Sound velocity peak driven by chiral partners in dense two-color QCD

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Suenaga-Murakami-Itou-Iida; Phys.Rev.D 107, 054001 (2023)
Kawaguchi-**Suenaga**; JHEP 08, 189 (2023)
Suenaga-Murakami-Itou-Iida; Phys.Rev.D 109, 074031 (2024)
Kawaguchi-**Suenaga**; Phys. Rev. D 109, 096034 (2024)
Fejos-**Suenaga**, in preparation,
etc.

my recent series of model study on dense two-color QCD

Introduction

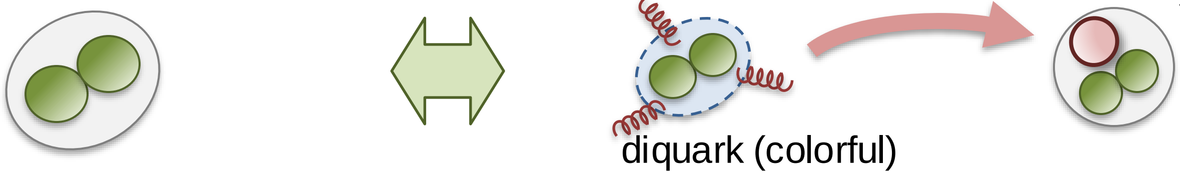
- **What is two-color QCD (QC_2D)?** $=$ **Strong interaction with** $N_c = 2$

then

- Diquarks turn to be color-singlet baryons → well-defined!

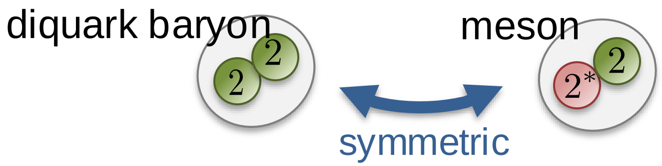
diquark (hadron for $N_c = 2$)

for $N_c = 3$



singly heavy baryon (SHB)
as a hadron

- Diquark baryons and mesons are treated in a unified way



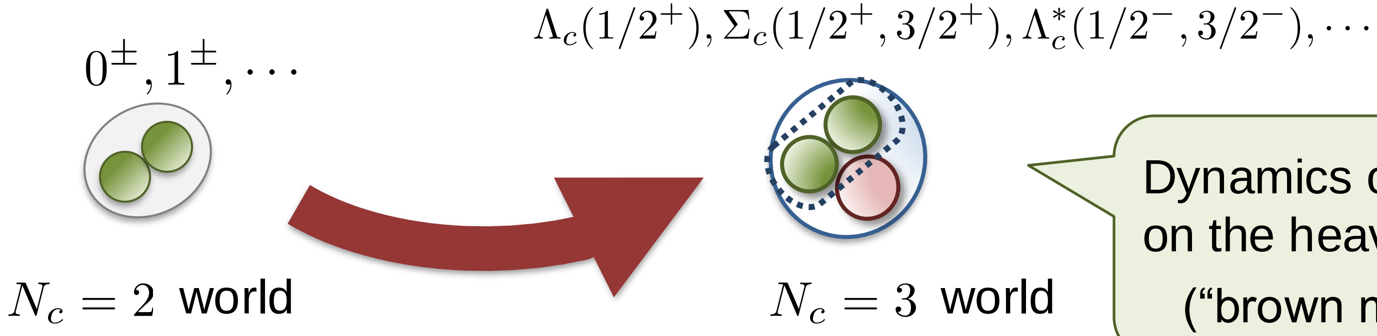
$2 \simeq 2^*$:pseudoreality of color SU(2)

→ Chiral symmetry (flavor structure) is extended to $SU(2N_f)$ from $SU(N_f)_L \times SU(N_f)_R$

⋮

• Why two-color QCD (QC_2D)?

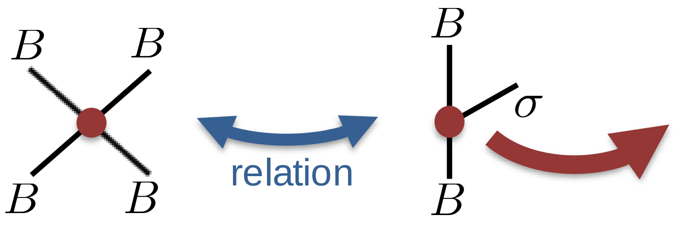
- Useful to extract information of **singly heavy baryon (SHB) spectrum** from the viewpoint of chiral symmetry and U(1) anomaly



Dynamics of only [diquark] is important based on the heavy-quark effective theory (HQET)
("brown mock" picture)

- The extended $SU(2N_f)$ symmetry doesn't matter for the above motivation, since it just relates couplings among diquarks and mesons

