CSQCD 2024

THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

香港科技大學

New Alternatives of Compact Stars And related related GW signatures

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IAS, HKUST

- Part 1: Up-down Quark Stars

C. Z, Phys.Rev.D 101 (2020) 4, 043003
J.Ren, C. Z, Phys.Rev.D 102 (2020) 8, 083003

- Part 2: Inverted Hybrid Stars

C. Z, Jing Ren *Phys.Rev.D* 108 (2023) 6, 063012
C. Z，Yudong Luo, Hongbo Li, Lijing Shao, Renxin Xu, *Phys.Rev.D* 109 (2024) 6, 063020

stable quark matter hypothesis

- Part 3: Hybrid Strangeon Stars

Strangeon Matter hypothesis

C. Z，Yong Gao, Cheng-jun Xia, Renxin Xu
Phys.Rev.D 108 (2023) 12, 12

Strange Quark Matter (SQM) hypothesis

[1984 E. Witten]

QM with nearly equal number of u,d,s quarks, **may be the most stable form of matter** even at zero temperature and zero pressure

However, assumed that

1. QCD vacuum is flavor-independent (a universal bag const)

2. stable u,d quark matter (udQM) can not exist, considering we don't see proton&neutron being converted to udQM.

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

Up-down Quark Matter (udQM) hypothesis

[B. Holdom, J. Ren, C. Z, Phys.Rev.Lett. 120 (2018) , 222001]

u,d quark matter could **be more stable than** SQM and ordinary nuclear matter at a sufficiently large baryon number A_{min}**>300**

Derived from an extended SU(3) chiral quark-meson model with the flavor dependence of the QCD vacuum naturally accounted and fits well with all the mass spectrum and decay widths of light scalar and pseudoscalar mesons.

PHYSICAL REVIEW LETTERS **120,** 222001 (2018)

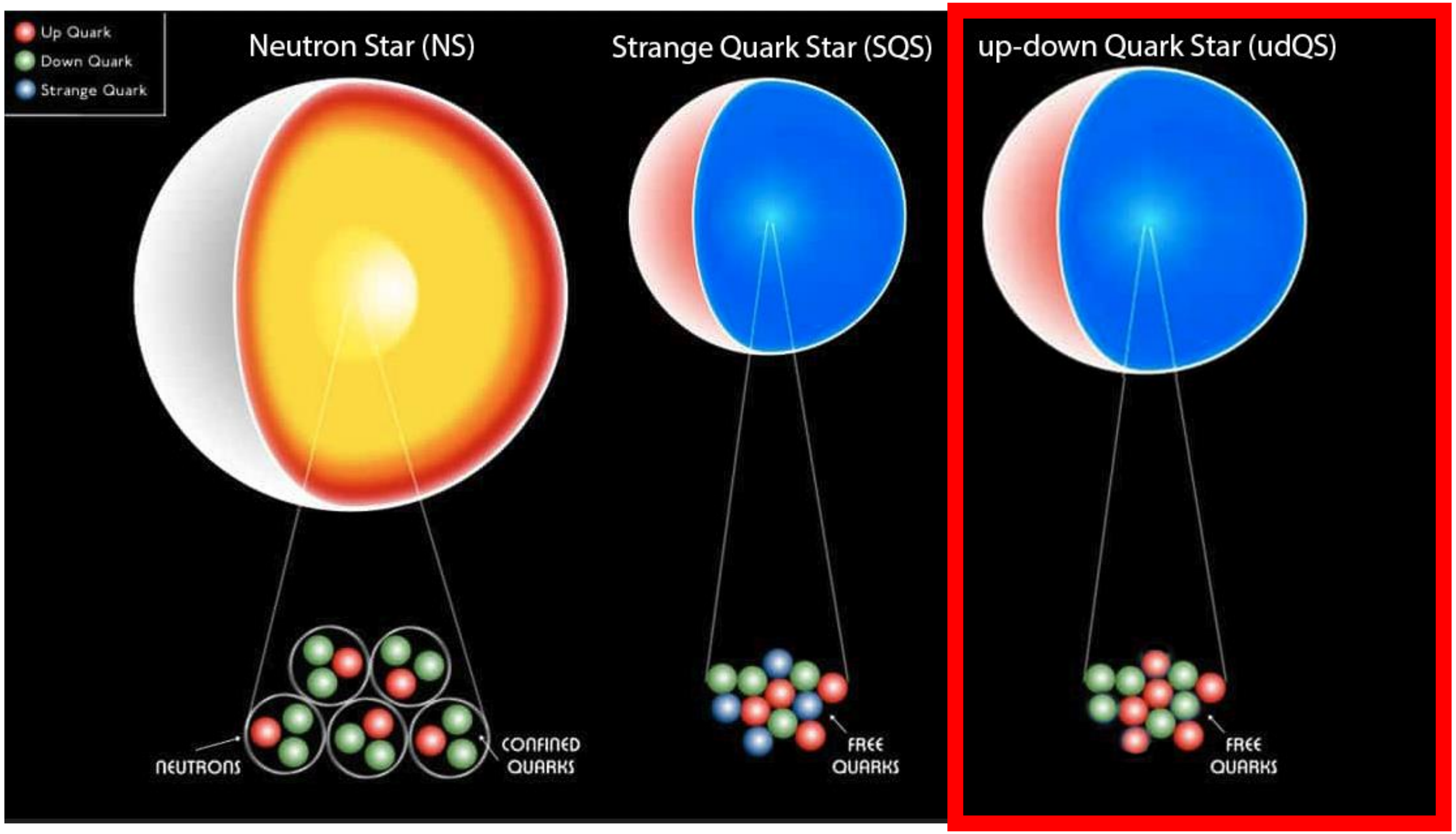
Quark Matter May Not Be Strange

Bob Holdom,* Jing Ren,† and Chen Zhang‡
Department of Physics, University of Toronto, Toronto, Ontario M5S1A7, Canada

(Received 30 July 2017; revised manuscript received 23 October 2017; published 31 May 2018)

If quark matter is energetically favored over nuclear matter at zero temperature and pressure, then it has long been expected to take the form of strange quark matter (SQM), with comparable amounts of u, d, and s quarks. The possibility of quark matter with only u and d quarks (udQM) is usually dismissed because of the observed stability of ordinary nuclei. However, we find that udQM generally has lower bulk energy per baryon than normal nuclei and SQM. This emerges in a phenomenological model that describes the spectra of the lightest pseudoscalar and scalar meson nonets. Taking into account the finite size effects, udQM can be the ground state of baryonic matter only for baryon number $A > A_{\rm min}$ with $A_{\rm min} \gtrsim 300$. This ensures the stability of ordinary nuclei and points to a new form of stable matter just beyond the periodic table.

DOI: 10.1103/PhysRevLett.120.222001



C. Z, *Phys.Rev.D* 101 (2020) 4, 043003

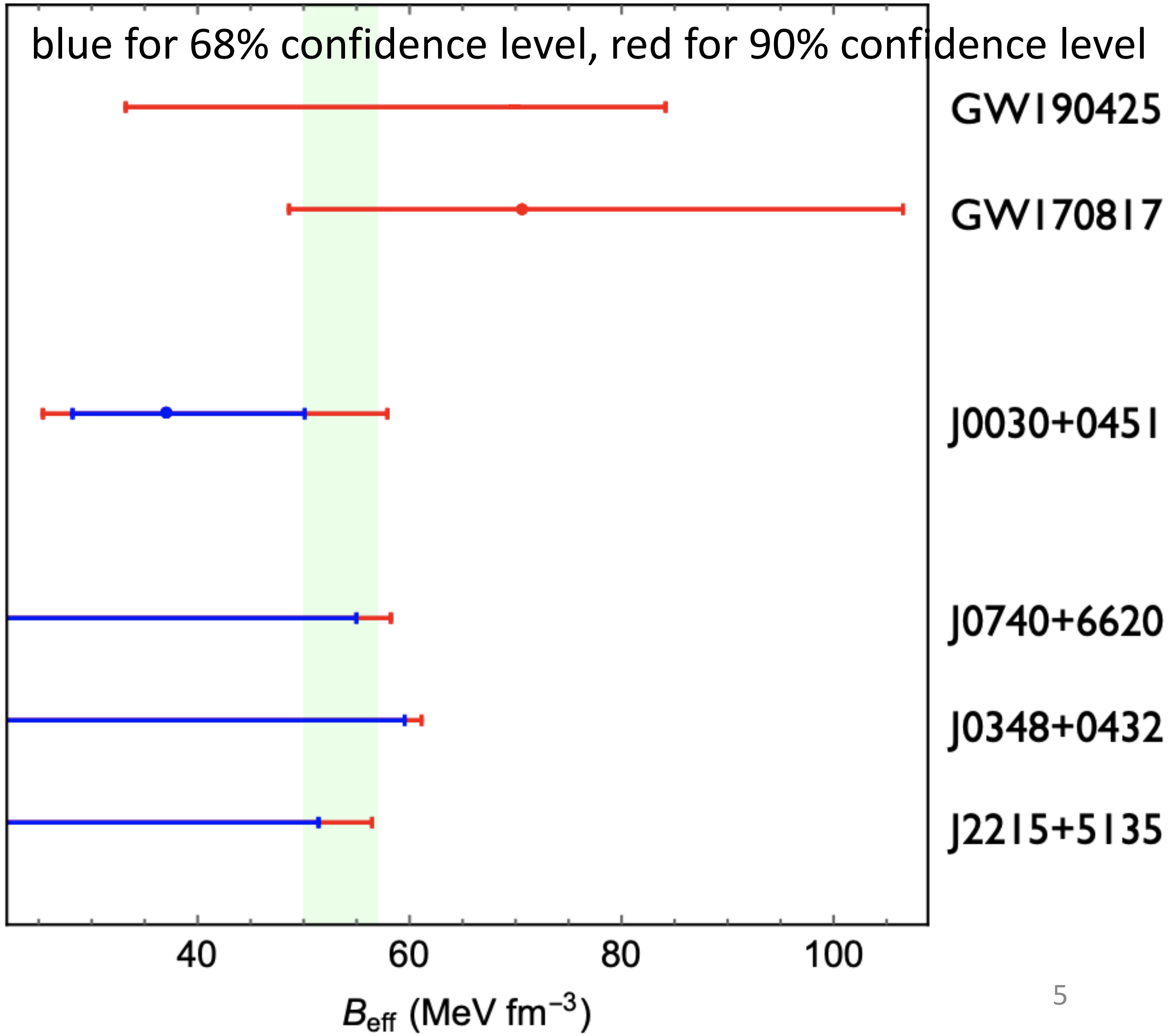
Up-down Quark Stars (udQS)

- *C. Zhang, **Phys.Rev.D 101 (2020) 4, 043003***
- *J.Ren, C. Zhang, **Phys.Rev.D*** **102 (2020) 8, 083003**

$$P = \frac{1}{3}(\rho - \rho_0) = \frac{1}{3}(\rho - 4B_{\rm eff})$$

$$\frac{E}{A} = 3\sqrt{2\pi}(\chi^3 B_{\rm eff})^{1/4} \in (900, 930)\ \text{MeV}$$

$\Rightarrow$ $50\,\text{MeV fm}^{-3} \lesssim \text{B}_{\text{eff}} \lesssim 57\,\text{MeV fm}^{-3}$



Part 2

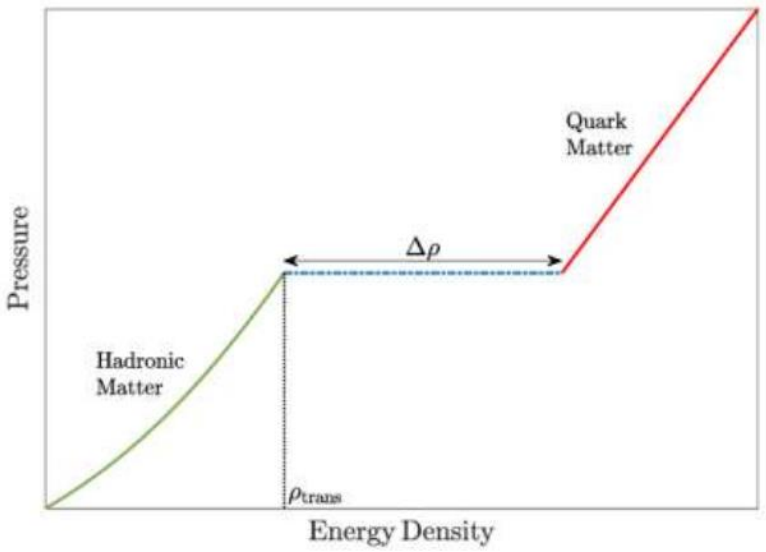
Inverted Hybrid Stars
and their GW asteroseismology

