Emulating neutron stars with dipolar supersolids

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Outline

- Dipolar supersolids
- Supersolids vs compact stars
- Emulating glitches
- Conclusions

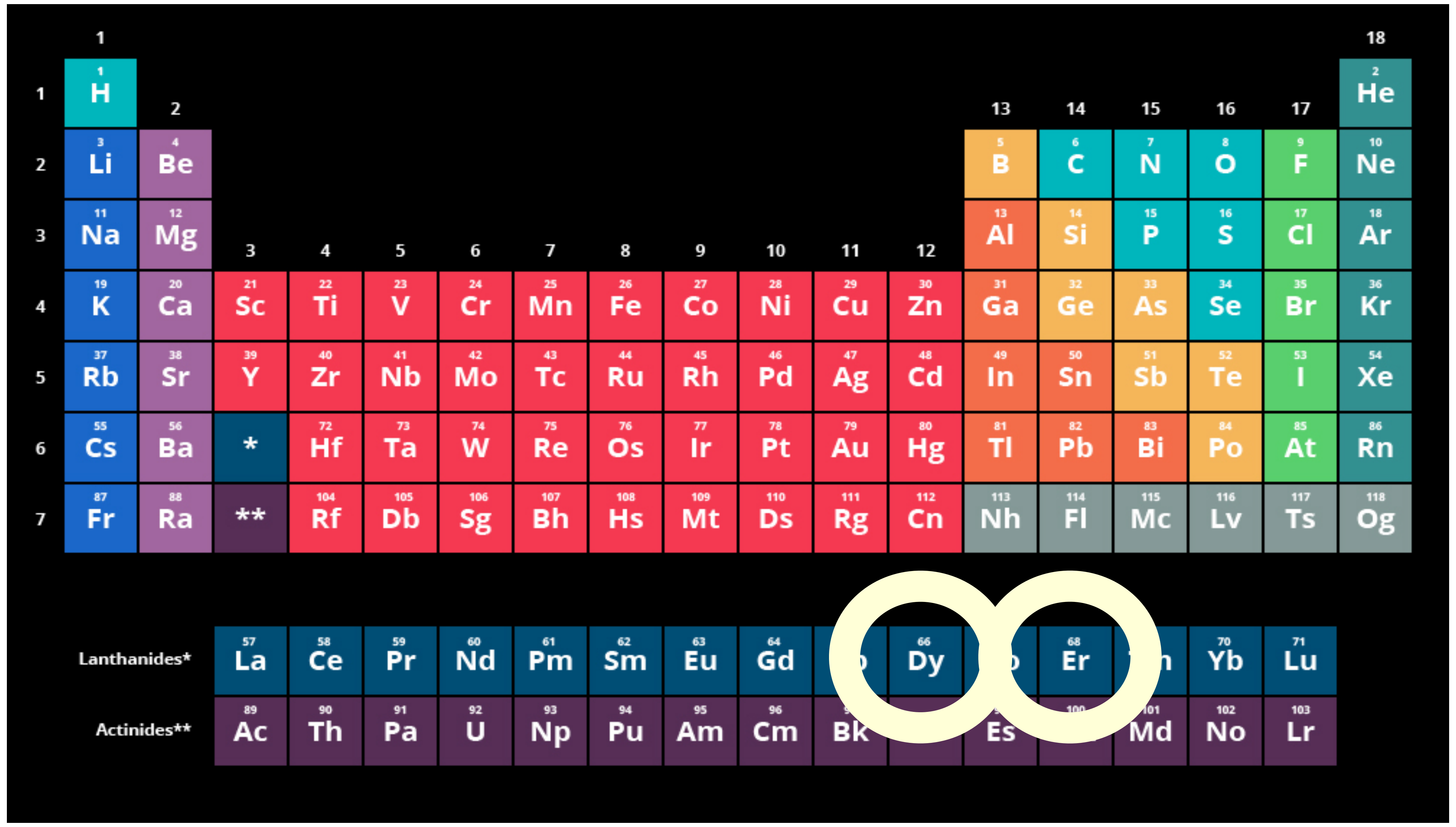
Dipolar supersolids

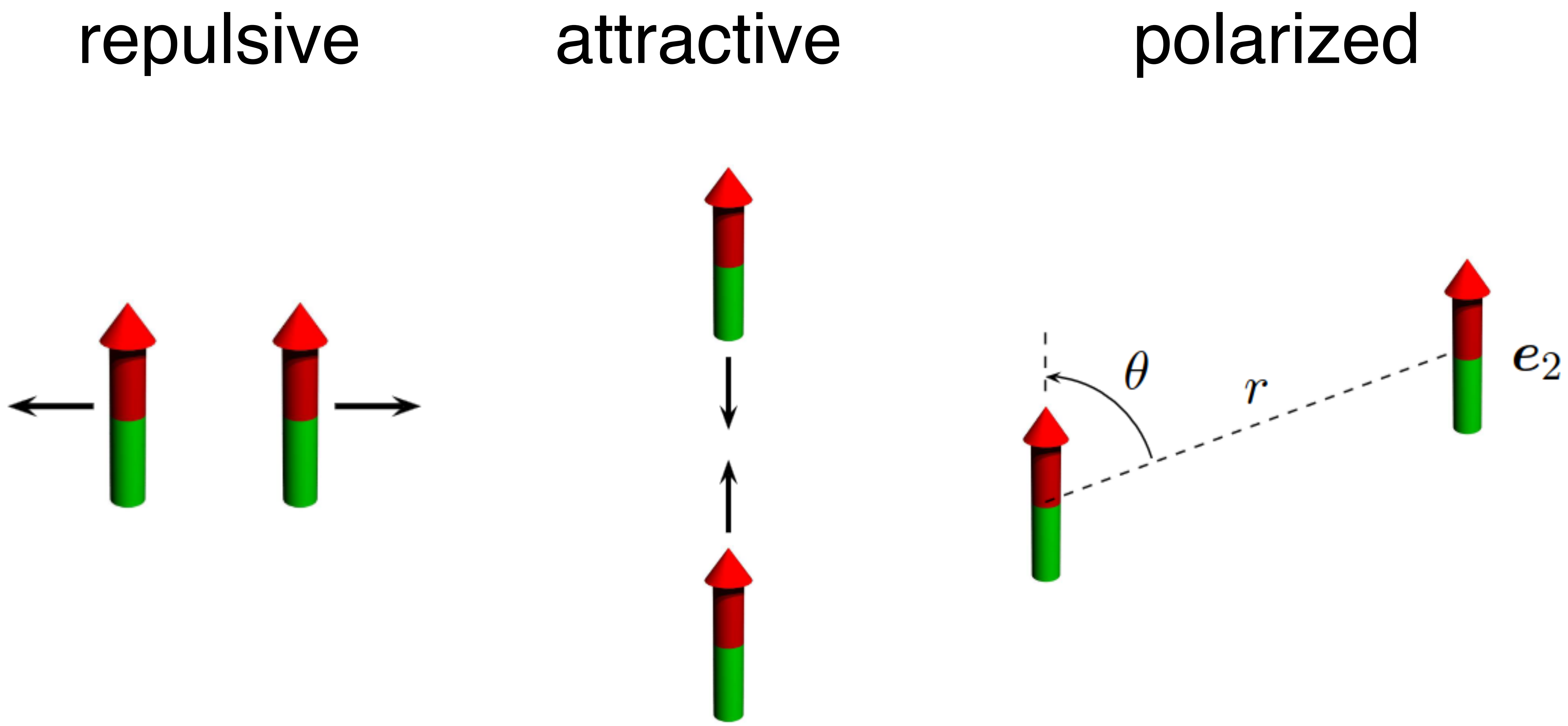
Superfluid **Rigid**

Review: M. Boninsegni and N. V. Prokof'ev, *Rev. Mod. Phys. 84, 759 (2012)*

- Currently realized with ultracold atoms in an optical trap
- Tool to emulate inhomogeneous hadronic matter in extreme conditions

