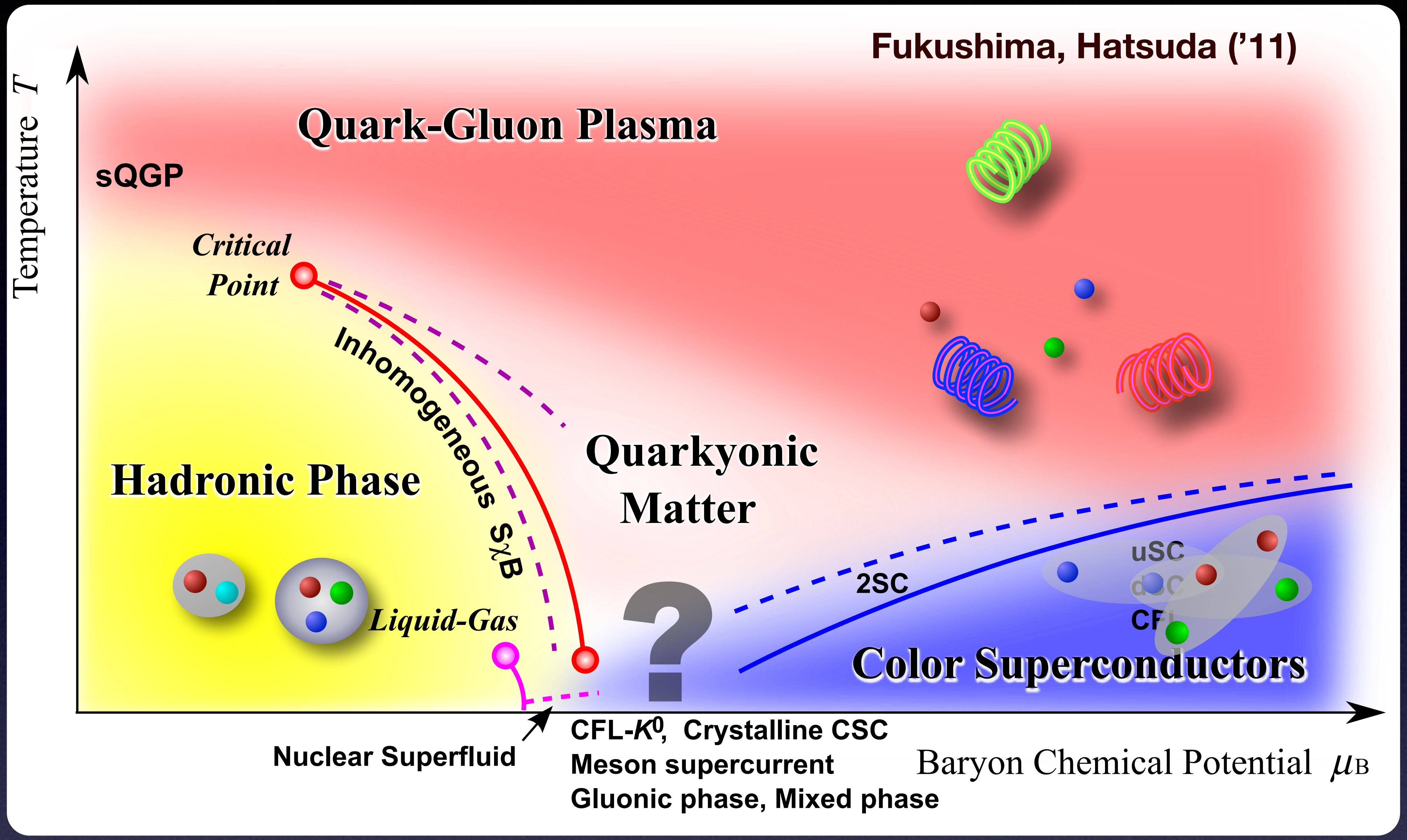
Exploring dense QCD through hamiltonian lattice simulations in (1+1) dimensions

Yoshimasa Hidaka
(YITP)

Based on
Hayata, YH, Nishimura, JHEP 07 (2024) 106, arXiv:2311.11643

QCD at finite density



- What is the equation of state for QCD at finite density?
- How does the quark distribution function change from baryonic matter to quark matter?
- What kind of phase is realized? An inhomogenous phase?

QCD_2

We study QCD_2 at density using density matrix renormalization group (DMRG)technique.

Properties of (1+1) dimensions

- Gauge fields are nondynamical
- Hilbert space is finite dimensional in Open Boundary Condition(OBC)

(dimensionless)QCD_2 Hamiltonian

$$J = \frac{ag_0}{2}, w = \frac{1}{2g_0 a}, m = m_0/g_0$$

We use $g_0 = 1$ unit

$$H/g_0 = J \sum_{n=1}^{N-1} E_i^2(n)$$

Electric field term

$$+w \sum_{n=1}^{N-1} \left(\chi^\dagger(n+1)U(n)\chi(n) + \chi^\dagger(n)U^\dagger(n)\chi(n+1) \right)$$

Hopping term

$$+m \sum_{n=1}^{N} (-1)^n \chi^\dagger(n)\chi(n)$$

Mass term

Elimination of Link variables U

Sala, Shi, Kühn, Bañuls, Demler, Cirac, Phys. Rev. D 98, 034505 (2018)
Atas, Zhang, Lewis, Jahanpour, Haase , Muschik , Nature Commun. 12, 6499 (2021)

$$\Theta\chi(n)\Theta^\dagger := U(n-1)U(n-2)\cdots U(1)\chi(n)$$

$$\Theta H\Theta^\dagger = J\sum_{n=1}^{N-1}\left(\sum_{m=1}^{n}\chi^\dagger(m)T_i\chi(m)\right)^2$$ **Electric fields term**

$$+w\sum_{n=1}^{N-1}\left(\chi^\dagger(n+1)\chi(n)+\chi^\dagger(n)\chi(n+1)\right)$$

Hopping term

$$+m\sum_{n=1}^{N}(-1)^n\chi^\dagger(n)\chi(n)$$ **mass term**

As a variational ansatz of wave function

- We employ a matrix product state

$$|\psi\rangle = \sum_{\{n_i\}} |n_1\rangle \cdots |n_N\rangle \mathrm{tr} M_1^{n_1} \cdots M_N^{n_N}$$

