Induced Domain Walls of QCD Axion, and Gravitational Waves

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Introduction

- **QCD axion** Peccei & Quinn (1977), Weinberg (1978), Wilczek (1978)

  { a solution to the strong CP problem
  { a (pseudo-)NG boson arising at SSB of the Peccei-Quinn symmetry

  The QCD axion acquires a potential from non-perturbative effects of QCD:

$$V_{\mathrm{QCD}} = \chi(T)\left[1 - \cos\left(\frac{a}{f_a}\right)\right]$$

$$\chi(T) = m_a^2(T) f_a^2 = \begin{cases} \chi_0 & (T < T_{\mathrm{QCD}}) \\ \chi_0 \left(\dfrac{T}{T_{\mathrm{QCD}}}\right)^{-n} & (T \geq T_{\mathrm{QCD}}) \end{cases}$$

$$\chi_0 \simeq (75.6\,\mathrm{MeV})^4$$
$$n \simeq 8.16$$
$$T_{\mathrm{QCD}} \simeq 153\,\mathrm{MeV}$$

Borsanyi *et. al.* (2016)

The QCD axion is a candidate of dark matter.

Introduction

- Two scenarios of QCD Axion

  - Pre-inflationary PQ breaking

    → The axion field becomes (nearly) uniform.

    Its coherent oscillation can explain dark matter via the misalignment mechanism.

    Preskill, Wise, Wilczek (1983), Abbott & Sikivie (1983), Dine & Fischler (1983)

    If $a_{\rm ini}/f_a = \mathcal{O}(1),\ f_a = \mathcal{O}(10^{12})\,\mathrm{GeV}$ for all DM.

  - Post-inflationary PQ breaking

    → The axion field value is randomly determined at the SSB.

    Topological defects are formed.

    "Domain wall problem" or DM from collapse of defects

    Davis (1986)

Introduction

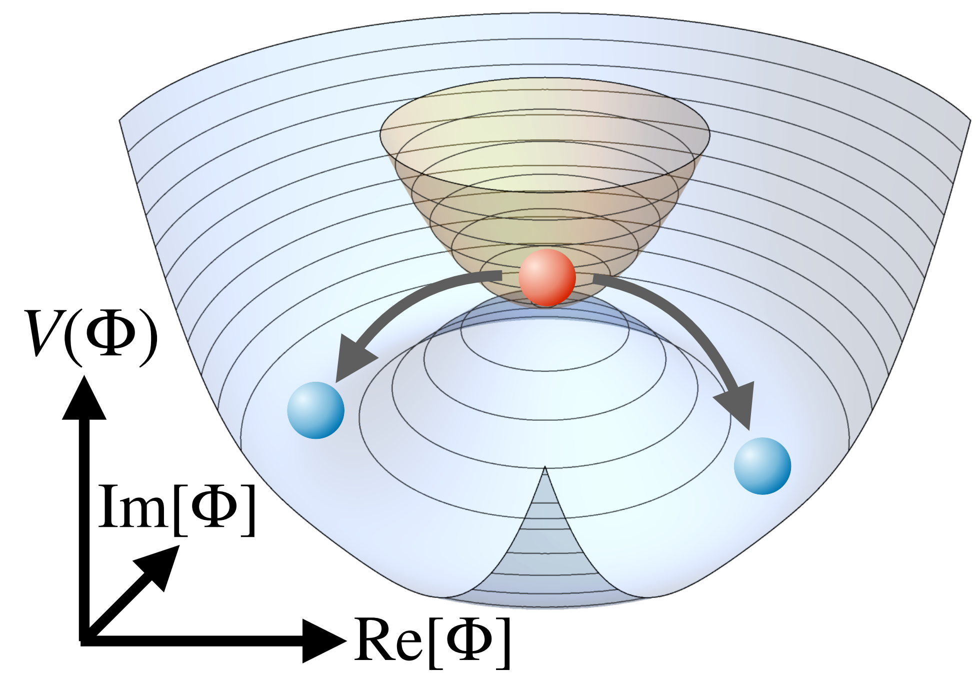
- Cosmic string

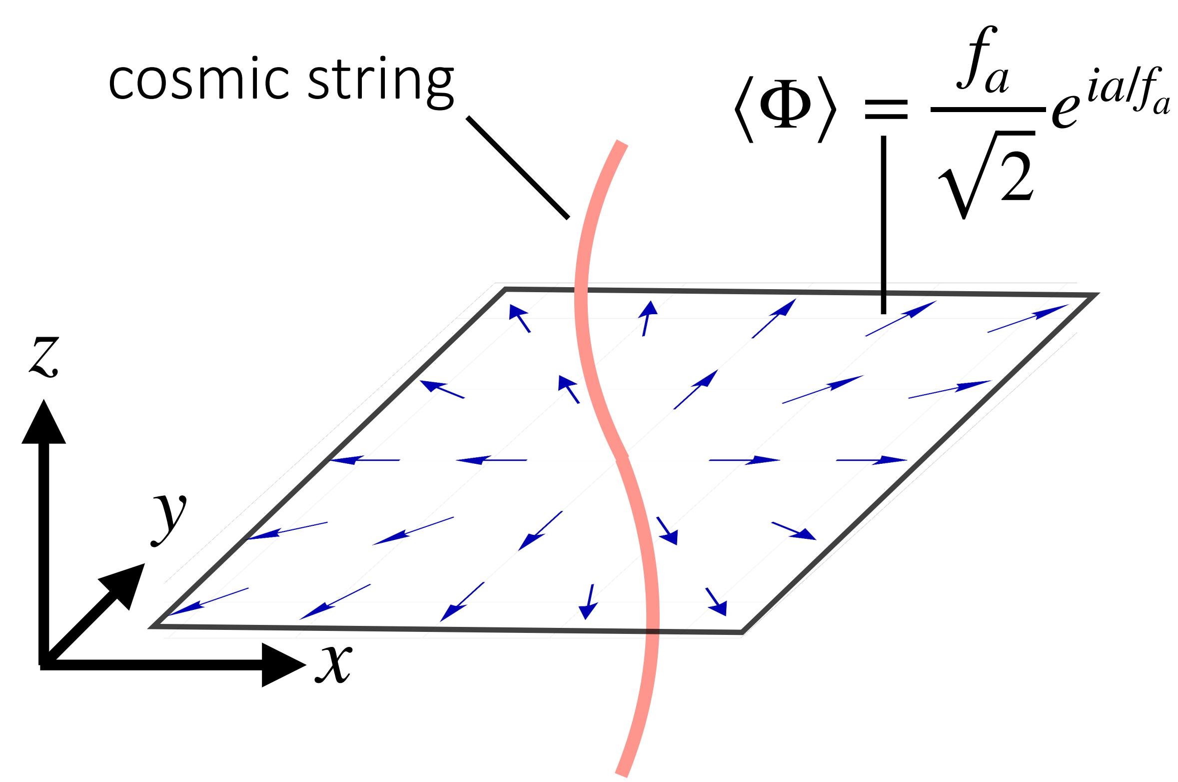
A complex scalar Φ with a wine-bottle $V(\Phi)$

At high temperatures,
Φ is stabilized at the origin.

At low temperatures,
the VEV, $\langle\Phi\rangle$, becomes nonzero.
An axion arises as a NG boson.

