Small Instanton Effects on Composite Axion Mass

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Strong CP Problem and **Axion**

Introduction

$\theta F\tilde{F} = \theta\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$ in QCD:

- θ violates CP symmetry.
- Too tiny neutron electric dipole moment $\rightarrow$ $|\boldsymbol{\theta}| \lesssim \mathbf{10^{-10}}$

Unnaturally tiny $|\theta|$ might be a hint for physics beyond the standard model:

Strong CP Problem

Peccei-Quinn (PQ) Mechanism & Axion [Peccei & Quinn (1977), Weinberg (1978), Wilczek (1978)]

1. **Anomalous** global $\mathrm{U}(1)$ symmetry in QCD, $\mathbf{U(1)_{PQ}}$ $\leftarrow$ "**PQ symmetry**,"

2. **Unbroken** (except for anomaly) $\mathbf{U(1)_{PQ}}$ **nullifies** $\boldsymbol{\theta}$.

3. However, **unbroken** (except for anomaly) $\mathbf{U(1)_{PQ}}$ $\rightarrow$ **massless colored fermion**.

4. Thus, $\mathrm{U}(1)_{\mathrm{PQ}}$: **spontaneously broken** $\rightarrow$ **axion** as a Nambu-Goldstone boson

5. Axion potential from QCD anomaly $\rightarrow$ $\boldsymbol{\theta = 0}$, dynamically.

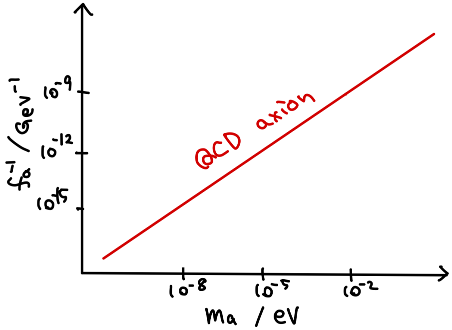
Introduction

Axion Mass

From chiral perturbation theory, $m_a f_a$ seems almost model-independent:

$$(m_a f_a)^2 \sim \frac{m_u m_d}{(m_u + m_d)^2} (m_\pi f_\pi)^2 \, .$$

m_a: axion mass, f_a: axion decay constant.



Constraint on f_a

- $f_a \gtrsim 10^9\mathbf{GeV}$: astrophysics
- $f_a \lesssim 10^{12}\mathbf{GeV}$: preferred, not to exceed dark matter abundance.

→ Far above electroweak scale,
far below Planck scale (and grand-unification scale).

An Example: The Simplest Composite Axion Model [Choi & Kim (1985)] :

New fermions

	$SU(N)$	$SU(3)_{QCD}$
ψ_1	**N**	$\bar{\mathbf{3}}$
ψ_1'	**N**	**1**
ψ_2	$\bar{\mathbf{N}}$	**3**
ψ_2'	$\bar{\mathbf{N}}$	**1**

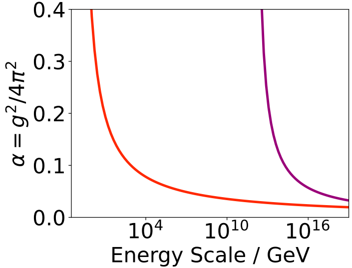
Maximal flavor symmetry for (3+1)-pairs of $(N, \bar{N})$:

(in vanishing coupling limit of $SU(3)_{QCD}$)

$$\mathbf{U(4)}_N \times \mathbf{U(4)}_{\bar{N}} = SU(4)_V \times U(1)_V \times SU(4)_A \times U(1)_A$$

$$SU(4)_V \supset \mathbf{SU(3)_{QCD}}, \qquad U(1)_A \supset \mathbf{U(1)_{PQ}}$$

No need for **fine-tuning of scalar potential**, in composite axion models.

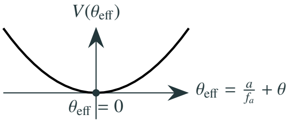


Two Topics from Energy Scale above PQ breaking

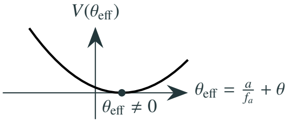
1. **“Axion Quality Problem”**
2. Possibility of **Axion Mass Enhancement**?

“Axion Quality Problem”

Suppose that **the global symmetry, $\mathrm{U}(1)_{\mathrm{PQ}}$, is slightly broken by some UV effects**.