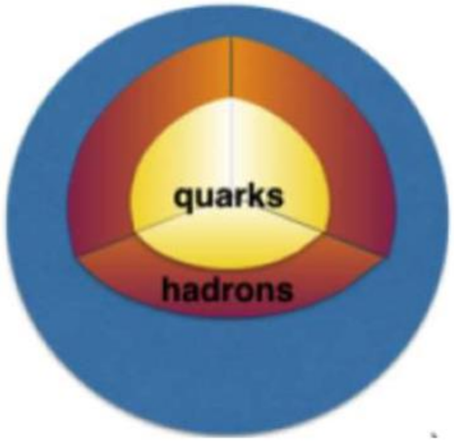
Based on

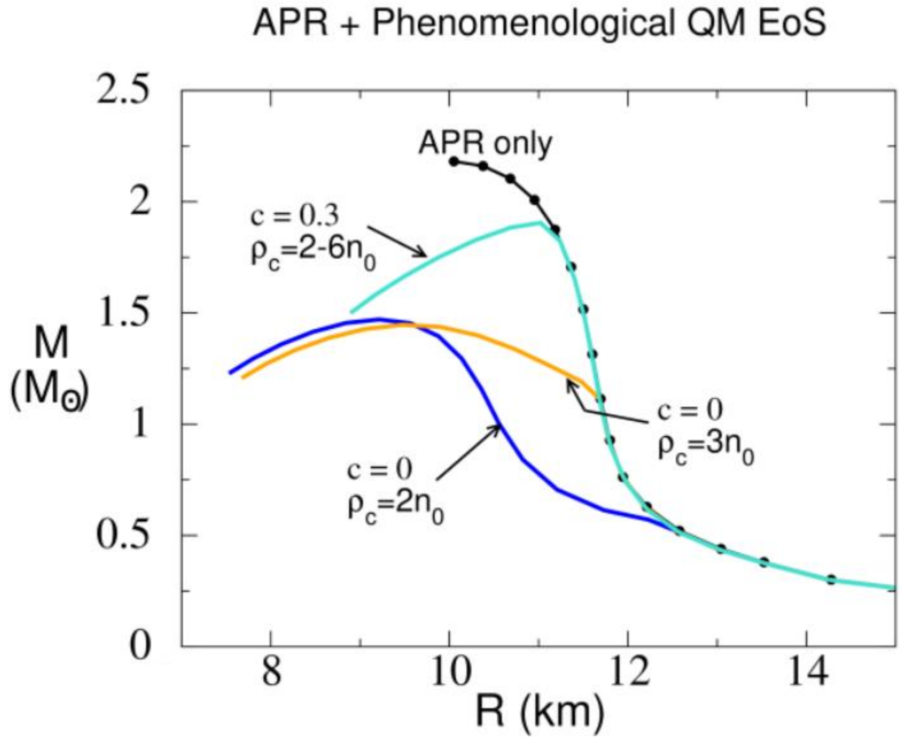
- ***C. Zhang***, *Jing Ren* ***Phys.Rev.D 108 (2023) 6, 063012***
- ***C. Zhang***, *Yudong Luo, Hongbo Li, Lijing Shao, Renxin Xu*, ***Phys.Rev.D*** **109 (2024) 6, 063020**

(Conventional) Hybrid Star

Basic picture: at high pressure, hadrons deconfine into quark matter



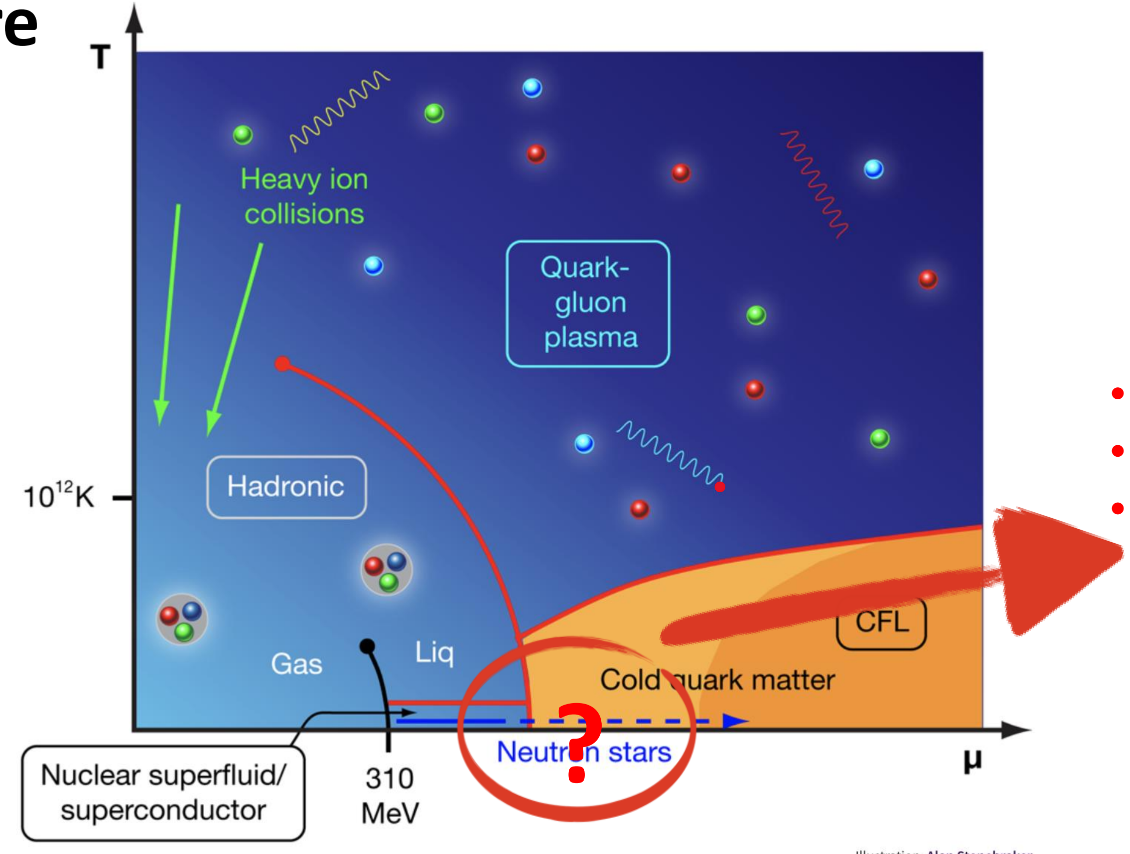




Graph adapted from arXiv:1612.09485 ***[nucl-th]****, arXiv:nucl-th/0411016*

However

- The picture of hybrid star aforementioned assumed that quark matter only become absolutely stable over hadronic matter at high pressure
- **But in context of stable QM hypothesis, QM is more stable than HM at low pressure**



- Non-perturbative QCD
- Inaccessible by Lattice and earth experiment
- With stable QM hypothesis, **cannot rule out the possibility of QM to HM transition**

Inverted hybrid stars

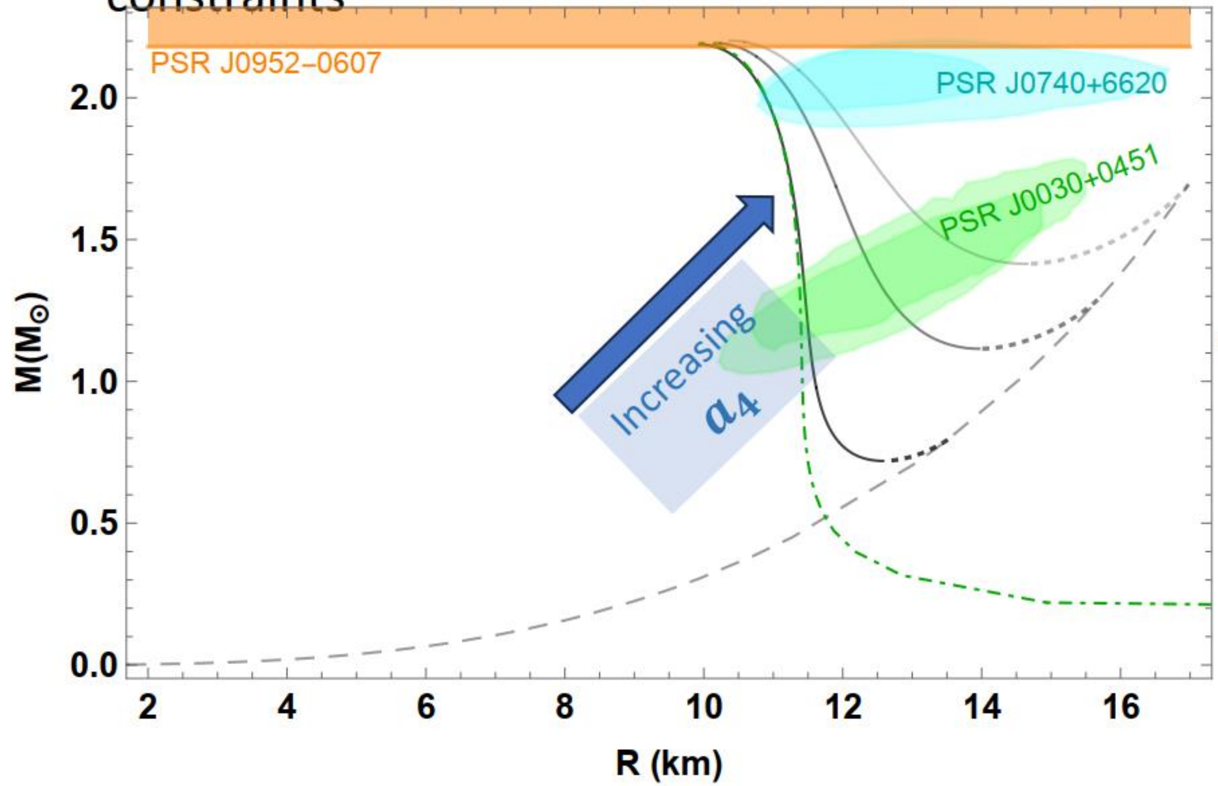
C. Zhang, J. Ren ***Phys.Rev.D 108 (2023) 6, 063012***

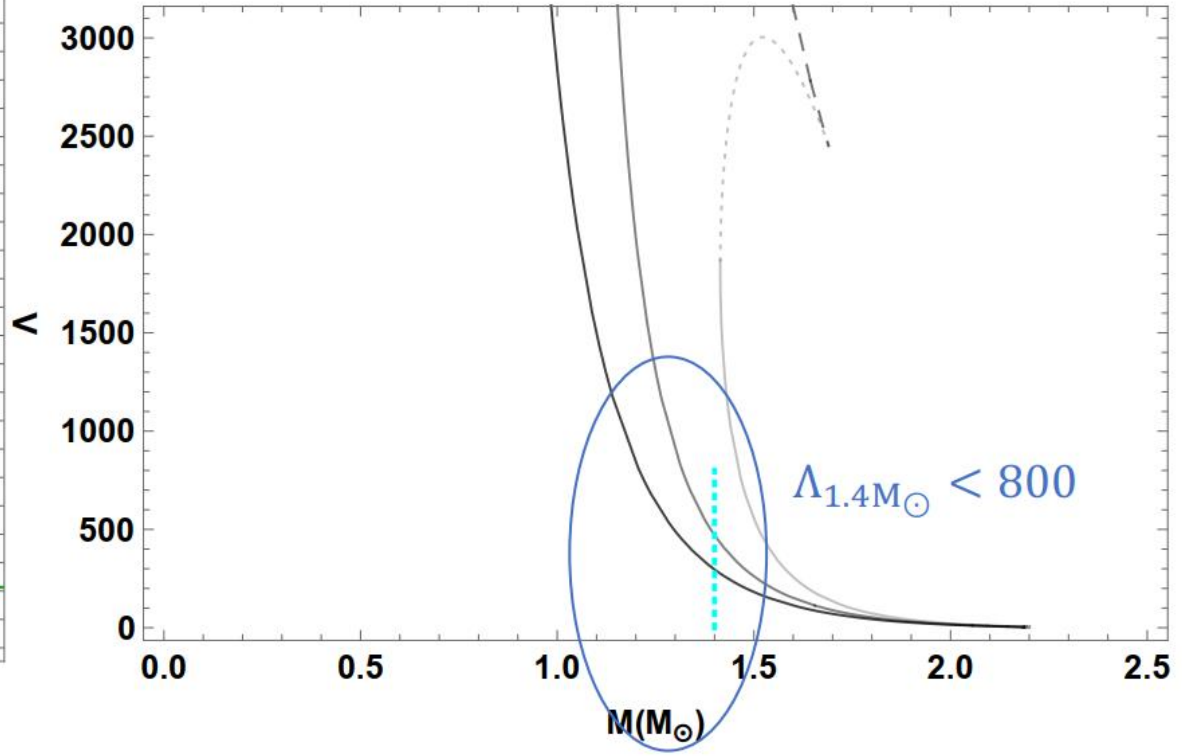
Motivation:

- At low pressure (crust), SQM/udQM hypothesis allows quark matter to be absolutely stable
- At high pressure (core), it's possible to have <**QM to HM transition**> associated **with the chemical potential crossing.**
- For brevity, we named it **Cross stars (CrSs).**

Complementarity of CrSs
an illustrative example

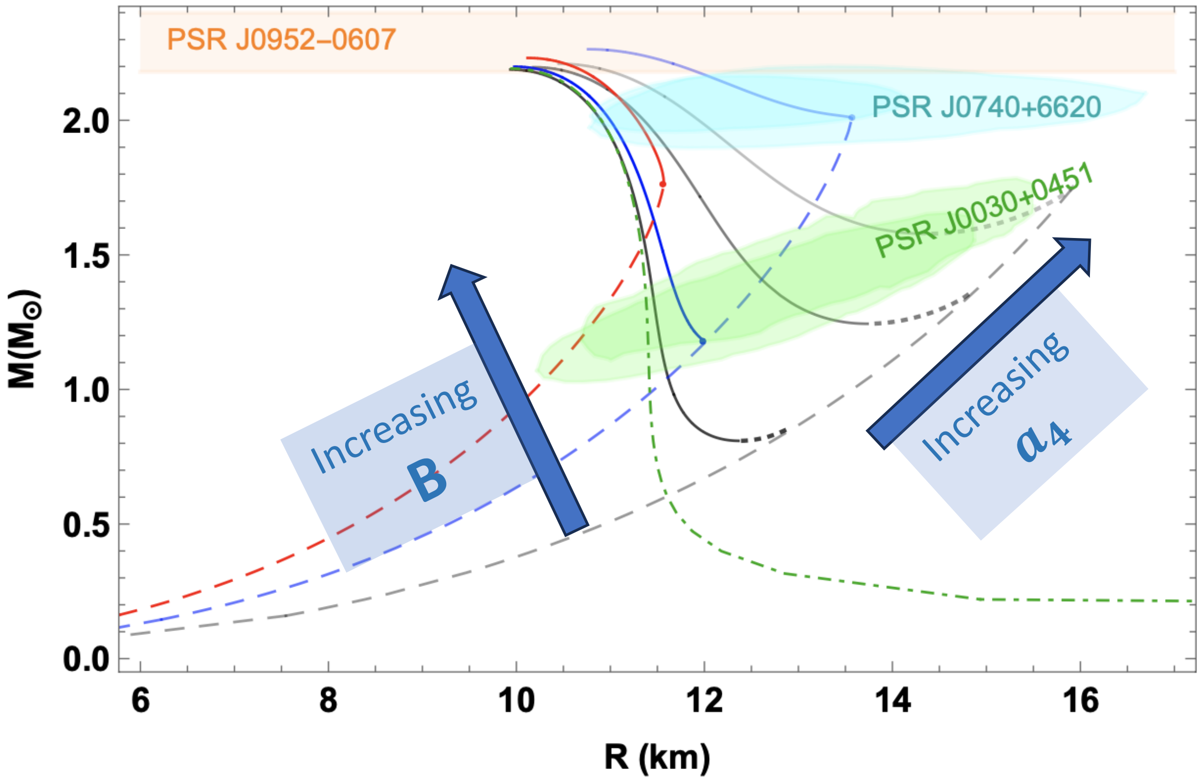
- NS with APR cannot well meet NICER PSR J0740+6620; udQS with B=20 MeV/fm^3 cannot meet GW170817. But inverted hybrid stars with (APR& udQM B=20 MeV/fm^3) can meet all constraints

