Introduction to the Feynman-Diagram Gauge

Kaoru Hagiwara

@ YITP meeting

on Aug. 23, 2024

based on works with

V. Barger, J.M Chen, J. Kanzaki, O. Mattelaer, K. Mawatari, Y. Yamada, Y.-J. Zheng

OUTLINE

Definition, or derivation of the Feynman-Diagram (FD) gauge

- By using the BRST invariance of S-matrix elements with one off-shell gauge boson

  QED/QCD : Feynman gauge ⇒ FD gauge   2003.03003 (KH-Kanzaki-Mawatari)

  EW (SM) : Unitary gauge ⇒ FD gauge   2203.10440 (Chen-KH-Kanzaki-Mawatari)

- As Green's function of the EOM (LC ⇒ FD)   2211.14562 (Chen-KH-Kanzaki-Mawatari-Zhong)

Results (what we have learned & what will be achieved soon)

- Tree
  - QED/QCD : MadGraph F vs FD gauge (2003.03003)
  - SM : MadGraph U vs FD gauge (2203.10440) LC vs FD gauge (2211.14562)
  - BSM : MadGraph F-gauge FeynRules ⇒ UFO ⇒ FD gauge amp. 2405.01256 (KH-Kanzaki-Mattelaer-Mawatari-Zheng)
  - Analytic : $e^+e^- \to W^+W^-$ (SM) 2406.08869 (Furusato-Mawatari-Suzuki-Zheng)

    $ub \to dtH$ (SMEFT) 2408.----- (Barger-KH-Zheng)
- Loop
  - QCD : asymptotic freedom from $q\bar{q} \to q'\bar{q}'$ @ 1-loop 2407.11527 (KH-Mawatari-Y.Yamada-Zheng)
  - SM : in progress

Why do FD gauge amplitudes allow physical interpretation for each Feynman Diagram ?

BRST identities for S-matrix elements with off-shell gauge boson:

Perturbative QFT : $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}_{INT}$ ⇒ solve EOM of $\mathcal{L}^{(0)}$

⇒ $\phi(x) \sim \int \frac{d^3p}{(2\pi)^3 2E} \left(e^{-ipx} a_p + e^{ipx} b_p^{+} \right)_{p^0 = E = \sqrt{\vec{p}^2 + m^2}}$

⇒ calculate $S_{fi} = \langle f | T e^{i\int d^4x \mathcal{L}_{Int}} | i \rangle$

perturbatively by using the Green's fuction method.

Gauge theories: $\mathcal{L} = \mathcal{L}^{(0)} + \mathcal{L}_{INT}$ is gauge inv. ⇒ EOM of $\mathcal{L}^{(0)}$ has no inversion

⇒ $\mathcal{L} + \mathcal{L}_{G.F.} = \mathcal{L}^{(0)} + \mathcal{L}_{G.F.} + \mathcal{L}_{INT}$ ⇒ EOM of $\mathcal{L}^{(0)} + \mathcal{L}_{G.F.}$ $\quad O_{\mu\nu} A^{a\nu} = 0$

⇒ $G_{\mu\nu} = (O_{\mu\nu})^{-1}$ gives the Green's function

⇒ calculate $S_{fi} = \langle f | T e^{i\int d^4x \mathcal{L}_{INT}} | i \rangle$ perturbatively

∵ $\mathcal{L} + \mathcal{L}_{G.F.}$ is not gauge invariant, but it can be made invariant under a global U(1) symmetry.

once the FP ghost term is added : $\mathcal{L} + \mathcal{L}_{G.F.} + \mathcal{L}_{F.P.}$ is BRST invariant

$$\mathcal{L}^{(0)} + \mathcal{L}_{G.F.} + \mathcal{L}_{F.P} = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu}) - \frac{1}{2\xi}(F^a[A])^2 + i\bar{c}^a \frac{\delta F^a[A]}{\delta A^b_\mu}(D_\mu c)^b$$

BRST transformation : $\bar{c}^a \xrightarrow[Q_{BRST}]{} F^a(A) \xrightarrow[Q_{BRST}]{} 0. \quad (Q^2_{BRST} = 0)$

↑ Fermionic

BRST invariance of $\mathcal{L} + \mathcal{L}_{G.F.} + \mathcal{L}_{F.P.}$ allows us to request that physical states be invariant under BRST transformation : $e^{iQ_{BRST}\eta} |phys\rangle = |phys\rangle$ or $Q_{BRST}|phys\rangle = 0$

$\Rightarrow$ S matrix elements $S_{fi} = \langle f | T e^{i\int d^4x \mathcal{L}_{INT}} | i \rangle$ are calculated perturbatively by using the Green's functions of EOM of $\mathcal{L}^{(0)} + \mathcal{L}_{G.F.} + \mathcal{L}_{F.P.}$.

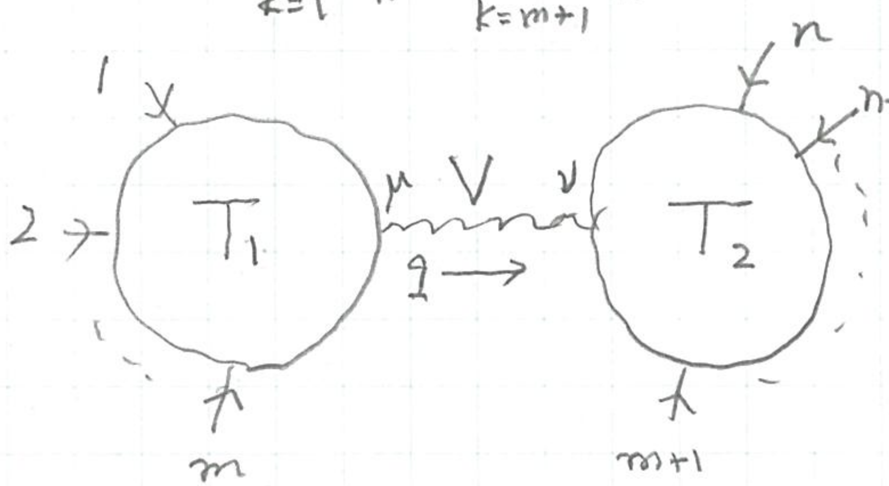
$\Rightarrow$ Because $Q_{BRST}|phys\rangle = 0$, matrix elements of $\langle phys| F^a[A] |phys'\rangle = \langle phys| \{Q_B, \bar{c}^a\} |phys'\rangle = 0$ at any oreder of perturbation theory.

$\Rightarrow$ Covariant R_ξ gauge : $F^a[A] = \partial^\mu A^a_\mu - \xi m_V \pi^a$ (π^a : Goldstone boson of $A^{a\mu}$)

$$\Rightarrow \langle phys| (\partial^\mu A^a_\mu - \xi m_V \pi^a) |phys'\rangle = 0$$

If the full S-matrix element with n-external on-shell particles has a gauge boson propator of momentum q^μ

$q^\mu = \sum_{k=1}^{m} p_k^\mu - \sum_{k=m+1}^{n} p_k^\mu$ (all momenta are flowing-in)