Axion potential from anomaly.



Additional PQ-breaking→ $\theta \neq 0$.

- It is belived that the quantum gravity yields such breaking.

With dimension-d, $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking operator ($\propto$ $[\text{dim}-d\ \text{operator}]/\Lambda_{\mathrm{UV}}^{d-4}$) ,

$$\delta\theta \sim \frac{f_a^d/\Lambda_{\mathrm{UV}}^{d-4}}{\Lambda_{\mathrm{QCD}}^4} \sim \frac{(10^9\mathrm{GeV})^d/(10^{19}\mathrm{GeV})^{d-4}}{(10^{-1}\mathrm{GeV})^4} \sim 10^{80-10d} . \quad (f_a \sim 10^9\mathrm{GeV})$$

→ **Roughly $d > 9$** for $|\delta\theta| \lesssim 10^{-10}$, assuming $\mathcal{O}(1)$ for dimensionless parameters.

Accidental Global Symmetry

An Example: "Baryon number" for N_f (vector-like) quarks.

1. Write all the Lorentz- and gauge-invariant terms with mass dimension ≤ 4.

$$\mathcal{L}_{\text{fermion}} = \sum_{f=1}^{N_f} \overline{\psi}_f i \not{D} \psi_f + \sum_{f,f'=1}^{N_f} \overline{\psi}_{f'} M_{f',f} \psi_f$$

2. This model has $\mathrm{U}(1)$ **symmetry**: $\psi \to e^{i\alpha}\psi \quad (\overline{\psi} \to e^{-i\alpha}\overline{\psi})$, **not imposed by hand**.

Not imposed by hand (realized by Lorentz- and gauge symmetry) = **"Accidental"**

One Way to Avoid Quality Problem

Forbid breaking of accidental $\mathrm{U}(1)_{\mathrm{PQ}}$ until **sufficiently high mass dimension**, by (Lorentz and) **gauge symmetries**.

Instanton

- Local minima of the action with $\int \mathrm{d}x^4 F\tilde{F} \neq 0$.
- Suppression factor: $\exp(-S) = \exp\left(-\frac{8\pi^2}{g^2}\right)$.

Instanton of Broken Gauge Symmetry [Affleck(1980)]

$$S = \frac{8\pi^2}{g^2} + \mathcal{O}\left([\rho v]^2\right)$$

(ρ: instanton size & v: symmetry-breaking scale.)

$\rightarrow$**"Small" instantons** with $\rho \lesssim v^{-1}$ can be relevant.

Product group model: Gauge $[\mathrm{SU}(3)]^{n_s}$ symmetry with n_s axions (by hand).

$$\mathcal{L} = \sum_{i=1}^{n_s} \left[-\frac{1}{4} F_i F_i + \left(\theta_i + \frac{a_i}{f_i} \right) \frac{g_i^2}{32\pi^2} F_i \tilde{F}_i \right] + \text{(scalars for symmetry breaking)}$$

Bi-fundamental scalars break gauge symmetry: $[\mathrm{SU}(3)]^{n_s} \to \mathrm{SU}(3)_{\mathrm{QCD}}$

Matching of Couplings: When $[\mathrm{SU}(3)]^{n_s}$ is broken to its diagonal $\mathrm{SU}(3)_{\mathrm{QCD}}$ subgroup,

$$\frac{1}{g_{\mathrm{QCD}}^2(v)} = \frac{1}{g_1^2(v)} + \ldots + \frac{1}{g_{n_s}^2(v)}$$

v: symmetry breaking scale, g_{QCD}: QCD coupling, g_i: coupling of each $\mathrm{SU}(3)_i$.

Axion mass enhancement by large couplings of $\mathrm{SU}(3)_i$. [Agrawal & Howe (2017), Csáki et al (2019)]

Introduction

1. We focus on axion models avoiding **axion quality problem**.
2. Our target is a model in which **axion mass enhancement** seems possible.
3. Question: **"axion mass enhancement" in the model?**

Model Addressing the **Quality Problem**,

in Which **Axion Mass Enhancement** “Seems” Possible

The Simple Model

New fermions

	$\mathrm{SU}(N)$	$\mathrm{SU}(3)_{\mathrm{QCD}}$
ψ_1	**N**	$\bar{\mathbf{3}}$
ψ_1'	**N**	**1**
ψ_2	$\bar{\mathbf{N}}$	**3**
ψ_2'	$\bar{\mathbf{N}}$	**1**

