# 余剰次元ゲージ理論における宇宙ひもの相互作用

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Work in progress (arXiv: 2409.XXXXX)

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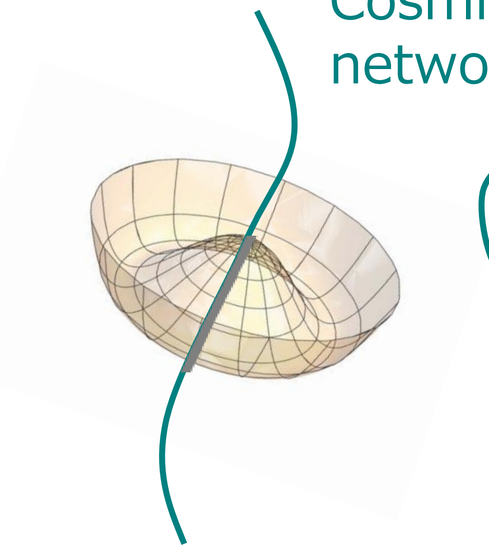
# Introduction (1/3)

Cosmic string : 1d topological defect produced when $\cancel{U(1)}$

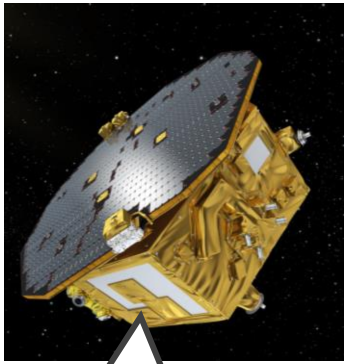
[Kibble (1976)]





GW observations

[Image: LISA website]



Energy of GW

$\Updownarrow$

Breaking scale of $U(1)$

Probing above $10^{11}$ GeV !

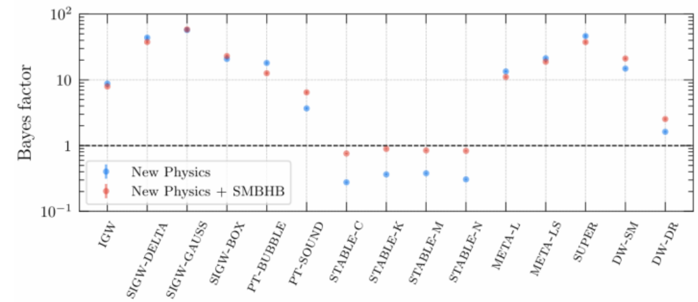
[Blanco-Pillado, Olum, Siemens (2018)]

Cosmic strings are good tools for probing $\cancel{U(1)}$ in BSM

# Introduction (2/3)

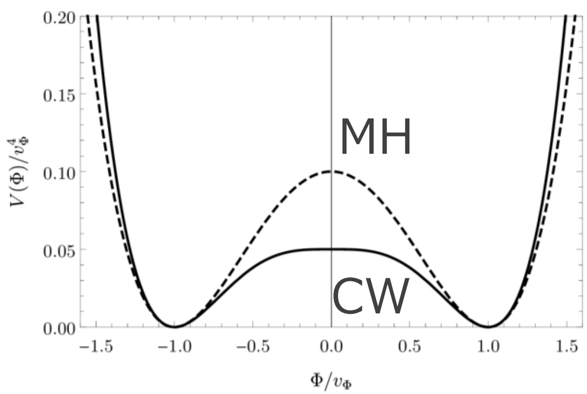
BSM → String properties → GW spectrum

cf. NANOGrav's result




[NANOGrav collaboration (2023)]

★ Scalar potential → Interaction between 2 strings



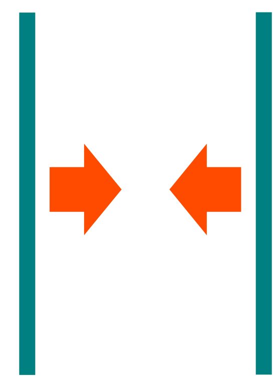

[Eto, Hamada, Jinno, Nitta, Yamada (2022)]

Mexican hat potential

$$V(\phi) = \lambda\left(|\phi|^2 - v_\phi^2\right)^2$$

Coleman-Weinberg potential

$$V(\phi) = \lambda\left(\log\frac{|\phi|^2}{v_\phi^2} - \frac{1}{2}\right)|\phi|^4 + \frac{\lambda v_\phi^4}{2}$$



Changed!

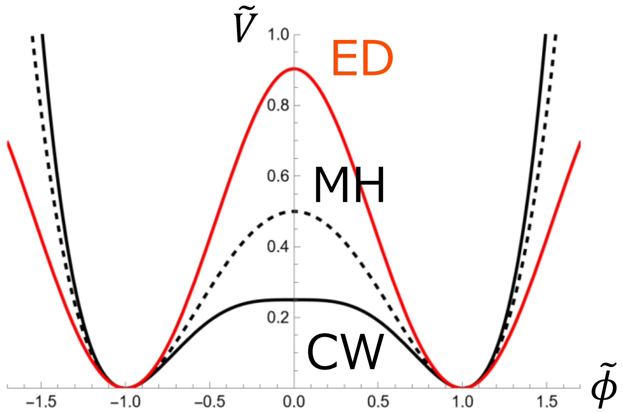
# Introduction (3/3)

Other mechanism (or scalar potential) for $\cancel{U(1)}$ ?

Hosotani mechanism in extra-dimensional models

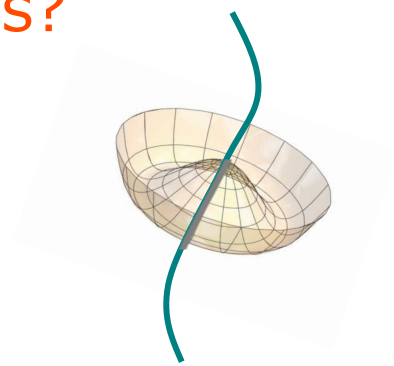
[Hosotani (1983)]

Scalar potential is different from the Mexican hat potential.




New cosmic strings?

New properties?



① Can we distinct the extra-dimensional models?