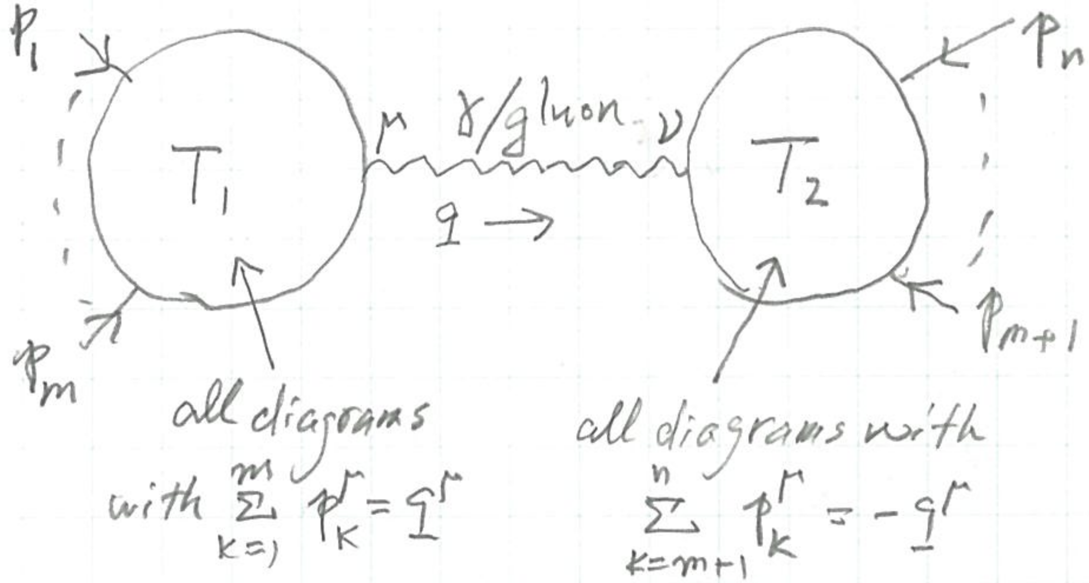
gauge-boson propagator

$$\underbrace{T(p_1, \cdots, p_n)}_{} = T_1^\mu(p_1, \cdots, p_m)\, G_V(q)_{\mu\nu}\, T_2^\nu(p_{m+1}, \cdots, p_n)$$

Sum of all the digrams which have a V propagator with q^μ.

$$\Rightarrow \begin{cases} q_\mu T_1^\mu = i m_V T_1^{\pi_V} \\ q_\mu T_2^\mu = -i m_V T_2^{\pi_V} \end{cases} \quad \text{when } q^0 > 0.$$

QED & QCD : Feynman gauge $\Rightarrow$ FD gauge   2003.03003 (KH-Kanzaki-Mawatari)



$$= T(p_1,\cdots,p_n) = T_1^\mu(p_1,\cdots,p_m)\,\frac{P^{\xi}_{\mu\nu}(q)}{q^2+i\epsilon}\,T_2^\nu(p_{m+1},\cdots,p_n)$$

$$P^{\xi}_{\mu\nu}(q) = -g_{\mu\nu} + (1-\xi)\frac{q_\mu q_\nu}{q^2}$$

$$\underbrace{q_\mu T_1^\mu = q_\nu T_2^\nu = 0}_{\text{(BRST)}} \Rightarrow \xi\text{-independence of } T.$$

$$P^{\xi}_{\mu\nu}(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - \xi\frac{q_\mu q_\nu}{q^2}$$

$$= \sum_{h=\pm1} \epsilon^*_\mu(q,h)\epsilon_\nu(q,h) + \mathrm{sgn}(q^2)\,\epsilon^*_\mu(q,0)\epsilon_\nu(q,0) - \xi\frac{q_\mu q_\nu}{q^2}$$

$$q^\mu = (q^0, 0, 0, |\vec{q}|)$$

$$q^0 > 0, \quad Q = \sqrt{|q^2|}$$

$$\epsilon^\mu(q,\pm1) = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$$

$$\epsilon^\mu(q,0) = \begin{cases} (\sinh y, 0, 0, \cosh y) & \text{when } q^2>0 \ : \ q^\mu = Q(\cosh y, 0, 0, \sinh y) \\ (\cosh y, 0, 0, \sinh y) & \text{when } q^2<0 \ : \ q^\mu = Q(\sinh y, 0, 0, \cosh y) \end{cases}$$

We introduce $\tilde{\epsilon}^\mu(q,0) = \epsilon^\mu(q,0) - \frac{q^\mu}{Q} = \left\{\begin{matrix} e^{-y}(-1,0,0,1) \\ e^{-y}(1,0,0,-1) \end{matrix}\right\} = -\mathrm{sgn}(q^2)\frac{Q\,n^\mu}{n\cdot q}$ when $q^0>0$

and define

$$P^{FD}_{\mu\nu}(q) = \sum_{h=\pm1}\epsilon^*_\mu(q,h)\epsilon_\nu(q,h) + \underbrace{\mathrm{sgn}(q^2)\,\tilde{\epsilon}_\mu(q,0)\tilde{\epsilon}_\nu(q,0)}_{= q^2 n^\mu n^\nu/(n\cdot q)^2} = -g_{\mu\nu} + \frac{q_\mu n_\nu + n_\mu q_\nu}{n\cdot q} \quad \text{with } n^\mu = \left(\mathrm{sgn}(q^0), -\frac{\vec{q}}{|\vec{q}|}\right)$$

BRST identities $q_\mu T_1^\mu = q_\nu T_2^\nu = 0 \Rightarrow T = T_1^\mu \dfrac{P^{\xi}_{\mu\nu}(q)}{q^2+i\epsilon} T_2^\nu = T_1^\mu \dfrac{P^{FD}_{\mu\nu}(q)}{q^2+i\epsilon} T_2^\nu$

[2003.03003] replaced all $P^{\xi=1}_{\mu\nu}(q)$ in the HELicity Amp. Subroutines (HELAS) by $P^{FD}_{\mu\nu}(q)$

and demonstrated that we obtain the same gauge-invariant S-matrix elements for a few $2\to3$ processes including $qq'\to qq'g$ & $gg\to ggg$ ($n=5$ external particles). --- (In QED & QCD, each diagram is gauge invariant for $2\to2$ processes.)

If N Feynman diagrams contribute, gauge-invariance tells

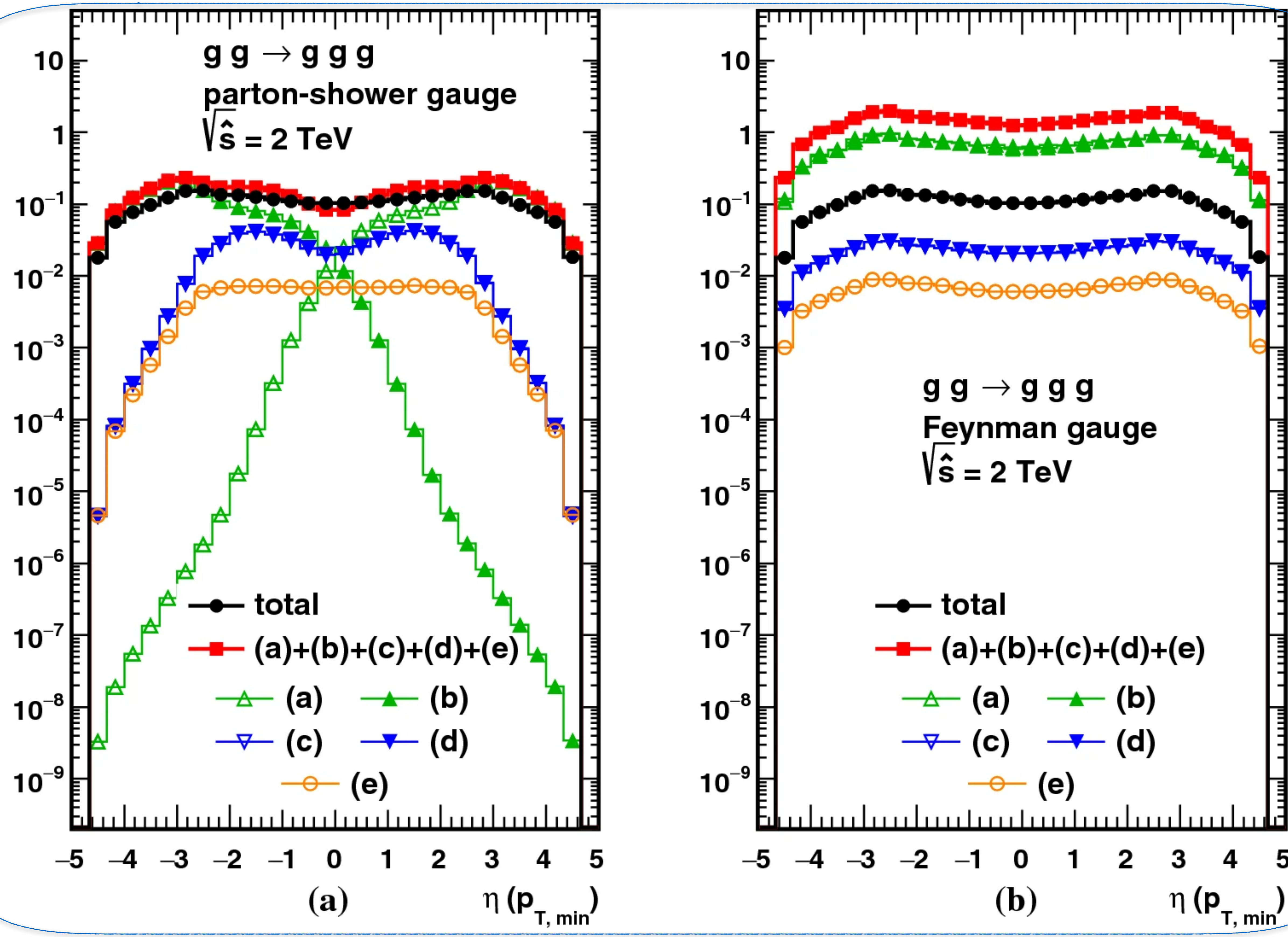
$$M = \sum_{\alpha=1}^{N} M_\alpha^{FD} = \sum_{\alpha=1}^{N} M_\alpha^{\xi=1} = \sum_{\alpha=1}^{N} M^{\xi}$$