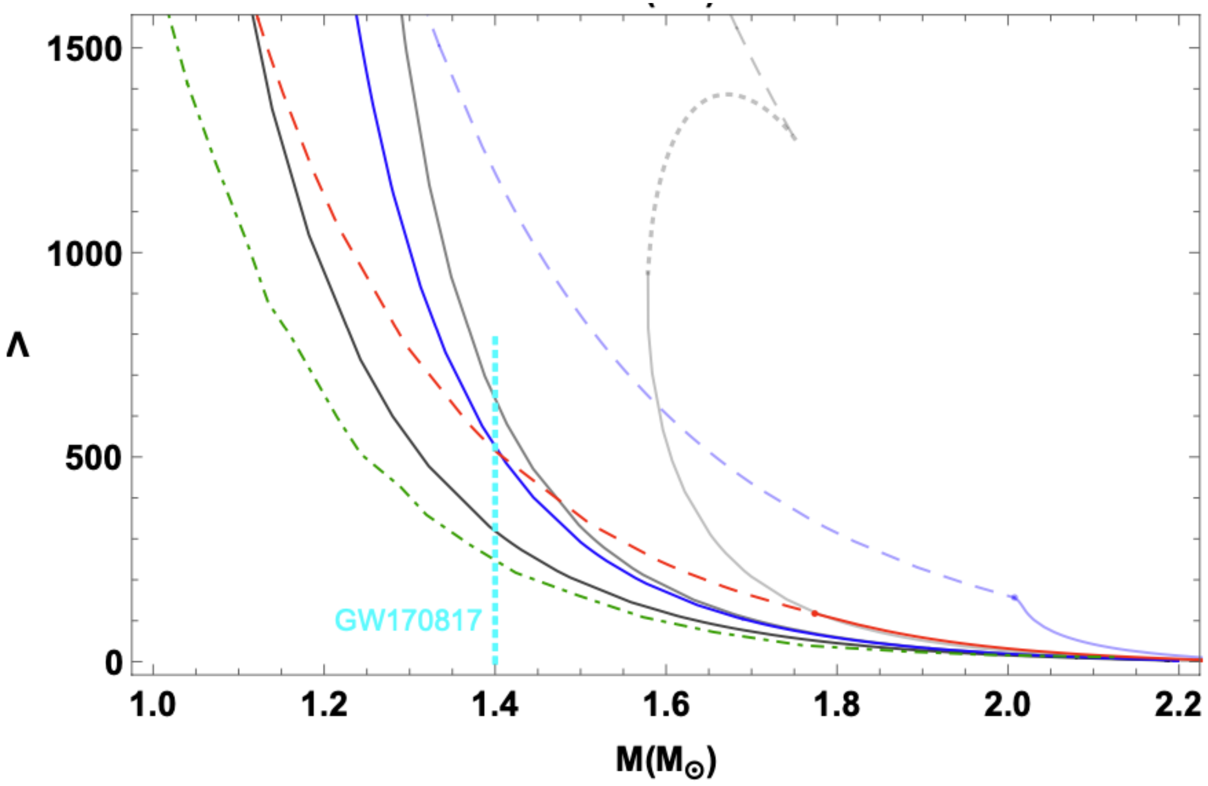




the interplay between the HM and QM compositions helps to reconcile astrophysical constraints at low and high masses, complementarily

CrSs with SQM





Radial and Non-Radial Oscillations of of Inverted Hybrid Stars

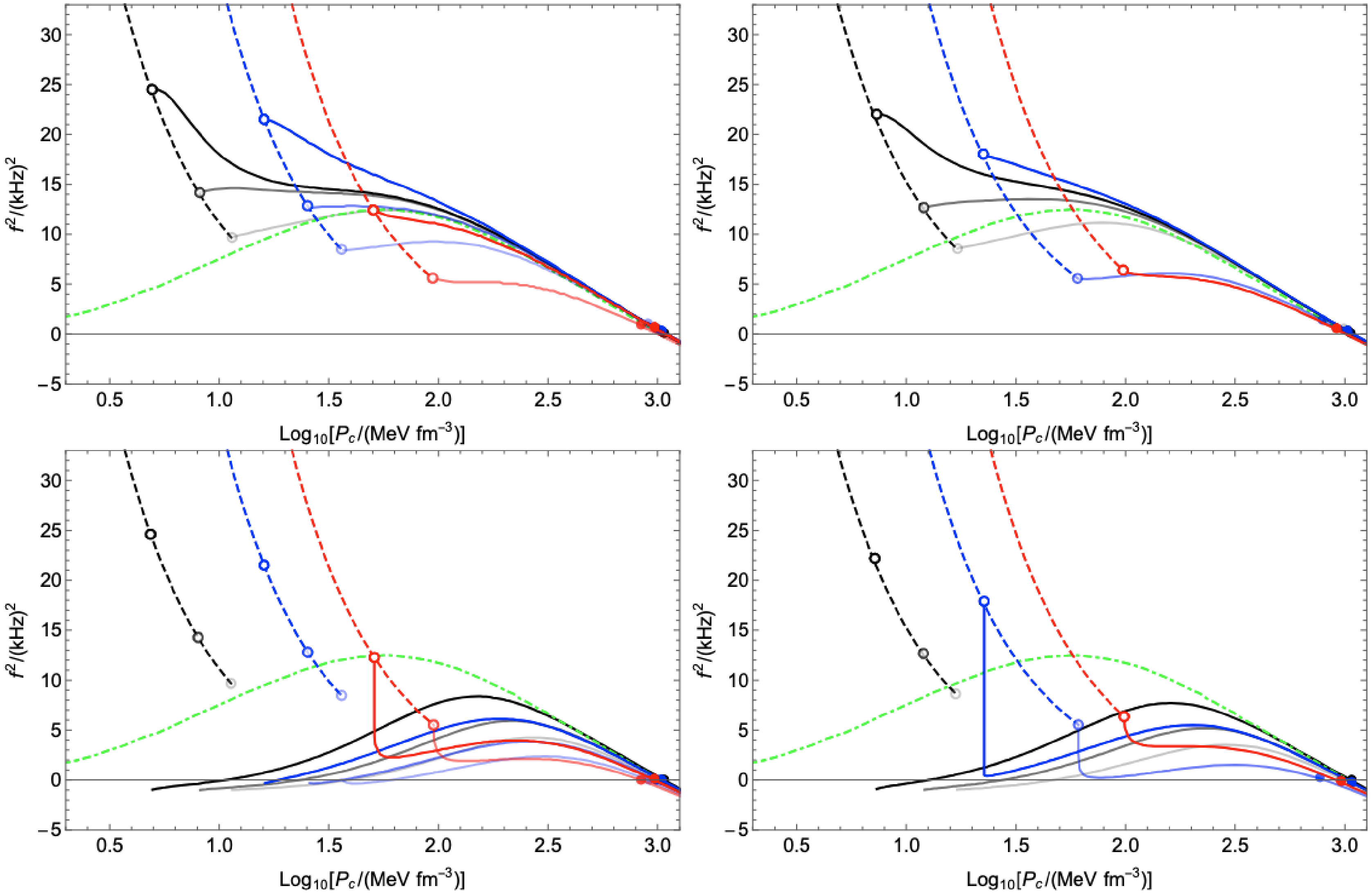
C. Zhang , *Yudong Luo, Hongbo Li, Lijing Shao, Renxin Xu*, ***Phys.Rev.D*** **109 (2024) 6, 063020**

Radial Oscillations

B=20 MeV/fm^3
B=35 MeV/fm^3
B=50 MeV/fm^3

Slow conversion:

Rapid conversion:



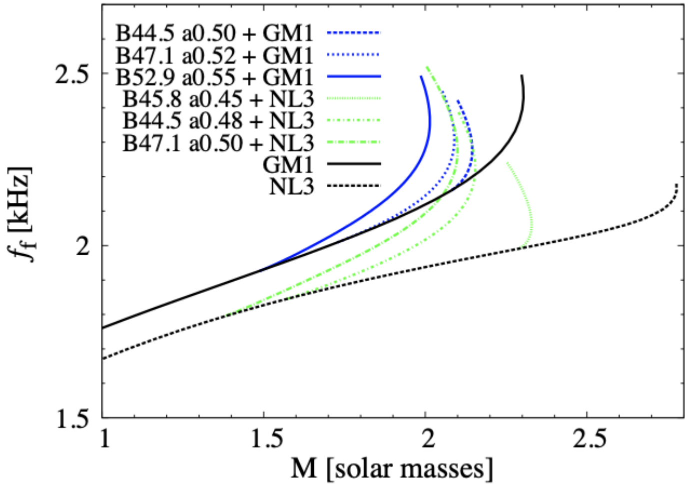
non-radial oscillations (Cowling Approx.)

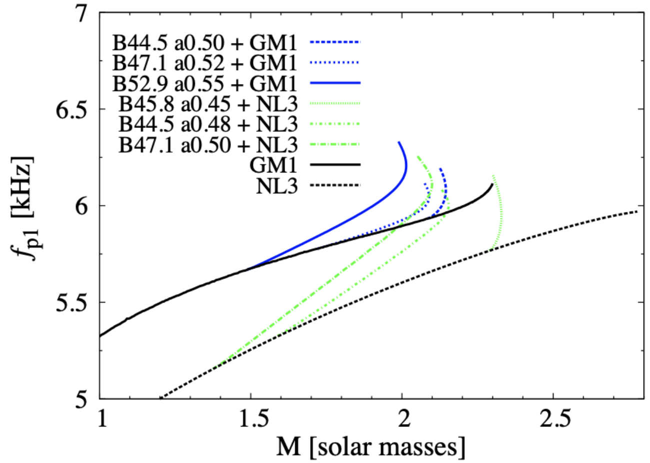
f (solid) and g (dotted) modes

p_1 modes

Conventional Hybrid Stars

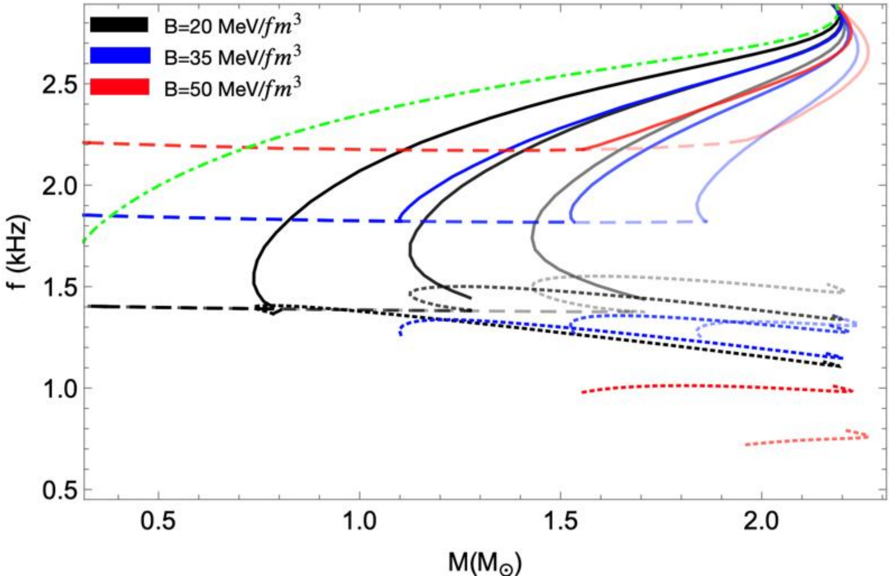
Flores et al (2014)

