Quark Confinement and Heavy Baryon Spectroscopy in the Quark Model

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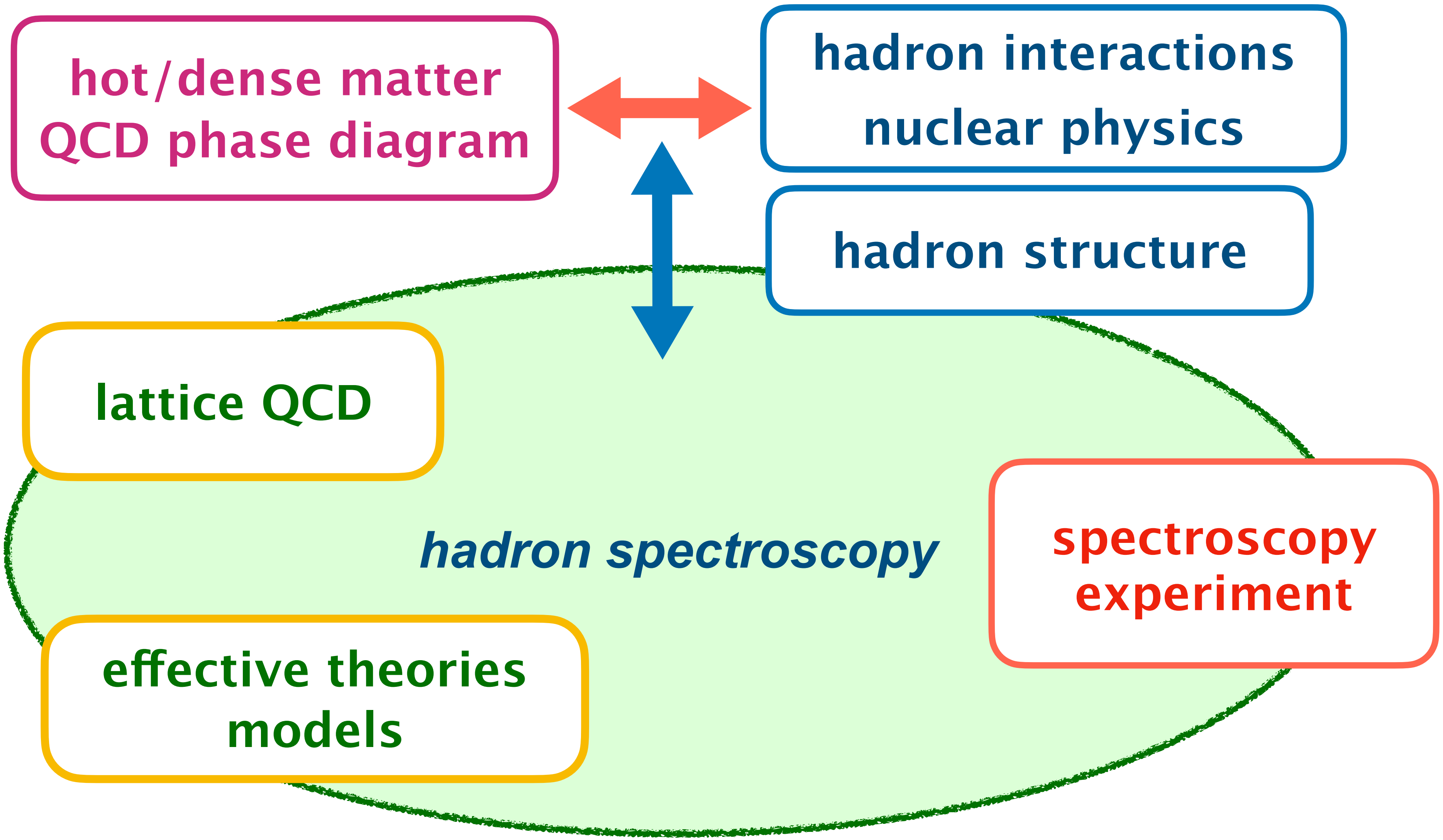
Compact Stars in the QCD Phase Diagram

October 10, 2024 @ YITP, Kyoto

Compact stars and QCD phase diagram



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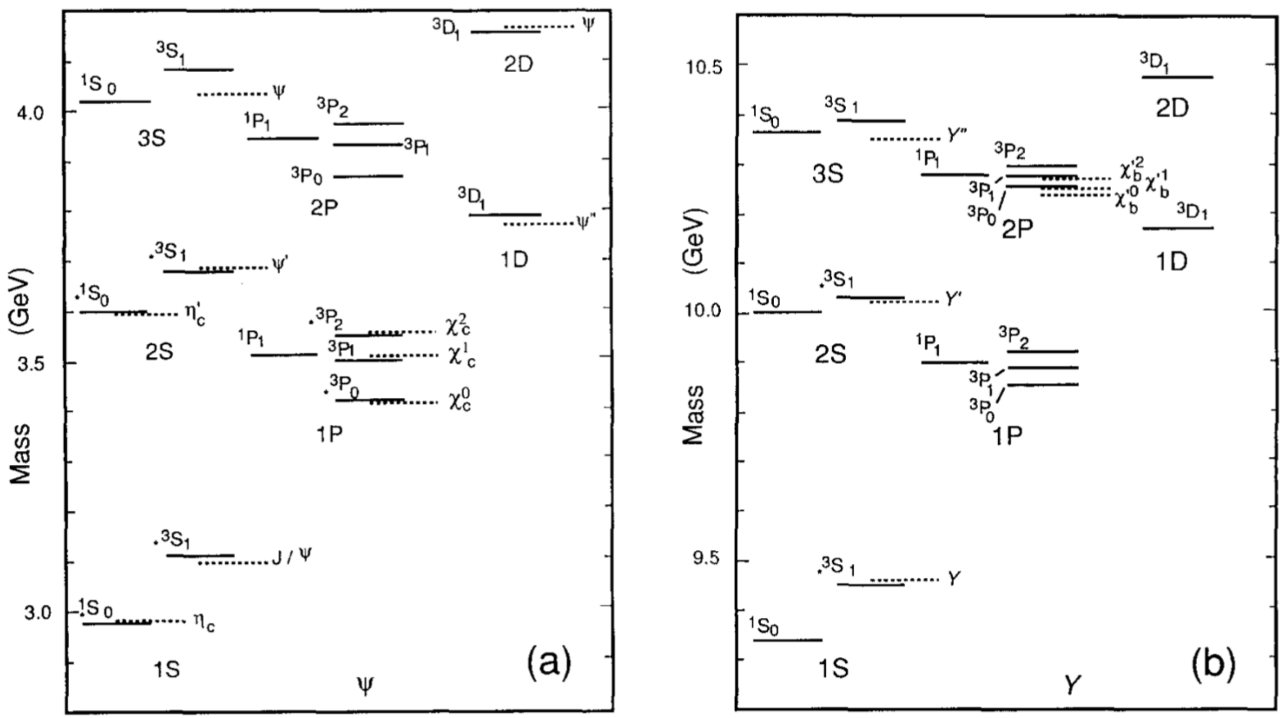
Current status of Quark Model

Quark model

- After 60 years since it was born, the Quark Model gives reliable guidelines to classify and interpret the hadron spectrum.
- The quarkonium ($c\bar{c}$, $b\bar{b}$) spectrum is *"hydrogen atom" in QCD.* A Linear + Coulomb potential fits 1S, 1P, 2S, . . $c\bar{c}$ and $b\bar{b}$ states.

$$V(r) = -\frac{e}{r} + \sigma r$$

E. Eichten, et al., PRL 34 (1975) 369



S.N. Mukherjee et al.
Phys. Rep. 231 (1993)

Constituent quark model

- **Quark model with constituent quark masses and Linear + Color Coulomb + Color Magnetic interactions ex. "AL1" potential by Silvestre-Brac, Few-Body Syst. 20, 1 (1996)**

$$H = \sum_i \left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i} \right) - K_G + \sum_{i<j} \frac{(\lambda_i \cdot \lambda_j)}{4} V_{ij}$$

$$V_{ij} = -\frac{3}{4} \left(\sigma r_{ij} - \frac{\alpha}{r_{ij}} + \frac{2\pi\alpha'}{3m_i m_j} f(r_{ij}, r_{0ij}) (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) - \Lambda \right)$$

$$f(r, r_0) = \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \qquad r_{0ij} = A \left(\frac{2m_i m_j}{m_i + m_j} \right)^{-B}$$

***10 parameters* (for *L*=0)**

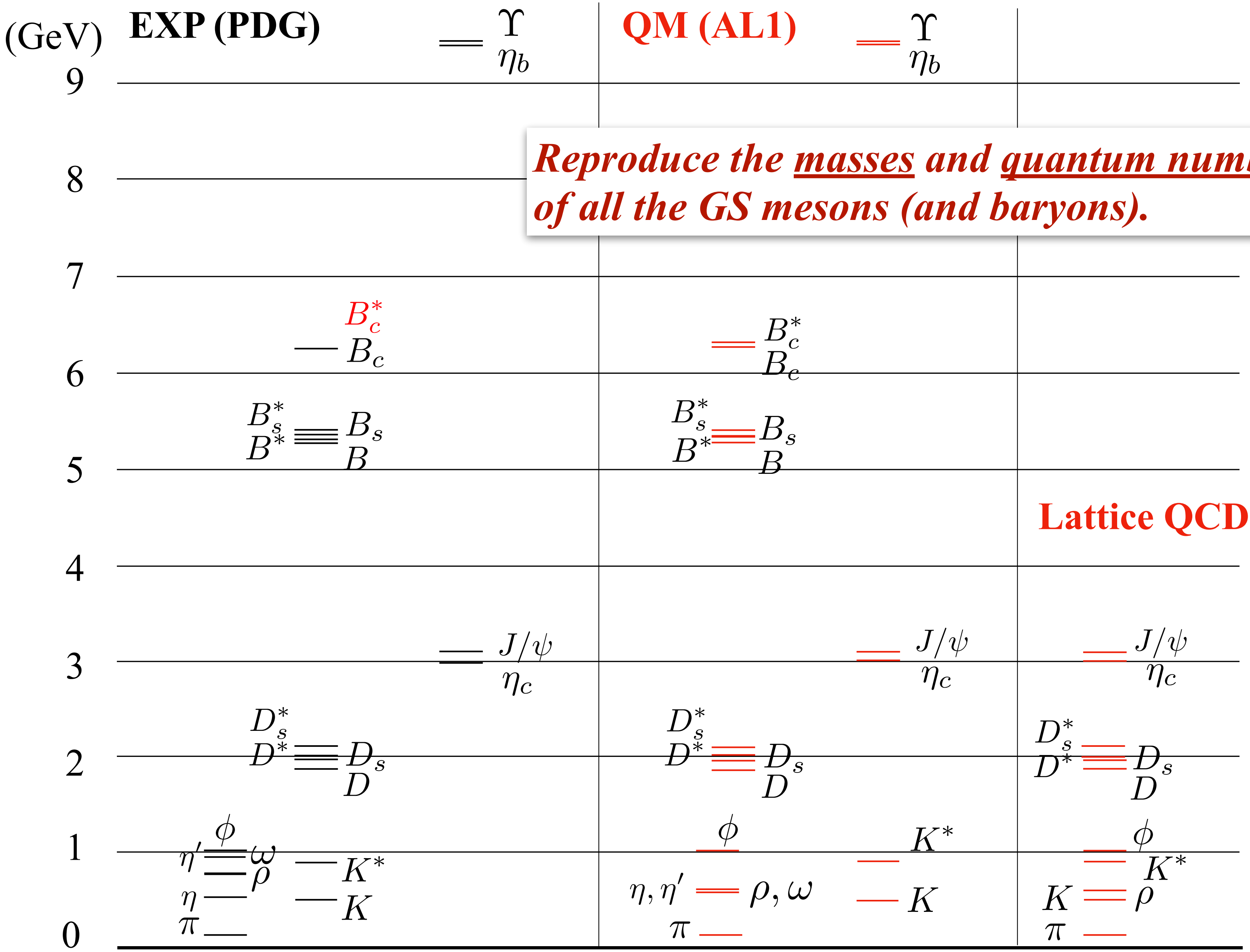
$$m_{u/d} = 0.315\text{GeV}, m_s = 0.577\text{GeV}, m_c = 1.836\text{GeV}, m_b = 5.227\text{GeV}$$

$$\sigma = 0.1653\text{GeV}^2, \alpha = 0.5069, \alpha' = 1.8609$$

$$B = 0.2204\text{GeV}, A = 1.6553\text{GeV}^{B-1}, \Lambda = 0.8321\text{GeV}$$

GS Meson Spectrum

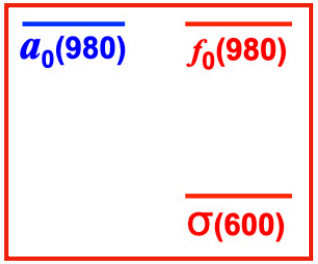
30 GS mesons (29 observed)



Reproduce the <u>masses</u> and <u>quantum numbers</u> of all the GS mesons (and baryons).