Degree of cancellation among diagrams is studied by $R = \dfrac{\sum_{\alpha=1}^{N} |M_\alpha^{(\mathrm{gauge})}|^2}{|\sum_{\alpha=1}^{N} M_\alpha|^2}$ ← gauge-dependent. ← gauge-invariant.

$\Rightarrow \begin{cases} R \text{ is often } \gg 1 \text{ in the Feynman gauge} \\ R \text{ is always } O(1) \text{ in the FD gauge} \end{cases}$ $\begin{matrix} \Rightarrow \text{subtle gauge cancellation among diagrams} \\ \Rightarrow \text{absence of} \quad \text{"} \end{matrix}$

In the kinematical region where $\frac{1}{|q^2|}$ is large, the diagrams with $G^{FD}_{\mu\nu}(q)$ dominates the full amplitudes in the FD gauge : $M = \sum_{\alpha=1}^{N} M_\alpha \approx M_k^{FD}$ (with $\underset{q\to}{\sim\sim\sim}$) $\Rightarrow |M_k^{FD}|^2 \approx |M|^2 \Rightarrow R \approx 1$

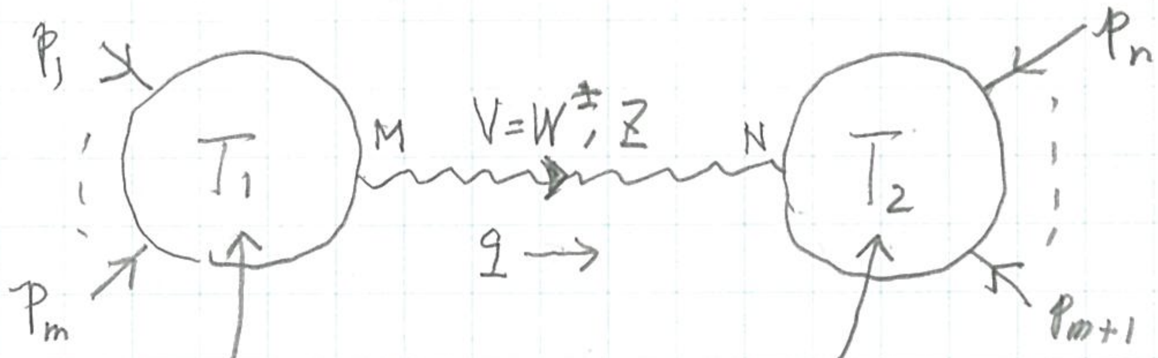
show Fig. 5 of [2003.03003]



Distributions of pseudorapidity of the final-state particle with minimum transverse momentum for the process $gg \to ggg$ in the PS gauge (**A**) and in the Feynman gauge (**B**). A line with filled circles denotes the total distribution, while lines with triangles (a–d) and open circles (e) show the distribution of the squared amplitudes of each type of the Feynman diagrams depicted in Fig. 4. A line with filled squares presents the distribution of the sum of the squared amplitudes of each diagram **[arXiv:2003.03003, EPJC2020, KH, Kanzaki & Mawatari]**

240821-7

EW ($W^\pm$ & Z of the SM) : $U_{\text{gauge}} \Rightarrow FD_{\text{gauge}}$ [2203.10440] Chen-KH-Kanzaki-Mawatari



all diagrams with $\sum_{k=1}^{m} p_k^\mu = q^\mu$

all diagrams with $\sum_{k=m+1}^{n} p_k^\mu = -q^\mu$

$$= T(p_1, \cdots, p_n) = T_1^M \frac{P_V^U(q)_{MN}}{q^2 - m_V^2} T_2^N \qquad M = \mu, 4 \quad N = \nu, 4$$

$$P_V^U(q)_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_V^2} \; ; \; P_V^U(q)_{\mu 4} = P_V^U(q)_{4\nu} = P_V^U(q)_{44} = 0$$

$$\text{BRST}: \begin{cases} q_\mu T_1^\mu = i m_V T_1^{\pi_V} \\ q_\nu T_2^\nu = -i m_V T_2^{\pi_V} \end{cases} \quad \text{when } q^0 > 0$$

$$\begin{aligned} P_V^U(q)_{\mu\nu} &= -g_{\mu\nu} + \frac{q_\mu q_\nu}{m_V^2} \\ &= -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} + \left(\frac{1}{m_V^2} - \frac{1}{q^2}\right) q_\mu q_\nu \\ &= \sum_{h=\pm 1} \epsilon_\mu^*(q,h)\,\epsilon_\nu(q,h) + \mathrm{sgn}(q^2)\,\epsilon_\mu^*(q,0)\,\epsilon_\nu(q,0) + \frac{q^2 - m_V^2}{m_V^2}\frac{q_\mu q_\nu}{q^2} \\ &= \sum_{h=\pm 1} \epsilon_\mu^*(q,h)\,\epsilon_\nu(q,h) + \mathrm{sgn}(q^2)\left[\left(\tilde{\epsilon}_\mu(q,0) + \frac{q_\mu}{Q}\right)\left(\tilde{\epsilon}_\nu(q,0) + \frac{q_\nu}{Q}\right) + \frac{q^2 - m_V^2}{m_V^2}\frac{q_\mu q_\nu}{Q^2}\right] \end{aligned}$$

Apply BRST identities

The same as QED & QCD

mixing

$$T = T_1^\mu \frac{P_V^U(q)_{\mu\nu}}{q^2 - m_V^2 + i\epsilon} T_2^\nu = T_1^M \frac{P_V^{FD}(q)_{MN}}{q^2 - m_V^2 + i\epsilon} T_2^N \; : \; (T_1^\mu, T_1^4) \begin{pmatrix} -g_{\mu\nu} + \frac{q_\mu n_\nu + n_\mu q_\nu}{n \cdot q} & i m_V \frac{n_\mu}{n \cdot q} \\ -i m_V \frac{n_\nu}{n \cdot q} & 1 \end{pmatrix} \begin{pmatrix} T_2^\nu \\ T_2^4 \end{pmatrix}$$

$T_1^4 = T_1^{\pi_V}$

$T_2^4 = T_2^{\pi_V}$

↑ mixing

↑ G.B. (π_V)

In [2203.10440] all the HELAS codes in the U gauge have been changed to FD gauge.