$[M_i^{n_i}]_{ij}$: $D \times D$ matrix

- Optimize the wave function by density matrix renormalization group technique

$$E = \min_{\psi} \langle \psi | H | \psi \rangle$$

We employ iTensor

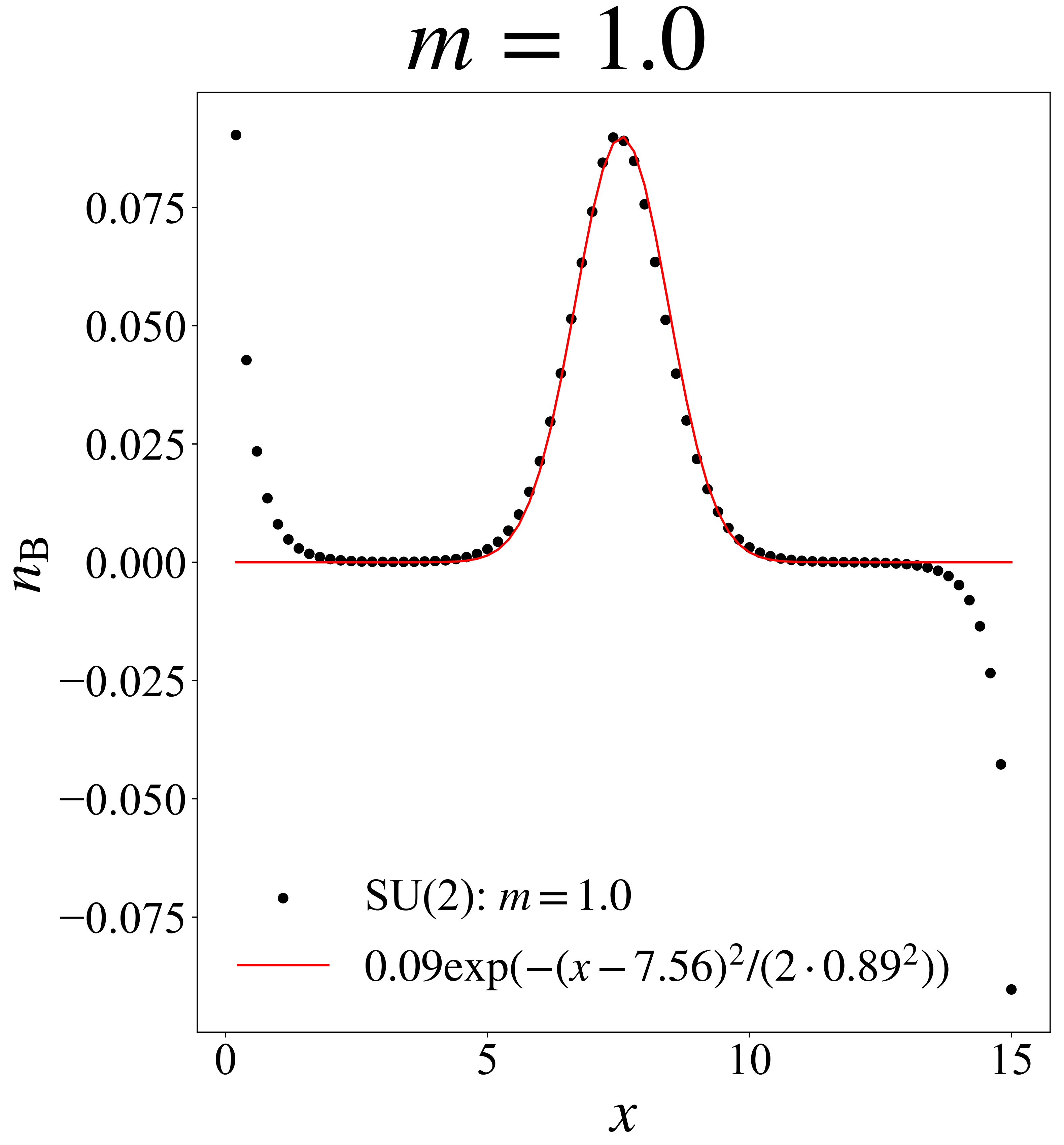
Numerical results

SU(2) QCD with $N_f = 1$

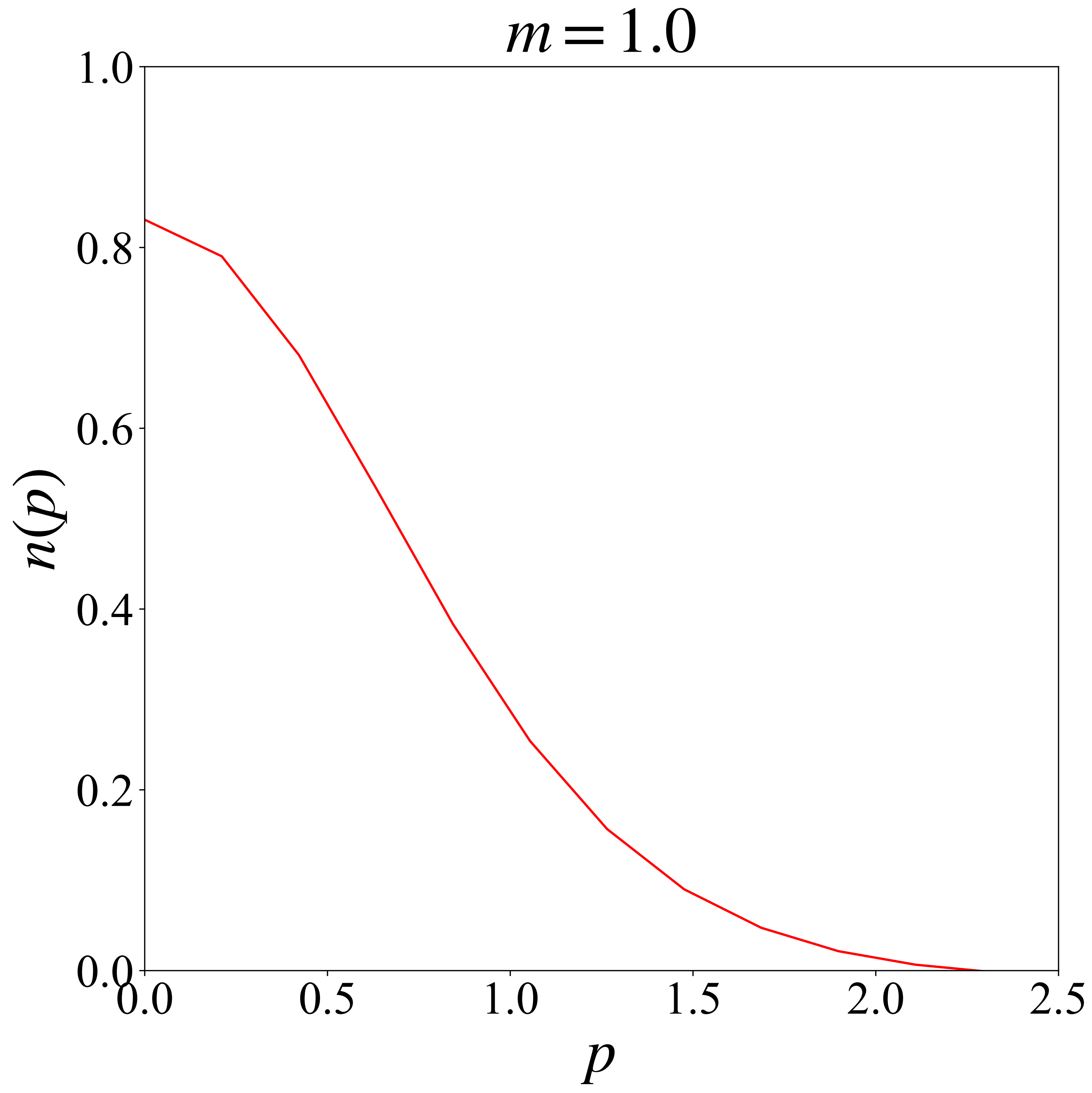
Color SU(2), 1 flavor, vacuum

single baryon state $\dim\mathscr{H} = 2^{300}$ $J = 1/20$ $w = 5$ volume $V = 15$