Difficulties in excited states

- **The lowest scalar (0^{++}) nonet has a hierarchy structure not consistent with the P-wave $q\bar{q}$ ($^3P_0$) excited states.**



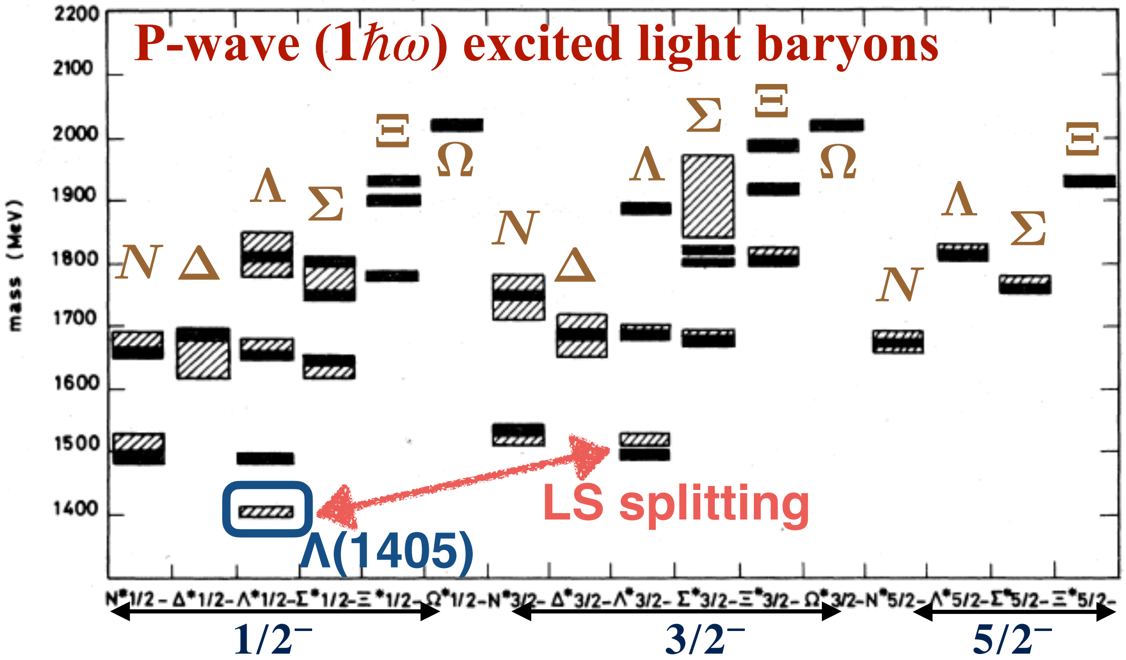
$$m(f_0) < m(a_0) \sim m(f_0')$$

Tetra-quark picture, R.L. Jaffe, PRD15, 267 (1977)

$$f_0 = ud\bar{u}\bar{d} \qquad f_0' = \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} + ds\bar{d}\bar{s}) \qquad a_0^0 = \frac{1}{\sqrt{2}}(us\bar{u}\bar{s} - ds\bar{d}\bar{s})$$

- **Λ(1405) (1/2-)**
  Isgur-Karl potential model
  "Shell model of hadrons"

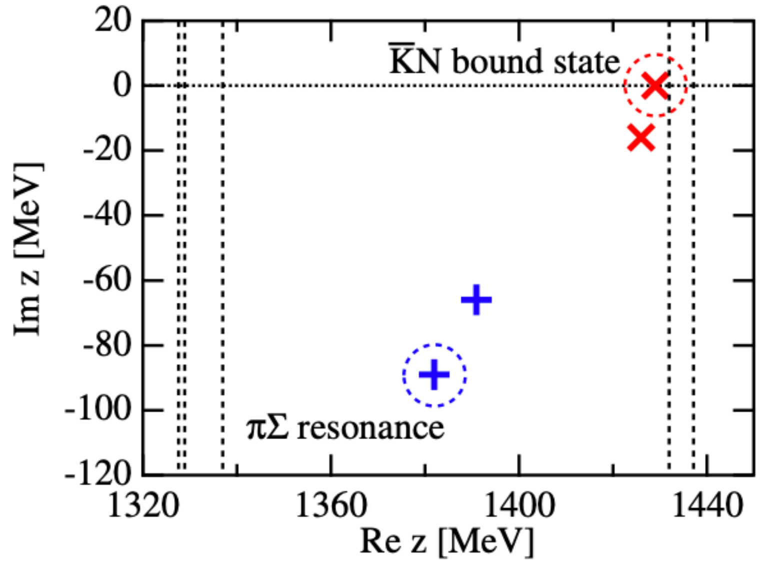
  Lowest P-wave baryon
  Large LS splitting
  (1/2- ↔ 3/2-)



Difficulties in excited states

- **Intense researches on the nature of $\Lambda(1405)$ conclude**
  $\Lambda(1405) \sim N\bar{K}\,(I=0, L=0)$ **bound state.**

  R.H. Dalitz, S.F. Tuan, PRL 2, 425 (1959)
  T. Hyodo, D. Jido, PPNP 67, 55 (2012)

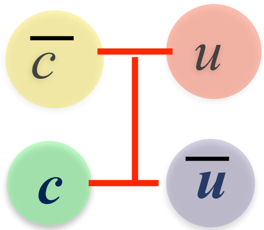


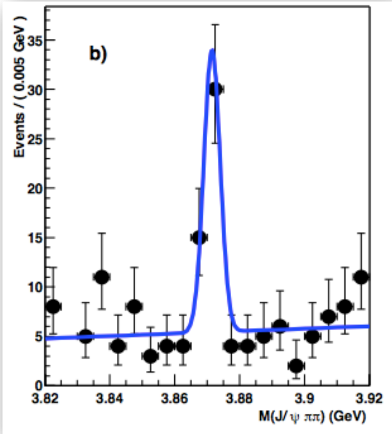
- **A *common origin* of difficulties**
  competition: orbital L=1 excitation *v.s.* extra $q\bar{q}$
  scalar meson nonet $q\bar{q}\,(L=1, 0^+)$ ***v.s.*** $qq\bar{q}\bar{q}\,(L=0, 0^+)$
  negative parity baryon $qqq\,(L=1, 1/2^-)$ ***v.s.*** $qqqq\bar{q}\,(L=0, 1/2^-)$

Exotic Multiquark Hadrons

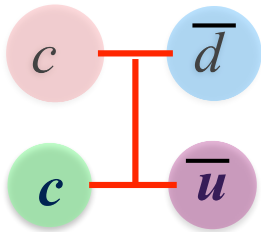
- Hadrons which do not fit the simple quark model picture

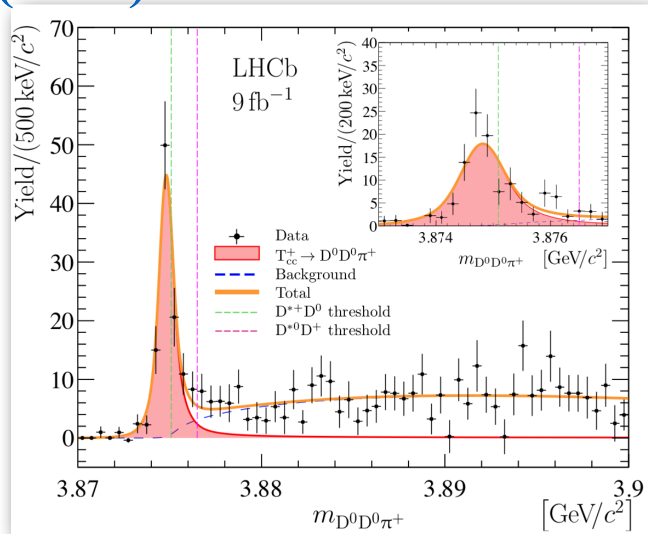
X(3872) ***Belle***
PRL 91 (2003)



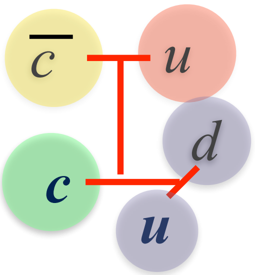


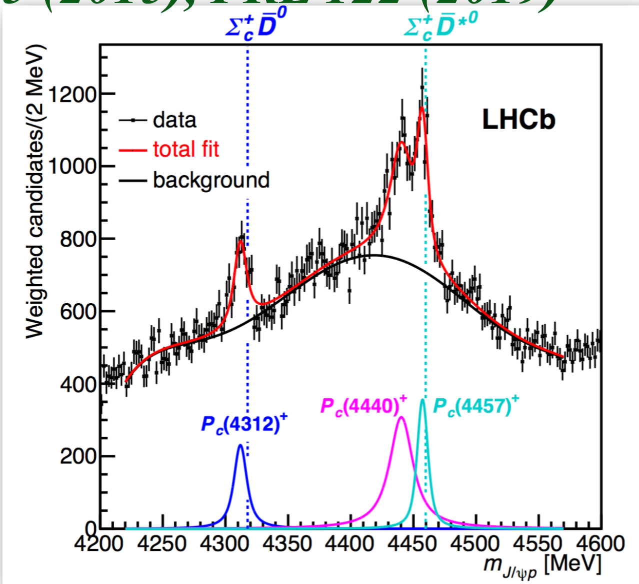
T_{cc} ***LHCb*** **(2022)**

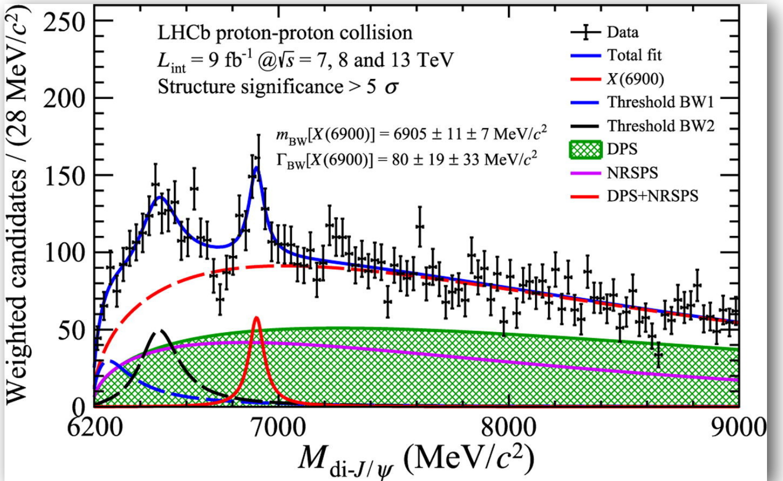




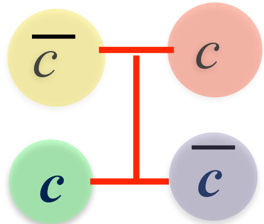
P_c (4312) (4440) (4457) ***LHCb***
PRL 115 (2015), PRL 122 (2019)







X **(6900)**
LHC **(2017-)**



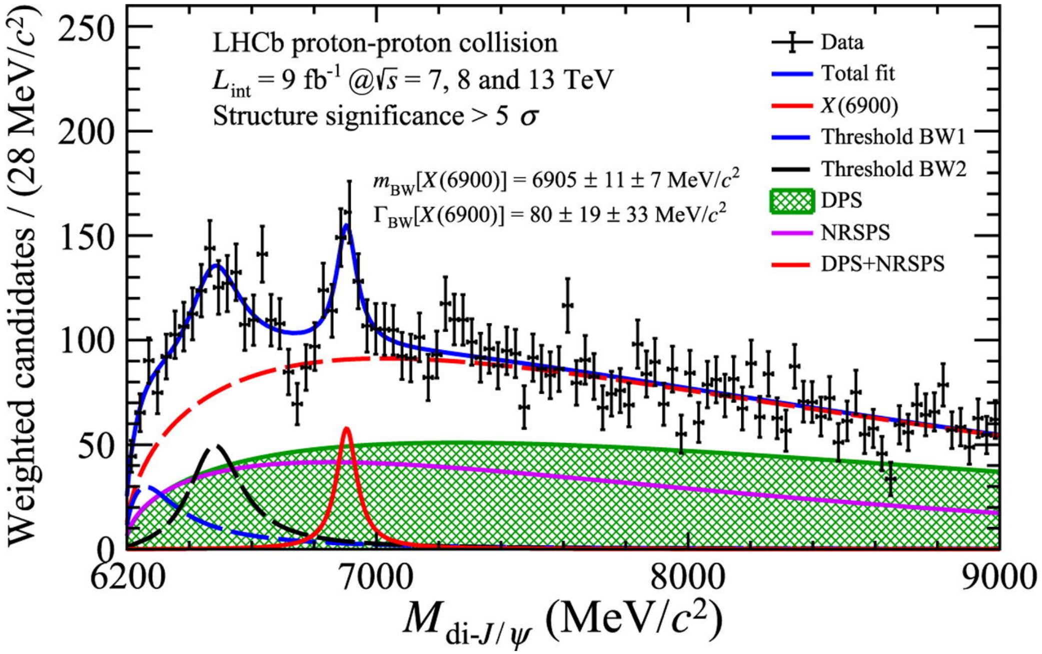
Exotic Multiquark Hadrons

- ***Fully charmed tetra-quark resonances*** $X(cc\bar{c}\bar{c}) \to J/\psi + J/\psi$

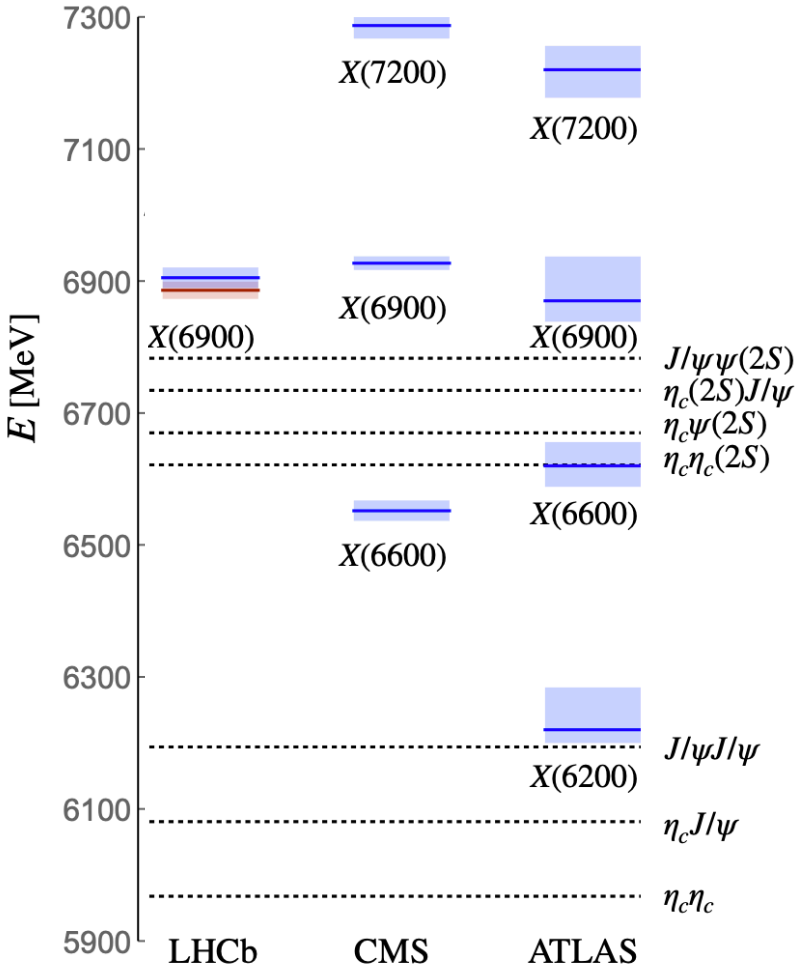
  observed at LHC (quantum numbers unknown)