eg



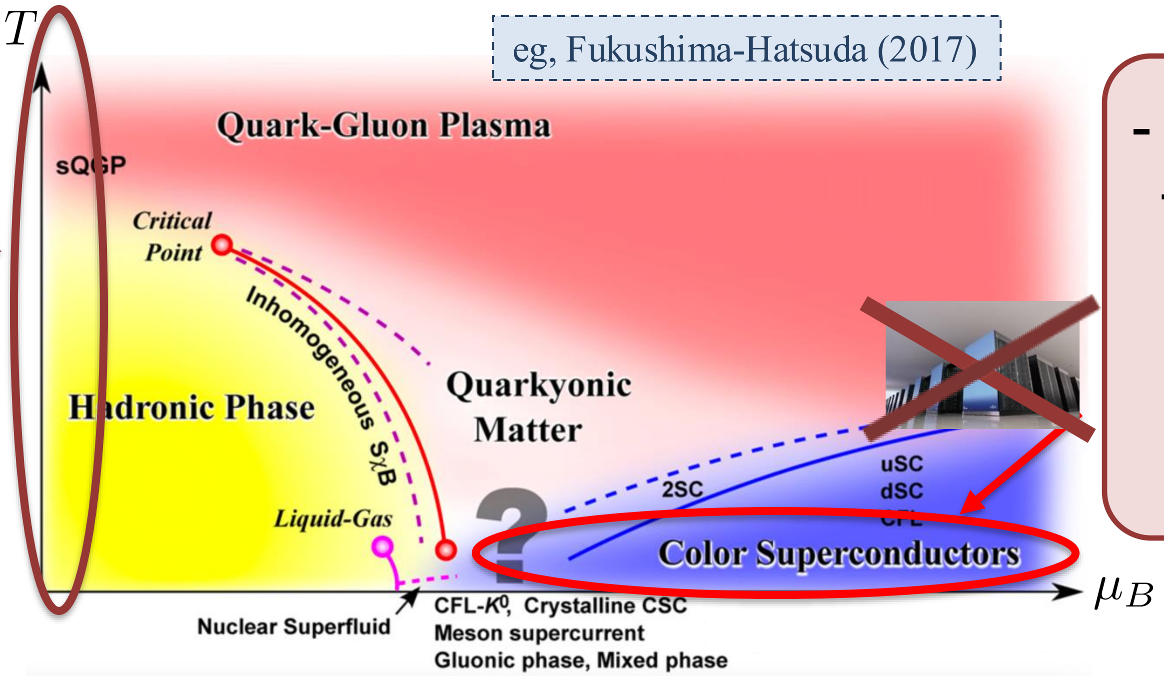
- From the viewpoint of mass generation, only this coupling is important *regardless of the couplings relations*
- U(1) anomaly *universally exists regardless of # of colors*

- **Why two-color QCD (QC_2D)?**

- The first-principles lattice simulation is straightforwardly applicable **in cold and dense regime**

eg, Fukushima-Hatsuda (2017)

Lattice QCD



- **Lattice QCD cannot apply due to the *sign problem* for** $N_c = 3$

⇨ **frontier of QCD** from lattice simulation point of view

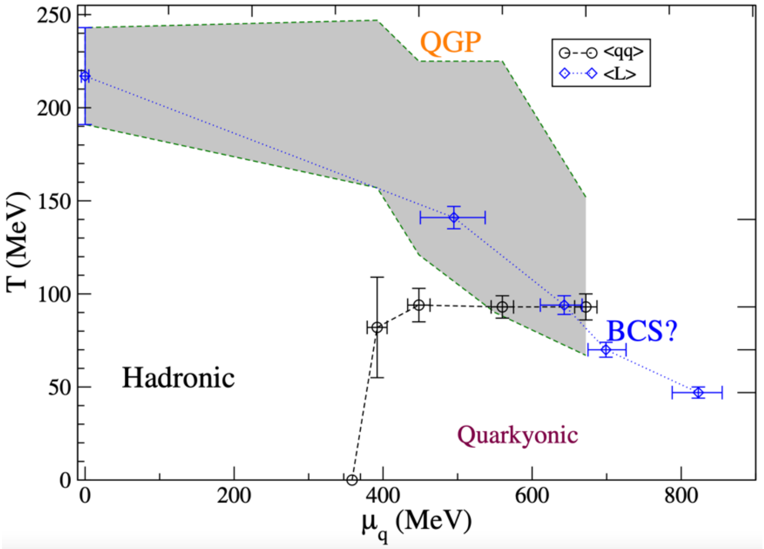
eg, G Aarts (2016), K. Nagata (2022).

In QC_2D world, the lattice simulation is possible thanks to the pseudoreality of $SU(2)_c$

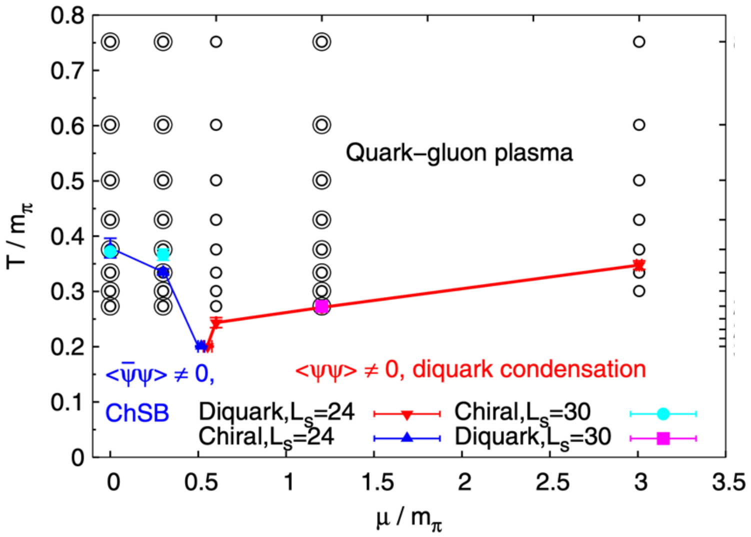
= **noteworthy advantage of QC_2D**

• Phase diagram in QC_2D

- Examples of simulation results of phase diagram in QC_2D



Boz-Cotter-Fister-Mehta-Skullerud (2013)