Baryon number density



Quark distribution function

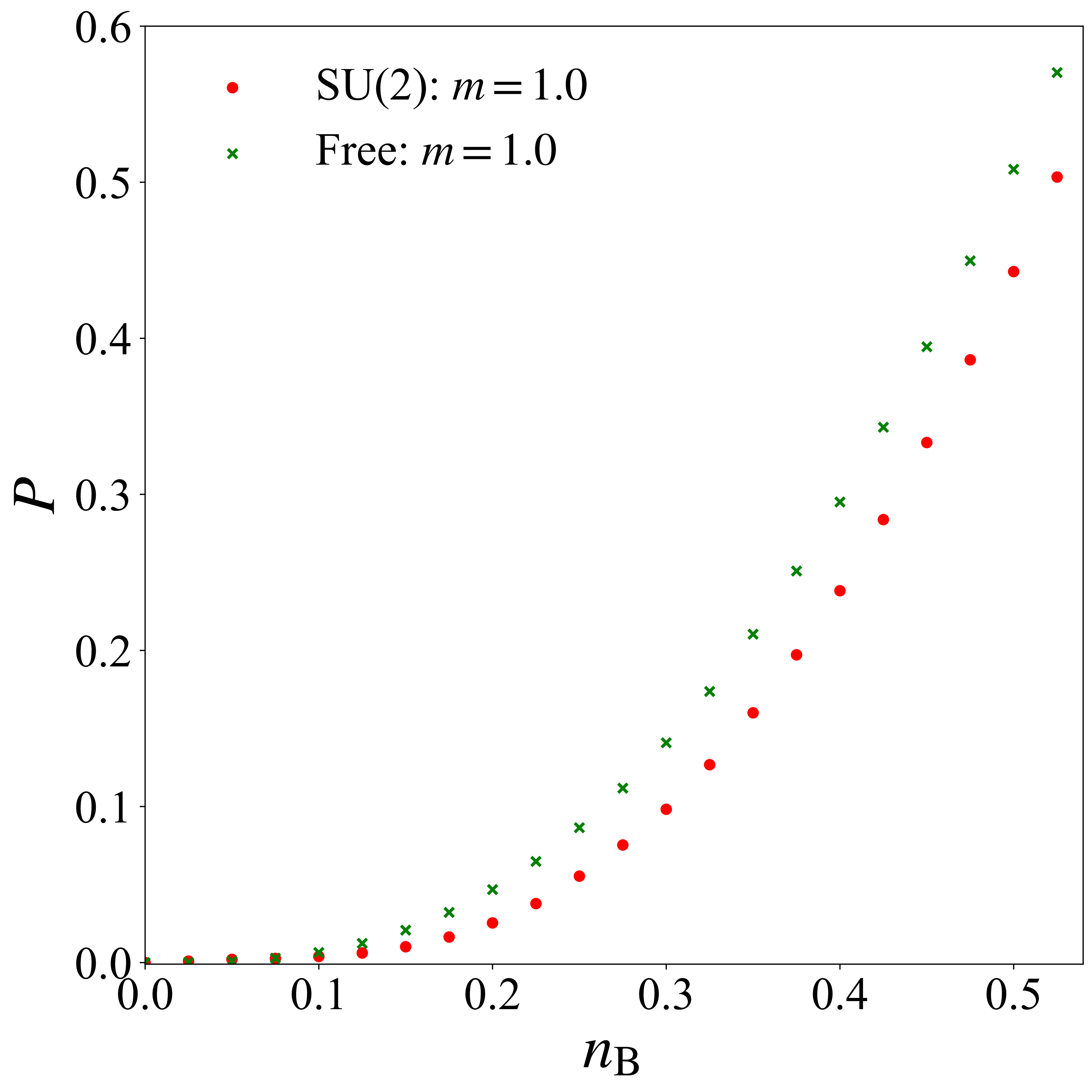


Baryon size ~ 1

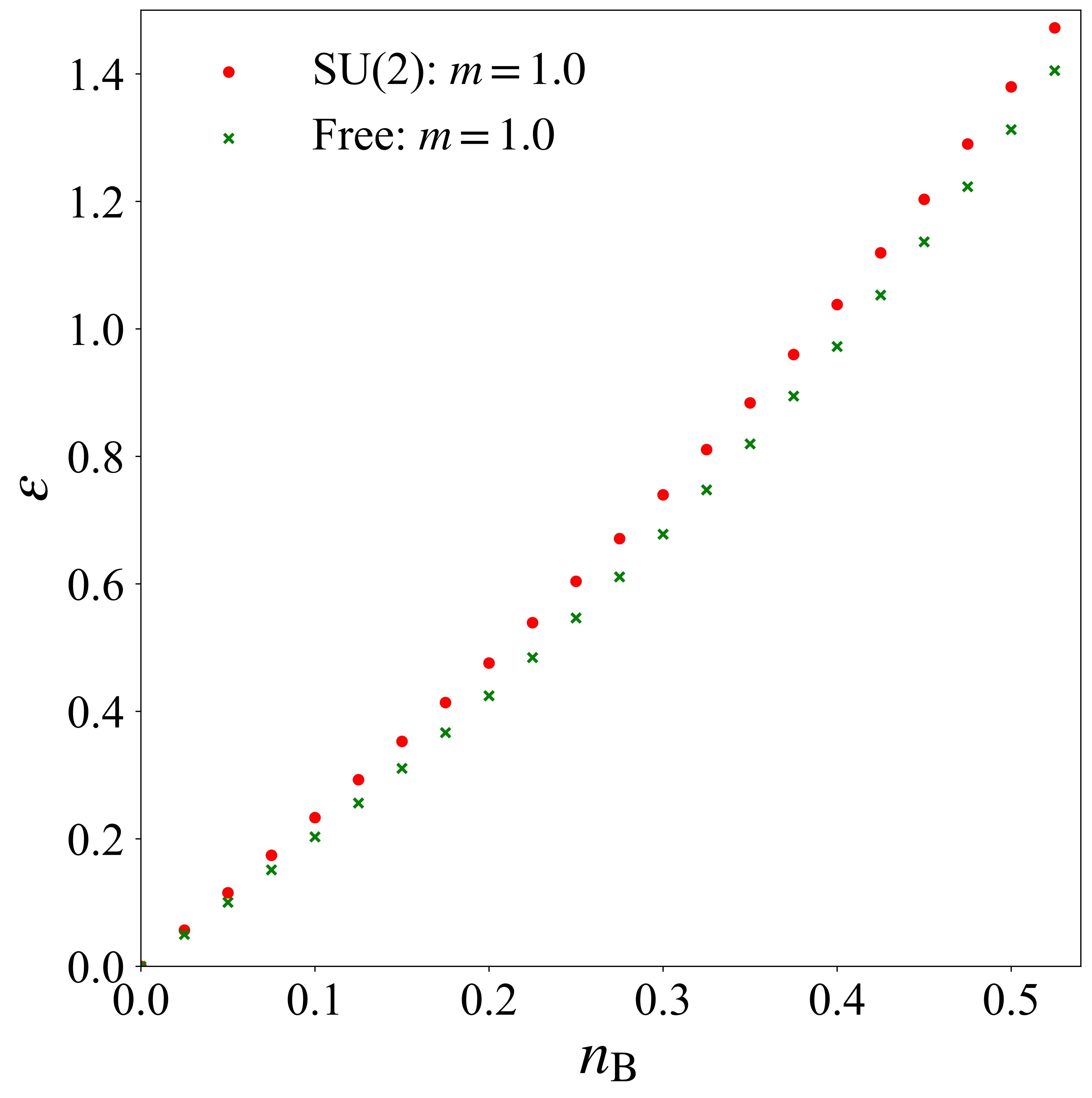
Color SU(2), 1 flavor, vacuum

$$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$$

Pressure



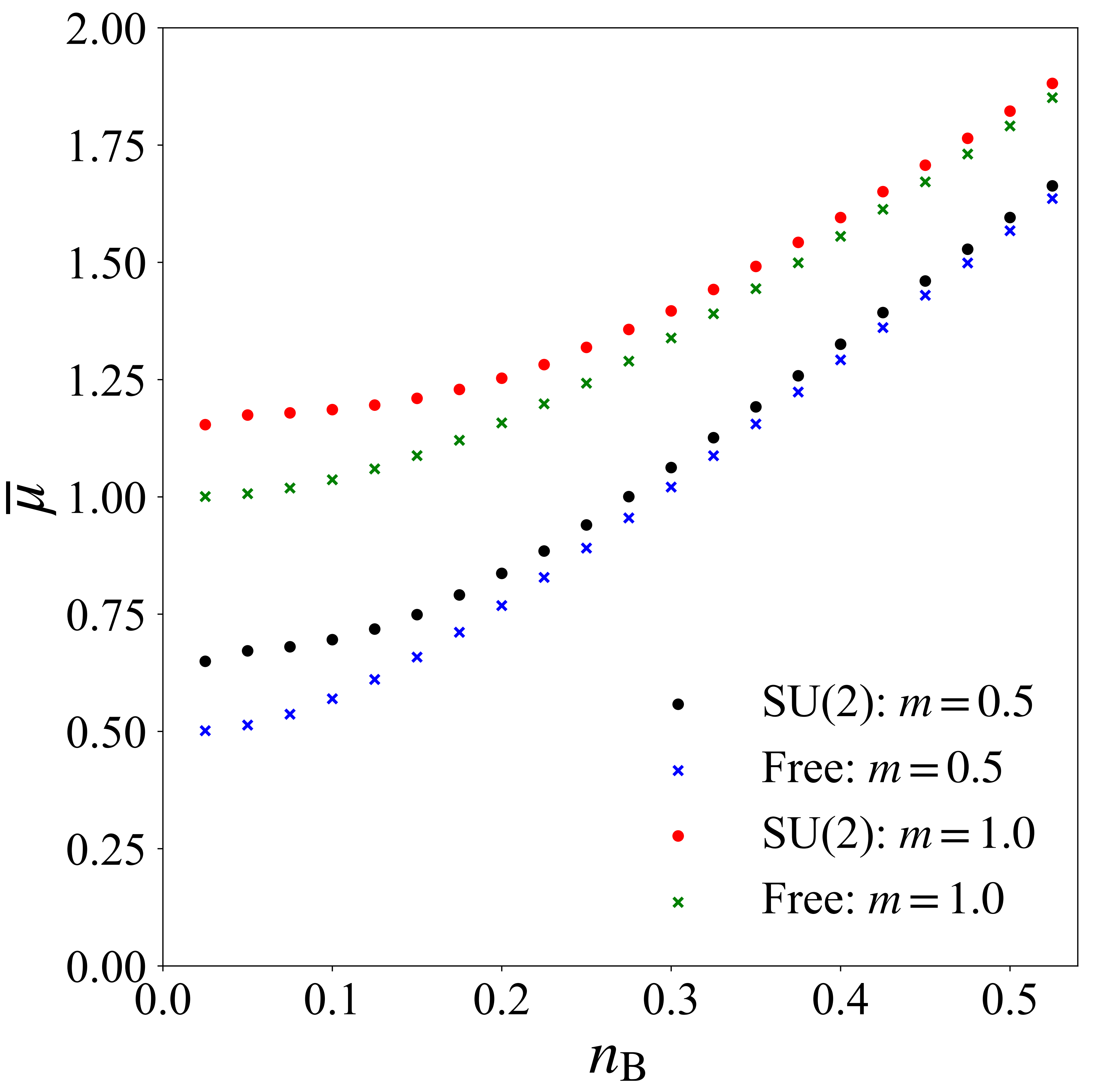
Energy density