→ It is difficult in our setup...

② How are the string properties changed?

→ Interesting interaction related to the potential's shape.

1. Introduction

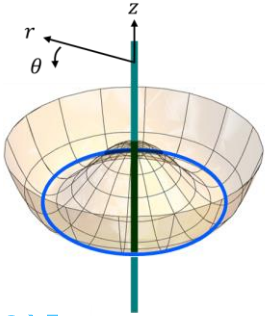
2. Review of string properties

3. String in extra-dim. gauge theory

4. Our results

# Abrikosov-Nielsen-Olesen string

Cosmic strings: classical solutions having 1d excited region




$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \left|D_\mu\phi\right|^2 - V(\phi) \qquad (D_\mu\phi = (\partial_\mu - igA_\mu)\phi)$$

Ansatz of string solution [Abrikosov (1957), Nielsen, Olesen (1973)]

$$\phi(x) = f(r)ve^{in\theta},\ \vec{A}(x) = \frac{na(r)}{gr}\,\vec{e}_\theta,\ (\text{others}) = 0 \qquad (n\text{: winding number})$$

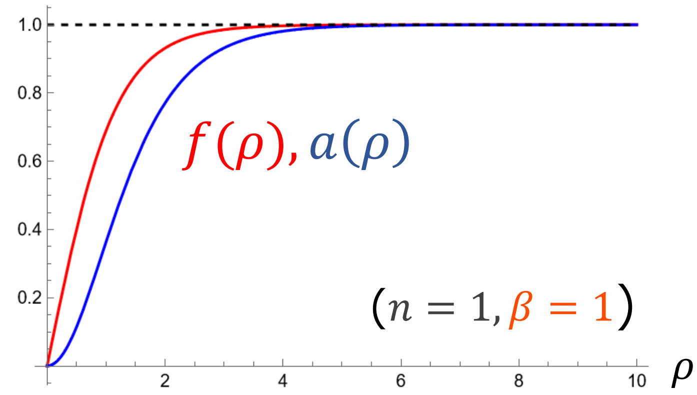
$$(f(0) = a(0) = 0, f(\infty) = a(\infty) = 1)$$ in the cylindrical coordinate $(r, \theta, z)$

$$V(\phi) = \lambda(|\phi|^2 - v^2)^2$$

EoM

$$f'' + \frac{f'}{\rho} - \frac{n^2(1-a)^2}{\rho^2}f^2 - \beta(f^2-1)f = 0$$

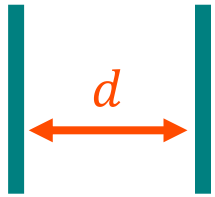
$$a'' - \frac{a'}{\rho} + 2f^2(1-a) = 0$$

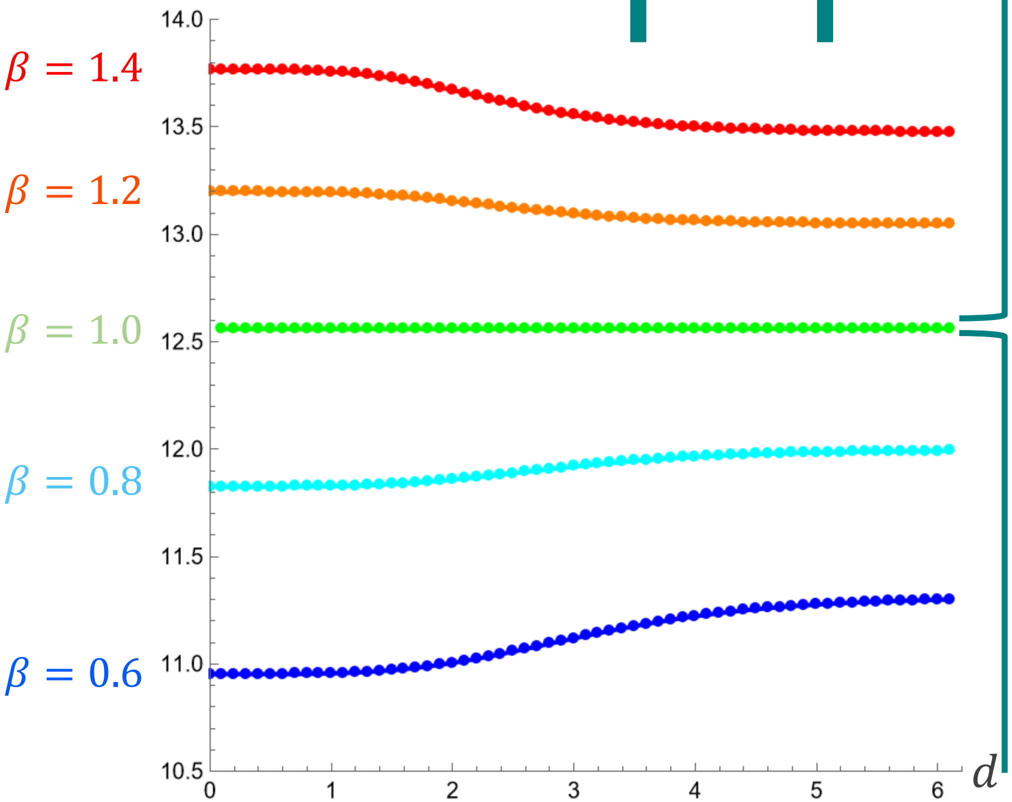



$$(\rho \equiv gvr, \beta \equiv m_\phi^2/m_A^2 = 2\lambda/g^2)$$

# Interaction between ANO strings

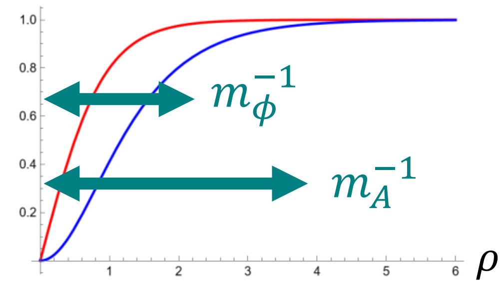
Energy of 2 strings
separated by $d$



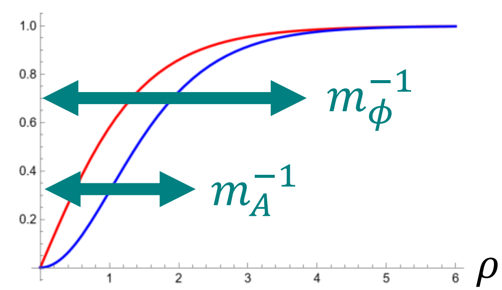

$\beta > 1$: Repulsive force (type-II)

