Impact of first-principles calculations on the QCD phase diagram and equation of state

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References:
[1] Y. Fujimoto, K. Fukushima, L. McLerran, M. Praszalowicz, PRL129 (2022) [2207.06753]
[2] Y. Fujimoto, S. Reddy, PRD 109 (2024) (selected as Editors' Suggestion) [2310.09427]
[3] Y. Fujimoto, PRD109 (2024) [2312.11443]; [2408.12514]

October 11, 2024 - CSQCD 2024 in Kyoto

Neutron stars: why do we study now?

Now is the most exciting period because of…

- Recent advances in astrophysics
- Recent advances in QCD — the first principle

Recent advances in QCD

- Higher-order computations of weak-coupling QCD EoS

Freedman,McLerran(1978); Baluni(1979); Kurkela,Romatschke,Vuorinen (2009);
Gorda,Säppi,Paatelainen,Seppänen,Österman,Schicho,Navarrete (2018-)

- Nuclear EoS from chiral effective field theory (χEFT)

Tews,Krüger,Hebeler,Schwenk(2013);Drischler,Furnstahl,Melendez,Philips(2020);
Keller,Hebeler,Schwenk(2022); … many others

- Lattice simulations of QCD at finite isospin density

Kogut,Sinclair (2002); NPLQCD collaboration (2007-);
Brandt,Chelnokov,Cuteri,Endrodi,… (2014-);

- Lattice simulations of two-color QCD at finite baryon density

e.g. Iida,Itou,Murakami,Suenaga (2024)

- Hadron-hadron interaction from the lattice QCD

HAL QCD collaboration (2006-)

- Hamiltonian lattice simulations of QCD in (1+1)-dimensions

Hayata,Hidaka,Nishimura (2023)

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Freedman,McLerran(1978); Baluni(1979); Kurkela,Romatschke,Vuorinen (2009);
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QCD at finite isospin density

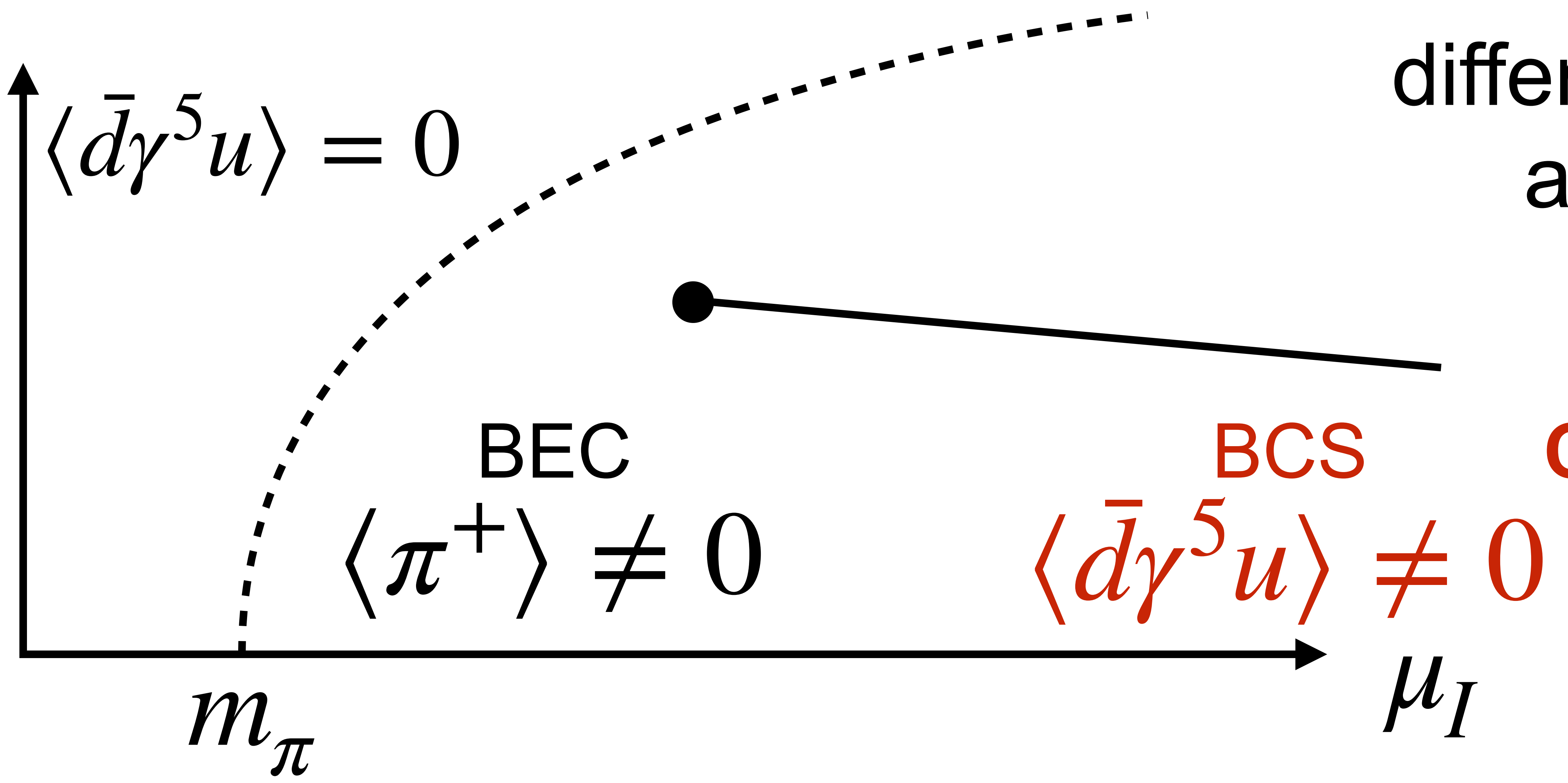
Alford,Kapustin,Wilczek (1999); Kogut,Sinclair (2002-);
Beane,Detmold,Savage et al. (2007-);
Endrodi et al. (2014-)…

- **No sign problem** → can be simulated on the lattice!
- Isospin chemical potential (conjugate to isospin density I_3):

$$\mu_u = \frac{\mu_I}{2}, \quad \mu_d = -\frac{\mu_I}{2}$$ … Fermi surface of u & $\bar{d}$

- Phase structure:

Son,Stephanov (2000)



different from phase structure at finite baryon density

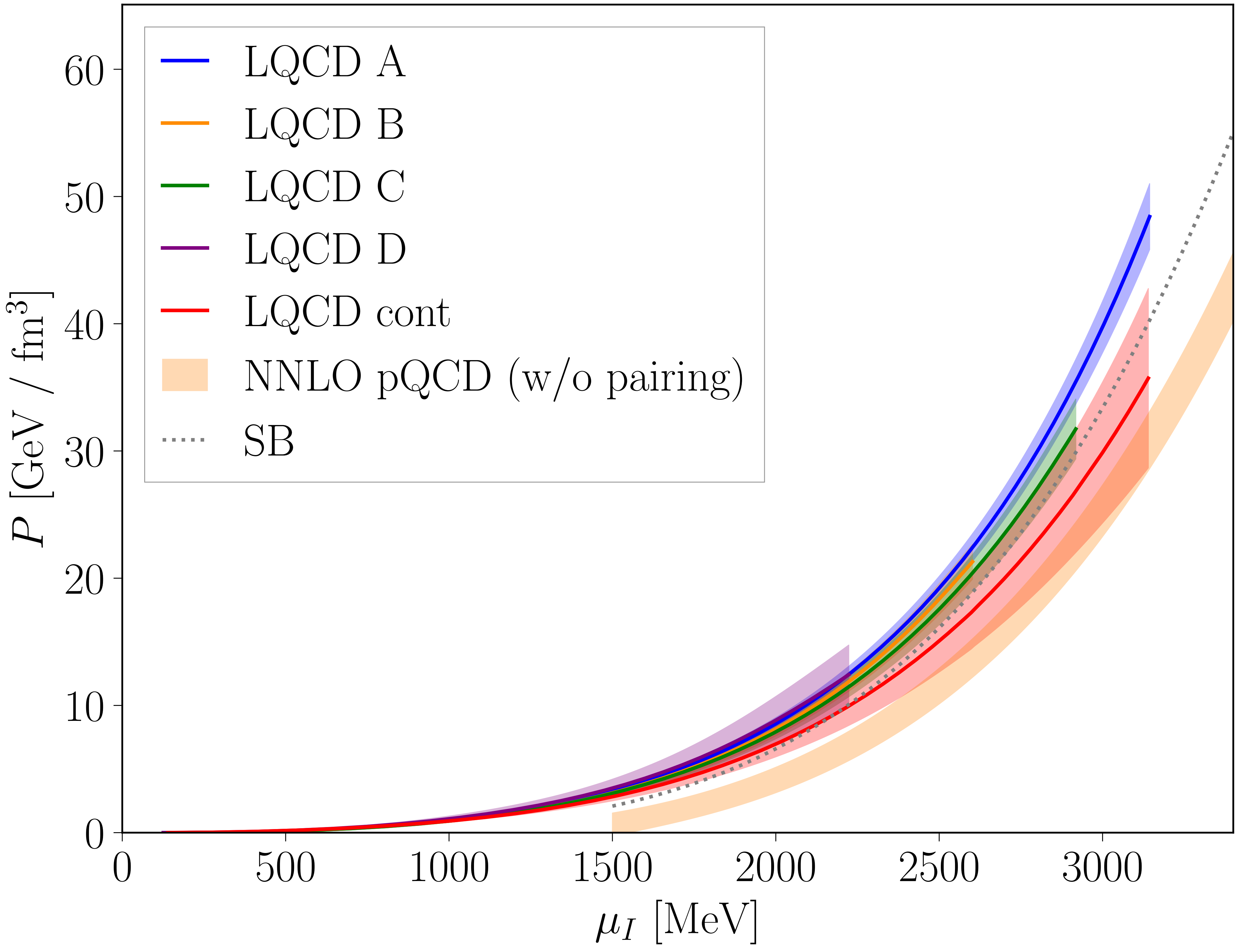
Cooper pairing

QCD at finite isospin density

Recent impact:

Abbott et al. (NPLQCD) (2023, 24)

