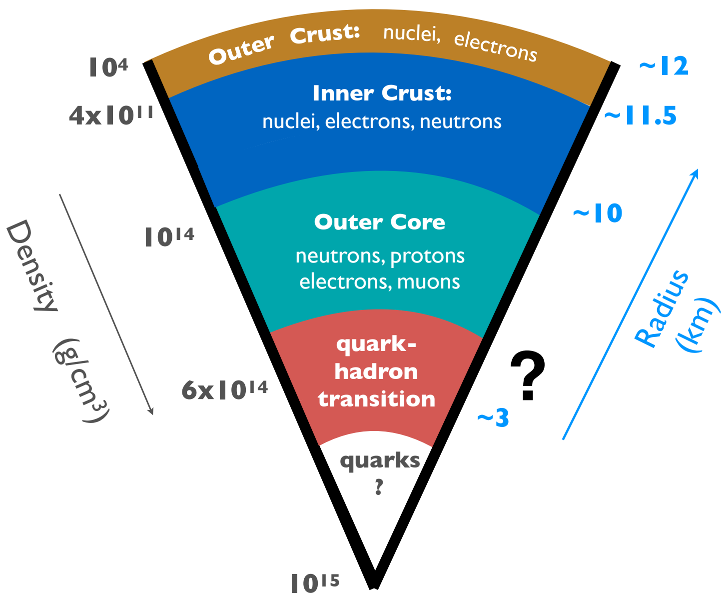


- our work: construct Gibbs mixed phase and crossover using ZL (nucleonic) + vMIT (quark) + KW model parameters

arXiv:2109.14091

Recap on the phase diagram





3G Science white paper

searching for QM in NSs

- has a phase transition already taken place in canonical-mass (cold) NSs before they merge in the binary system?
- are quarks only able to appear temporarily in the (warm) massive, transient remnant of mergers or supernovae?
- when and how do they emerge - **the onset density, temperature, nature** of PT?
- imprints in observations? possible links to e.g. HICs?

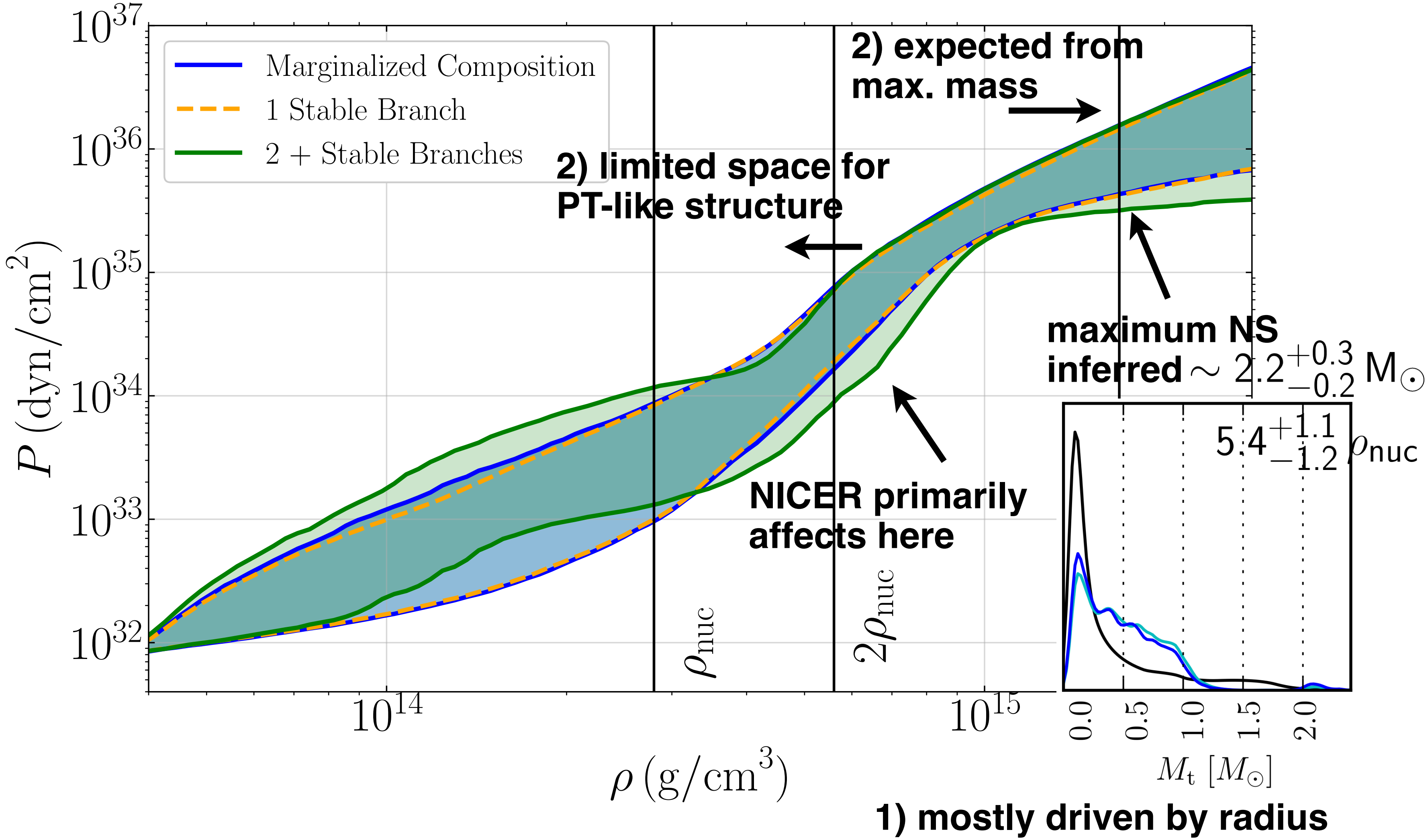
THANK YOU!