Buividovich-Smith-Smekal (2020)

・・・

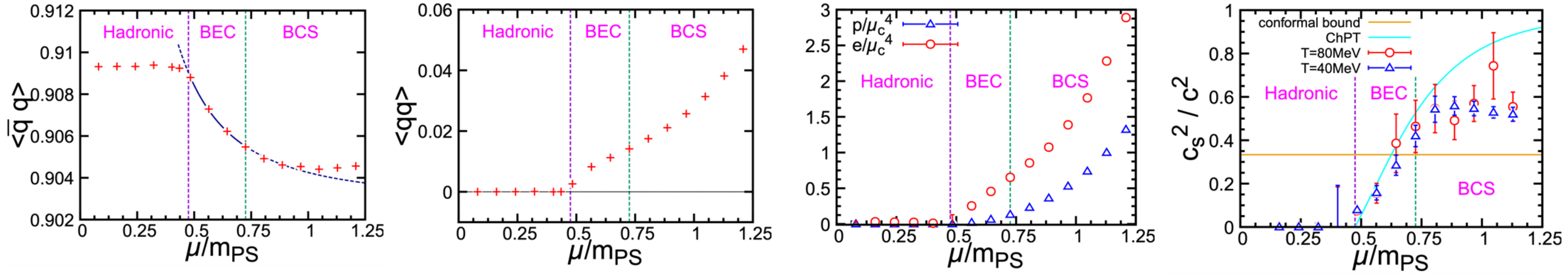
- Ireland/UK group (Hands, Skullerud, …)
- UK group (Buividovich, ...)
- Russian group (Bornyakov, ...)
- Japanese group (Iida-san, Itou-san, …), (+Nonaka-san)

etc.

Introduction

- **Lattice results**
  - In addition to phase diagram, hadron mass spectrum, gluon propagator, transport coefficient, EoS, sound velocity, $\langle\bar{\psi}\psi\rangle, \langle\psi\psi\rangle, \langle L\rangle$, etc. have been simulated

eg, Japanese group results



. . .

My approach

− (i) Regard QC_2D lattice simulations as useful "numerical experiments" of cold and dense QCD, then (ii) give interpretation from symmetry viewpoints based on effective models

My publications on QC_2D

Gluon propagator: **Suenaga**-Kojo(2019), Kojo-**Suenaga**(2021), CSE effect: **Suenaga**-Kojo(2021), Sound velocity: Kojo-**Suenaga**(2022), Kawaguchi-**Suenaga**(2024), Topological susceptibility: Kawaguchi-**Suenaga**(2023), Hadron mass: **Suenaga**-Murakami-Itou-Iida (2023, 2024), and in-preparations.

Q: What is your ultimate goal?

A: To provide information on NS physics

⋮

in a broad sense

A: To unveil $SU(N_c)$ Yang-Mills theory in many-body system of quarks/hadrons!

message of this talk:

→ There is no reason to ignore fruitful QC_2D numerical experiments!

Model

- **Pauli-Gursey SU(4) symmetry**

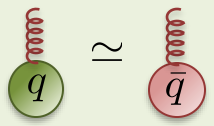
  - Pseudo reality of $SU(2)_c$ allows us to rewrite QC_2D Lagrangian with massless quarks as

$$\mathcal{L}_{\mathrm{QC_2D}} = \bar{\psi} i \partial\!\!\!/ \psi - g_s \bar{\psi} A\!\!\!/^a T_c^a \psi = \Psi^\dagger i \partial_\mu \sigma^\mu \Psi - g_s \Psi^\dagger A_\mu^a T_c^a \sigma^\mu \Psi$$

pseudoreality: $\sigma^2 \sigma^a \sigma^2 = -(\sigma^a)^*$

In two-flavor: $\Psi = (\psi_R, \tilde{\psi}_L)^T = (u_R, d_R, \tilde{u}_L, \tilde{d}_L)^T$ with $\tilde{\psi}_L = \sigma^2 \tau_c^2 \psi_L^*$

Four-dimensional Pauli matrix: $\sigma^\mu = (1, \sigma^i)$



gluons are blind

  - $\mathcal{L}_{\mathrm{QC_2D}}$ is obviously invariant under $\Psi \to g\Psi \ [g \in SU(4)]$

$SU(2)_L \times SU(2)_R$ chiral symmetry ⟹ (enlarged) Pauli-Gursey $SU(4)$ symmetry   Pauli (1957), Gursey (1958)

- All low-energy effective model of QC_2D is constructed to satisfy this symmetry

eg, Kogut-Stephanov-Toublan (1999), Kogut-Stephanov-Toublan-Verbaarschot-Zhitnitsky (2000)

• Linear sigma model (LSM)

- LSM is an effective model describing not only NG bosons (pions etc.) but also their P-wave excitations

⟺ extended model including all order of ChPT

Black-Fariborz-Jora-Park-Schechter-Shahid (2009)