➔ Formation of cosmic string





Introduction

- Domain wall

Explicit breaking of U(1) symmetry

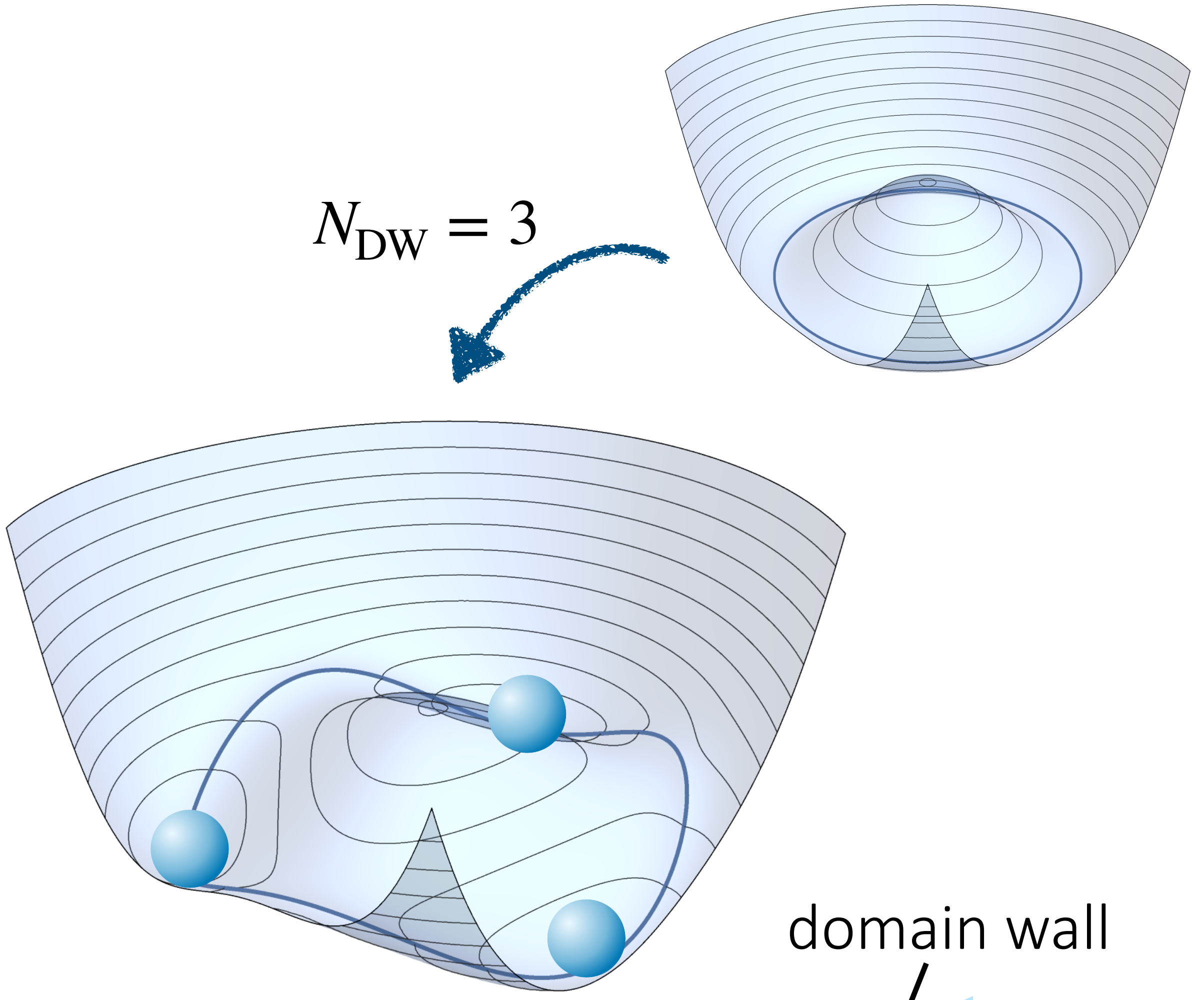
$$\mathrm{U}(1) \to \mathbb{Z}_{N_{\mathrm{DW}}}$$

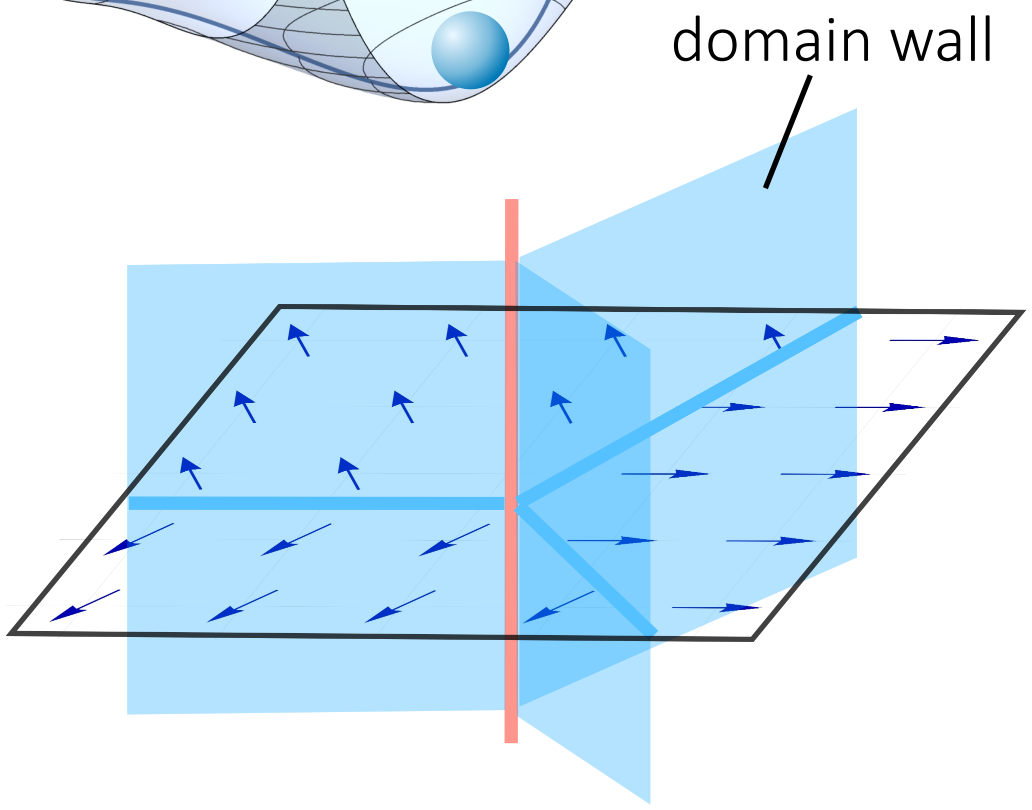
➜ Potential for the axion, $m_a \neq 0$

$$V_{\mathrm{QCD}} = \chi \left[1 - \cos\left(N_{\mathrm{DW}} \frac{a}{f_a} \right) \right]$$

When $m_a > H$,
the axion rolls down the potential.

➜ Formation of domain walls





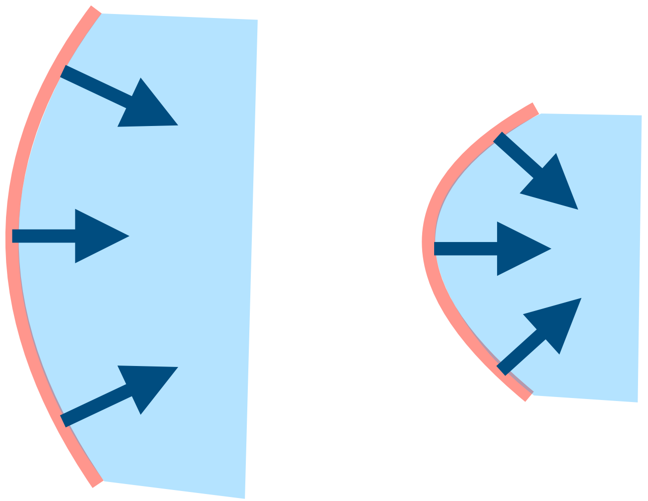
Introduction

Domain wall number

- $N_{\mathrm{DW}} = 1$

  The network rapidly decay due to the DW tension.

  The emitted axions can account for DM.



- $N_{\mathrm{DW}} > 1$

  The DW tension balance with each other.

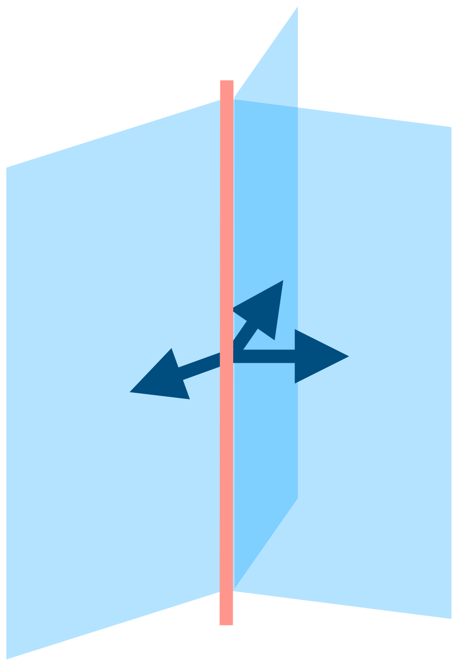
  The network becomes long-lived.