Ultracold dipolar atoms





Long-range dipolar interaction

$$U_{\rm dd}(\boldsymbol{r}) = \frac{3\hbar^2 a_{\rm dd}}{m}\frac{1-3\cos^2\theta}{r^3}$$

$$a_{dd} = \frac{\mu_0\,\mu_m^2\,m}{12\pi\hbar^2}$$

^{165}Dy, $a_{dd} \simeq 131 a_0$

Short-range repulsion (Feshbach resonance)

$$U_c(\mathbf{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\mathbf{r})$$

a_s tunable

Numerical simulation

Evolution of the macroscopic wavefunction

Dissipation

Angular rotation of the trap

$$i\hbar\frac{\partial\Psi}{\partial t} = (1 - i\gamma)\left[\mathscr{H}[\Psi; a_s, a_{\rm dd}, \omega] - \Omega(t)\hat{L}_z\right]\Psi$$

Hamiltonian

$$\mathscr{H}[\Psi; a_s, a_{\rm dd}, \boldsymbol{\omega}] = -\frac{\hbar^2\nabla^2}{2m} + \frac{1}{2}m\left[\omega_r^2(x^2 + y^2) + \omega_z^2 z^2\right] + \int {\rm d}^3\mathbf{r}'\, U(\mathbf{r} - \mathbf{r}')\left|\Psi(\mathbf{r}', t)\right|^2 + \gamma_{\rm QF}|\Psi(\mathbf{r}, t)|^3 - \mu$$

Trapping potential (pancake-like)

Self-interaction

LHY correction

Relevant parameters

Relative interaction strength $\epsilon_{dd} = \frac{a_{dd}}{a_s}$ — a_{dd}: fixed; a_s: tunable

Number density $n = \frac{N}{V}$ tunable

Ultracold dipolar atoms:	Number density and interaction strengths can be separately tuned
Nuclear matter:	The interaction strengths are given functions of density

Tuning the relative interaction strength

The Fano-Feshbach resonance allows to change a_s and thus ϵ_{dd}

BEC	Supersolid or Superglass	Crystal or glass
Contact interaction dominates **Homogeneous Superfluid**	Competition between interactions **Inhomogeneous Superfluid**	Dipolar interaction dominates **Solid**
$\epsilon_{dd} \ll 1$	$\epsilon_{dd} \sim 1$	$\epsilon_{dd} \gg 1$

Density matters!

Energy density scaling

kinetic
(single particle)

$\mathscr{E}_{sp} \propto n$

Interaction
(mean field)