A Model Addressing the Quality Problem

[M. Redi & R. Sato (2016)]

New fermions

	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_{\mathrm{W}}$
ψ_1	**N**	$\bar{\mathbf{3}}$		
ψ_1'	**N**	**1**		
ψ_2		**3**	$\bar{\mathbf{N}}$	
ψ_2'		**1**	$\bar{\mathbf{N}}$	
ψ_3			**N**	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			**4**

Accidental $[\mathrm{U}(1)]^4$ Global Symmetry

$$\mathrm{U}(4)_{N,1} \times \mathrm{U}(4)_{\bar{N},1} \times \mathrm{U}(4)_{N,2} \times \mathrm{U}(4)_{\bar{N},2} \supset \mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(4)_{\mathrm{W}} \times \mathbf{U(1)_{PQ} \times U(1)_1} \times \mathrm{U}(1)_2 \times \mathrm{U}(1)_3$$

(anomalous)

	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_{\mathrm{W}}$	$\mathrm{U}(1)_{\mathrm{PQ}}^{(\mathrm{SSB})}$	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_3$
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$			1	1	1	1
ψ_1'	$\mathbf{N}$	$\mathbf{1}$			−3	1	−3	1
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$		1	1	−1	−1
ψ_2'		$\mathbf{1}$	$\bar{\mathbf{N}}$		−3	1	3	−1
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$	0	−1	0	1
ψ_4	$\bar{\mathbf{N}}$			$\mathbf{4}$	0	−1	0	−1

Two θ-angles of $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ $\leftarrow$ "nullified" by $\mathbf{U(1)_{PQ}}$ and $\mathbf{U(1)_1}$.

Axion Mass Enhancement?

Axion Mass Enhancement?

Small instantons: Axion mass enhancement as in $[\mathrm{SU}(3)]^{n_s}$ product group model?

Axion potential from $\mathrm{SU}(3)_\mathrm{W}$ and $\mathrm{SU}(4)_\mathrm{W}$ small instantons are

$$\propto \Lambda^4 \exp\left(-\frac{8\pi^2}{g^2_{\mathrm{SU}(3)_\mathrm{W}}(\Lambda)}\right) \quad \text{and} \quad \propto \Lambda^4 \exp\left(-\frac{8\pi^2}{g^2_{\mathrm{SU}(4)_\mathrm{W}}(\Lambda)}\right).$$

$\mathrm{SU}(3)_\mathrm{W}$, $\mathrm{SU}(4)_\mathrm{W}$ small instantons: **coupling much stronger than QCD** is possible.
(Note that $\frac{1}{g^2_{\mathrm{QCD}}(\Lambda)} = \frac{1}{g^2_{\mathrm{SU}(3)_\mathrm{W}}(\Lambda)} + \frac{1}{g^2_{\mathrm{SU}(4)_\mathrm{W}}(\Lambda)}$ permits $g_{\mathrm{SU}(3)_\mathrm{W}}(\Lambda) \gg g_{\mathrm{QCD}}(\Lambda)$)

However...

We will see that **indivisual** $\mathrm{SU}(3)_\mathrm{W}$ **or** $\mathrm{SU}(4)_\mathrm{W}$ **small instantons do not contribute** to the axion potential ! $\rightarrow$ **No axion mass enhancement**

Axion Mass Enhancement?

Fermion zero modes around instantons nullify instanton effects.

Vanishing physical quantity

$$\mathcal{L} = \xi^{+} i\sigma^{\mu} D_{\mu}\xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu}\eta$$

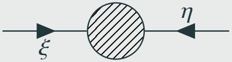
Situation: $\sigma^{\mu} D_{\mu}$ has normalizable zero mode wavefunction $\psi^{(0)}$.

Decomposing ξ and η as

$$\xi = \xi_0 \psi^{(0)} + \text{(non-zero modes)}, \quad \eta = \eta_0 \psi^{(0)} + \text{(non-zero modes)},$$

Path integral is

$$\int \mathrm{d}\xi^{\dagger}\mathrm{d}\xi\mathrm{d}\eta^{\dagger}\mathrm{d}\eta \ \exp[-S]\mathcal{O} \propto \int d\xi_0 d\eta_0 \ \exp[-0]\mathcal{O} = \int d\xi_0 d\eta_0 \ \mathcal{O}.$$



This is vanishing, if $\mathcal{O}$ does not include ξ and η.