$m_\phi > m_A$




$\beta < 1$: Attractive force (type-I)

$m_A > m_\phi$




※ $\beta = 1$: No force (BPS state)

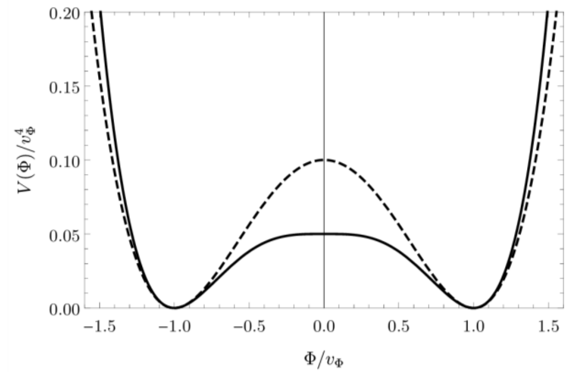
[Bogomolnyi (1976)]
[Parasad, Sommerfield (1975)]
[Goodband, Hindmarsh (1995)]

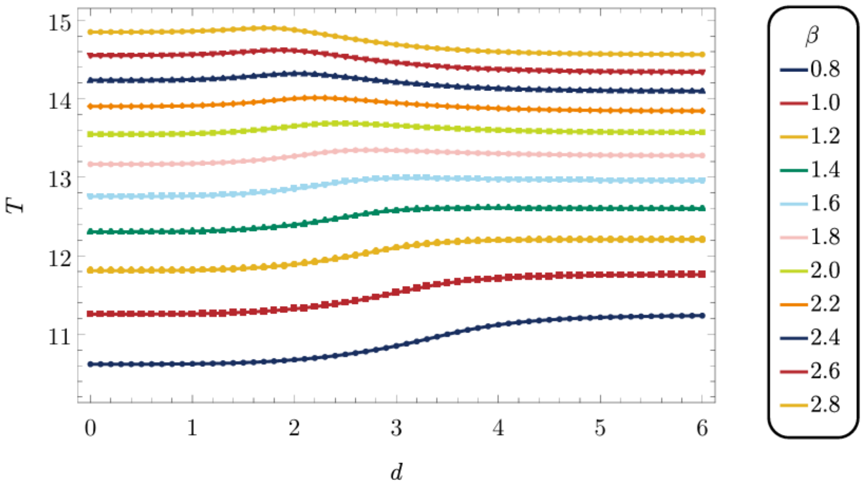
# The CW potential case

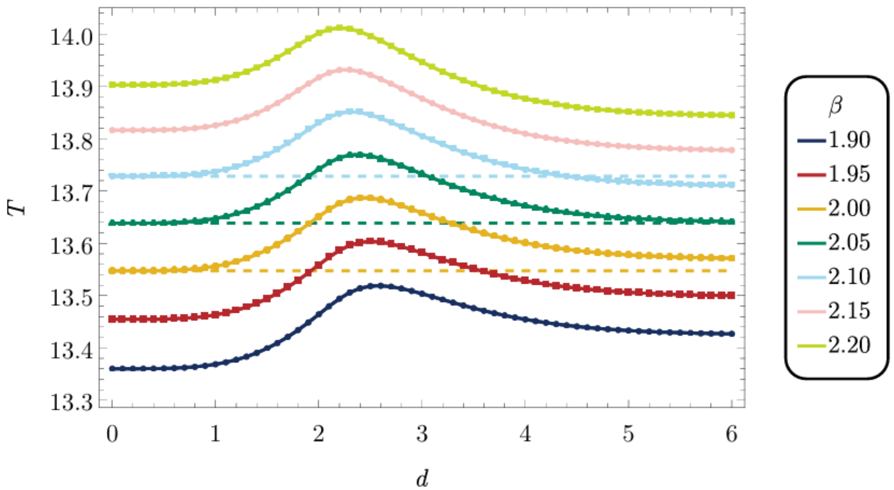
[Eto, Hamada, Jinno, Nitta, Yamada (2022)]

ANO strings with Coleman-Weinberg potential

$$V(\phi) = \lambda\left(\log\frac{|\phi|^2}{v_\phi^2} - \frac{1}{2}\right)|\phi|^4 + \frac{\lambda v_\phi^4}{2}$$




- Repulsive at large $d$, attractive at small $d$ ($\beta\sim2.0$)

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# Extra-dimensional model

$SU(2)$ gauge theory on $M^4 \times S^1/\mathbb{Z}_2$ [Kubo, Lim, Yamashita (2002)]

$$S = \int d^4x \int_0^{\pi R} dy \left[ -\frac{1}{4} F^a_{MN} F^{a,MN} \right] \qquad \left( \begin{array}{ll} M = 0,1,2,3,5 & x^5 = y \\ a = 1,2,3 & \end{array} \right)$$

$\mathbb{Z}_2$ parity: $A_\mu(x,-y) = P_0 A_\mu(x,y) P_0^{-1}, A_y(x,-y) = -P_0 A_y(x,y) P_0^{-1}$

$A_\mu(x,\pi R - y) = P_1 A_\mu(x,y) P_1^{-1}, A_y(x,-y) = -P_1 A_y(x,\pi R + y) P_1^{-1}$

If $P_0 = P_1 = \mathrm{diag}(1,-1)$,
only $A^3_\mu, A^1_y$ and $A^2_y$ have Kaluza-Klein 0 modes.

