Non-invertible flavor symmetries in magnetized extra dimensions

Hajime Otsuka (九州大学)

Reference :

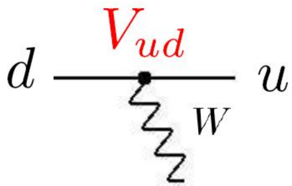
T. Kobayashi (Hokkaido U.) and H.O.,, 2408.13984 [hep-th]

素粒子物理学の進展２０２４（PPP2024）@ YITP, Aug. 21, 2024

Flavor puzzle

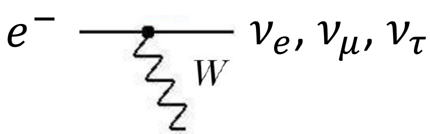
- Origin of flavor and CP : important issue in the SM

PDG ('20)



$$V_{\rm CKM} = \begin{pmatrix} 0.97401 \pm 0.00011 & 0.22650 \pm 0.00048 & 0.00361^{+0.00011}_{-0.00009} \\ 0.22636 \pm 0.00048 & 0.97320 \pm 0.00011 & 0.04053^{+0.00083}_{-0.00061} \\ 0.00854^{+0.00023}_{-0.00016} & 0.03978^{+0.00082}_{-0.00060} & 0.999172^{+0.000024}_{-0.000035} \end{pmatrix}$$

NuFIT 5.0 (2020)



$$|U|^{\rm w/o\ SK\text{-}atm}_{3\sigma} = \begin{pmatrix} 0.801 \to 0.845 & 0.513 \to 0.579 & 0.143 \to 0.156 \\ 0.233 \to 0.507 & 0.461 \to 0.694 & 0.631 \to 0.778 \\ 0.261 \to 0.526 & 0.471 \to 0.701 & 0.611 \to 0.761 \end{pmatrix}$$

- Hierarchical structure of quarks/lepton masses

2 5	100	1270	4180	172000	Mass [MeV]
u d	*s*	*c*	*b*	*t*	

→ Non-trivial flavor structure of quarks and leptons

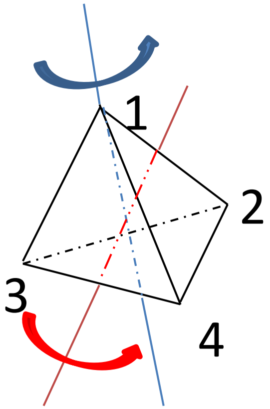
“Traditional” flavor symmetry

- 場の変換:

$$\phi_i \xrightarrow{g} \rho_{ij}(g)\phi_j \qquad g \in G_{\rm flavor}$$

- $G_{\rm flavor}$ として、
  Abelian and/or non-Abelian symmetries

Ex., $U(1)_{\rm FN}$
A_4 : Tetrahedral sym.



問い :

通常はInvertible symmetryを考えるが、non-invertible symmetry？

Non-invertible symmetry

Many examples in 2D CFT
E.P.Verlinde (1988),
G.W.Moore and N. Seiberg (1989),...

- Ordinary symmetry :

  - Fusion rule of symmetry operator :

$$U_{g_1} U_{g_2} = U_{g_1 g_2} \qquad g_1, g_2 \in G$$

Ex., $U(1) : e^{i\theta_1} e^{i\theta_2} = e^{i(\theta_1+\theta_2)}$

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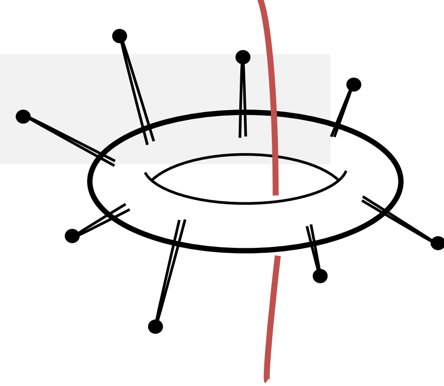
- **Non-invertible symmetry :**

  - Fusion rule of symmetry operator :

Ex., $U_{\theta_1} U_{\theta_2} = U_{\theta_1+\theta_2} + U_{\theta_1-\theta_2}$

$U_\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ except for $\theta = 0, \pi$

Short summary



- ６次元Yang-Mills理論 on $T^2/\mathbb{Z}_N$
  (IIB型超弦理論のmagnetized D-branes）
  におけるnon-invertible symmetries

  - Fusion rules of $\mathbb{Z}_N$-invariant operators :

    For $T^2/\mathbb{Z}_2, \quad U_{\lambda_1}U_{\lambda_2} = U_{\lambda_1+\lambda_2} + U_{\lambda_1-\lambda_2}$

  - Representations of chiral zero-modes

- Chiral zero-modes will correspond to
  3 generations of quarks/leptons
  Non-invertible symmetries → flavor symmetries

Outline

✓ Introduction

▪ ６次元QED on T^2におけるフレーバー対称性

▪ Non-invertible symmetry on $T^2/\mathbb{Z}_N$

▪ Conclusion

6D QED on $R^{1,3} \times T^2$ with magnetic flux

$$\mathcal{L} = -\frac{1}{4} F_{mn} F^{mn} + i\bar{\Psi}\Gamma^m D_m \Psi \qquad m,n = x^\mu, y_1, y_2$$

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$

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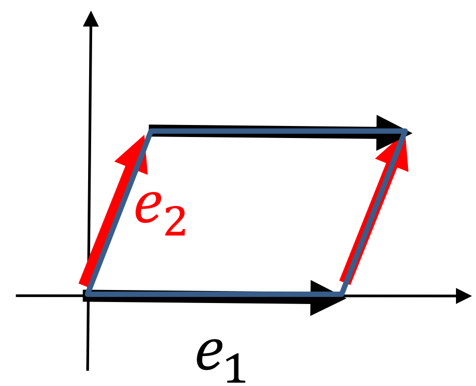

