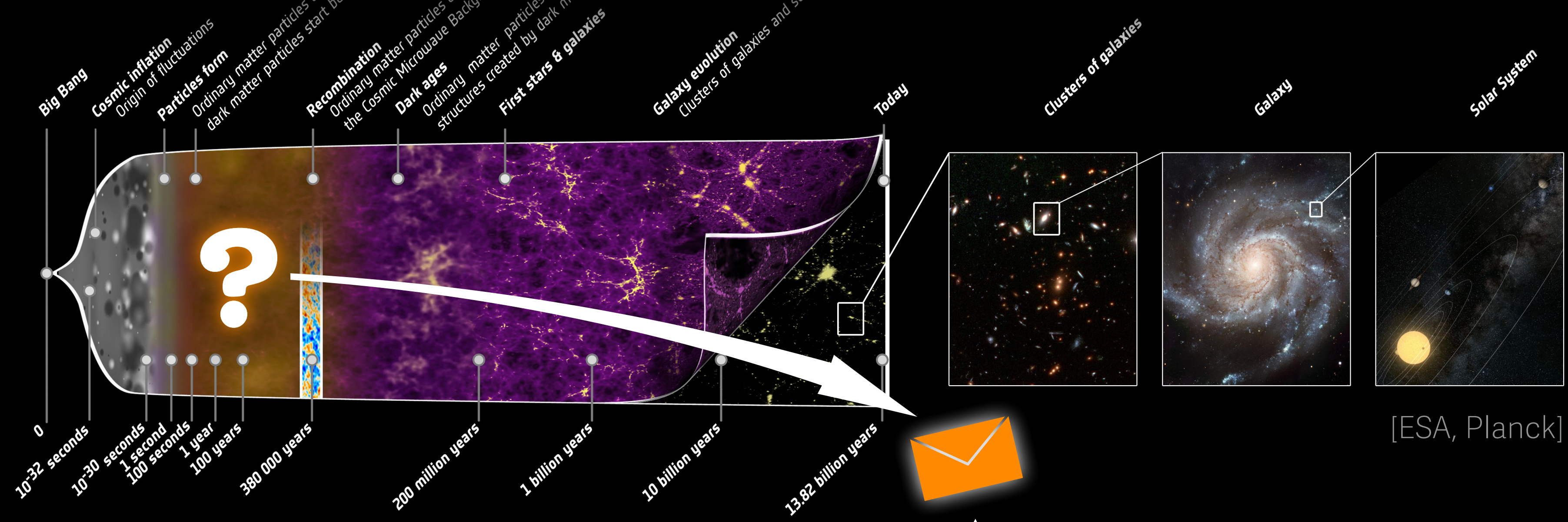
Hosking integral and its implications for constraining primordial magnetic fields

2024/8/20, PPP2024

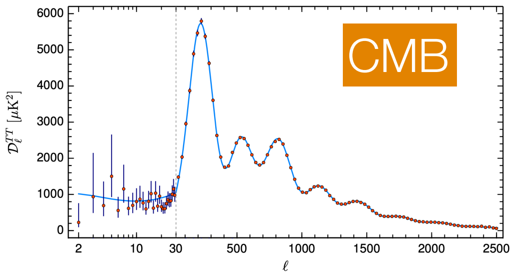
Fumio Uchida (KEK)

[FU, Fujiwara, Kamada, Yokoyama 2023]
[FU, Fujiwara, Kamada, Yokoyama 2024]
[FU, Kamada, Tashiro ongoing]

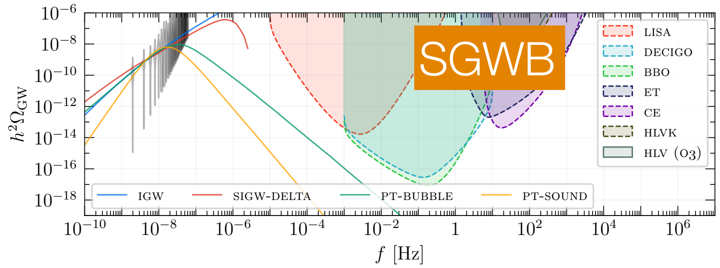
PMF as a messenger from the early universe



[ESA, Planck]



[Planck, 2020]



[NANOGrav, 2023]

… primordial magnetic field (PMF)