Q & A

BACKUP

SLIDES

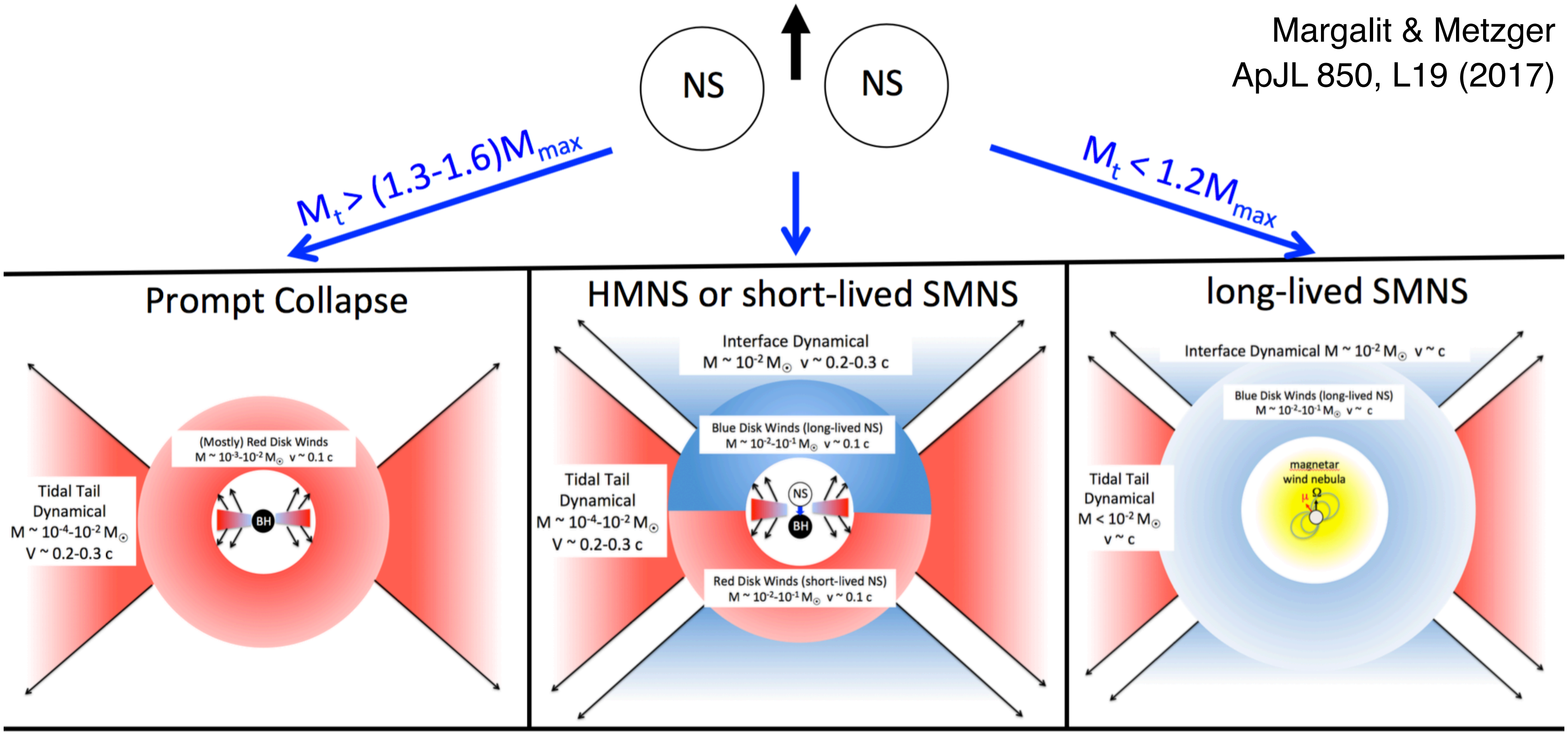
Single branch (minimal) vs. multiple branches