$\mathscr{E}_{contact} \propto n^2$ $\mathscr{E}_{dipolar} \propto U_{dd} n^2$

Quantum fluctuations

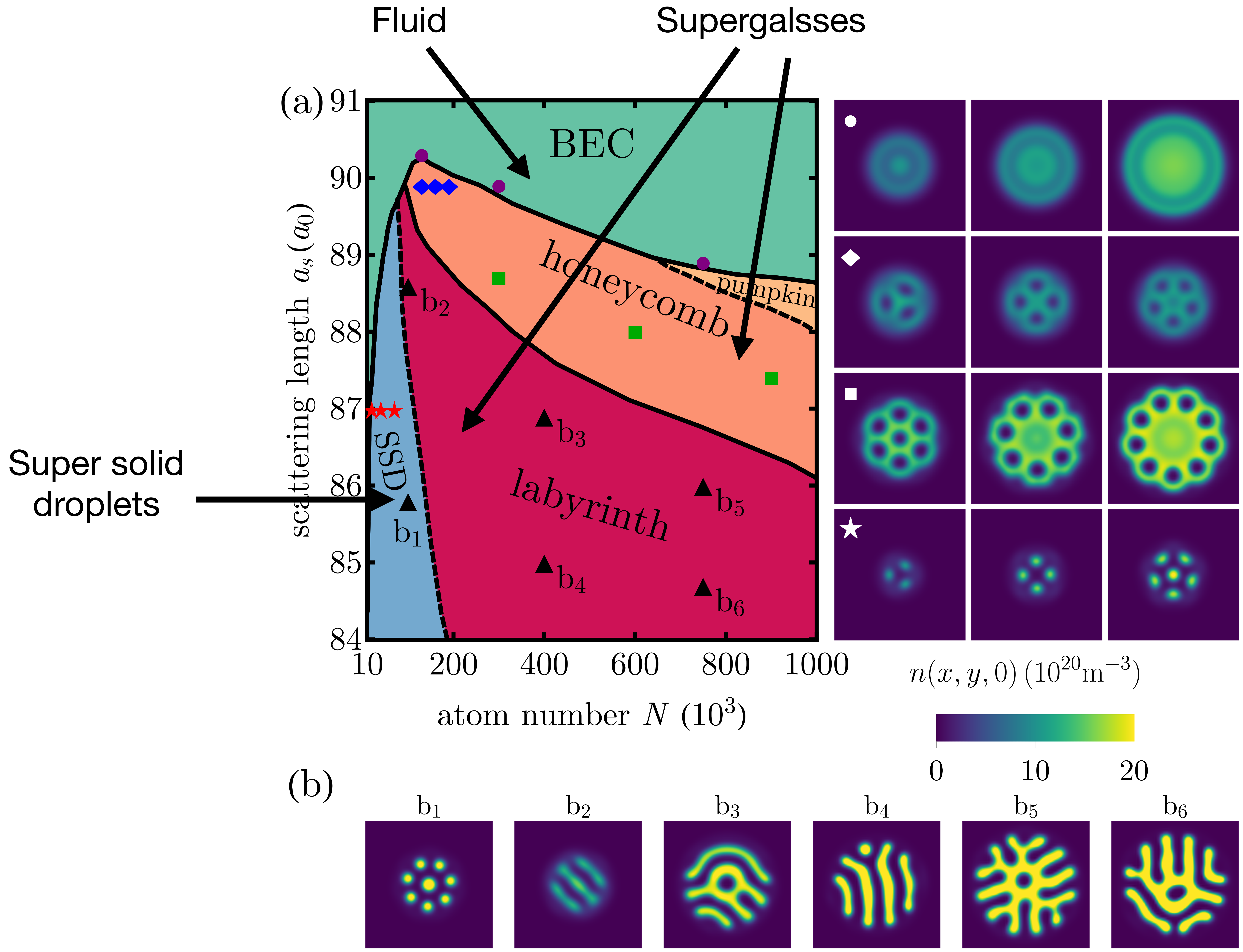
$\mathscr{E}_{LHY} \propto n^{5/2}$

Repulsive

Long-range inhomogeneous

Repulsive "channel" changes with density

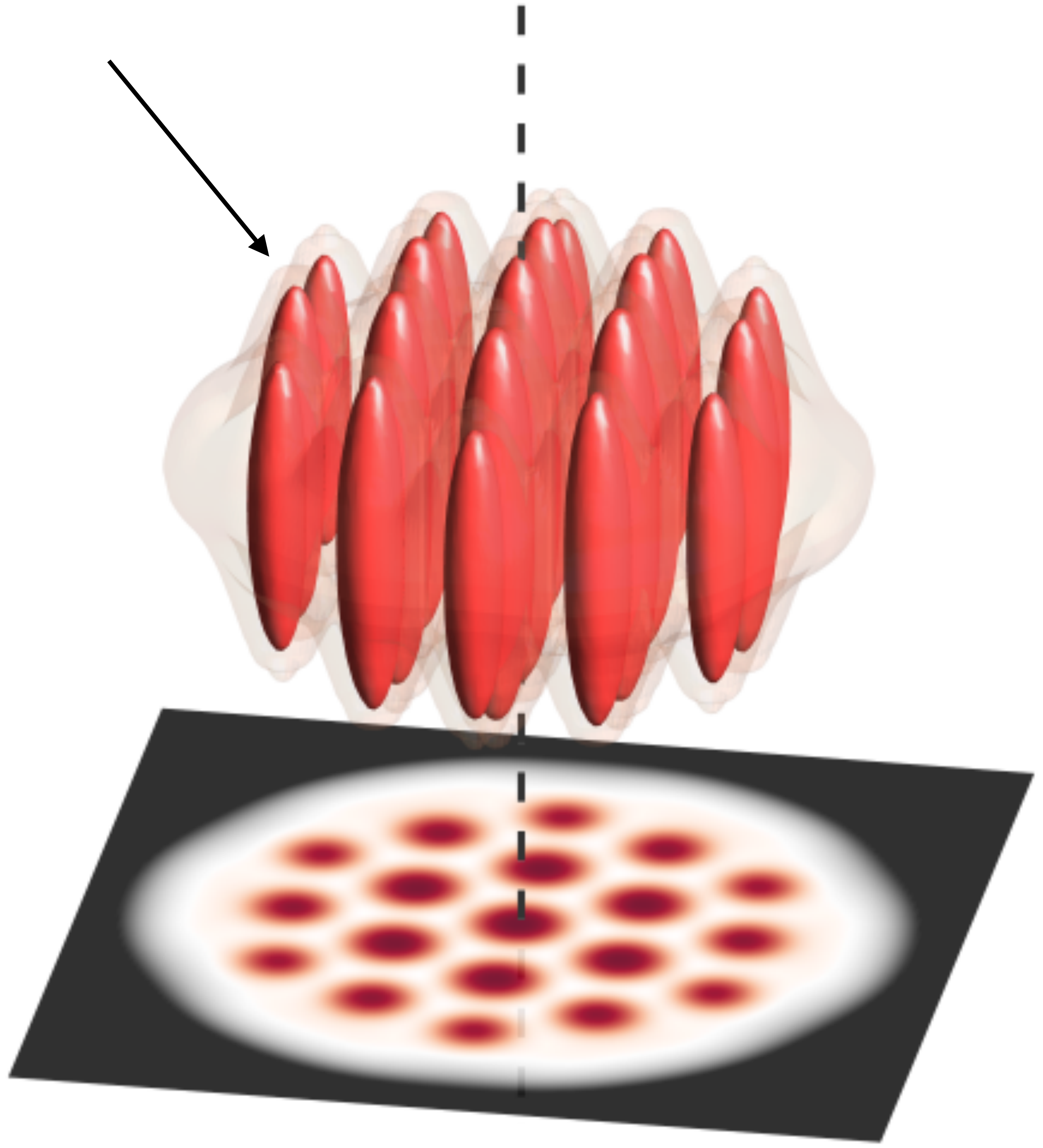
Phase diagram



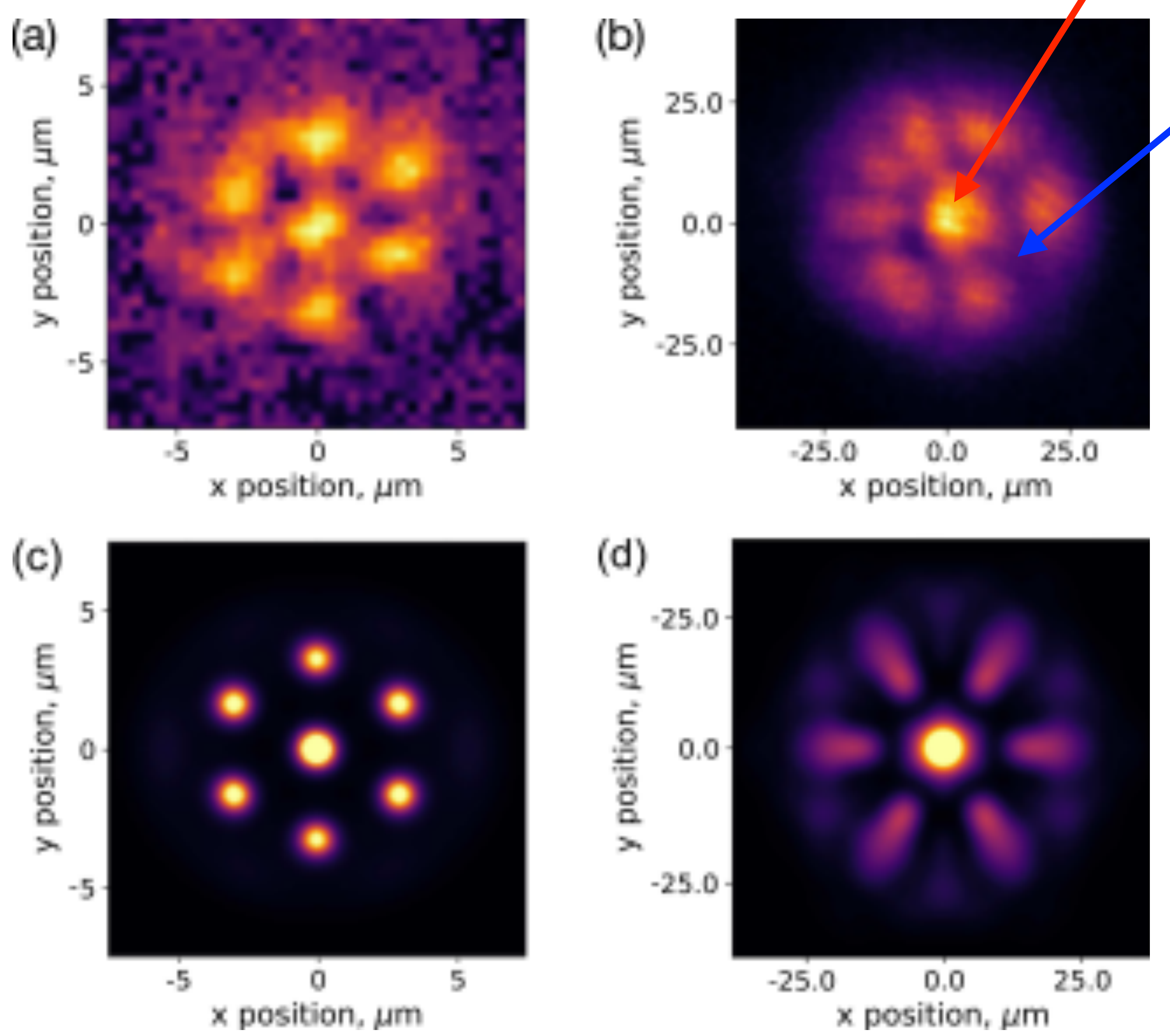
J. Hertkorn et al.,
Phys. Rev. Research 3, 033125 (2021