Focus of this talk

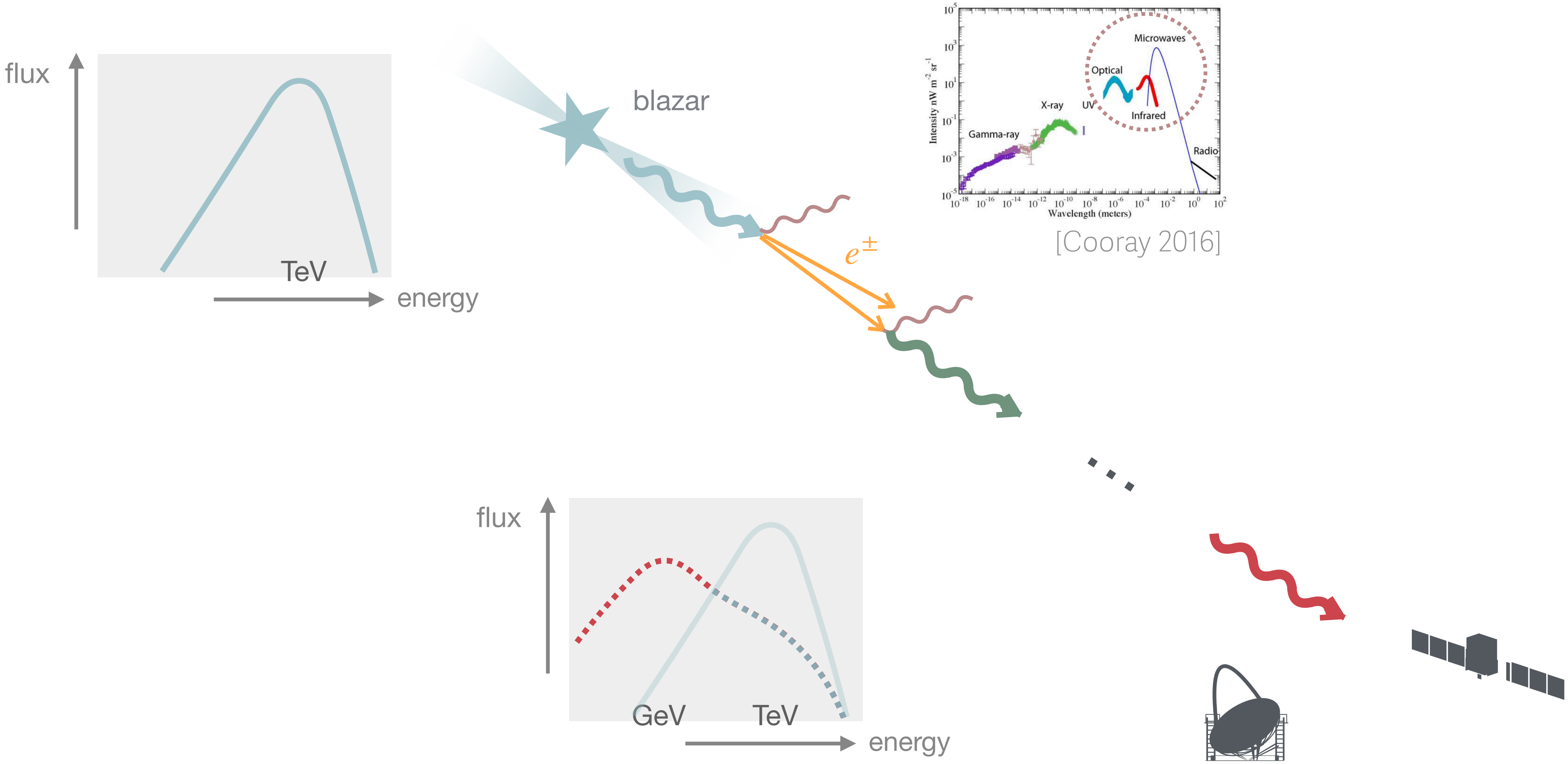
PMFs are small.
- standard cosmology $\not\ni$ PMFs
- most observations → upper bounds on PMF strengths: $B_0 \lesssim 1\text{-}10\,\mathrm{nG}$

Nevertheless,
- blazar observations suggest **the existence of PMFs**: $B_0 \gtrsim 10^{-17}\,\mathrm{G}$

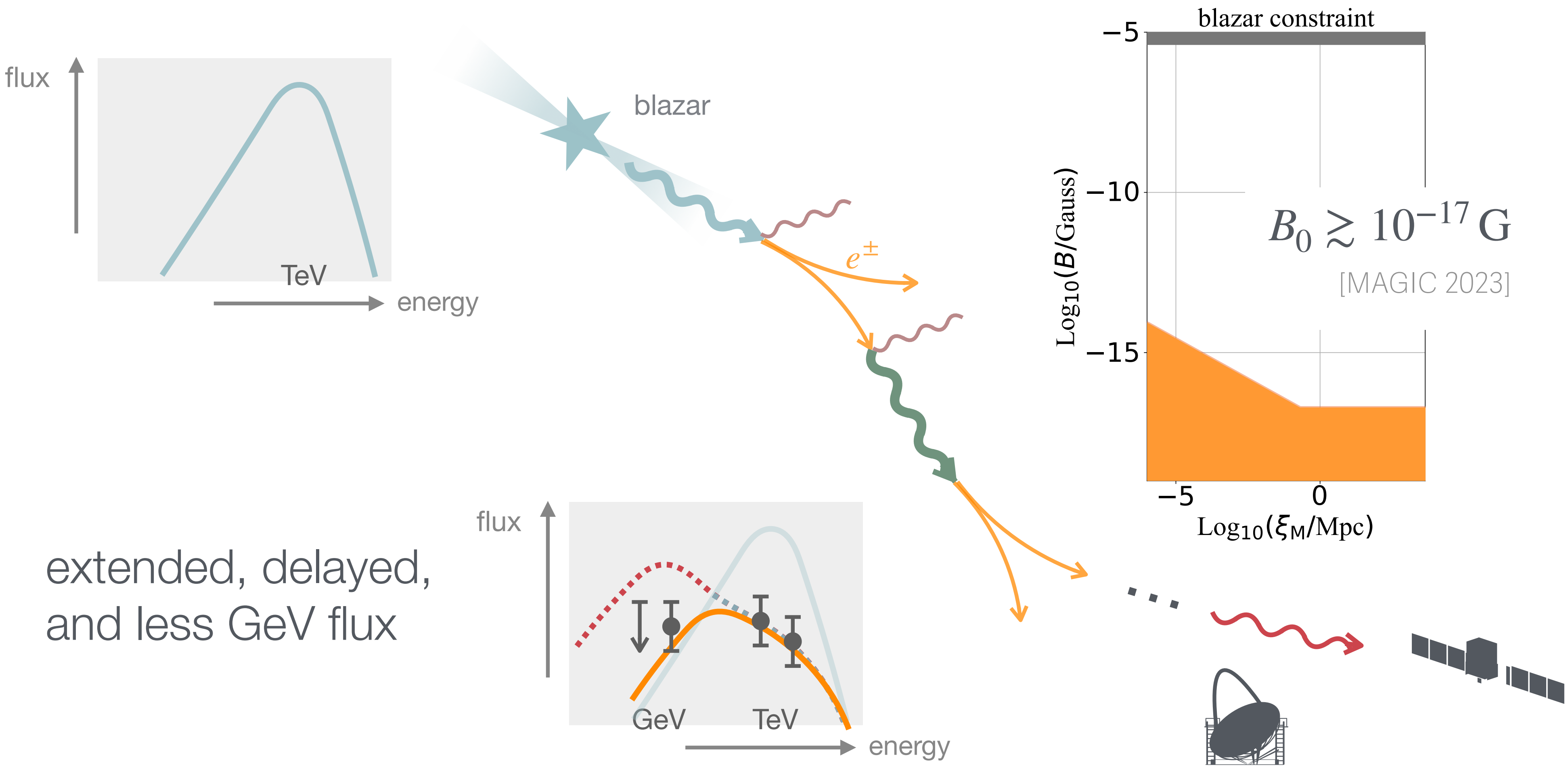
To read out information of the early universe from PMFs..
- **evolution of PMFs is crucial**.
- **Hosking integral helps us understand the evolution**!