- $P_V^U(q)_{\mu\nu}$ is 4x4 matrix $\Rightarrow$ $P_V^{FD}(q)_{MN}$ is 5x5 matrix
- All the SM G.B. (π_V) vertices are coded from scratch because they are not used in U gauge HELAS.
- BRST identities are used at all steps to check the HELAS codes & the Feynman rules.
- We assign $(V^\mu, \pi_V) = (V^\mu, V^4) = V^M$ as the same particle when generating Feynman diagrams.

We spent 2 years to code new HELAS for the SM, mainly because we don't have super-graduate students!

like Hitoshi Murayama & Isamu Watanabe who coded the original HELAS.

Results reported for

$W^+W^- \to W^+W^-, ZZ$; $ZZ \to ZZ$

$e^+e^- \to W^+W^- \to \mu^+\nu_\mu e^-\bar{\nu}_e$ etc

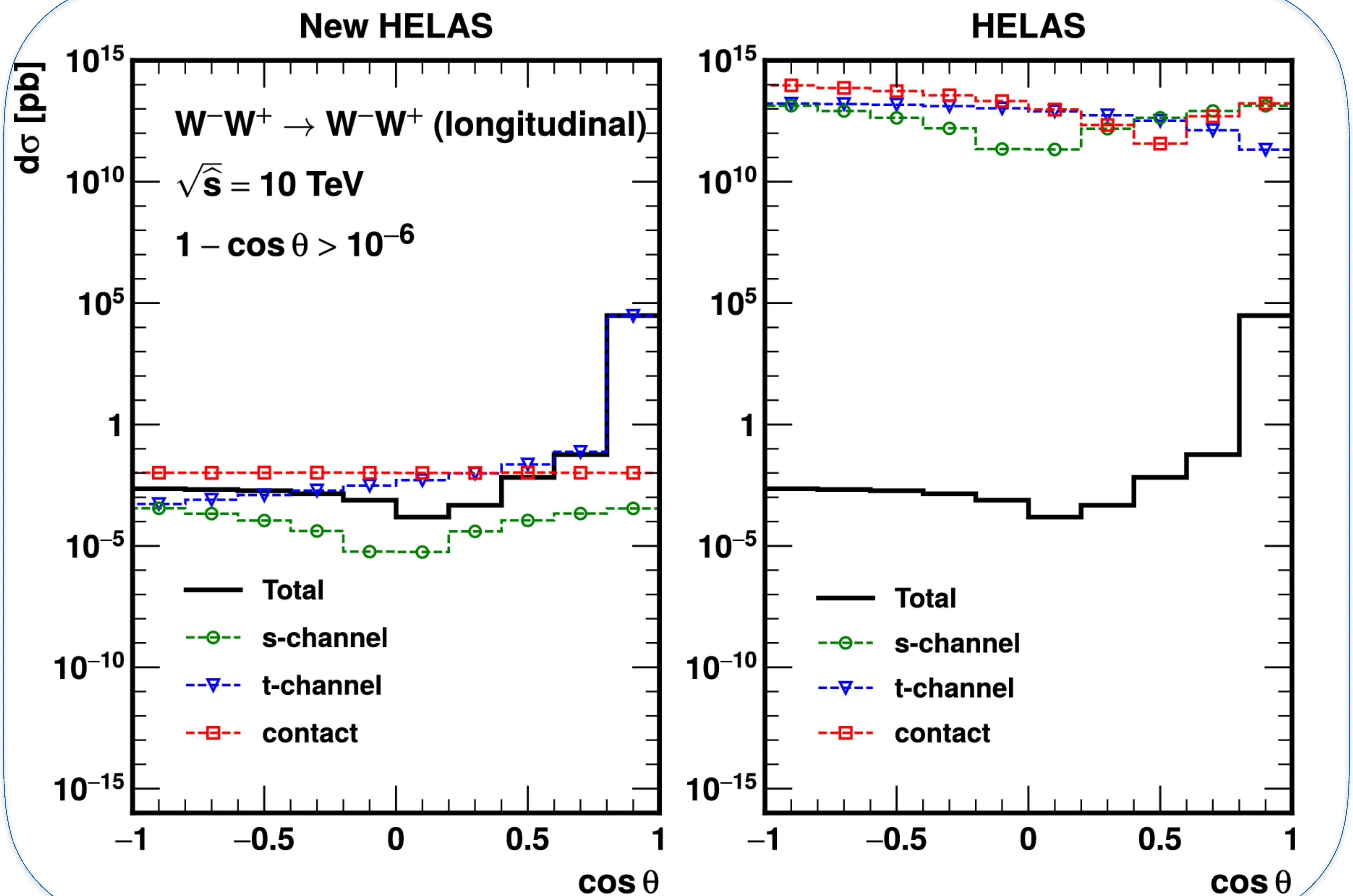
$\ell\bar{\ell}' \to \nu_\ell\bar{\nu}_{\ell'} W^+W^-, \nu_\ell\bar{\nu}_{\ell'} ZZ$

$\ell\bar{\ell}' \to \ell\bar{\ell}' W^+W^-, \ell\bar{\ell}' ZZ$

$\Rightarrow$ show [2203.10440] Fig. 7 for $W_L^-W_L^+ \to W_L^-W_L^+$

$\Rightarrow$
- absence of subtle cancellation among diagrams in the FD gauge ($R \sim O(1)$ always)
- In the kinematical region where $\frac{1}{|q^2 - m_V^2|}$ is large, diagram with $G_V^{FD}(q)_{MN}$ dominates the full amplitude.

  $\Rightarrow$ EW parton shower
- G.B.E.T. ($V_L \to \pi_V$) is manifest, confirming earlier results by
  - Wulzer 1309.6055
  - Chen-Han-Tweedie 1611.00788



[arXiv:2203.10440, EPJC2023,Chen, KH, Kanzaki & Mawatari]

Light Cone gauge ⇒ 5×5 EOM ⇒ 5×5 Green's function ⇒ FD gauge propagator

[2211.14562] Chen-KH-Kanzaki-Mawatari-Zheng

$$\mathcal{L}^{(0)} = -\frac{1}{4}(\partial^\mu V^\nu - \partial^\nu V^\mu)^2 + \frac{1}{2}m_V^2 V^\mu V_\mu + \frac{1}{2}(\partial_\mu \pi_V)^2 - m_V(\partial^\mu V_\mu)\pi_V$$

$$+\mathcal{L}^{LC}_{G.F.} = -\frac{1}{2\xi}(n^\mu V_\mu)^2$$

EOM:
$$\begin{bmatrix} (\partial^2+m_V^2)g_{\mu\nu} - \partial_\mu\partial_\nu - \frac{n_\mu n_\nu}{\xi} & m_V\partial_\mu \\ -m_V\partial_\nu & -\partial^2 \end{bmatrix}\begin{bmatrix} V^\nu(x) \\ \pi_V(x)\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} (-q^2+m_V^2)g_{\mu\nu} + q_\mu q_\nu - \frac{n_\mu n_\nu}{\xi} & -im q_\mu \\ +im q_\nu & q^2 \end{bmatrix}\begin{bmatrix} \tilde V^\nu(q) \\ \tilde\pi_V(q)\end{bmatrix} = 0$$