Supersolid droplets

Observations of supersolids@ MIT, Pisa/LENS, Stuttgart, Innsrbuck



$N = 3 \times 10^5$



Bland *et al* *PRL 128, 195302 (2022)*

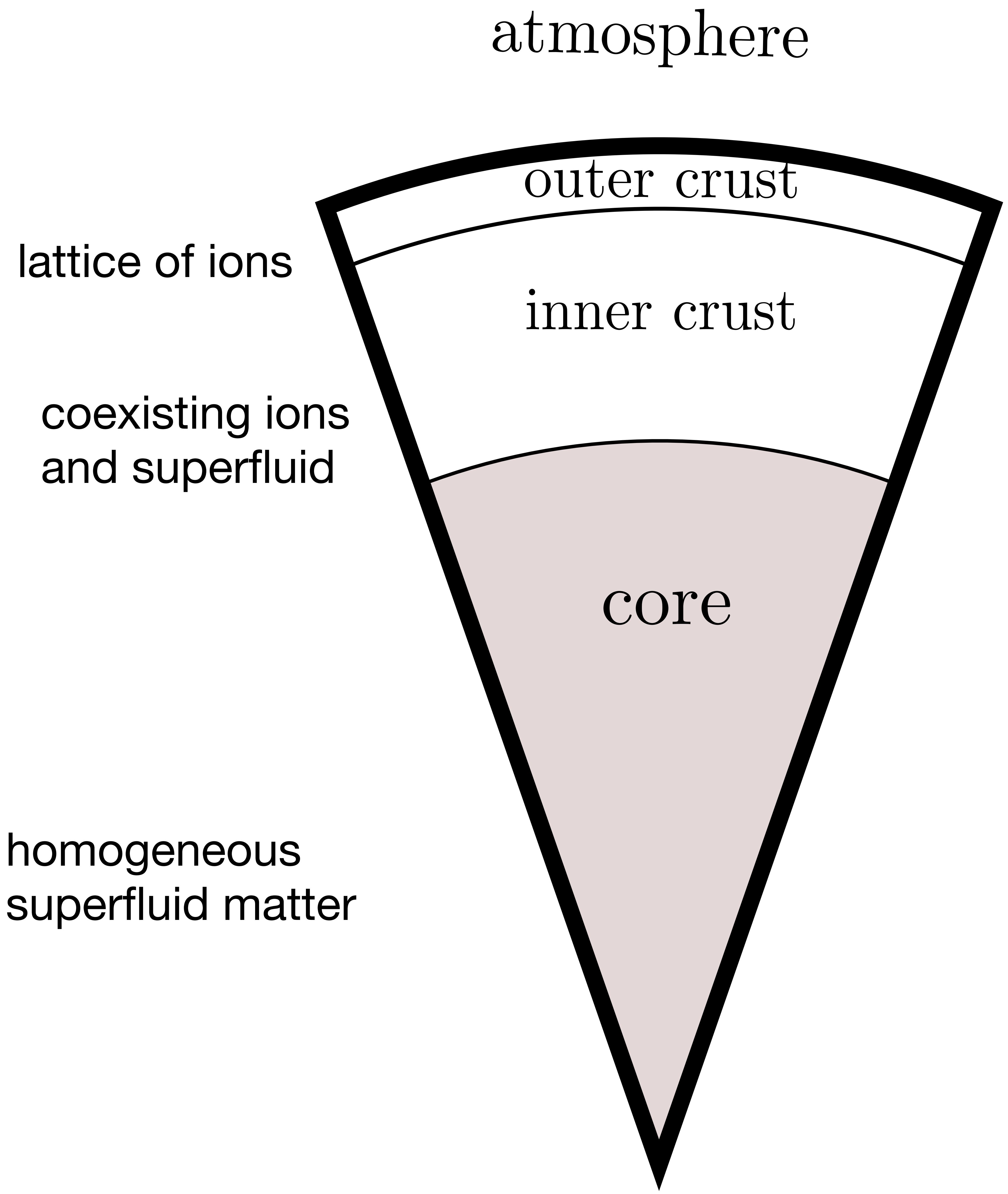
Compact stars vs Supersolids

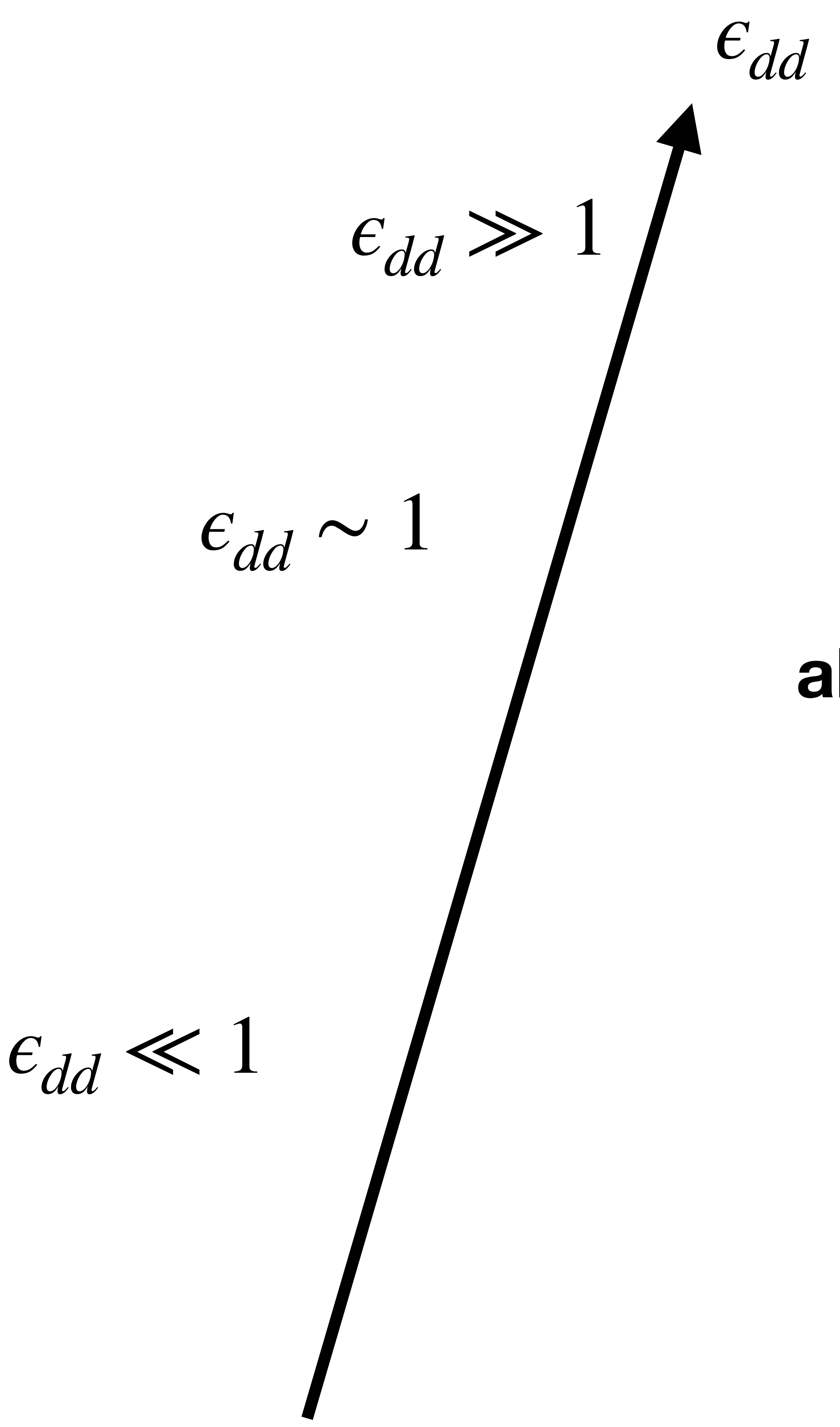
Nuclear matter	*Supersolids*
Fermions	**Bosons** **(fermions can in principle be used)**
Long range attraction **Short range repulsion**	**Long range attraction** **Short range repulsion**
Scalar, vector and tensor forces	**Dipole-dipole + s-wave scattering**
High density $\rho \sim 10^{14}$**g/cm**^{3}	**Diluted** $\rho \sim 10^{-5}$**g/cm**^{3}
Density and interactions given by nature	**Density and interactions tunable**