[Hosking, Schekochihin 2021, 2023], [FU, Fujiwara, Kamada, Yokoyama 2023, 2024]

Observation of blazars behind a void



Observation of blazars behind a void



extended, delayed,
and less GeV flux

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Nevertheless,

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To read out information of the early universe from PMFs..

- **evolution of PMFs is crucial.**
- **Hosking integral helps us understand the evolution.**

[Hosking, Schekochihin 2021, 2023], [FU, Fujiwara, Kamada, Yokoyama 2023, 2024]

Outline

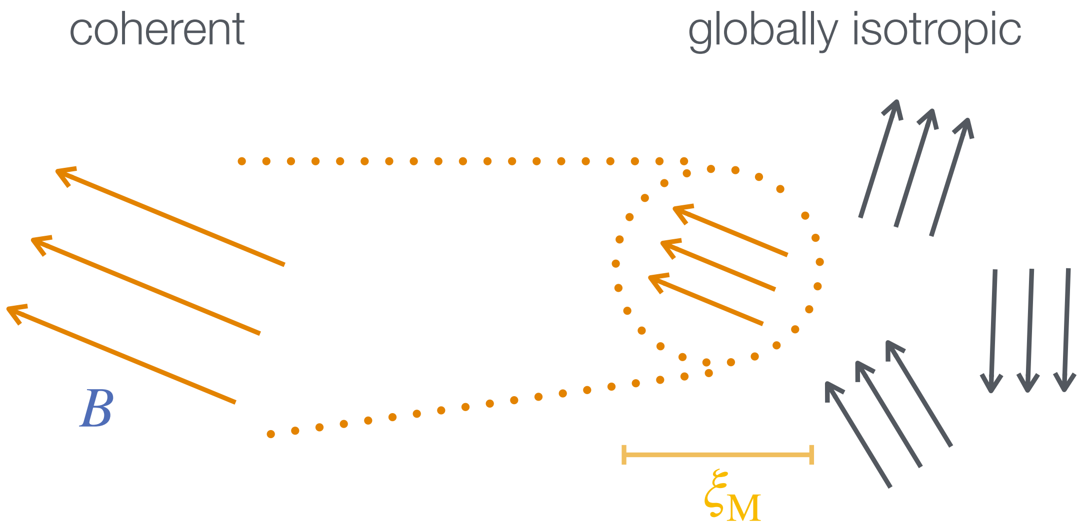
introduction

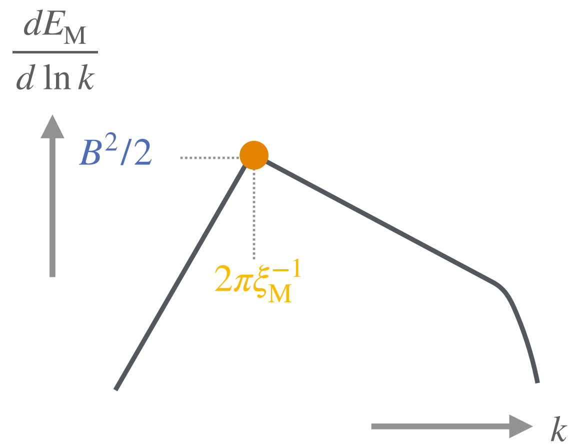
Hosking integral

CMB distortion constraint

Summary

Parametrization of a random magnetic field





- comoving strength B $(= a^2 B_p)$
- comoving coherence length ξ_M $(= a^{-1} \xi_{M,p})$

  a: scale factor

The dynamics is governed by MHD

dynamics in terms of physical quantities – expansion of the universe

= dynamics in terms of comoving quantities [Brandenburg, Enqvist, Olesen 1996]

= **fluid dynamics** + **electromagnetism** in a flat spacetime

magneto-hydrodynamics (MHD)

electric conductivity

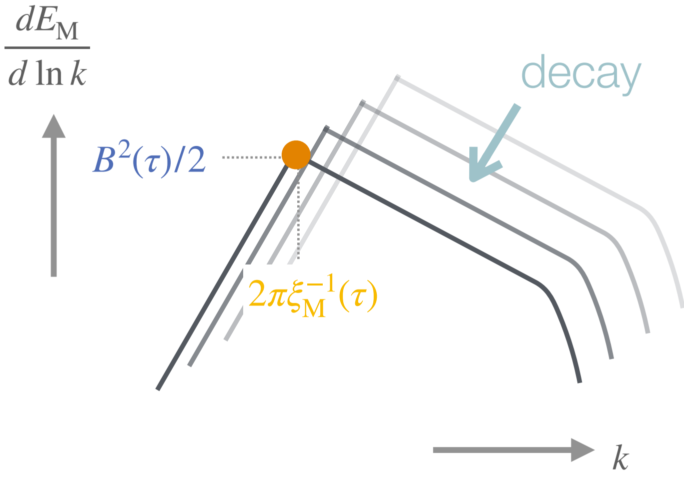
$$\frac{\partial \boldsymbol{B}}{\partial \tau} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = \sigma^{-1} \nabla^2 \boldsymbol{B}$$

viscosity

friction (drag force)

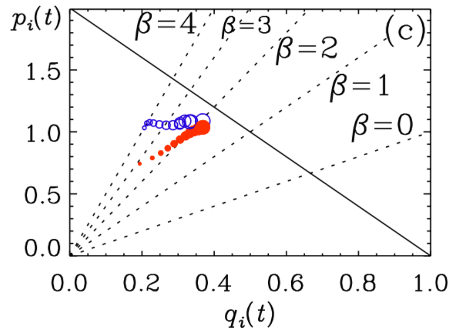
$$\frac{\partial \boldsymbol{v}}{\partial \tau} + (\boldsymbol{v} \cdot \nabla)\, \boldsymbol{v} - \frac{(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}}{\rho + p} + \frac{\nabla p}{\rho + p} = \eta \left(\nabla^2 \boldsymbol{v} + \frac{1}{3} \nabla (\nabla \cdot \boldsymbol{v}) \right) - \alpha \boldsymbol{v}$$

Parametrizing magnetic energy decay



$$B^2(\tau) \propto \tau^{-p_M}, \quad \xi_M(\tau) \propto \tau^{q_M}$$

- direct numerical simulations



[Brandenburg+ 2017]

- **analytic determination**?