- Introduce a 4×4 Σ matrix

$$\Sigma_{ij} = \frac{1}{2}\begin{pmatrix} 0 & -\frac{B'-iB}{2\sqrt{2}} & \frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & \frac{a_0^+-i\pi^+}{2\sqrt{2}} \\ \frac{B'-iB}{2\sqrt{2}} & 0 & \frac{a_0^--i\pi^-}{2\sqrt{2}} & \frac{\sigma-i\eta-a_0^0+i\pi^0}{4} \\ -\frac{\sigma-i\eta+a_0^0-i\pi^0}{4} & -\frac{a_0^--i\pi^-}{2\sqrt{2}} & 0 & -\frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} \\ -\frac{a_0^+-i\pi^+}{2\sqrt{2}} & -\frac{\sigma-i\eta-a_0^0+i\pi^0}{4} & \frac{\bar{B}'-i\bar{B}}{2\sqrt{2}} & 0 \end{pmatrix}_{ij} \sim \Psi_j\sigma^2\tau_c^2\Psi_i$$

cf, $\Sigma = \sigma + i\pi^a\tau^a$ for $N_c = 3$

$\Sigma \to g\Sigma g^T$

$[g \in SU(4)]$

- Assignment of hadron fields

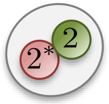
$$B \sim -\frac{i}{\sqrt{2}}\psi^T C\gamma_5\tau_c^2\tau_f^2\psi \quad B' \sim -\frac{1}{\sqrt{2}}\psi^T C\tau_c^2\tau_f^2\psi \quad \sigma \sim \bar{\psi}\psi$$

$$a_0^a \sim \bar{\psi}\tau_f^a\psi \quad \eta \sim \bar{\psi}i\gamma_5\psi \quad \pi^a \sim \bar{\psi}i\gamma_5\tau_f^a\psi$$

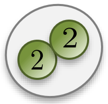
⟺

Hadron	J^P	Quark number	Isospin
σ	0^+	0	0
a_0	0^+	0	1
η	0^-	0	0
π	0^-	0	1
B $(\bar{B})$	0^+	$+2(-2)$	0
B' $(\bar{B}')$	0^-	$+2(-2)$	0

} mesons



} baryons



Model

- **Lagrangian of Linear sigma model (LSM)**

  - (approximately) $SU(4)$-invariant LSM Lagrangian is given by

$$\mathcal{L} = \mathrm{tr}[D_\mu \Sigma^\dagger D^\mu \Sigma] - m_0^2 \mathrm{tr}[\Sigma^\dagger \Sigma] - \lambda_1 \left(\mathrm{tr}[\Sigma^\dagger \Sigma]\right)^2 - \lambda_2 \mathrm{tr}[(\Sigma^\dagger \Sigma)^2] + \mathrm{tr}[H^\dagger \Sigma + \Sigma^\dagger H] + c(\mathrm{det}\Sigma + \mathrm{det}\Sigma^\dagger)$$

$D_\mu \Sigma = \partial_\mu \Sigma - i\mu_q \delta_{\mu 0}\{J, \Sigma\}$ with $J = \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}$

<u>chemical potential effect</u>

$H = h_q E$ with $E = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}$

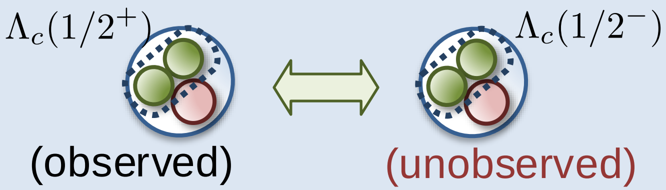
<u>current-quark mass effect</u>

$U(1)_A$ anomaly

- Advantage of LSM:

eg, Exploration of mass generation of parity (chiral) partners

<u>in $N_c = 3$ world</u>



<u>My hope</u>

Hints from QC_2D analysis for the unobserved HQS-singlet $\Lambda_c(1/2^-)$?

connection with SHB in $N_c = 3$

Mean fields

- **Mean field**

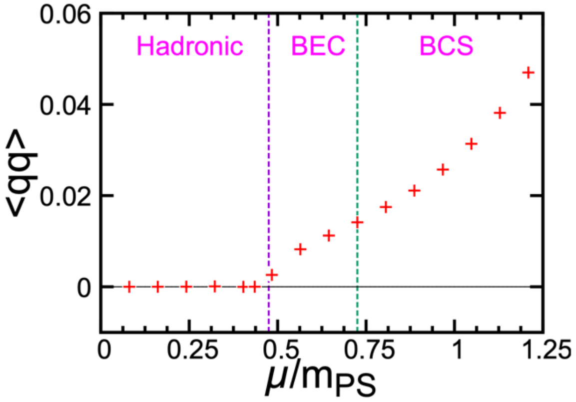
- The mean fields are $\sigma_0 \equiv \langle \sigma \rangle$ and $\Delta \equiv \left\langle \frac{B+\bar{B}}{\sqrt{2}} \right\rangle$

$\sigma_0 \sim \langle \bar{\psi}\psi \rangle$: chiral condensate

$\Delta \sim -\frac{i}{2}\langle \psi^T C\gamma_5 \tau_c^2 \tau_f^2 \psi \rangle + \mathrm{h.c.}$: diquark condensate