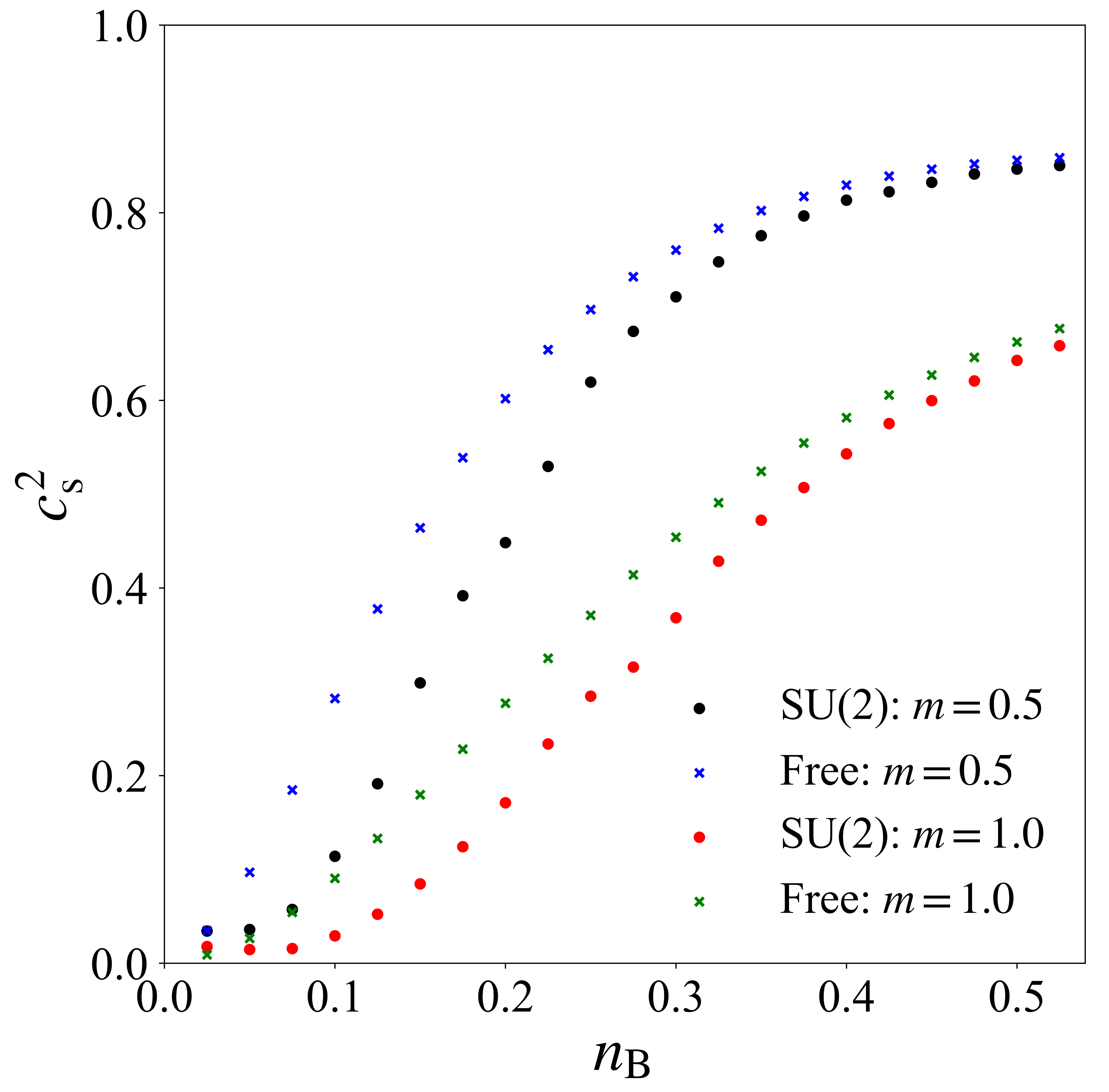
Color SU(2), 1 flavor, vacuum

$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$

Chemical potential

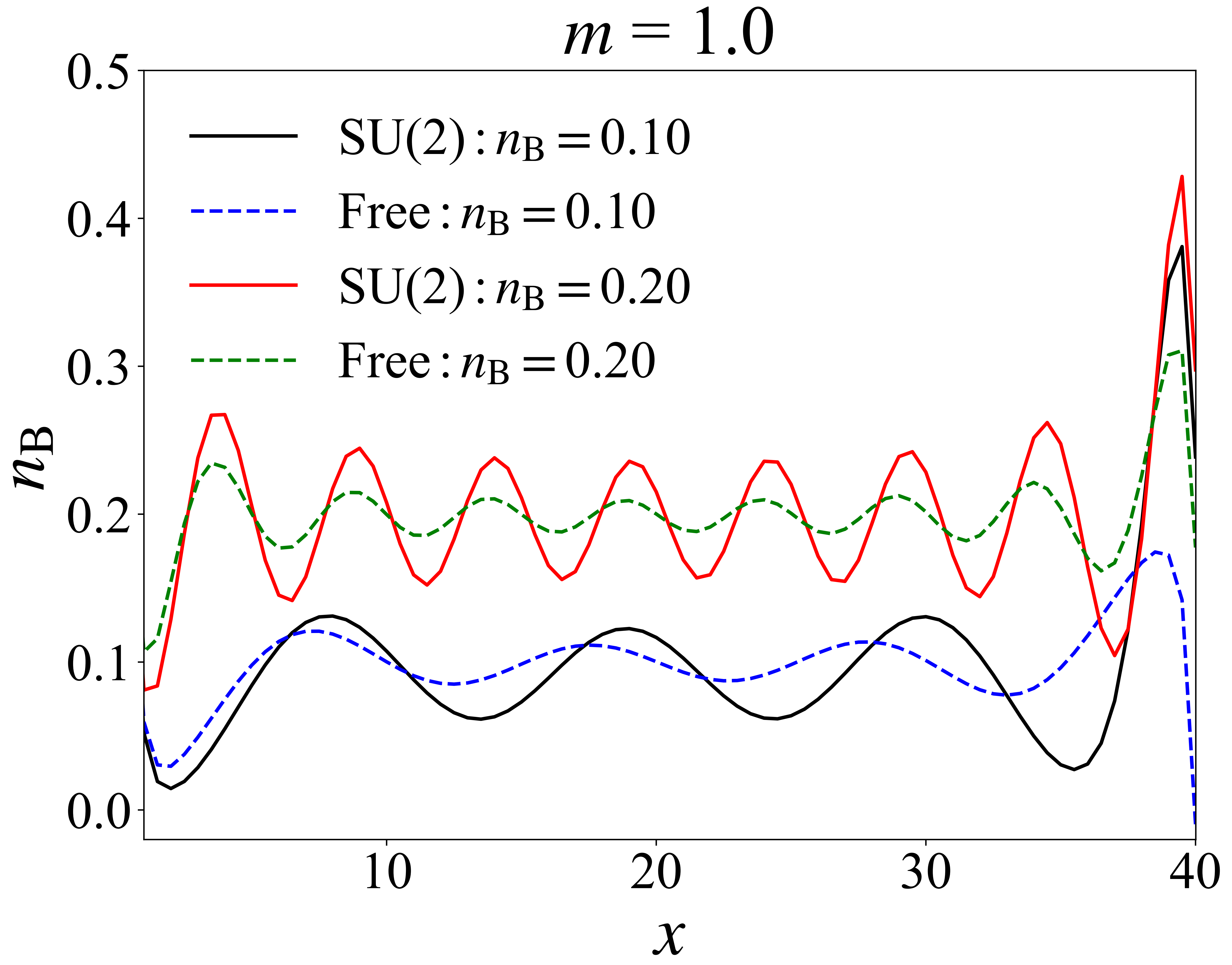


Sound velocity



Inhomogeneous phase (density wave)

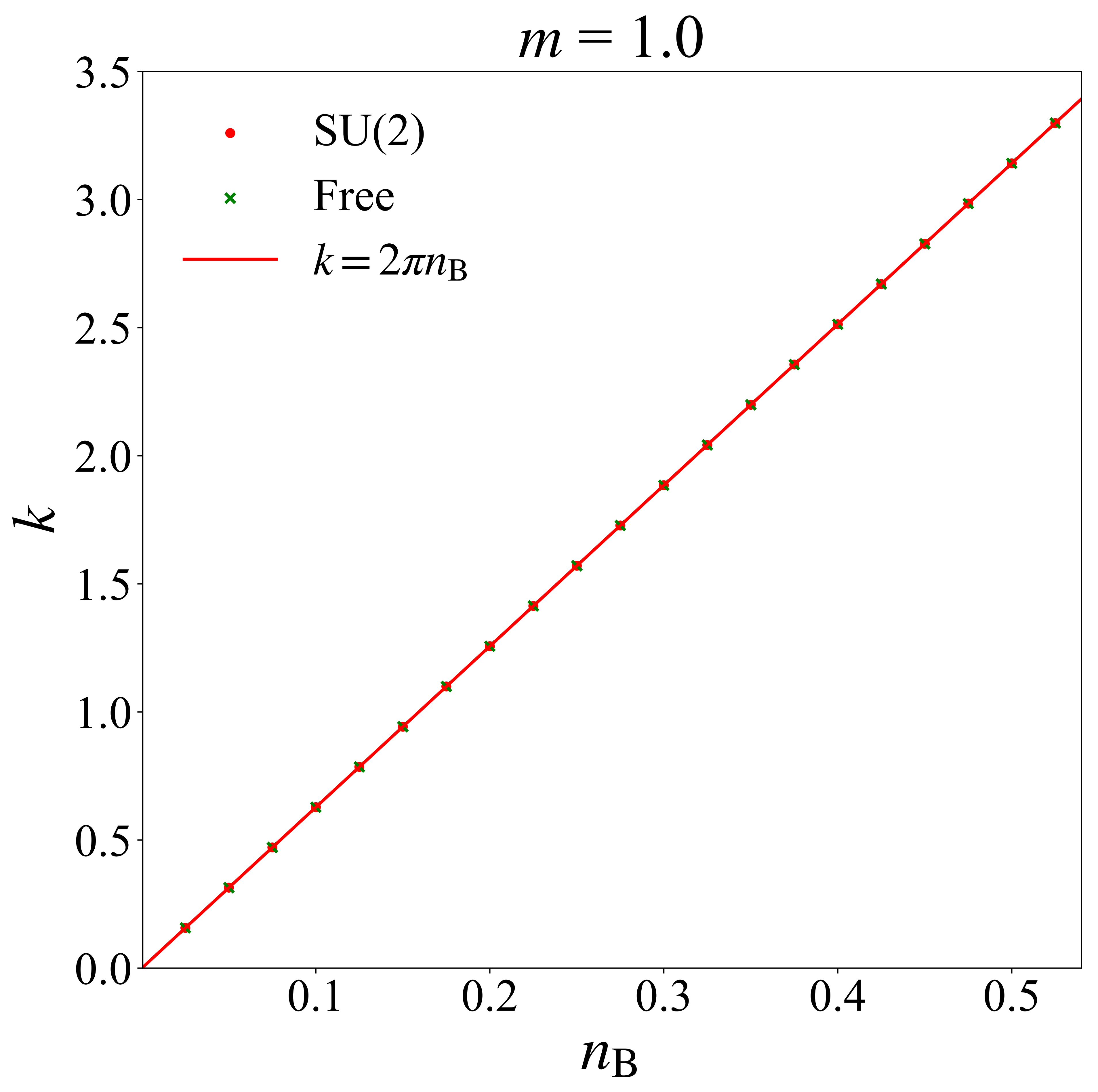
$J = 1/8 \;\; w = 2 \;\; V = 40 \;\; \dim\mathscr{H} = 2^{320}$