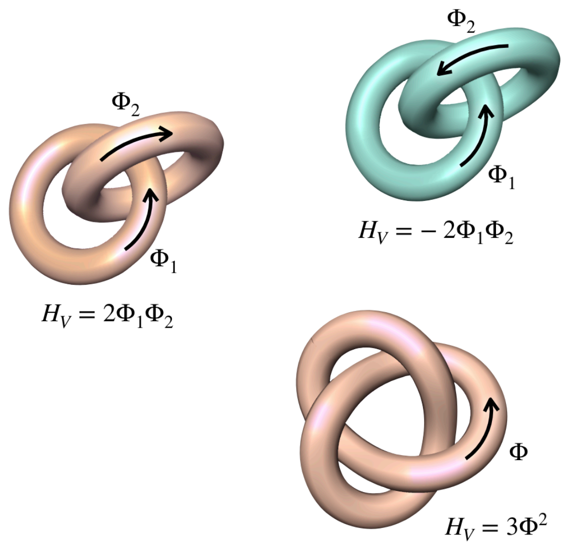
Magnetic helicity conservation

Magnetic helicity is conserved in the infinite-conductivity limit

$$H_V = \int_V d^3x\, h(\boldsymbol{x}) = \text{const}\,., \quad h := \boldsymbol{A}\cdot\boldsymbol{B},$$

$$\langle h \rangle := \lim_{V\to\infty} \frac{H_V}{V} = \epsilon B^2 \xi_{\rm M} = \text{const}\,.$$

ϵ: helicity fraction



Maximally helical magnetic field: $\epsilon = \pm 1$

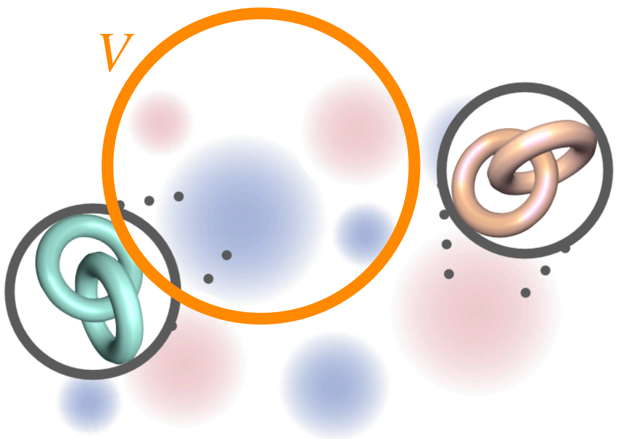
$$B^2 \xi_{\rm M} = \text{const}\,.$$ [Frisch+ 1975]

- magnetic field lines: frozen in the fluid [Alfven 1942]
- magnetic helicity = linking number of field lines [Moffatt 1969]

Non-helical magnetic field

non-helical: $\epsilon = 0 \quad \Leftrightarrow \quad \langle \boldsymbol{A} \cdot \boldsymbol{B} \rangle = 0$

- zero magnetic helicity **on average**
- **spatial fluctuation** → finite sub-volumes have non-zero magnetic helicity



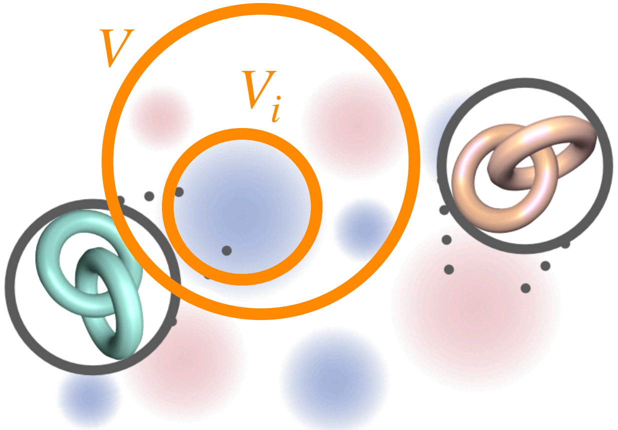
magnetic helicity density

$$H_V = \int_V d^3x \, h(\boldsymbol{x}) = \text{const} \,. \neq 0$$

Idea of the Hosking integral [Hosking, Schekochihin 2021]

random walk → $H_V = \sum_i H_{V_i} \propto \sqrt{V}$ → $I_{\rm H} \simeq \lim_{V\to\infty} \frac{H_V^2}{V}$ = finite and const.

$$I_{\rm H} := \int d^3r \, \langle h(\boldsymbol{x}+\boldsymbol{r}) h(\boldsymbol{x}) \rangle$$



magnetic helicity density

$$H_V = \int_V d^3x \, h(\boldsymbol{x}) = \text{const} . \neq 0$$

Non-helical magnetic field decay

Hosking integral conservation

$$I_{\mathrm{H}} = \int d^3r \, \langle h(\boldsymbol{x}+\boldsymbol{r}) h(\boldsymbol{x}) \rangle$$

$$\langle h(\boldsymbol{x}+\boldsymbol{r}) h(\boldsymbol{x}) \rangle \overset{\text{approx.}}{=} \begin{cases} \langle h^2 \rangle, & r < \xi_{\mathrm{M}} \quad \text{correlation within } \xi_{\mathrm{M}} \\ 0, & r > \xi_{\mathrm{M}} \quad \text{no correlation beyond } \xi_{\mathrm{M}} \end{cases}$$