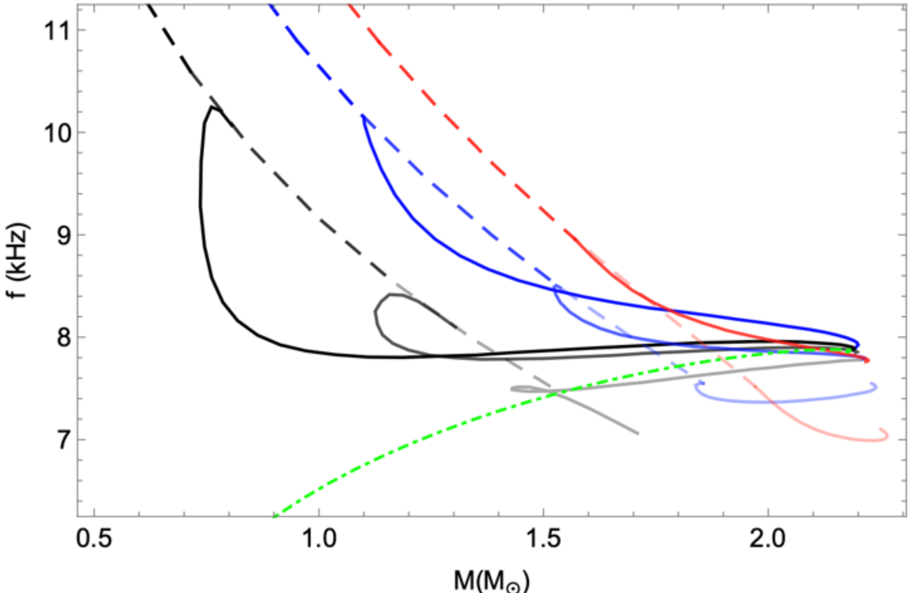




Inverted Hybrid Stars

C. Z, Y. Luo, H. Li, L. Shao, R. Xu, **Phys.Rev.D 109 (2024) 6, 063020**





Part 3

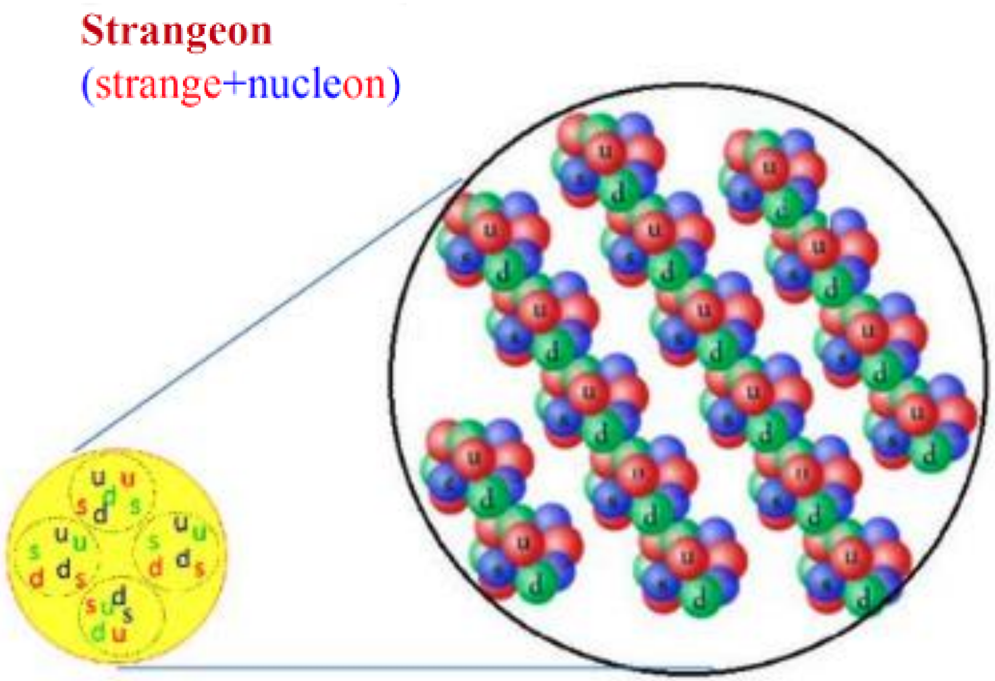
Hybrid Strangeon Stars

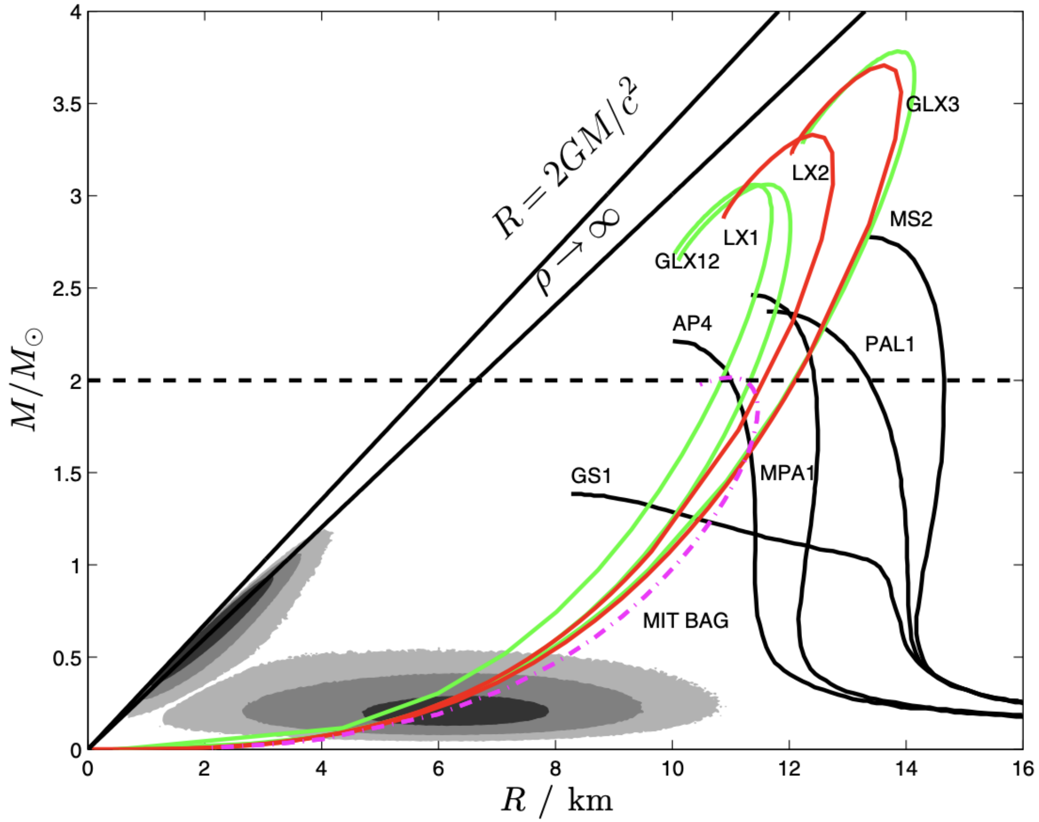
Based on

- ***C. Z, Yong Gao , Cheng-jun Xia, Renxin Xu Phys.Rev.D 108 (2023) 12, 12***

Strangeon (Strange-Cluster) matter

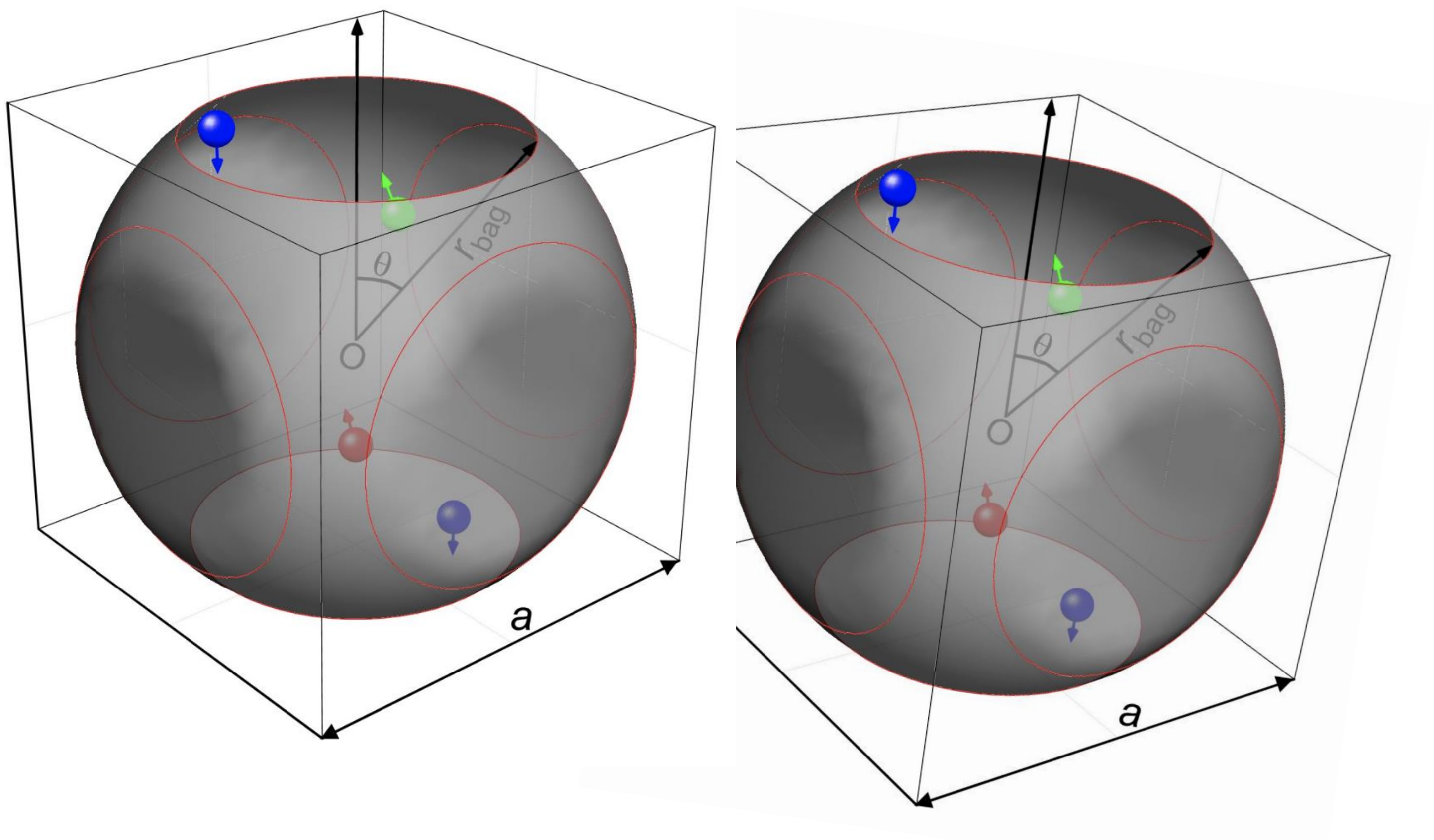
Renxin Xu 2003 ApJ 596 L59





Graph adapted from Xiaoyu Lai "Strangeon matter in kilonova" and LI, Z, QU, Z, CHEN, L, et al. Astrophys J, 2015, 798: 56

Strangeons to SQM transition?



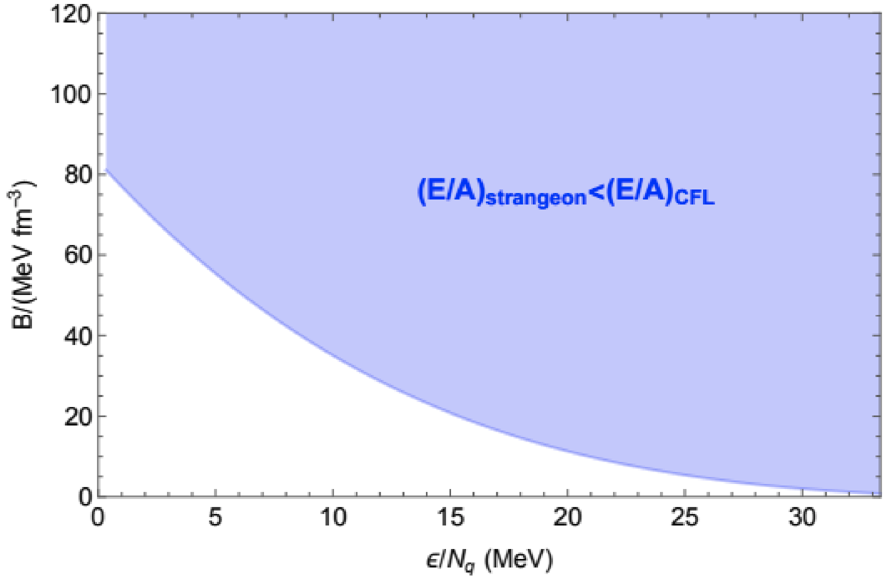
Z. Q. Miao, C. J. Xia, X. Y. Lai, T. Maruyama, R. X. Xu and E. P. Zhou, Int. J. Mod. Phys. E 31, no.04, 2250037 (2022)

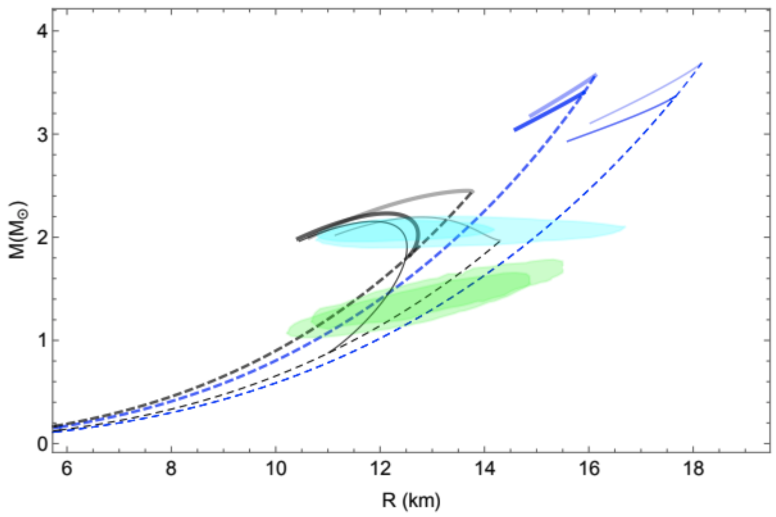
A increasing pressure shrink of strangeon lattice spacing

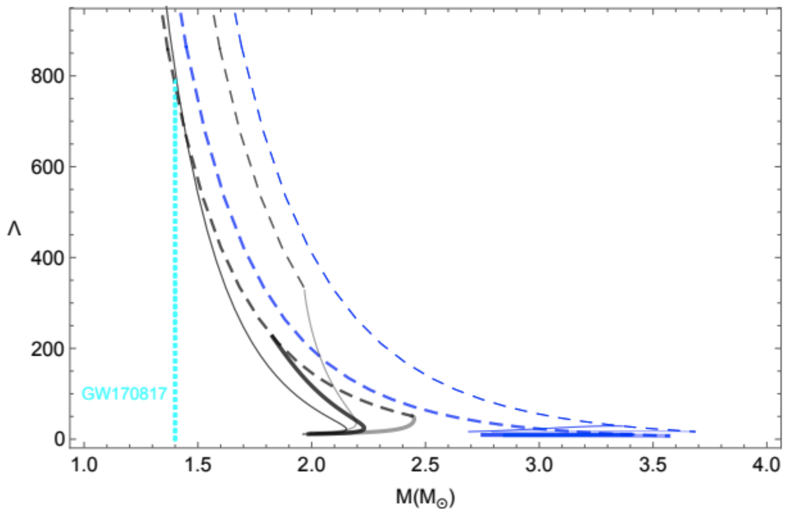
→the lattice constant becomes smaller than the radius of individual quark bags