- full posterior is dominated by EoSs with a single stable branch
- **onset** for the unstable branch **i.e. extra softening** pushed to two ends

Remnant dynamics

Margalit & Metzger
ApJL 850, L19 (2017)



- **GW + EM** constraints from 170817 seem to favor Mmax<2.16~2.3 solar masses   Ruiz et al. (2018), Rezzolla et al. (2018), Shibata et al. (2019)
- **NS radius >10.68 km** to prevent prompt collapse   Bauswein et al. (2017)

NSBH mergers

LVK collaboration
arXiv:2106.15163

THE ASTROPHYSICAL JOURNAL LETTERS, 915:L5 (24pp), 2021 July 1 https://doi.org/10.3847/2041-8213/ac082e

Observation of Gravitational Waves from Two Neutron Star–Black Hole Coalescences

Table 2
Source Properties of GW200105 and GW200115

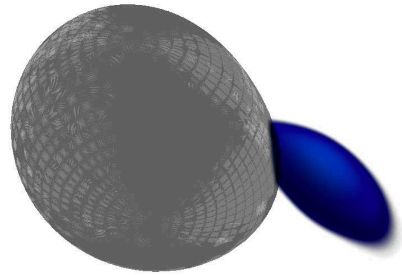
	GW200105		GW200115	
	Low Spin ($\chi_2 < 0.05$)	High Spin ($\chi_2 < 0.99$)	Low Spin ($\chi_2 < 0.05$)	High Spin ($\chi_2 < 0.99$)
Primary mass $m_1/M_\odot$	$8.9^{+1.1}_{-1.3}$	$8.9^{+1.2}_{-1.5}$	$5.9^{+1.4}_{-2.1}$	$5.7^{+1.8}_{-2.1}$
Secondary mass $m_2/M_\odot$	$1.9^{+0.2}_{-0.2}$	$1.9^{+0.3}_{-0.2}$	$1.4^{+0.6}_{-0.2}$	$1.5^{+0.7}_{-0.3}$
Mass ratio q	$0.21^{+0.06}_{-0.04}$	$0.22^{+0.08}_{-0.04}$	$0.24^{+0.31}_{-0.08}$	$0.26^{+0.35}_{-0.10}$
Total mass $M/M_\odot$	$10.8^{+0.9}_{-1.0}$	$10.9^{+1.1}_{-1.2}$	$7.3^{+1.2}_{-1.5}$	$7.1^{+1.5}_{-1.4}$
Chirp mass $\mathcal{M}/M_\odot$	$3.41^{+0.08}_{-0.07}$	$3.41^{+0.08}_{-0.07}$	$2.42^{+0.05}_{-0.07}$	$2.42^{+0.05}_{-0.07}$
Detector-frame chirp mass $(1+z)\mathcal{M}/M_\odot$	$3.619^{+0.006}_{-0.006}$	$3.619^{+0.007}_{-0.008}$	$2.580^{+0.006}_{-0.007}$	$2.579^{+0.007}_{-0.007}$
Primary spin magnitude χ_1	$0.09^{+0.18}_{-0.08}$	$0.08^{+0.22}_{-0.08}$	$0.31^{+0.52}_{-0.29}$	$0.33^{+0.48}_{-0.29}$
Effective inspiral spin parameter $\chi_{\rm eff}$	$-0.01^{+0.08}_{-0.12}$	$-0.01^{+0.11}_{-0.15}$	$-0.14^{+0.17}_{-0.34}$	$-0.19^{+0.23}_{-0.35}$
Effective precession spin parameter χ_p	$0.07^{+0.15}_{-0.06}$	$0.09^{+0.14}_{-0.07}$	$0.19^{+0.28}_{-0.17}$	$0.21^{+0.30}_{-0.17}$
Luminosity distance D_L/Mpc	280^{+110}_{-110}	280^{+110}_{-110}	310^{+150}_{-110}	300^{+150}_{-100}
Source redshift z	$0.06^{+0.02}_{-0.02}$	$0.06^{+0.02}_{-0.02}$	$0.07^{+0.03}_{-0.02}$	$0.07^{+0.03}_{-0.02}$