Axion Mass Enhancement?

Interactions between **fermion zero modes** yield non-zero instanton effects.

An example of non-vanishing physical quantity

$$L = \xi^{+} i\sigma^{\mu} D_{\mu}\xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu}\eta - m(\eta\xi + \xi^{\dagger}\eta^{\dagger})$$

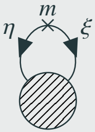
Situation: $\sigma^{\mu} D_{\mu}$ has normalizable zero mode wavefunction $\psi^{(0)}$.

Decomposing ξ and η as

$$\xi = \xi_0 \psi^{(0)} + \text{(non-zero modes)}, \quad \eta = \eta_0 \psi^{(0)} + \text{(non-zero modes)},$$

Path integral is

$$\int \mathrm{d}\xi^{\dagger}\mathrm{d}\xi\mathrm{d}\eta^{\dagger}\mathrm{d}\eta \, \exp[-S]\mathcal{O} \propto \int d\xi_0 d\eta_0 \, \exp\left[\int \mathrm{d}^4 x \, m\left(\xi_0\psi^{(0)}\right)\left(\eta_0\psi^{(0)}\right)\right]\mathcal{O} \neq 0.$$



This is non-vanishing, due to the interactions of Weyl fermions.

Axion Mass Enhancement?

't Hooft Vertex: Effective vertex corresponding to fermion zero mode integration.

An example

$$\mathcal{L} = \xi^{+} i\sigma^{\mu} D_{\mu}\xi + \eta^{\dagger} i\sigma^{\mu} D_{\mu}\eta - m(\eta\xi + \xi^{\dagger}\eta^{\dagger})$$

Situation: $\sigma^{\mu} D_{\mu}$ has normalizable zero mode wavefunction $\psi^{(0)}$.

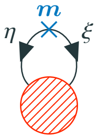
Zero mode integration

$$\int \mathrm{d}\xi^{\dagger}\mathrm{d}\xi\mathrm{d}\eta^{\dagger}\mathrm{d}\eta\ \exp[-S]$$

$$\propto \int d\xi_0 d\eta_0\ \exp\left[\int \mathrm{d}^4x\, m\left(\xi_0\psi^{(0)}\right)\left(\eta_0\psi^{(0)}\right)\right]$$

$$= m\int \mathrm{d}^4x\psi^{(0)}(x)\,\psi^{(0)}(x) = m\rho$$

't Hooft vertex



$$\propto m \times \frac{1}{\rho} \times \int^{\rho^{-1}} \frac{\mathrm{d}^4\ell}{(2\pi)^4}\frac{1}{\ell^2}$$

$$\propto m\rho$$

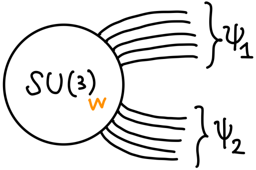
Zero mode path integration $\simeq$ **massless fermion lines in 't Hooft vertex**.

[See e.g. 't Hooft (1976), Csaki et al (2023)]

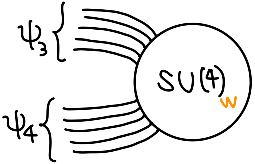
Axion Mass Enhancement?

't Hooft vertex for $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ small instantons:

Effect of fermion zero modes around instantons are captured by 't Hooft vertex



$\mathrm{SU}(3)_{\mathrm{W}}$ instanton



$\mathrm{SU}(4)_{\mathrm{W}}$ instanton

	$\mathrm{SU}(N)_{\mathrm{ST}\,2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}\,1}$	$\mathrm{SU}(4)_{\mathrm{W}}$
ψ_1	$\mathbf{N}$	$\overline{\mathbf{3}}$		
ψ_1'	$\mathbf{N}$	$\mathbf{1}$		
ψ_2		$\mathbf{3}$	$\overline{\mathbf{N}}$	
ψ_2'		$\mathbf{1}$	$\overline{\mathbf{N}}$	
ψ_3			$\mathbf{N}$	$\overline{\mathbf{4}}$
ψ_4	$\overline{\mathbf{N}}$			$\mathbf{4}$

We will find that **a single $\mathrm{SU}(3)_{\mathrm{W}}$ or $\mathrm{SU}(4)_{\mathrm{W}}$ instanton does not contribute**.

