Collective advantages in qubit reset: coherent qubits effect

Yue Liu

Email: yueliu@xmu.edu.cn

Department of Physics, Xiamen University, China

arXiv:2407.03096

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Outline

1. Introduction

2. Dicke state

3. Main result

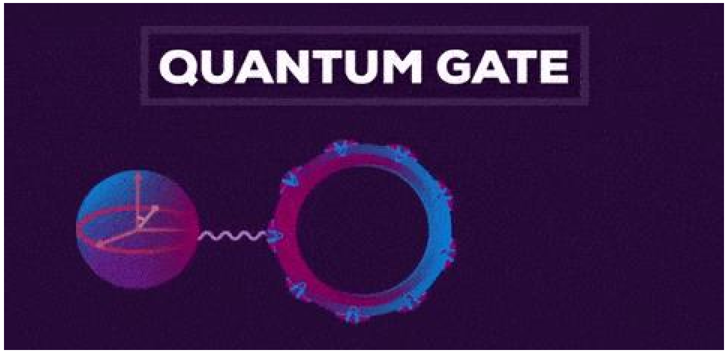
    3.1 First main result —— the thermodynamic limit

    3.2 Second main result —— the reset factor

4.Summary

Introduction

Quantum computing



the next year

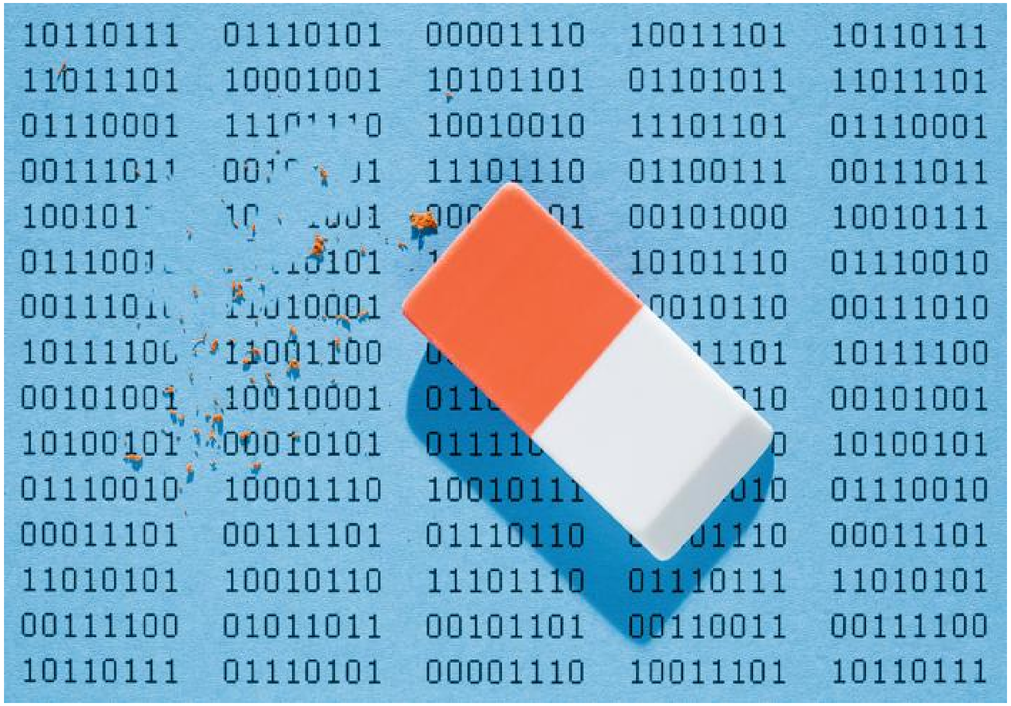
rapid development

energy consumption?

will use one-fifth of global energy consumption

need urgent attention

Landauer principle



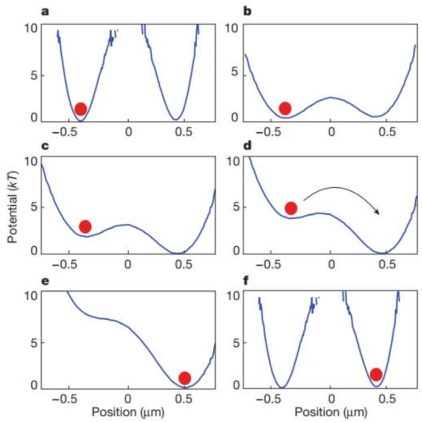
The minimal thermodynamic cost of a single qubit reset

$$Q = k_{\mathrm{B}} T \ln 2$$

R. Landauer, IBM J. Res. Dev. 5, 183–191 (1961).
J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, Nat. Phys. 11, 131 (2015).
I. Georgescu, Nat. Rev. Phys. 3, 770 (2021)