$$\parallel$$

$$O_{MN}\tilde V^N(q) = 0 \quad \text{with} \quad \tilde V^4 = \tilde\pi_V$$

Green's function : $G^{RM} O_{MN} = \delta^R{}_N \quad (m = m_V)$

$$G^{RM} = \frac{1}{q^2-m^2}\begin{bmatrix} -g^{\rho\mu} + \frac{q^\rho n^\mu + n^\rho q^\mu}{n\cdot q} + \xi(q^2-m^2)\frac{q^\rho q^\mu}{(n\cdot q)^2} & i\frac{m n^\rho}{n\cdot q} + i\xi(q^2-m^2)\frac{m q^\rho}{(n\cdot q)^2} \\ -i\frac{m n^\mu}{n\cdot q} - i\xi(q^2-m^2)\frac{m q^\mu}{(n\cdot q)^2} & 1+\xi(q^2-m^2)\frac{m^2}{(n\cdot q)^2} \end{bmatrix}$$

$$\Rightarrow \xi = 0 \quad \& \quad n^\mu = \left(\mathrm{sgn}(q^0), -\frac{\vec q}{|\vec q|}\right) \Rightarrow G_V^{FD}(q)^{RM}$$

$$+\mathcal{L}^{R_\xi}_{G.F.} = -\frac{1}{2\xi}(\partial^\mu V_\mu - \xi m_V \pi_V)^2$$
$$= -\frac{1}{2\xi}(\partial^\mu V_\mu)^2 + m_V(\partial^\mu V_\mu)\pi_V - \frac{1}{2}\xi m_V^2 \pi_V^2$$

EOM:
$$\begin{cases} [(\partial^2+m_V^2)g_{\mu\nu} - \partial_\mu\partial_\nu - \frac{1}{\xi}\partial^2 g_{\mu\nu}]\, V^\nu(x) = 0 \\ [\partial^2 + \xi m_V^2]\, \pi_V(x) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} [(-q^2+m_V^2)g_{\mu\nu} + q_\mu q_\nu + \frac{1}{\xi}q^2 g_{\mu\nu}]\, \tilde V^\nu(q) = 0 \\ [-q^2 + \xi m_V^2]\, \tilde\pi_V(q) = 0 \end{cases}$$

Green's functions :

$$\begin{cases} G_V^{R_\xi}(q)^{\mu\nu} = \dfrac{-g^{\mu\nu} + \frac{q^\mu q^\nu}{m^2}}{q^2-m^2} - \dfrac{\frac{q^\mu q^\nu}{m^2}}{q^2-\xi m^2} \\ G_{\pi_V}^{R_\xi}(q) = \dfrac{1}{q^2-\xi m^2} \end{cases}$$

In a recent paper [2408.------] Barger-KH-Zheng, in which we calculate the amplitudes for single top + H production process, $ub \to dtH$, in SMEFT that gives CPV top Yukawa, we introduce 5-dim momentum $q^M = (q^\mu, im)$ and the metric $g^{MN} = g_{MN} = (1,-1,-1,-1,-1)$, which turn out to be very useful in organizing the weak boson propagators in all gauges.

$$G_V^{FD}(q)_{MN} = \frac{1}{q^2-m_V^2}\left[-g_{MN} + \frac{q_M n_N + n_M q_N^*}{n\cdot q}\right] \qquad \text{with } n^M = (n^\mu, 0),\ q^2-m_V^2 = q^M q_M^*$$

By noting that the BRST identies for the sub-amplitudes connected by a gauge boson propagator with q^μ

$$T(p_1,\cdots,p_n) = T_1(p_1,\cdots,p_m)^M G_V(q)_{MN} T_2^N(p_{m+1},\cdots,p_n) \quad \text{can be expressed as} \quad q_M T_1^M = q_N^* T_2^N = 0 \text{ when } q^0>0$$

we can immediately tell

$$T = T_1^M G_V^{FD}(q)_{MN} T_2^N = T_1^M \frac{-g_{MN}}{q^2-m_V^2} T_2^N \quad \Rightarrow \frac{-g_{MN}}{q^2-m_V^2} \text{ is the Feynman gauge propagator!}$$

In the same token, the R_ξ gauge propagator can be expressed as

$$G_V^{R_\xi}(q)_{MN} = G_V^U(q)_{\mu\nu} - \frac{1}{q^2-\xi m_V^2}\,\frac{q_\mu q_N^* + q_M q_\nu - q_M q_N^*}{m_V^2} \quad \Rightarrow \text{cancellation of } \xi m_V^2 \text{ pole (Kugo-Ojima) is manifest!}$$

$$\Rightarrow T = T_1^M G_V^{R_\xi}(q)_{MN} T_2^N = T_1^\mu G_V^U(q)_{\mu\nu} T_2^\nu = T_1^M G_V^{FD}(q)_{MN} T_2^N \quad ///$$

Together with Barger & Zheng, we have studied consequences of CPV top Y-kawa, by using the phenomenological Lagrangian, $\mathcal{L}_{ttH} = -\frac{y}{\sqrt{2}}\bar{t}(\cos\xi + i\sin\xi\,\gamma_5)t$ [1807.00281], [1912.11795]. Recently, we studied $\mu\bar{\mu} \to \nu\bar{\nu} t\bar{t} H$ by using the above coupling, and found the cross section grows with energies when $y \neq y_{SM}$ or $\xi \neq 0$ [2310.10852].

In order to understand the origin of the apparently unitarity violating amplitudes, we employed SMEFT operator; $\mathcal{L} = \mathcal{L}_{SM} + \left\{ \frac{c}{\Lambda^2} Q^\dagger \phi\, t_R \left(\phi^\dagger \phi - \frac{v^2}{2}\right) + h.c. \right\}$; $Q = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$, $\phi = \begin{pmatrix} \frac{v + i\pi^0}{\sqrt{2}} \\ i\pi^- \end{pmatrix}$

which gives manifestly gauge invariant form of the complex Yukawa when $\frac{c}{\Lambda^2} = \frac{y_{SM} - y e^{i\xi}}{v^2}$

The $\frac{y_{SM}}{v^2}$ term of the dim-6 operator cancels the SM top Yukawa in $\mathcal{L}_{SM}$ and replace y_{SM} by $y e^{i\xi}$, giving the phenomenological $\mathcal{L}_{ttH}$. There ard additional couplings in the operator, which are all proportional to $(y_{SM} - y e^{i\xi})$. We identify the origin of the rising cross section as a consequence of dim-6 $\pi^+\pi^- t\bar{t}H$ coupling, and used the G.B.E.T. to show $\hat{\sigma}(W_L^- W_L^+ \to t\bar{t}H) \approx \hat{\sigma}(\pi^+\pi^- \to t\bar{t}H)$ at high energies.