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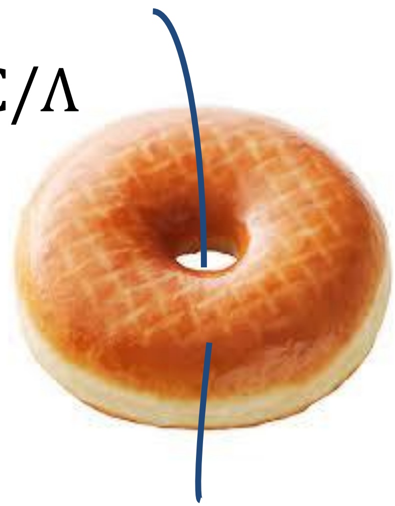

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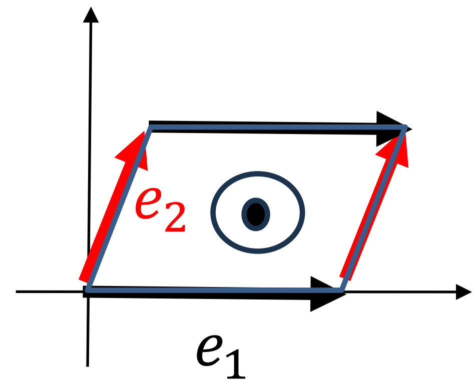

- *U(1)* magnetic flux

$$F_{y_1 y_2} = 2\pi i M \qquad (M \in \mathbb{Z}_{>0})$$

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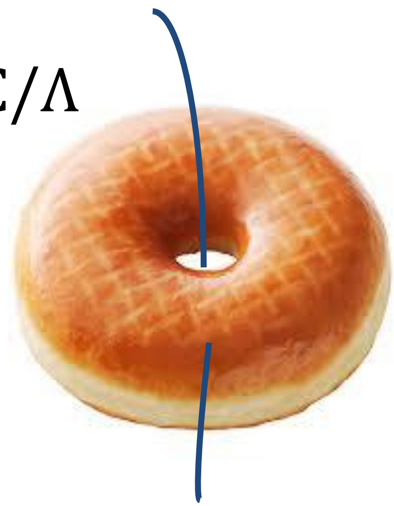

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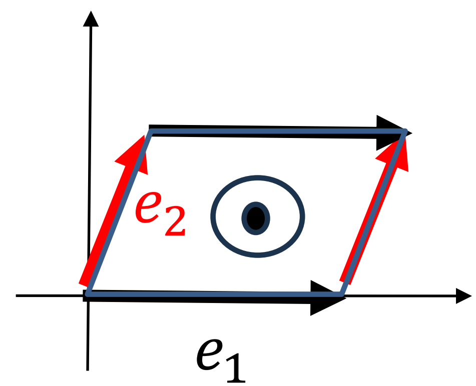

- *U(1)* magnetic flux

$$F_{y_1 y_2} = 2\pi i M \qquad (M \in \mathbb{Z}_{>0})$$

- *Index theorem*

$$\frac{1}{2\pi} \int \mathrm{tr} F = M$$

M個のZero-modes

6D QED on $R^{1,3} \times T^2$ with magnetic flux

$$\mathcal{L} = -\frac{1}{4} F_{mn} F^{mn} + i \bar{\Psi} \Gamma^m D_m \Psi \qquad m, n = x^\mu, y_1, y_2$$

- *Kaluza-Klein展開*

$$\Psi(x, y) = \chi(x) \psi(y) + (\text{KK modes})$$

- *Dirac方程式 (for zero-modes)*

$$\Gamma^m D_m \psi(y) = 0 \qquad \psi(y) = \begin{pmatrix} \psi_+(y) \\ \psi_-(y) \end{pmatrix}$$