$m_{BW}[X(6900)] = 6905 \pm 11 \pm 7\ \mathrm{MeV}/c^2$

$\Gamma_{BW}[X(6900)] = 80 \pm 19 \pm 33\ \mathrm{MeV}/c^2$

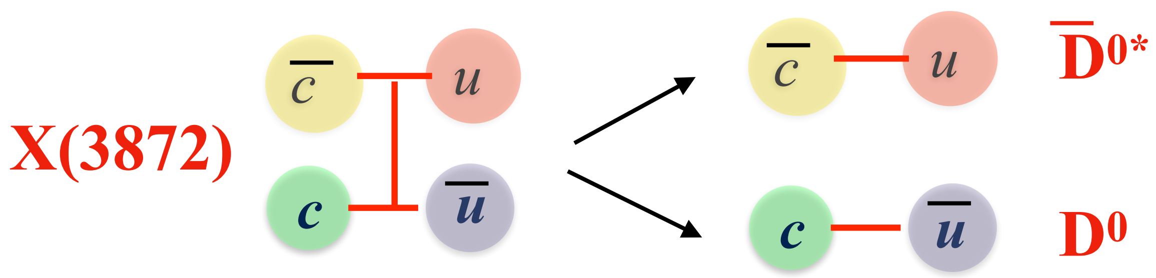


Two J/ψ spectrum



Current status summary

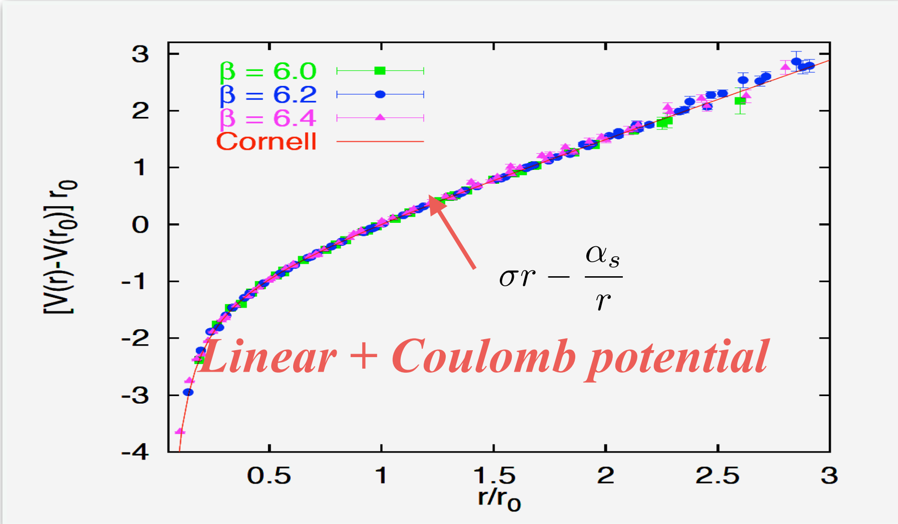
- Quark model works excellently for the ground states.
- Many "exotic" resonances appear at around the threshold of meson production. Couplings to two-hadron thresholds are critically important in understanding hadron spectra and structures.
- How is a multi-quark state coupling to two-hadron scattering states?
  We focus on the *Confinement Mechanism of Multi-Quark Systems.*



Confinement in Multiquark systems

Quark Confinement

- Heavy $Q\bar{Q}$ potential
  Linear confinement
  + Coulomb potential
  $V(r) = \sigma r - \frac{\alpha_s}{r}$
  well reproduced by lattice QCD



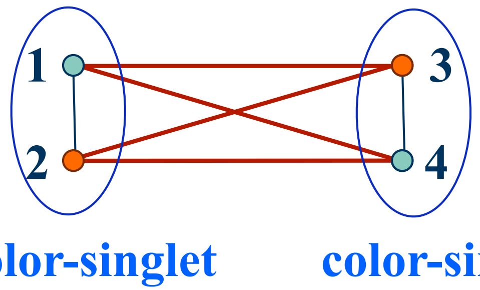
G.S. Bali / Phys. Rep. 343 (2001) 1

- *Naive generalization* to multiquark systems
  Sum of two-body linear potential with color saturation

$$V = \sum_{i<j} (\lambda_i \cdot \lambda_j)(-a r_{ij}) \qquad (\lambda_i \cdot \lambda_j) = \sum_\alpha \lambda_i^\alpha \lambda_j^\alpha$$

string tension: $\sigma = \frac{16}{3} a \sim 1 \text{ GeV/fm}$

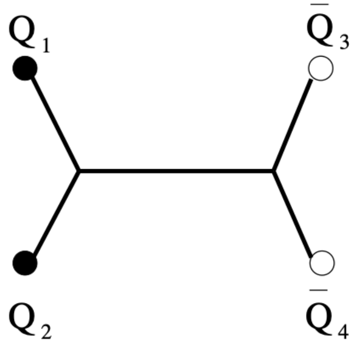
No confinement force between color singlet hadrons

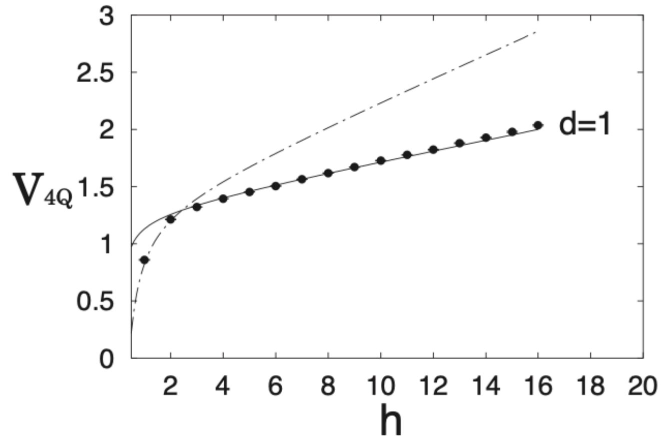


Quark Confinement

- **Lattice QCD gives "string confinement" for tetraquarks**
  The string with the minimal length connects quarks into color singlet hadron.

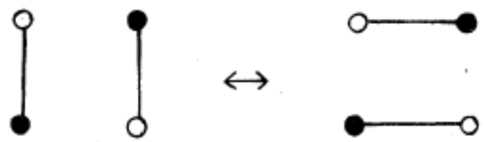
Lattice QCD Wilson loop for tetra quarks
F. Okiharu, et al., J. Mod. Phys. 7 ,774–789 (2016)





- **"String Flipflop and Quark Matter",**
  H. Miyazawa, Phys. Rev. D20, 2953 (1979)
  with reconnections of strings according to the spatial configuration of the quarks

$$V_{\text{string}} = \sigma \times \underset{\text{links}}{\text{Min}} \sum r_{\text{link}}$$

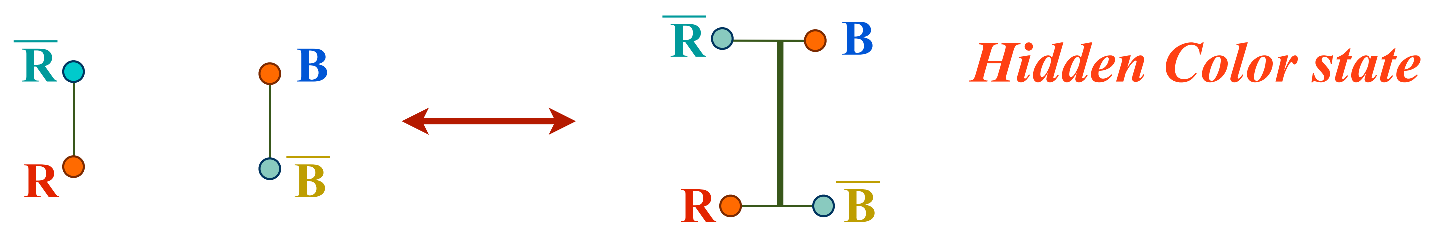


String confinement model

- Similar "string-type" confinement potential models for multi-quark systems were discussed by

  O.W. Greenberg, J. Hietarinta, Phys. Lett. B 86, 309 (1979)
  N. Isgur, J. E. Paton, Phys. Lett. B 124, 247 (1983)
  M. Oka, Phys. Rev. D 31, 2274 (1985).
  F. Lenz, et al., Annals Phys. 170, 65 (1986).
  Y. Koike et al., Nucl. Phys. A449, 635 (1986), PTP S137, 21 (2000).
  G.A. Miller, Phys. Rev D37, 2431 (1998).
  J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 85, 014019 (2012).

- The string FF model works well for the U(1) charge, but for the color SU(3) theory, color recombination becomes nontrivial. Introduction of hidden color states is unavoidable.



Quark confinement for tetraquarks

- **Reconsider the quark confinement potential for the quark model from the QCD viewpoints.**
- **We propose a "string-like confinement" by extending the color SU(3) configuration space of the conventional quark model. Roles of hidden-color (HC) states are examined.**
- **We test the model at the *fully charmed* ($cc\bar{c}\bar{c}$) *tetraquarks*.**

PHYSICAL REVIEW D **108,** L071501 (2023)

Letter

Quark confinement for multiquark systems: Application to fully charmed tetraquarks

Guang-Juan Wang,[1,2,*] Makoto Oka,[3,2,†] and Daisuke Jido[4,‡]

Quark model - color configurations

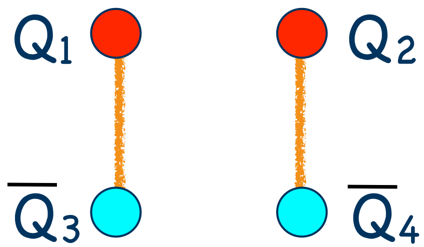
- **Only two independent color states are allowed in Quark Model.**

$$\mathbf{3}\otimes\mathbf{3}\otimes\bar{\mathbf{3}}\otimes\bar{\mathbf{3}} = 2\times\mathbf{1}\oplus 4\times\mathbf{8}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

- **Color singlet $Q_1\, Q_2\, Q_3^{bar}\, Q_4^{bar}$ system is described by**

Two singlets (mesons) states:

$$|1\rangle = |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle \quad |1'\rangle = |(Q_1\bar{Q}_4)_{\mathbf{1}}(Q_2\bar{Q}_3)_{\mathbf{1}}\rangle$$

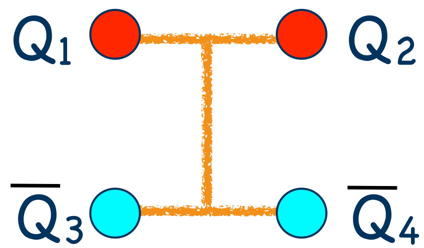


Singlet + hidden color states:

$$|1\rangle = |(Q_1\bar{Q}_3)_{\mathbf{1}}(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle \quad |8\rangle = |(Q_1\bar{Q}_3)_{\mathbf{8}}(Q_2\bar{Q}_3)_{\mathbf{8}}\rangle$$

Diquarks with color 3^{bar} and 6:

$$|3\rangle = |(Q_1Q_2)_{\bar{\mathbf{3}}}(\bar{Q}_3\bar{Q}_4)_{\mathbf{3}}\rangle \quad |6\rangle = |(Q_1Q_2)_{\mathbf{6}}(\bar{Q}_3\bar{Q}_4)_{\bar{\mathbf{6}}}\rangle$$



- ***These bases are all equivalent.***

$$|1\rangle = \sqrt{\frac{1}{3}}|3\rangle + \sqrt{\frac{2}{3}}|6\rangle \qquad |8\rangle = -\sqrt{\frac{2}{3}}|3\rangle + \sqrt{\frac{1}{3}}|6\rangle$$

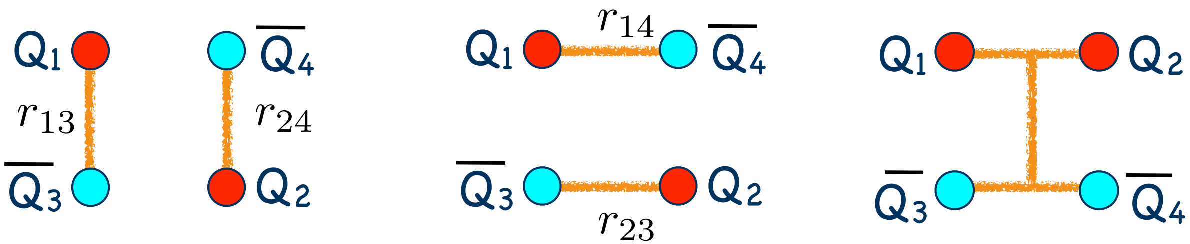
- **Two-meson states are not orthogonal.** $\langle 1|1'\rangle = \dfrac{1}{3}$

Novel string-like potential

- **If the quarks are the only carriers of color charges, the quark model does not have enough *freedom for color configurations*.**
- **We propose to extend the color Hilbert space of the quark model that can describe the color dynamics for multiquark systems.**

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)

- **For a tetra-quark systems, we choose 3 color basis states:**



$$|\mathbf{1}\rangle\rangle \equiv |(Q_1 \to \bar{Q}_3)_1(Q_2 \to \bar{Q}_4)_1\rangle$$
$$|\mathbf{1}'\rangle\rangle \equiv |(Q_1 \to \bar{Q}_4)_1(Q_2 \to \bar{Q}_3)_1\rangle$$
$$|\mathbf{hc}\rangle\rangle \equiv |(Q_1 \leftrightarrow Q_2)_{\bar{3}} \leftarrow (\bar{Q}_3 \leftrightarrow \bar{Q}_4)_3\rangle$$

orthogonal bases

Novel string-like potential

- **The string confinement potential**

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{1}\rangle\rangle = \sigma(r_{13}+r_{24}),$$
$$\langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{1}'\rangle\rangle = \sigma(r_{14}+r_{23}).$$

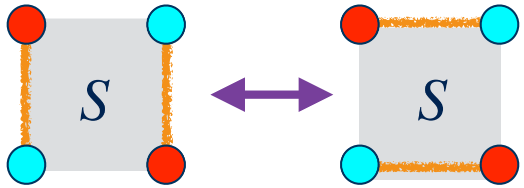
σ: **string tension**

- **Transitions by quantum tunneling filled the area by gauge field**

$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{1}'\rangle\rangle = \kappa e^{-\sigma S}$$

S: **Minimal surface area**

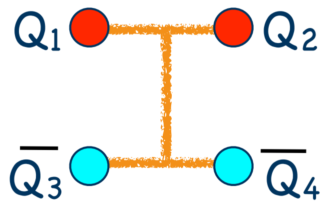
Y. Koike, O. Morimatsu, K. Yazaki, PTP S137, 21 (2000)