Wave number dependence

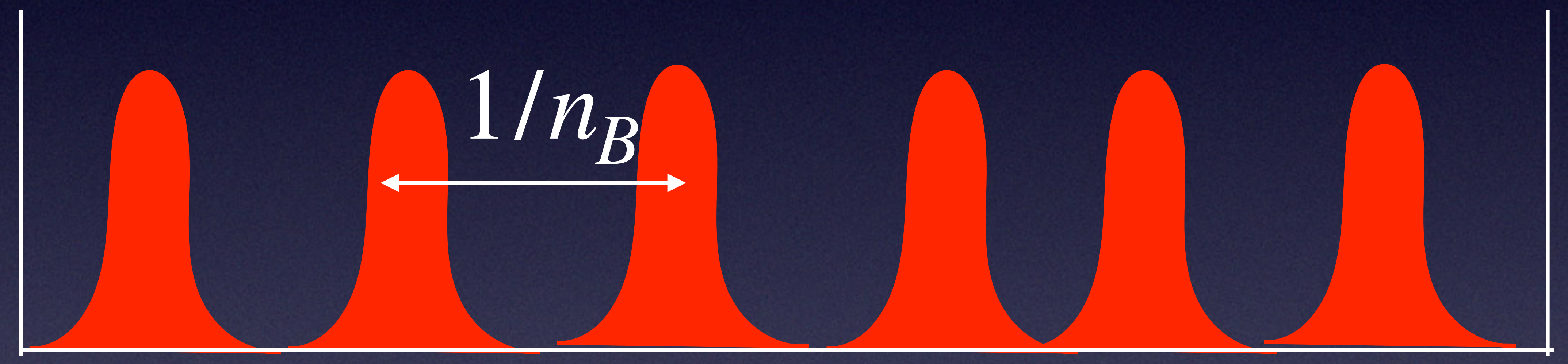
$$J = 1/8 \quad w = 2 \quad V = 40 \quad \dim\mathscr{H} = 2^{320}$$

Wave number dependence



Hadronic picture

If hadron interactions are repulsive



distance $1/n_B \Rightarrow k = 2\pi n_B$

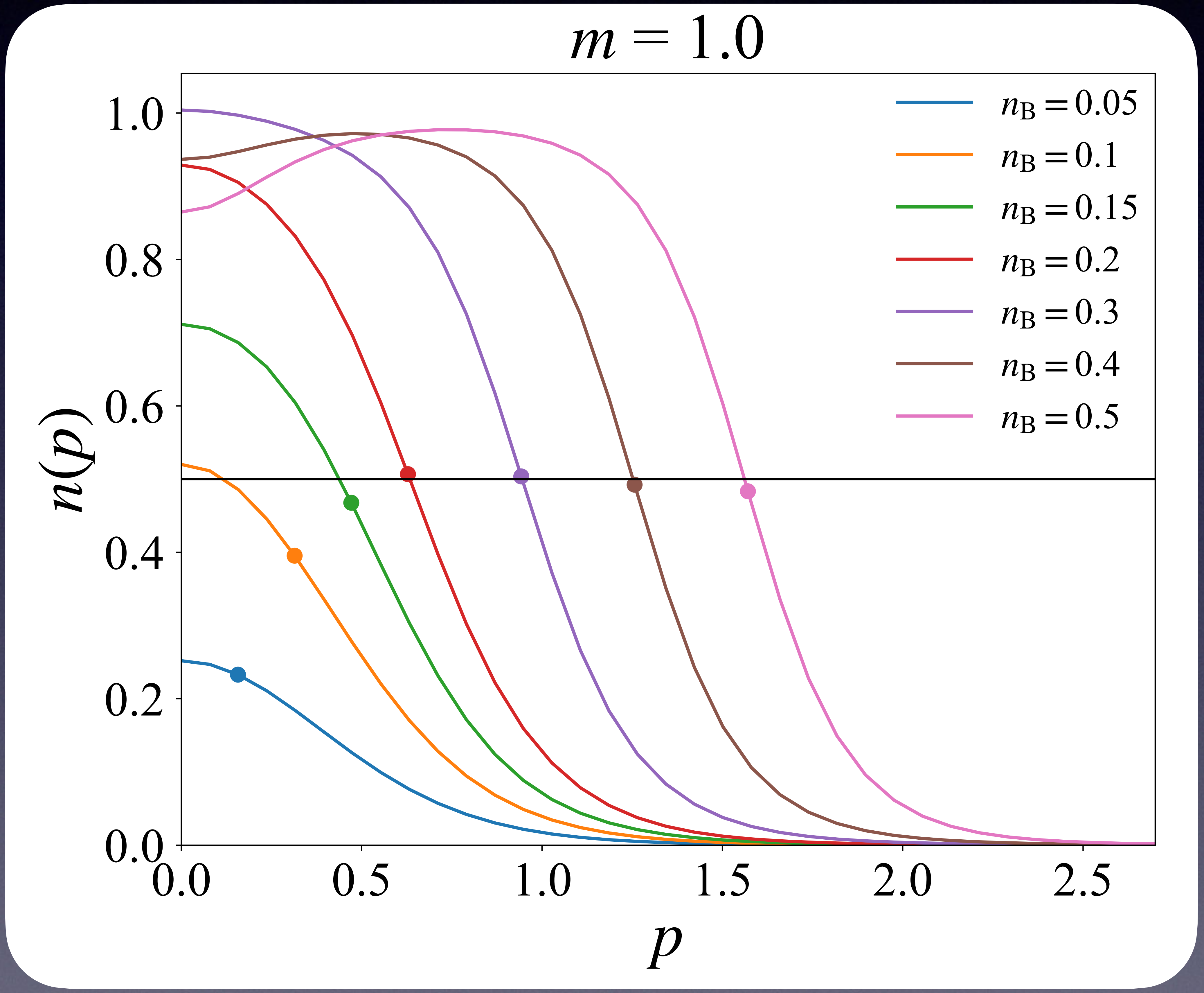
Quark picture

If interactions between quarks
Fermi surface is unstable

$\Rightarrow$density wave $k = 2p_F = 2\pi n_B$

Quark distribution function

$J = 1/8 \;\; w = 2 \;\; V = 60 \quad \dim\mathscr{H} = 2^{480}$



- **Low density**
  No Fermi sea
- **High density**
  Fermi-sea
  +BCS like pairing
  (density wave)