- *M個の縮退した zero-modes*

If $M > 0$, $\psi_+^{j,M}(y)$ $\qquad j = 0,1, \dots, M-1$

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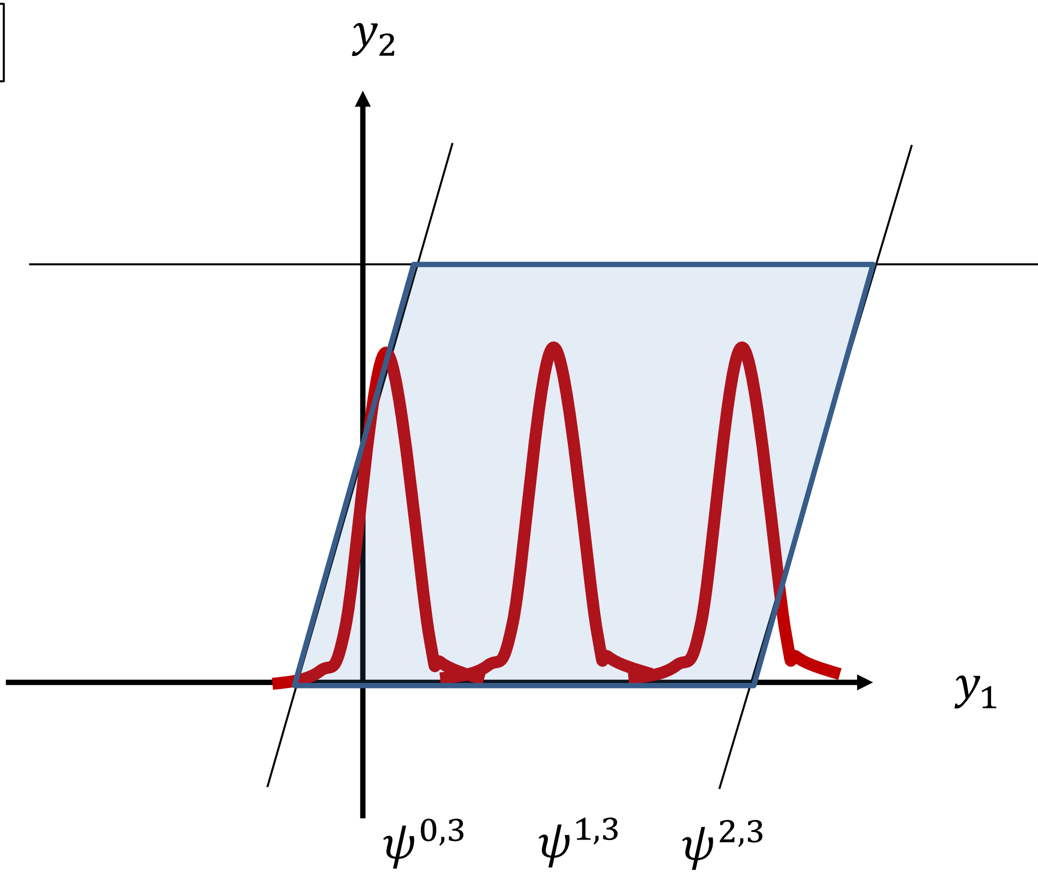
M個の縮退したzero modes on T^2

D.Cremades, L.E.Ibanez, F.Marchesano (2004),
H. Abe, K.Choi, T.Kobayashi, H.Ohki (2009),
M.Berasaluce-Gonzalez, P.G.Camara, F.Marchesano, (2012)

$M = 3$



3 chiral zero-modes 〜 3 generations of quarks/leptons

湯川結合

$$\mathcal{L} = -\frac{1}{4}F_{mn}F^{mn} + i\bar{\Psi}\Gamma^{m}(\partial_m - iqA_m)\,\Psi$$

$$m, n = x^{\mu}, y_1, y_2$$

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$$A_{y_1}(x,y) = \varphi_1(x)\phi_1(y) + (\text{KK modes})$$

$$A_{y_2}(x,y) = \varphi_2(x)\phi_2(y) + (\text{KK modes})$$

湯川結合

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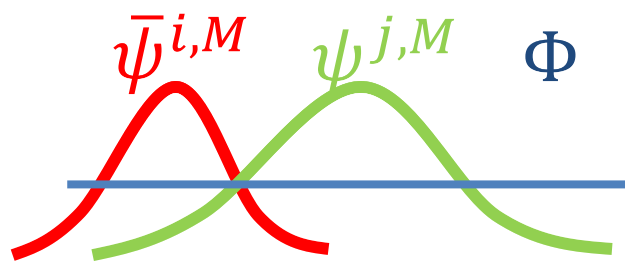
$$A_{y_2}(x,y) = \varphi_2(x)\phi_2(y) + (\text{KK modes})$$

- *湯川結合(波動関数の重なり積分)*

$$Y_{ij} = \int_{T^2}$$



U(N) Yang-Mills理論に拡張可能

$$\mathcal{L} = -\frac{1}{4} F_{mn} F^{mn} + i \bar{\Psi} \Gamma^m (\partial_m - iqA_m)\, \Psi$$

$$m, n = x^\mu, y_1, y_2$$

- *U(1)* magnetic flux

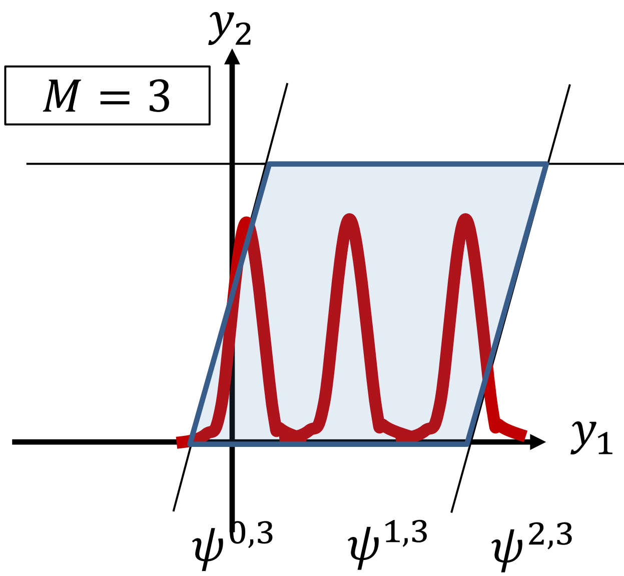
$$F_{y_1 y_2} = 2\pi i \begin{pmatrix} M_a \mathbb{I}_{N_a \times N_a} & 0 & 0 \\ 0 & M_a \mathbb{I}_{N_b \times N_b} & 0 \\ 0 & 0 & M_c \mathbb{I}_{N_c \times N_c} \end{pmatrix} \quad (M_a, M_b, M_c \in \mathbb{Z})$$