- **Confinement in $|\mathbf{hc}\rangle\rangle$ channel**

$$\langle\langle \mathbf{hc}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \sigma\left[\frac{r_{13}+r_{24}+r_{14}+r_{23}}{4}+\frac{r_{12}+r_{34}}{2}\right]$$
$$\langle\langle \mathbf{1}|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \langle\langle \mathbf{1}'|V_{\rm ST}|\mathbf{hc}\rangle\rangle = \pm\kappa'\exp(-\sigma S)$$

$\kappa' = \sqrt{8}\kappa$



- **3-channel confinement potential (full 4-body potential)**

$$V_{\rm ST} = \begin{pmatrix} \sigma(r_{13}+r_{24}) & \kappa e^{-\sigma S} & \kappa' e^{-\sigma S} \\ \kappa e^{-\sigma S} & \sigma(r_{14}+r_{23}) & -\kappa' e^{-\sigma S} \\ \kappa' e^{-\sigma S} & -\kappa' e^{-\sigma S} & \frac{\sigma}{4}[r_{13}+r_{24}+r_{14}+r_{23}+2(r_{12}+r_{34})] \end{pmatrix} \begin{matrix} |\mathbf{1}\rangle\rangle \\ |\mathbf{1}'\rangle\rangle \\ |\mathbf{hc}\rangle\rangle \end{matrix}$$

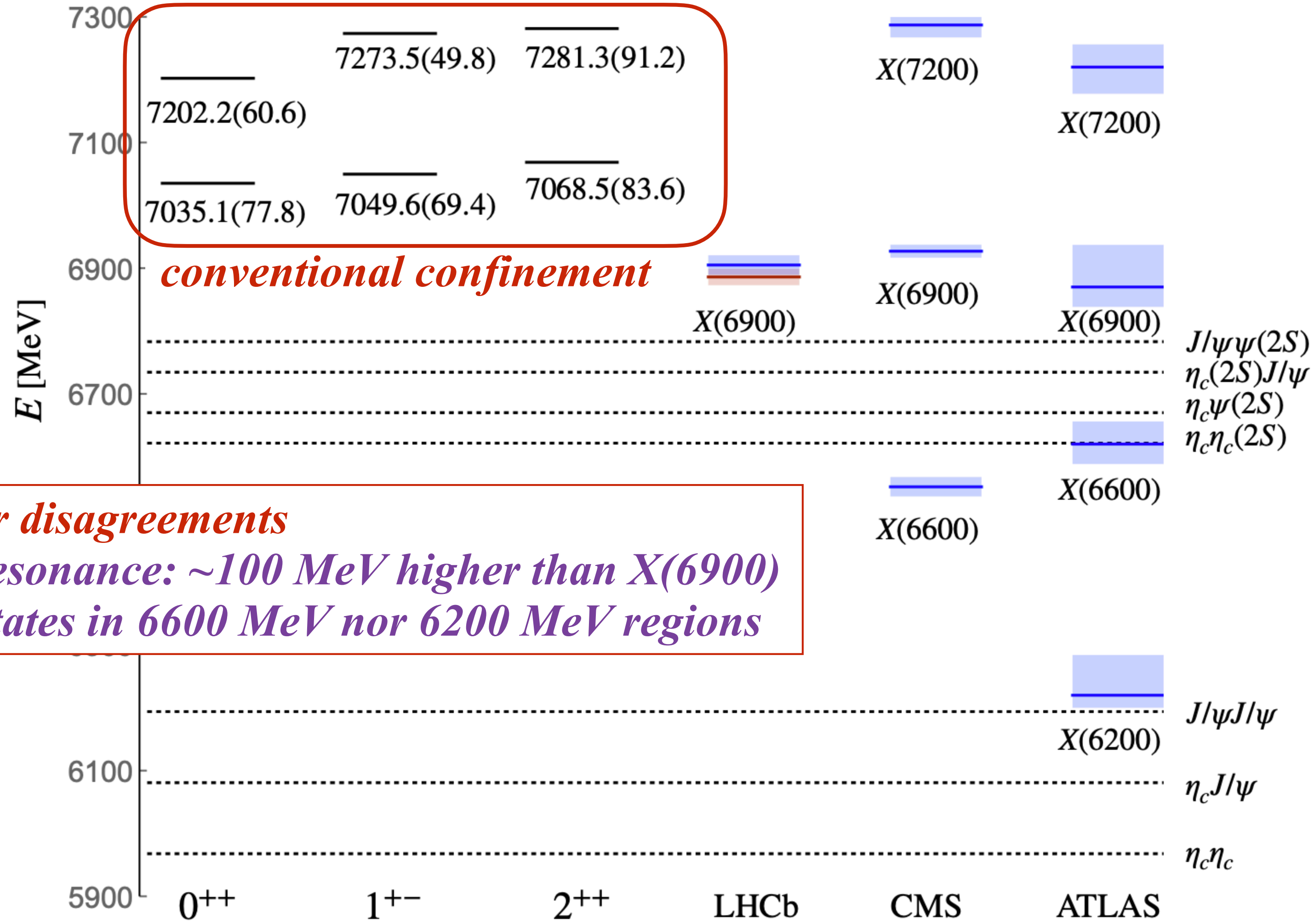
Application to Fully-charmed Tetraquarks

Full solutions (bound and resonance states) in complex scaling method are obtained for the S-wave $cc\bar{c}\bar{c}$ tetra-quark systems (0^{++},1^{+-},2^{++}).

Tetra-charms with conventional confinement

- **Conventional confinement for $cc\bar{c}\bar{c}$ with complex scaling**

G.J. Wang, Q. Meng, MO, PR D106 (2022) 096005



Clear disagreements

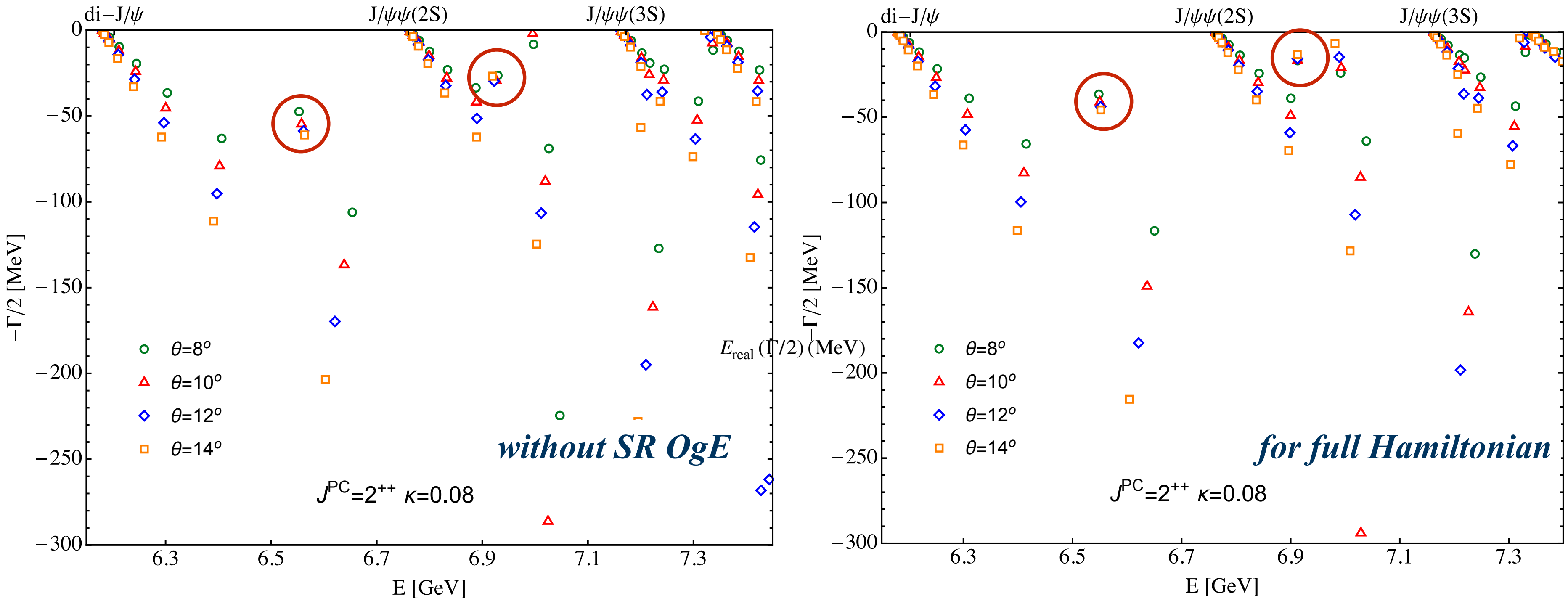
1st resonance: ~100 MeV higher than X(6900)

No states in 6600 MeV nor 6200 MeV regions

Tetra-charms with novel confinement

- Complex scaling method is employed for resonances

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)

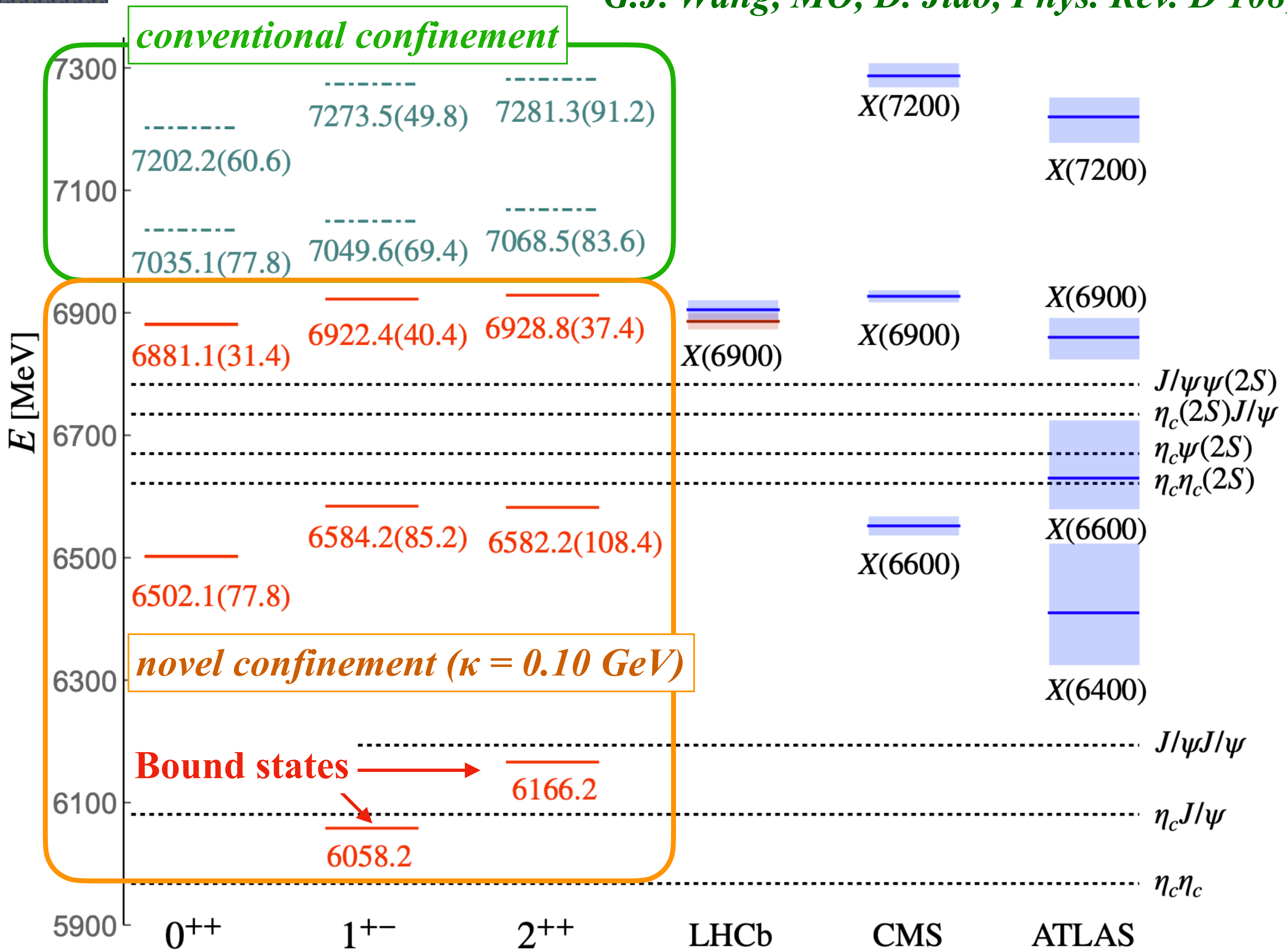


- **The SR OgE interactions play minor roles.**

$$V_{\rm cen}(r_{ij}) = V_{\rm Coul} + V_{\rm hyp} = \frac{\alpha_s}{r_{ij}} - \frac{8\pi\alpha_s}{3m_i m_j}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r_{ij}^2}\mathbf{s}_i\cdot\mathbf{s}_j$$