  DWs follow the scaling solution:

  $\mathcal{O}(1)$ DWs in each Hubble volume.

  → Overclosure of the universe.

  "Domain wall problem"

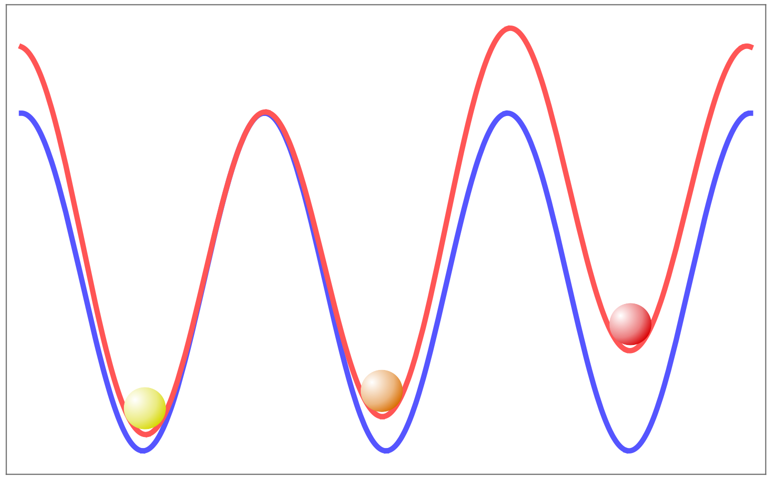


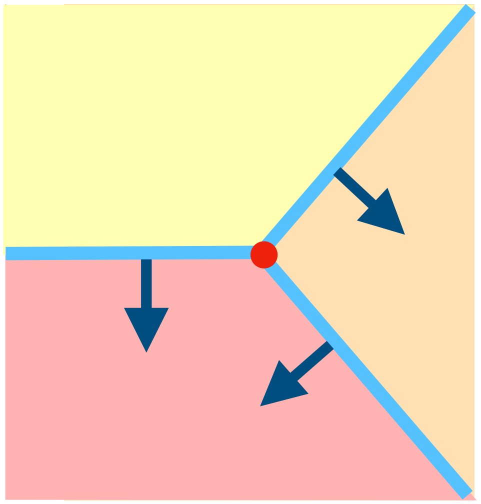
- Domain wall collapse

To avoid the overclosure, we can introduce a bias term:

$$V(a) = \underline{\underline{\chi\left[1-\cos\left(N_{\rm DW}\frac{a}{f_a}\right)\right]} + \epsilon\chi\left[1-\cos\left(N_{\rm b}\frac{a}{f_a}+\theta\right)\right]}$$

→ DWs collapse when the bias overcomes the DW tension.





When the DWs disappear, axions and gravitational waves are emitted.

Introduction

Axion-like particles

- There can also be many axion-like particles (e.g., the axiverse).
- Similar couplings and periodicity to the QCD axion
- Their scales of fields and potentials are free in general.

QCD axion:
Potential of $\mathcal{O}(100)\,\mathrm{MeV}$ scale with temperature dependence

In general, two or more axions can mix in the potential.
One combination of them work as the QCD axion.

In this talk, I discuss the domain walls of mixed axions.

Main topic

Model

- Potential

We consider two axions (a, ϕ) with the potential of

$$V(a,\phi) = \chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]+\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]$$

$$\chi(T) \equiv \frac{m_a^2(T) f_a^2}{N_a^2}, \quad \Lambda^4 \equiv \frac{m_\phi^2 f_\phi^2}{N_{\rm DW}^2}$$

We assume

$$m_\phi \gg m_{a0} \equiv m_a(T=0)$$ → Order of DW formation

$$\frac{m_\phi f_\phi^2}{N_{\rm DW}^2} \gg \frac{m_{a0} f_a^2}{N_a^2}$$ → Hierarchy of DW tension

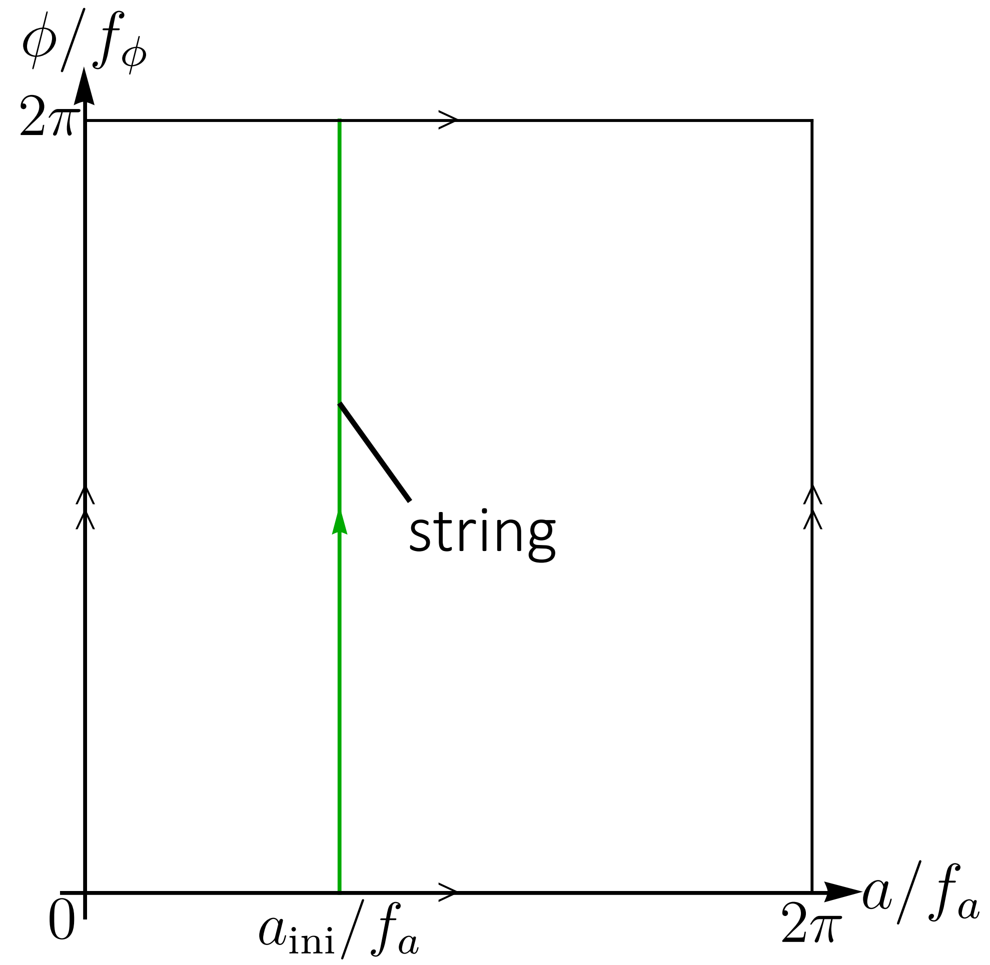
Model

- Initial condition

As the initial condition, we consider

$\begin{cases} \text{the pre-inflationary for } a \\ \text{the post-inflationary for } \phi \end{cases}$

- the pre-inflationary for a → Initially homogeneous, no winding of a
- the post-inflationary for ϕ → Cosmic strings of ϕ



Induced domain walls

- Domain walls of ϕ

$$\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]+\underbrace{\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]}_{V_{\rm DW}(\phi)}$$