no information on matter effects
no significant EM detections

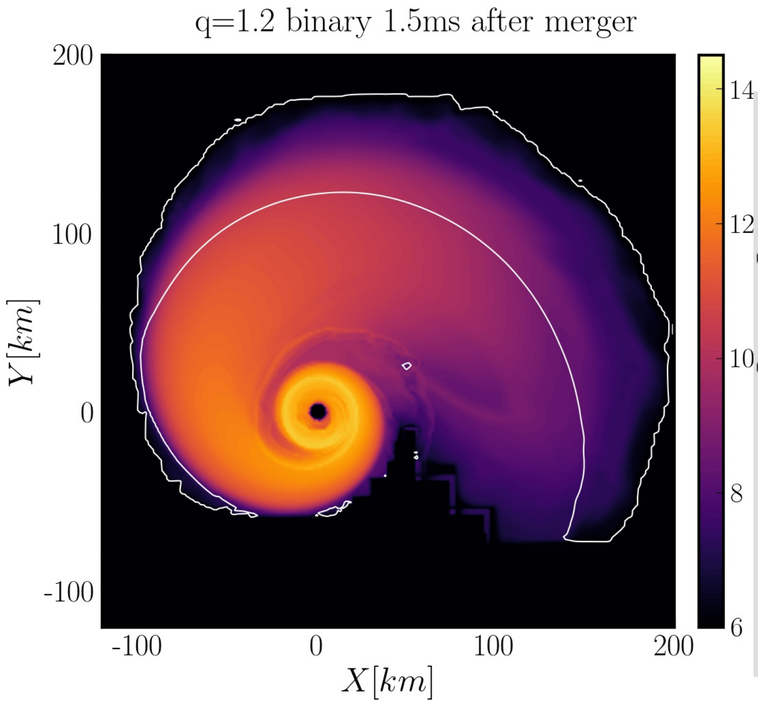
- GW200105: ~1.9 + ~9 solar masses
- GW200115: ~1.5 + ~6 solar masses