$$\sim \xi_{\mathrm{M}}^3 \langle h^2 \rangle$$

$$\sim B^4 \xi_{\mathrm{M}}^5 = \text{const}\,.$$

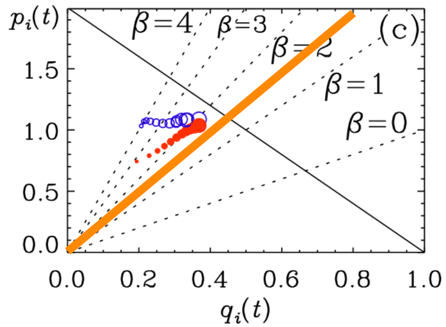
↑ assuming Gaussianity

Conserved quantity explains the inverse transfer

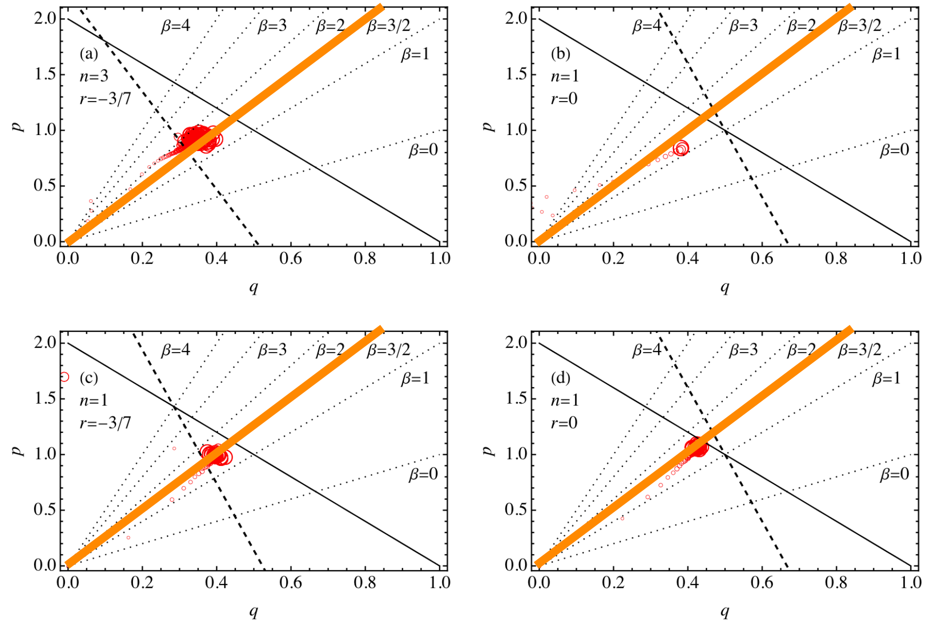
non-helical

$$B^4 \xi_{\rm M}^5 = \text{const}\,.$$

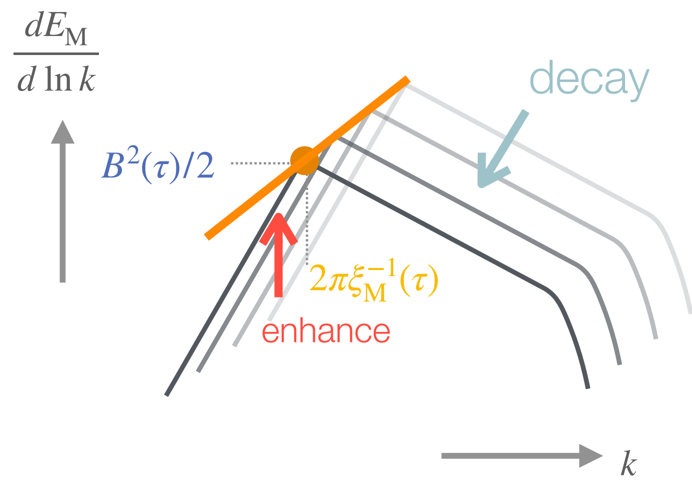
$\to -2p_{\rm M} + 5q_{\rm M} = 0$



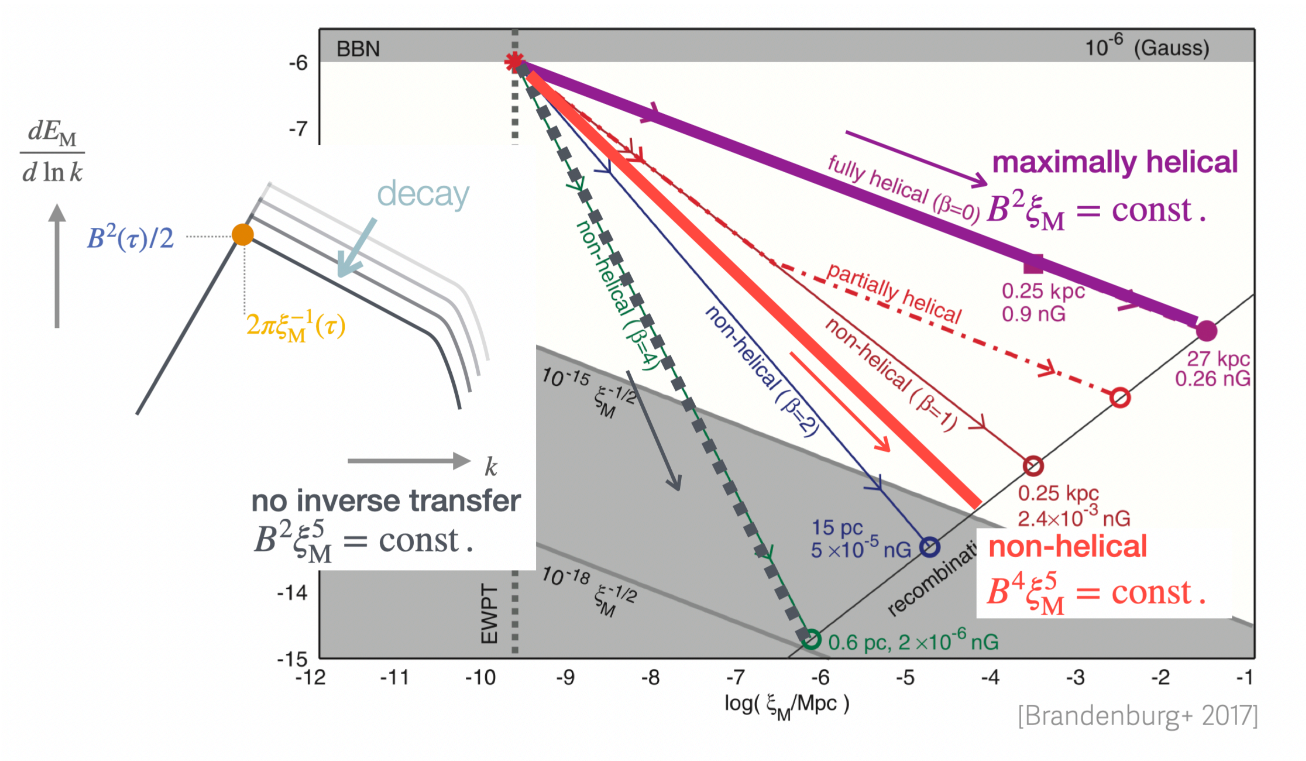
[Brandenburg+ 2017]