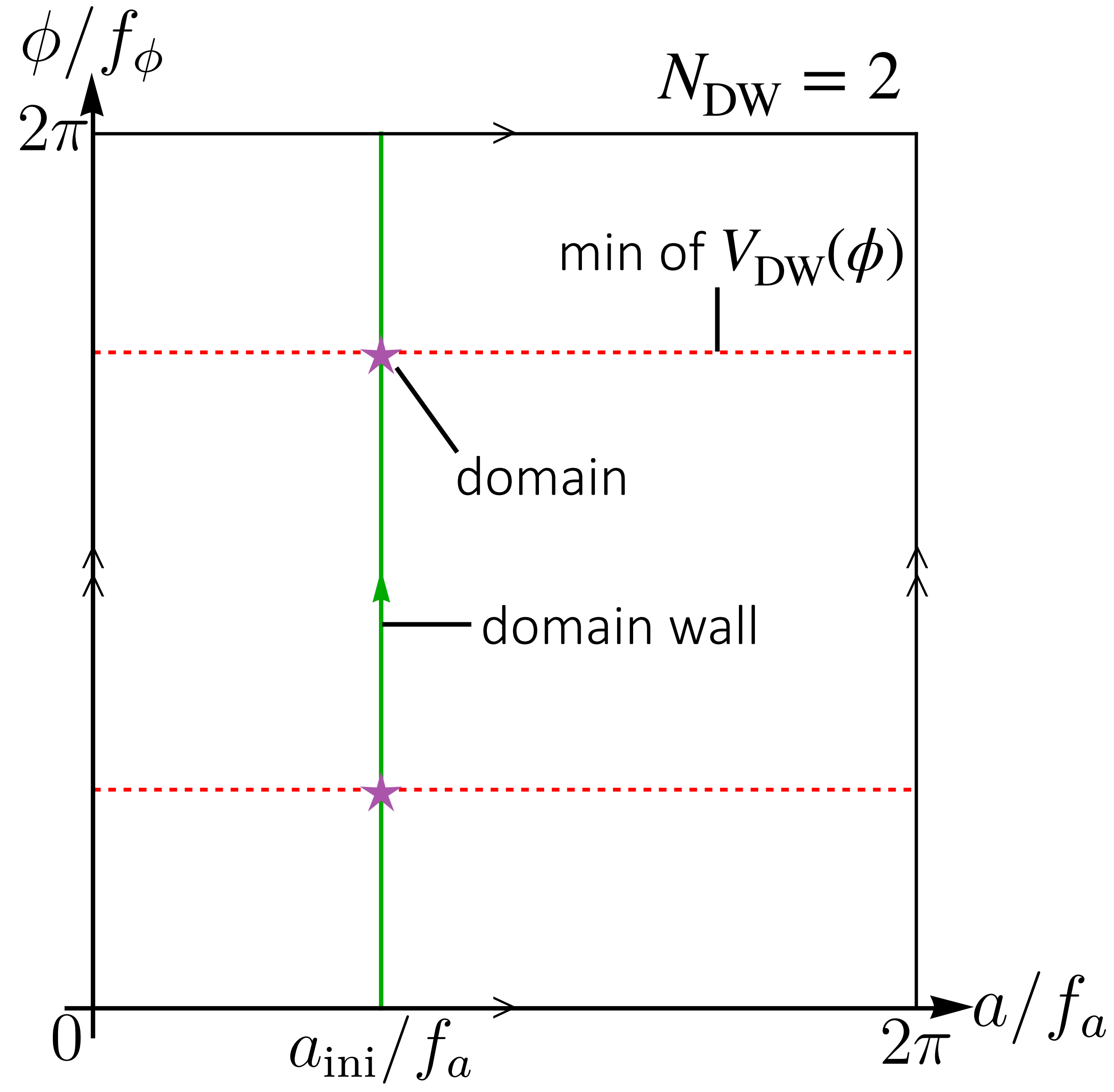
When $m_\phi \gtrsim 3H$, DWs of ϕ are formed.

In each domain, $\phi_k = 2\pi k\dfrac{f_\phi}{N_{\rm DW}}$

Potential height: $\Lambda^4 = \dfrac{m_\phi^2 f_\phi^2}{N_{\rm DW}^2}$

DW width: $\sim m_\phi^{-1}$

Tension: $\sigma_\phi = \dfrac{8 m_\phi f_\phi^2}{N_{\rm DW}^2}$



Induced domain walls

- Domain walls of ϕ

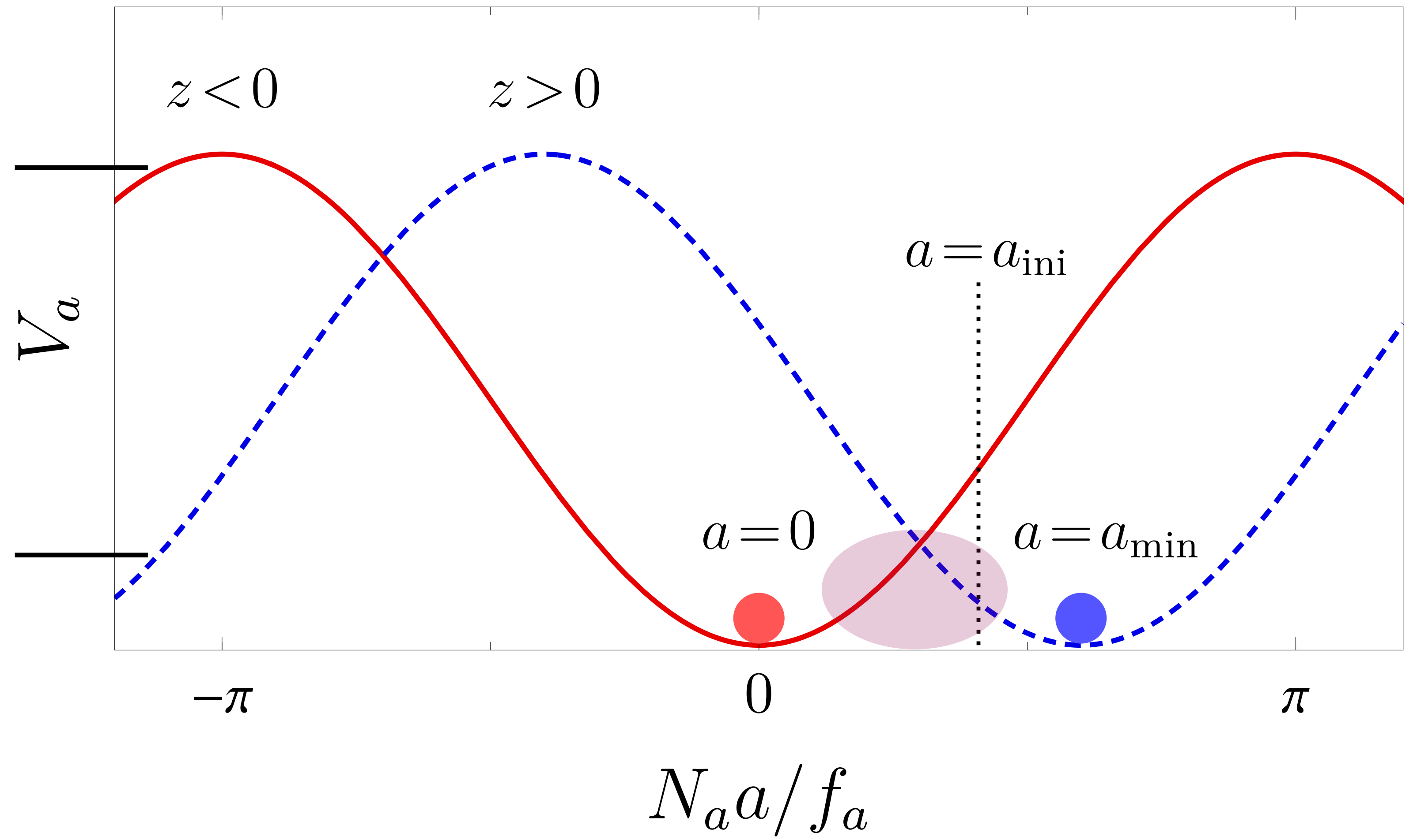
$$\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right], \quad \phi_k = 2\pi k\frac{f_\phi}{N_{\rm DW}}$$

When $m_a \gtrsim 3H$, a starts to oscillate in the potential.