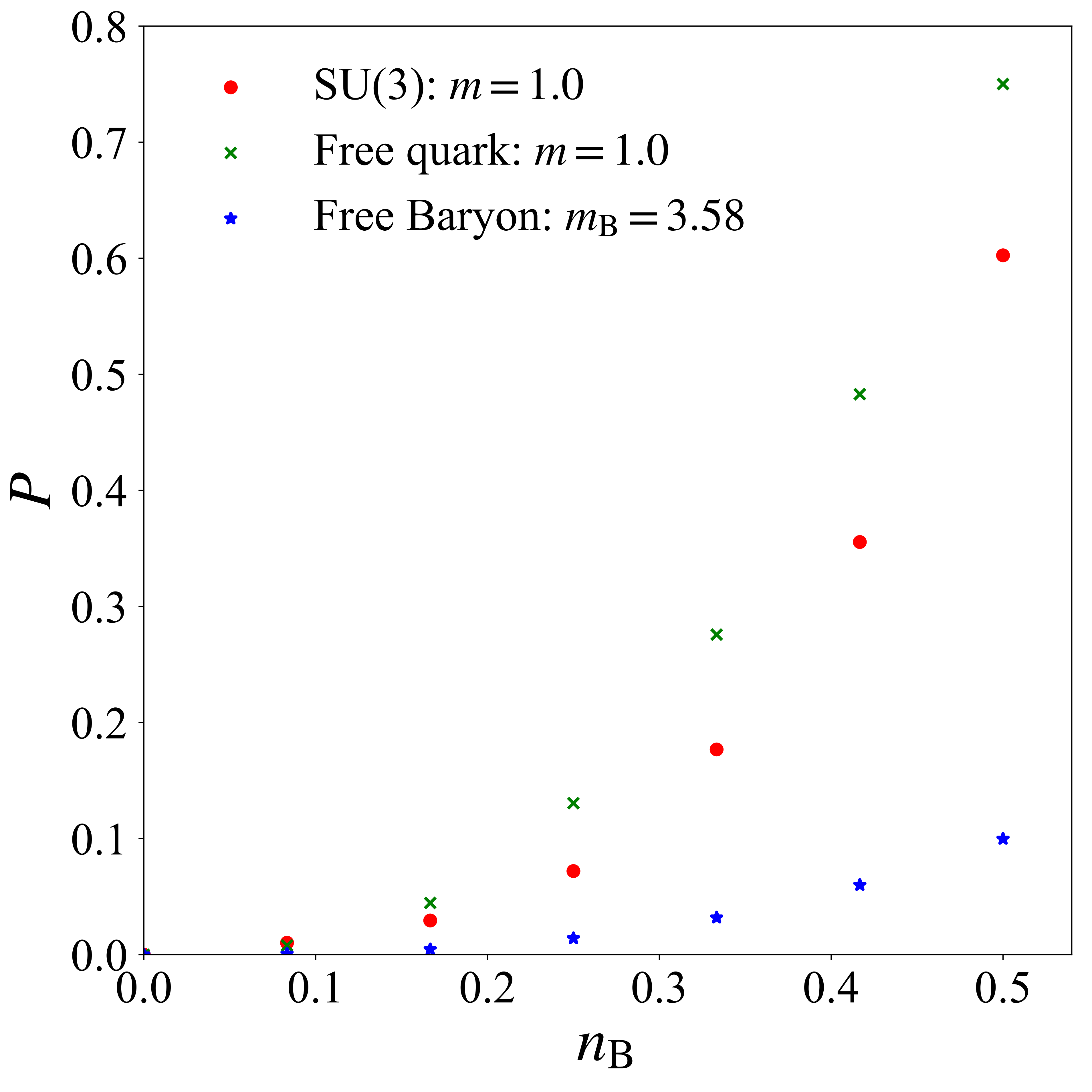
baryon quark transition around $n_B \sim 0.2$