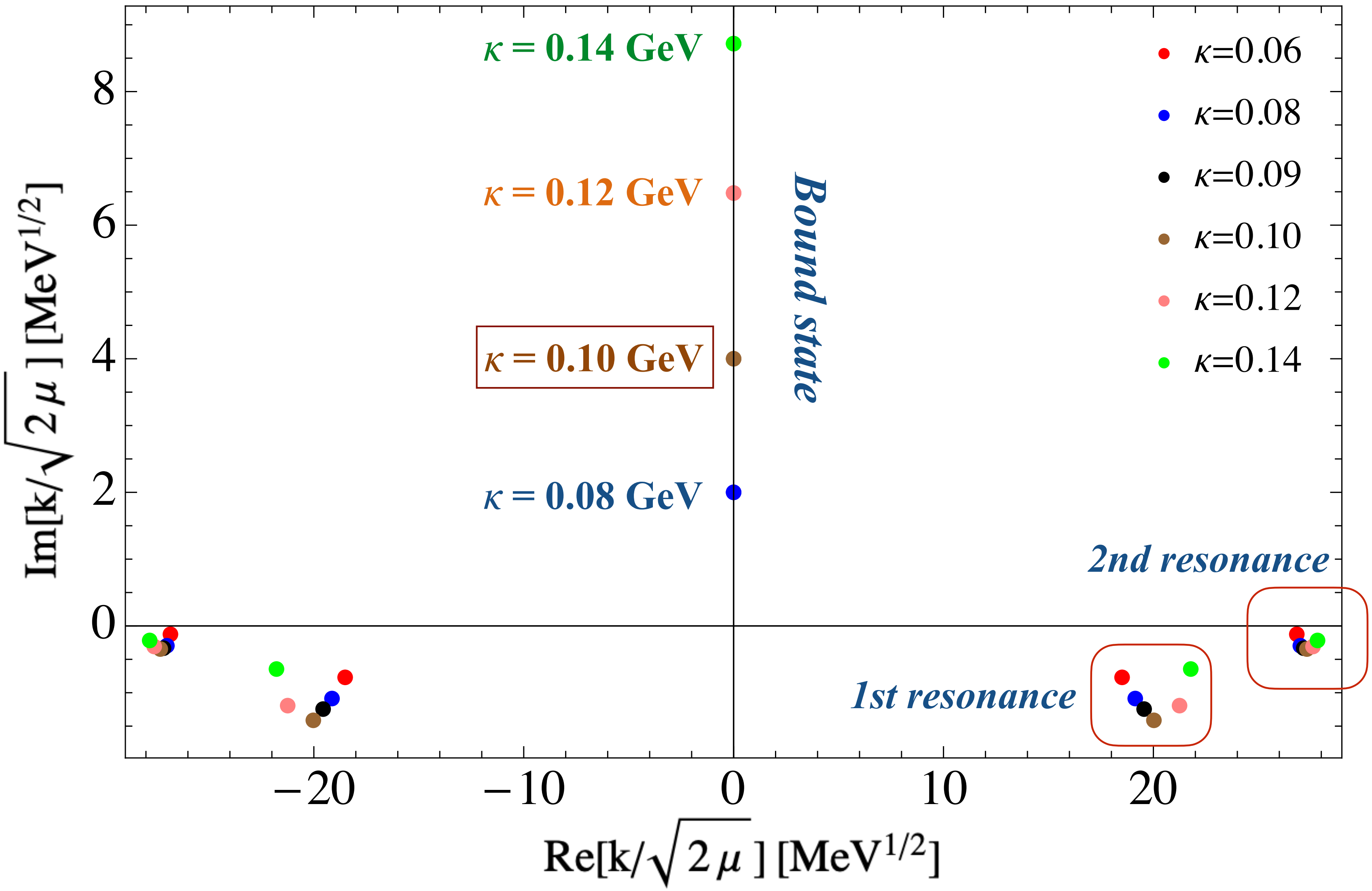
$cc\bar{c}\bar{c}$ spectrum with novel confinement

G.J. Wang, MO, D. Jido, Phys. Rev. D 108, L071501 (2023)



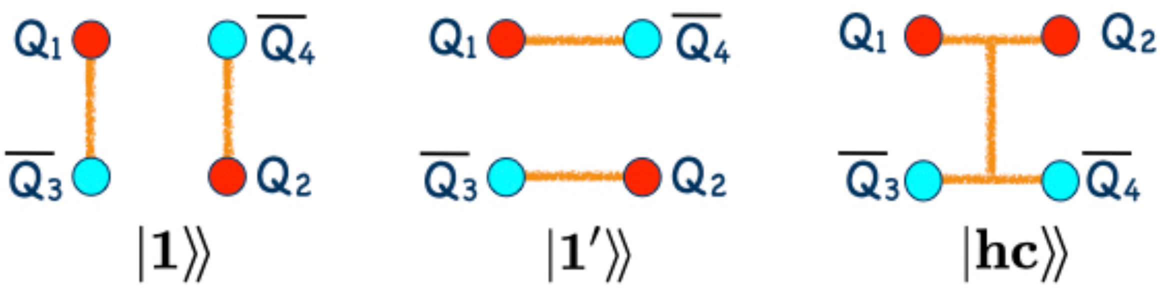
$cc\bar{c}\bar{c}$ spectrum with novel confinement

- **Pole positions of the 2^{++} charmed tetraquark in complex k plane**



Further thoughts

- Confinement in the multi-quark systems is not trivial nor well established from the quark model viewpoints.
- We propose string-like confinement potential with a compact hidden-color state.



- The model can be extended to N=5, 6, . . ., but the multi-body forces are getting complicated. To extend the model to quark matter, we will need to consider "clustering" of quarks within which color confinement is at work.

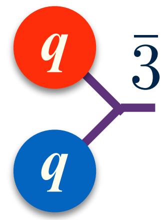
Diquarks in Heavy Baryons

Diquark

- **Diquark:** strong color correlation between quarks

  S-wave color $\bar{3}$ diquarks: S(0^+) and A(1^+)

  color $3 \otimes 3 = \bar{3} \oplus 6$ **spin :** $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$



- **Spin dependent force from magnetic gluon exchange predicts strong attraction in S(0^+).**

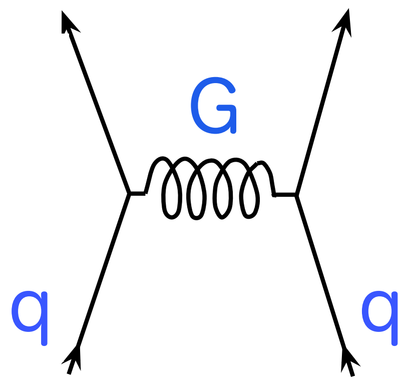
  Color-Magnetic Interaction $\Delta_{\rm CM} \equiv \langle -\sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \rangle$

  S(0^+) color $\bar{3}$ $\Delta_{\rm CM} = -8$ ***aka good diquark***

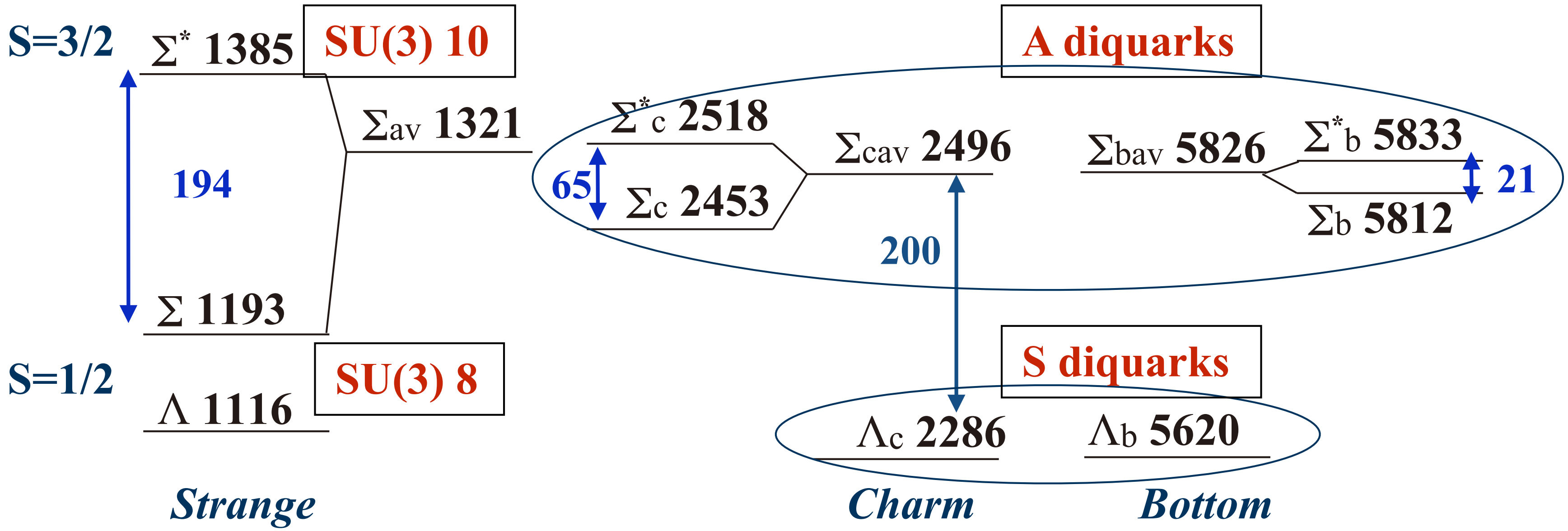
  A(1^+) color $\bar{3}$ $\Delta_{\rm CM} = +8/3$ ***aka bad diquark***

  M(A)-M(S) = (2/3) [M(Δ)- M(N)] ~ 200 MeV

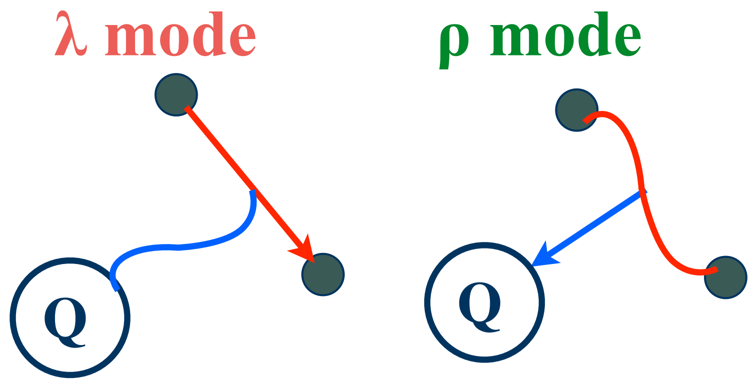
  consistent with the splitting of $\Lambda_c - \Sigma_c$



Diquarks in Heavy Baryons



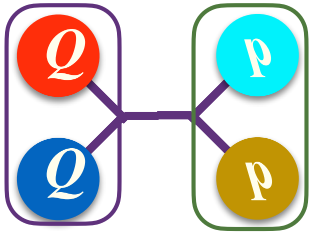
- **Excited states have two distinct modes**
  These two modes are separated
  The λ modes are favored
  in the singly heavy baryons

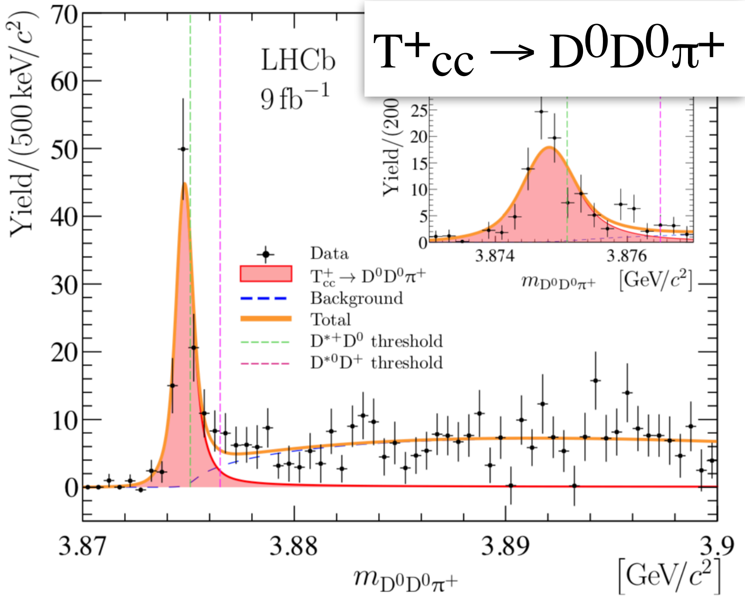


Quark model calculation by Yoshida, et al., PRD 92, 114029 (2015)

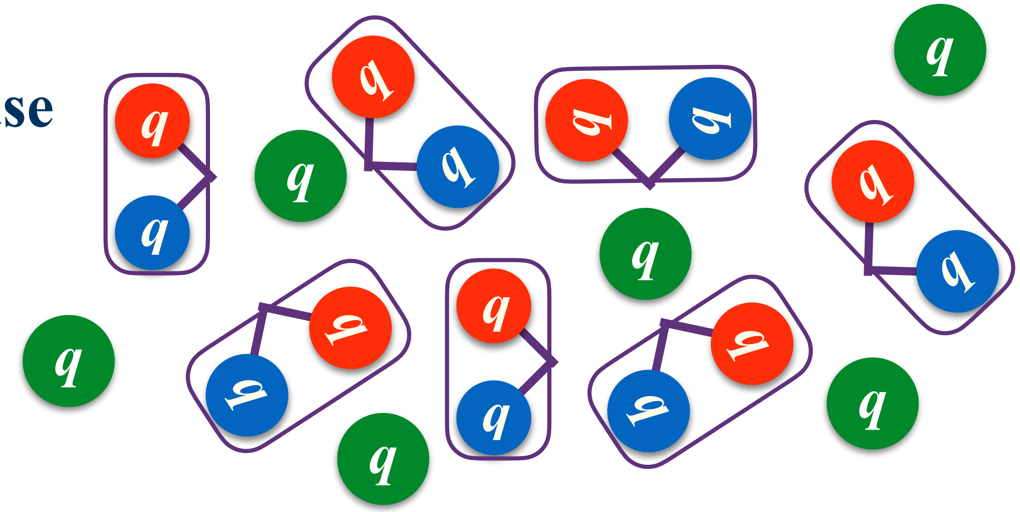
Diquarks in exotic hadrons/matter

- **Diquark may form doubly heavy tetraquark bound states.**
  $T_{QQ} = QQ\bar{q}\bar{q}$





- **Diquarks may form BE condensate in dense hadronic matter.**
  => color-superconducting phase



Diquark in Lattice QCD

- **Hess, Karsch, Laermann, Wetzorke, PR D58, 111502 (1998)**
  quench, Landau gauge fixed
  $M(0^+) \sim 694$ MeV, $M(1^+) \sim 810$ MeV
- **Alexandrou, de Forcrand, Lucini, PRL 97, 222002 (2006)**
  From Qqq system, quench, gauge invariant
  $M(1^+) - M(0^+) \sim 200\text{-}220$ MeV, $R(S) \sim 1$ fm
- **Babich, et al., PR D76, 074021 (2007)**
  quench, Landau gauge
  $M(1^+) - M(0^+) \sim 162$ MeV, $M(0^+) - 2m_q \sim -200$ MeV
- **Yujiang Bi, et al., Chinese Physics C40 (2016) 073106**
  full QCD, Landau gauge
  $M(1^+) - M(0^+) \sim 290$ MeV, $M(0^+) - m_q \sim 310$ MeV
- **K. Watanabe, Phys. Rev. D105 (2022) 074510**
  quark-diquark potential and diquark mass

Chiral effective theory of Diquarks

Chiral Effective Theory of Diquarks

- Goal: to explore properties of *light diquarks* under SU(3)×SU(3) chiral symmetry and answer questions such as

What are the chiral partners of diquarks and their implications to hadron spectroscopy?