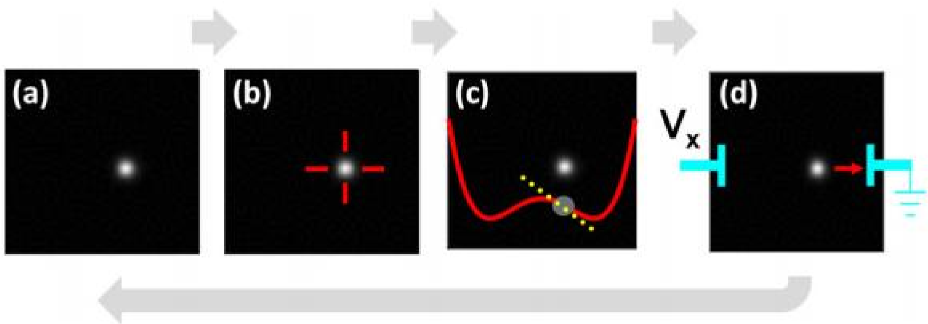
Experimental verification

Optical tweezers



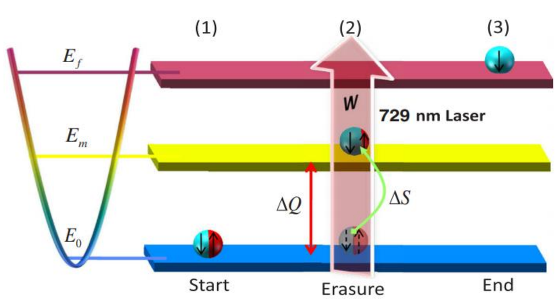
A. Berut *et al.*, Nature (London) 483, 187 (2012).

Virtual potential



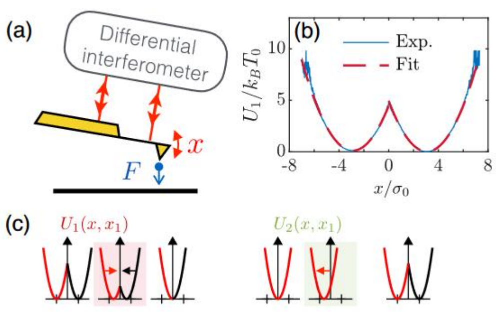
Y. Jun *et al.*, Phys. Rev. Lett. 113, 190601 (2014).

Trapped ultracold ion



L. L. Yan *et al.*, Phys. Rev. Lett. 120, 210601 (2018).

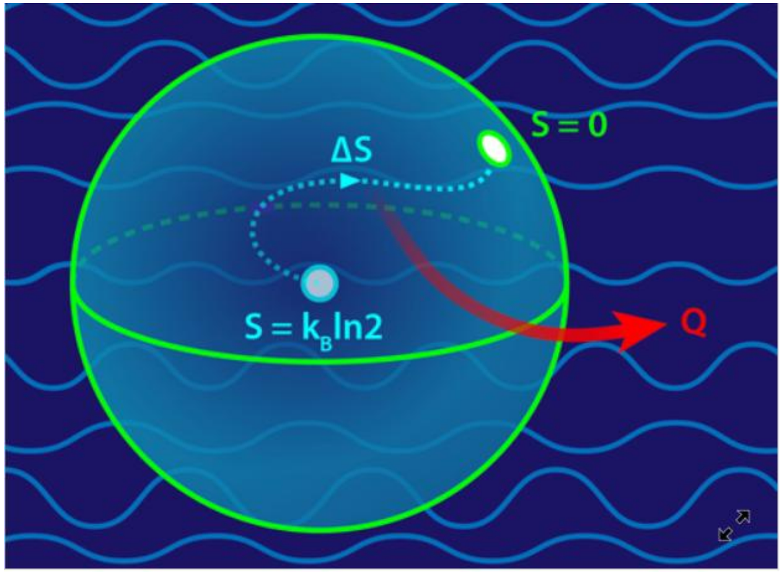
Conductive cantilever



S. Dago *et al.*, Phys. Rev. Lett. 126, 170601 (2021).

Finite-time Landauer bound

Landauer bound is unattainable since it requires an infinitely long quasistatic process and perfect reset.



In real life, devices are required to be high-speed and high-fidelity, which has led to a lot of research aimed at obtaining the finite-time Landauer bound.

