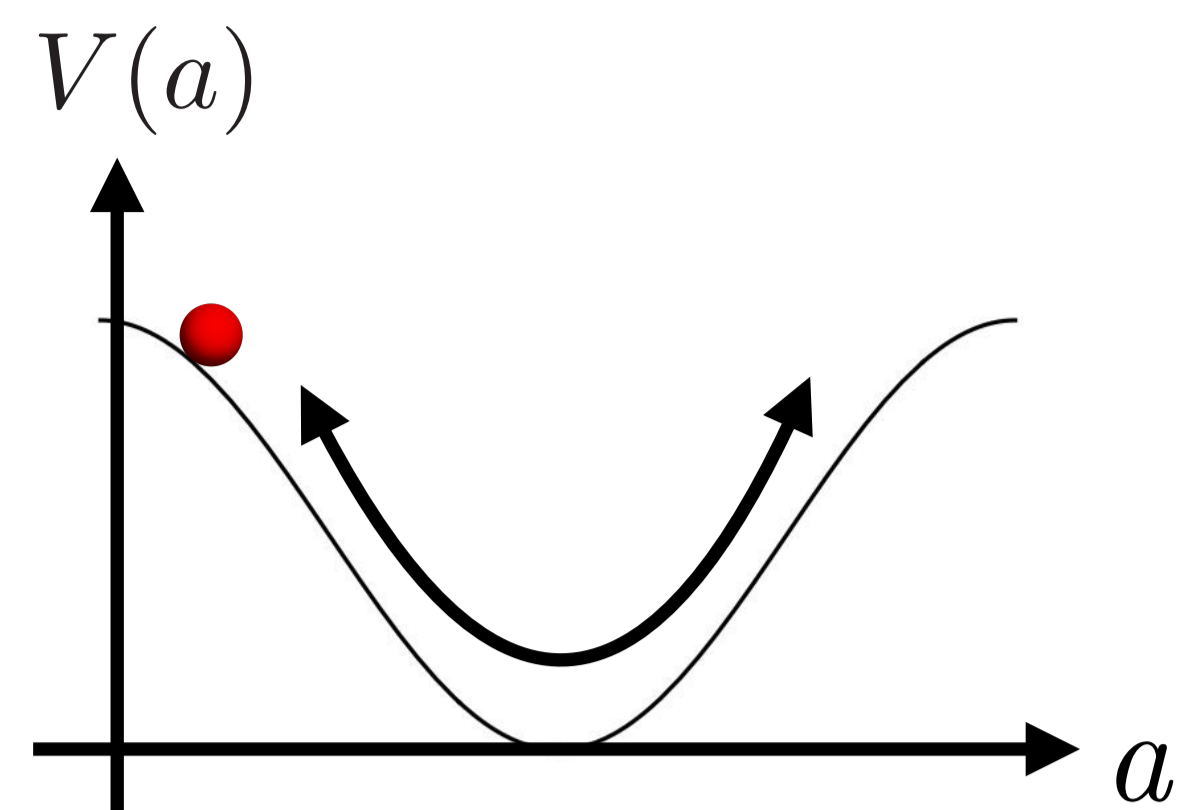


1. Introduction

QCD axion can solve the Strong CP problem. It is also one of leading candidates for dark matter (DM) through the misalignment mechanism.



In scenarios with multiple axions, it is possible that **the QCD axion a mix with another axion**, e.g. ALP ϕ .

For instance, we can suppose that the potential of these axions is

$$V(a, \phi) = m_a^2(T) f_a^2 \left[1 - \cos \left(\frac{a}{f_a} + N_f \frac{\phi}{f_\phi} \right) \right] + m_\phi^2 f_\phi^2 \left[1 - \cos \left(\frac{\phi}{f_\phi} \right) \right].$$