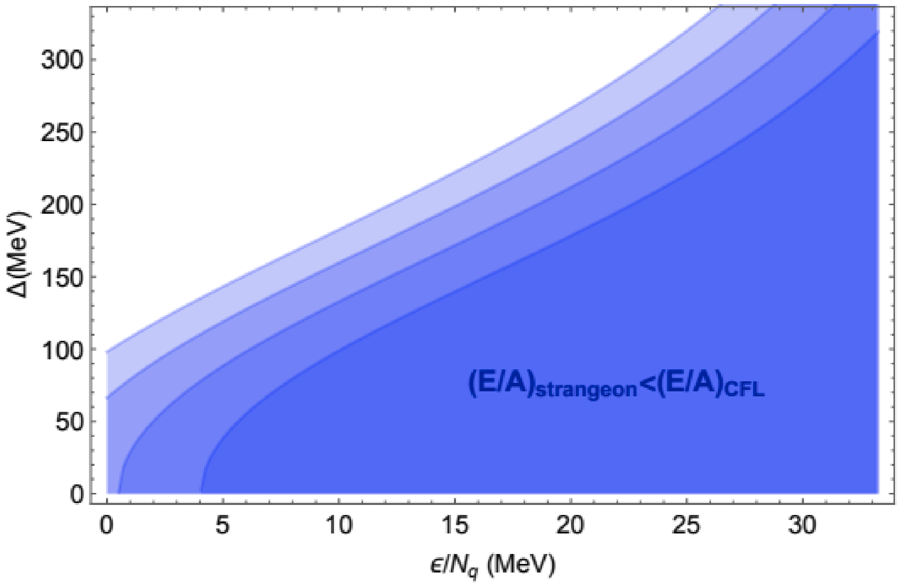
→Strangeons to SQM transition

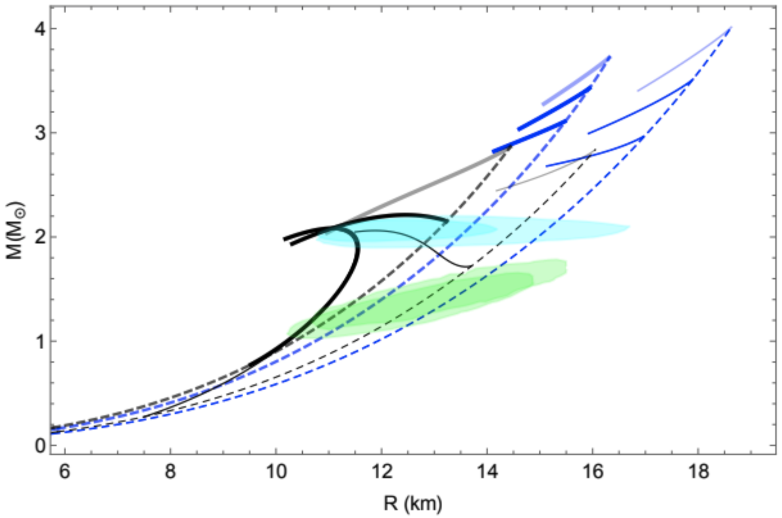
- Allowed parameter space for the existence of hybrid strangeon stars from stability consideration

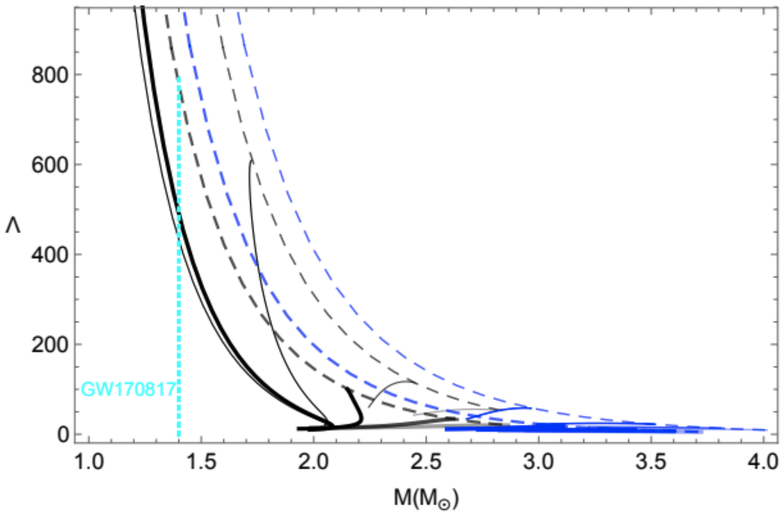




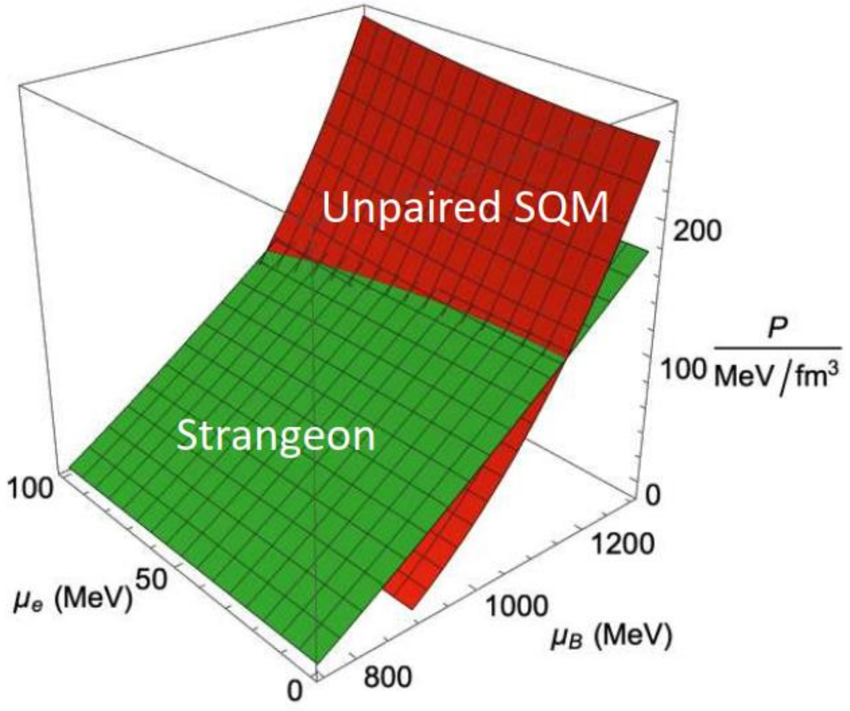


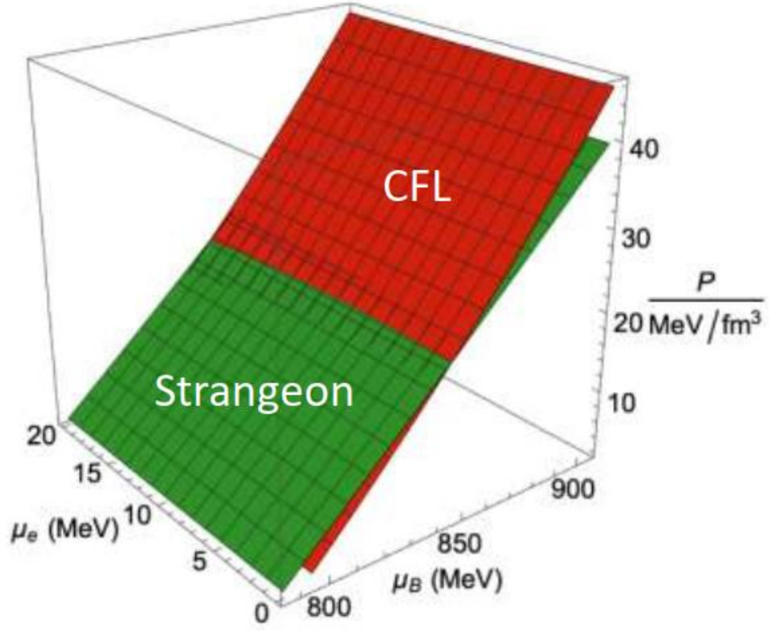






Mixed Phase?





$\Delta\mu_B \sim$ only 1 MeV

=> Mixed phase is not prefered

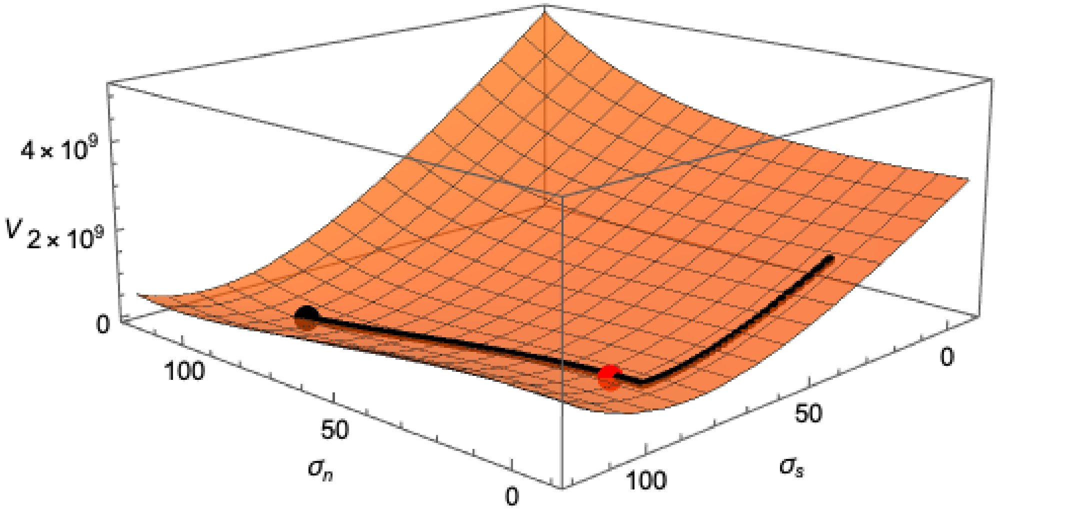
Summary

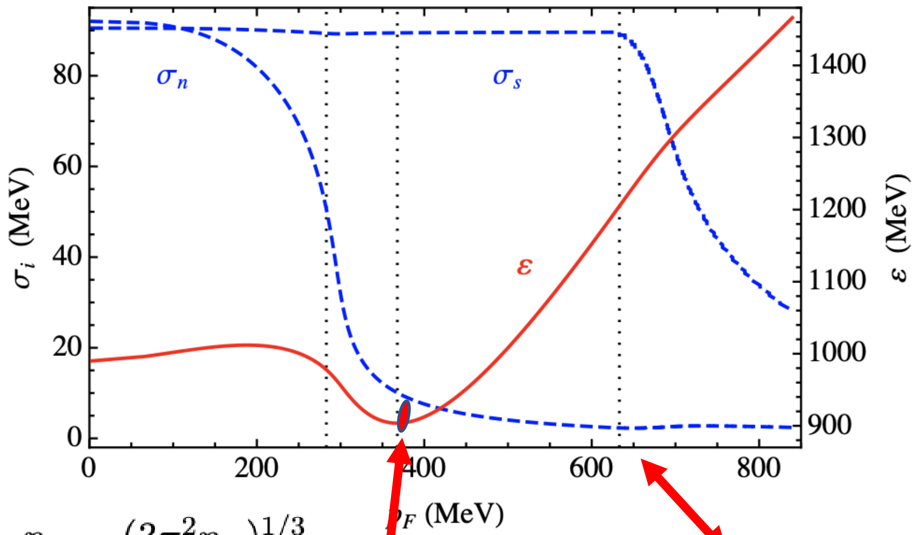
- Up-down quark matter can be the ground state of matter. They can form up-down quark stars that can naturally have large M_TOV.
- Both theory and experiment can not exclude the possibility of **inverted hybrid stars.** We have shown their consistency with recent observations. They have distinct p-mode behaviour compared to conventional hybrid stars.
- Strangeon matter can transit to SQM at high pressure, form **Hybrid strangeon stars.** They are consistent with recent observations.
- Open questions: search for udQM? microscopic picture of inverse phase transition? radiation signals? Seismology of hybrid strangeon stars?....