Finite-time Landauer bound

by thermodynamical bound

such as speed limit

$$\frac{L(\boldsymbol{p}(0),\boldsymbol{p}(\tau))^2}{2\Sigma\langle A\rangle_\tau} \leq \tau. \quad \Longrightarrow \quad Q \geq -T\Delta S_{\mathrm{sys}} + \frac{\mathcal{T}(p_0,p_\tau)^2}{\tau\beta\langle a\rangle_\tau/2}$$

N. Shiraishi *et al.*, Phys. Rev. Lett 121, 070601 (2018).

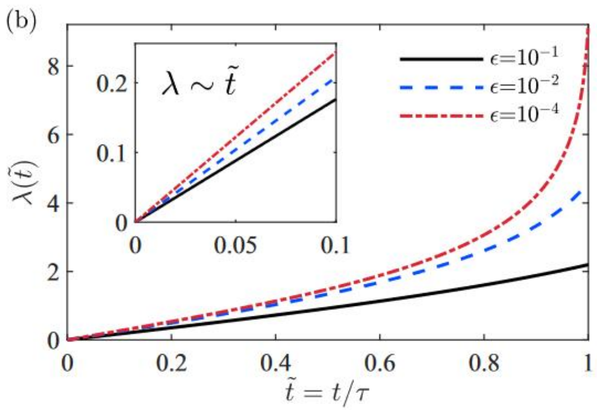
T. Van Vu and K. Saito, Phys. Rev. X 13, 011013 (2023).
T. Van Vu and K. Saito, Phys. Rev. Lett 128, 010602 (2022).

$$\frac{l}{2A_{\mathrm{tot}}} \leq f\left(\frac{\Sigma}{A_{\mathrm{tot}}}\right) \quad \Longrightarrow \quad \frac{Q}{T} \geq \ln 2 + B_S = \ln 2 + \tanh^{-1} v,$$

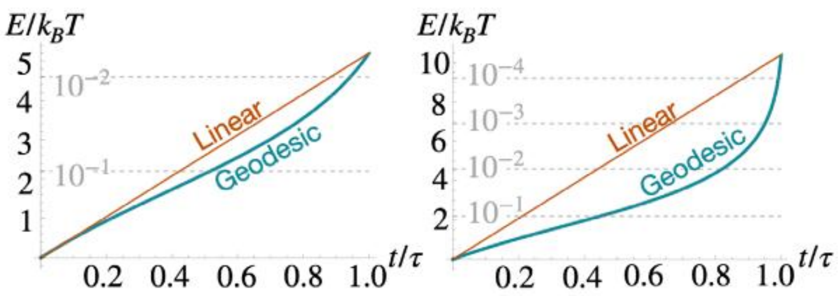
J. Lee *et al.*, Phys. Rev. Lett 129, 120603 (2022).

Optimal protocol

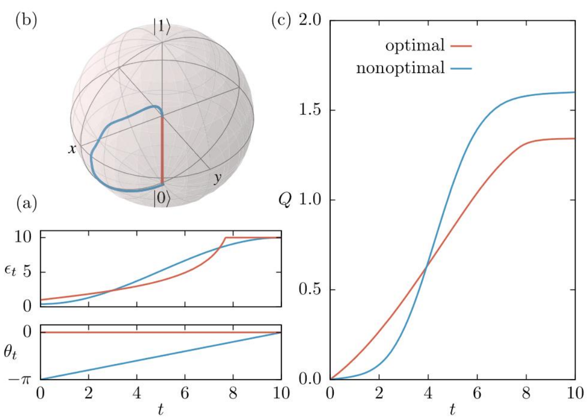
the minimal thermodynamical cost



Y. Ma *et al.*, Phys. Rev. E 106, 034112 (2022)



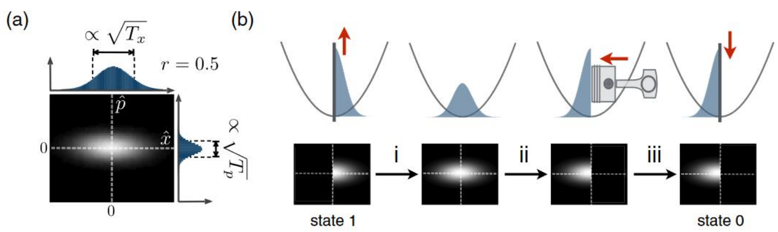
M. Scandi *et al.*, Phys. Rev. Lett 129, 270601 (2022)