show [2310.10852] Fig. 3

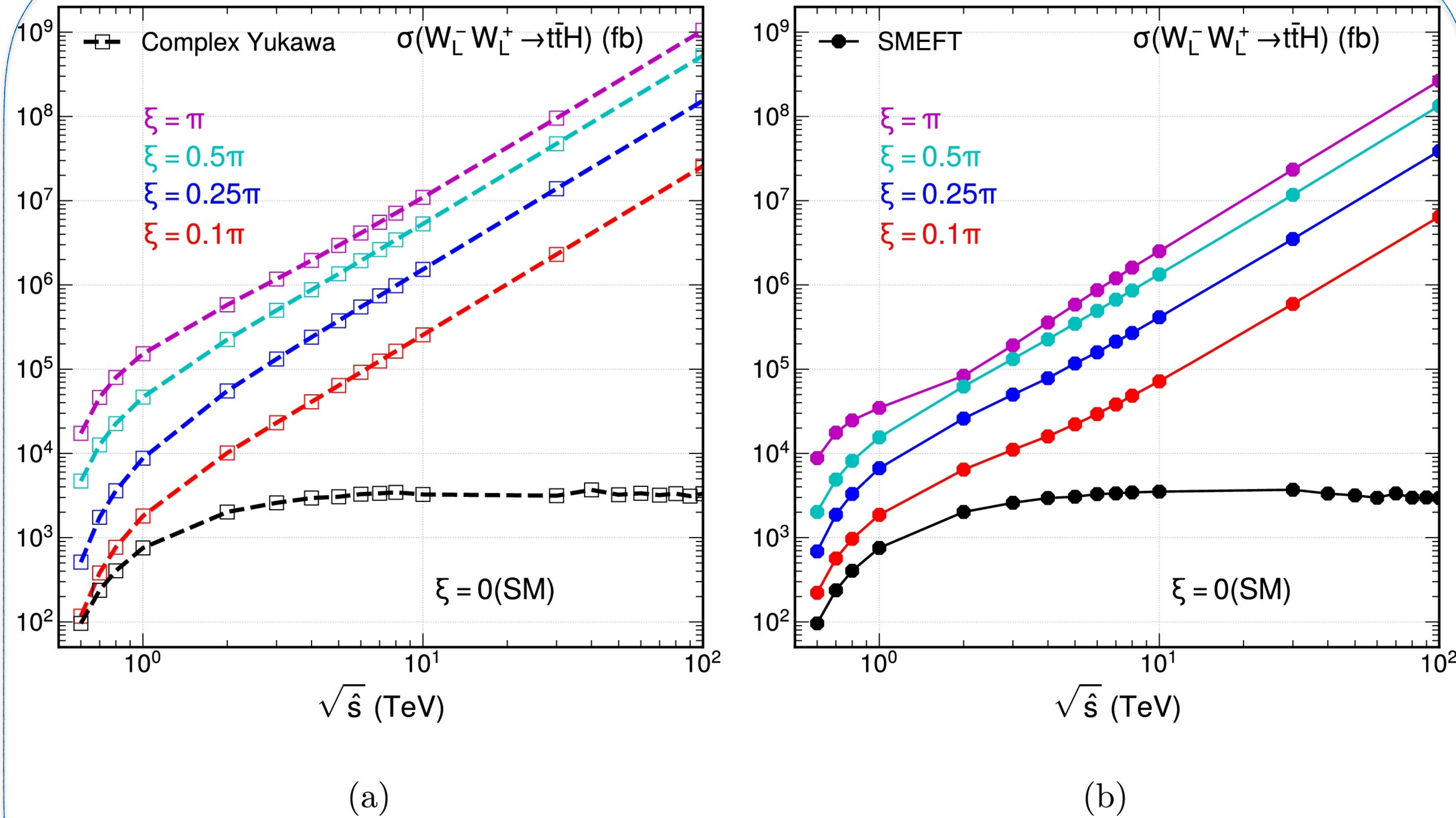


Fig. 3. $W_L^- W_L^+ \to t\bar{t}H$ cross section vs. the colliding W^-W^+ energy $\sqrt{\hat{s}}$: (a) complex Yukawa model and (b) SMEFT.

[arXiv:2310.10852, PLB2024, Barger, KH & Zheng]

It is desirable that we can obtain FD gauge amplitudes for BSM, because in the U gauge, only the total sum of all the diagrams is useful and we cannot tell which diagram or which kinematical region is sensitive to the BSM couplings. However,

- We coded all the FD gauge subroutines (new HELAS) by hand for the SM [2203.10440].
- Hand coding of FD gauge subroutines for BSM can be very hard, and many BSM's!

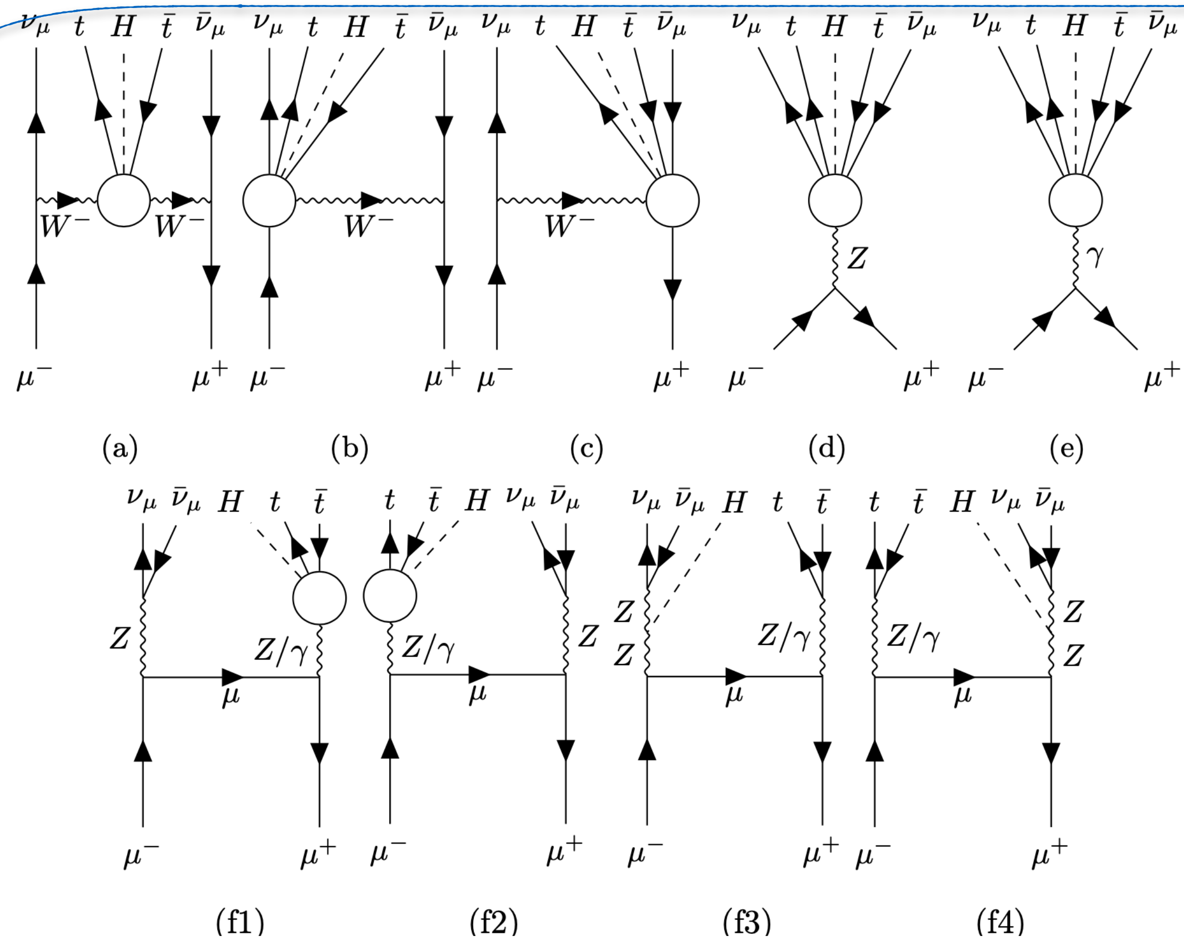
We find a solution to obtain all the FD gauge subroutines automatically by assembling the Feynman gauge vertices [2405.01256] KH-Kanzaki-Mattelaer-Mawatari-Zheng.

- All the gauge boson and the Goldstone boson couplings are gauge-independent for all BSM.
- Gauge-dependence arises only for the gauge-boson and the Goldstone boson propagators.
- All the vertex functions can be generated automatically for BSM in the Feynman gauge.