diquark cond. by lattice

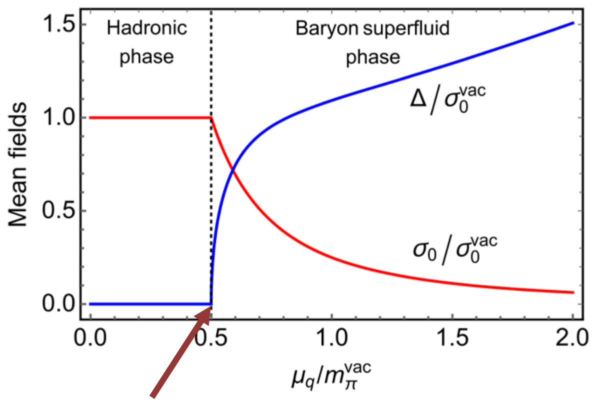
Iida et al, 2405.20566



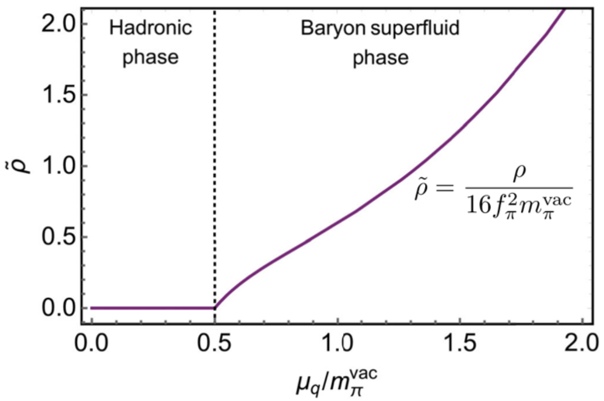
$\langle qq \rangle = 0$: hadronic phase

$\langle qq \rangle \neq 0$: baryon superfluid phase

σ_0 and Δ vs μ_q



density ρ vs μ_q



2[nd] order phase transition at $\mu_q = m_\pi^{\mathrm{vac}}/2$

<u>Input here</u>

$\sigma_0^{\mathrm{vac}} = 250\,\mathrm{MeV}$ (put by hand)

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{\mathrm{vac}} = 738\,\mathrm{MeV}$

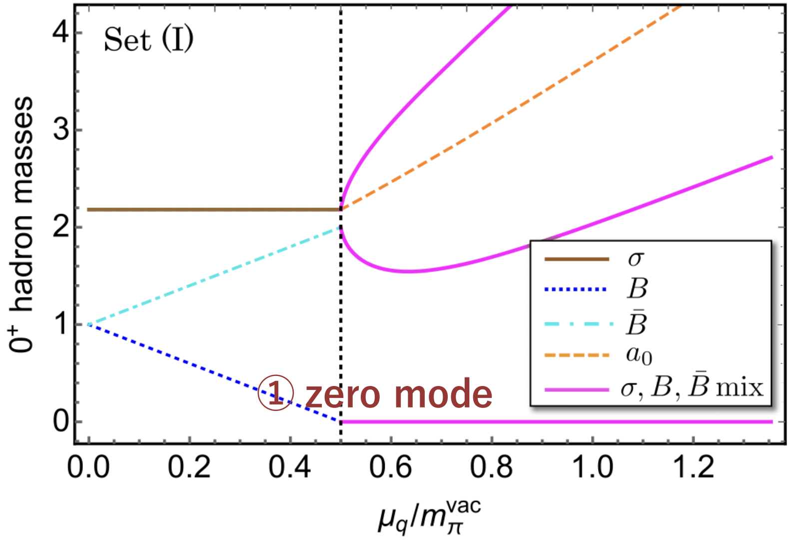
$m_{a_0}^{\mathrm{vac}}/m_\pi^{\mathrm{vac}} = 2.18$

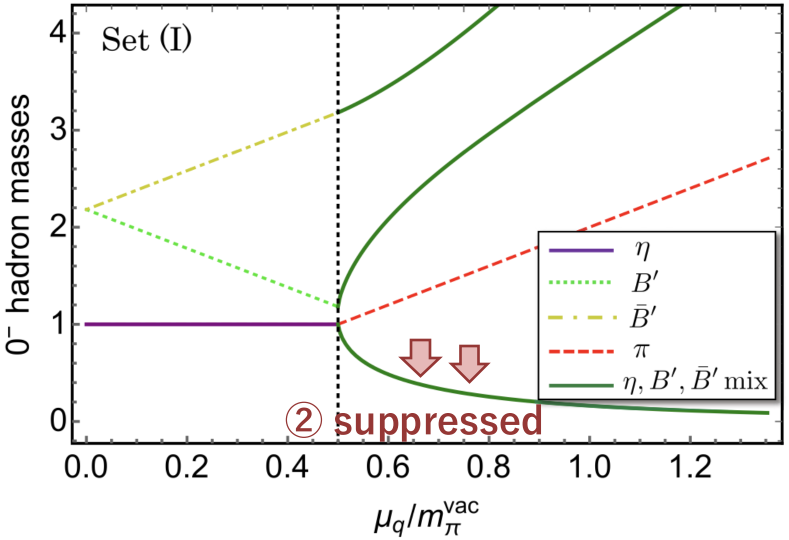
} lattice
Murakami et al

Hadron masses

- **Results on hadron mass at finite μ_q $(T=0)$**

(normalized by $m_\pi^{\rm vac}$)





Input here

$\sigma_0^{\rm vac} = 250\,{\rm MeV}$ (put by hand)

$\lambda_1 = c = 0$ (large Nc)

$m_\pi^{\rm vac} = 738\,{\rm MeV}$, $m_{a_0}^{\rm vac}/m_\pi^{\rm vac} = 2.18$ } lattice (Murakami et al)

- Baryon number violation in superfluid phase

$\sigma \leftrightarrow B \leftrightarrow \bar{B}$ mixing (0+ sector)