- *ゲージ対称性の破れ*

$$U(N) \to U(N_a) \times U(N_b) \times U(N_c)$$

- *様々なBSMが議論されている*

Flavor symmetry of chiral matters on T^2



H. Abe, K.Choi, T.Kobayashi, H.Ohki (2009),
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Flavor symmetry :

M 個の Chiral matters $\psi^{j,M}$を入れ替える対称性

$$H_M \simeq \left(\mathbb{Z}_M \times \mathbb{Z}_M^{(Z)}\right) \rtimes \mathbb{Z}_M^{(C)}$$

$$\mathbb{Z}_M = e^{\frac{2\pi i}{M}} \mathrm{diag}(1,1,\dots)$$

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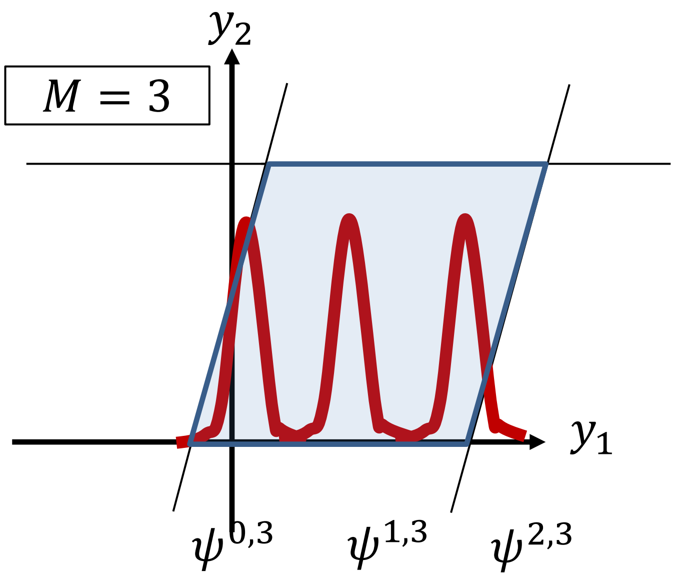

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Under the discrete translations :

$$y_1 \to y_1 + \frac{1}{M}, \qquad y_2 \to y_2 + \frac{1}{M}$$

$\psi^{j,M}$ の変換 (Generators : $e^{\frac{1}{M}D_{y_1}}$ and $e^{\frac{1}{M}D_{y_2}}$) :

$$Z : e^{\frac{1}{M}D_{y_1}} \psi^{j,M} = e^{2\pi i \frac{j}{M}} \psi^{j,M} \qquad (Z)^M = (C)^M = \mathbb{I}$$

$$C : e^{\frac{1}{M}D_{y_2}} \psi^{j,M} = \psi^{j+1,M} \qquad CZ = e^{\frac{2\pi i}{M}} ZC$$

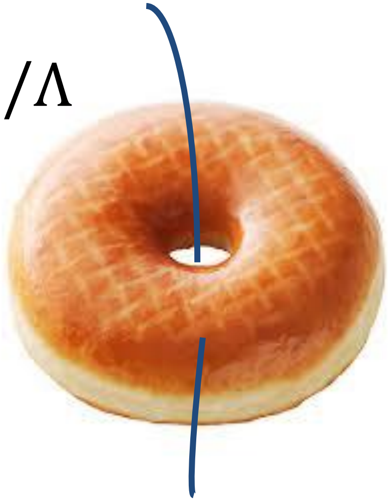
６次元Yang-Mills理論 on T^2 with magnetic fluxes

→ 磁場中の量子力学

Quantum mechanical system

D.Cremades, L.E.Ibanez, F.Marchesano (2004)
T.h.Abe, Y.Fujimoto, T.Kobayashi, T.Miura, K.Nishiwaki, M. Sakamoto (2014)

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$



- *Hamiltonian*

$$\hat{H} = (\hat{p} - A)^2 = \left(\hat{p} - \frac{1}{2}F\hat{y}\right)^2$$

$$[\hat{y}_i, \hat{p}_j] = i\delta_{ij} \qquad i, j = 1,2$$

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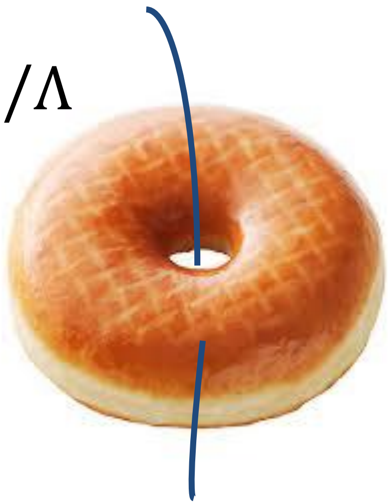

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磁場中の量子力学

$$\{\hat{y}_1, \hat{y}_2, \hat{p}_1, \hat{p}_2\} \to \{\hat{Y}, \hat{P}, \hat{T}_1, \hat{T}_2\}$$

Harmonic oscillator : $\hat{H} = \frac{1}{2}\hat{P}^2 + \frac{\omega}{2}\hat{Y}^2$

$$[\hat{Y}, \hat{P}\,] = i\ ,\ [\hat{T}_1, \hat{T}_2] = 2\pi i M\ , \text{(others=0)}$$

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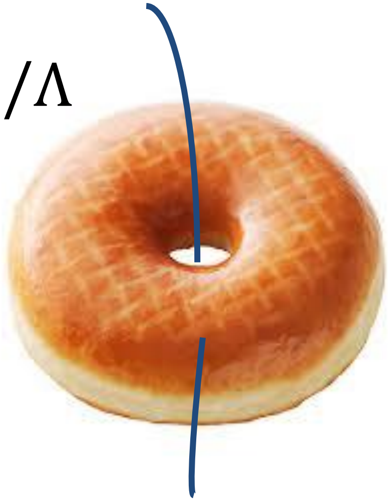