Axion Mass Enhancement?

Reason for no contribution from $\mathrm{SU}(3)_\mathrm{W}$ or $\mathrm{SU}(4)_\mathrm{W}$ Instantons:

- Every fermion leg from $\mathrm{SU}(3)_\mathrm{W}$ instantons: $\mathrm{U}(1)_1$ charge is $+1$
- Every fermion leg from $\mathrm{SU}(4)_\mathrm{W}$ instantons: $\mathrm{U}(1)_1$ charge is -1
- Every interaction (except for 't Hooft verteces): $\mathrm{U}(1)_1$ charge is 0

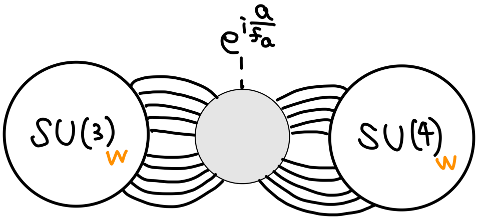
	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_\mathrm{W}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_\mathrm{W}$	$\mathrm{U}(1)_{\mathrm{PQ}}^{(\mathrm{SSB})}$	$\mathrm{U}(1)_1$	$\mathrm{U}(1)_2$	$\mathrm{U}(1)_3$
ψ_1	**N**	$\bar{\mathbf{3}}$			1	**+1**	1	1
ψ_1'	**N**	**1**			-3	**+1**	-3	1
ψ_2		**3**	$\bar{\mathbf{N}}$		1	**+1**	-1	-1
ψ_2'		**1**	$\bar{\mathbf{N}}$		-3	**+1**	3	-1
ψ_3			**N**	$\bar{\mathbf{4}}$	0	**-1**	0	1
ψ_4	$\bar{\mathbf{N}}$			**4**	0	**-1**	0	-1

$\rightarrow$ It is impossible to **close all fermion legs** from a single $\mathrm{SU}(3)_\mathrm{W}$ or $\mathrm{SU}(4)_\mathrm{W}$ instanton.

(On the other hand, a pair of $\mathrm{SU}(3)_\mathrm{W}$ and $\mathrm{SU}(4)_\mathrm{W}$ does contribute.)

Fermion legs cannot be closed around a single $\mathrm{SU}(3)_{\mathrm{W}}$ or $\mathrm{SU}(4)_{\mathrm{W}}$ instanton,

while a pair of $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ instantons can contribute.



No Axion Mass Enhancement!

Small instanton effects are always from "pairs"

A pair of $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ instanton:

$$\exp\left(-\frac{8\pi^2}{g^2_{\mathrm{SU}(3)_{\mathrm{W}}}(\Lambda)}\right)\times\exp\left(-\frac{8\pi^2}{g^2_{\mathrm{SU}(4)_{\mathrm{W}}}(\Lambda)}\right)=\exp\left(-8\pi^2\left[\frac{1}{g^2_{\mathrm{SU}(3)_{\mathrm{W}}}(\Lambda)}+\frac{1}{g^2_{\mathrm{SU}(4)_{\mathrm{W}}}(\Lambda)}\right]\right)$$

$$=\exp\left(-\frac{8\pi^2}{g^2_{\mathrm{QCD}}(\Lambda)}\right)$$

- The last "=" is from the matching of couplings: $\frac{1}{g^2_{\mathrm{QCD}}(\Lambda)}=\frac{1}{g^2_{\mathrm{SU}(3)_{\mathrm{W}}}(\Lambda)}+\frac{1}{g^2_{\mathrm{SU}(4)_{\mathrm{W}}}(\Lambda)}$

→ **No axion mass enhancement by large coupling of $\mathbf{SU(3)_{W}}$.**

(Even when $g_{\mathrm{SU}(3)_{\mathrm{W}}}\gg g_{\mathrm{QCD}}$, effects are accompanied by $g_{\mathrm{SU}(4)_{\mathrm{W}}}\sim g_{\mathrm{QCD}}$ suppression.)

Summary

- PQ mechanism and the axion solve the strong CP problem.

- **Axion quality problem** is a severe restriction for axion models.

- Possibility of **axion mass enhancement** is argued.

★ We discussed the axion mass in an axion model addressing the quality problem with an accidental PQ symmetry [Redi & Sato (2016)].
This model possesses product gauge group broken into QCD.
$\rightarrow$ axion mass enhancement?