The effective potential $V_{\mathrm{eff}}(A^1_y, A^2_y)$ arises in 4d lagrangian.

$$S = \int d^4x \left[ -\frac{1}{4} F^3_{\mu\nu} F^{3,\mu\nu} - \left| D_\mu \left( \underbrace{\frac{A^1_y - iA^2_y}{\sqrt{2}}}_{\equiv \phi} \right) \right|^2 + V_{\mathrm{eff}}(A^1_y, A^2_y) \right]$$

Abelian Higgs model

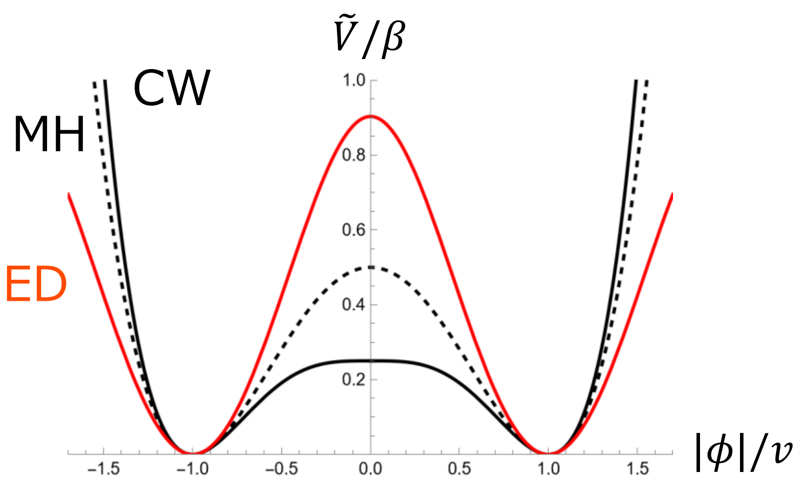
# String in Hosotani mechanism

We can add fermions so that $\langle\phi\rangle \neq 0$.

*e.g.* Adding an adjoint fermion $\psi_+^a$

$$(\psi_\pm^a(x,-y) = \eta P_0 \psi_\pm^a(x,y) P_0^{-1},\ \psi_\pm^a(x,\pi R - y) = \eta' P_1 \psi_\pm^a P_1^{-1},\ \eta\eta' = \pm)$$

$$\tilde{V}(\phi) = \beta \frac{16}{3\zeta(3)\pi^2}\left[\sum_{k=1}^{\infty} \frac{1}{k^5}\cos\left(\frac{\pi|\phi|}{v}\right) + \frac{15}{16}\zeta(5)\right] \qquad \begin{pmatrix} \beta \equiv m_\phi^2/m_A^2 \\ v \equiv 1/(2\sqrt{2}g_4 R) \end{pmatrix}$$




We can consider string solutions by using the ansatz

$$\phi(x) = f(r) v e^{i\theta},\ \vec{A}(x) = \frac{a(r)}{g_4 r}\vec{e}_\theta$$

Other 2 examples $(\psi_+^a, \psi_+^i)$ and $(\psi_+^a, \psi_-^i)$ will be in our paper.

※ $\beta \sim g_4^2 \times 0.0075$, but we consider any value of $\beta$.

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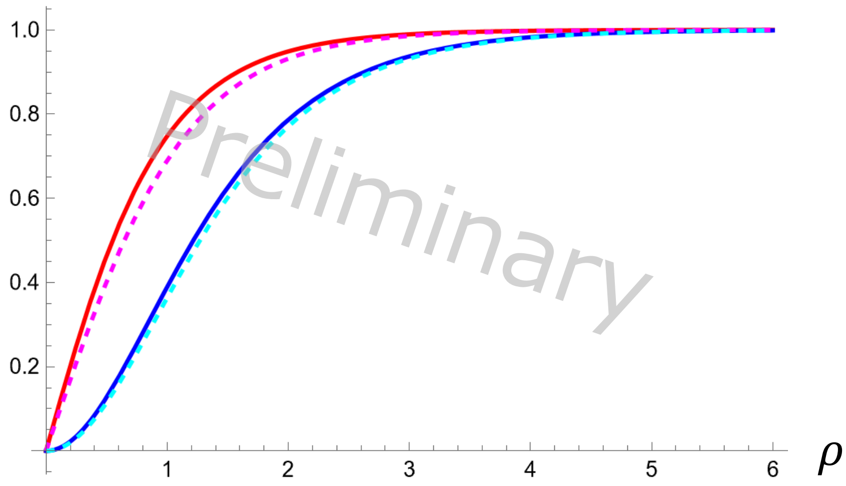
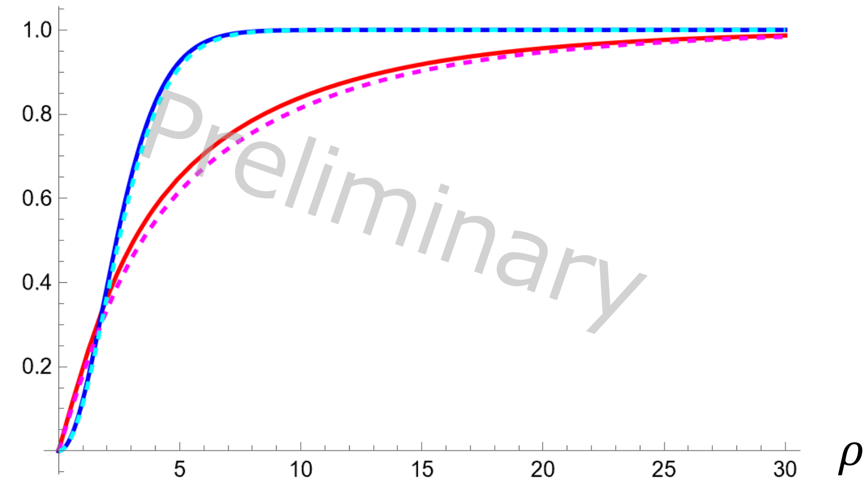
# Behavior of string solution

$\beta = 1, n = 1$




$\beta = 0.005, n = 1$




(Solid lines: $f(\rho), a(\rho)$ in the ex.-dim. model, dashed lines: $f(\rho), a(\rho)$ of the ANO string)

Half width

$f(\rho)$: 85% of that of MH

$a(\rho)$: 95% of that of MH

$f(\rho)$: 90% of that of MH

$a(\rho)$: 96% of that of MH

$f, a$ becomes more slender, but they do not change significantly.