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磁場中の量子力学

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$[\hat{Y}, \hat{P}] = i$, $[\hat{T}_1, \hat{T}_2] = 2\pi i M$, (others=0)

$\hat{T}_1, \hat{T}_2$は、Hamitonianに現れない保存量だが、statesに制限を与える

Quantum mechanical system

- *状態ベクトルに対する制限 (～ 波動関数に対する境界条件)*

$$e^{i\hat{T}_a}|\psi\rangle = |\psi\rangle$$

Operators $\hat{T}_1, \hat{T}_2$ *は、M個の縮退した状態* $\left|\psi^{j,M}\right\rangle = \left|\frac{j}{M}\right\rangle$ *を区別*

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- *We choose the eigenstate of $\hat{T}_2$ for the state s.t.*

$$e^{i\hat{T}_2}\left|\psi^{j,M}\right\rangle = e^{2\pi i\, j}\left|\psi^{j,M}\right\rangle$$

$$\left|\psi^{j,M}\right\rangle = e^{-i\frac{j}{M}\hat{T}_1}|0\rangle$$

$\hat{T}_1$: T_2 方向の並進に対する momentum operator

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$\hat{T}_1$: T_2 方向の並進に対する momentum operator

- *Schrödinger eq.*

$$\hat{H}\left|\psi^{j,M}\right\rangle = E_n\left|\psi^{j,M}\right\rangle$$

For two-dimensional spinor,

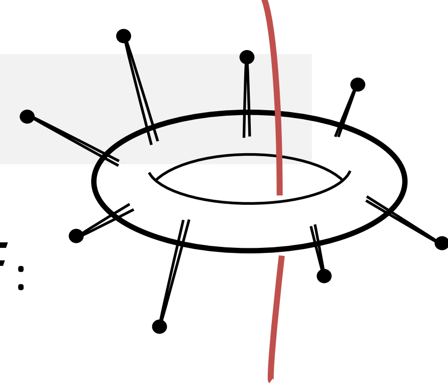
$$E_n = \omega\, n$$

$n = 0$: zero modes
$n \neq 0$: KK modes

$T^2/\mathbb{Z}_N$ orbifoldsへ拡張

→ Non-invertible symmetries

Orbifolding



- *2次元空間における原点周り（$y_1 = y_2 = 0$）の回転*：

$$\boldsymbol{y} \to R_\theta \boldsymbol{y} \qquad R_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

- *In the operator formalism:*

T.h.Abe, Y.Fujimoto, T.Kobayashi, T.Miura, K.Nishiwaki, M. Sakamoto (2014)

$$\hat{\boldsymbol{y}} \to \hat{U}_\theta \hat{\boldsymbol{y}} \hat{U}_\theta^\dagger = R_\theta \hat{\boldsymbol{y}}$$
$$\hat{\boldsymbol{p}} \to \hat{U}_\theta \hat{\boldsymbol{p}} \hat{U}_\theta^\dagger = R_\theta \hat{\boldsymbol{p}}$$
$$\hat{H} \to \hat{H}$$

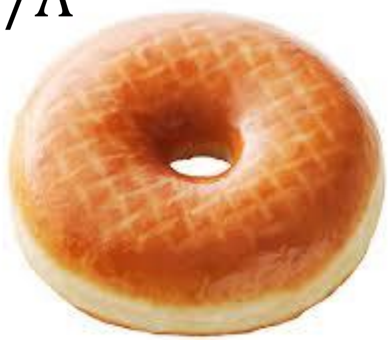
due to $[R_\theta, F] = 0$

- $\{\hat{T}_1, \hat{T}_2\}$ によって指定される*境界条件は修正*：

$$\hat{T}_a \to \hat{U}_\theta \hat{T}_a \hat{U}_\theta^\dagger = \left(R_\theta^T e_a\right)^T \left(\hat{\boldsymbol{p}} - \frac{1}{2} F^T \hat{\boldsymbol{y}}\right)$$

Ex., $\mathbb{Z}_2$ - twist

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$



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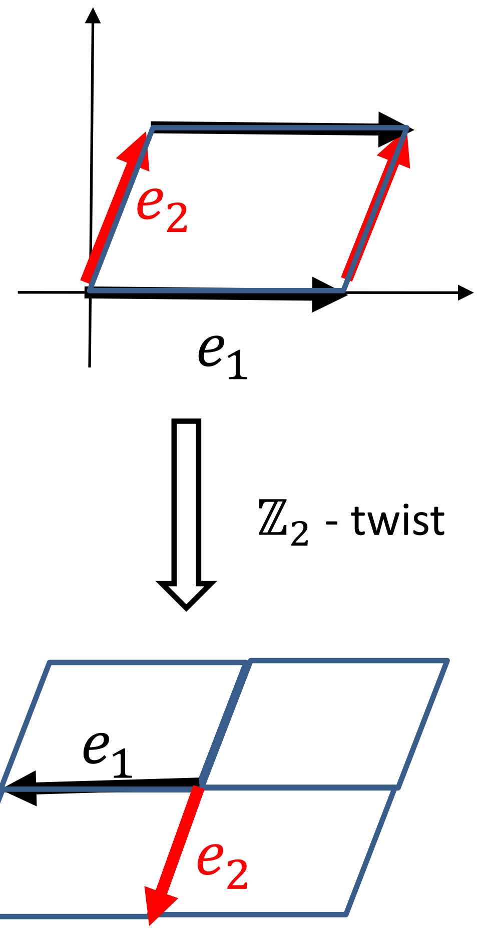
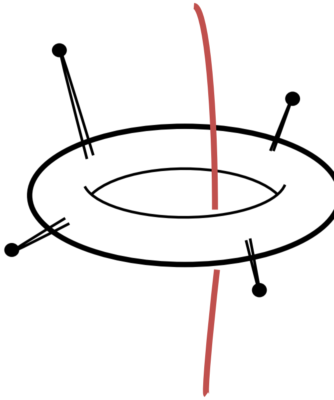