SU(3) QCD with $N_f = 1$

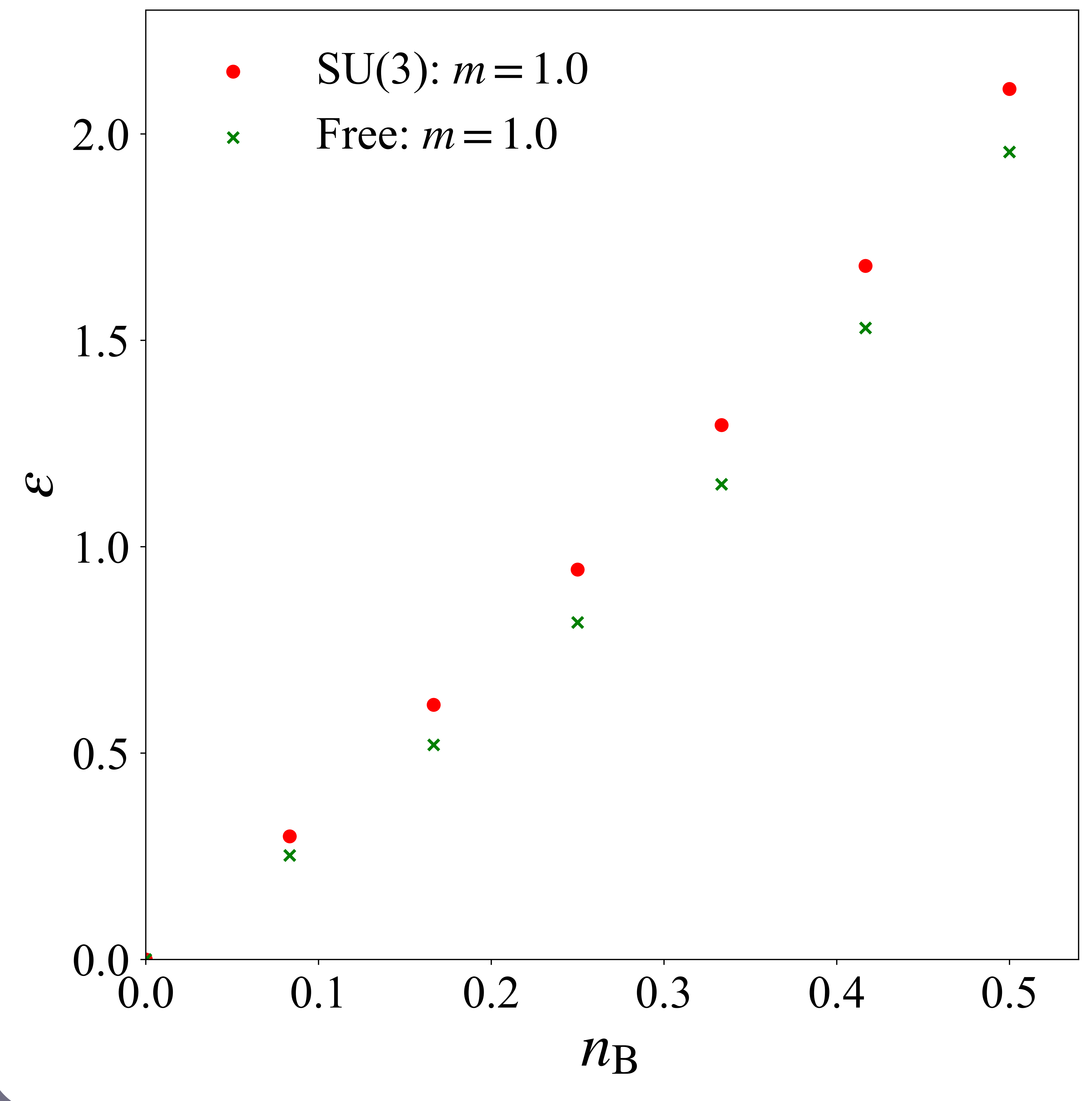
Color SU(3), 1 flavor

$$J = 1/8 \quad w = 2 \quad V = 12 \quad \dim\mathscr{H} = 2^{144}$$

Pressure



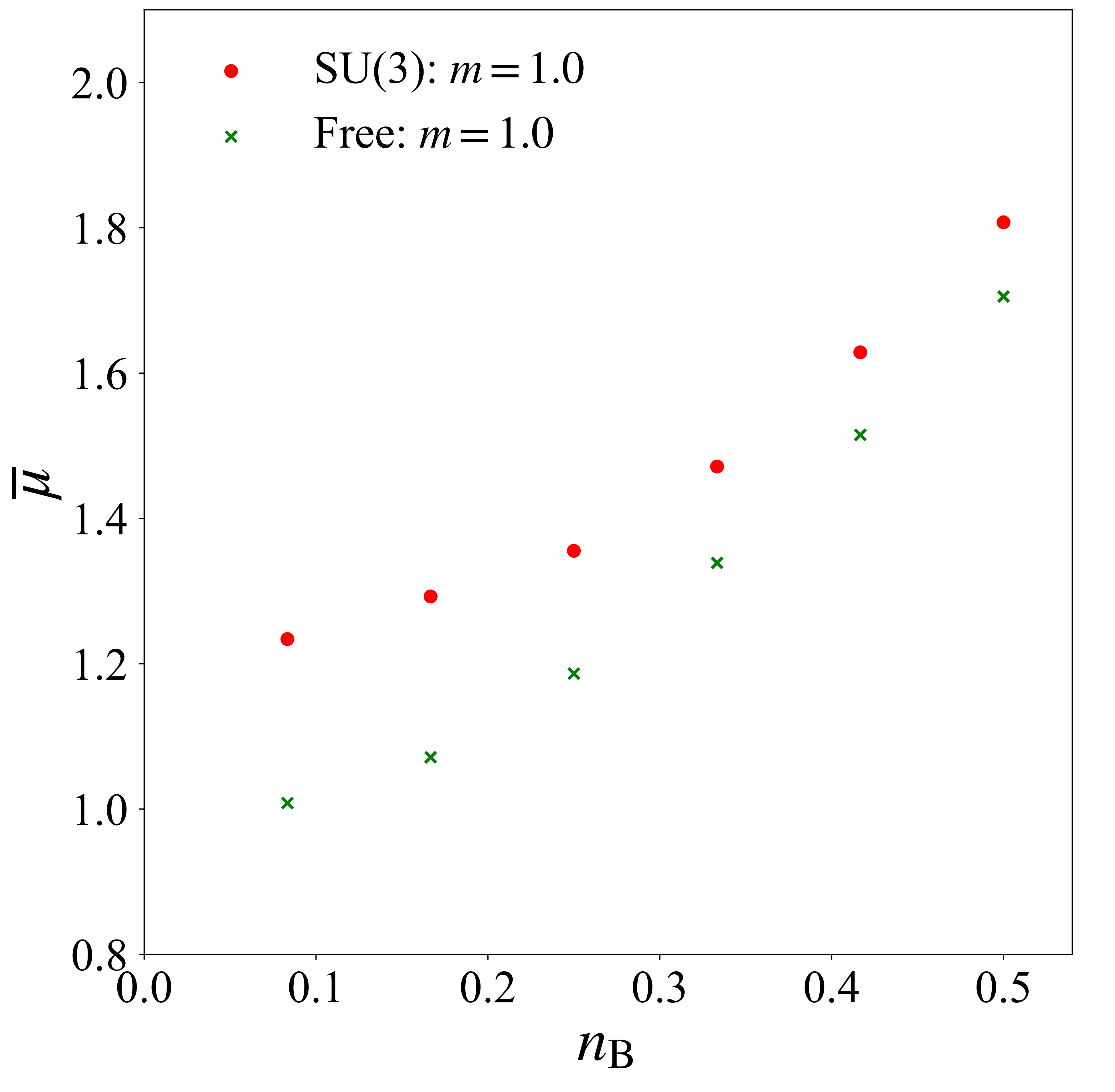
Energy density



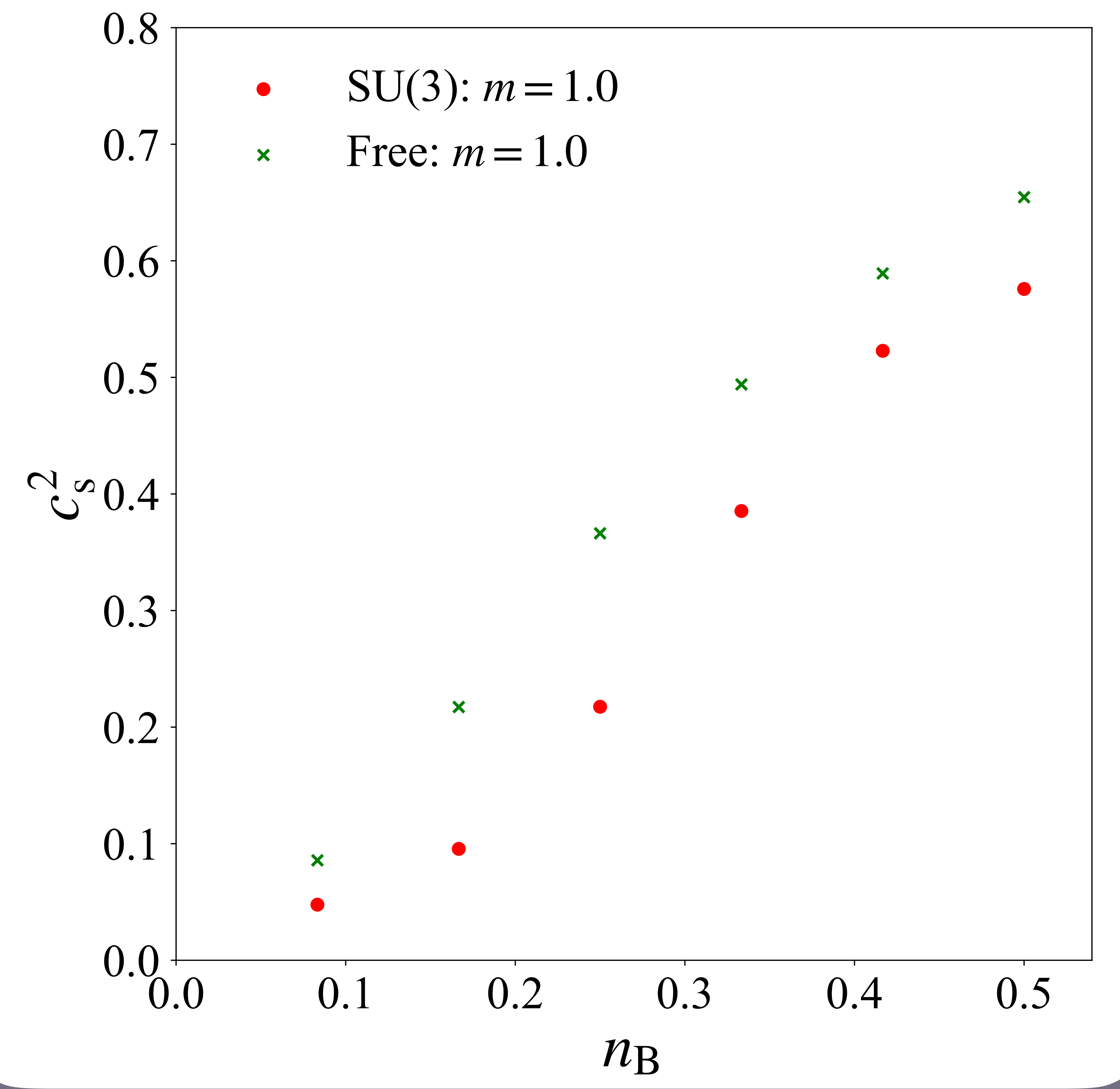
Color SU(3), 1 flavor, vacuum

$J = 1/8 \;\; w = 2 \;\; V = 12 \;\; \dim\mathscr{H} = 2^{144}$

Chemical potential



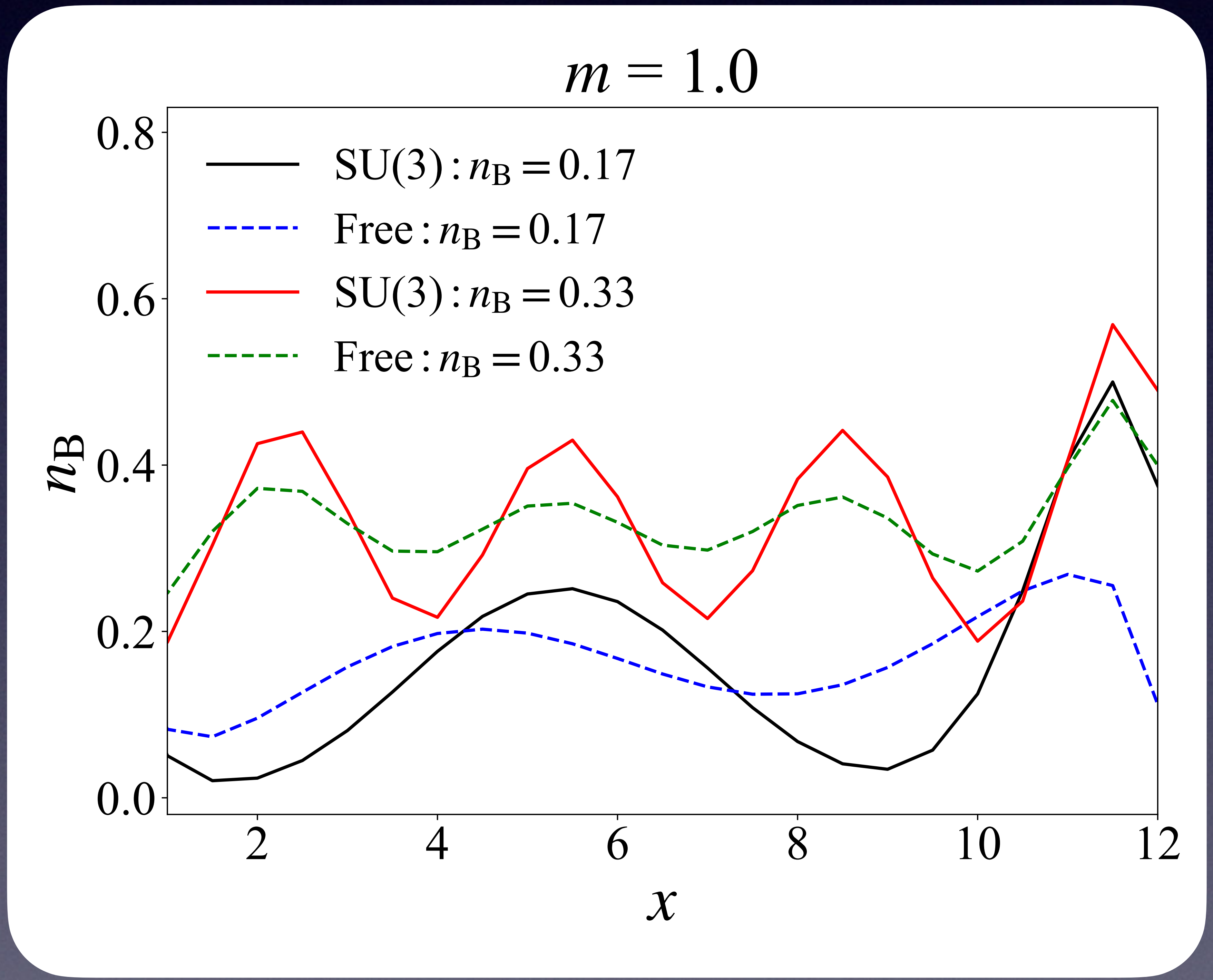
Sound velocity

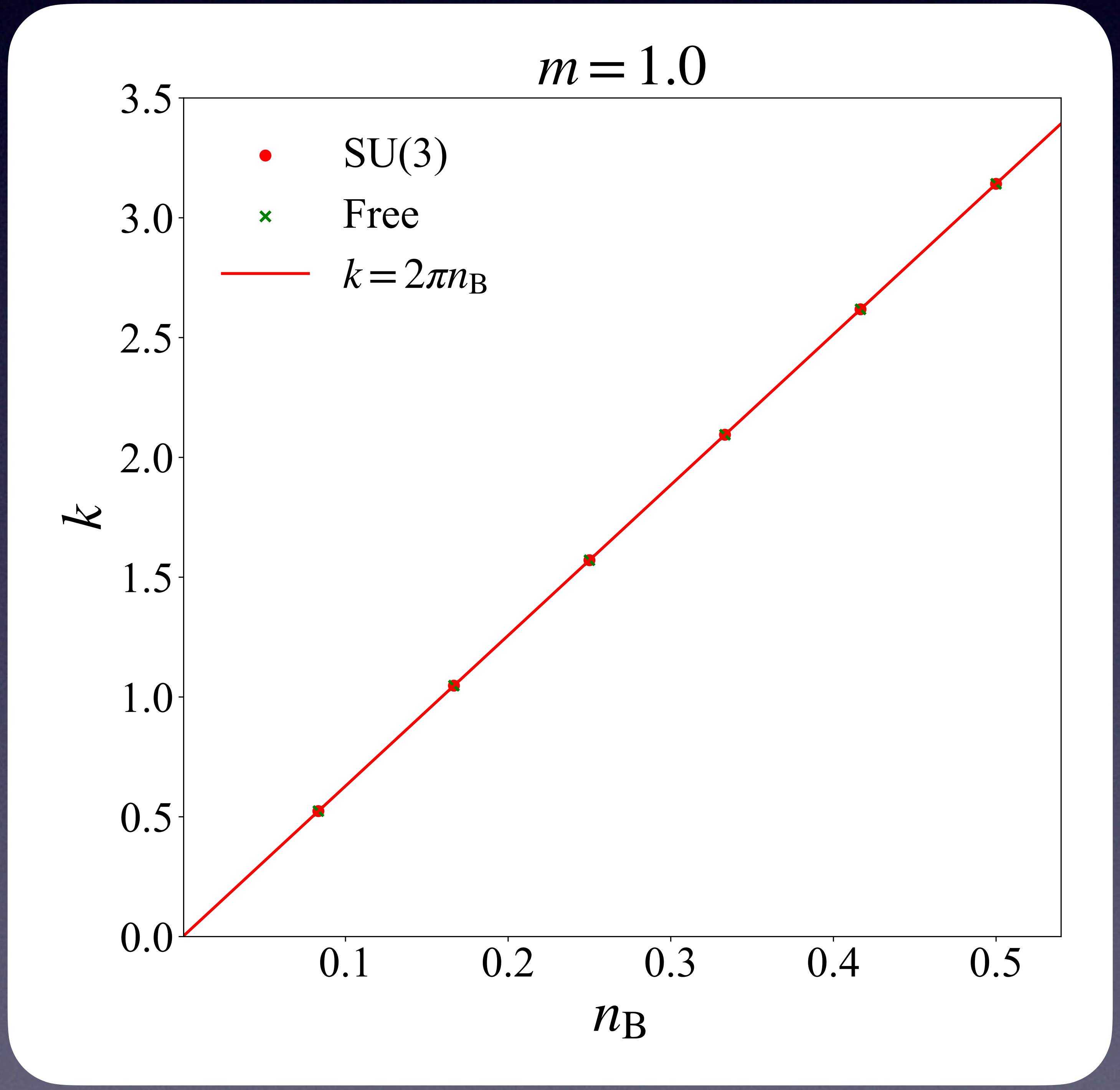