EoS is calculated up to $\mu_I \sim 3$ GeV by lattice QCD in the continuum limit



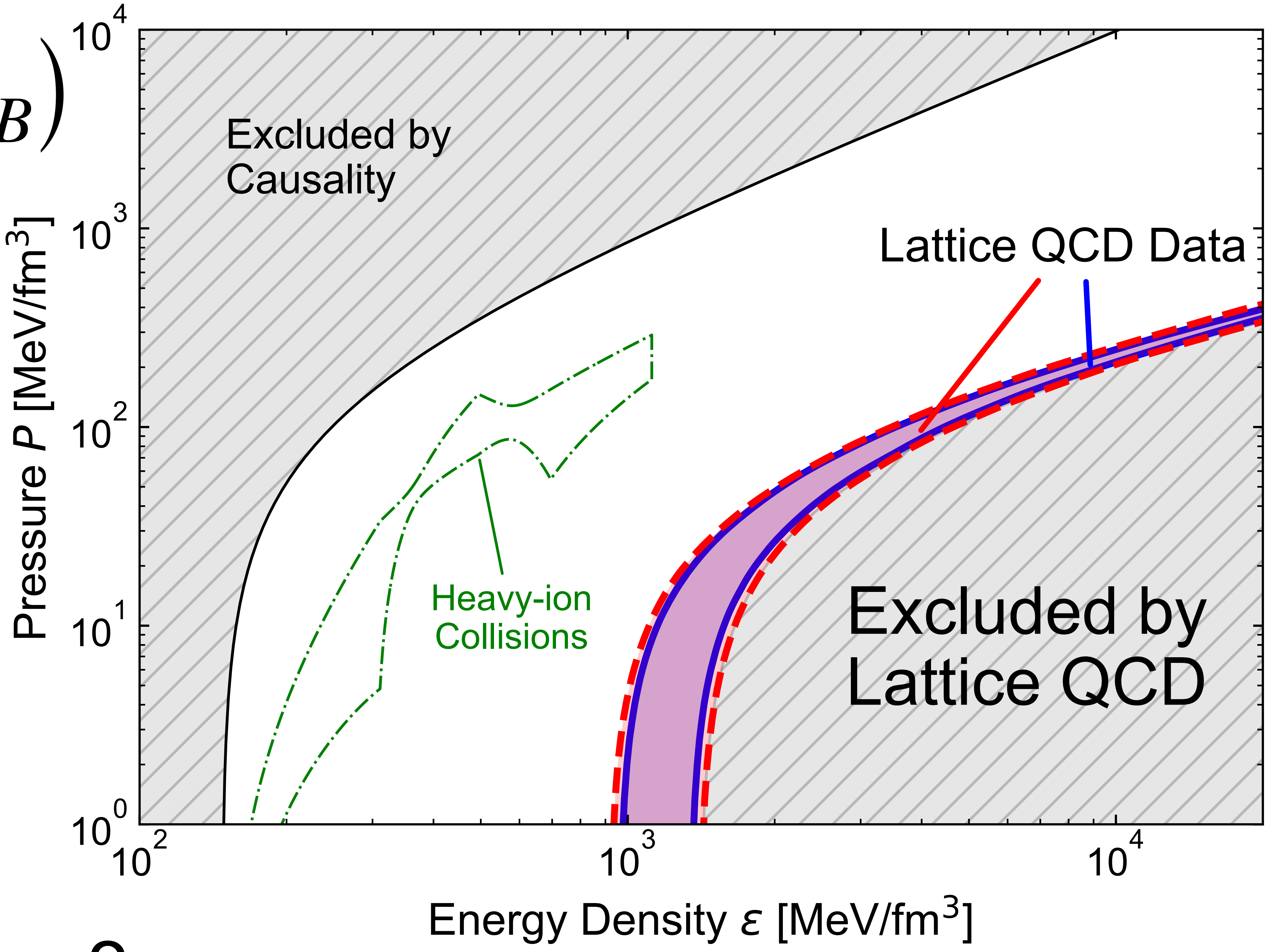
→ comparison with the weak-coupling results feasible

QCD at finite isospin density

Abbott et al. (NPLQCD) (2023, 24

What can we learn from this?
→ Bounds on the symmetric nuclear matter EoS from QCD inequality

$$P(\mu_B) \leq P_{\text{lattice}}\left(\mu_I = \frac{2}{N_c}\mu_B\right)$$



Cohen (2003);
Fujimoto,Reddy, PRD 109 (2023)

cf. Moore,Gorda (2023)

What else can we learn?
→ Weak-coupling results can be constrained by comparison w/ lattice QCD results

Outline

1. Conformal EoS in neutron star cores

- useful observables: trace anomaly
- Bayesian inference from astrophysical data
- role of the pQCD constraint

2. Interplay between weak-coupling and lattice QCD

Following issues about the weak-coupling results:

- applicability
- fixing the undetermined parameter in pQCD
- color-superconducting gap

Part 1: Conformal EoS in NS cores

Trace anomaly equation

Related to scale/conformal nature of matter:

$$j_D^\nu = x_\mu T^{\mu\nu} \;\to\; \partial_\nu j_D^\nu = T^\mu_\mu \begin{cases} = 0 & \text{Classical YM} \\ \neq 0 & \text{in QFT (RG effect)} \end{cases}$$

Expectation value: $\langle T^\mu_\mu \rangle = \underbrace{\langle T^\mu_\mu \rangle_{\mu_B}}_{\text{matter}} + \underbrace{\langle T^\mu_\mu \rangle_0}_{\text{vacuum}}$

matter (μ_B-dependent)

Finite-μ_B part of the trace anomaly (interaction measure):

$$\langle T^\mu_\mu \rangle_{\mu_B} = \varepsilon - 3P$$ **… can be read out from NS EoS**

Normalized trace anomaly: $\Delta_{\mathrm{tr}} \equiv \dfrac{\langle T^\mu_\mu \rangle_{\mu_B}}{3\varepsilon} = \dfrac{\varepsilon - 3P}{3\varepsilon}$

cf. Gavai,Gupta,Mukherjee (2004)

Conformal limit

Limit $\alpha_s \to 0$ (conformal theory) is achieved when $\varepsilon \to \infty$.

The weak-coupling EoS in this **conformal limit** is:

Trace anomaly: $\varepsilon - 3P \;\sim -\beta_0 \mu^4 \left(\frac{\alpha_s}{\pi}\right)^2 \to 0^+$

Sound speed: $v_s^2 = \frac{dP}{d\varepsilon} \;\sim \frac{1}{3}\frac{1}{1-\beta_0\left(\frac{\alpha_s}{\pi}\right)^2} \to \frac{1}{3} - 0^+$ $\quad(\beta_0 < 0)$

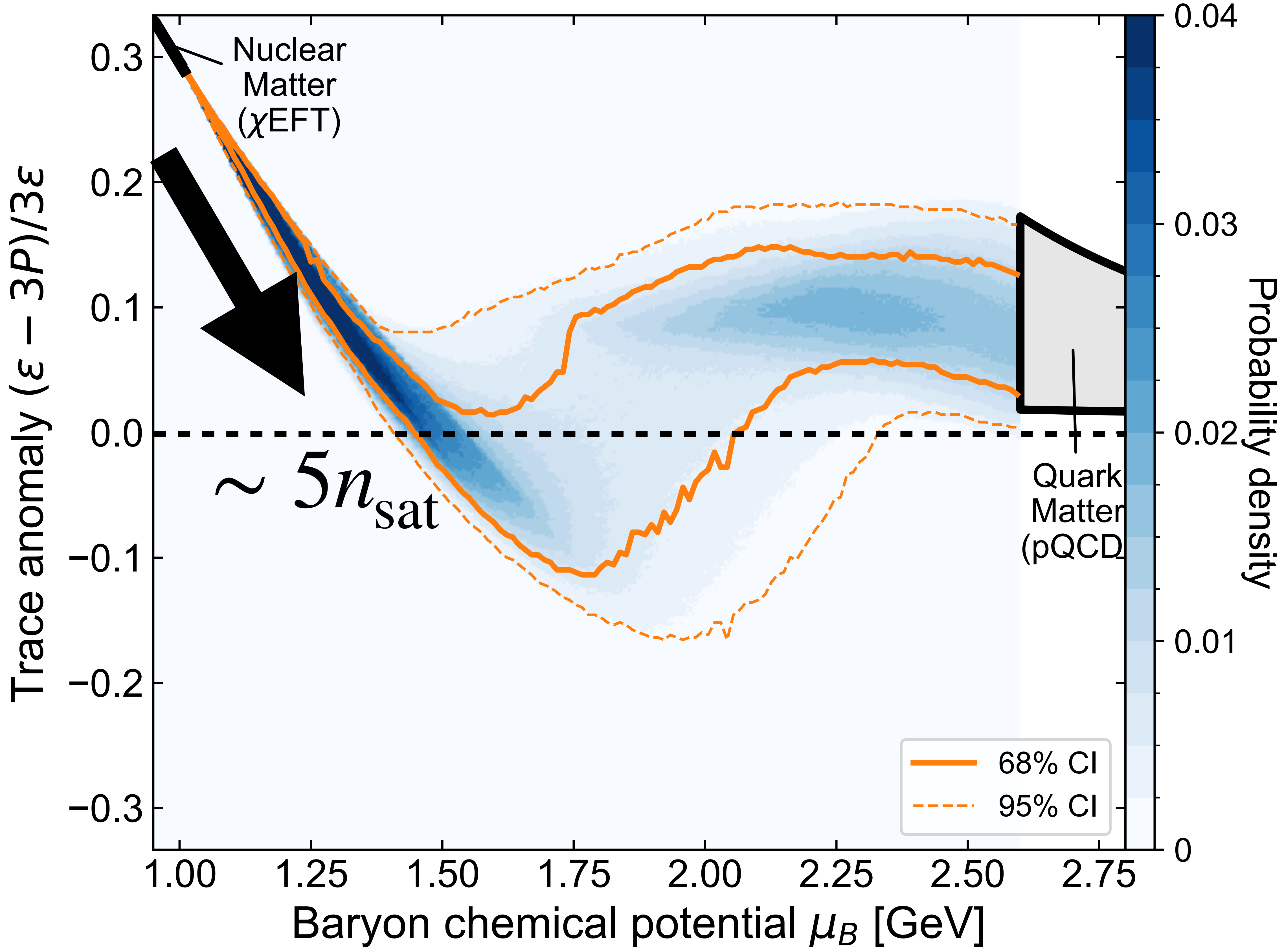
At the intermediate density, $\varepsilon - 3P = 0$ and $v_s^2 = 1/3$ are **different conditions**

Trace anomaly from neutron star data

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

Normalized trace anomaly: $\Delta_{\rm tr} \equiv \dfrac{\langle T^\mu_{\ \mu} \rangle_{\mu_B}}{3\varepsilon} = \dfrac{\varepsilon - 3P}{3\varepsilon}$

Results of Bayesian inference: Method: Annala et al. (2019); Altiparmak,Ecker,Rezzolla (2022)



$\Delta_{\rm tr} \sim 0$ already at $\sim 5n_{\rm sat}$

$\rightarrow$ **rapid approach to conformal EoS $P \approx \varepsilon/3$**

Suggests the existence of strongly-coupled conformal matter?