- *境界条件を決定する演算子* $\hat{T}_1, \hat{T}_2$:

$$\hat{T}_1 \to -\hat{T}_1 \qquad\qquad \hat{T}_2 \to -\hat{T}_2$$

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux

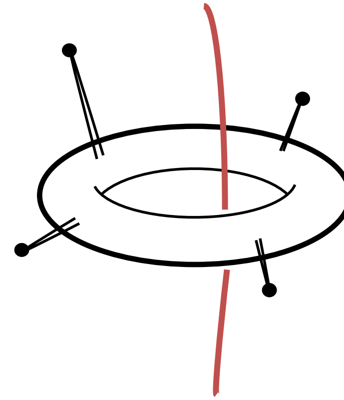


- $\mathbb{Z}_2$ のgauge化 *(leave all operators invariant under* $\mathbb{Z}_2$*)*
  2つの $\mathbb{Z}_2$-invariant operators:

$$\hat{U}_1^{(\lambda_1)} \equiv e^{i\lambda_1 \hat{T}_1} + e^{-i\lambda_1 \hat{T}_1}$$

$$\hat{U}_2^{(\lambda_2)} \equiv e^{i\lambda_2 \hat{T}_2} + e^{-i\lambda_2 \hat{T}_2}$$

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux



- $\mathbb{Z}_2$ のgauge化 *(leave all operators invariant under* $\mathbb{Z}_2$*)*
  2つの $\mathbb{Z}_2$-invariant operators:

$$\hat{U}_1^{(\lambda_1)} \equiv e^{i\lambda_1 \hat{T}_1} + e^{-i\lambda_1 \hat{T}_1}$$

$$\hat{U}_2^{(\lambda_2)} \equiv e^{i\lambda_2 \hat{T}_2} + e^{-i\lambda_2 \hat{T}_2}$$

- *Fusion rule :*

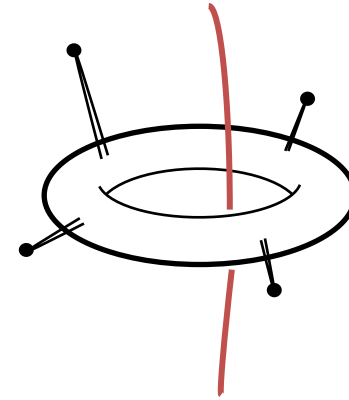
$$\hat{U}_1^{(\lambda)} \hat{U}_1^{(\lambda')} = \hat{U}_1^{(\lambda+\lambda')} + \hat{U}_1^{(\lambda-\lambda')}$$

$$\hat{U}_2^{(\lambda)} \hat{U}_2^{(\lambda')} = \hat{U}_2^{(\lambda+\lambda')} + \hat{U}_2^{(\lambda-\lambda')}$$

$$\hat{U}_2^{(\lambda_2)} \hat{U}_1^{(\lambda_1)} = e^{i\rho} \hat{U}_1^{(\lambda_1)} \hat{U}_2^{(\lambda_2)}$$

If $e^{i\rho} = 1$: Abelian structure
$e^{i\rho} = -1$: Non-abelian structure

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux



- $\mathbb{Z}_2$ のgauge化 *(leave all operators invariant under $\mathbb{Z}_2$)*
  2つの $\mathbb{Z}_2$-invariant operators:

$$\hat{U}_1^{(\lambda_1)} \equiv e^{i\lambda_1 \hat{T}_1} + e^{-i\lambda_1 \hat{T}_1}$$

$$\hat{U}_2^{(\lambda_2)} \equiv e^{i\lambda_2 \hat{T}_2} + e^{-i\lambda_2 \hat{T}_2}$$

- *Representations of chiral matters:*

$\lambda_1 = \lambda_2 = 1/M$

$$\hat{U}_1 \ |\psi\rangle_+^{j,M} = |\psi\rangle_+^{j+1,M} + |\psi\rangle_+^{j-1,M}$$

$$\hat{U}_2 \ |\psi\rangle_+^{j,M} = 2\cos\left(\frac{2\pi j}{M}\right)|\psi\rangle_+^{j,M}$$

$\mathbb{Z}_2$ *-even state :* $|\psi\rangle_+^{j,M} = \frac{1}{2}\left(|\psi\rangle^{j,M} + |\psi\rangle^{-j,M}\right)$

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux

$\mathbb{Z}_2$ *-even state :* $|\psi\rangle_+^{j,M} = \frac{1}{2}\left(|\psi\rangle^{j,M} + |\psi\rangle^{-j,M}\right)$

- *M=2 :*

$\lambda = 1/2$

$$\hat{U}_1 : \begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix} \to 2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix}$$

$$\hat{U}_2 : \begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix} \to 2\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix}$$

- $D_4 \simeq \left(\mathbb{Z}_2 \times \mathbb{Z}_2^{(\hat{U}_2/2)}\right) \rtimes \mathbb{Z}_2^{(\hat{U}_1/2)}$ *の2次元既約表現*

$\mathbb{Z}_2 : \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux

$\mathbb{Z}_2$ *-even state :* $|\psi\rangle_+^{j,M} = \frac{1}{2}\left(|\psi\rangle^{j,M} + |\psi\rangle^{-j,M}\right)$