Back Up Slides

up down Quark Matter (*ud*QM)

- At large particle number (bulk) li





$p_F = (3\pi^2 n_A)^{1/3}$

Strangeness starts to appear

$<\sigma_n>$ give masses to u,d quarks

$<\sigma_s>$ give masses to s quarks

$$\left(\frac{E}{A}\right)_{udQM} \approx 903\,\mathrm{MeV} < \left(\frac{\mathrm{E}}{\mathrm{A}}\right)_{^{56}\mathrm{Fe}} \approx 930\,\mathrm{MeV}$$

Equation of States $\Longrightarrow P \approx \frac{1}{3}(\rho - \rho_0)$

Model inverted hybrid stars
QM sector

- Interacting Quark matter (IQM)

C. Z, R.B. Mann, Phys.Rev.D 103 (2021) 6, 063018

$$\Omega = -\frac{\xi_4}{4\pi^2}\mu^4 + \frac{\xi_4(1-a_4)}{4\pi^2}\mu^4 - \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\pi^2}\mu^2 - \frac{\mu_e^4}{12\pi^2} + B_{\rm eff}$$

$$(\xi_4, \xi_{2a}, \xi_{2b}) = \begin{cases} ((\frac{1}{3})^{\frac{4}{3}} + (\frac{2}{3})^{\frac{4}{3}})^{-3}, 1, 0) & \text{2SC phase} \\ (3, 1, 3/4) & \text{2SC+s phase} \\ (3, 3, 3/4) & \text{CFL phase} \end{cases}$$

$$\lambda = \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\sqrt{\xi_4 a_4}}$$

$$\bar\lambda = \frac{\lambda^2}{4B_{\rm eff}}$$

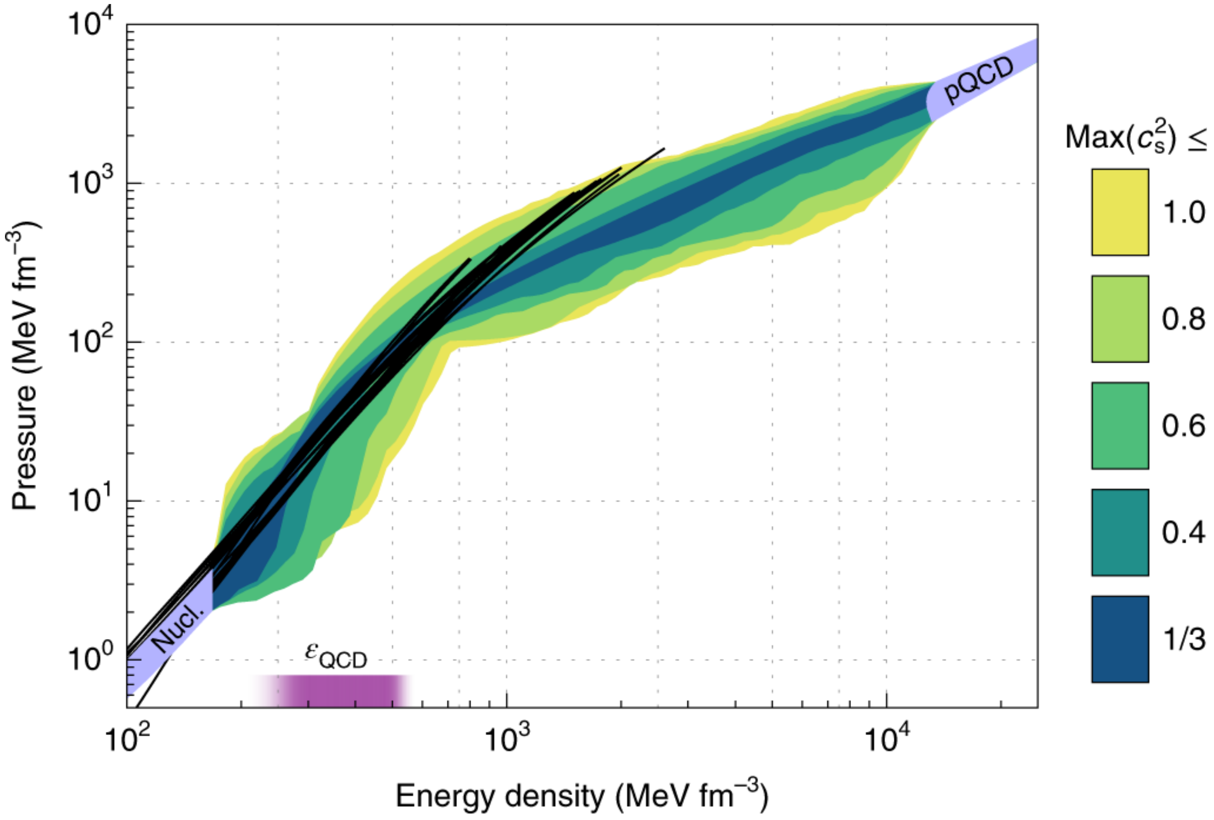
$$p = \frac{1}{3}(\rho - 4B_{\rm eff}) + \frac{4\lambda^2}{9\pi^2}\left(-1 + \sqrt{1 + 3\pi^2\frac{(\rho - B_{\rm eff})}{\lambda^2}}\right) \Rightarrow \bar p = \frac{1}{3}(\bar\rho - 1) + \frac{4}{9\pi^2}\bar\lambda\left(-1 + \text{sgn}(\lambda)\sqrt{1 + \frac{3\pi^2}{\bar\lambda}(\bar\rho - \frac{1}{4})}\right)$$

$$\mu_{\rm QM} = \frac{3\sqrt{2}}{(a_4\xi_4)^{1/4}}\sqrt{[(P+B)\pi^2 + \lambda^2]^{1/2} - \lambda} \Rightarrow \left(\frac{E}{A}\right)_{\rm QM} = \frac{3\sqrt{2}\pi}{(\xi_4 a_4)^{1/4}}\frac{B^{1/4}}{\sqrt{(\lambda^2/B + \pi^2)^{1/2} + \lambda/\sqrt{B}}}$$

For simplicity, in this work we ignore color-superconductivity, thus $\lambda = 0$ for udQM $\quad \lambda = -\sqrt{3}m_s^2/(4\sqrt{a_4})$ for SQM

Model inverted hybrid stars
HM sector

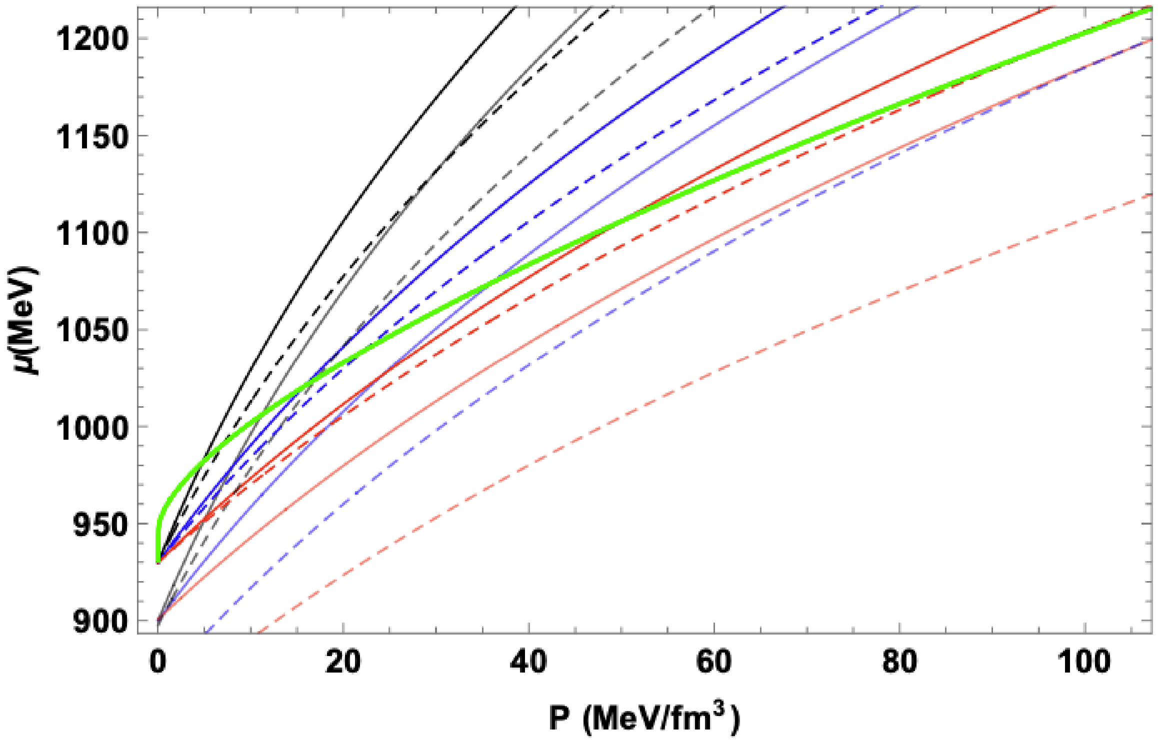
- HM: large uncertainties above saturation density

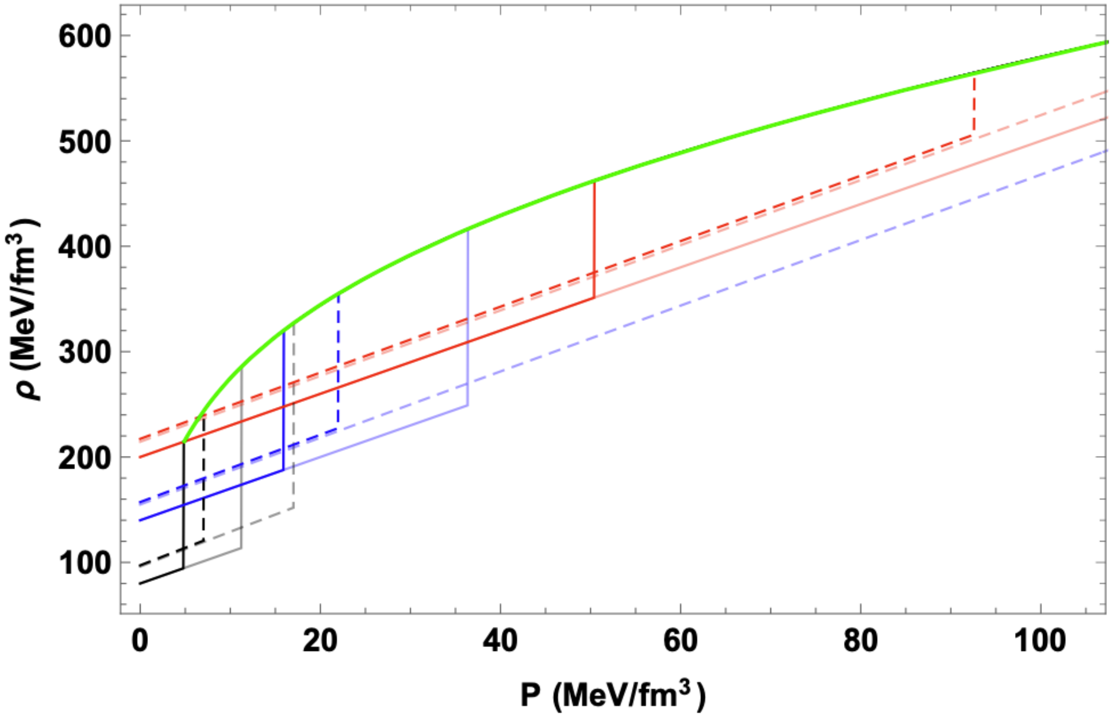


Variations of QM EOSs

Next, we fix HM EOS (APR), and vary the QM EOSs

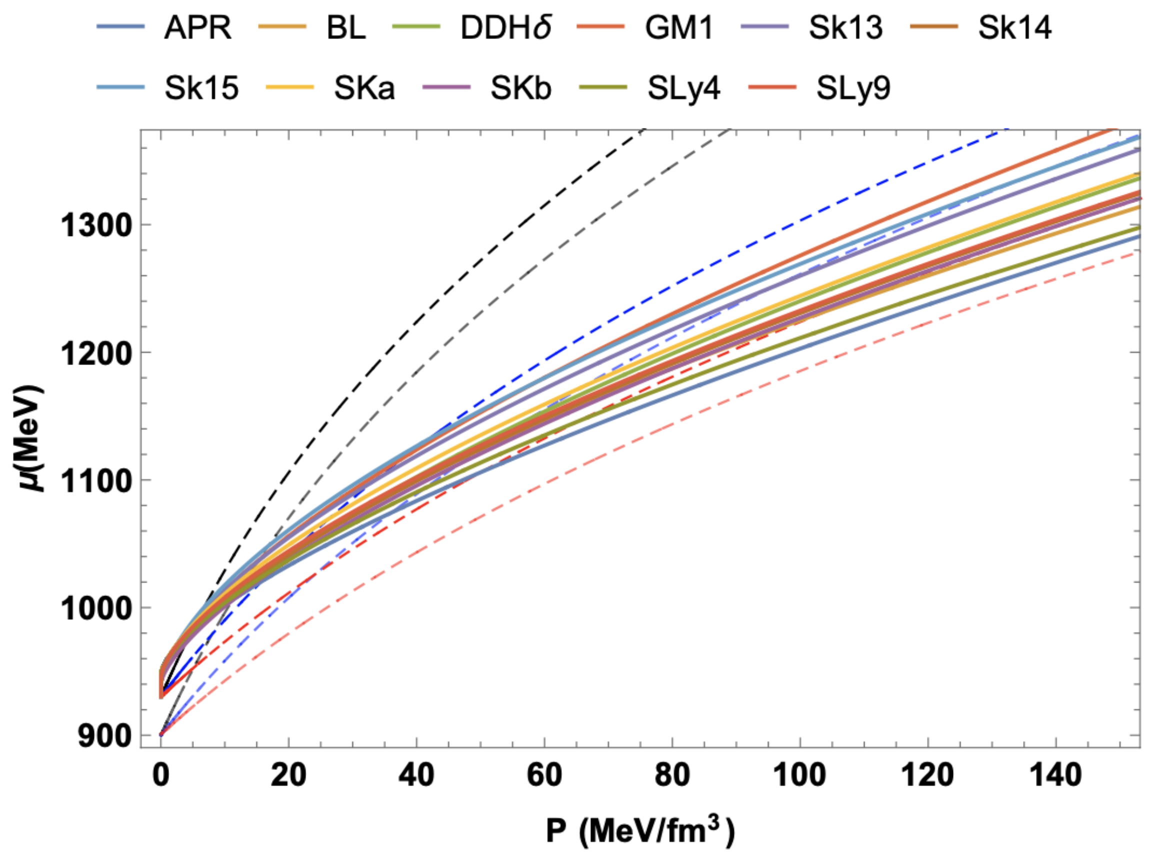
- Solid lines are for udQM
- Dashed line are for SQM
- darker=$a_{4,min}$, lighter= $a_{4,\max}$