Trace anomaly, effective d.o.f. & role of pQCD

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

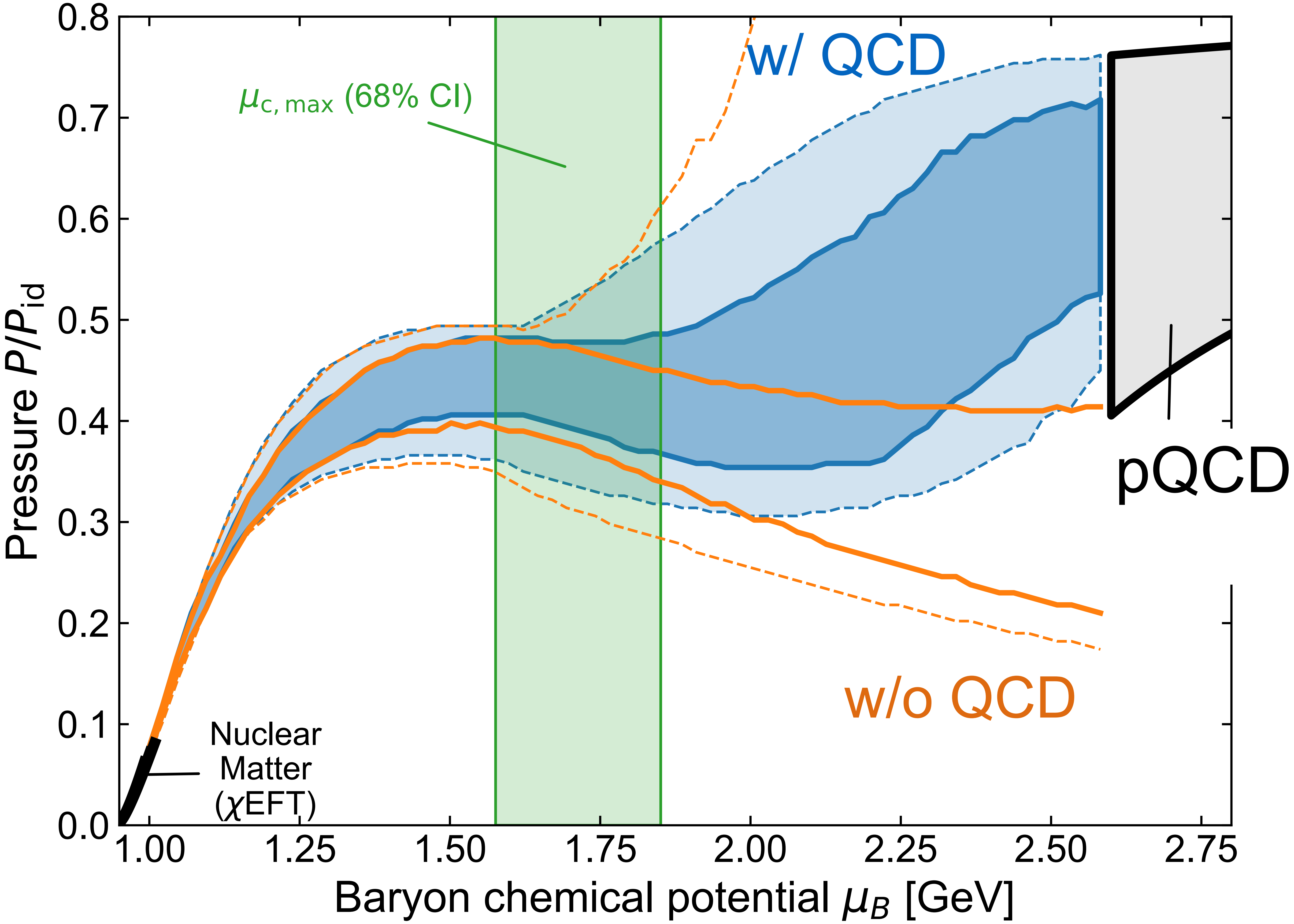
- Trace anomaly:
  related to the changes in the effective degrees of freedom ν

$$\varepsilon - 3P \propto \frac{d\nu}{d\ln\mu}$$

$$(\nu = P/P_{\mathrm{ideal}})$$

- It is natural to assume ν is increasing
  → positive $\varepsilon - 3P$

- **Indeed, positive $\varepsilon - 3P$ favored by QCD effect**



See also: Annala,Gorda,Hirvonen,Komoltsev,Kurkela,Nättilä,Vuorinen (2023)

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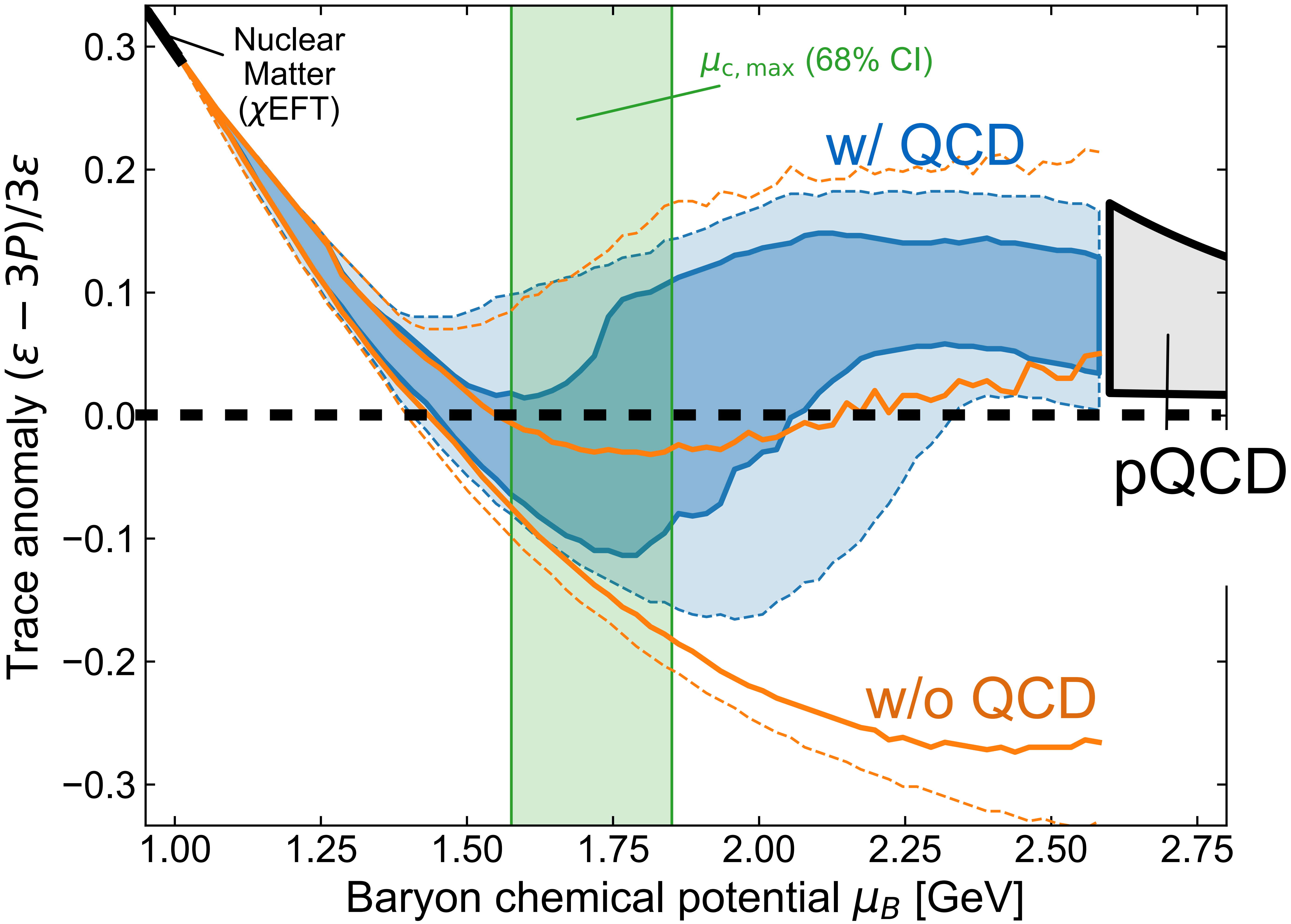


Trace anomaly and speed of sound

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

Speed of sound can be decomposed into Δ_{tr} and its derivative:

$$v_s^2 = \varepsilon \frac{d}{d\varepsilon}\left(\frac{P}{\varepsilon}\right) + \frac{P}{\varepsilon}$$

$$= \underbrace{\varepsilon \frac{d\Delta_{\mathrm{tr}}}{d\varepsilon}}_{\text{Derivative component}} + \underbrace{\left(\frac{1}{3} - \Delta_{\mathrm{tr}}\right)}_{\text{Non-derivative component}}$$

This explains why $\varepsilon - 3P = 0$ and $v_s^2 = \dfrac{1}{3}$ are the different conditions