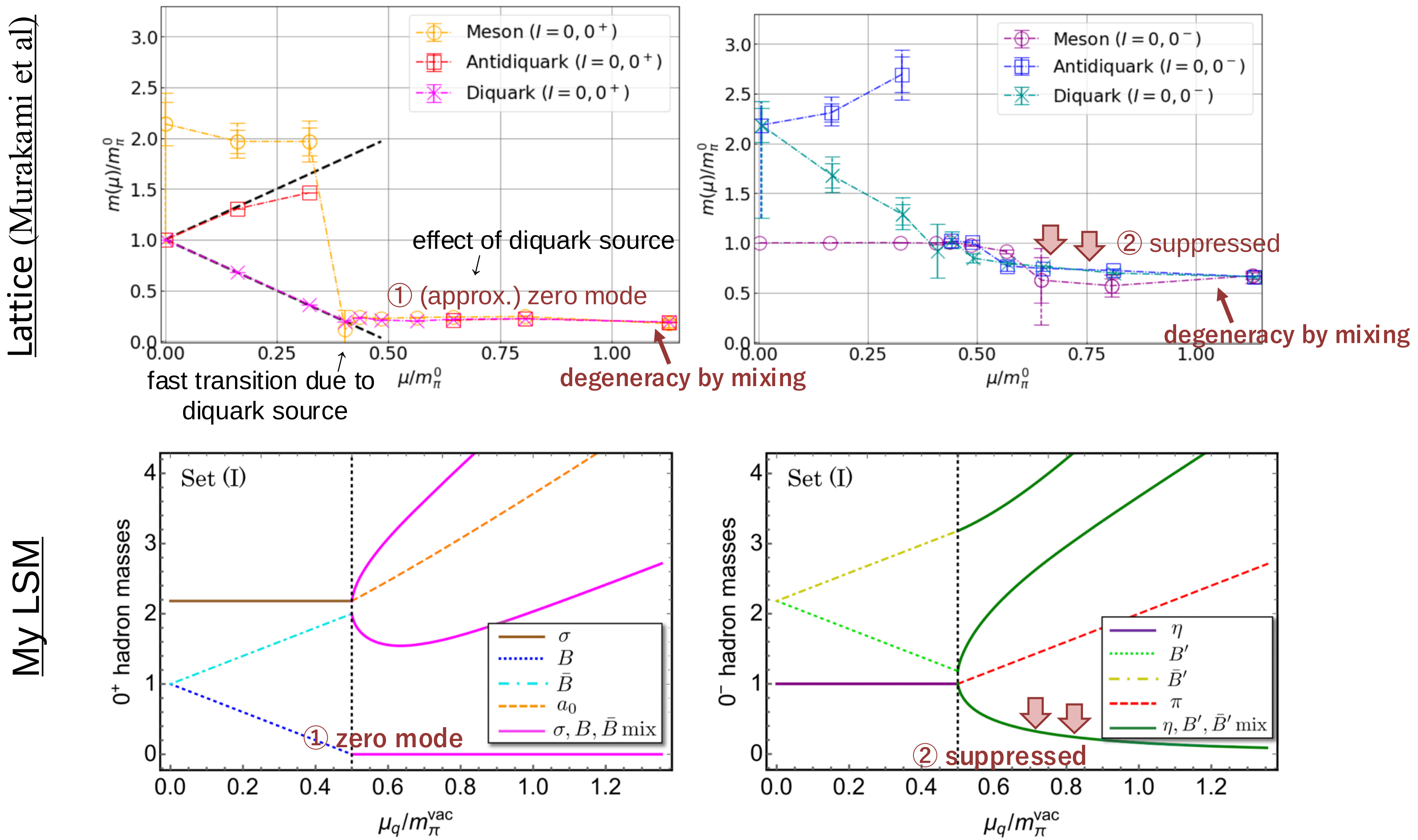
$\eta \leftrightarrow B' \leftrightarrow \bar{B}'$ mixing (0- sector)

① zero mode (NG mode of $U(1)$ baryon-number breaking)