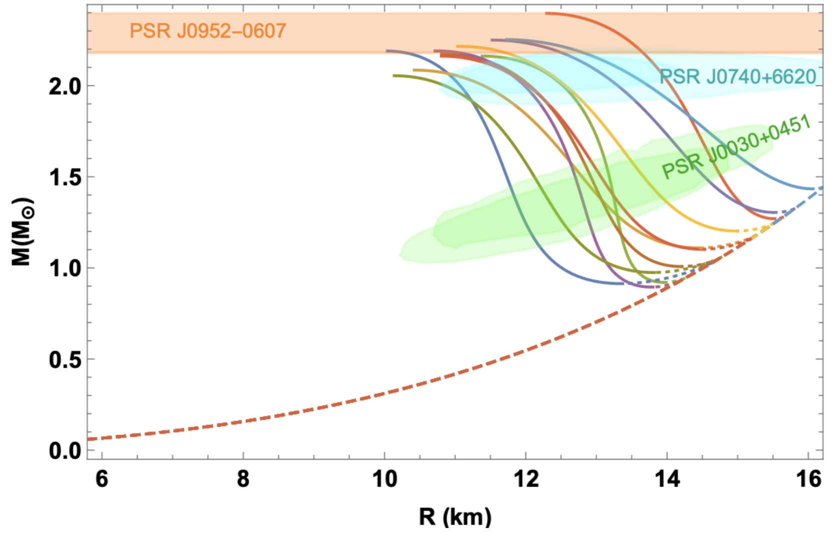


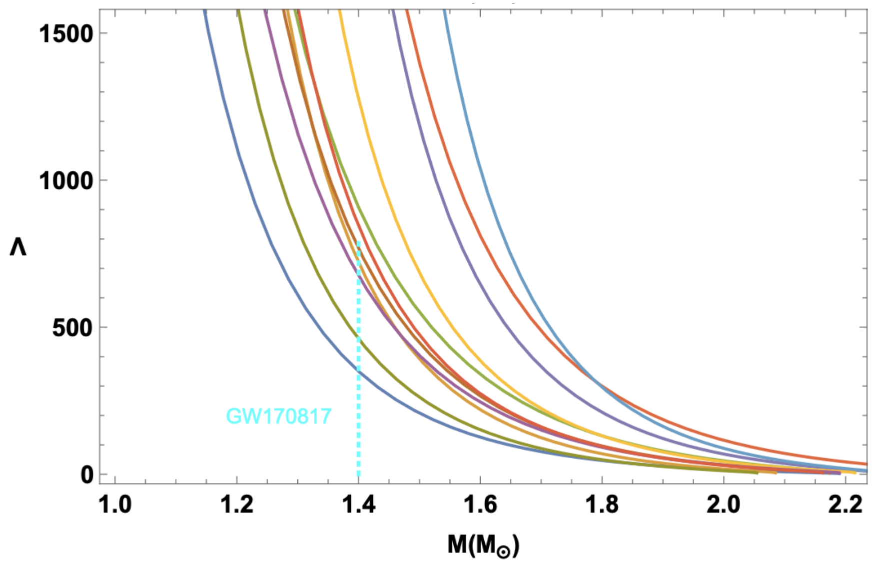


Variations of HM EOSs: Non-hyperonic

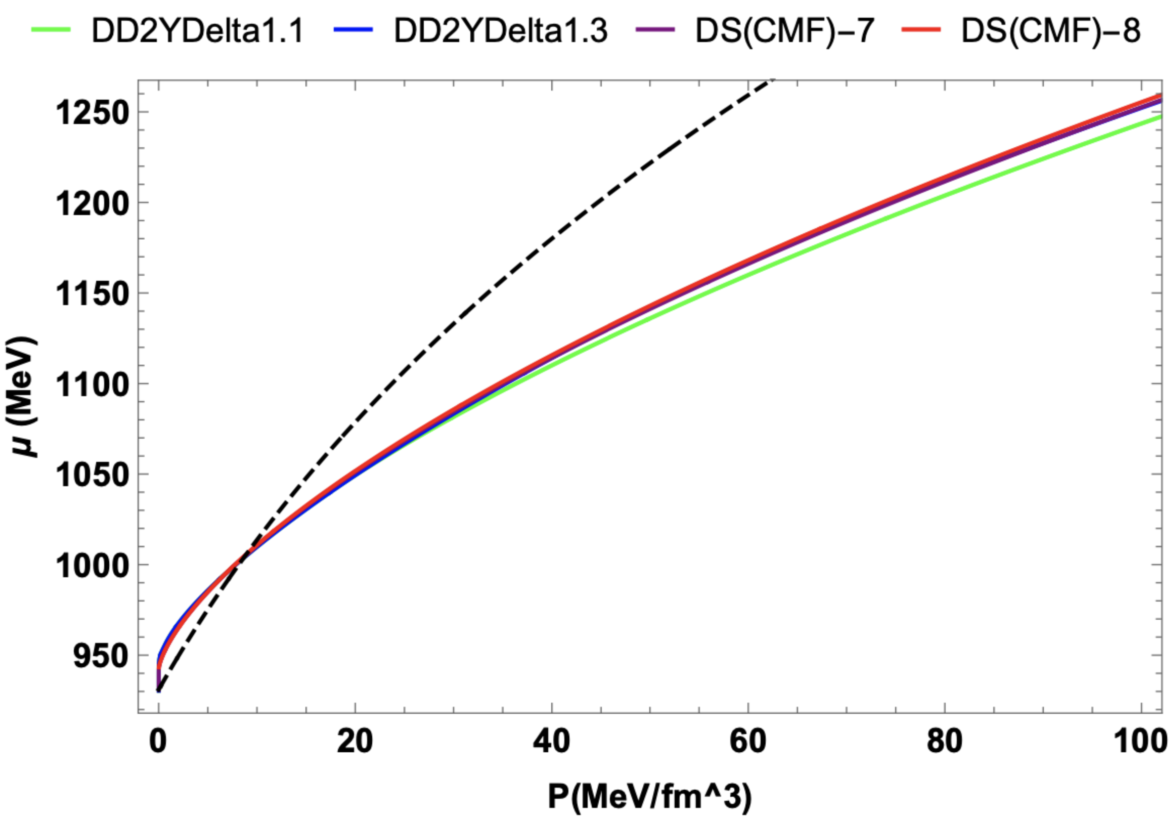
- Fix QM EOS, but vary HM EOSs

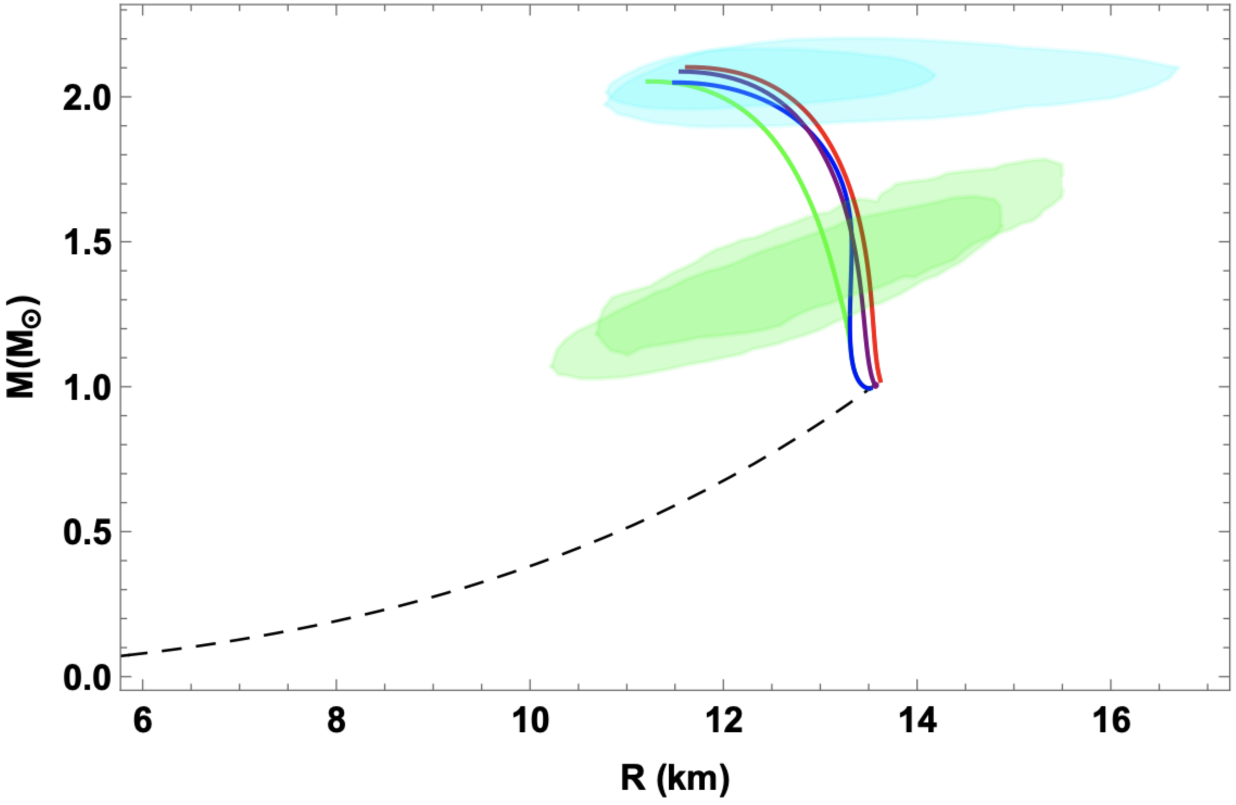






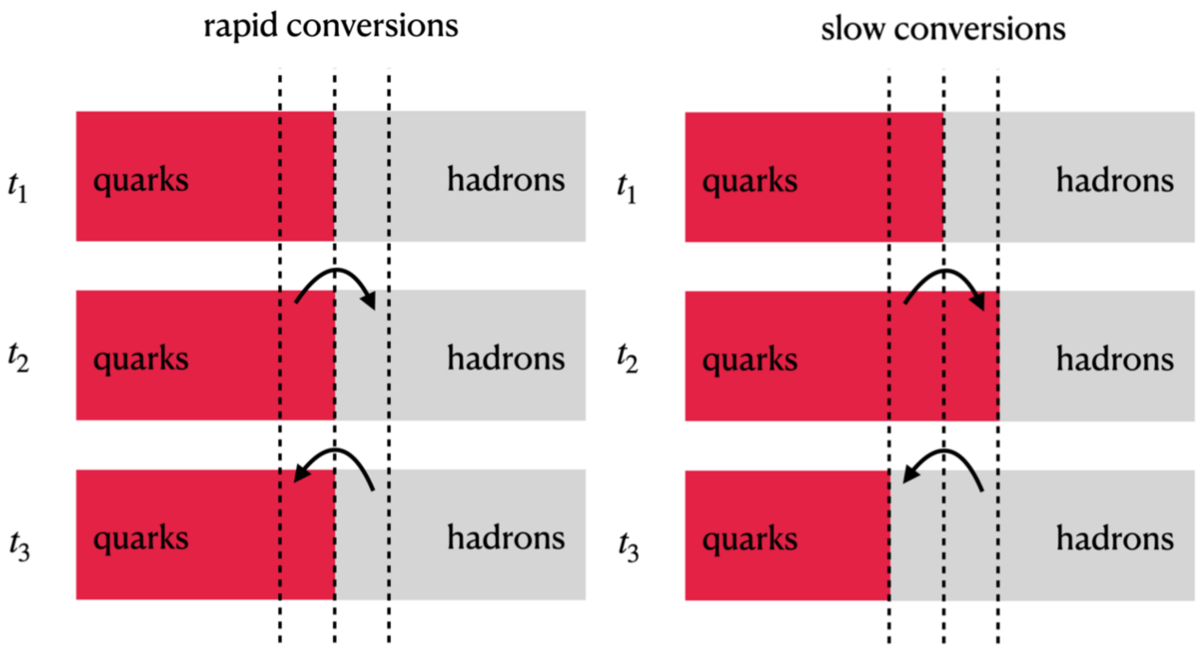
Variations of HM EOSs: Hyperonic
(with Delta resonances, as expected for core region)

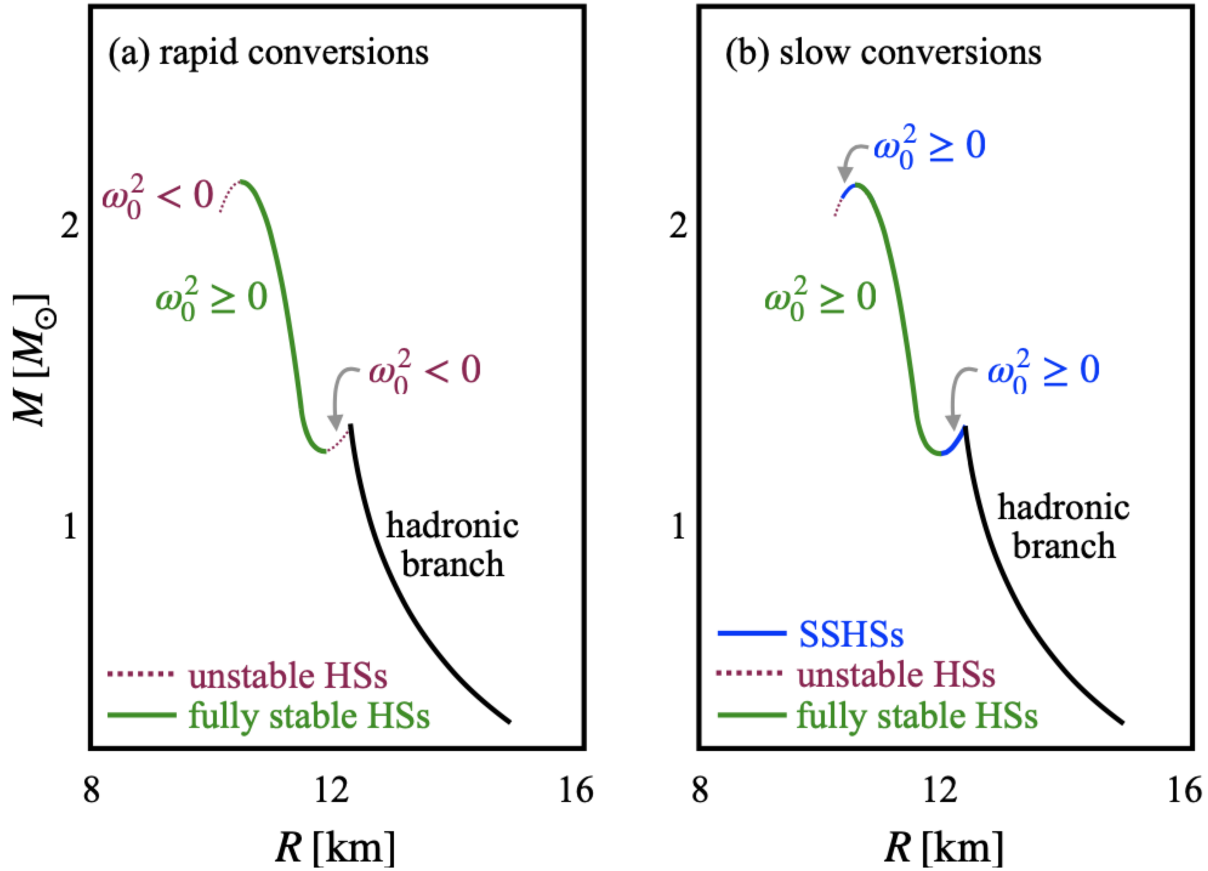




Radial Oscillations

- Conventional Hybrid stars





Graph adapted from Lugones, G., & Grunfeld, A. G. (2021). *Universe*, *7*(12), 493.