★ **No enhancement**, due to $\mathrm{U}(1)$ symmetry **not spontaneously broken** and **nullify θ parameter in hidden sector**.

BACKUP

Model – Symmetries

	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_{\mathrm{W}}$
ψ_1	**N**	$\bar{\mathbf{3}}$		
ψ_1'	**N**	**1**		
ψ_2		**3**	$\bar{\mathbf{N}}$	
ψ_2'		**1**	$\bar{\mathbf{N}}$	
ψ_3			**N**	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			**4**

Maximal Flavor Symmetry (in vanishing coupling limit of $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$)

$$\mathbf{U(4)}_{N,1} \times \mathbf{U(4)}_{\bar{N},1} \times \mathbf{U(4)}_{N,2} \times \mathbf{U(4)}_{\bar{N},2} \supset \mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(4)_{\mathrm{W}}$$

Model – Symmetries

	$SU(N)_{ST2}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_W$
ψ_1	**N**	$\bar{\mathbf{3}}$		
ψ'_1	**N**	**1**		
ψ_2		**3**	$\bar{\mathbf{N}}$	
ψ'_2		**1**	$\bar{\mathbf{N}}$	
ψ_3			**N**	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			**4**

Symmetries remaining with non-vanishing $SU(3)_W$ and $SU(4)_W$ couplings:

$$U(4)_{N,1} \times U(4)_{\bar{N},1} \times U(4)_{N,2} \times U(4)_{\bar{N},2} \supset SU(3)_W \times SU(4)_W \times [\mathbf{U(1)}]^6$$

1. Two $U(1)$'s are anomalous w.r.t. $SU(N)_{ST1} \times SU(N)_{ST2}$, aligning their θ angles.
2. The other $[\mathbf{U(1)}]^4$ will be discussed next.

Fermion Condensations

$\mathrm{SU}(3)_{\mathrm{W}}$, $\mathrm{SU}(4)_{\mathrm{W}}$ couplings $\to 0$.

Then, $[\mathrm{SU}(N)_{\mathrm{ST}}]^2$: independent vector-like theories.

$$\langle \psi_1^{(\prime)} \psi_4 \rangle \sim \Lambda^3 \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\langle \psi_2^{(\prime)} \psi_3 \rangle \sim \Lambda^3 \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

	$\mathrm{SU}(N)_{\mathrm{ST}2}$	$\mathrm{SU}(3)_{\mathrm{W}}$	$\mathrm{SU}(N)_{\mathrm{ST}1}$	$\mathrm{SU}(4)_{\mathrm{W}}$
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$		
ψ_1'	$\mathbf{N}$	$\mathbf{1}$		
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$	
ψ_2'		$\mathbf{1}$	$\bar{\mathbf{N}}$	
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			$\mathbf{4}$

Gauge symmetries: $\mathrm{SU}(3)_{\mathrm{W}} \times \mathrm{SU}(4)_{\mathrm{W}} \to \mathrm{SU}(3)_{\mathrm{QCD}}$: vector-part

Global symmetries: Axial $[\mathrm{U}(4)]^2 = [\mathrm{U}(1)^2] \times [\mathrm{SU}(4)]^2 \supset \mathrm{U}(1)_{\mathrm{PQ}}$

$[\mathrm{SU}(N)_{\mathrm{ST}}]^2$-anomalous $[\mathrm{U}(1)]^2$, 15 massive gauge bosons, other $\mathbf{15} = \mathbf{8} \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \mathbf{1}$

Quality Problem and Larger Model

Lowest-Dimensional PQ-Breaking Operator

PQ-breaking, dimension-6 operator:

$$\psi_1\psi_2\psi_3\psi_4$$

Not enough to avoid **quality problem**.

	$SU(N)_{ST2}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_W$
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$		
ψ'_1	$\mathbf{N}$	$\mathbf{1}$		
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$	
ψ'_2		$\mathbf{1}$	$\bar{\mathbf{N}}$	
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$
ψ_4	$\bar{\mathbf{N}}$			$\mathbf{4}$

In larger model with $[SU(N)_{ST}]^{n_s} \times SU(3)_W \times [SU(4)_W]^{n_s-1}$ symmetry, dimension-$3n_s$ is the lowest-dimension PQ breaking.