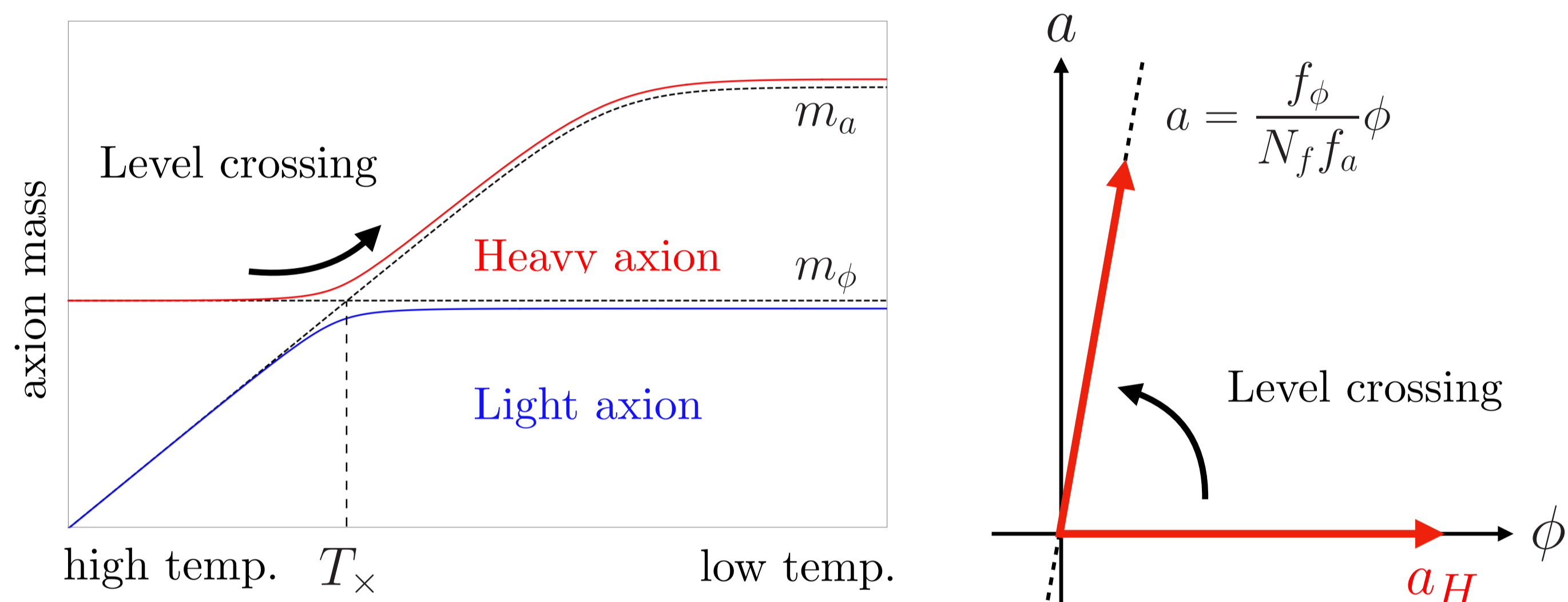
a mix with ϕ .

As the universe cools down to the QCD scale, the mass of the QCD axion grows,

$$m_a^2(T) = \begin{cases} m_{a,0}^2 & (T < T_{\text{QCD}}) \\ m_{a,0}^2 \left(\frac{T}{T_{\text{QCD}}} \right)^{-n} & (T \geq T_{\text{QCD}}) \end{cases}. \quad (T_{\text{QCD}} \approx 153 \text{MeV}, n \approx 8.16)$$

From this feature of QCD axion, one of the striking phenomena is **level crossing** between the axion mass eigenvalues.

Kitajima, Takahashi 2015; Daido, et al 2015; Ho, et al 2018; Cyncynates, Thompson 2023



If this process is adiabatic, nearly all the energy of the ALP transfers to the QCD axion, $\rho_\phi \rightarrow \rho_a$.

➔ DM abundance would increase compared to the absence of the level crossing.

2. Adiabatic condition

Intuitively, the adiabatic condition is if both axions oscillate many times during a time scale the level crossing lasts.

Condition 1

$$\Delta t_\times < C_{\text{ad}} \frac{1}{m_\phi}$$

Cyncynates, Thompson 2023

On the other hand, the adiabatic condition that includes **beat frequency** have been also considered.

Condition 2

$$\Delta t_\times < C'_{\text{ad}} \max \left[\frac{2\pi}{m_L(T_\times)}, \frac{2\pi}{m_H(T_\times) - m_L(T_\times)} \right]$$

Ho, et al 2018

However, we did not clearly know which condition is reliable.

3. Numerical validation

For convenience, we introduce $r_f = f_\phi / N_f f_a$, $r_m = m_\phi / m_{a,0}$.