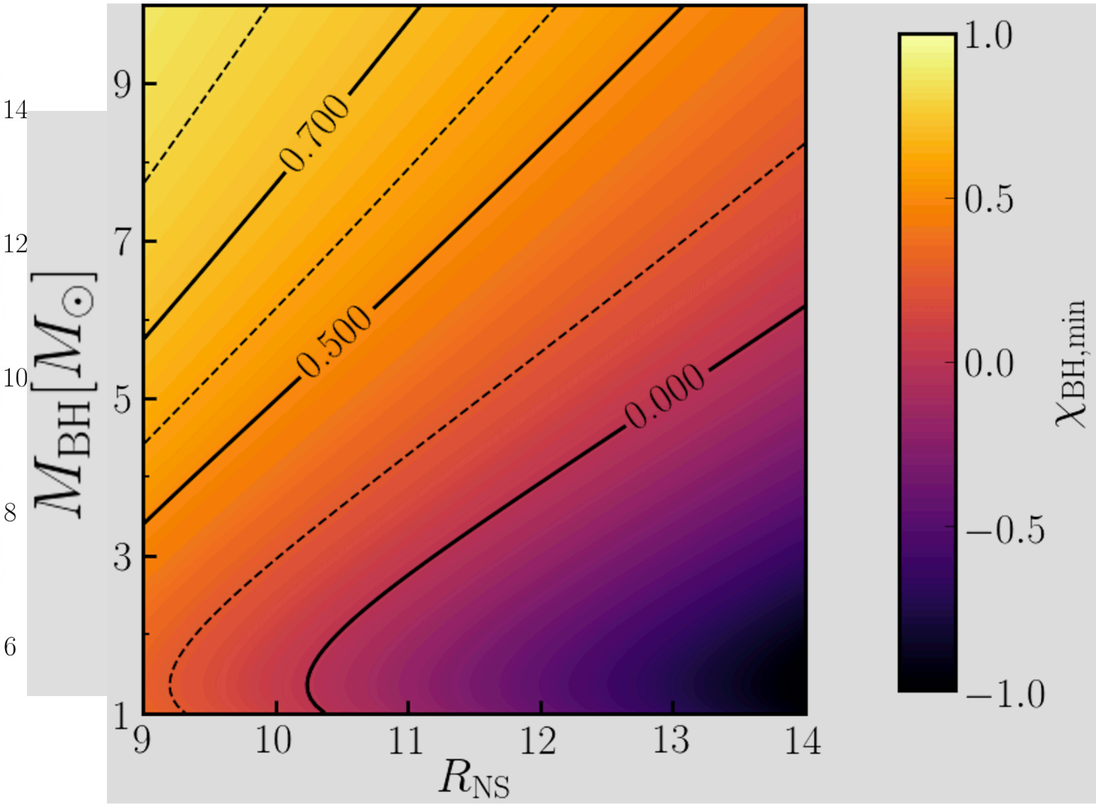
see events of GWTC-3: arXiv:2111.03606

Outcome of a NSBH merger



Foucart et al. (2018)



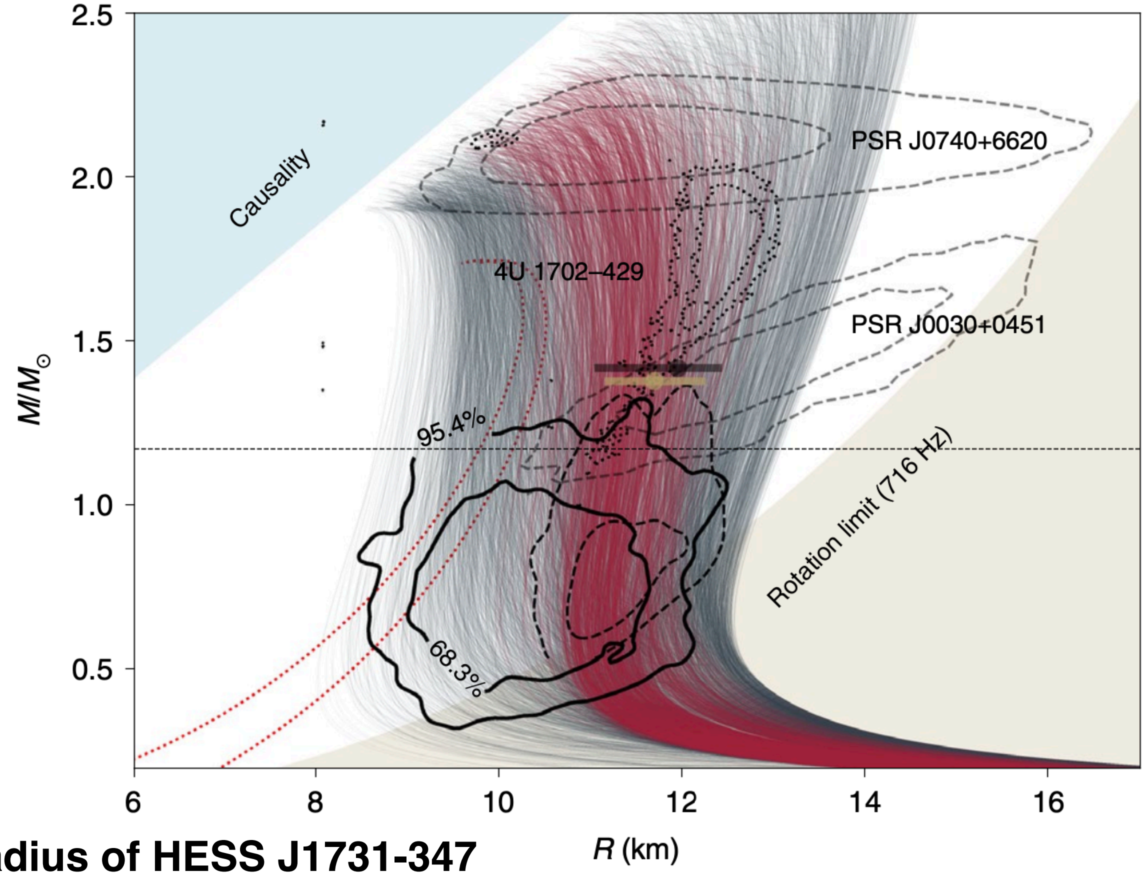


- NS is either tidally disrupted or plunges into the BH - mass ratio, spin, EoS
- radius determines if tides are **measurable** & if **EM** signals can be produced

Alternative x-ray probes of NS radii

Article

potential outlier candidate?