# Calculation of interaction energy

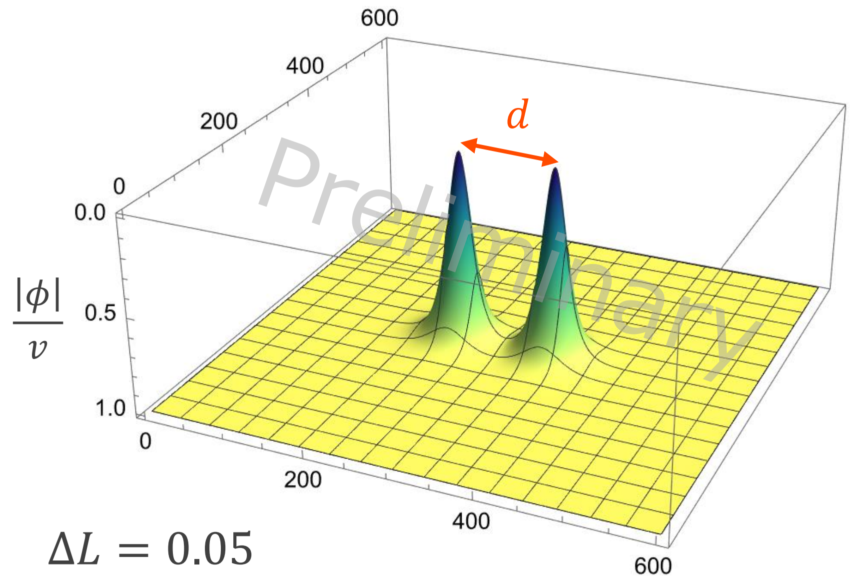
We calculate the energy of parallel 2-strings system on 2d lattice by using the gradient flow method.

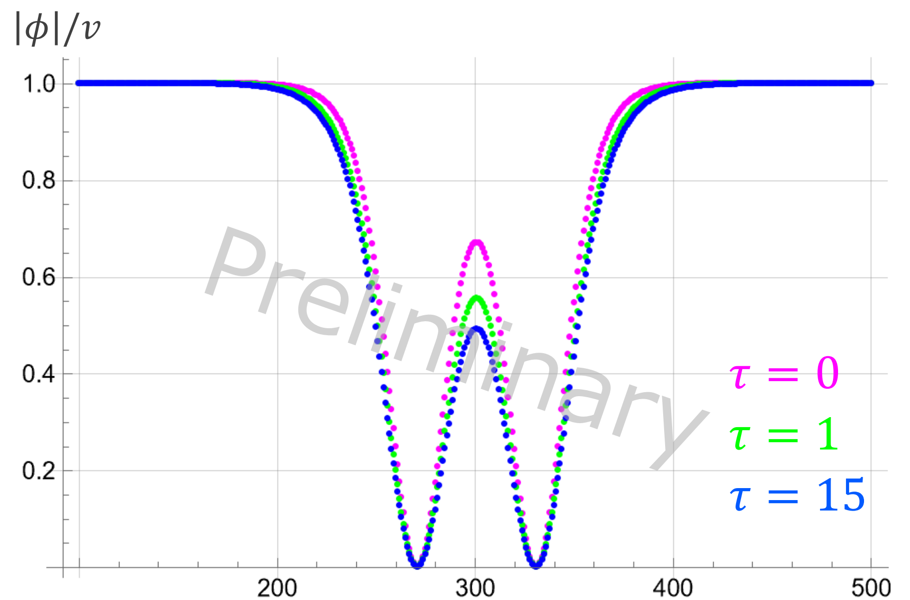
[Eto, Hamada, Jinno, Nitta, Yamada (2022), Fujikura, Li, Yamaguchi (2023)]

Gradient flow eq.

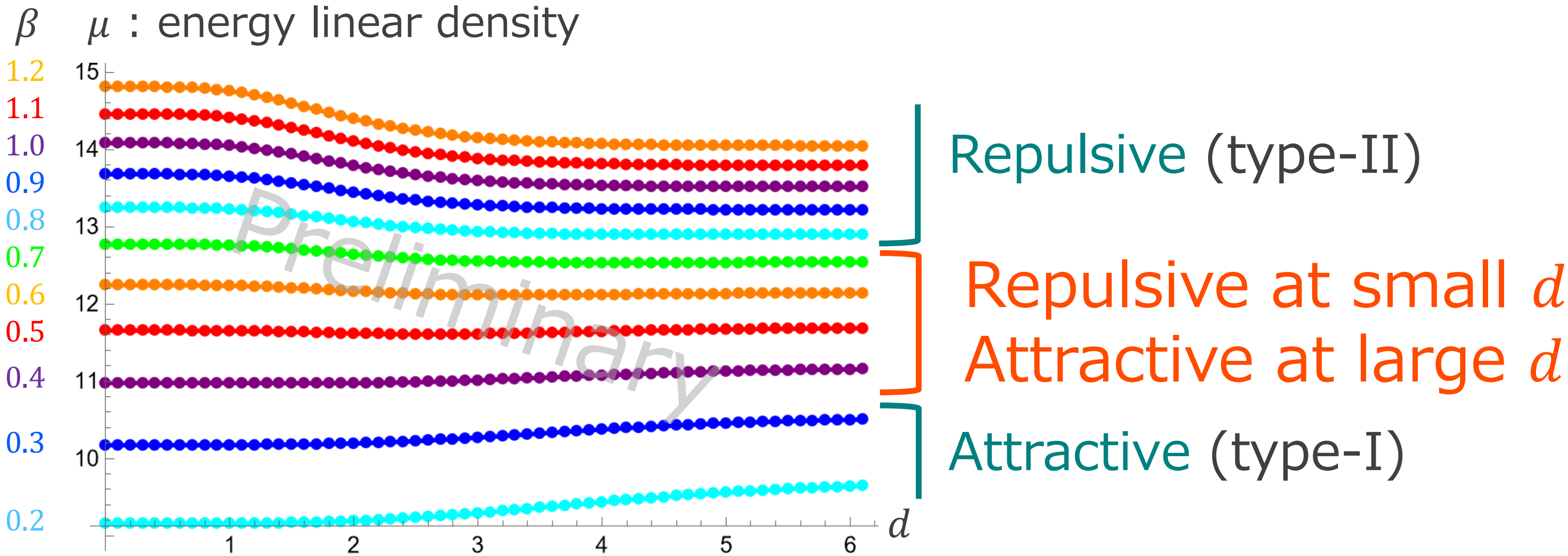
$$\frac{\partial \phi_i(\rho, \tau)}{\partial \tau} = -\frac{\delta \mu}{\partial \phi_i} \quad (\tau\text{: flow time})$$