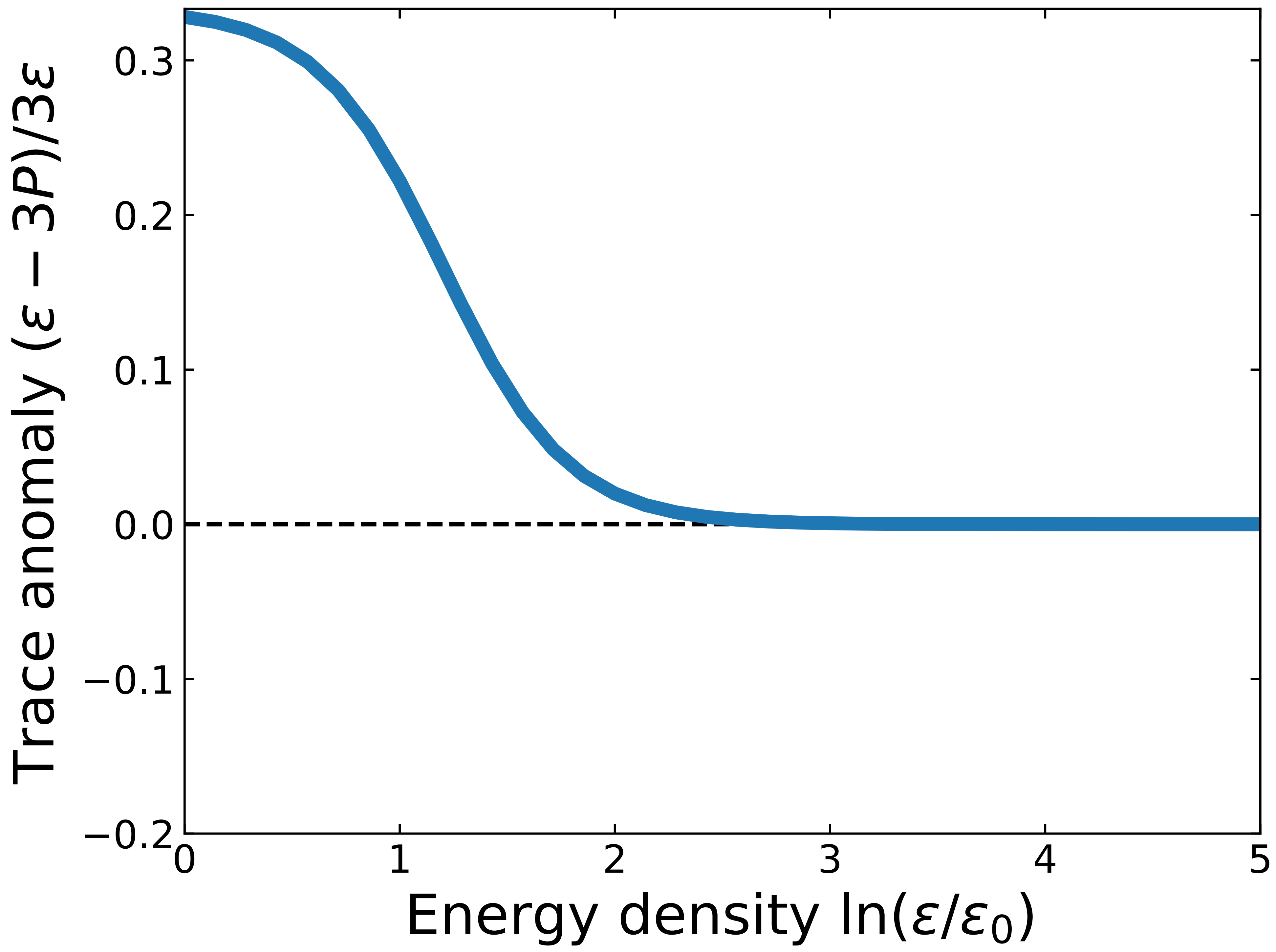
Trace anomaly and speed of sound

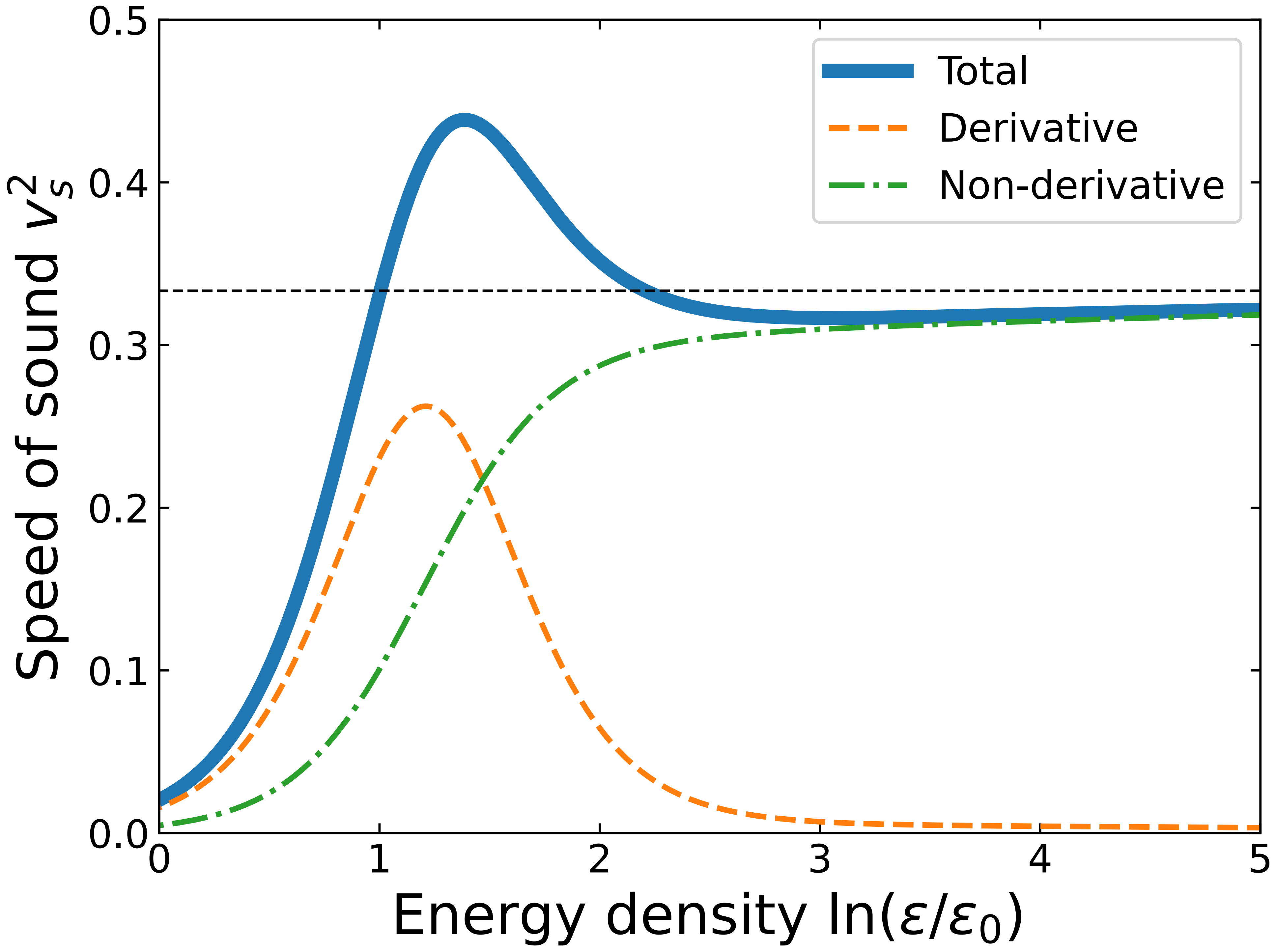
Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

Rapid approach to $\Delta_{\mathrm{tr}} \to 0$ naturally spikes v_s^2

Trace anomaly $\Delta_{\mathrm{tr}} = \dfrac{\varepsilon - 3P}{3\varepsilon}$

Sound velocity $v_s^2 = \underbrace{\varepsilon \dfrac{d\Delta_{\mathrm{tr}}}{d\varepsilon}}_{\text{Derivative}} + \underbrace{\left(\dfrac{1}{3} - \Delta_{\mathrm{tr}}\right)}_{\text{Non-derivative}}$



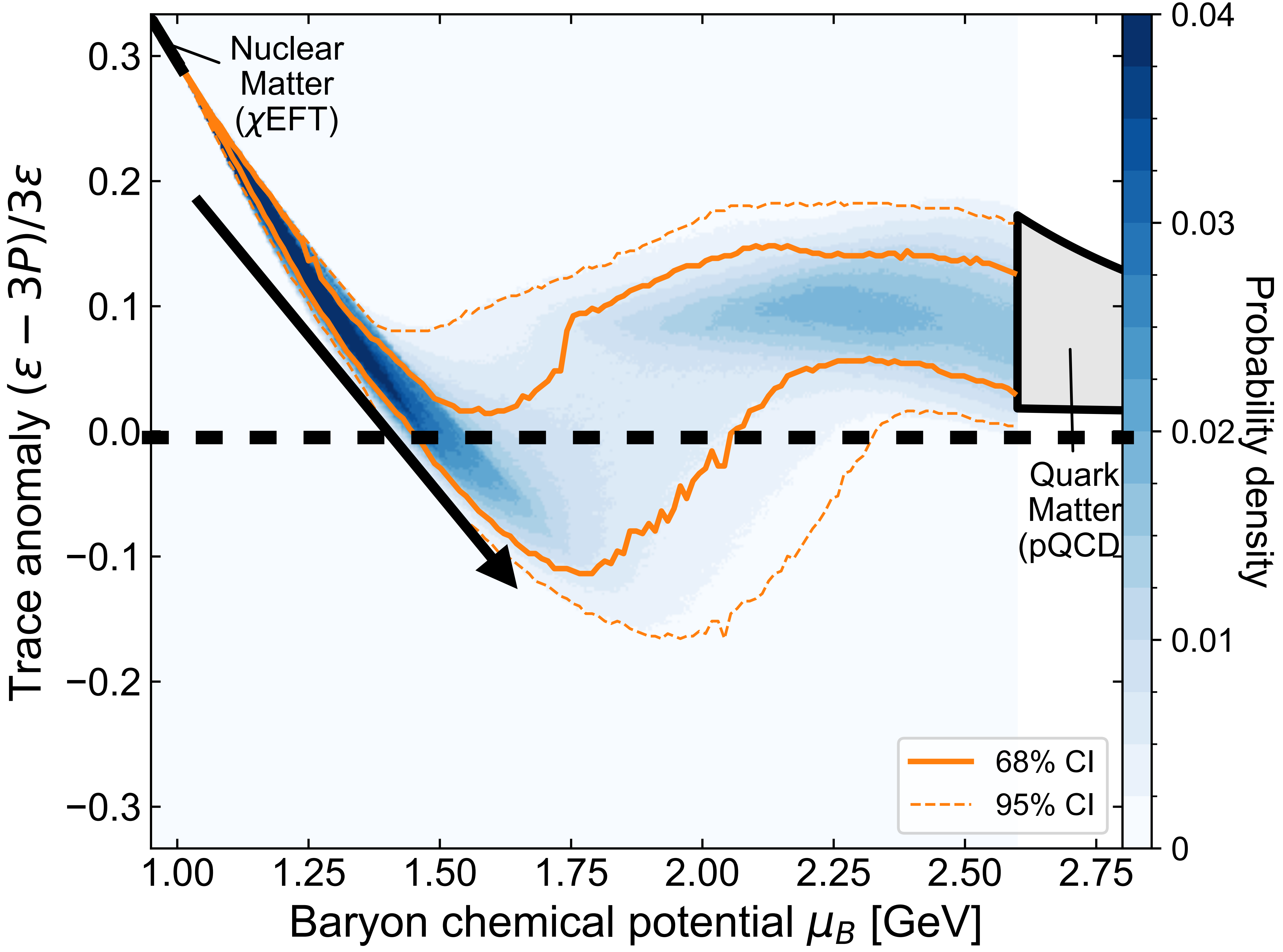


Trace anomaly and speed of sound

Fujimoto, Fukushima, McLerran, Praszalowicz, PRL129 (2022)