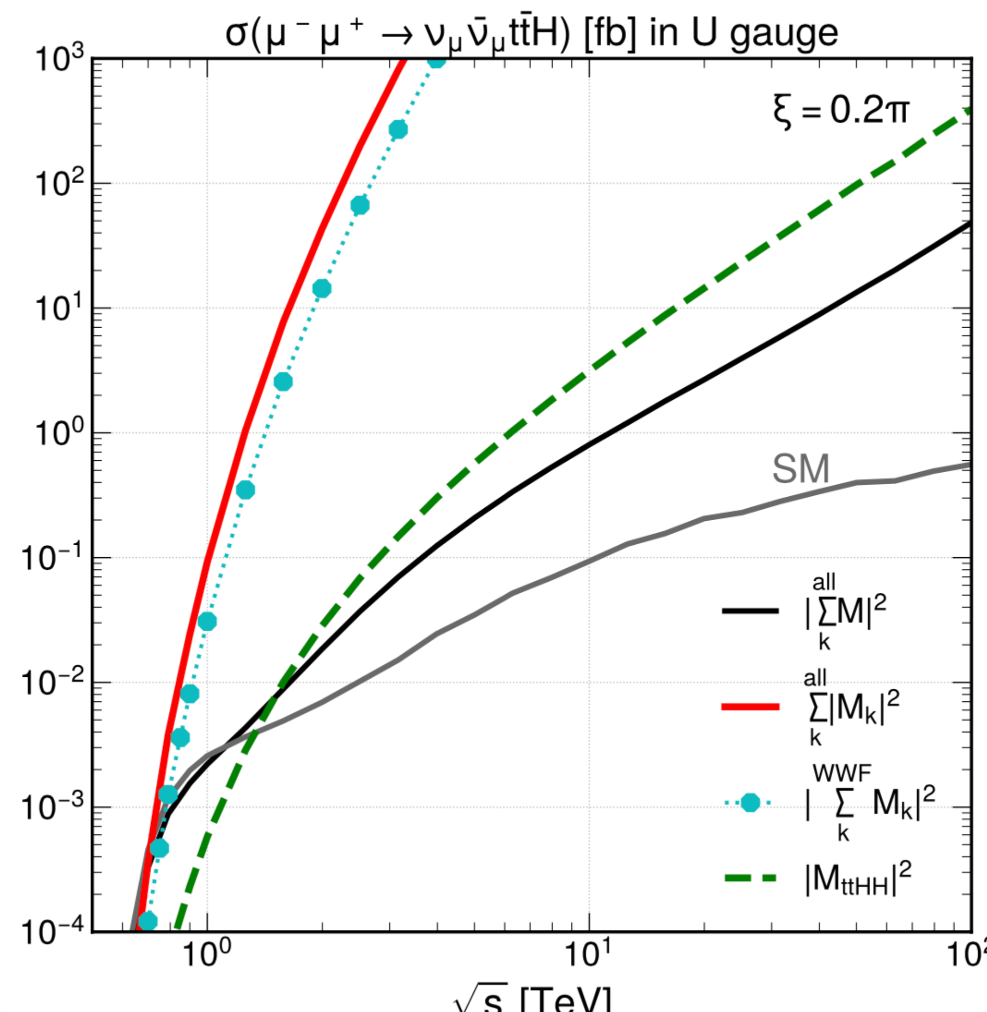
FeynRules ⇒ UFO

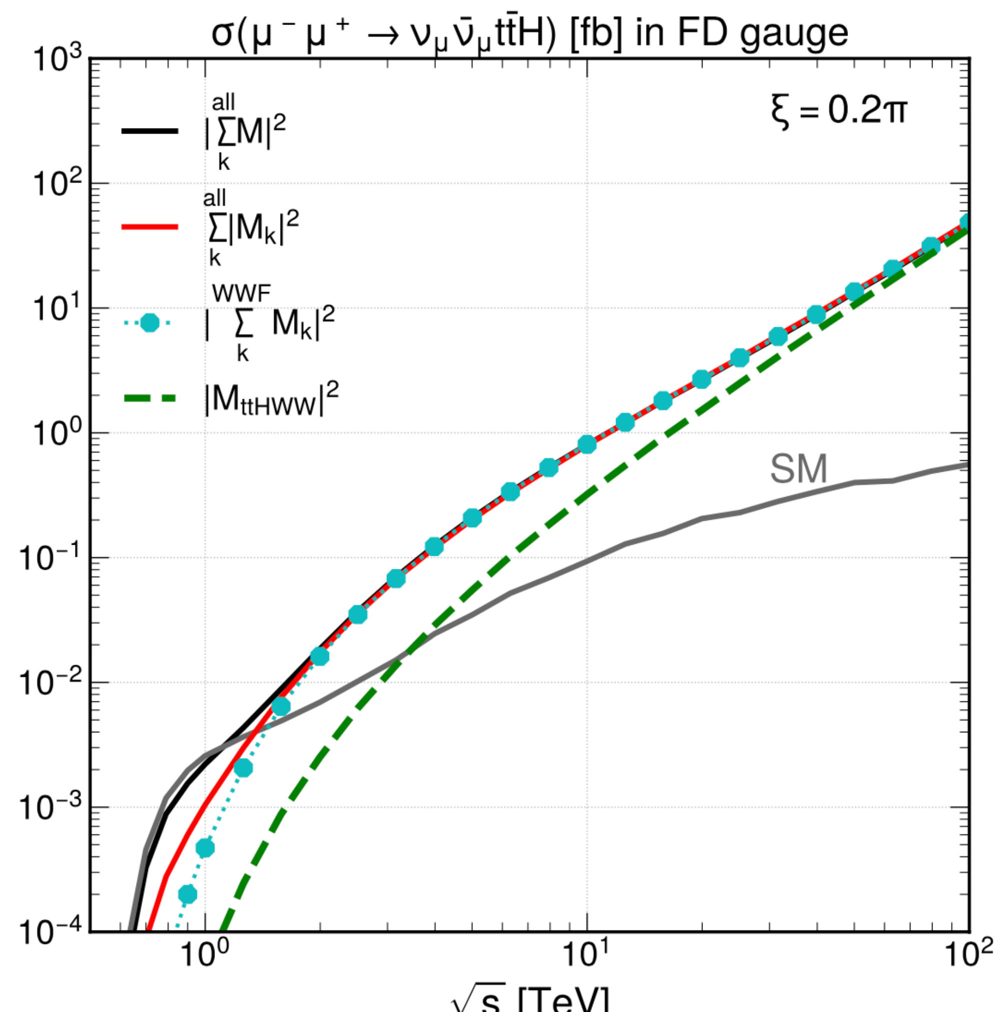
We created an option in MadGraph, which allows users to obtain FD gauge amplitudes for an arbitrary BSM, once it is given in the Feynman gauge in FeynRules. ⇒ tested by using $\mathcal{L}_{SM} + \left\{ \frac{y_{SM} - y e^{i\xi}}{v^2} Q^\dagger \phi\, t_R \left(\phi^\dagger\phi - \frac{v^2}{2}\right) + h.c. \right\}$

⇒ show [2405.01256] Fig. 1, Table 1, Fig. 4



No. of diagrams	SM U	SM FD	SMEFT U	SMEFT FD
a) WWF	19	21	20	30
b) $\mu^- W^+$	11	11	11	13
c) $W^- \mu^+$	11	11	11	13
d) anni-Z	24	24	25	36
e) anni-γ	8	8	8	10
f) anni-μ	14	14	14	16
Total	87	89	89	118



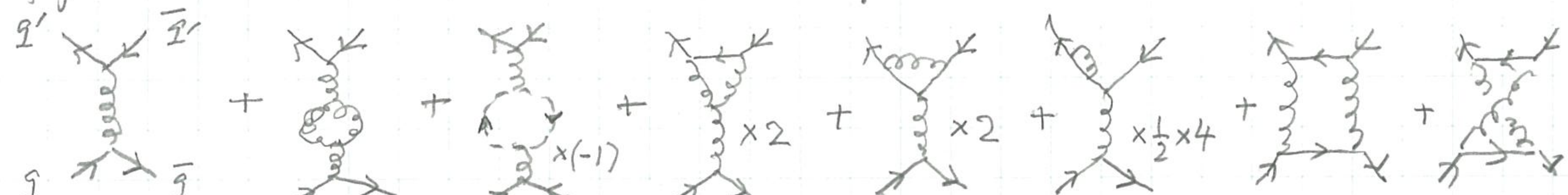


[arXiv:2405.01256, PRD2024, KH, Kanzaki, Mattelaer, Mawatari & Zheng]

QCD loops in the FD gauge [2407.11527] KH-Mawatari-Y.Yamada-Zheng

Because FD gauge amplitudes at the tree level have attractive properties which allow us to interpret each Feynman diagram contribution separately, I hoped that we might obtain more physical picture of the asymptotic freedom by performing loop integrals in the FD gauge. ⇒ I was wrong !