How can we observe the chiral properties of diquarks?

What are the roles of $U(1)_A$ anomaly in diquark interactions?

How do diquarks decay strongly?

How do diquarks behave in matter, where chiral symmetry is partially restored?

Chiral Effective Theory of Diquarks

M. Harada, Y.R. Liu, M.O., K. Suzuki, "*Chiral effective theory of diquarks and U_A(1) anomaly*", Phys. Rev. D 101, 054038 (2020)

Y. Kim, E. Hiyama, M.O., K. Suzuki, "*Spectrum of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 102, 014004 (2020)

Y. Kawakami, M. Harada, M.O., K. Suzuki, "*Suppression of decay widths in singly heavy baryons induced by the U_A(1) anomaly*", Phys. Rev. D 102, 114004 (2020)

Y. Kim, Y.R. Liu, M.O., K. Suzuki, "*Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*", Phys. Rev. D 104, 054012 (2021)

Y. Kim, M.O., K. Suzuki, "*Doubly heavy tetraquarks in a chiral-diquark picture*", Phys. Rev. D 105, 074021 (2022)

Y. Kim, M.O., D. Suenaga, K. Suzuki, "*Strong decays of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D 107, 074015 (2023)

D. Suenaga, M.O., "*Axial anomaly effect to the chiral-partner structure of diquarks at high temperature*", Phys. Rev. D 108, 014030 (2023)

H. Takada, D. Suenaga, M. Harada, A. Hosaka, M.O., "*Axial anomaly effect on three-quark and five-quark singly heavy baryons*", Phys. Rev. D 108, 054033 (2023)

Chiral Effective Theory of Diquarks

- Some previous works on ChET for diquarks:
  D.K. Hong, Y.J. Sohn, I. Zahed, Phys. Lett. B596 (2004) 191, Int. J. Mod. Phys. A27, 1250051 (2012), *Non-linear chiral diquark effective theory for penta/tetraquarks*
  T. Hatsuda, M. Tachibana, N. Yamamoto, G. Baym, Phys. Rev. Lett. 97, 122001 (2006), Phys. Rev. D76, 074001 (2007), *Chiral effective theory and the axial anomaly in dense QCD*
- Chiral effective theories of single heavy baryons
  D. Ebert, T. Feldmann, C. Kettner, H. Reinhardt, Zeit. Phys. C71, 329–335 (1996), *Diquark model for single heavy baryons*
  Y. Kawakami, M. Harada, Phys. Rev. D97, 114024 (2018), Phys. Rev. D99, 094016 (2019), *Chiral effective theory of single heavy baryons*
  D. Suenaga, A. Hosaka, PR D104 (2021) 034009, Phys. Rev. D105, 074036 (2022), *Pentaquark picture for singly heavy baryons*

Chiral Effective Theory of Diquarks

⌗ Linear representation of chiral SU(3)$_R$ × SU(3)$_L$

$q^a_{\alpha i}$ $\quad a$ (color), α (Dirac), i (flavor)

$$q^a_{iR} = P_R\, q^a_i, \quad q^a_{iL} = P_L\, q^a_i \qquad P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}$$

$$q_R \to U_R\, q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R$$

$$q_L \to U_L\, q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L$$

⌗ Scalar diquarks (color $\bar{3}$)

$d^a_{iR} \equiv \epsilon_{ijk}(q^T_{jR}\, C\, q_{kR})^{\bar{3}}$ Right scalar diquark, chiral $(\bar{3},1)$, color $\bar{3}$

$d^a_{iL} \equiv \epsilon_{ijk}(q^T_{jL}\, C\, q_{kL})^{\bar{3}}$ Left scalar diquark, chiral $(1, \bar{3})$, color $\bar{3}$

⌗ Parity eigenstates: 0^+, 0^- diquarks

$$S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\gamma_5\, q_k)^{\bar{3}}$$

$$P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\, q_k)^{\bar{3}}$$

$$(\bar{3}, 1) + (1, \bar{3})$$

Scalar/Pseudoscalar Diquarks

- **The effective Lagrangian with $SU(3)_R \times SU(3)_L$ symmetry**

M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)

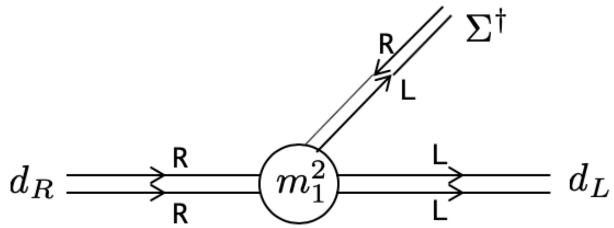
$$\mathcal{L} = \mathcal{D}_\mu d_{R,i} (\mathcal{D}^\mu d_{R,i})^\dagger + \mathcal{D}_\mu d_{L,i} (\mathcal{D}^\mu d_{L,i})^\dagger$$

$$-m_0^2 (d_{R,i} d_{R,i}^\dagger + d_{L,i} d_{L,i}^\dagger)$$ **chiral invariant**

$$-\frac{m_1^2}{f} (d_{R,i} \Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i} \Sigma_{ij} d_{R,j}^\dagger)$$ **$U_A(1)$ anomaly**

$$-\frac{m_2^2}{2f^2} \epsilon_{ijk} \epsilon_{\ell mn} (d_{R,k} \Sigma_{\ell i} \Sigma_{mj} d_{L,n}^\dagger + d_{L,k} \Sigma_{\ell i}^\dagger \Sigma_{mj}^\dagger d_{R,n}^\dagger)$$ **SCSB mass**

$$+\frac{1}{4} \mathrm{Tr} \left[\partial^\mu \Sigma^\dagger \partial_\mu \Sigma \right] + V(\Sigma).$$



$$\Delta Q_5 = \pm 6 \; (= \pm 2 N_f)$$

Scalar and PS nonets mesons

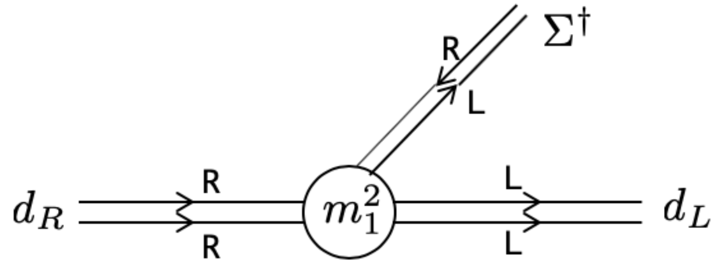
$$\Sigma_{ij} \equiv \sigma_{ij} + i \pi_{ij}$$

$U_A(1)$ anomaly

- **$U_A(1)$ anomaly in the diquark effective theory**

$$-\frac{m_1^2}{f}(d_{R,i}\Sigma^\dagger_{ij}d^\dagger_{L,j} + d_{L,i}\Sigma_{ij}d^\dagger_{R,j}) \qquad \Delta Q_5 = \pm 6\,(= \pm 2N_f)$$

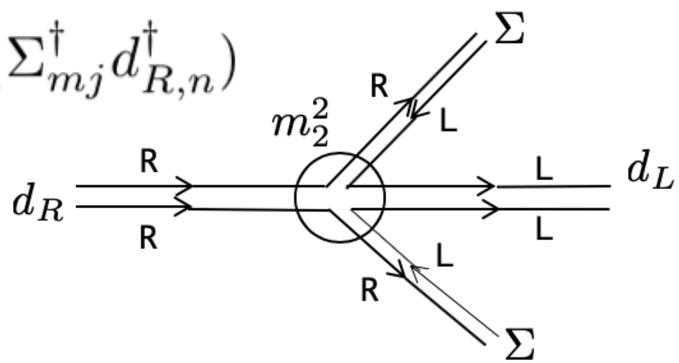
3 left quarks and 3 right antiquarks
flavor antisymmetric induces anomalous singlet current ($\Delta Q_5 = 6$)



$$\partial_\mu J_A^{\mu 0} = \frac{3m_1^2}{2}(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger)$$

- **non-anomalous term**

$$-\frac{m_2^2}{2f^2}\epsilon_{ijk}\epsilon_{\ell mn}(d_{R,k}\Sigma_{\ell i}\Sigma_{mj}d^\dagger_{L,n} + d_{L,k}\Sigma^\dagger_{\ell i}\Sigma^\dagger_{mj}d^\dagger_{R,n})$$



Scalar/Pseudoscalar Diquarks

- **For the SSB vacuum $\langle \Sigma_{ij} \rangle = f\delta_{ij}$ the mass term of the right and left diquarks are given by**

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 + m_2^2 \\ m_1^2 + m_2^2 & m_0^2 \end{pmatrix}$$

- **The mass eigenstates are**
  Scalar diquark

$$S_i^a = \frac{1}{\sqrt{2}}(d_{R,i}^a - d_{L,i}^a)$$

$$\longrightarrow M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2},$$

Pseudo-scalar diquark

$$P_i^a = \frac{1}{\sqrt{2}}(d_{R,i}^a + d_{L,i}^a)$$

$$\longrightarrow M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

Scalar/Pseudoscalar Diquarks

SU(3) breaking and inverse mass hierarchy

$$A \equiv 1+\epsilon \equiv \frac{f_s}{f_\pi}\left(1+\frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$$

***i*=3 (ud)**

$$M_3(0^+) = \sqrt{m_0^2 - Am_1^2 - m_2^2}, \qquad M_3(0^-) = \sqrt{m_0^2 + Am_1^2 + m_2^2}.$$

***i*=1,2 (ds), (us)**

$$M_1(0^+) = M_2(0^+) = \sqrt{m_0^2 - m_1^2 - Am_2^2}, \quad M_1(0^-) = M_2(0^-) = \sqrt{m_0^2 + m_1^2 + Am_2^2},$$

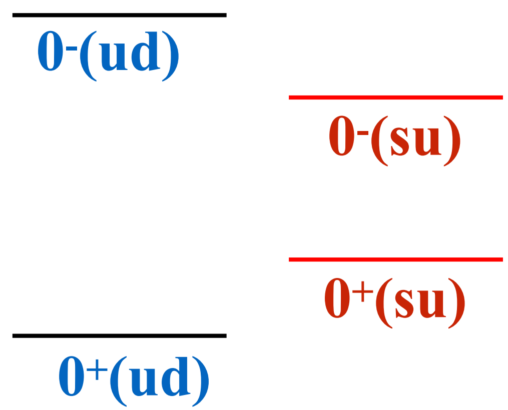
Inverse Mass Hierarchy due to the $U_A(1)$ anomaly

$$M_1(0^+)^2 + M_1(0^-)^2 = M_3(0^+)^2 + M_3(0^-)^2$$

$$M_1(0^+)^2 - M_3(0^+)^2 = M_3(0^-)^2 - M_1(0^-)^2 > 0$$

$$M_3(0^-) > M_1(0^-)$$

(ds), (us) **(ud)**

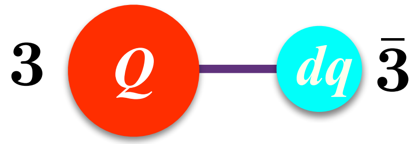


M. Harada, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D101, 054038 (2020)

Diquark-Heavy-Quark model

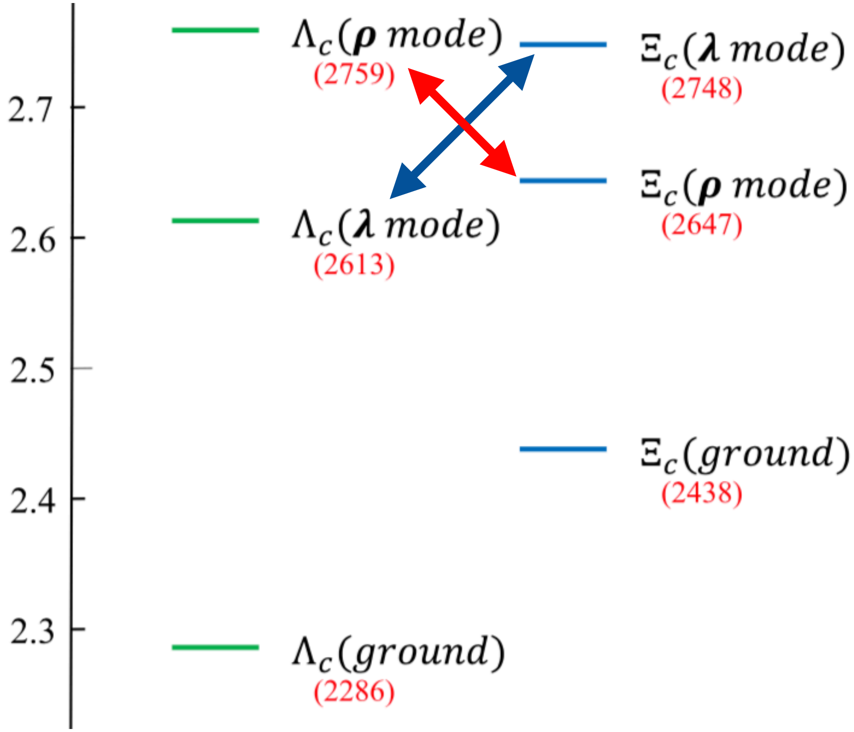
Single-Heavy-Baryon with a *Q-dq* potential:

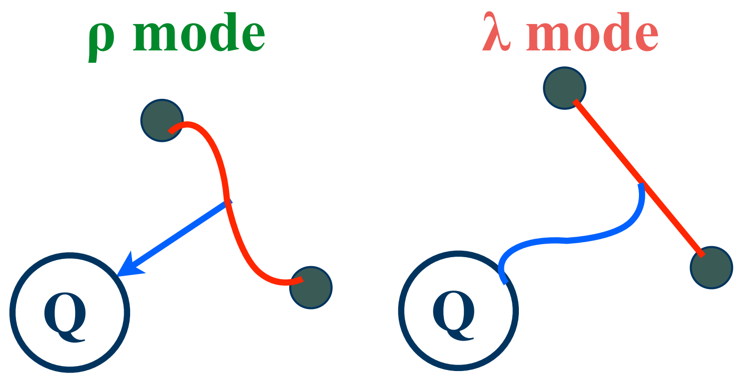
$$V(r) = -\frac{\alpha}{r} + \lambda r + C,$$



B. Silvestre-Brac, C. Semay, Z. Phys. C 59, 457 (1993)
T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, PR D 92, 114029 (2015)

Y. Kim, E. Hiyama, M. O., K. Suzuki, Phys. Rev. D 102, 014004 (2020)





Inverse mass hierarchy in the ρ mode due to the $U(1)_A$ anomaly.