② nonlinear suppression of " η " mass due to the mixing

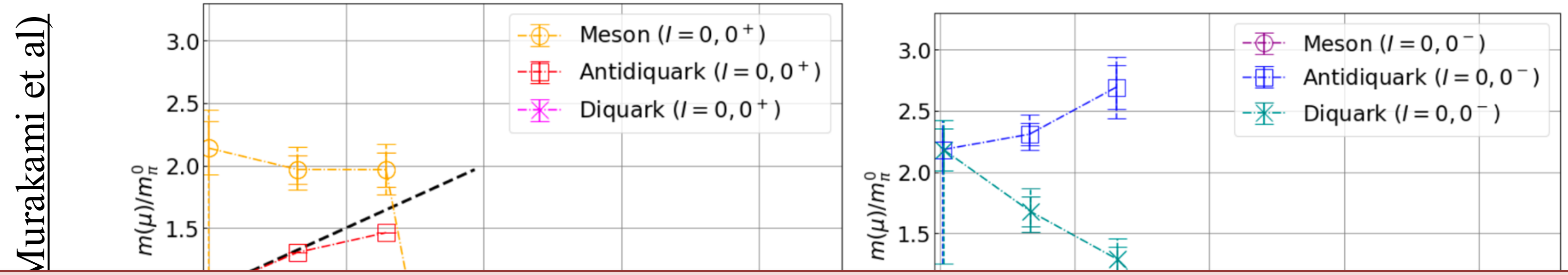
Hadron masses

- **Comparison with lattice**



Hadron masses

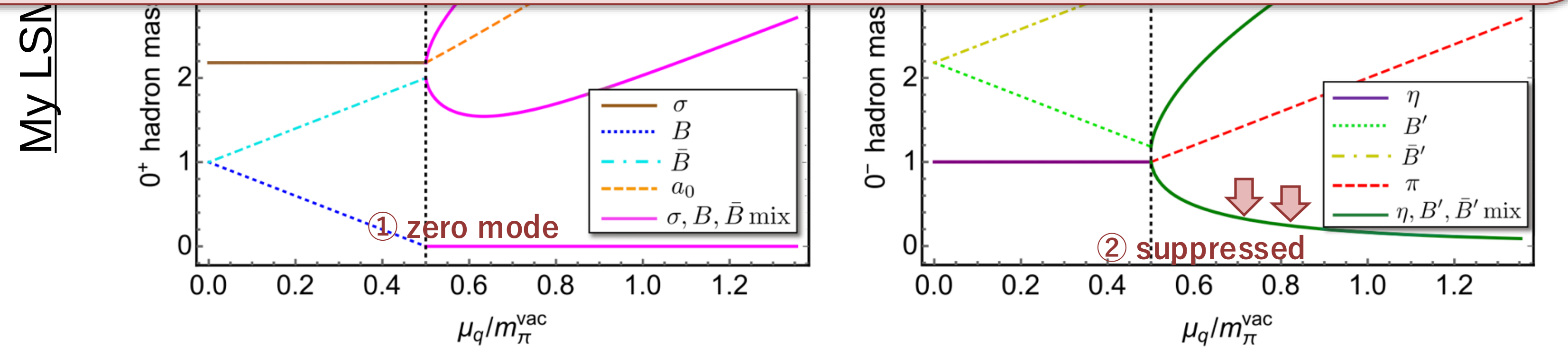
・**Comparison with lattice**



Succeeded in qualitative understanding!

→ **We are now ready to explore other quantities of dense QC_2D with my LSM**

mixing



Sound velocity

- **Sound velocity at mean-field level within the LSM**

$\bar{\mu} = \mu/\mu_{\rm cr} = 2\mu/m_\pi$

$\delta\bar{m}^2_{\sigma-\pi} = (m_\sigma^2 - m_\pi^2)/\mu_{\rm cr}^2$

pressure: $$p = \underbrace{f_\pi^2 m_\pi^2 \left(\bar{\mu}^2 + \frac{1}{\bar{\mu}^2}\right)}_{\text{ChPT result}} + f_\pi^2 m_\pi^2 \left[\frac{4}{\delta\bar{m}^2_{\sigma-\pi}}(\bar{\mu}^2-1)^2\right]$$

energy: $$\epsilon = \underbrace{f_\pi^2 m_\pi^2 \left[\frac{(\bar{\mu}^2+3)(\bar{\mu}^2-1)}{\bar{\mu}^2}\right]}_{\text{ChPT result}} + f_\pi^2 m_\pi^2 \left[\frac{4}{\delta\bar{m}^2_{\sigma-\pi}}(3\bar{\mu}^2+1)(\bar{\mu}^2-1)\right]$$

sound velocity: $$c_s^2 = \frac{(1-1/\bar{\mu}^4) + 8(\bar{\mu}^2-1)/\delta\bar{m}^2_{\sigma-\pi}}{(1+3/\bar{\mu}^4) + 8(3\bar{\mu}^2-1)/\delta\bar{m}^2_{\sigma-\pi}}$$

= chiral partner contribution

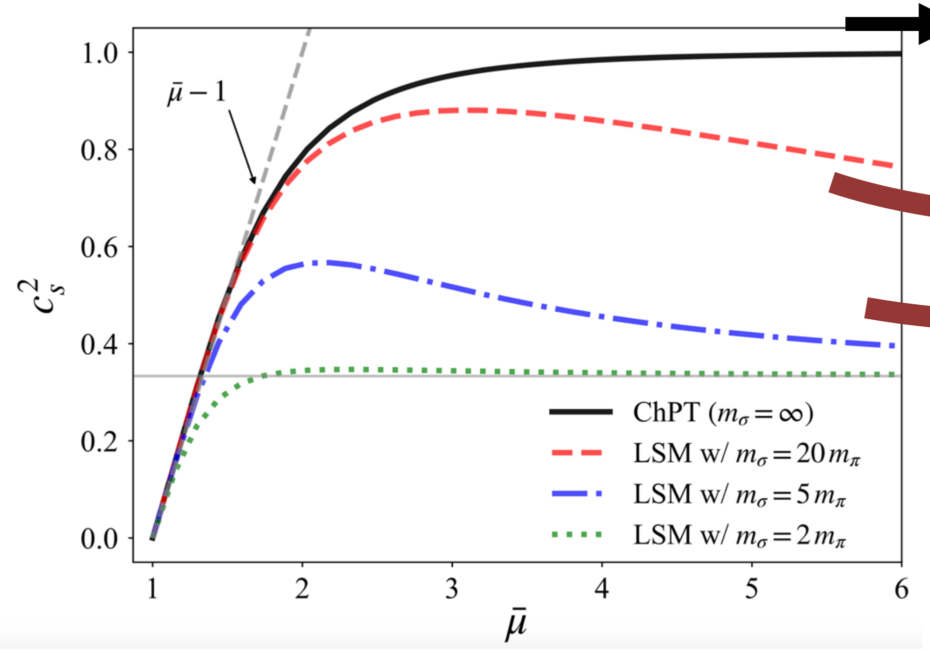
Universal structure: $(\text{LSM result}) = (\text{ChPT result}) + (1/\delta\bar{m}^2_{\sigma-\pi}\ \text{contribution})$