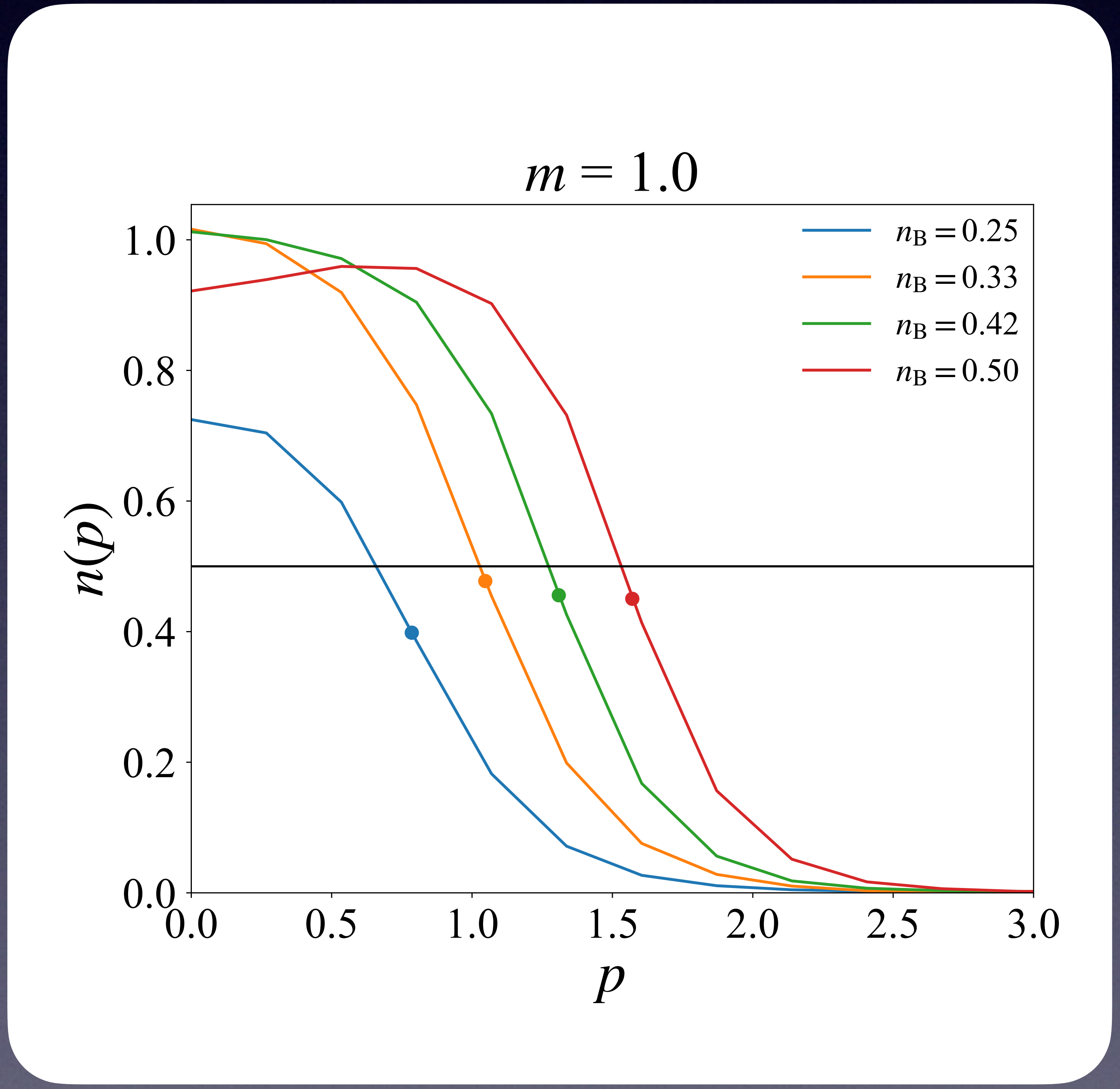
Color SU(3), 1 flavor

$$J = 1/8 \;\; w = 2 \;\; V = 12 \;\; \dim\mathscr{H} = 2^{144}$$

density wave **Wave number dependence** **Quark distribution**







Baryon quark transition around $n_B = 0.3$?

Summary

- We study QCD_2 at finite density with one flavor for two and three colors.

- We employ Hamiltonian formalism and density matrix renormalization group techniques

- We find inhomogeneous phases both two and three colors.