Due to the difference in ϕ, a has different potential minima.

$$\frac{m_a^2 f_a^2}{N_a^2}\left[1-\cos\left(N_a\frac{a}{f_a}\right)\right]$$

$$\frac{m_a^2 f_a^2}{N_a^2}\left[1-\cos\left(N_a\frac{a}{f_a}+\frac{2\pi N_\phi}{N_{\rm DW}}\right)\right]$$



a has a potential barrier between the domains. → "Induced domain walls"

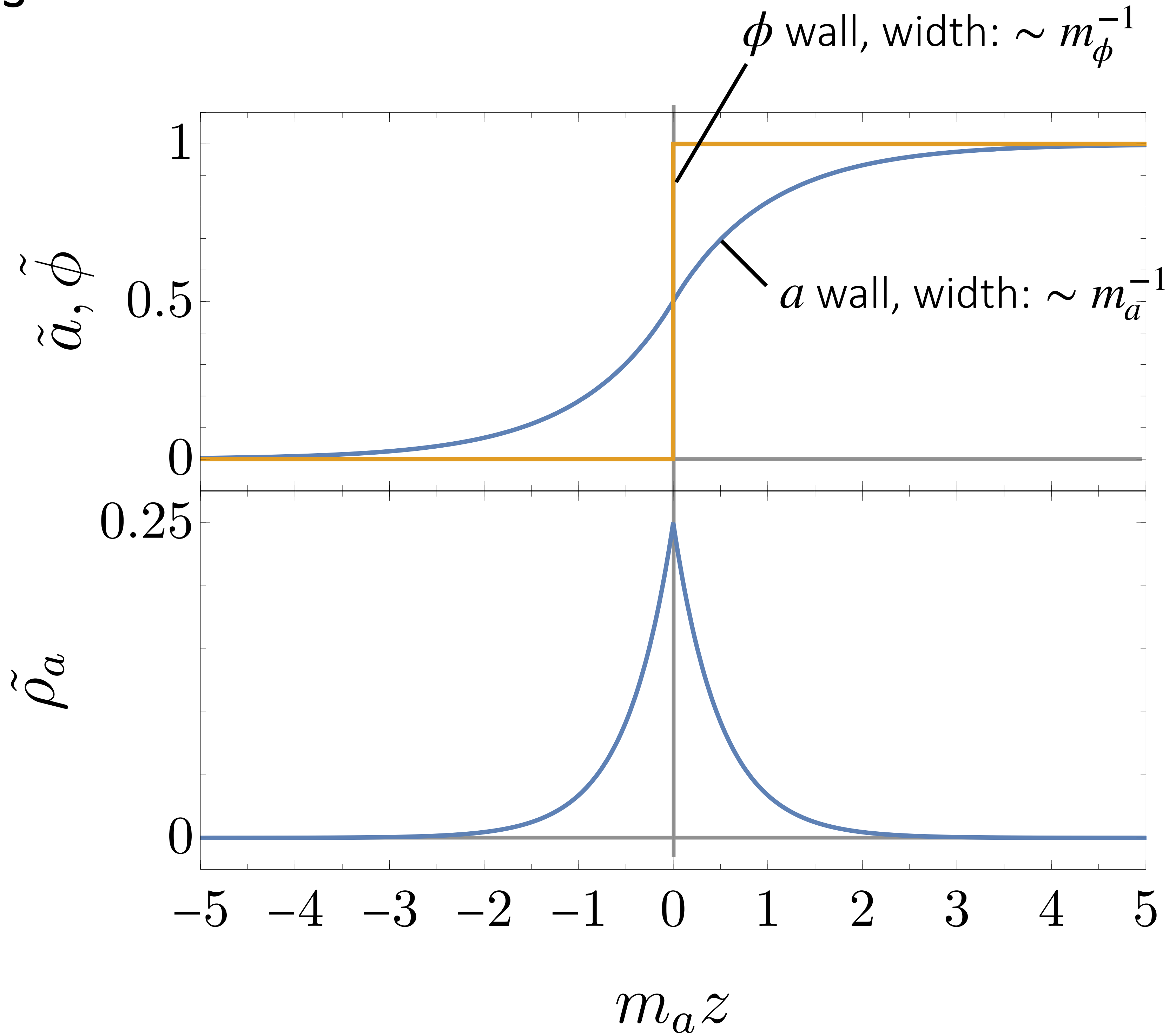
Induced domain walls

- Induced domain walls

The DW of a is induced around that of ϕ.

The width is much larger for the indued DW.

The energy is localized around the induced DW.



Induced domain walls

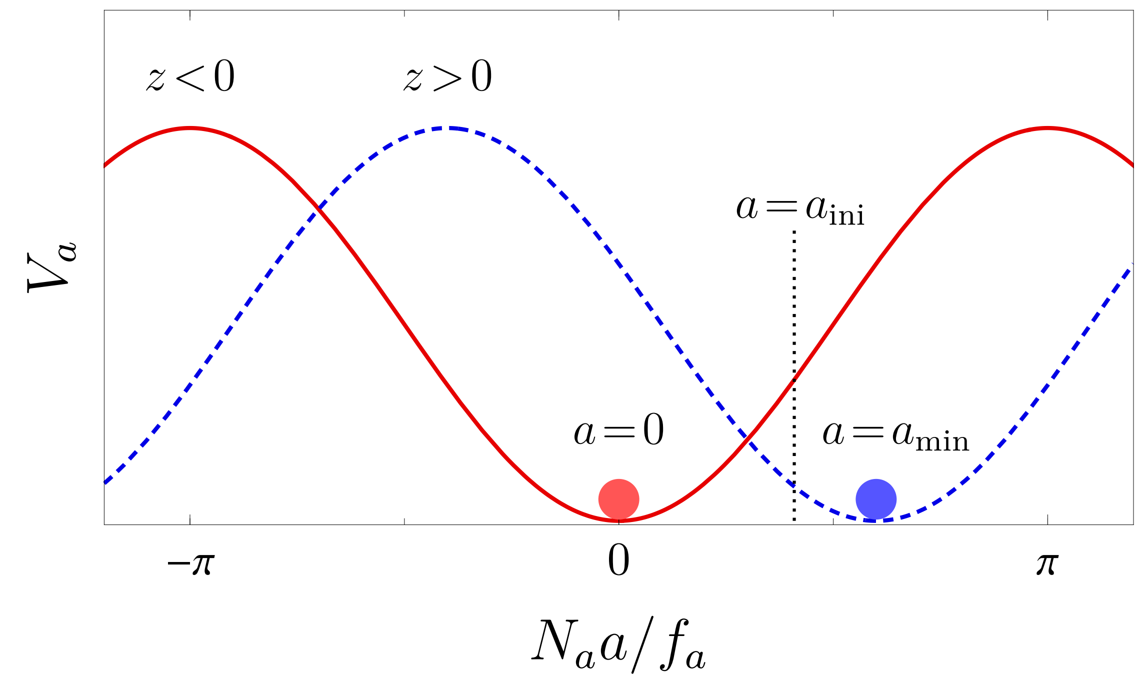
$$\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]+\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]$$

- Induced domain walls

Difference in a between domains:

$$a_{\rm min}=2\pi f_a\left(-\frac{N_\phi}{N_a N_{\rm DW}}+\frac{l}{N_a}\right)$$