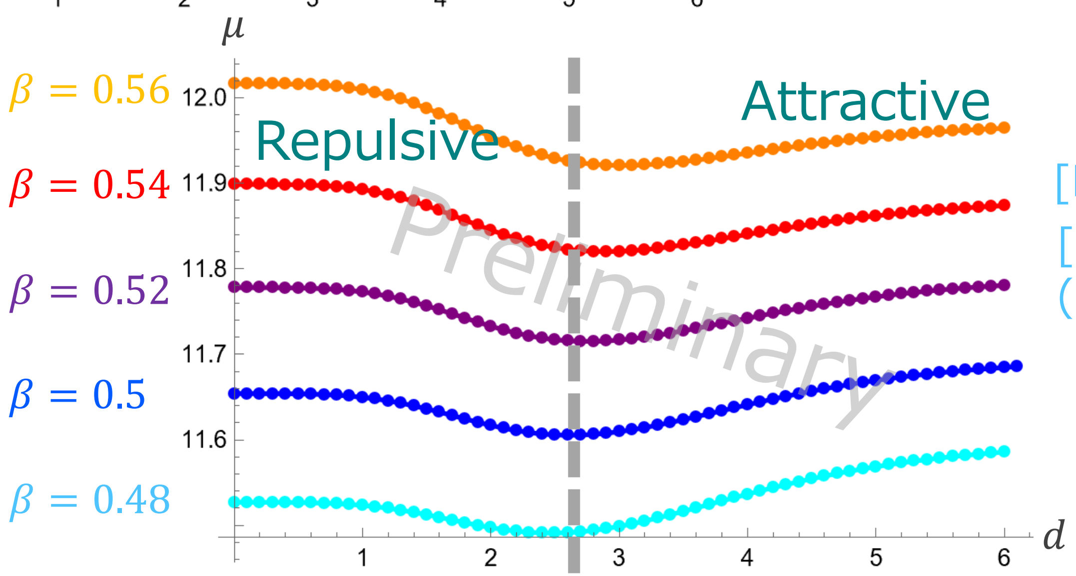
Setting initial configuration as $\phi_i(\rho, 0)$, we solve GF eq. from $\tau = 0$ to $15$.



# Numerical results




"type-1.5"

[Babaev, Speight (2005)]

[Moshchalkov, *et. al* (2009)]

2 scalar fields with MH potentials. ($\beta_1 > 1$ and $\beta_2 < 1$)

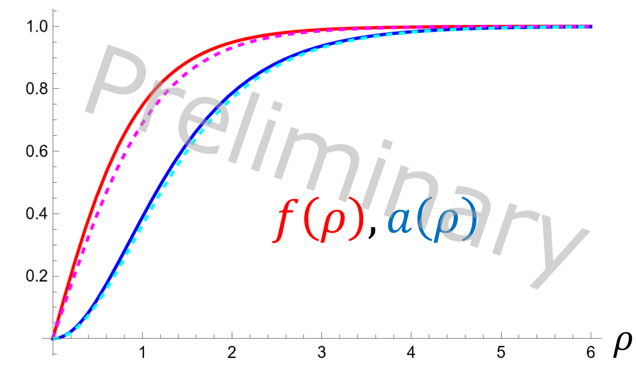
# Interpretation of type-1.5 ①

## Large $d$: Scalar field vs. gauge field

Point source formalism [Speight (1997)]

$$f(\rho) \sim 1 - c_\phi K_0\left(\sqrt{2\beta}\rho\right)$$
$$a(\rho) \sim 1 - c_A \rho K_1\left(\sqrt{2}\rho\right)$$

at $\rho \gg 1$ $\left(c_\phi, c_A \in \mathbb{R}\right)$




$$\Rightarrow \frac{d\mu}{dd} \propto c_\phi^2 \sqrt{\beta} K_1\left(\sqrt{2\beta}d\right) - c_A^2 K_1\left(\sqrt{2}d\right)$$

+: attractive, −: repulsive

(In the MH case, + at $\beta < 1$, − at $\beta > 1$, and 0 at $\beta = 1$)