Neutron star vs dipolar superfluids

$$\epsilon_{dd} = \frac{a_{dd}}{a_s}$$

dipolar superfluids

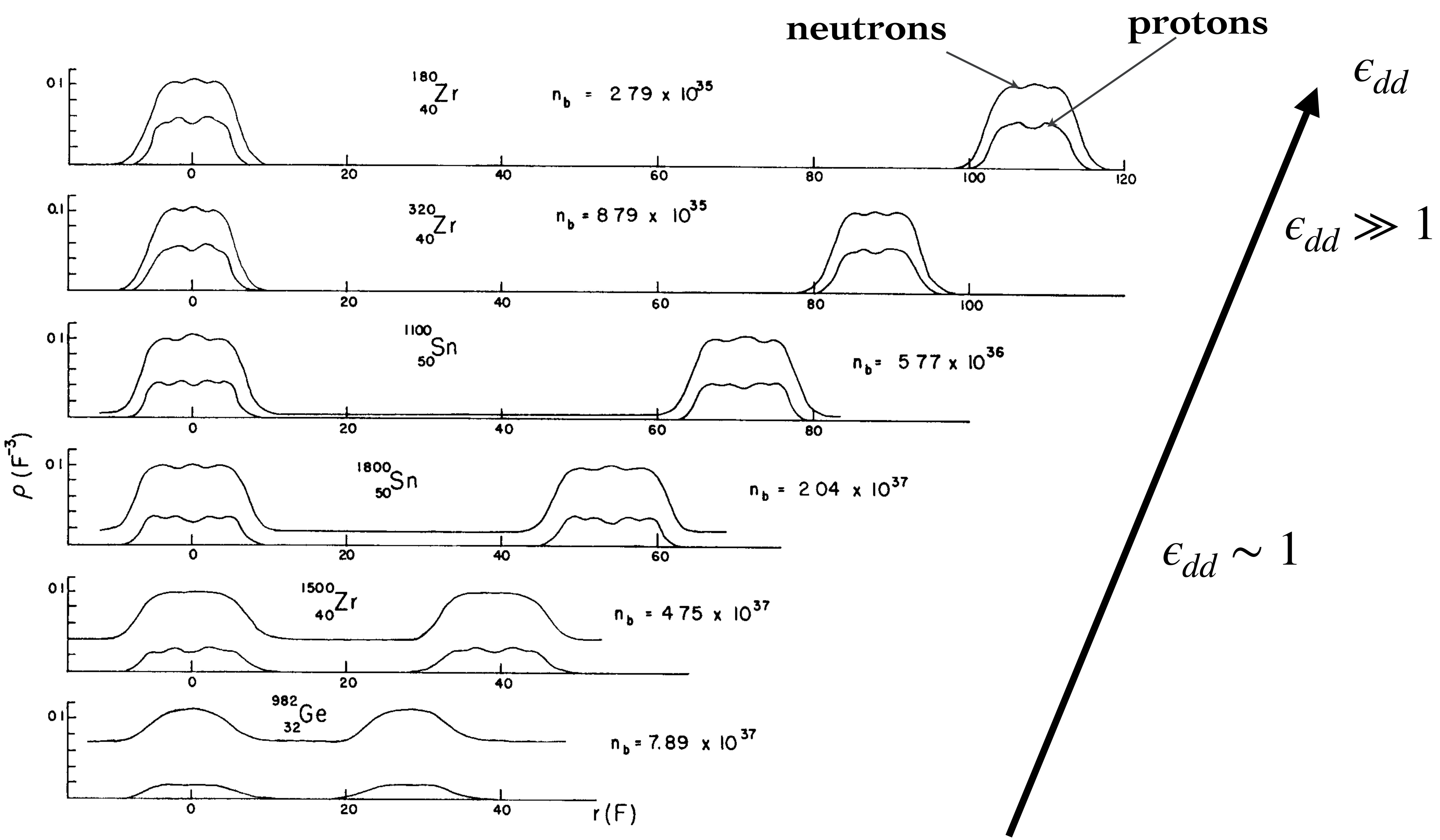




In principle
all layers could be emulated !

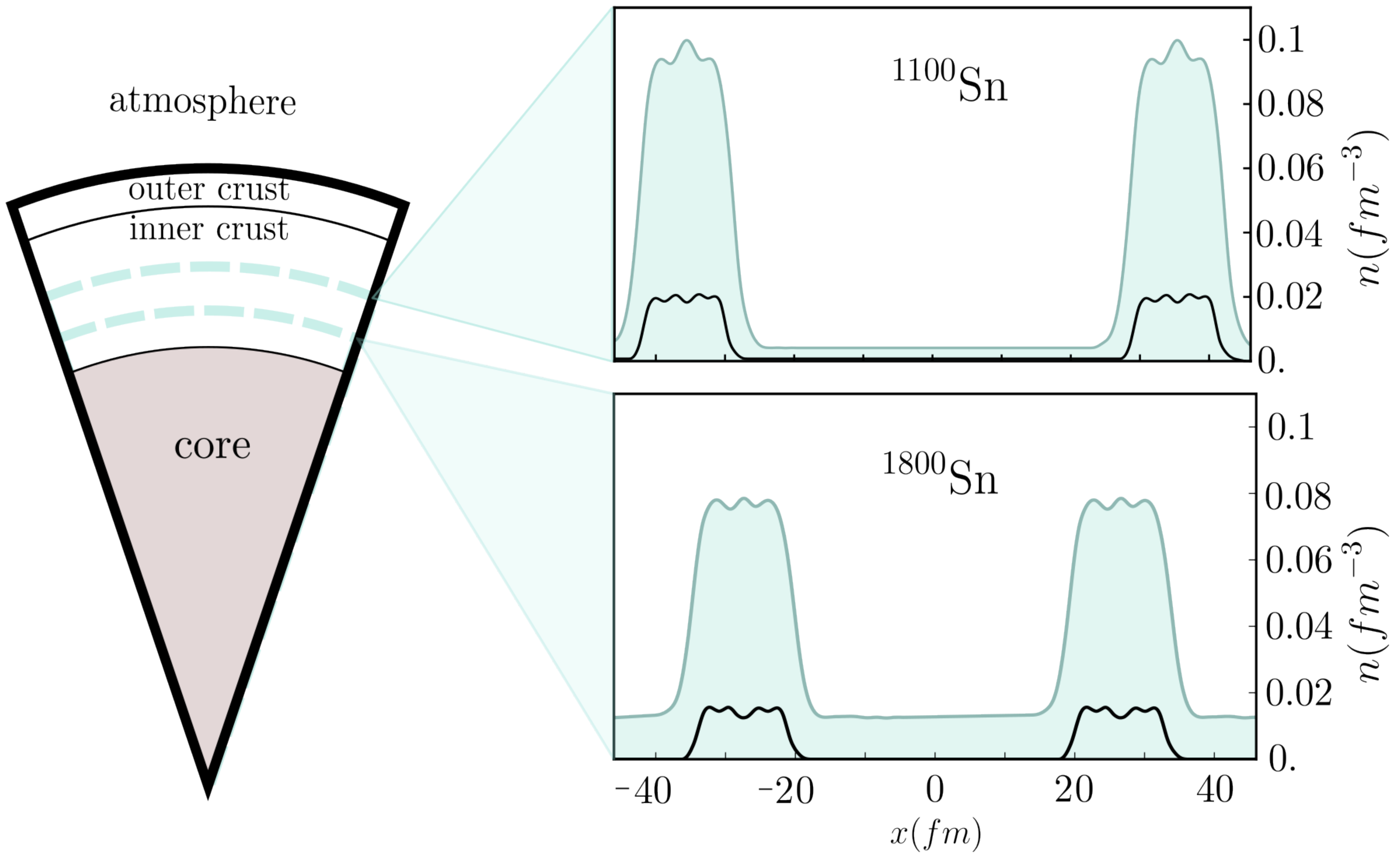
Inner crust

Particularly relevant for glitches: inner crust provides the pinning of superfluid vortices