integer



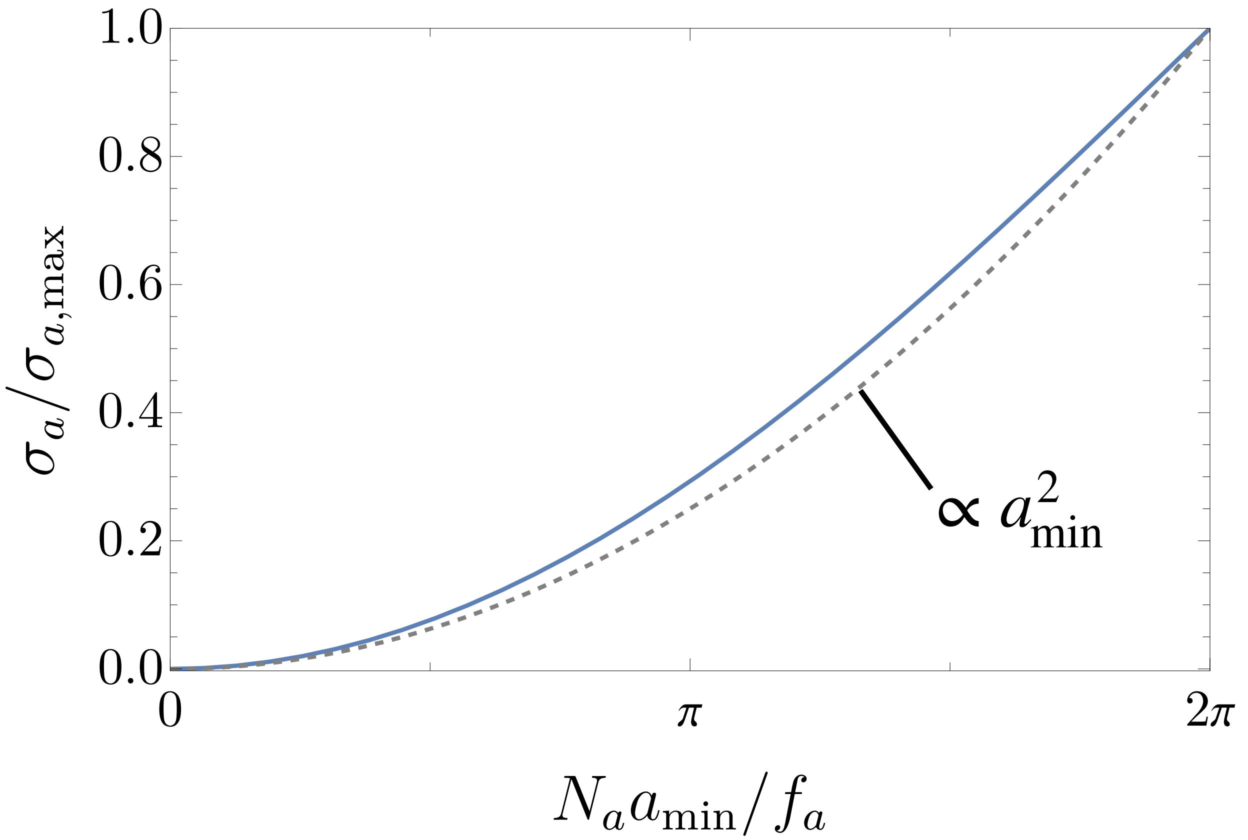
The DW tension is approximately

$$\sigma_a \propto m_a a_{\rm min}^2 \simeq 2.34\, m_a f_a^2$$

$(N_{\rm DW}=2,\ N_a=N_\phi=1)$

Smaller than in the conventional case:

$$\sigma_{a,\rm max}=\frac{8m_a f_a^2}{N_a^2}$$

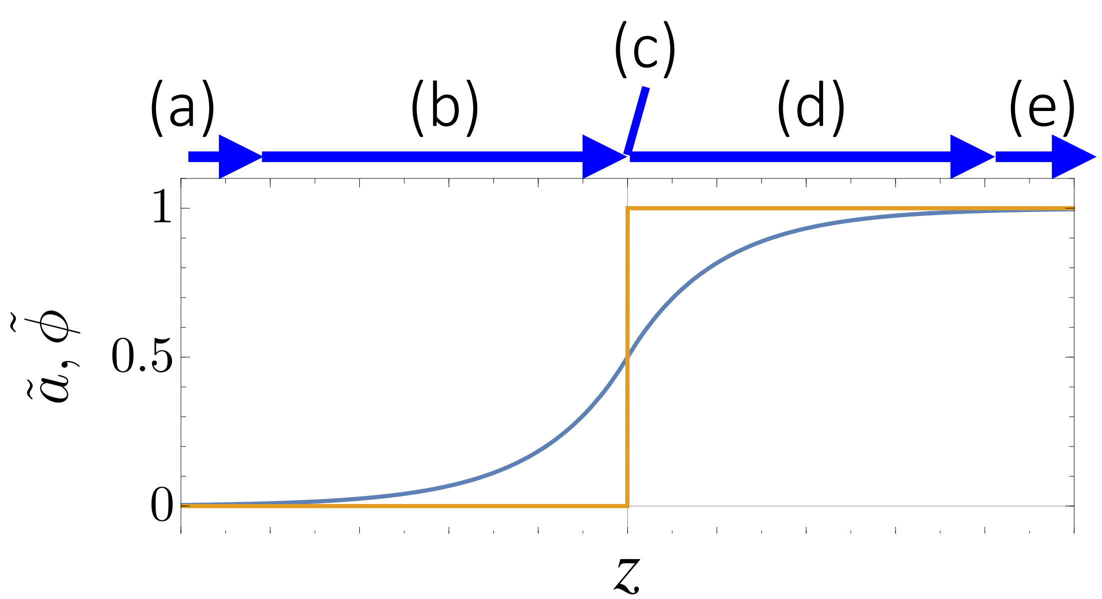


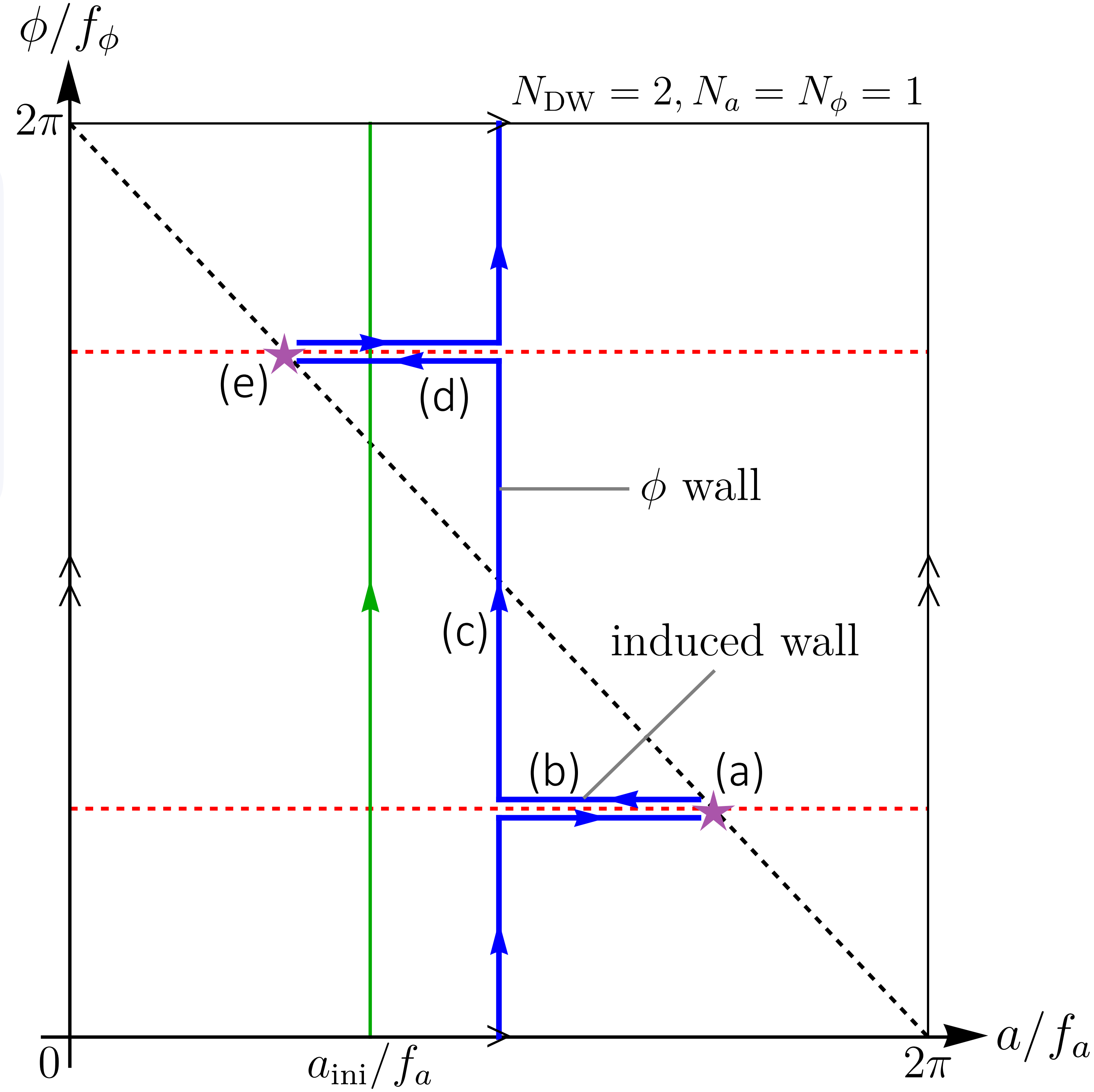
Induced domain walls

- Induced domain walls

- Winding only for ϕ
- Potential min. in each domain
- ϕ wall + induced wall

$$\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]+\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]$$