Numerical result

At $d = 5$,

+ (⇔ attractive) at $\beta \leq 0.7$

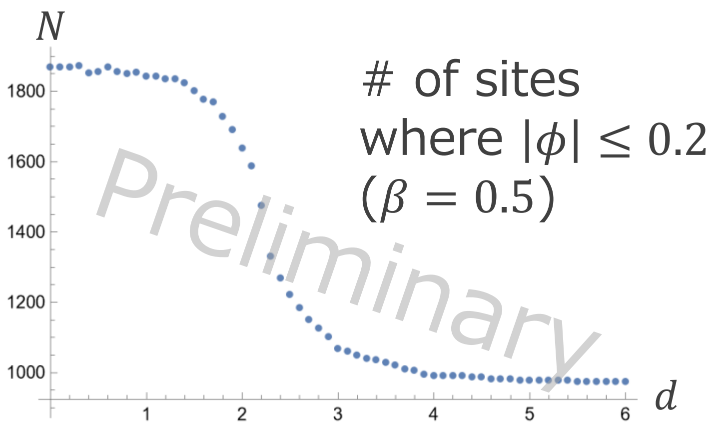
− (⇔ repulsive) at $\beta \geq 0.8$

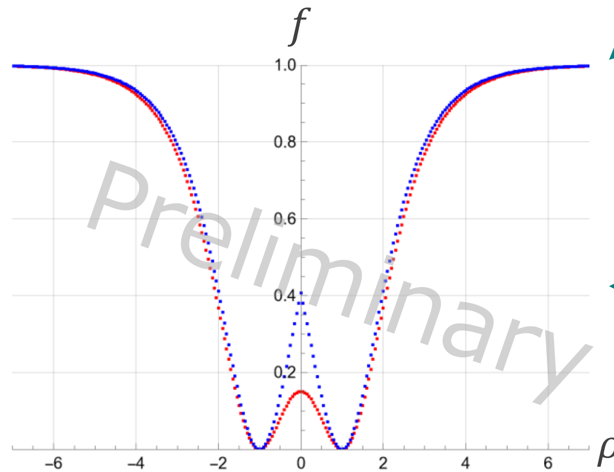
Consistent with results on 2d lattice!

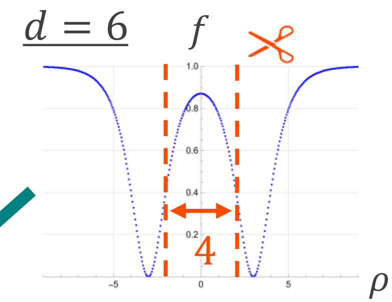
# Interpretation of type-1.5 ②

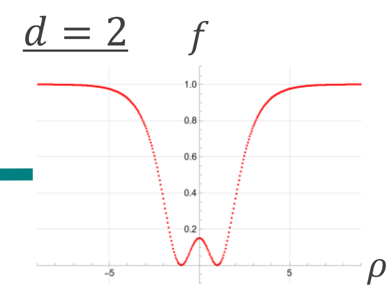
## Small $d$: Change of the shape of strings

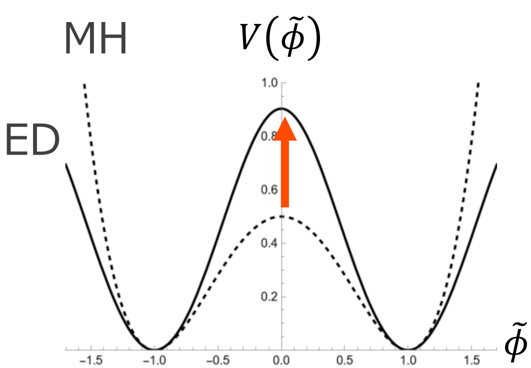
When $d \to 0$, the excited region of $\phi$ increases.



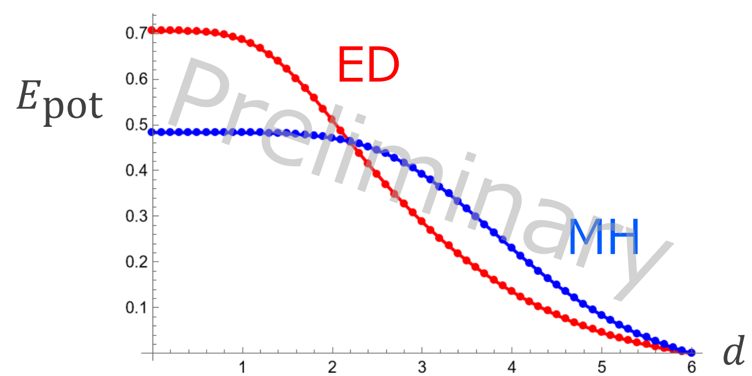



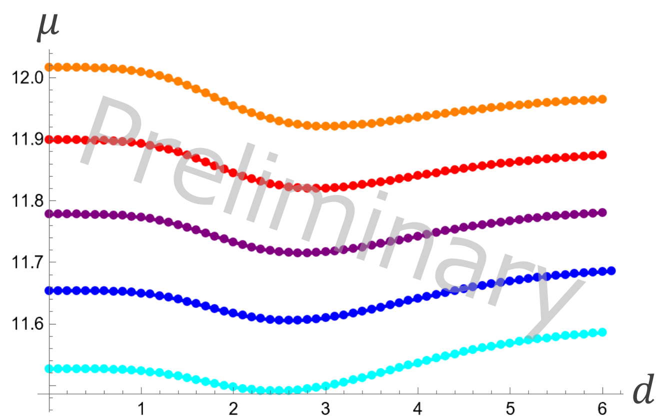

The value of $V(0)$ seems to affects a contribution from the potential energy.



# Interpretation of type-1.5 ③

Small $d$

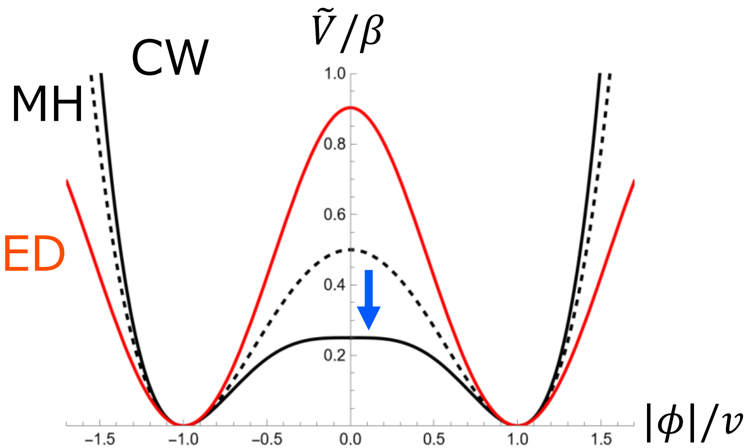
Increase of
the excited region
&
Value of $V(0)$