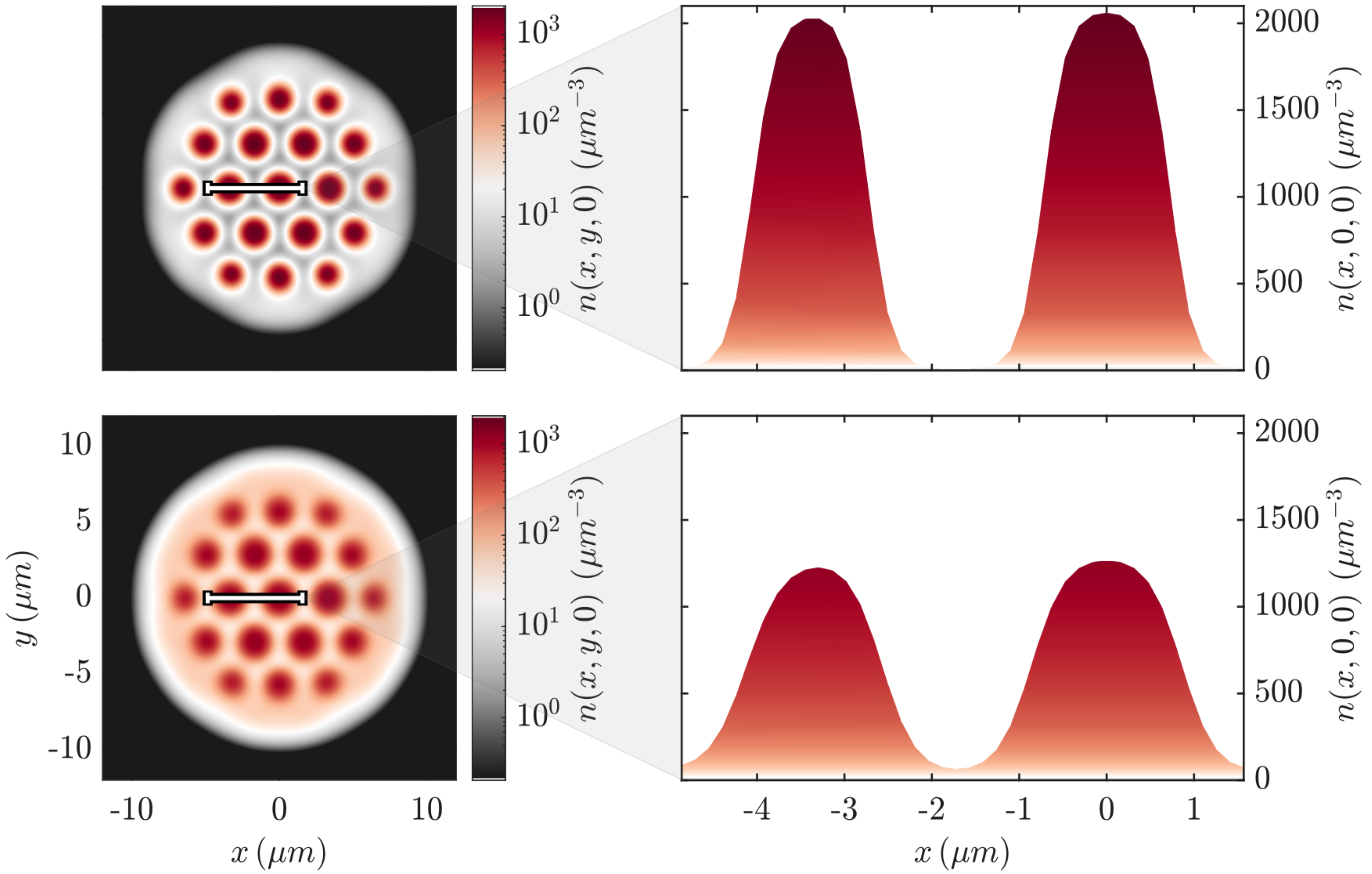


J. W. Negele and D. Vautherin, *Nucl. Phys. A207, 298 (1973)*

Neutron Star inner crust



Dipolar Supersolid



$$\epsilon_{dd} \sim 1$$

Poli, Bland, White, Mark, Ferlaino, Trabucco, MM
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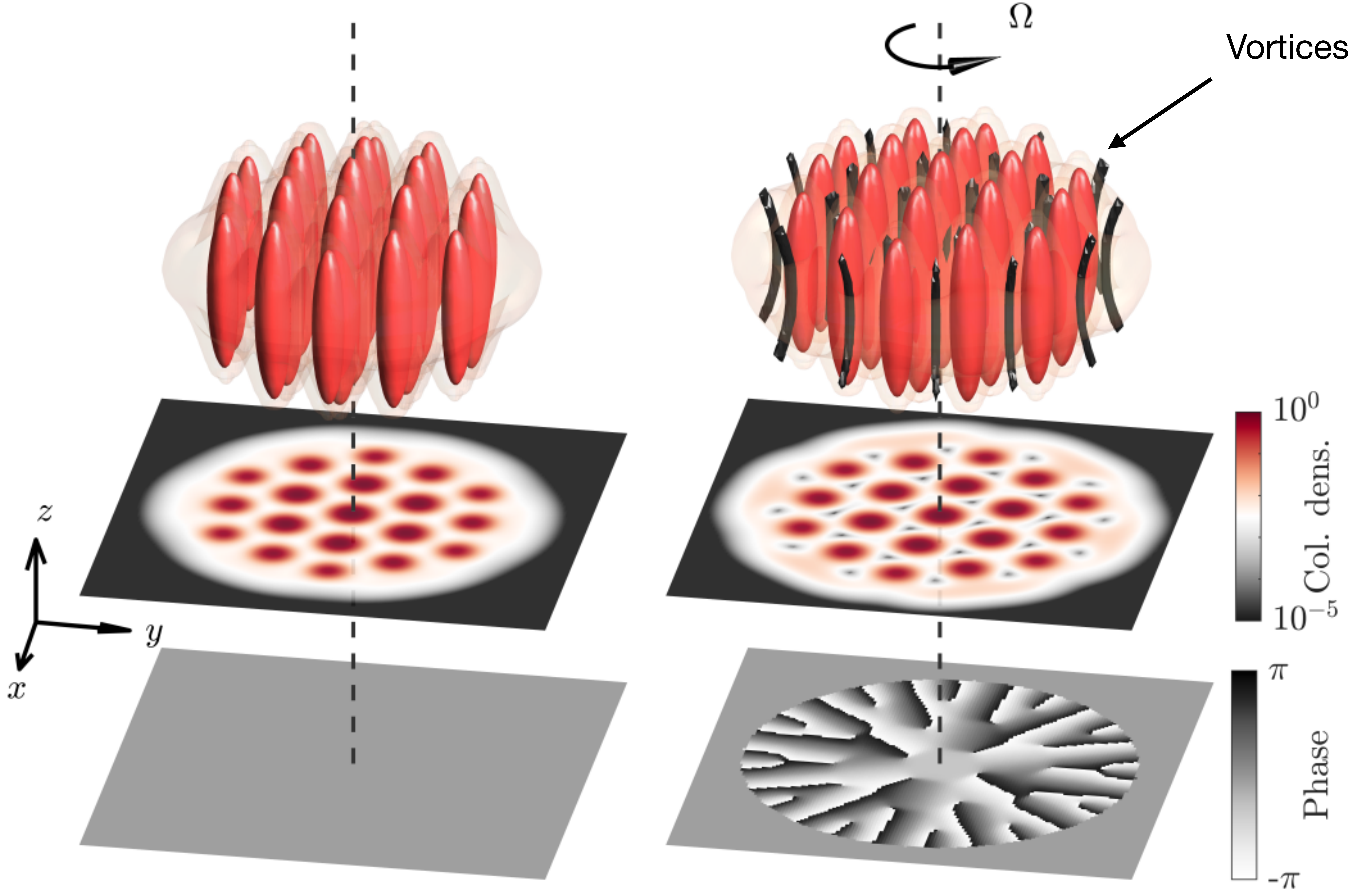
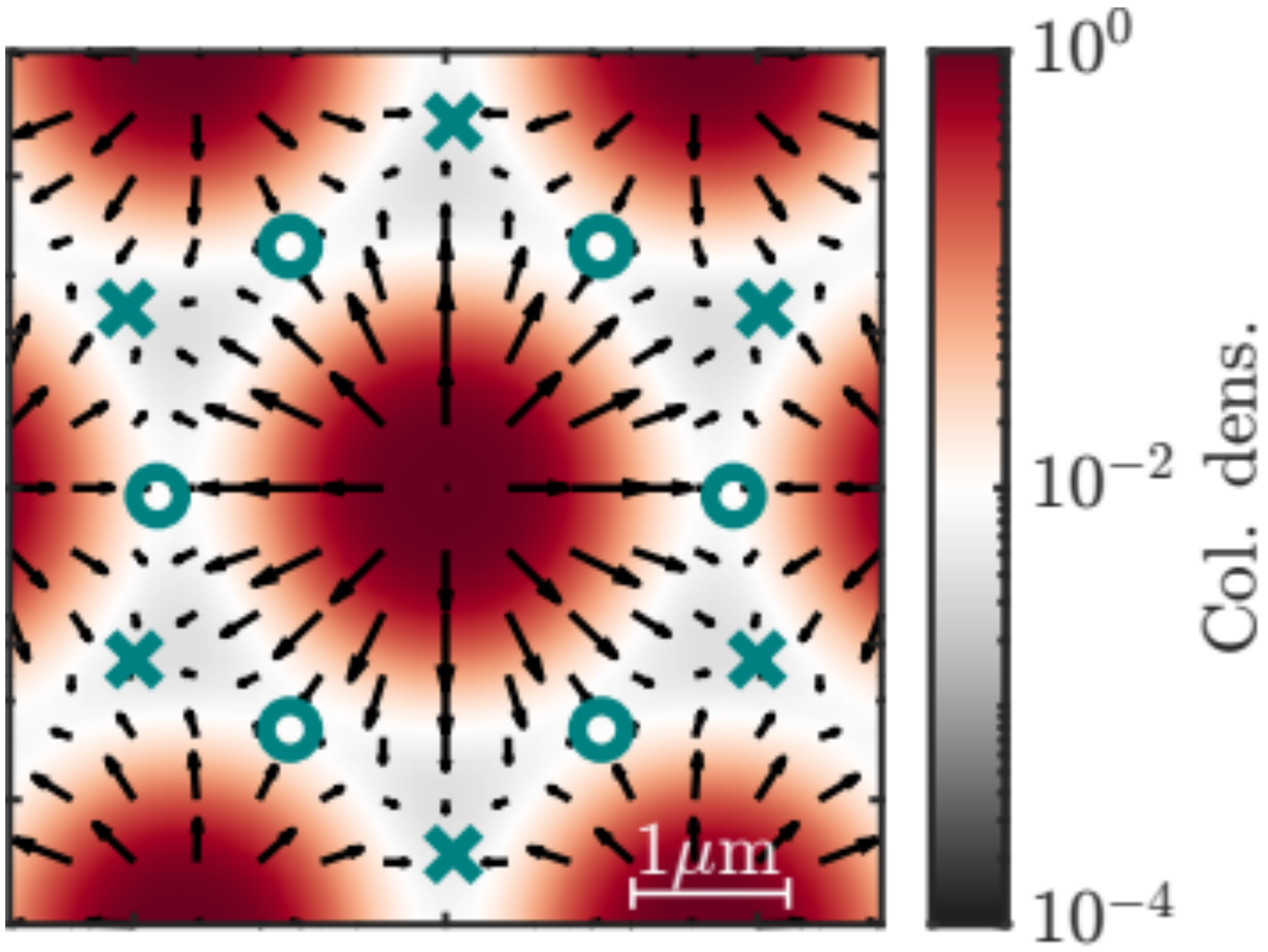
Emulating neutron star glitches