Domain wall collapse

- Scaling solution

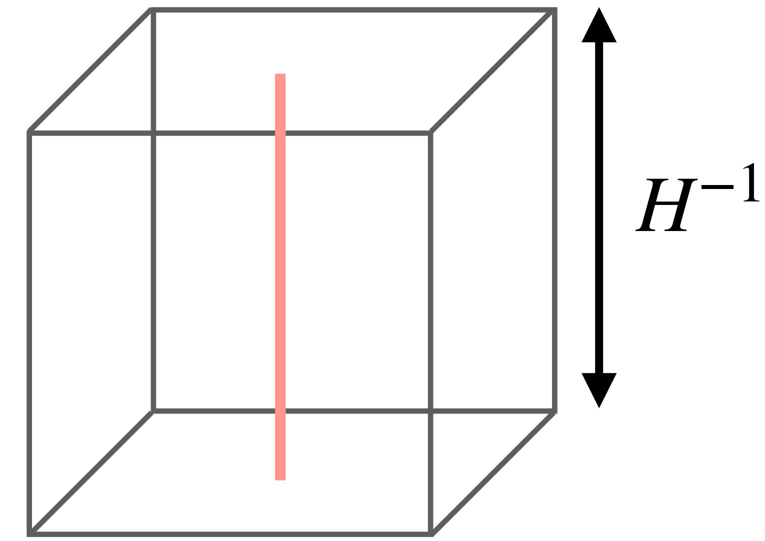
For $N_{\mathrm{DW}} > 1$, string-wall network is long-lived.

→ Scaling solution:

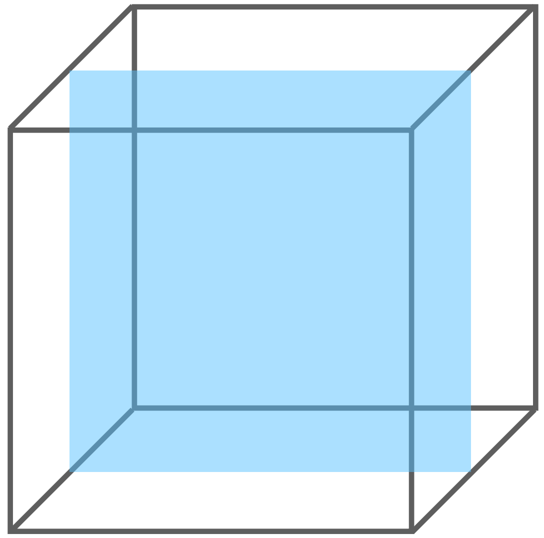
$\mathcal{O}(1)$ strings/walls in each Hubble patch

$$\rho_{\mathrm{str}} \sim \mu H^2 \propto \rho_{\mathrm{tot}}$$

$$\rho_{\mathrm{wall}} \sim \sigma H$$



→ DWs dominates the universe.
Bias potential to avoid the overclosure



Domain wall collapse

$$\chi(T)\left[1-\cos\left(N_a\frac{a}{f_a}+N_\phi\frac{\phi}{f_\phi}\right)\right]+\Lambda^4\left[1-\cos\left(N_{\rm DW}\frac{\phi}{f_\phi}\right)\right]$$

- Bias term

We introduce a bias potential:

$$V_{\rm bias}(\phi)\equiv\epsilon\Lambda^4\left[1-\cos\left(N_{\rm b}\frac{\phi}{f_\phi}+\theta\right)\right]$$

We assume $\epsilon \ll 1$. → DW formation is not affected.

The potential difference:

$$\Delta V_k = |V_{\rm bias}(\phi_{k-1})-V_{\rm bias}(\phi_k)|$$
$$= 2\epsilon\Lambda^4|\cos\theta|\equiv c_{\rm b}\epsilon\Lambda^4 \quad (N_{\rm DW}=2,\ N_a=N_\phi=N_{\rm b}=1)$$

Note: This bias potential does not spoil the PQ solution.

Domain wall collapse

- DW collapse

When the bias overcomes the tension, the network collapses:

$$\rho_{\phi,\mathrm{DW}}(t_{\mathrm{dec}}) = \frac{\mathcal{A}\sigma_\phi}{t_{\mathrm{dec}}} \simeq \langle \Delta V \rangle \qquad (\mathcal{A}\text{: area parameter})$$

From this condition, the decay temperature becomes

$$H_{\mathrm{dec}} = \frac{c_{\mathrm{b}}\epsilon m_\phi}{16}, \qquad T_{\mathrm{dec}} = \left(\frac{90}{\pi^2 g_*(T_{\mathrm{dec}})}\right)^{1/4} \sqrt{\frac{c_{\mathrm{b}}\epsilon M_{\mathrm{Pl}} m_\phi}{16}}$$

We require that the network collapse before BBN:

$$T_{\mathrm{dec}} \gtrsim 10\,\mathrm{MeV}$$

Domain wall collapse

- Production of QCD axions Kawasaki, Saikawa, Sekiguchi (2014)

In the scaling regime, DWs decrease their energy emitting axions:

$$\left.\frac{d\rho_{a,\mathrm{wall}}}{dt}\right|_{\mathrm{emit}} = -\frac{\sigma_a}{2t^2}$$

The number density of the emitted axion:

$$n_a(t) = \frac{\sigma_a}{\tilde{\epsilon}_a m_a t}$$ (averaged axion energy: $\bar{\omega}_a = \tilde{\epsilon}_a m_a$)

$$\frac{\rho_{a,\mathrm{dec}}}{s} \simeq \frac{0.43\,\mathrm{eV}}{\tilde{\epsilon}_a} \times \left(\frac{g_{*,\mathrm{dec}}}{10.75}\right)^{1/2} \left(\frac{g_{*s,\mathrm{dec}}}{10.75}\right)^{-1} \left(\frac{f_a}{4\times 10^9\,\mathrm{GeV}}\right) \left(\frac{T_{\mathrm{dec}}}{12\,\mathrm{MeV}}\right)^{-1}$$

Note: We assume ϕ decays to the Standard Model particles.

Domain wall collapse

- Coherent contribution

Coherent oscillations of a also contribute to DM.

The evaluation is the same as the misalignment mechanism except for the initial field value:

$$\frac{\rho_a(T)}{s(T)} \simeq \frac{m_{a0} m_{a,\mathrm{osc}}}{s(T_{\mathrm{osc}})} \underline{\frac{\sum (a_{\mathrm{ini}} - a_{\mathrm{min},k})^2}{N_{\mathrm{DW}}}}$$