Chiral effective theory

- **An additional term for the scalar diquark**

Y. Kim, M.O. K. Suzuki, in preparation

$$\begin{aligned}\mathcal{L}_S &= \mathcal{D}_\mu d_{R,i}(\mathcal{D}^\mu d_{R,i})^\dagger + \mathcal{D}_\mu d_{L,i}(\mathcal{D}^\mu d_{L,i})^\dagger - m_{S0}^2(d_{R,i}d_{R,i}^\dagger + d_{L,i}d_{L,i}^\dagger) \\ &\quad - \frac{m_{S1}^2}{f_\pi}(d_{R,i}\Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i}\Sigma_{ij}d_{R,j}^\dagger) \\ &\quad - \frac{m_{S2}^2}{2f_\pi^2}\epsilon_{ijk}\epsilon_{lmn}(d_{R,k}\Sigma_{li}\Sigma_{mj}d_{L,n}^\dagger + d_{L,k}\Sigma_{li}^\dagger\Sigma_{m,j}^\dagger d_{R,n}^\dagger) \\ &\quad + \frac{\mu_0^2}{f_\pi^2}\left(d_R\{\Sigma^\dagger\Sigma - \frac{1}{3}\mathrm{Tr}[\Sigma^\dagger\Sigma]\}d_R^\dagger + d_L\{\Sigma\Sigma^\dagger - \frac{1}{3}\mathrm{Tr}[\Sigma\Sigma^\dagger]\}d_L^\dagger\right).\end{aligned}$$

parameter sets with *varied $U(1)_A$ anomaly* effect

Cases	Baryon/Diquark masses (MeV)			$\mathcal{L}_S$ parameters (MeV^2)			
(Name)	$M_0 = M_\rho(\Xi_c; 1/2^-)$	$M_\rho(\Xi_b; 1/2^-)$	$M_{ns}(0^-)$	m_{S0}^2	m_{S1}^2	m_{S2}^2	μ_0^2
Case: L	2623	5946	1115	$(1062)^2$	$(762)^2$	$-(491)^2$	$-(324)^2$
Case: N	2765	6084	1271	$(1119)^2$	$(690)^2$	$-(258)^2$	0
Case: E	2890	6207	1406	$(1171)^2$	$(612)^2$	$(321)^2$	$(319)^2$
Case: H	3279	6589	1819	$(1347)^2$	0	$(852)^2$	$(690)^2$

Axialvector/Vector Diquarks

Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

- **The $1^+/1^-$ diquarks in (3,3) representation**

$$d^{\mu a}_{ij} \equiv \epsilon_{abc}(q^{bT}_{iL}\, C\gamma^\mu\, q^c_{jR}) = \epsilon_{abc}(q^{bT}_{jR}\, C\gamma^\mu\, q^c_{iL}) \quad \text{chiral (3,3) vector diquark}$$

$$d^{\mu a}_{V[ij]} = d^{\mu a}_{ij} - d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_i\, C\gamma^\mu\gamma^5\, q^c_j) \quad \text{Vector } 1^- \text{ diquark, flavor } \bar{3}$$

$$d^{\mu a}_{A\{ij\}} = d^{\mu a}_{ij} + d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_i\, C\gamma^\mu\, q^c_j) \quad \text{Axial-vector } 1^+ \text{ diquark, flavor } 6$$

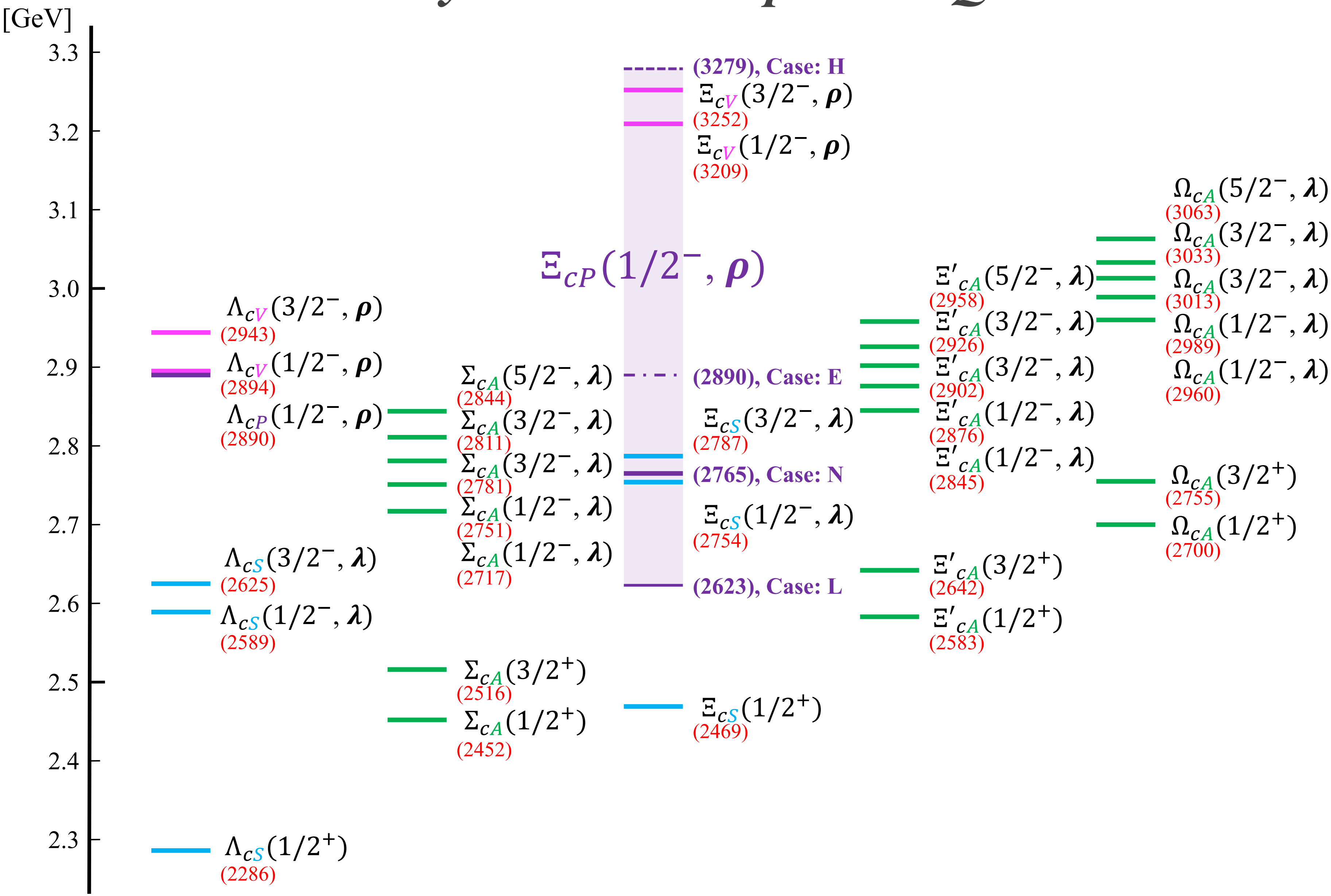
$$d^\mu \longrightarrow U_L d^\mu U_R^T, \quad (3,3) \qquad d^{\mu\dagger} \longrightarrow U_R^{T\dagger} d^\mu U_L^\dagger \quad (\bar{3},\bar{3})$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F^\dagger_{\mu\nu}] + m_0^2\mathrm{Tr}[d^\mu d^\dagger_\mu] + \frac{m_1^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d^{\dagger T}_\mu] + \frac{2m_2^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger\Sigma d^{\mu T} d^{\dagger T}_\mu]$$

$$F^{\mu\nu} = D^\mu d^\nu - D^\nu d^\mu$$

- **All the terms are chiral and $U_A(1)$ invariant.**

Charm Baryons in the Diquark-HQ model



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Y. Kim, M.O. K. Suzuki, in preparation

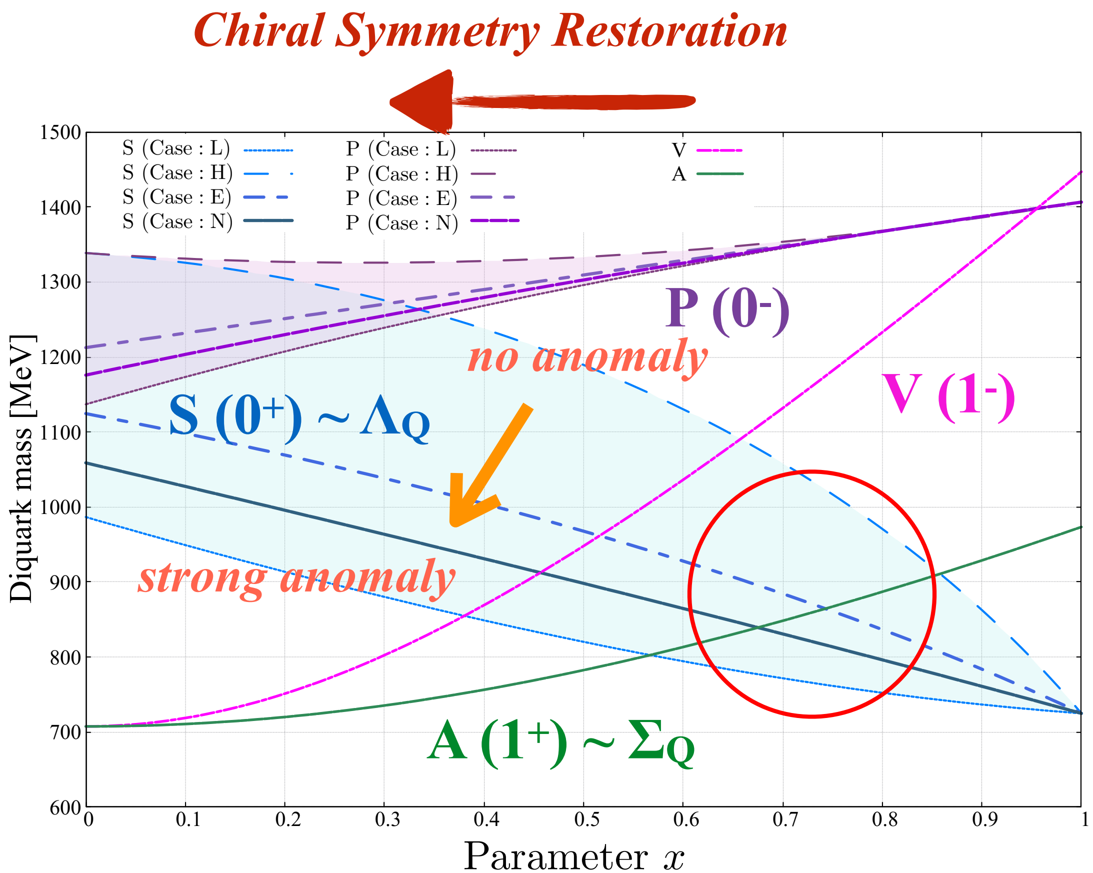
Heavy Baryons under Chiral Restoration

Diquark masses under chiral restoration

- **Simulation of diquark masses for decreasing chiral condensate**

$$\langle \bar{q}q \rangle \to x \langle \bar{q}q \rangle_0$$

$$x = 1 \to x = 0$$

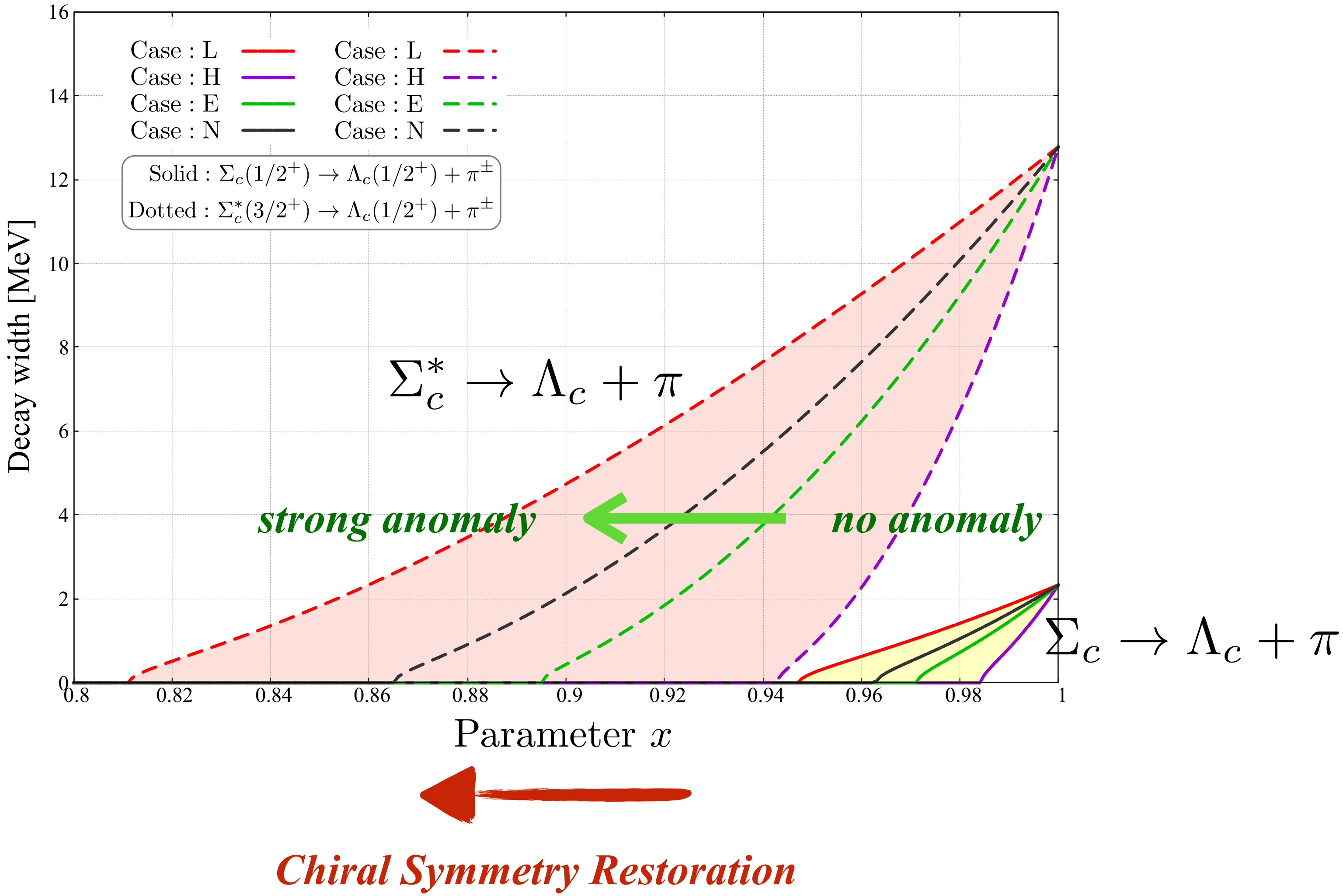


Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Y. Kim, M.O. K. Suzuki, in preparation