- *M=2 :*

$\lambda = 1/2$

$$\hat{U}_1 : \begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix} \to 2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix}$$

$$\hat{U}_2 : \begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix} \to 2\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,2} \\ |\psi\rangle_+^{1,2} \end{pmatrix}$$

- $D_4 \simeq \left(\mathbb{Z}_2 \times \mathbb{Z}_2^{(\hat{U}_2/2)}\right) \rtimes \mathbb{Z}_2^{(\hat{U}_1/2)}$ *の2次元既約表現*

- *M=3 :*

$\mathbb{Z}_2 : \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\lambda = 1/3$

$$\hat{U}_1 : \begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix} \to 2\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix}$$

$$\hat{U}_2 : \begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix} \to 2\begin{pmatrix} 1 & 0 \\ 0 & \cos(2\pi/3) \end{pmatrix}\begin{pmatrix} |\psi\rangle_+^{0,3} \\ |\psi\rangle_+^{1,3} \end{pmatrix}$$

- *No "invertible" symmetry remains*

Non-invertible symmetry on $T^2/\mathbb{Z}_2$ with magnetic flux

$\mathbb{Z}_2$ *-even state :* $|\psi\rangle_+^{j,M} = \frac{1}{2}(|\psi\rangle^{j,M} + |\psi\rangle^{-j,M})$

- *M=2k :*

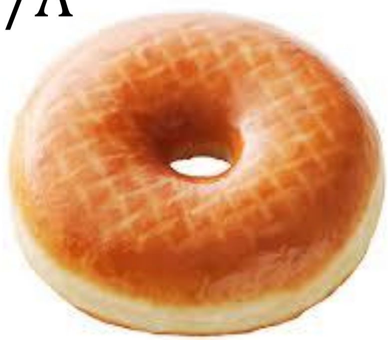
  - *Similarly, we discuss transformations of chiral matters under the non-invertible symmetries.*

  - *They belong to 2-dim. or singlet irreducible rep. of*

$$D_4 \simeq \left(\mathbb{Z}_2 \times \mathbb{Z}_2^{(\hat{U}_2/2)}\right) \rtimes \mathbb{Z}_2^{(\hat{U}_1/2)}$$

Ex., $\mathbb{Z}_3$ - twist

$\boldsymbol{T^2} = \mathbb{C}/\Lambda$



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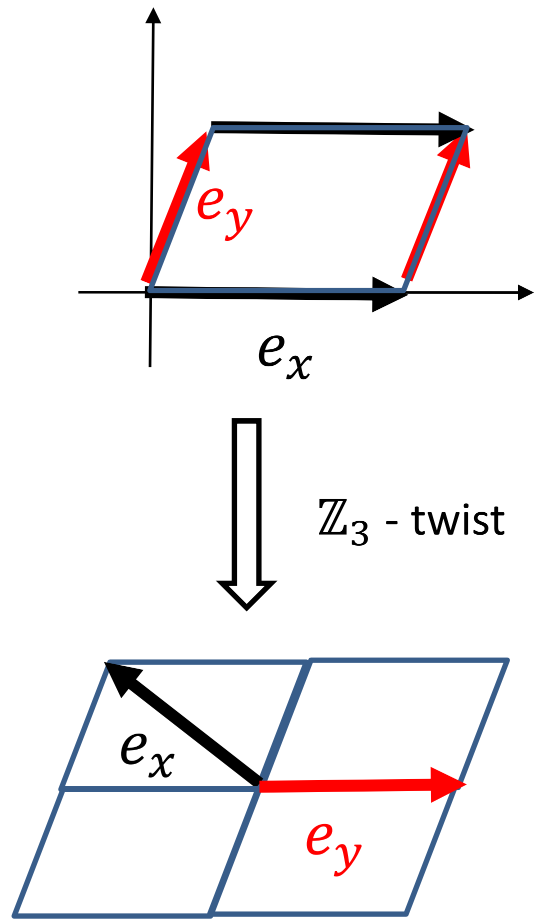
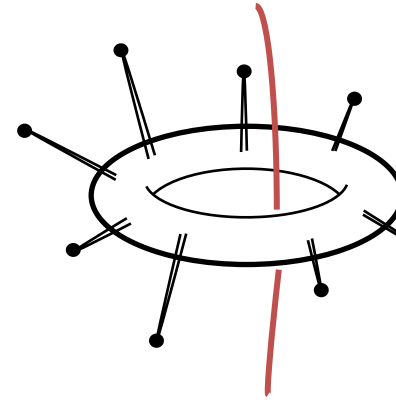


- *境界条件を決定する演算子* $\hat{T}_1$ and $\hat{T}_2$:

$$\hat{T}_1 \to -\hat{T}_1 - \hat{T}_2, \qquad \hat{T}_2 \to \hat{T}_1$$

Non-invertible symmetry on $T^2/\mathbb{Z}_3$ with magnetic flux

- $\mathbb{Z}_3$ のgauge化 *(leave all operators invariant under $\mathbb{Z}_3$)*
  $\mathbb{Z}_3$-invariant operator :

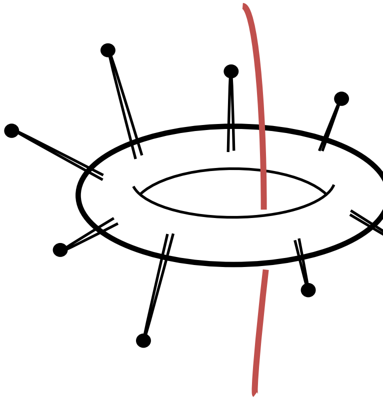


$$\hat{U}_{\mathbb{Z}_3}^{(\lambda_1,\lambda_2)} \equiv e^{i\lambda_1\hat{T}_1+i\lambda_2\hat{T}_2} + e^{i\lambda_1\hat{T}_2-i\lambda_2(\hat{T}_1+\hat{T}_2)} + e^{-i\lambda_1(\hat{T}_1+\hat{T}_2)+i\lambda_2\hat{T}_1}$$