Average over domains

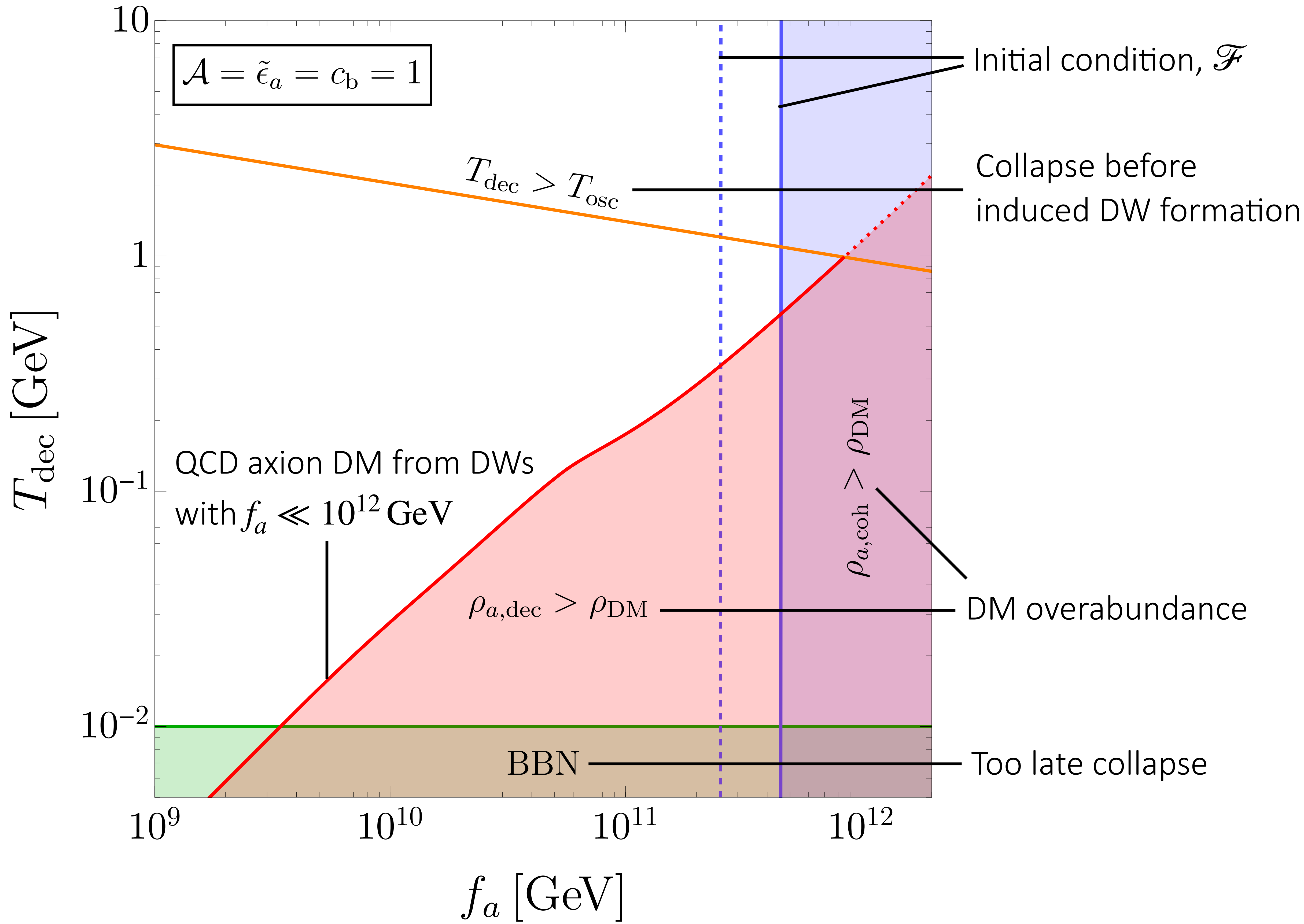
For $N_{\mathrm{DW}} = 2,\ N_a = N_\phi = 1,$

$$\frac{\rho_a(T)}{s(T)} \simeq \frac{m_{a0} m_{a,\mathrm{osc}}}{s(T_{\mathrm{osc}})} \underline{\frac{a_{\mathrm{ini}}^2 + (\pi f_a - |a_{\mathrm{ini}}|)^2}{2}}$$

$$\equiv$$

$$\frac{\pi^2 f_a^2}{4} \leq \mathcal{F}(a_{\mathrm{ini}}) \leq \frac{\pi^2 f_a^2}{2}$$

Domain wall collapse



Domain wall collapse

- **Gravitational wave production** Hiramatsu, Kawasaki, Saikawa (2013)

DWs also emit gravitational waves.

The main contribution is produced at the DW collapse:

$$f_{\rm peak,dec} \sim H_{\rm dec}$$

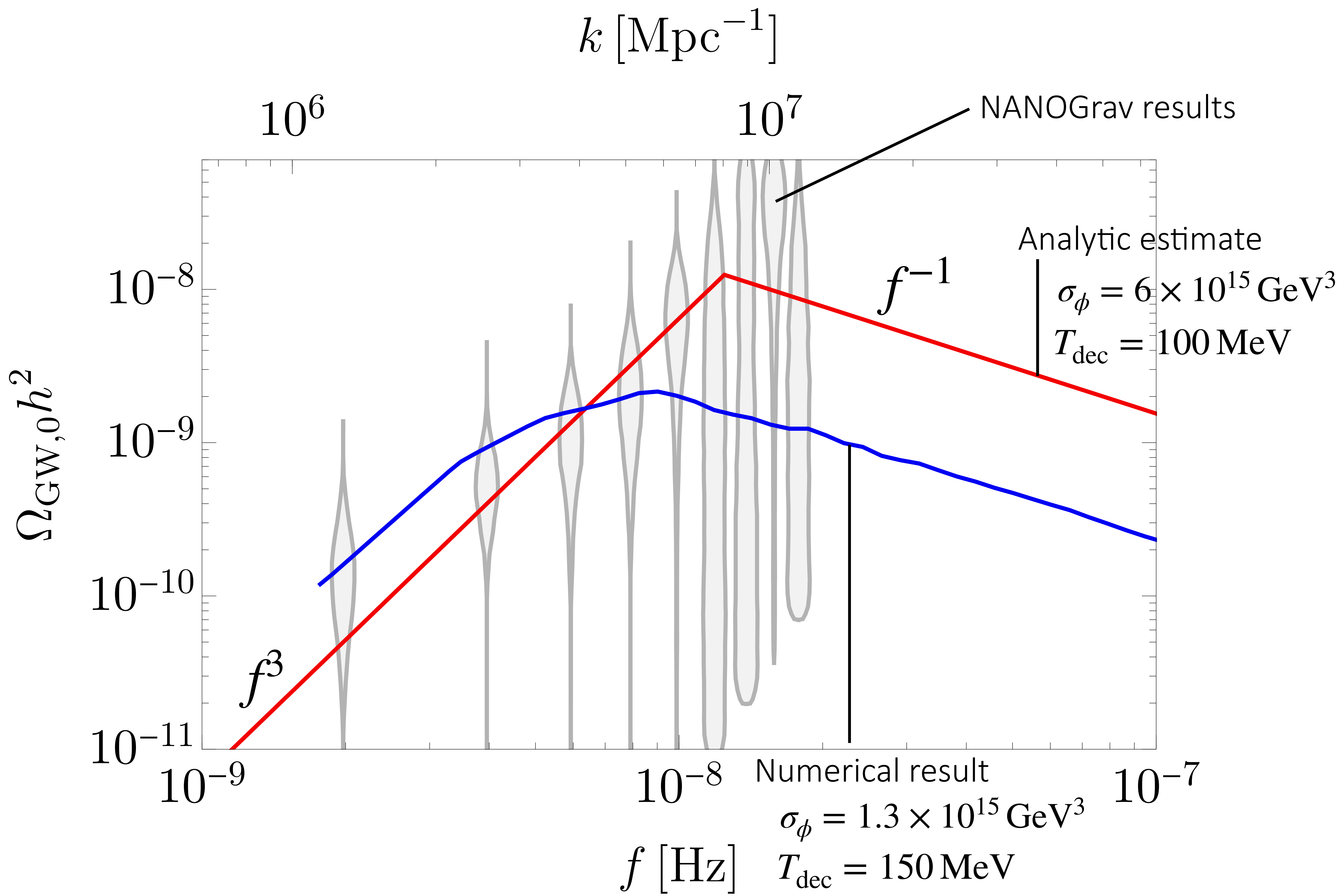
$$\Omega^{\rm peak}_{\rm GW,dec} = \frac{\epsilon_{\rm GW}\sigma_\phi^2}{24\pi M_{\rm Pl}^4 H_{\rm dec}^2} \quad (\epsilon_{\rm GW}\text{: GW emission efficiency})$$

The current peak of the GW spectrum is given by

$$f_{\rm peak,0} \sim 11\,{\rm nHz} \times \left(\frac{g_{*s,\rm dec}}{10.75}\right)^{-1/3} \left(\frac{g_{*,\rm dec}}{10.75}\right)^{1/2} \left(\frac{T_{\rm dec}}{100\,{\rm MeV}}\right)$$

$$\Omega^{\rm peak}_{\rm GW,0} h^2 \sim 2.6 \times 10^{-9} \times \epsilon_{\rm GW} \left(\frac{g_{*s,\rm dec}}{10.75}\right)^{-4/3} \left(\frac{T_{\rm dec}}{100\,{\rm MeV}}\right)^{-4} \left(\frac{\sigma_\phi}{2\times 10^{15}\,{\rm GeV}^3}\right)^{-2}$$

Domain wall collapse



Kitajima, Lee, KM, Takahashi, Yin (2023)

Summary

- Heavy axion DWs can induce QCD axion DWs via the mixing even with a pre-inflationary initial condition for the QCD axion.
- By introducing a bias potential to ϕ, the network collapses while solving the strong CP problem.
- QCD axions from induced DWs can account for all dark matter with f_a as small as $\mathcal{O}(10^9)\,\mathrm{GeV}$.
- GWs from induced DWs can account for the NANOGrav result.