T. Van Vu and K. Saito, Phys. Rev. Lett. 128, 010602 (2022)
T. Van Vu and K. Saito, Phys. Rev. X 13, 011013 (2023).

Quantum effect for qubit reset

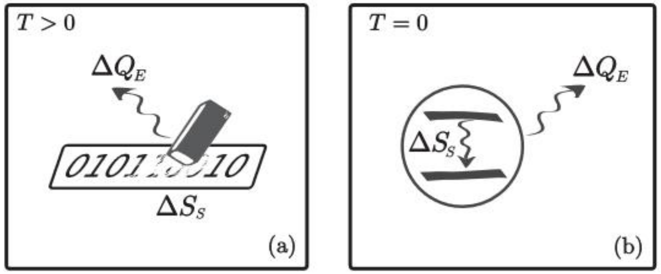
Squeezed reservoir



$$W = k_B T e^{-2r} \ln 2$$

J. Klaers, Phys. Rev. Lett. 122, 040602 (2019)

Zero temperature

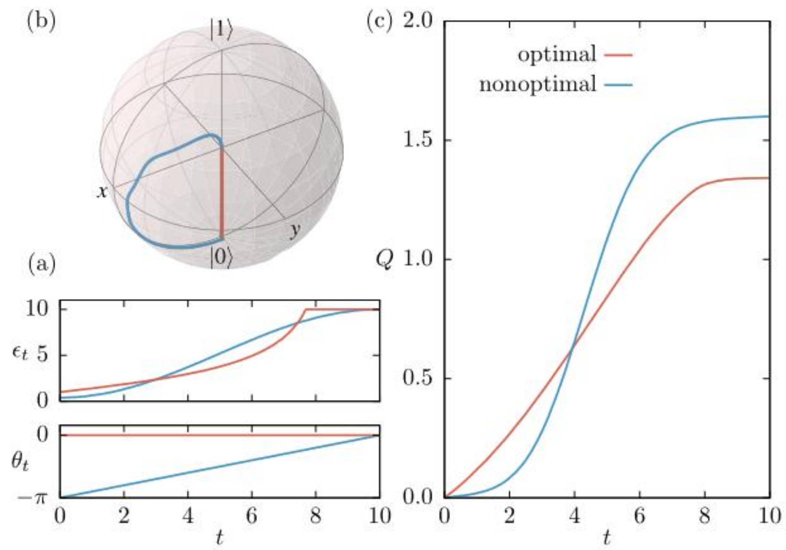


$$\Delta Q_E \geq -T\Delta S_S + \frac{3\hbar c}{\pi L}\Delta S_S^2,$$

A. M. Timpanaro, et al., Phys. Rev. Lett. 124, 240601 (2020)

Quantum effect for qubit reset

quantum coherence

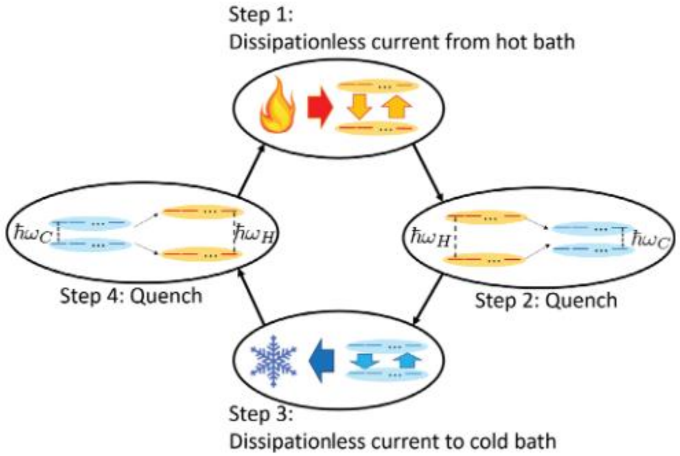


$$\beta Q \geq \underbrace{\Delta S_{\mathrm{cl}} + \frac{\|\Lambda(\varrho_0) - \Lambda(\varrho_\tau)\|_1^2}{2\tau\bar{\gamma}_\tau}}_{\text{classical}} + \underbrace{C_{\mathrm{rel}} + \frac{C_{\mathrm{res}}}{2\tau\bar{\gamma}_\tau}}_{\text{quantum}}.$$