In this paper, we calculated $q\bar{q} \to q'\bar{q}'$ @ 1-loop

$M =$ 

(a) (b) (c) (d) (e) (f) (g) (h)

where all gluon propagators are in the FD gauge : $\sim\sim\sim = i\,G^{FD}_g(q)_{\mu\nu} = i\,\dfrac{-g_{\mu\nu} + \dfrac{q_\mu n_\nu + n_\mu q_\nu}{n\cdot q}}{q^2 + i\epsilon}$

A few results are worth reporting :

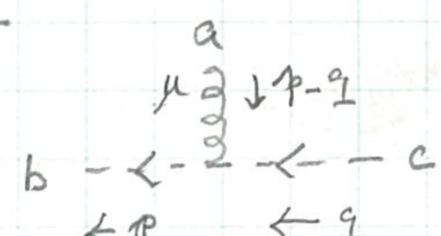
• non-decoupling of F.P. ghosts : $\mathcal{L}_{G.F.} = -\frac{1}{2\xi}\left(\hat{n}^\mu(\partial) A^a_\mu\right)^2 \Rightarrow \mathcal{L}_{F.P.} = i\,\bar{c}^a \hat{n}^\mu(\partial)(D_\mu c)^a$

$= i\,\bar{c}^a \hat{n}^\mu(\partial)\partial_\mu c^a$

$- i g f^{abc} \bar{c}^a \hat{n}^\mu(\partial) A^b_\mu c^c$

$\Rightarrow$ a - - ← - - b $= \dfrac{-\delta^{ab}}{n(q)\cdot q} = \dfrac{-\delta^{ab}}{|q^0| + |\vec{q}|}$

← q



$= -i g f^{abc} n^\mu(p)$

$$= f^{cad} f^{dbc} g^2 \int \frac{d^D k}{(2\pi)^D} \frac{n_\mu(k)\, n_\nu(k+q)}{(|k^0|+|\vec{k}|)(|k^0+q^0|+|\vec{k}+\vec{q}|)}$$

$\Rightarrow$ In the LC gauge, $n_\mu n_\nu$ doesn't depend on k, & hence decouple because of $n_\mu P^{LC}(q)_{\mu\nu} = 0$.

$\Rightarrow$ In the FD gauge, the integral is non-analytic (no Feynman param, etc.)

$\Rightarrow$ We evaluate the integrals for $(\mu\nu) = (0\,0), (i, 0), (0, j), (i, j)$ separately, and only for UV singular $\left(\frac{1}{D-4}\right)$ piece.

- We reproduce the known results (the QCD β function & finite terms) in three methods:

I: $P_g^{FD}(q)_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu n_\nu + n_\mu q_\nu}{n \cdot q}$ &

$g_{\mu\nu}$ terms only $\Rightarrow$ F-gauge

$(g_{\mu\nu})\left(\frac{q_\mu n_\nu + n_\mu q_\nu}{n\cdot q}\right) \Rightarrow$ UV by $\frac{\partial}{\partial q^\mu} Int \big|_{q^\mu=0}$ } pinch technique

$\left(\frac{q_\mu n_\nu + n_\mu q_\nu}{n\cdot q}\right)(\quad " \quad) \Rightarrow$ UV by $\quad "$

II: $P_g^{FD}(q)_{\mu\nu} = P_T(q)_{\mu\nu} + P_L(q)_{\mu\nu}$

$$= \delta_i^\mu \delta_j^\nu \left(\delta_{ij} - \frac{q^i q^j}{|\vec{q}|^2}\right) + q^2 \frac{n_\mu(q)\, n_\nu(q)}{(n_\mu(q)\cdot q)^2}$$

TT terms are the same as Coulomb/Landau

TL & LL terms give non-analytic terms

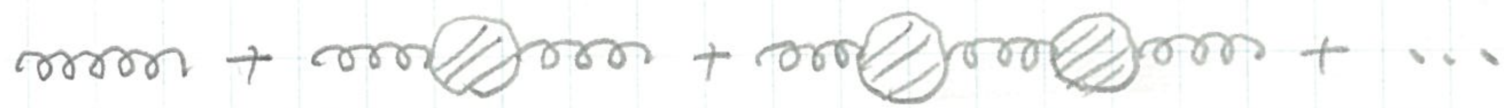
$\Rightarrow$ cancel out with F.P. loop

III: Feynman gauge on the FD gauge gluon background (BG gauge quantization with classical gluons in the FD gauge)

From a practical point of view, the results III is most important.

- We find that the Feynman gauge $\Pi_{\mu\nu}(q)$ from (b)+(c) when loop gluons are quantized on the classical FD gauge gluon background satisfies $q^\mu \Pi_{\mu\nu}(q) = q_\nu \Pi_{\mu\nu}(q) \Rightarrow \Pi_{\mu\nu}(q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\Pi(q^2)$ and that $\Pi(q^2)$ has full QCD β function. $\Pi(q^2) = q^2 \Pi'(q^2) = 0$ @ $q^2 = 0$.

- Schwinger-Dyson summation of all the 1PI correction then gives



$$= \frac{P^{FD}(q)_{\mu\nu}}{q^2} + \frac{P^{FD}(q)_{\mu\alpha}}{q^2}\left(-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{q^2}\right)\Pi(q^2)\frac{P^{FD}(q)_{\beta\nu}}{q^2} + \cdots$$

$$= \frac{P^{FD}(q)_{\mu\nu}}{q^2}\left\{1 - \frac{\Pi(q^2)}{q^2} + \left(\frac{\Pi(q^2)}{q^2}\right)^2 - \cdots\right\} = \frac{P^{FD}(q)_{\mu\nu}}{q^2 + \Pi(q^2)} = \frac{P^{FD}(q)_{\mu\nu}}{q^2(1+\Pi'(q^2))}$$

$\Rightarrow$ Improved Born amplitudes in the FD gauge.

$\Rightarrow$ We need to check 3- & 4-point vertices on the FD gauge BG in order to complete NLO. (We welcome your contributions!)

$\Rightarrow$ EW loops on the FD gauge BG. (in progress)

Why do FD gauge amplitudes allow physical interpretation for each Feynman Diagram?

I don't know the answer.

Speculation ?

FD gauge is not 'a gauge' because it gives the non-removal $h=0$ polarization component of the off-shell gauge boson.

$$\tilde{\epsilon}^{\mu}(q,0) = -\,\mathrm{sgn}(q^2)\frac{Q\, n^{\mu}}{n\cdot q} \quad \begin{cases} \longrightarrow 0 \quad \text{as} \quad Q\to 0 \ \& \ e^{-y} \ (y\to\infty) \\ \tilde{\epsilon}(q,0)\cdot j = \underbrace{\epsilon(q,0)\cdot j}_{} \ \text{in the rest frame!} \end{cases}$$

↑ conserved current ⇒ Wigner's d-functions!

It is exactly this $h=0$ off-shell component of the gauge boson which couples with the massless Goldstone boson to form a massive gauge boson in the EW theory. ///