Normalized trace anomaly:

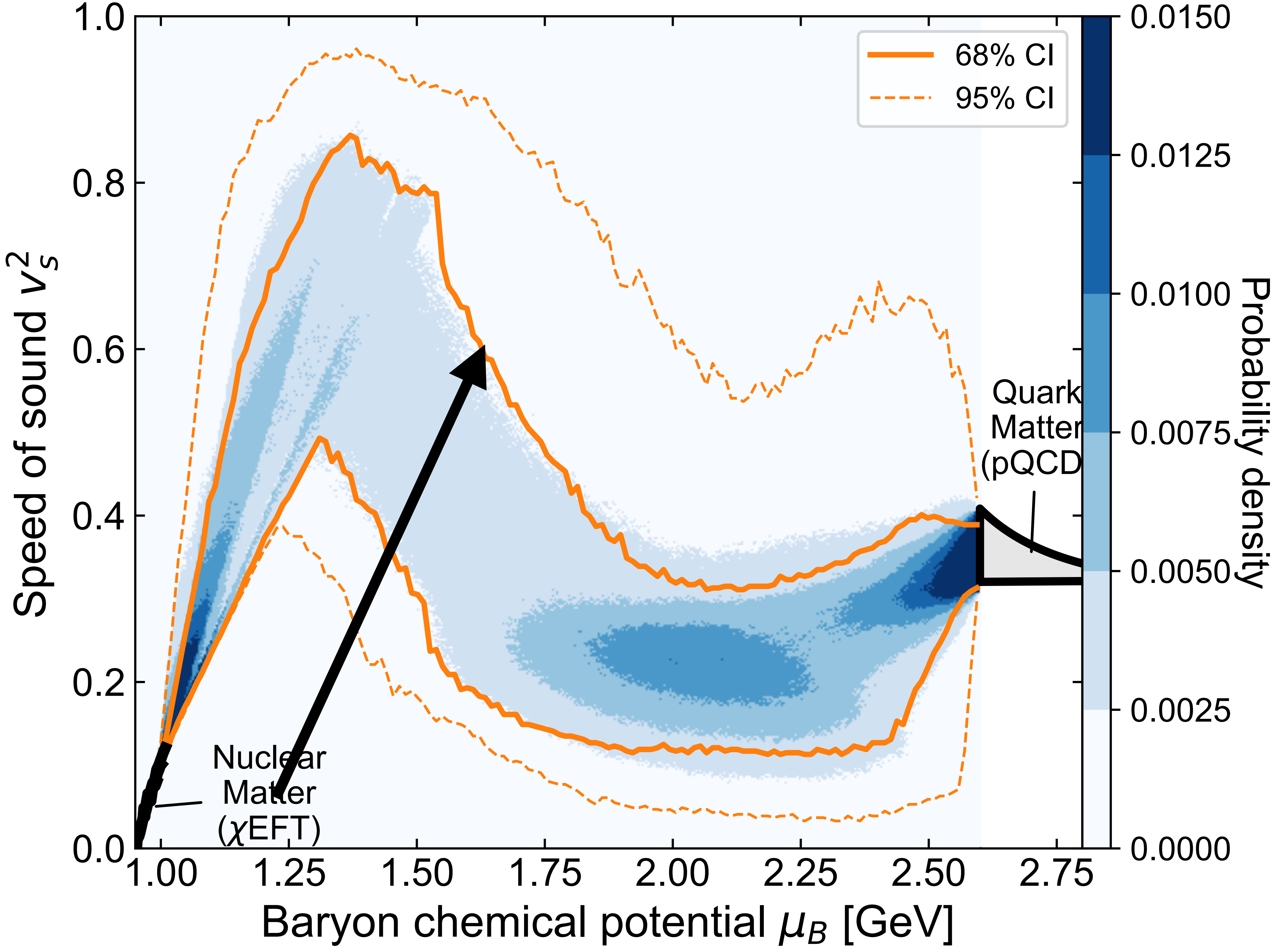
$$\Delta_{\rm tr} = (\varepsilon - 3P)/3\varepsilon$$



Sound speed:

$$v_s^2 = dP/d\varepsilon$$

see also: Bedaque,Steiner (2014);
Tews,Carlson,Gandolfi,Reddy (2018);
Fujimoto,Fukushima,Murase (2019)



Rapid approach to $\varepsilon - 3P \to 0$ drives the peak in v_s^2 in actual data

Part 2: Interplay between weak-coupling and lattice QCD at high density

Notation

- QCD_I: QCD at finite μ_I and zero μ_B
- QCD_B: QCD at finite μ_B and zero μ_I

Weak-coupling results in high-density QCD

Freedman,McLerran (1978); Kurkela et al. (2009-)

QCD EoS in weak-coupling α_s expansion:

$$P_{\mathrm{QCD}}(\mu) = \frac{3\mu^4}{4\pi^2}\left[1 - \mathcal{O}(\alpha_s)\right] + \frac{3\mu^2\Delta^2}{2\pi^2}\left[1 + \mathcal{O}(\alpha_s^{1/2})\right]$$

where

$$\ln\left(\frac{\Delta_{\mathrm{gap}}}{\mu}\right) = -b_{-1}\left(\frac{\alpha_s}{\pi}\right)^{-1/2} - b_0$$

Son (1999), Brown,Liu,Ren (1999); Wang,Rischke (2001)
Review: Alford,Rajagopal,Schafer,Schmitt (2008);

Applicability at low μ?

- Usually, it is used down to $\mu \sim 0.9$ GeV for the input of neutron stars

Kurkela,Fraga,Vuorinen (2014)

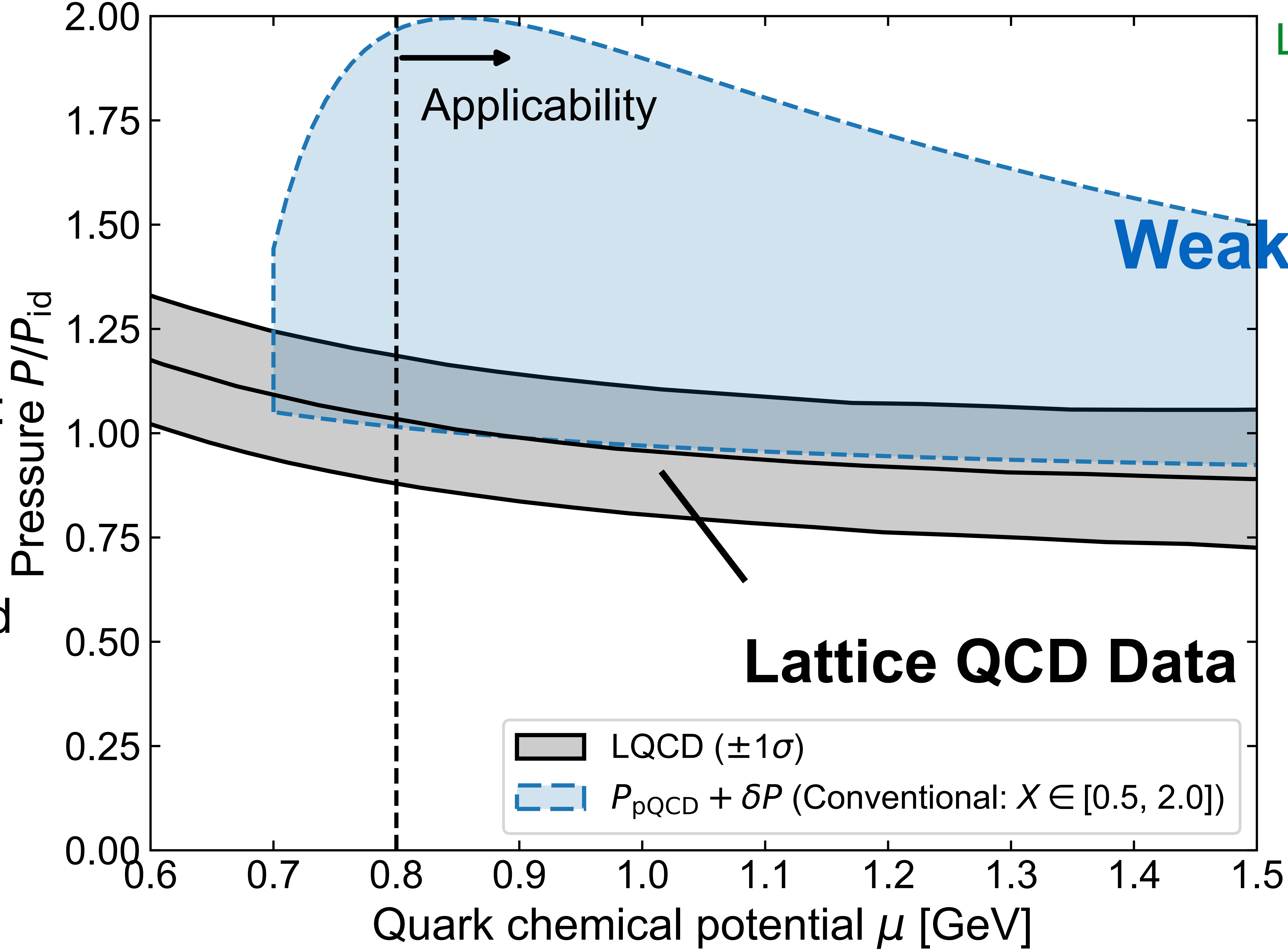
Weak-coupling formula is **universal** for QCD$_B$ and QCD$_I$ up to $\mathcal{O}(\alpha_s^2)$