Rotating supersolids



Vortex pinning

X = stable
O= metastable



$a_s = 90a_0\,, \quad \boldsymbol{\omega}_{\text{trap}} = 2\pi \times (50, 130)\text{Hz}$

Inertia of supersolids

Momenta of inertia can be defined in two ways:

From the mass distribution

$$I_{mass} = < x^2 + y^2 >_\psi$$

From the response to rotation

$$L_{solid} = I_{solid}\,\Omega$$

$$I_{solid} = \alpha I_{mass}$$

Legget *PRL, 25, 1543 (1970)*

related to the superfluid fraction

$$0 \le \alpha \le 1$$

$\alpha = 1$ completely solid

$\alpha = 0$ completely superfluid

Total angular momentum

$$L_{total} = < \hat{L}_z >_\psi \neq L_{solid}$$

Ansatz

$$L_{total} = L_{solid} + L_{vortices}$$

to get

Evolution

Optical trap → Outer crust

Dipolar atoms → Inner crust

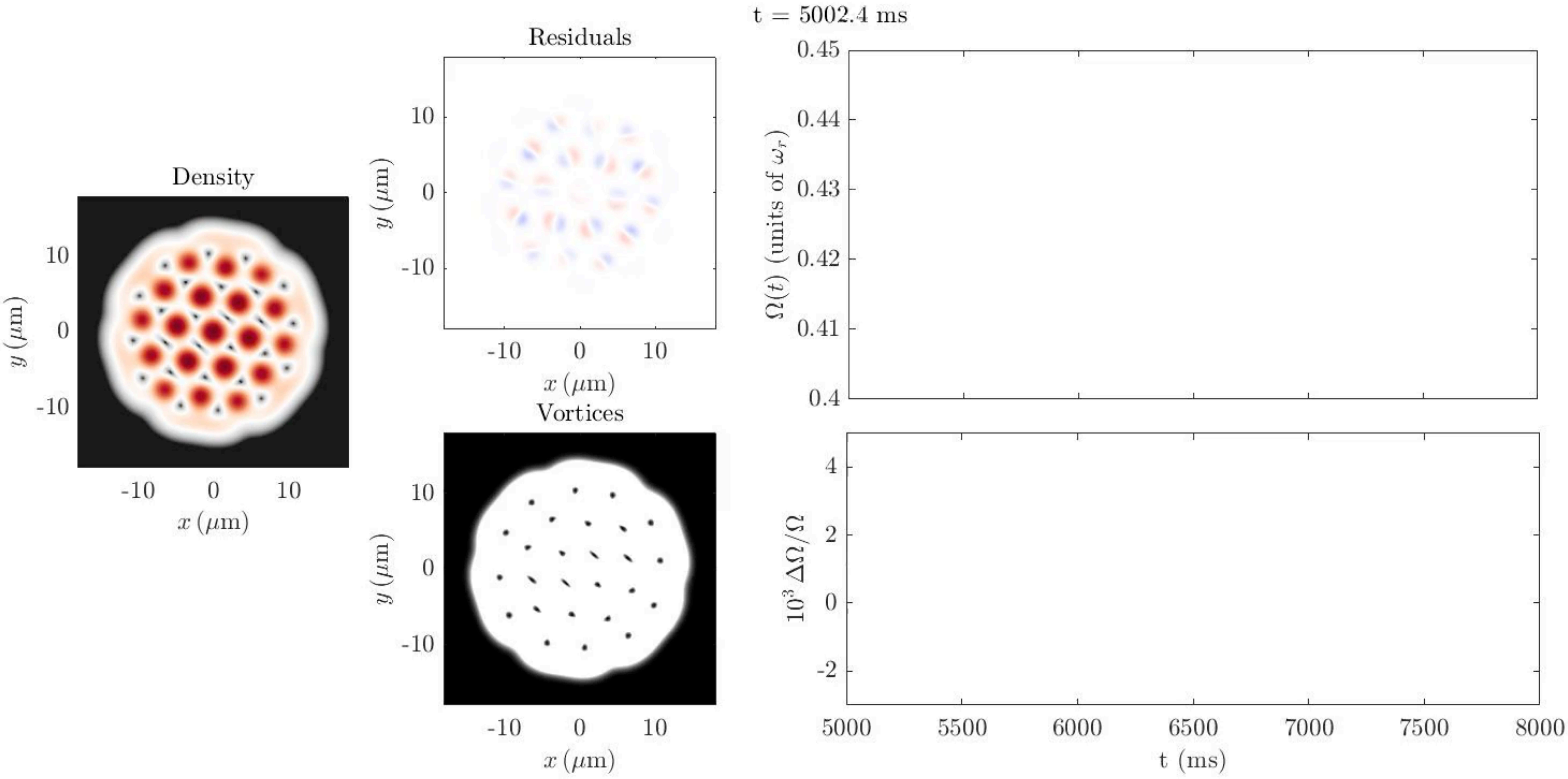
To emulate the NS spin down, we put a “break” on the optical trap $\dot{L}_{total} = -N_{em}$

System of equations solved recursively

$$I_{solid}\dot{\Omega} = -N_{em} - \dot{L}_{vortices} - \dot{I}_{solid}\Omega$$

$$i\hbar\frac{\partial\Psi}{\partial t} = (1 - i\gamma)\left[\mathscr{H}[\Psi; a_s, a_{\rm dd}, \omega] - \Omega(t)\hat{L}_z\right]\Psi$$