To examine the validity of the adiabatic condition, we focus on the upper bounds of f_a required to satisfy each condition.

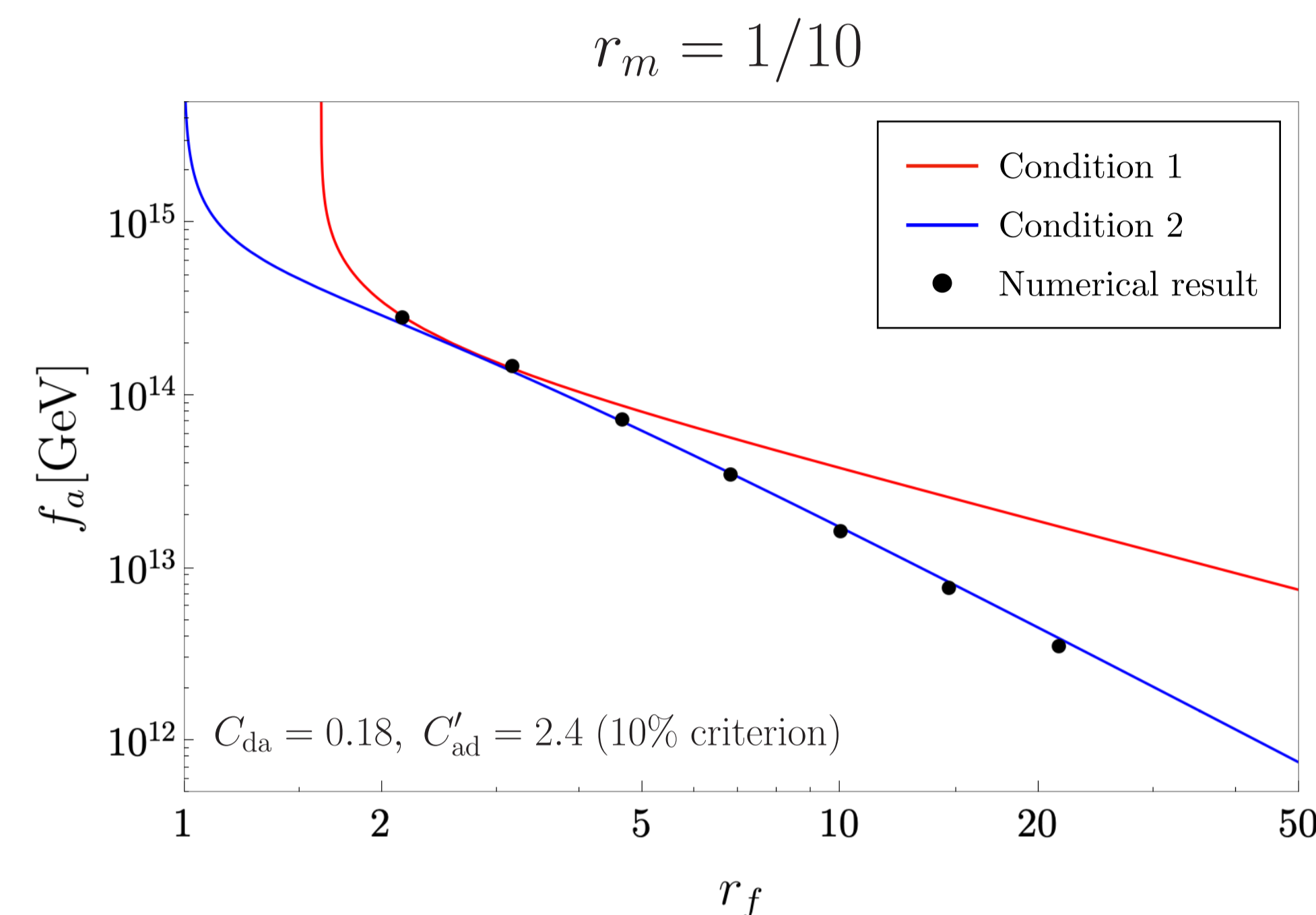
Condition 1

$$f_a < \frac{r_m}{C_{\text{ad}} \sqrt{\chi_0}} \Delta t_\times \propto r_m^{1+4/n} r_f^{-1}$$

Condition 2

$$f_a < \frac{r_m r_f}{2\pi C'_{\text{ad}} \sqrt{\chi_0}} \Delta t_\times \propto r_m^{1+4/n} r_f^{-2}$$

in an approximation for $r_f \gg 1$.