→ Lattice QCD$_I$ can be used as a benchmark

Weak-coupling results vs lattice QCD$_I$ data

Lattice data: Abbott et al. (2023, 24);
Fujimoto (2023, 24)

Uncertainty in weak-coupling results: varying the renormalization scale $\bar{\Lambda}$ by a factor 2 around its typical scale $\bar{\Lambda} = 2\mu$



Empirical evidence for the dense-QCD weak-coupling results to be applicable down to $\mu \sim 0.8\ \mathrm{GeV}$

At least the magnitude is correct

"Uncertainty" in pQCD

Fraga,Pisarski,Schaffner-Bielich(2001);
Kurkela,Romatschke,Vuorinen (2009)

$$\alpha_s \simeq \frac{1}{\beta_0 \ln\left(\frac{\bar{\Lambda}}{\Lambda_{\overline{\rm MS}}}\right)}$$

- **$\bar{\Lambda}$: renormalization scale**

  … only ambiguity in pQCD from perturbative series truncation

- Canonical choice: $\bar{\Lambda} = 2\mu$ (typical hard interaction scale)

- "Uncertainty" quantified by varying by factor 2

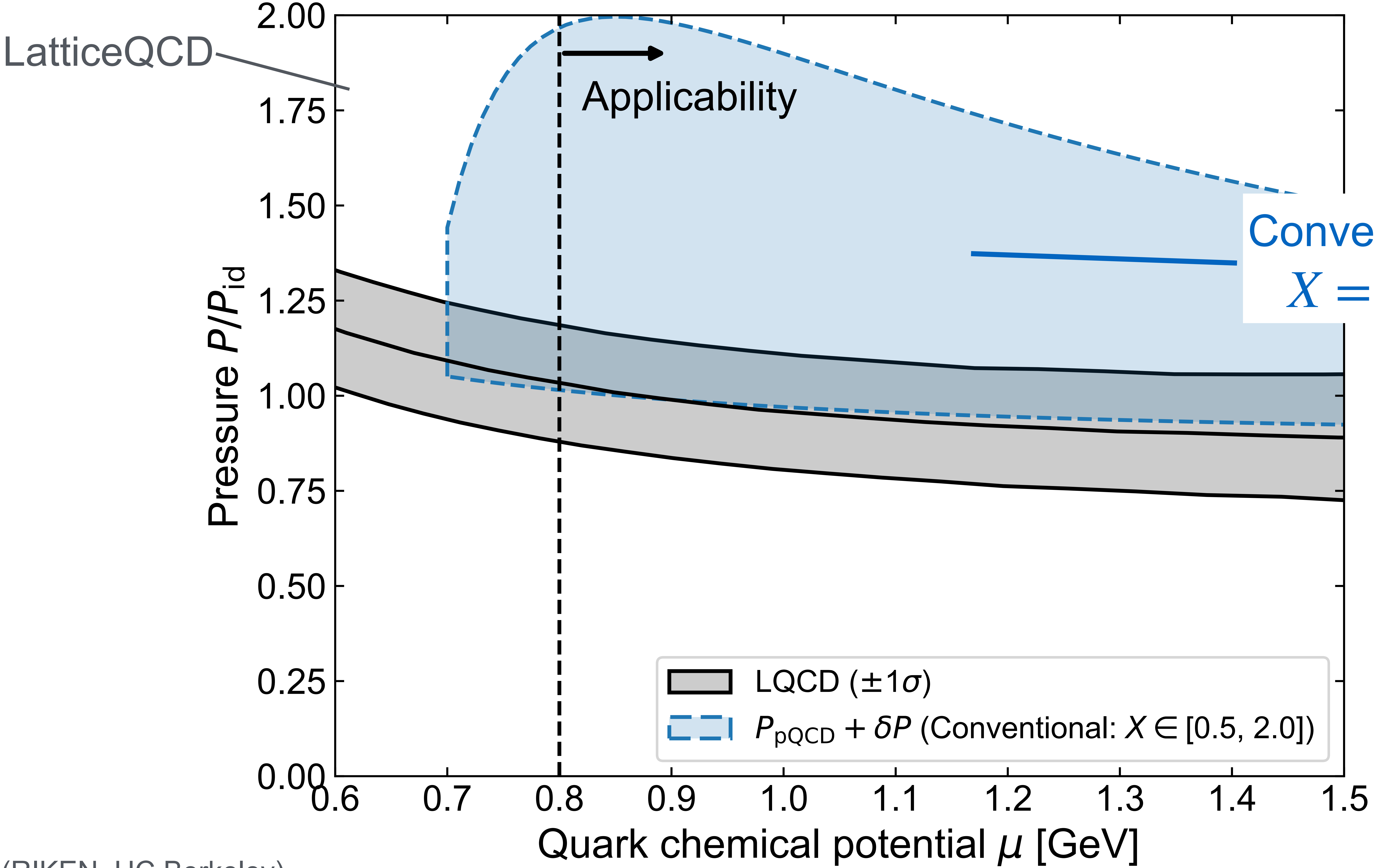
  i.e. $X \in [0.5,\ 2]$ with $X \equiv \bar{\Lambda}/(2\mu)$

  … ad hoc procedure, purely based on historical practice

cf. Gorda,Komoltsev,Kurkela,Mazeliauskas (2022)

Prescription for $\bar{\Lambda}$ determination

Fujimoto, 2408.12514



Conventional choice for $\bar{\Lambda}$

$$X = \bar{\Lambda}/2\mu \in [0.5, 2]$$

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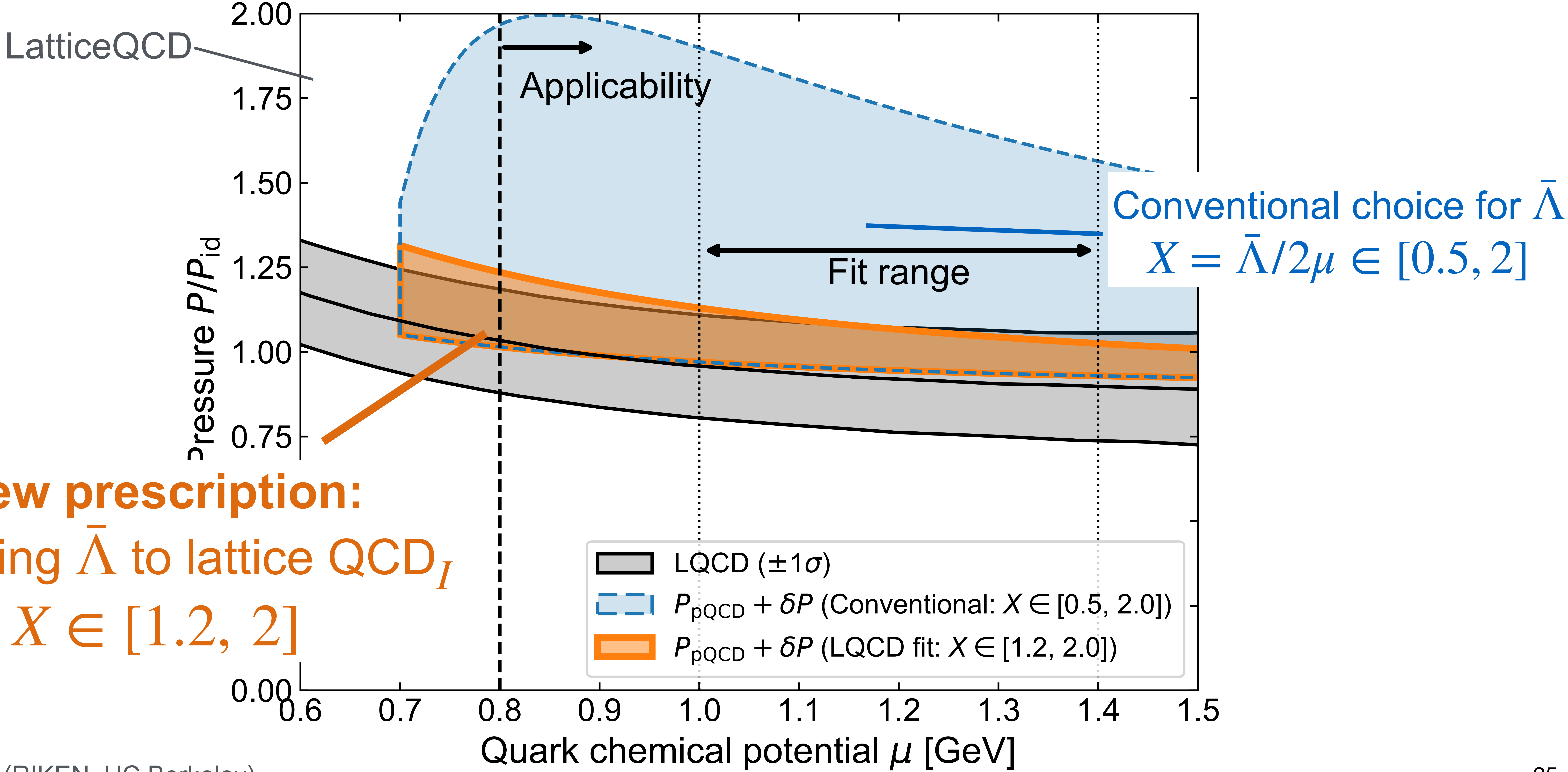

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$X = \bar{\Lambda}/2\mu \in [0.5, 2]$