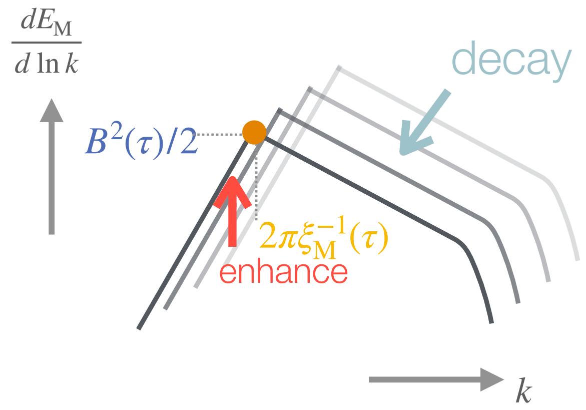


[Zhou+ 2022]



Update: non-helical magnetic field decay





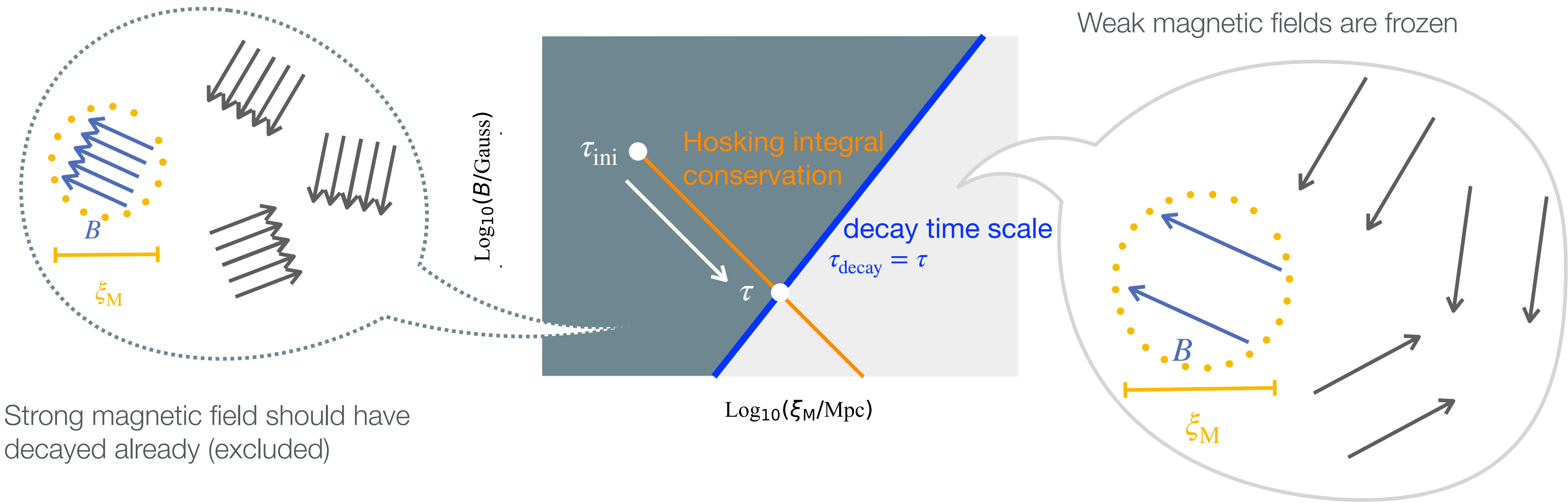
non-helical magnetic field decay: **slower** than previously expected

no inverse transfer

Outline

If we know timescales for B to decay..

$$\begin{cases} B^4 \xi_M^5 = B_{ini}^4 \xi_{M,ini}^5 & \text{Hosking integral conservation} \\ \tau_{decay}(B, \xi_M, \tau) \geq \tau & \text{yet to decay} \end{cases}$$



We have to understand decay timescales

Common ideas:

- Alfvén crossing scale (eddy turnover time) $\tau_A = v_A^{-1} \xi_M,$ $v_A := B/\sqrt{\rho + p}$
  [Banerjee+ 2004]

  non-linear regime

- Damping of slow magnetosonic and the Alfvén mode $\tau_{\rm damp} \sim (l_\gamma \rho_b / \rho_\gamma)^{-1} \tau_A^2$
  [Jedamzik+ 1998] [Subramanian+ 1998]

  linear analysis of the MHD waves

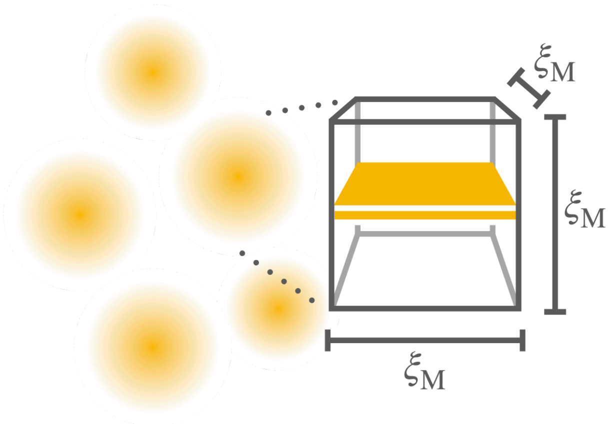
Recently proposed idea: **non-linear** regimes