Radial Oscillations

- Solving Scheme:

$\xi = \Delta r / r$

$$\frac{d\xi}{dr} = \mathcal{V}(r)\xi + \mathcal{W}(r)\Delta P,$$
$$\frac{d\Delta P}{dr} = X(r)\xi + Y(r)\Delta P,$$

$$\mathcal{V}(r) = -\frac{3}{r} - \frac{dP}{dr}\frac{1}{(P+\rho)},$$
$$\mathcal{W}(r) = -\frac{1}{r}\frac{1}{\Gamma P}, \qquad \Gamma = \frac{n}{P}\frac{dP}{dn} = \frac{\rho+P}{P}\frac{dP}{d\rho}$$
$$X(r) = \omega^2 e^{2\Lambda - 2\Phi}(P+\rho)r - 4\frac{dP}{dr} + \left(\frac{dP}{dr}\right)^2 \frac{r}{(P+\rho)} - 8\pi e^{2\Lambda}(P+\rho)Pr,$$
$$Y(r) = \frac{dP}{dr}\frac{1}{(P+\rho)} - 4\pi(P+\rho)re^{2\Lambda},$$

Initial Conditions:

$(\Delta P)_{r=0} = -3(\xi\Gamma P)_{r=0}$ and $\xi(0) = 1$

Solve the eigenfrequencies via shooting method,
Until match the boundary Condition: $(\Delta P)_{r=R} = 0$

Junction conditions: Slow: $[\xi]_-^+ = 0, \quad [\Delta P]_-^+ = 0$; Rapid: $[\Delta P]_-^+ = 0, \quad \left[\xi - \frac{\Delta P}{rP'}\right]_-^+ = 0$

For small discontinuity, slow≈rapid

Non-Radial treatment via Cowling Approximation

$$\xi^i = \left(e^{-\Lambda}W, -V\partial_\theta, -V\sin^{-2}\theta\partial_\phi\right)$$

$$\frac{dW}{dr} = \frac{d\rho}{dP}\left[\omega^2 r^2 e^{\Lambda-2\Phi}V + \frac{d\Phi}{dr}W\right] - \ell(\ell+1)e^{\Lambda}V$$

$$\frac{dV}{dr} = 2\frac{d\Phi}{dr}V - e^{\Lambda}\frac{W}{r^2}$$

Initial Conditions:

$$W(r)|_{r\to 0} = Cr^{\ell+1} \text{ and } V(r)|_{r\to 0} = -Cr^{\ell}/\ell$$

Solve the eigenfrequencies via **shooting method,**
Until match **the boundary Condition**:

$$\omega^2 r^2 e^{\Lambda-2\Phi}V + \Phi' W = 0$$

$$\omega_g < \omega_f < \omega_{p1}$$

Junction conditions

$$W_+ = W_-$$

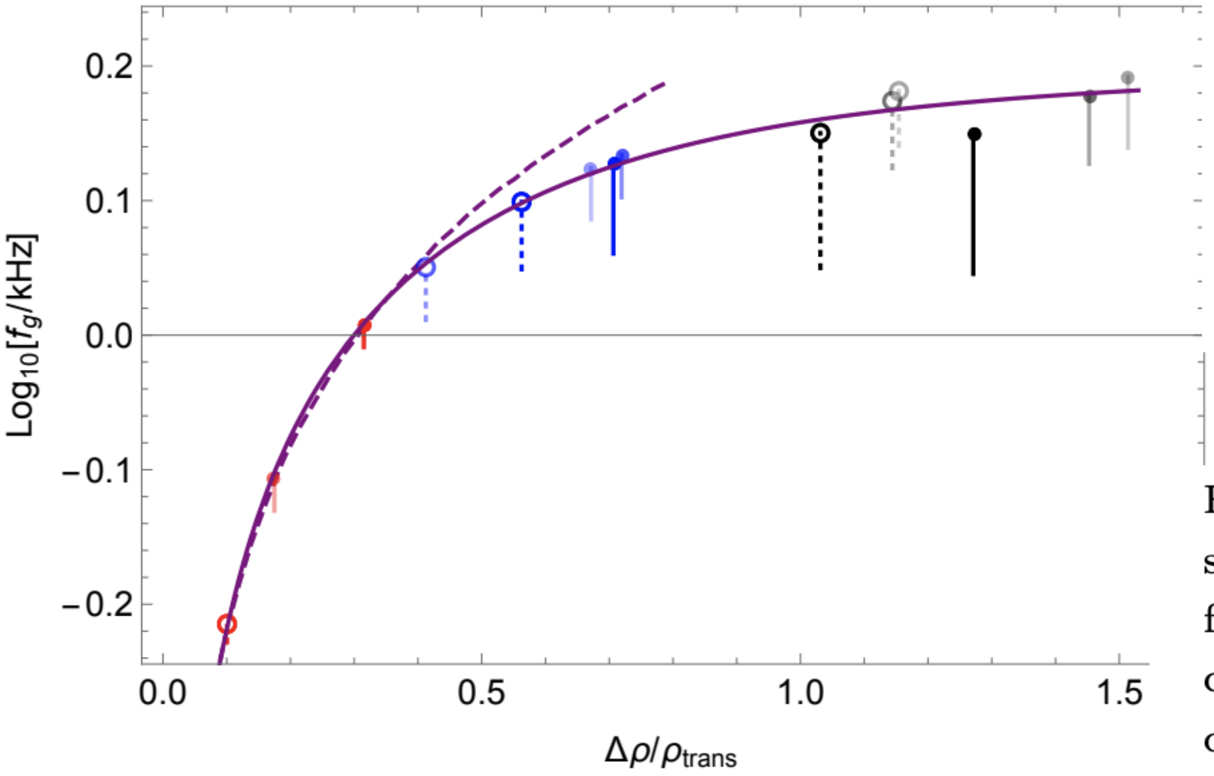
$$V_+ = \frac{e^{2\Phi}}{\omega^2 {r_d}^2}\left[\frac{\rho_- + P}{\rho_+ + P}\left(\omega^2 {r_d}^2 e^{-2\Phi}V_- + e^{-\Lambda}\Phi' W_-\right) - e^{-\Lambda}\Phi' W_+\right],$$

Sotani, H., Yasutake, N., Maruyama, T. and Tatsumi, T., 2011 Physical Review D, 83(2), p.024014

Non-Radial Oscillations

Cowling Approximation

- Assumes that the fluid would oscillate on a fixed background metric with the metric perturbation (ω mode) being neglected
- there is no damping (the eigenfrequencies of oscillation modes only have the real parts)
- compared to full treatments:
  - differences by less than 20% for f modes, around 10% for p-modes [66], and 5%–10% for g-modes



$\frac{\Delta\rho}{\rho_{\rm trans}}$	$a_{4,min}$		$\frac{a_{4,\min}+a_{4,\min}}{2}$		$a_{4,max}$	
	udQM	SQM	udQM	SQM	udQM	SQM
B_{20}	1.27	1.03	1.45	1.14	1.51	1.15
B_{35}	0.71	0.56	0.72	0.41	0.67	NA
B_{50}	0.32	0.10	0.17	NA	NA	NA

Figure 4: $\Delta\rho/\rho_{\rm trans}$ vs g-mode frequencies for different (B,a_4) sets. Filled and empty circles denote the maximum g-mode frequencies of CrSs with udQM and SQM, respectively, with the color convention being same as Fig. 1, and the vertical bars denote the frequencies ranges. The dashed purple line represents the fit for conventional hybrid stars adapted from Ref. [109], while the solid purple line denotes our fit for CrSs.

A New Rescaling Scheme

C. Z, Yong Gao , Cheng-jun Xia, Renxin Xu Phys.Rev.D 108 (2023) 6, 063002

$$\rho = \frac{a}{9}\epsilon\left(\frac{{N_q}^4}{18{n_s}^4}n^5 - \frac{{N_q}^2}{{n_s}^2}n^3\right) + m_q N_q n$$

$$p = \frac{2a}{9}\epsilon\left(\frac{{N_q}^4}{9{n_s}^4}n^5 - \frac{{N_q}^2}{{n_s}^2}n^3\right).$$

$$\bar{\rho} = \frac{\rho}{m_q n_s},\ \bar{p} = \frac{p}{m_q n_s},\ \bar{n} = \frac{N_q n}{n_s},\ \bar{\epsilon} = \frac{\epsilon}{N_q m_q}$$

$$\bar{\rho} = \frac{a}{9}\bar{\epsilon}\left(\frac{1}{18}\bar{n}^5 - \bar{n}^3\right) + \bar{n},$$

$$\bar{p} = \frac{2a}{9}\bar{\epsilon}\left(\frac{1}{9}\bar{n}^5 - \bar{n}^3\right)$$

At zero pressure, $\bar{n} = 3$.

Requiring $\bar{\rho} \geq 0$ at zero pressure thus yield $\bar{\epsilon} \leq \frac{2}{a} \approx 0.1757$ ($\bar{\epsilon}_{max}^{theo}$)

We take $\bar{\epsilon}_{max}^{em} = \frac{120\ \mathrm{MeV}}{930\ \mathrm{MeV}} = 0.13$

Model hybrid strangeon stars: Strangeon sector

- Absorb Nq:

$$\frac{\rho}{n_s} = \frac{a}{9}\tilde{\epsilon}\left(\frac{1}{18}\bar{n}^5 - \bar{n}^3\right) + m_q\bar{n},$$
$$\frac{P}{n_s} = \frac{2\,a}{9}\tilde{\epsilon}\left(\frac{1}{9}\bar{n}^5 - \bar{n}^3\right), \qquad (10)$$

where $a = A_6^2/A_{12} = 8.4^2/6.2 \approx 11.38$, $\tilde{\epsilon} = \epsilon/N_q$ and $\bar{n} = N_q\, n/n_s$. Note that $\bar{n} = 3$ at star surface where $P = 0$.

$$\mu_{\text{strangeon}} = \frac{3\mu}{N_q} = 3\frac{\rho/n_s + P/n_s}{\bar{n}} = 3m_q + a\tilde{\epsilon}(\frac{5}{54}\bar{n}^4 - \bar{n}^2).$$

$$\left(\frac{E}{A}\right)_{\text{strangeon}} = 3m_q - \frac{3a}{2}\tilde{\epsilon}$$

Model Hybrid strangeon stars: QM sector

- Interacting Quark matter (IQM) *C. Z, R.B. Mann, Phys.Rev.D 103 (2021) 6, 063018*

$$\Omega = -\frac{\xi_4}{4\pi^2}\mu^4 + \frac{\xi_4(1-a_4)}{4\pi^2}\mu^4 - \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\pi^2}\mu^2 - \frac{\mu_e^4}{12\pi^2} + B_{\rm eff}$$

$$(\xi_4, \xi_{2a}, \xi_{2b}) = \begin{cases} ((\frac{1}{3})^{\frac{4}{3}} + (\frac{2}{3})^{\frac{4}{3}})^{-3}, 1, 0) & \text{2SC phase} \\ (3, 1, 3/4) & \text{2SC+s phase} \\ (3, 3, 3/4) & \text{CFL phase} \end{cases}$$

$$p = -\Omega,\ n_q = -\frac{\partial\Omega}{\partial\mu},\ n_e = -\frac{\partial\Omega}{\partial\mu_e},\ \rho = \Omega + n_q\mu + n_e\mu_e$$

$$\lambda = \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\sqrt{\xi_4 a_4}}$$

$$\bar{\lambda} = \frac{\lambda^2}{4B_{\rm eff}}$$

$$p = \frac{1}{3}(\rho - 4B_{\rm eff}) + \frac{4\lambda^2}{9\pi^2}\left(-1 + \sqrt{1 + 3\pi^2\frac{(\rho - B_{\rm eff})}{\lambda^2}}\right)$$

→

$$\bar{p} = \frac{1}{3}(\bar{\rho} - 1) + \frac{4}{9\pi^2}\bar{\lambda}\left(-1 + \text{sgn}(\lambda)\sqrt{1 + \frac{3\pi^2}{\bar{\lambda}}(\bar{\rho} - \frac{1}{4})}\right)$$

$$\mu_{\rm QM} = \frac{3\sqrt{2}}{(a_4\xi_4)^{1/4}}\sqrt{[(P+B)\pi^2 + \lambda^2]^{1/2} - \lambda}$$

→

$$\left(\frac{E}{A}\right)_{\rm QM} = \frac{3\sqrt{2}\pi}{(\xi_4 a_4)^{1/4}}\frac{B^{1/4}}{\sqrt{(\lambda^2/B + \pi^2)^{1/2} + \lambda/\sqrt{B}}}$$