Decays of SHB in matter

- **Decays of Σ_c, Σ_c^* baryons under chiral symmetry restoration**



Diquarks in NJL for chiral restoration

- **D. Suenaga and MO, *Phys. Rev. D 108, 014030 (2023)***
  Analysis of the chiral-partner structure of diquarks in the $N_f = 3$ Nambu-Jona-Lasinio model

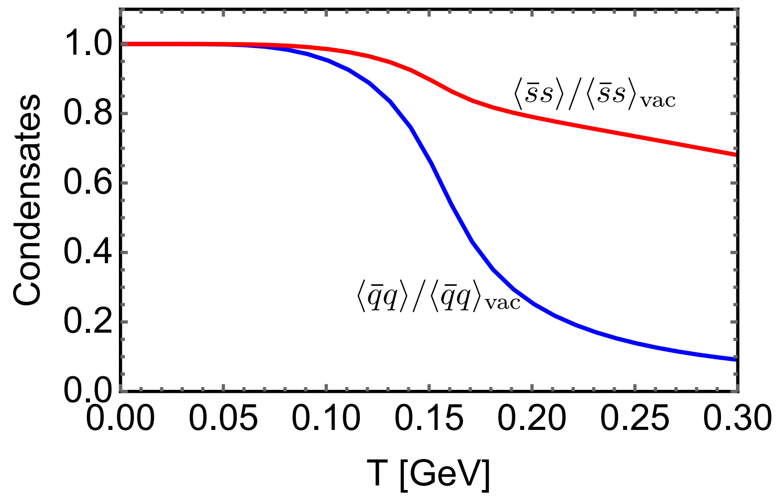
mesons $\phi_{ij} = (\bar{\psi}_R)^a_j (\psi_L)^a_i$

diquarks
$$(\eta_L)^a_i = \epsilon_{ijk}\epsilon^{abc}(\psi_L^T)^b_j C(\psi_L)^c_k$$
$$(\eta_R)^a_i = \epsilon_{ijk}\epsilon^{abc}(\psi_R^T)^b_j C(\psi_R)^c_k$$

$$\mathcal{L}_{4q} = 8G\mathrm{tr}[\phi^\dagger\phi] + 2H(\eta_L^T\eta_L^* + \eta_R^T\eta_R^*)$$

$$\mathcal{L}_{6q}^{\mathrm{anom.}} = -8K(\det\phi + \det\phi^\dagger) + K'(\eta_L^T\phi\eta_R^* + \eta_R^T\phi^\dagger\eta_L^*)$$

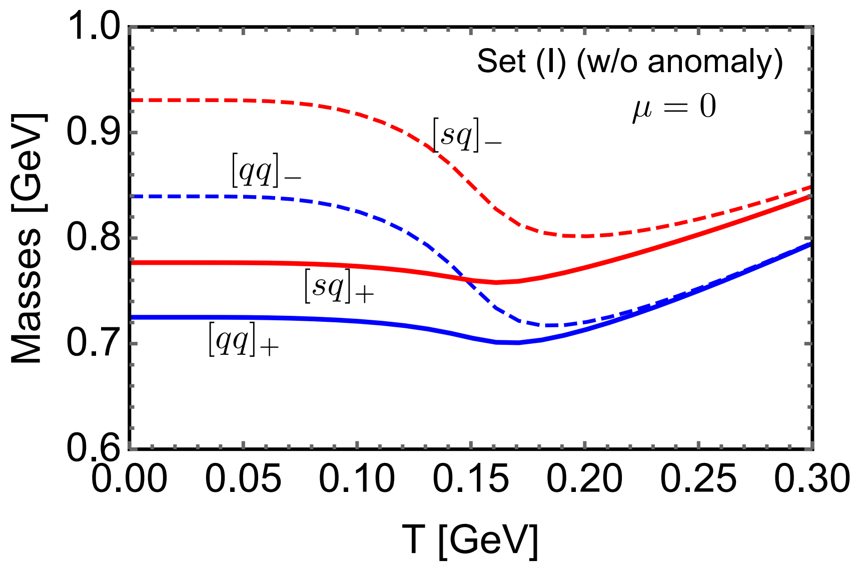
- **Chiral order parameters at finite T**

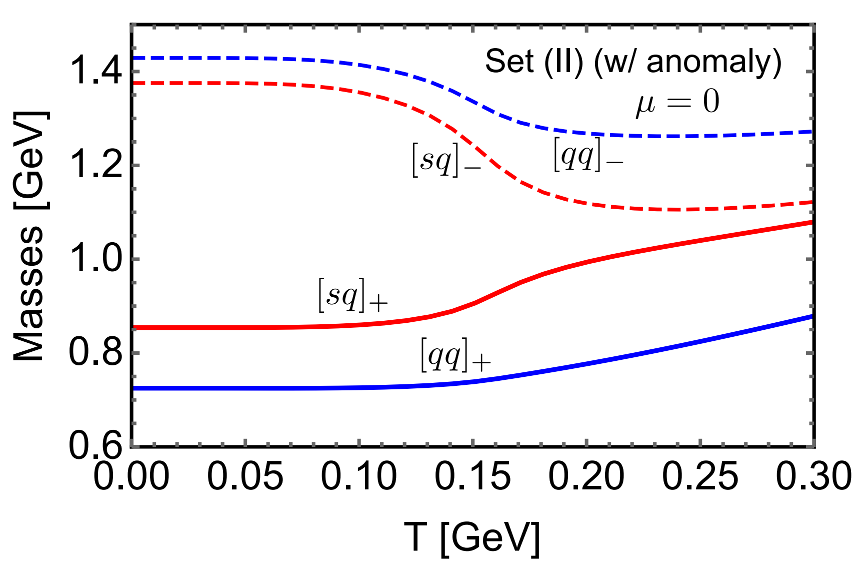


Scalar-Pseudoscalar Diquarks

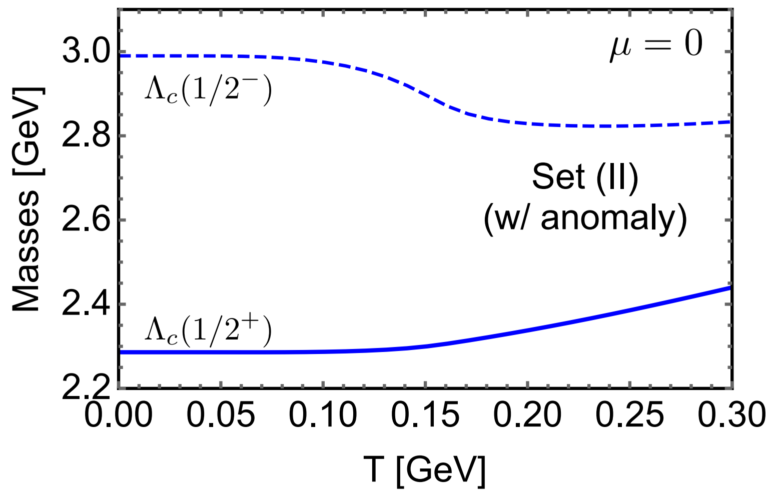
- **Finite temperature behaviors of the diquark masses**

without anomaly ***with anomaly***

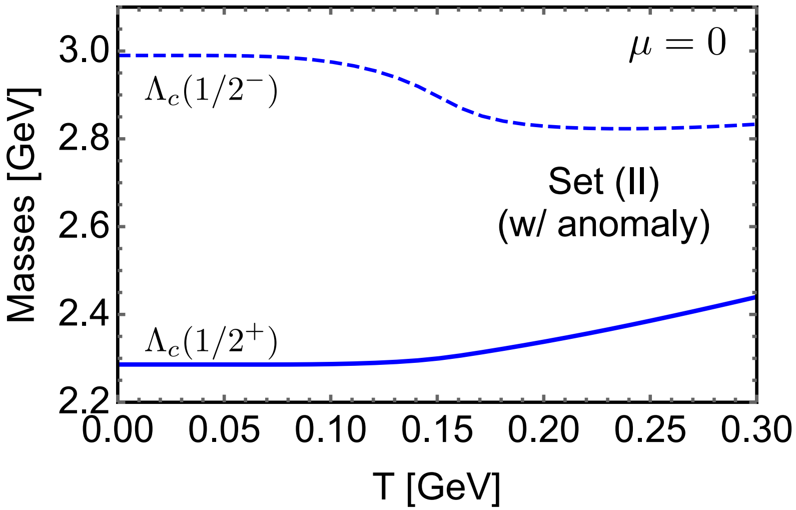


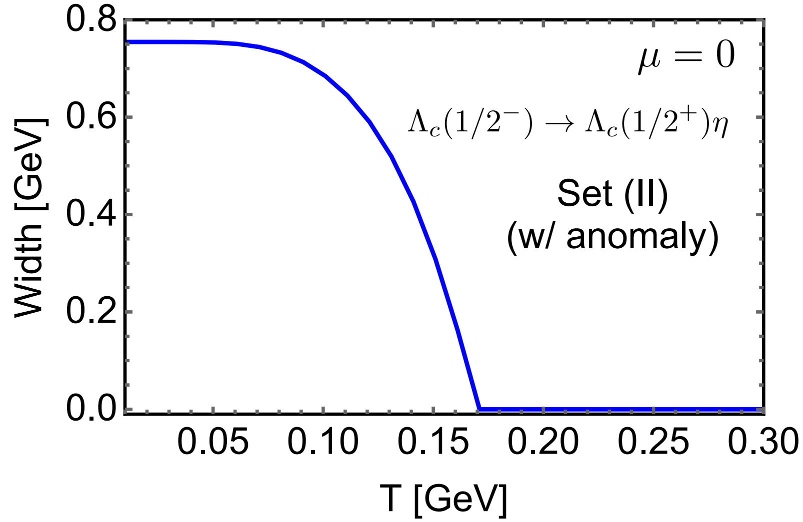


- **Confirm that U(1) anomaly leads to the inverse hierarchy.**
  We expect significant effects of U(1) anomaly on the decay widths of the ρ-mode negative-parity states.

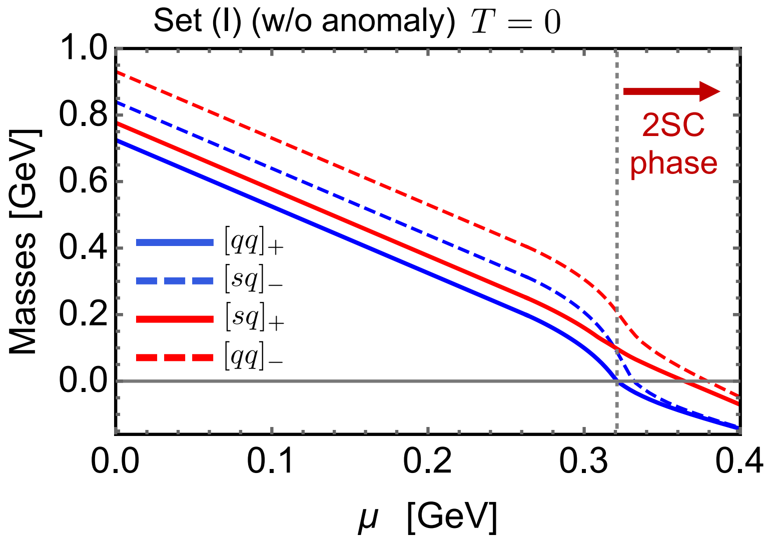


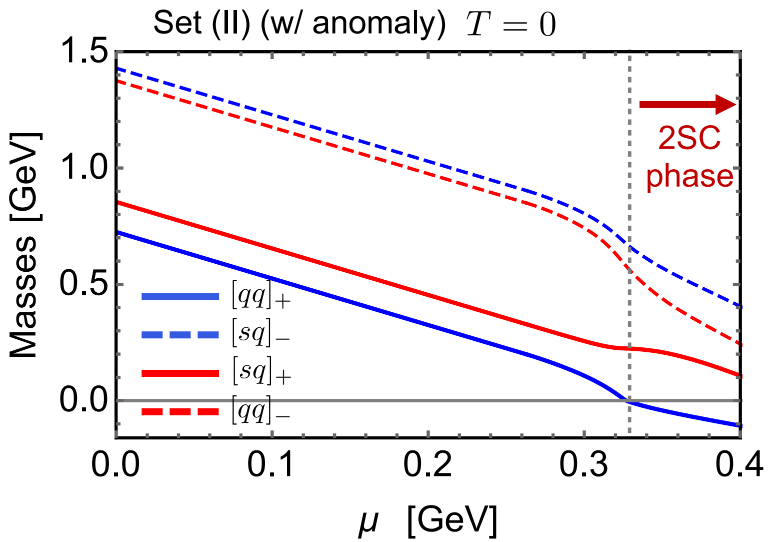
Diquark at finite T and μ





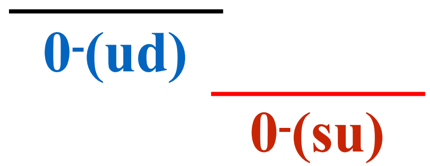
For finite chemical potential





Summary

- **Chiral effective theories of Scalar/Pseudoscalar diquarks and Axialvector/Vector diquarks are formulated.**
- **$U_A(1)$ anomaly is found to give the inverse mass hierarchy in the pseudoscalar diquark spectrum.**



- **Spectrum of Single Heavy Baryon (SHB) is calculated based on the chiral picture of diquarks. Inverse mass hierarchy appears in ρ-mode excited states, and make the Λ_Q and Ξ_Q spectra largely different.**
- **Under chiral restoration, we find the mass crossing of the 1^+ and 0^+ diquarks. Effects of $U_A(1)$ anomaly are significant in the behaviors of diquarks at finite temperature. It is interesting to observe behaviors of SHB in hot/dense matter.**