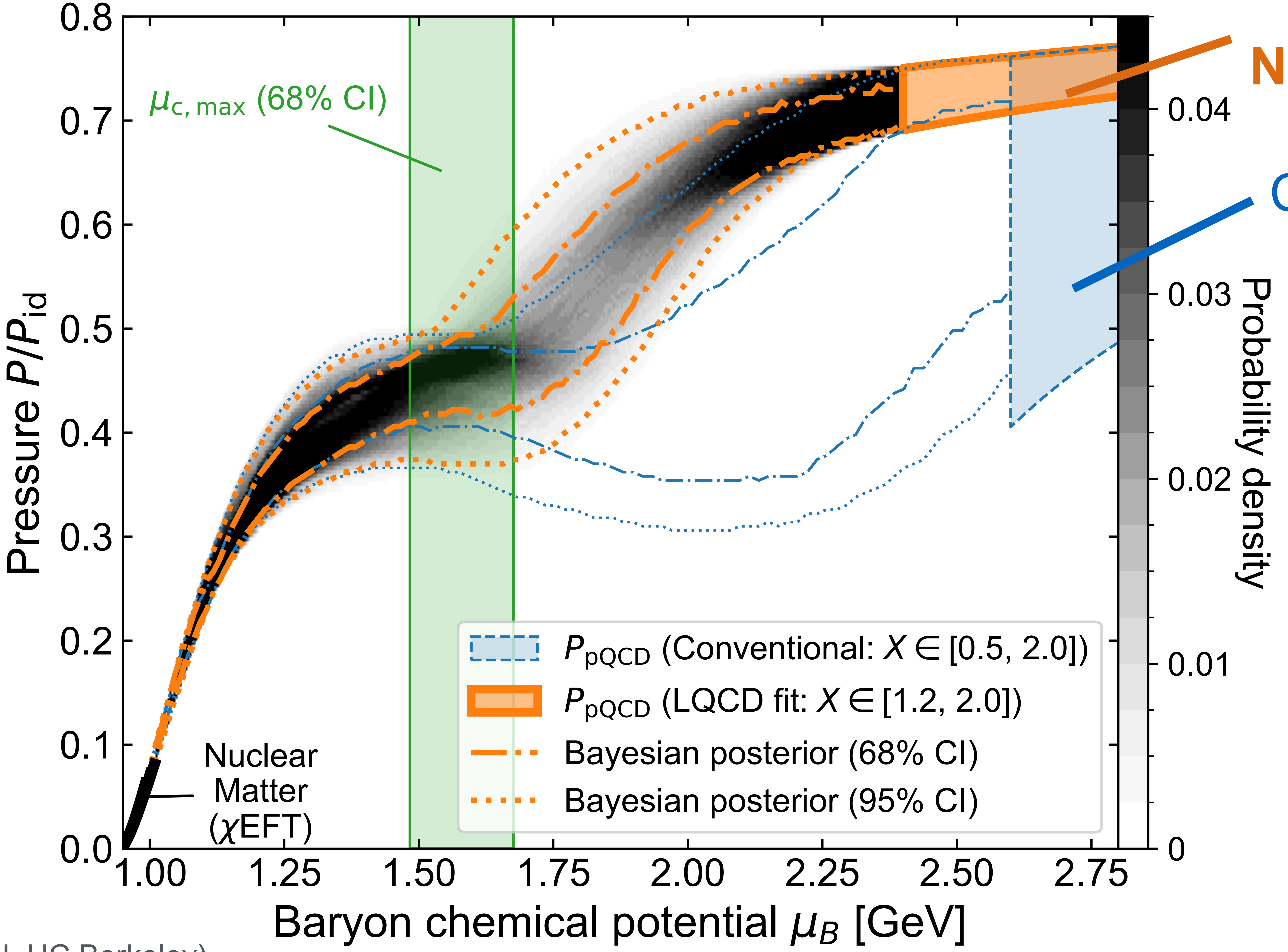
New prescription:

Matching $\bar{\Lambda}$ to lattice QCD_I

$X \in [1.2,\, 2]$

Effect on QCD$_B$: NS phenomenology

Fujimoto, 2408.12514



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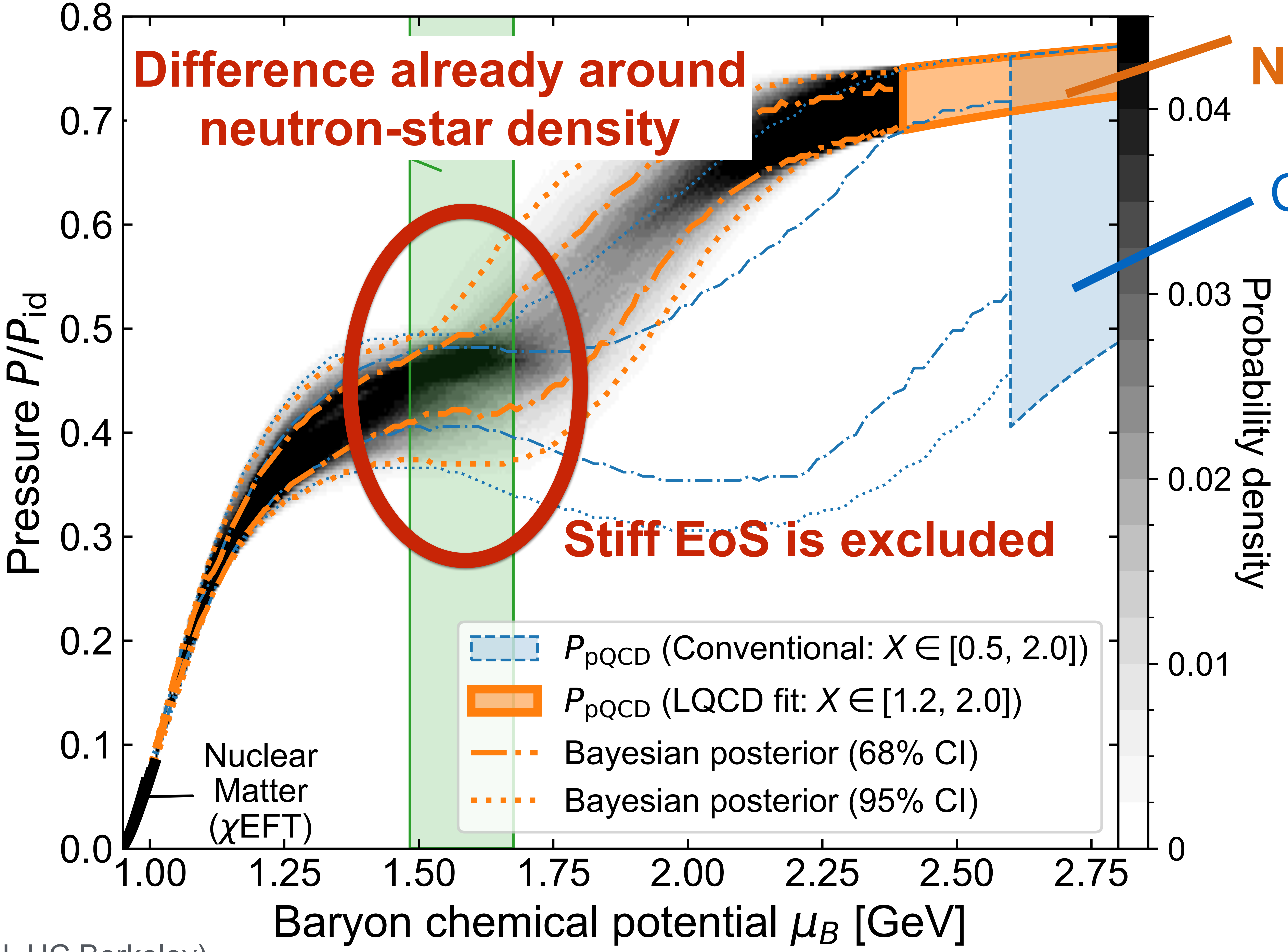


New choice

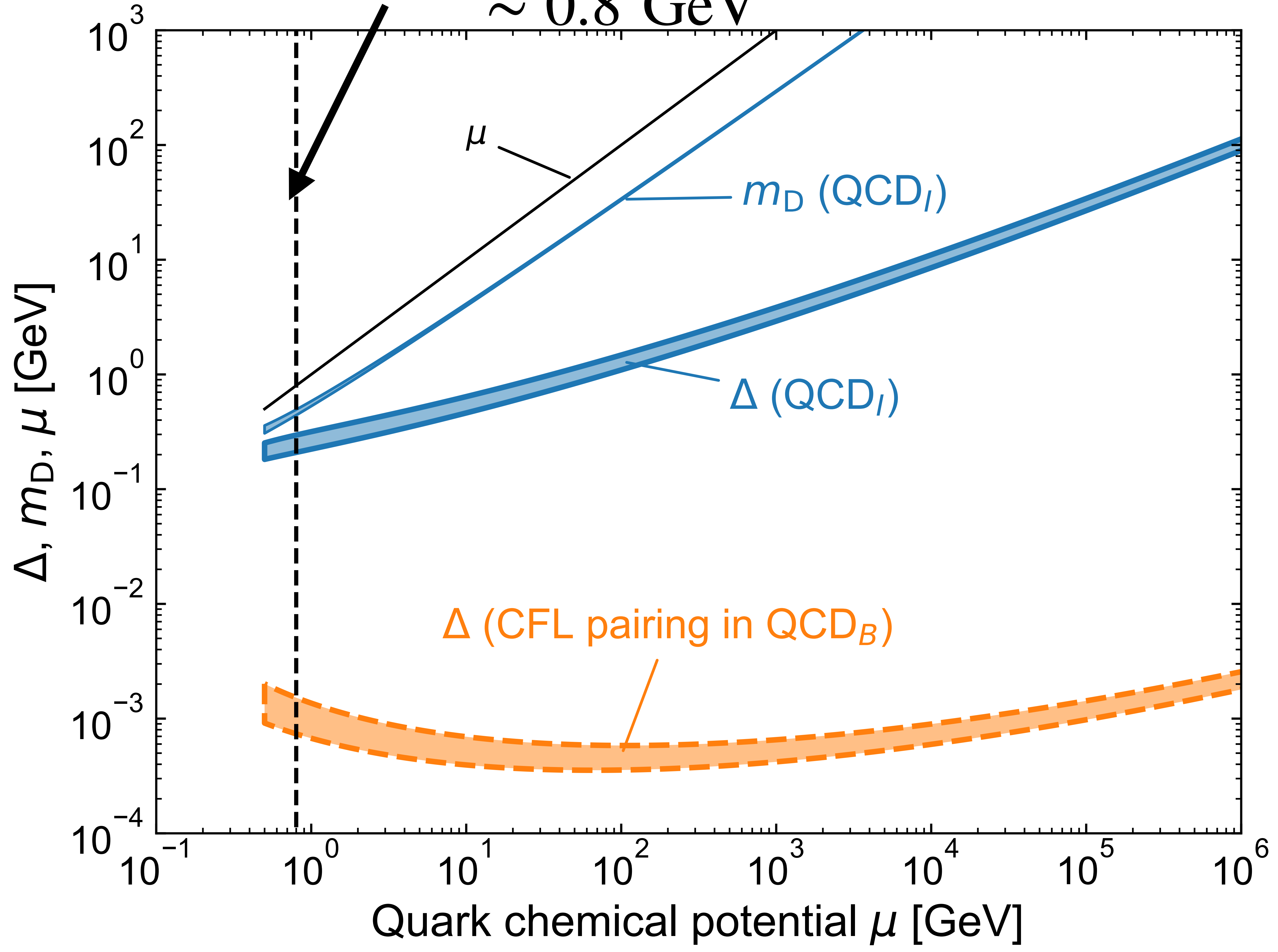
Conventional choice

Color superconductivity in weak coupling

Fujimoto, 2408.12514

Plot of weak-coupling Cooper pairing gap:

Lower limit of applicability implied from lattice
$\sim 0.8\ \mathrm{GeV}$



$\Delta_{\rm CFL} \sim 1\ \mathrm{MeV}$ at $\mu = 0.8\ \mathrm{GeV}$

cf. Kurkela,Rajagopal,Steinhorst (2024)

- A negligibly **small** contribution to bulk thermodynamics in weak-coupling

$$P_{\rm QCD}(\mu) = \frac{3\mu^4}{4\pi^2}\left[1 - \mathcal{O}(\alpha_s)\right]$$

$$\xcancel{+\frac{3\mu^2\Delta^2}{2\pi^2}\left[1 + \mathcal{O}(\alpha_s)\right]}$$

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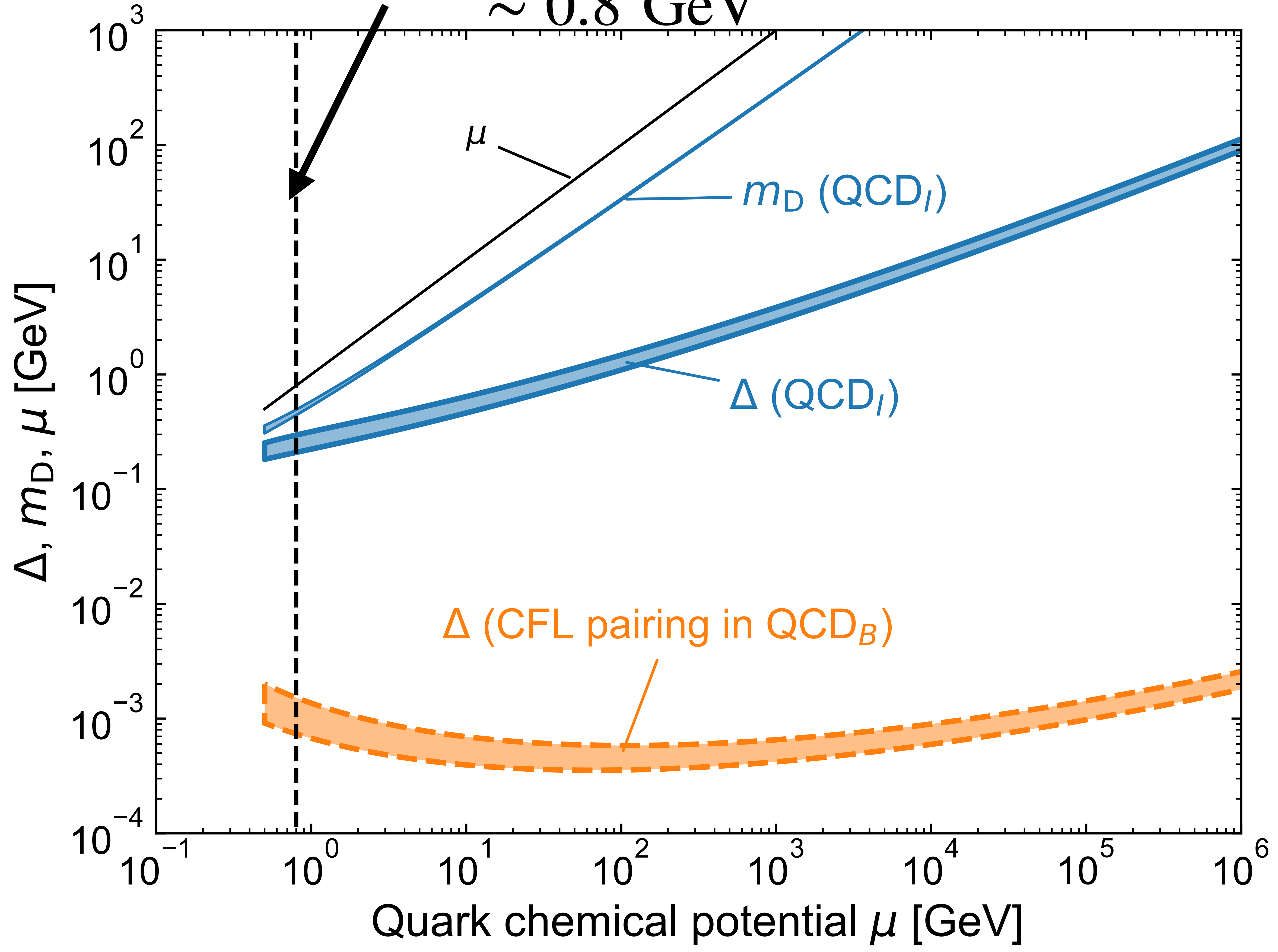


Lower limit of applicability implied from lattice
~ 0.8 GeV

$$\Delta_\mathrm{CFL} \sim 1\ \mathrm{MeV} \text{ at } \mu = 0.8\ \mathrm{GeV}$$

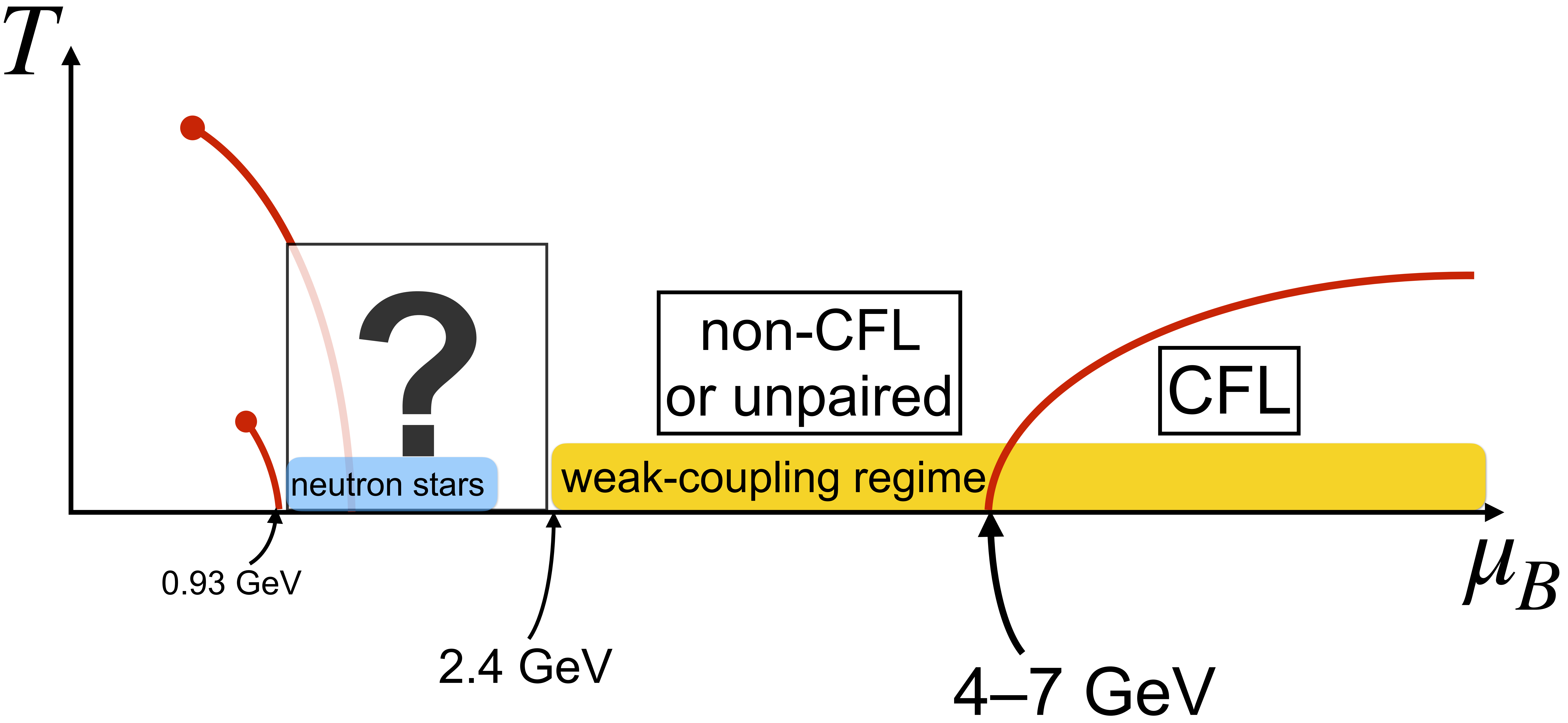
- Comparable to the stress by strange quark mass: $\Delta_\mathrm{CFL} \sim m_s^2/4\mu$

→ **CFL may not be the ground state even at $\mu_B = 2.4$ GeV**

NB: CFL and color superconductor may still be realized in NSs due to the nonperturbative enhancement from instantons

Alford,Rajagopal,Wilczek (1997);
Rapp,Schafer,Shuryak,Velkovsky (1997)

Effect on the QCD phase diagram



Summary

- **Trace anomaly:** useful measure for conformal nature of matter.

- The behavior of the trace anomaly may signify that the conformal matter may be realized in the core of neutron stars (possibly quark matter?)

- **QCD at finite isospin density:** useful nonperturbative piece of information

- **Weak-coupling results:** Matches with lattice data at finite isospin density
  - Empirical evidence for the validity down to $\mu \sim 0.8\ \mathrm{GeV}$.
  - Color-superconducting gap is negligible in the weak coupling limit.
  - CFL phase may be unstable against unpairing induced by the stress from strange quark mass