- Sweet—Parker **magnetic reconnection** timescale (for weak magnetic field) [Hosking+ 2021]
- fast **magnetic reconnection** timescale $\tau_{\rm fast} \sim 10^2 (\sigma\eta)^{1/2} \tau_A$ [Hosking+ 2023]
  [FU, Fujiwara, Kamada, Yokoyama 2023, 2024]

Numerical study (no photon drag, constant dissipation coefficients, Prandtl number $\lesssim 10^3$)

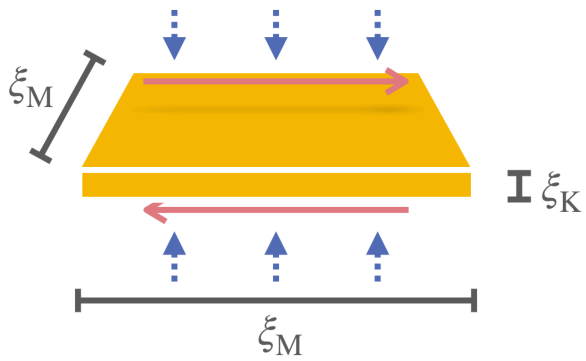
$\tau_{\rm num} \sim \mathcal{O}(10)\, \tau_A$ [Brandenburg+ 2024]

Magnetic reconnection

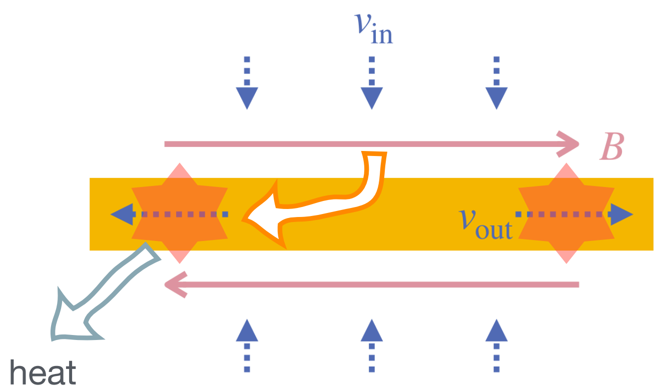


a current sheet in every volume ξ_M^3

[Hosking, Schekochihin 2021]



antiparallel $\boldsymbol{B}$s inflowing onto both sides of each sheet



Zero magnetic field inside the sheet.

magnetic energy inflow
→ outflow velocity
→ dissipated into heat

[Sweet 1958], [Parker 1957], [Park 1984]

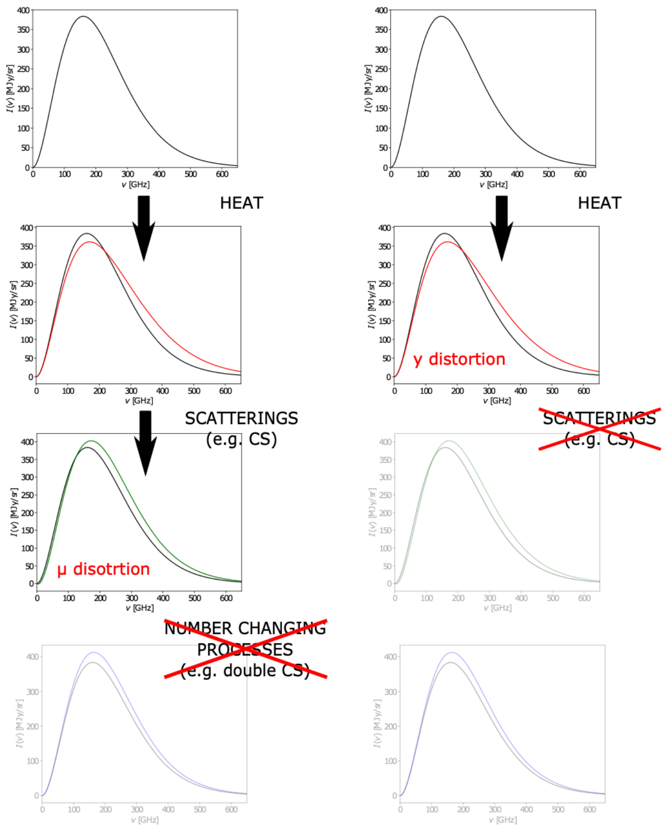
→ energy transfer from magnetic to kinetic fields

hierarchy between magnetic and kinetic coherence length

Application: CMB spectral distortion

[Jedamzik+ 2000]
[Kunze, Komatsu 2017]

$z < 2 \times 10^6$ $\qquad$ $z < 1 \times 10^4$



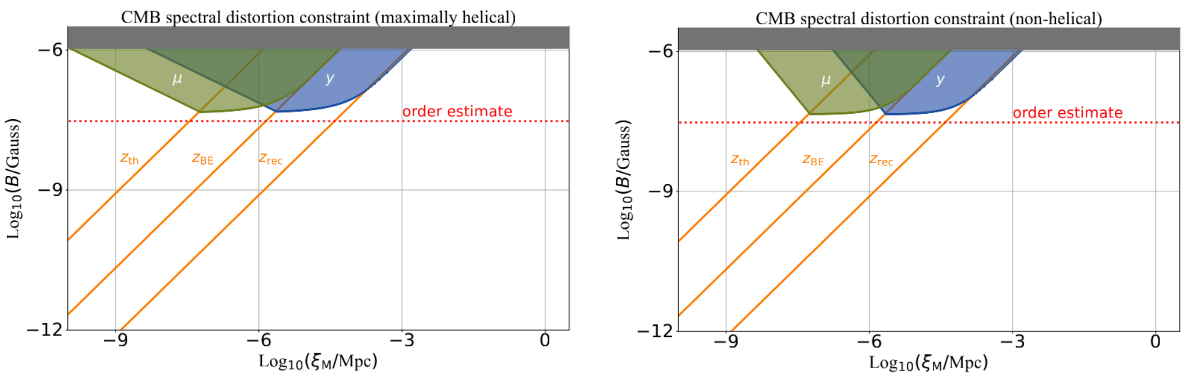
[Lucca 2023]