Large $d$

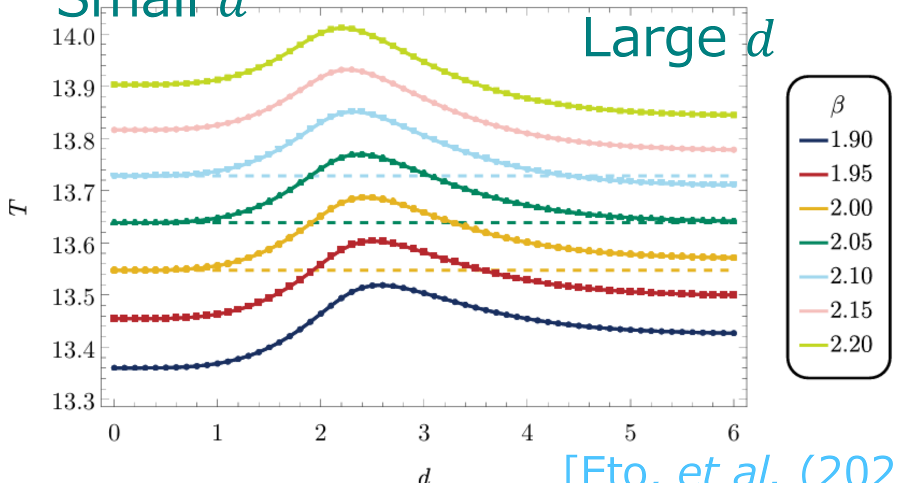
Scalar field
vs.
gauge field

cf. the CW potential case




Small $d$

Large $d$

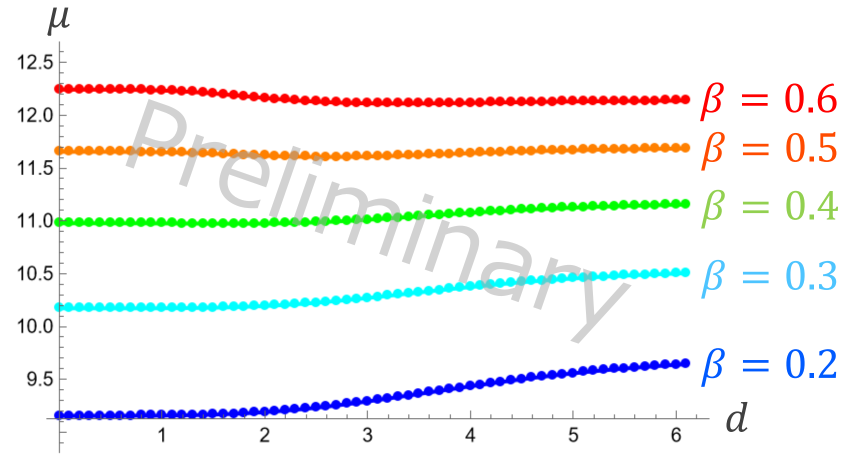



[Eto, *et al.* (2022)]

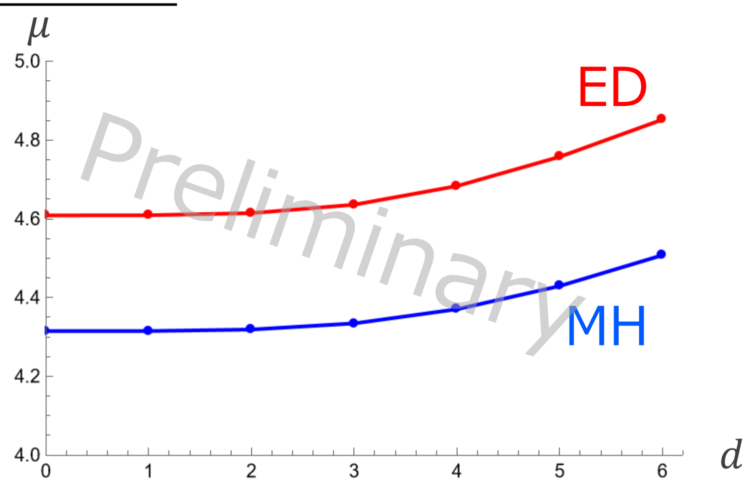
# Result on small $\beta$

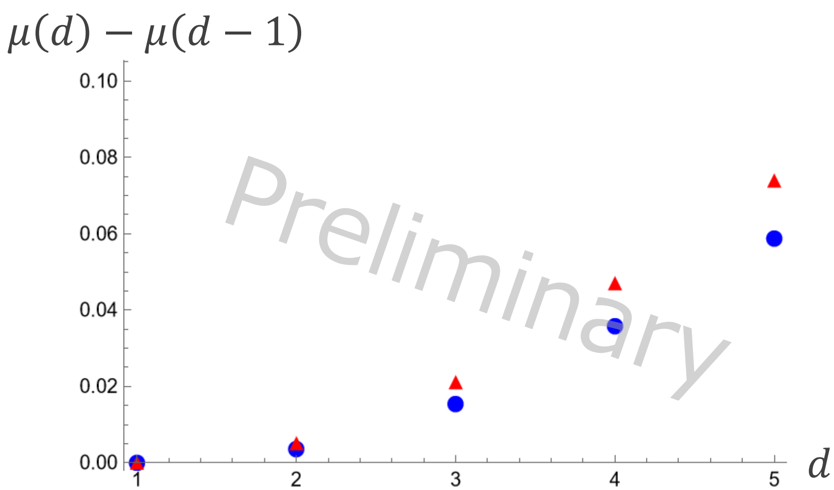
In our setup, $\beta \sim g_4^2 \times 0.0075$, but calculation is difficult on this value.

However, the interaction tends to be attractive (type-I) at small $\beta$.




$\underline{\beta = 0.01}$




The interactions are almost the same.

# Summary

- We consider the string solutions in the $SU(2)$ gauge theory on $M^4 \times S^1/\mathbb{Z}_2$, which has Abelian Higgs model as a 4d effective model.
- In such the model, the 2 strings show type 1.5 like interaction.
- We interpret the type 1.5 like interaction by considering increase of the excited region. This interpretation is related to the value of $V(0)$, and consistent with the Coleman-Weinberg potential case.
- In our setup, $\beta$ becomes very small. At small $\beta$, the interaction becomes attractive (type-I), and it is difficult to distinguish between the Abelian Higgs model and the extra-dimensional model in this study through cosmic strings.