We show that the allowed value of f_a decrease as r_f^{-2} .

The adiabaticity of the level crossing is strongly related to the beat frequency.

4. Axion dark matter

For $f_a < 10^{12} \text{GeV}$, the misalignment mechanism in the single QCD axion cannot produce a sufficient DM abundance,

$$\Omega_a h^2 \sim 0.12 \theta_{a,i}^2 \left(\frac{f_a}{10^{12} \text{GeV}} \right)^{1.17},$$

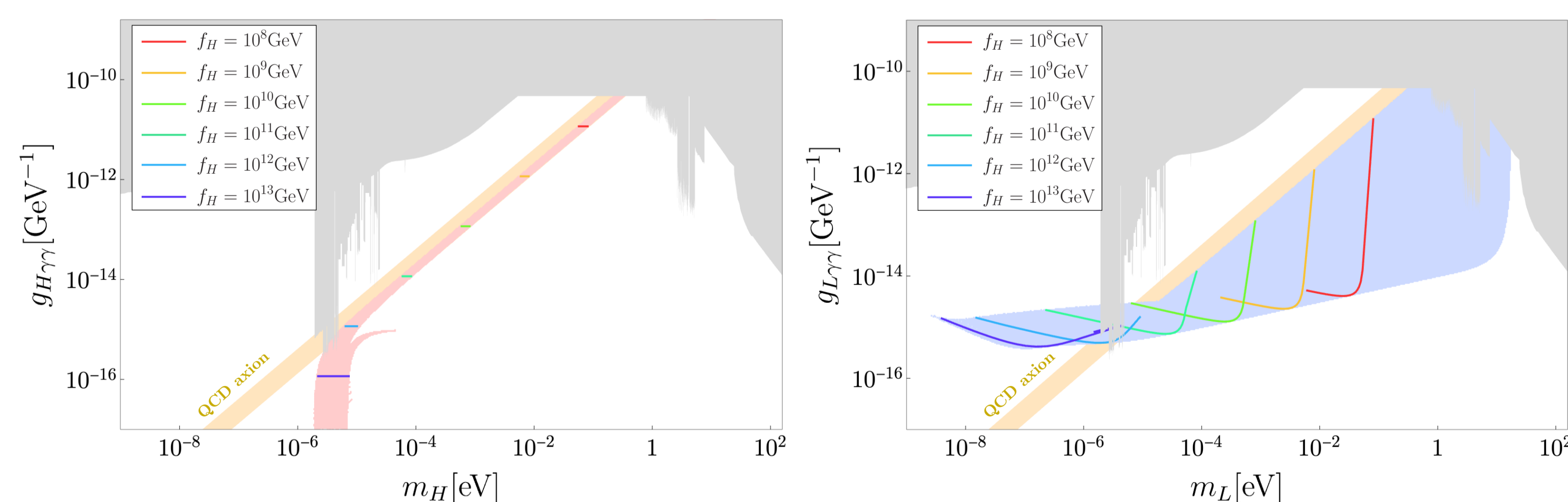
without fine-tuning $\theta_{a,i}$.

However, if a large amount of the ALP entirely converted into the QCD axion throughout the level crossing, it would be possible to explain dark matter as the QCD axion for f_a smaller than 10^{12}GeV .

Thanks to the refined adiabatic condition, we can analytically discuss the DM abundance through the adiabatic level crossing.

If the level crossing is adiabatic and $r_f \gg 1$, the QCD axion can entirely contribute to the DM abundance:

$$\Omega_a h^2 \sim 0.12 \theta_{a,i}^2 \left(\frac{r_m}{0.1} \right)^{-\frac{1}{2}} \left(\frac{r_f}{100} \right)^2 \left(\frac{f_a}{10^{10} \text{GeV}} \right)^{\frac{3}{2}}.$$



The viable parameter regions to explain the observed DM are extensive.

Summary

- We have studied the level crossing phenomenon of the two axions in detail.
- We have shown that the refined adiabatic condition has a significant relation with the beat frequency.
- Interestingly, we have the parameter region that both of the heavy and light axions contribute to dominant DM. In this case, with the similar decay constant, both of them can be probed.