Doroshenko et al. Nature Astronomy 6, 1444 (2022)

mass & radius of HESS J1731-347

- “A strangely light neutron star within a supernova remnant”
- relies on specific assumptions about EoS prior, atm-models, temp. distribution etc.

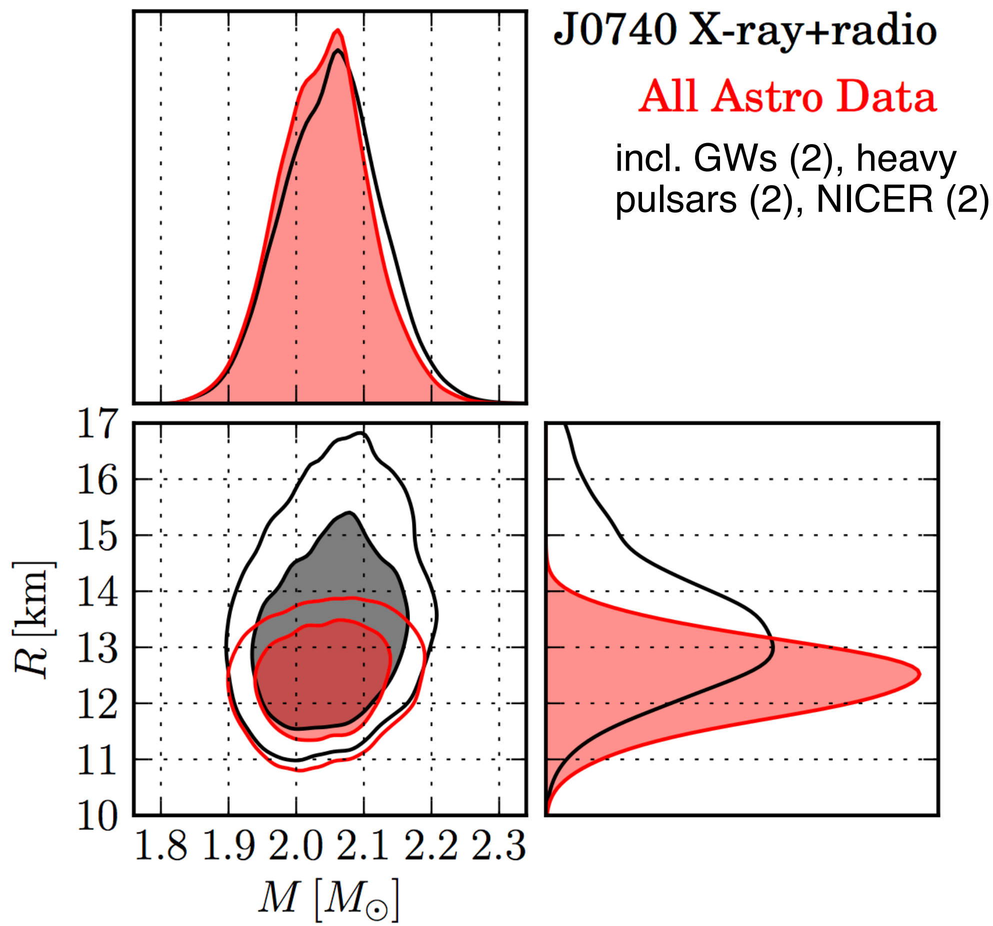
Impact of the J0740 radius constraint

Bayesian analyses

- hierarchical inference scheme and the **nonparametric** priors (not assuming a specific functional form but correlations within a function - GP processes)

  Landry et al., arXiv:2003.04880

- ***NICER*** data + XMM-Newton: Riley+Watts (Amsterdam) and Miller+ (Maryland/Illinois),
- $12.4^{+1.3}_{-1.0}$ km vs. $13.7^{+2.6}_{-1.5}$ km
- **other** astrophysical observations overall **reduce** the inferred radius of J0740+6620 from ~13.34 km to ~12.47 km at 90% credibility