Supersolid glitches



More quark matter phases

Supersolids are **versatile**.
Possible applications

Crystalline color superconductors

Zoey Dong talk,
MM et al. *Rev.Mod.Phys. 86 (2014) 509-561*

Pion crystals

Geraint Evans talk,
MM et al. *Phys.Rev.D* 103 (2021) 7, 076003

More inhomogeneous superfluid phases

Vivian Incera and Will Gyory talks

Conclusion

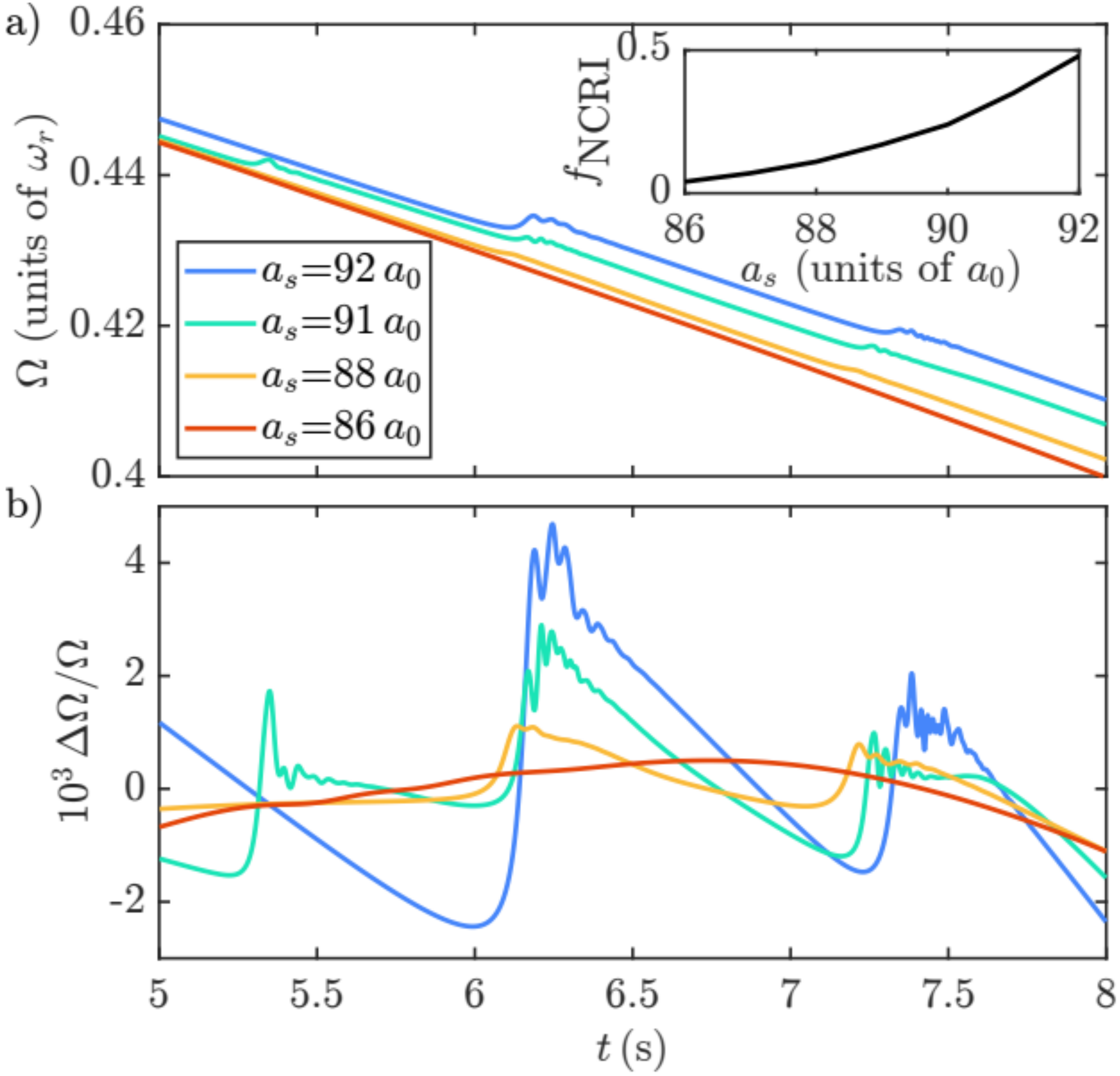
- Dipolar atoms offer the opportunity to study inhomogeneous superfluids
- They mimic some aspects of nuclear matter
- Toy model for the interior of neutron stars
- Emulation of the glitch mechanism

Outlook

- Layered superfluids to have layered structure
- Vortices in glasses
- Scaling

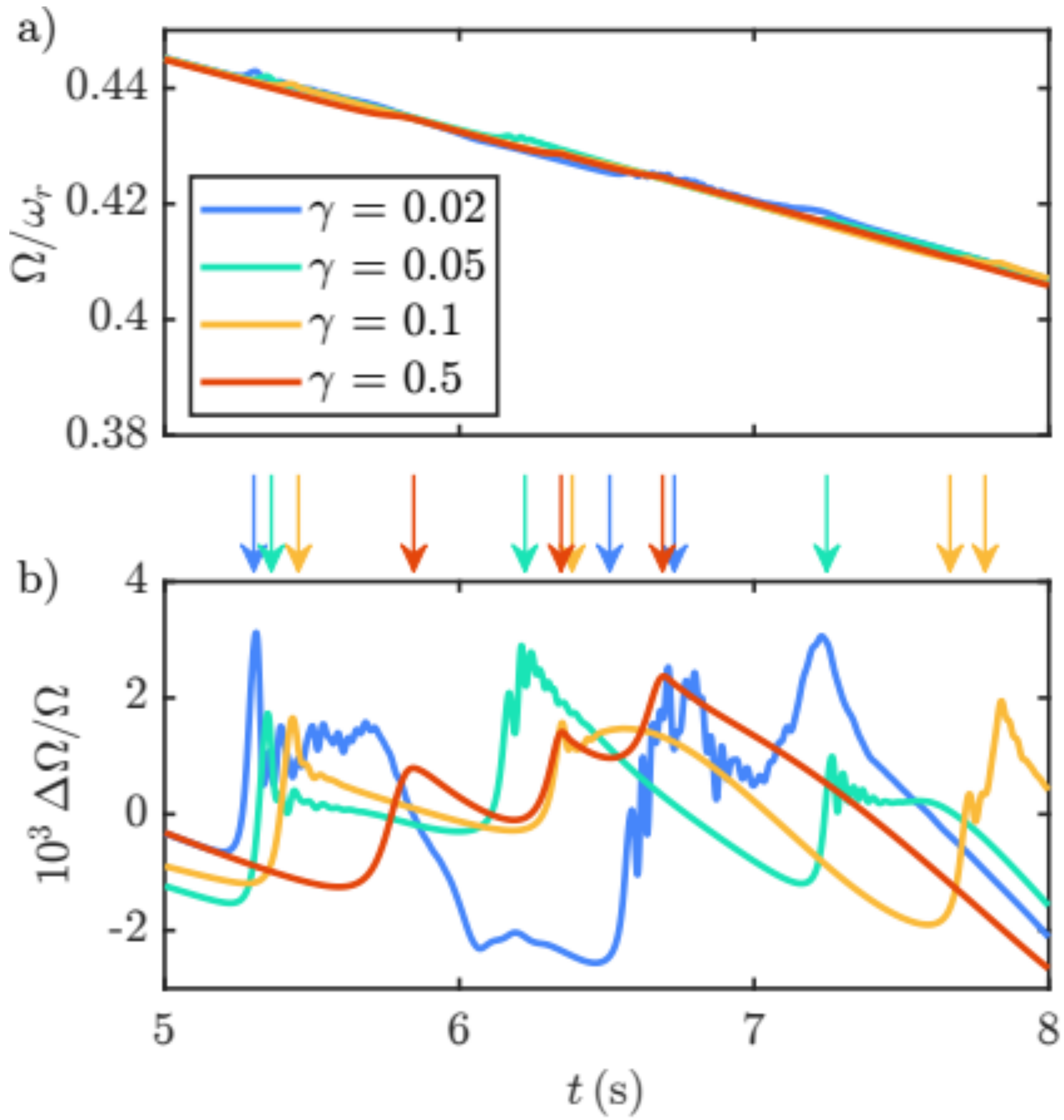
Thank you!

Backup



$$\alpha = 1 - f_{\mathrm{NCRI}}$$

- Parameters $N_{\mathrm{em}} = 4.3 \times 10^{-35}\mathrm{kg\ m^2/s^2}\,, \quad \gamma = 0.05\,, \quad \Omega_{\mathrm{init}} = 0.5\omega_r\,.$



- Different values of γ mimic different coupling with the outer crust

- Parameters $N_{\rm em} = 4.3 \times 10^{-35}\text{kg m}^2/\text{s}^2\,,\ a_s = 91 a_0\,,\quad \Omega_{\rm init} = 0.5\omega_r\,.$

Testing the angular momentum decomposition