- *Fusion rule :*

$$\begin{aligned}\hat{U}_{\mathbb{Z}_3}^{(\lambda_1,\lambda_2)}\hat{U}_{\mathbb{Z}_3}^{(\lambda_3,\lambda_4)} = \ & e^{\pi i M(\lambda_2\lambda_3-\lambda_1\lambda_4)}\hat{U}_{\mathbb{Z}_3}^{(\lambda_1+\lambda_3,\lambda_2+\lambda_4)} \\ & + e^{\pi i M(\lambda_1\lambda_3-\lambda_2\lambda_3+\lambda_2\lambda_4)}\hat{U}_{\mathbb{Z}_3}^{(-\lambda_1+\lambda_2-\lambda_4,-\lambda_1+\lambda_3-\lambda_4)} \\ & + e^{\pi i M(\lambda_1\lambda_4-\lambda_2\lambda_4-\lambda_1\lambda_3)}\hat{U}_{\mathbb{Z}_3}^{(\lambda_1-\lambda_4,\lambda_2+\lambda_3-\lambda_4)}\end{aligned}$$

Non-invertible symmetry on $T^2/\mathbb{Z}_{4,6}$ with magnetic flux



- *Symmetry operators:*

$$\hat{U}_{\mathbb{Z}_4}^{(\lambda_1,\lambda_2)} \equiv e^{i\lambda_1\hat{T}_1+i\lambda_2\hat{T}_2} + e^{-i\lambda_1\hat{T}_2+i\lambda_2\hat{T}_1} + e^{-i\lambda_1\hat{T}_1-i\lambda_2\hat{T}_2} + e^{i\lambda_1\hat{T}_2-i\lambda_2\hat{T}_1}$$

$$\hat{U}_{\mathbb{Z}_6}^{(\lambda_1,\lambda_2)} \equiv e^{i\lambda_1\hat{T}_1+i\lambda_2\hat{T}_2} + e^{i\lambda_1(\hat{T}_1-\hat{T}_2)+i\lambda_2\hat{T}_1} + e^{-i\lambda_1\hat{T}_2+i\lambda_2(\hat{T}_1-\hat{T}_2)} + e^{-i\lambda_1\hat{T}_1-i\lambda_2\hat{T}_2} + e^{-i\lambda_1(\hat{T}_1-\hat{T}_2)-i\lambda_2\hat{T}_1} + e^{i\lambda_1\hat{T}_2-i\lambda_2(\hat{T}_1-\hat{T}_2)}$$

Conclusion

- ６次元Yang-Mills理論 on $T^2/\mathbb{Z}_N$
  (IIB型超弦理論のmagnetized D-branes ）
  におけるNon-invertible symmetries

  - Fusion rules of $\mathbb{Z}_N$-invariant operators :

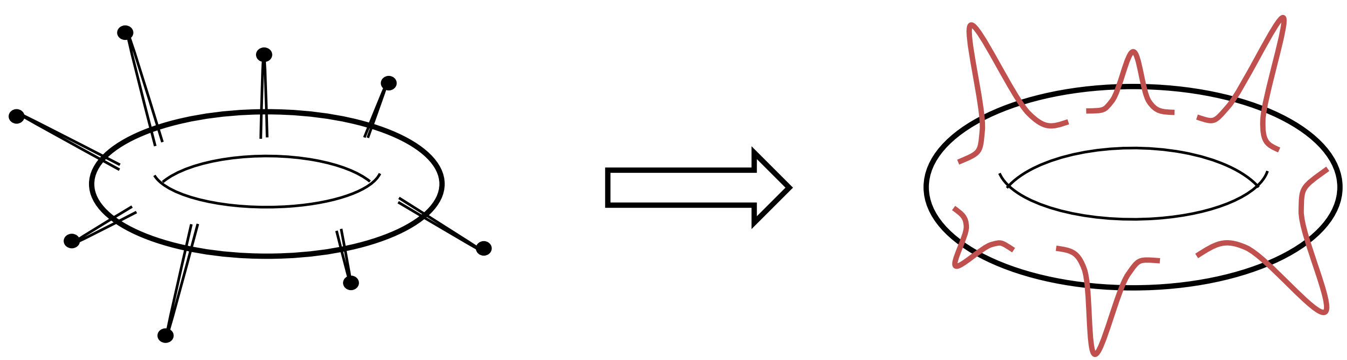
    For $T^2/\mathbb{Z}_2$, $\quad U_{\lambda_1} U_{\lambda_2} = U_{\lambda_1+\lambda_2} + U_{\lambda_1-\lambda_2}$

  - Representations of chiral zero-modes

- Chiral zero-modes will correspond to
  3 generations of quarks/leptons
  Non-invertible symmetries → flavor symmetries

Discussion

- n点結合（高次演算子）のSelection rule
- $T^6/\mathbb{Z}_N$ or $T^6/\mathbb{Z}_M \times \mathbb{Z}_N$への応用
- Non-invertible symmetriesの破れ
  - $T^{2n}/\mathbb{Z}_N$ orbifoldsの特異点解消によって実現
  - blowup modes がnon-invertible symmetriesの破れを決定



$Thank\ you!$