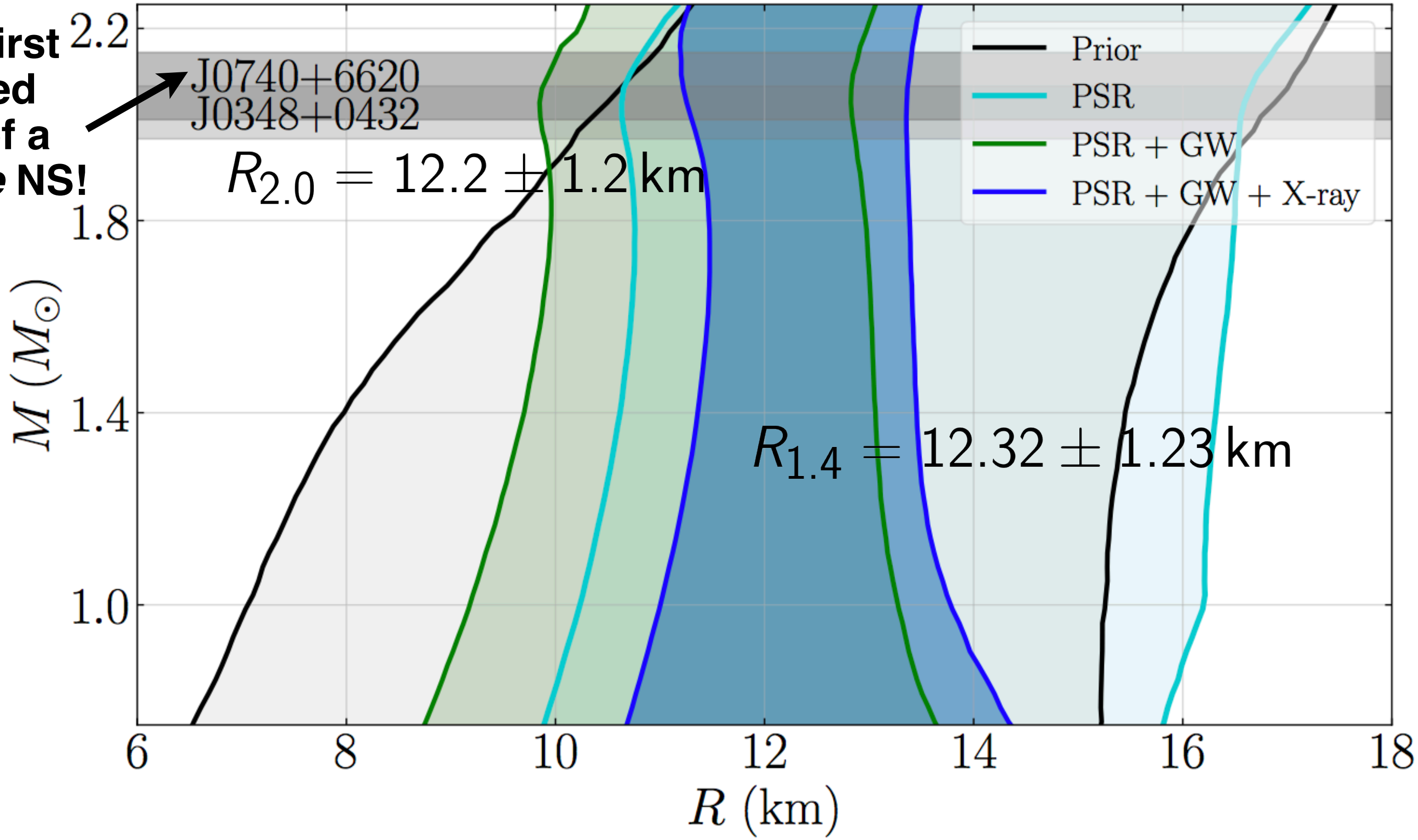


inferred central density ~ $3.0^{+1.6}_{-1.6}\,\rho_{\rm nuc}$

Multimessenger constraints

Legred et al.
(including **SH**),
arXiv:2106.05313

NICER first measured radius of a *massive* NS!