Order estimate

$$\mu \simeq \frac{B^2_{z=2\times10^6}/2}{\rho_\gamma} < 9 \times 10^{-5}, \qquad y \simeq \frac{B^2_{z=1\times10^4}/2}{\rho_\gamma} < 1.5 \times 10^{-5}$$

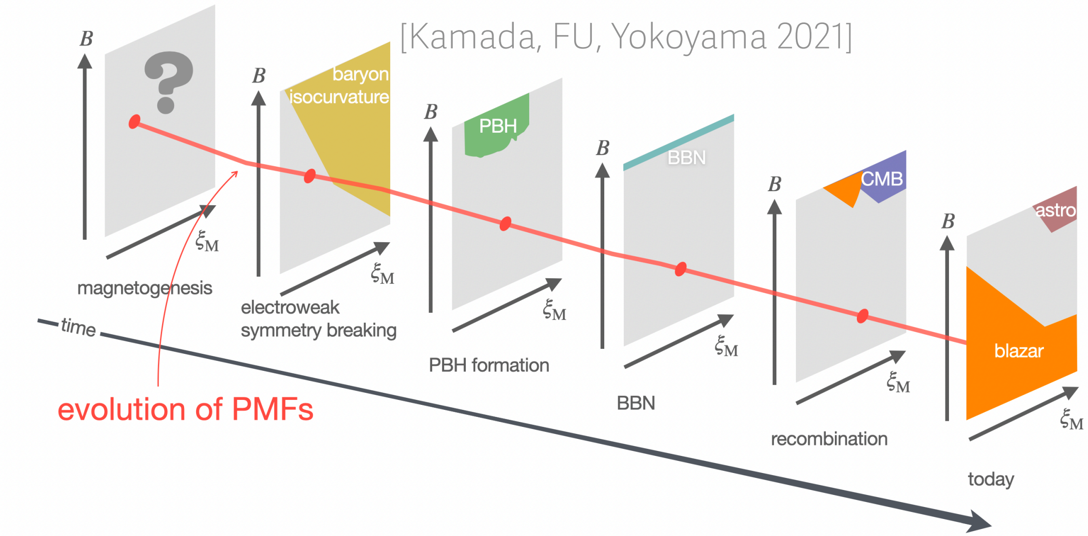
[Fixen+ 1996]

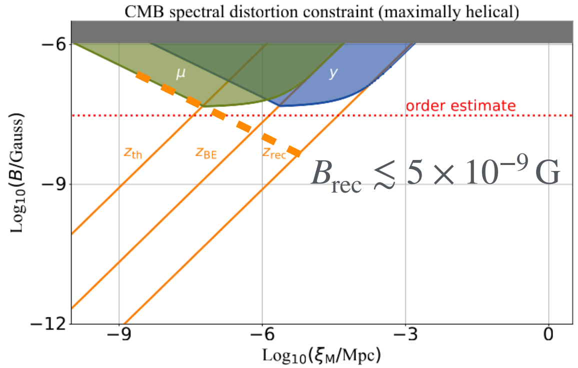
$$\rightarrow B \lesssim 3 \times 10^{-8}\,\mathrm{G}$$

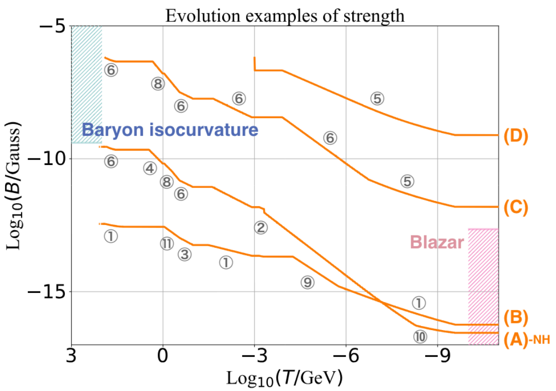


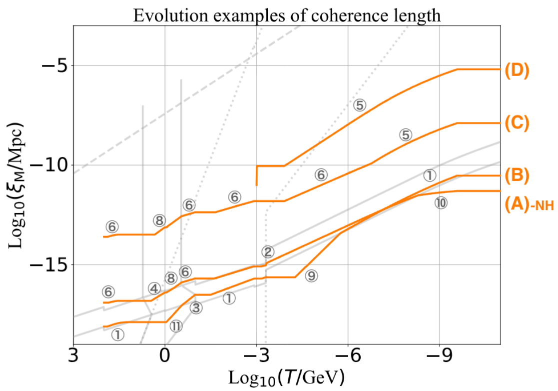
[FU, Kamada, Tashiro, ongoing]

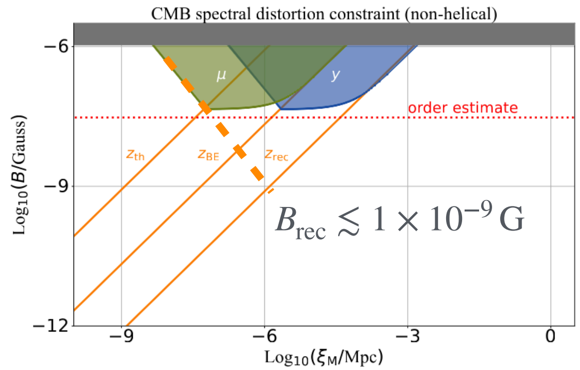
Once we assume relevant decay timescales, . . .











[FU, Kamada, Tashiro, ongoing]

[FU, Fujiwara, Kamada, Yokoyama 2024]

Outline

introduction

Hosking integral

CMB distortion constraint

Summary

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- PMFs evolution: governed by MHD
- Hosking integral
  - fluctuation of magnetic helicity
  - explains non-helical inverse transfer
  - constrains evollution of $B(\tau)$ and $\xi_{\mathrm{M}}(\tau)$
- Once we identify decay timescales, we can analytically describe PMF decay.

  magnetic reconnection timescale?
- application example: CMB distortion constraint on PMF $B_{\mathrm{rec}} \lesssim 5 \times 10^{-9}\,\mathrm{G}$