- Integrating out the chiral partners $(m_\sigma \to \infty)$ yields the ChPT results $(1/\delta\bar{m}^2_{\sigma-\pi} \to 0)$

Sound velocity

• Sound velocity peak

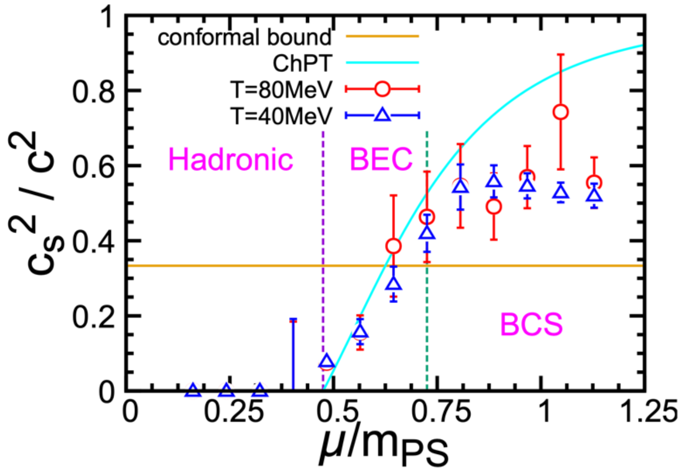
$$\bar{\mu} = \mu/\mu_{\rm cr} = 2\mu/m_\pi$$

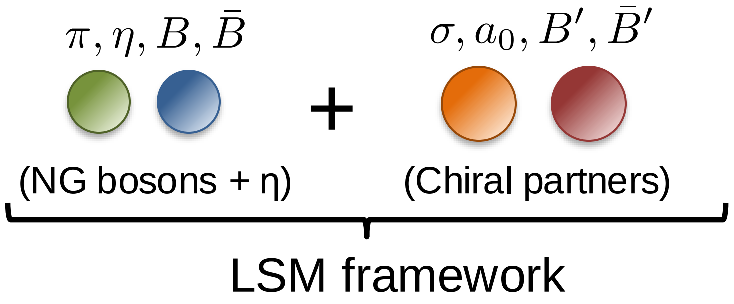


ChPT result $(c_s^2 \to 1)$: no peak

peak when $m_\sigma < \infty$
$(c_s^2 \to 1/3)$

cf, lattice: Iida-Itou-Murakami-Suenaga(2024)





- The peak structure is driven by contributions from chiral partners

- Any connection with crossover to quark matter ?

- Fluctuation and spin-1 hadron effect are needed for more quantitative comparison

Conclusions

- I constructed the LSM as an effective model of cold and dense QC_2D

  - Not only NG bosons but also their chiral partners are described
  - → Extended model of ChPT

- Qualitative understanding of $0^{\pm}$ hadron spectrum measured on the lattice

  → Good benchmark to explore dense QC_2D

- The sound velocity peak is realized when $\sqrt{3}m_\pi < m_\sigma < \infty$

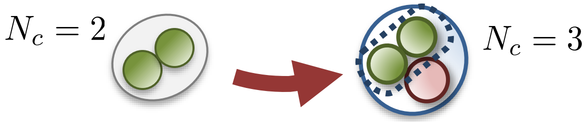
  → "Active" contribution from chiral partners is important

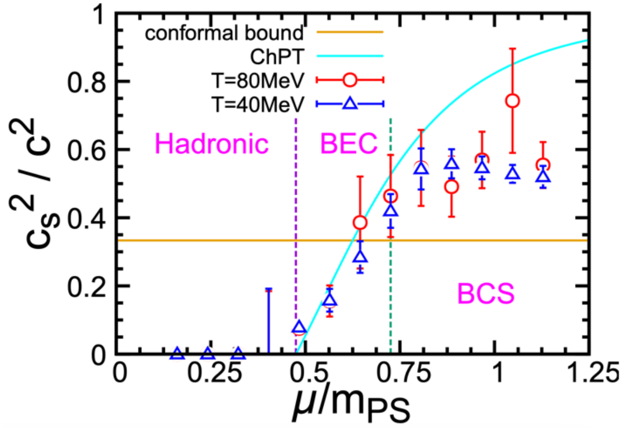
- Q: Any connection with crossover to quark matter ? Kojo-Suenaga (2022)

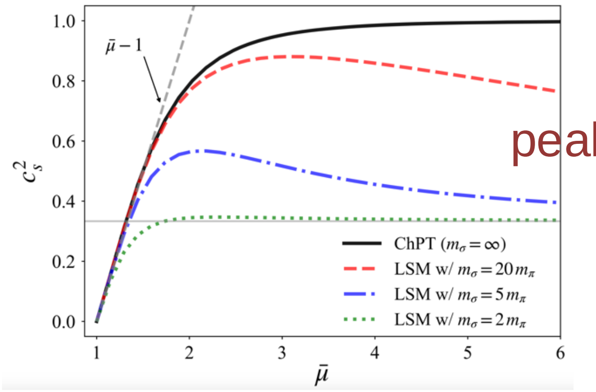
- We are ready to explore other dense QC_2D quantities with my LSM

  Topological susceptibility: Kawaguchi-Suenaga(2023),
  Extension with spin-1 hadrons: Suenaga-Murakami-Itou-Iida(2024), ...

study on SHB spectrum from diquarks in QC_2D







peak