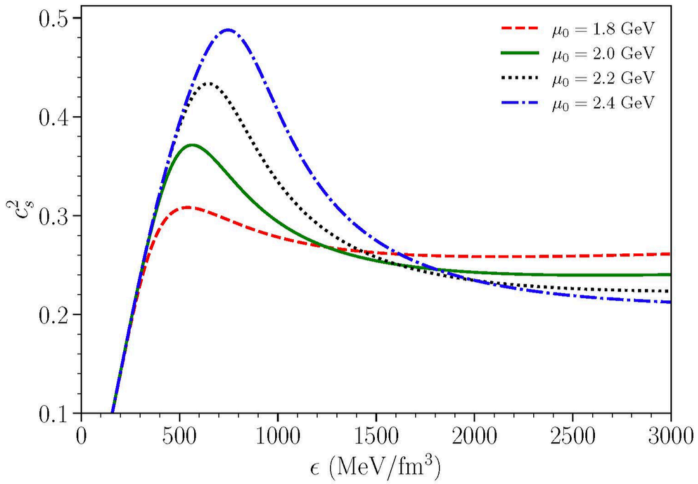
- nonparametric survey conditioned on ensembles of existing model EoSs
- GW170817+190425, NICER J0030 & J0740, and massive pulsars

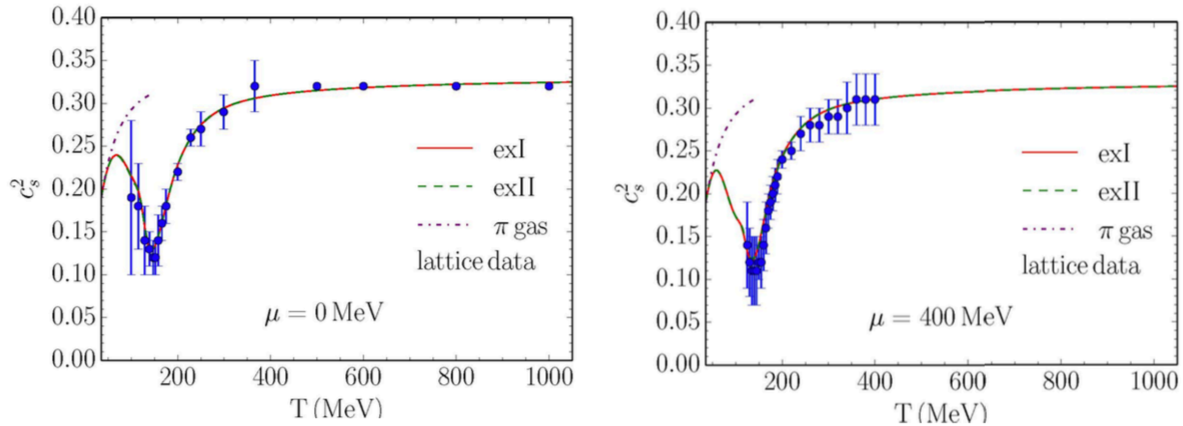
Crossover matter

- Kapusta-Welle approach: switching function of baryon chemical potential (arXiv:2103.16633)

$$P_B = (1 - S)P_H + SP_Q$$

$$S = \exp\left[-(\mu_0/\mu)^4\right]$$

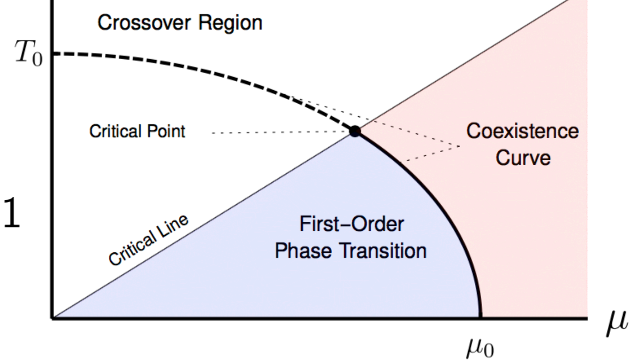




$$P(T,\mu) = SP_{\rm pQCD} + (1-S)P_{\rm hadron} \qquad S = \exp\left[-\frac{1}{(T/T_0)^n + (\mu/3\pi T_0)^n}\right]$$

Albright, Kapusta & Young 2015.

arXiv:1812.01684



$$S = 1/2$$

$$\left(\frac{T}{T_0}\right)^2 + \left(\frac{\mu}{\mu_0}\right)^2 = 1$$

- analogy: lattice QCD shows a crossover at finite temperature