$(n_s = 3 \;\rightarrow)$

	$SU(N)_{ST3}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_{W1}$	$SU(N)_{ST2}$	$SU(4)_{W2}$
ψ_1	$\mathbf{N}$	$\bar{\mathbf{3}}$				
ψ'_1	$\mathbf{N}$	$\mathbf{1}$				
ψ_2		$\mathbf{3}$	$\bar{\mathbf{N}}$			
ψ'_2		$\mathbf{1}$	$\bar{\mathbf{N}}$			
ψ_3			$\mathbf{N}$	$\bar{\mathbf{4}}$		
ψ_4				$\mathbf{4}$	$\bar{\mathbf{N}}$	
ψ_5					$\mathbf{N}$	$\bar{\mathbf{4}}$
ψ_6	$\bar{\mathbf{N}}$					$\mathbf{4}$

We find **no enhancement similarly in larger models with $n_s \geq 3$**

Larger Model ($n_s = 3$)

	$SU(N)_{ST3}$	$SU(3)_W$	$SU(N)_{ST1}$	$SU(4)_{W1}$	$SU(N)_{ST2}$	$SU(4)_{W2}$
ψ_1	**N**	$\bar{\mathbf{3}}$				
ψ'_1	**N**	**1**				
ψ_2		**3**	$\bar{\mathbf{N}}$			
ψ'_2		**1**	$\bar{\mathbf{N}}$			
ψ_3			**N**	$\bar{\mathbf{4}}$		
ψ_4				**4**	$\bar{\mathbf{N}}$	
ψ_5					**N**	$\bar{\mathbf{4}}$
ψ_6	$\bar{\mathbf{N}}$					**4**

	$U(1)_{PQ}^{(SSB)}$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$
ψ_1	1	1	1	1	0
ψ'_1	−3	1	−3	1	0
ψ_2	1	1	−1	−1	0
ψ'_2	−3	1	3	−1	0
ψ_3	0	−1	0	1	0
ψ_4	0	0	0	−1	−1
ψ_5	0	0	0	1	1
ψ_6	0	−1	0	−1	0

- Only $U(1)_{PQ}$ is spontaneously broken, also for larger n.
- Additional $n - 2$ anomalous (and unbroken) $U(1)$s, cancelling the additional θ angles.

Axion Mass Enhancement?

Another Explanation: Directly from symmetry, without relying on 't Hooft vertex.

Vacuum amplitude with fixed axion field value a and background $\mathrm{SU}(3)_{\mathrm{W}}$ and $\mathrm{SU}(4)_{\mathrm{W}}$ gauge field:

$$W(a)|_{m,n} = \int \prod \mathcal{D}A_{\mathrm{ST}} \mathcal{D}\psi^\dagger \mathcal{D}\psi \; e^{-S[\psi, A_{\mathrm{ST}}, a]}$$

- m, n are winding number for each sectors.
- The amplitude = contribution to the axion potential.

We can redefine (rename) the fermions in path integral by $\mathrm{U}(1)_1$ rotation $e^{i\alpha}$.

$\rightarrow W(a)|_{m,n}$ changes its phase by anomaly, without shifting the axion a.

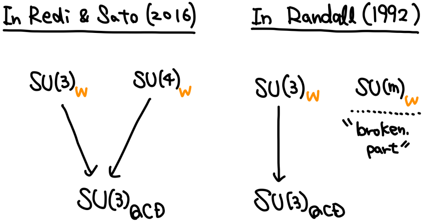
$$W(a)|_{m,n} = \exp\left[2i\alpha(m-n)\right] W(a)|_{m,n}.$$

The amplitude (effects on the axion potential) vanishes, unless $m = n$

Other Composite Axion Models

In enlarged model ($n_s > 2$): Discussion is similar. No axion mass enhancement.

Randall (1992): Gauge symmetry of the model is $\mathrm{SU}(N)_{\mathrm{ST}} \times \mathrm{SU}(m)_{\mathrm{w}} \times \mathrm{SU}(3)_{\mathrm{w}}$.



In many models including Randall (1992),

$$\mathrm{SU}(3)_{\mathrm{QCD}} \times [\text{broken part}] = \mathrm{SU}(3)_{\mathrm{QCD}} \times [\text{hidden gauge group}]$$

i.e. QCD is **just the spectator** of the PQ breaking.

→ **No hidden small instanton effects on axion mass.**