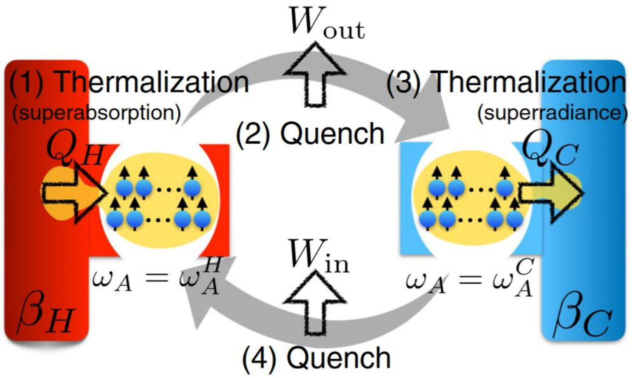
"quantum friction"

H. J. D. Miller, G. Guarnieri, M. T. Mitchison, and J. Goold, Phys. Rev. Lett. 125, 160602 (2020)
Tan Van Vu and Keiji Saito, Phys. Rev. Lett. 128, 010602 (2022)

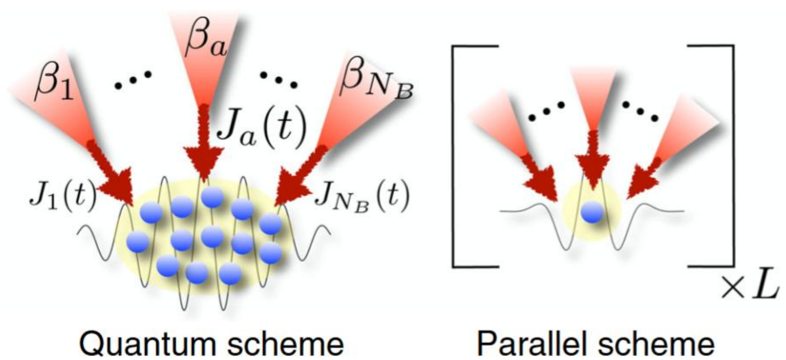
Collective effect for heat engine and heat transfer



H. Tajima and K. Funo, Phys. Rev. Lett. 127, 190604 (2021)

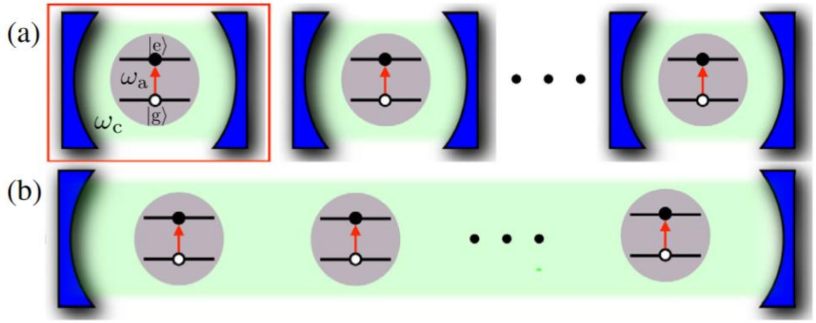


S. Kamimura, et al, Phys. Rev. Lett. 128, 180602 (2022).

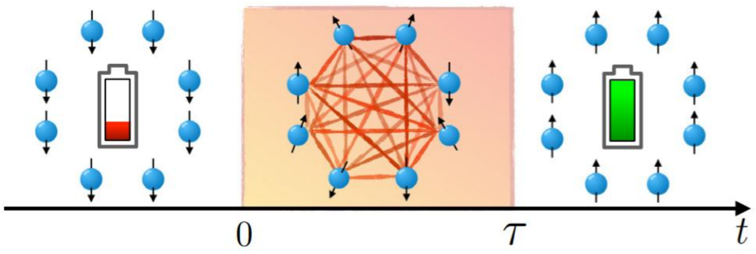


Shunsuke Kamimura, et al, Phys. Rev. Lett. 131, 090401 (2023)

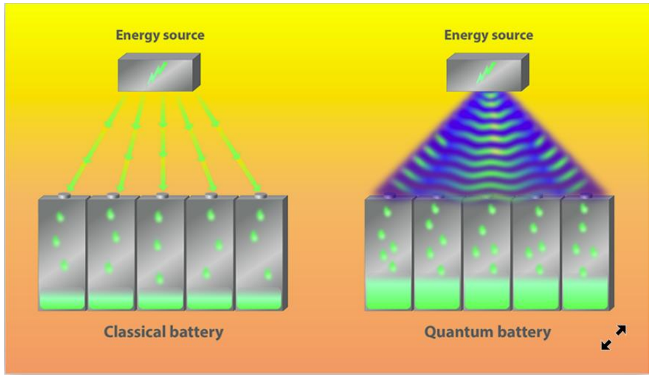
Collective effect for quantum battery



D. Ferraro, et al., Phys. Rev. Lett. 120, 117702 (2018).

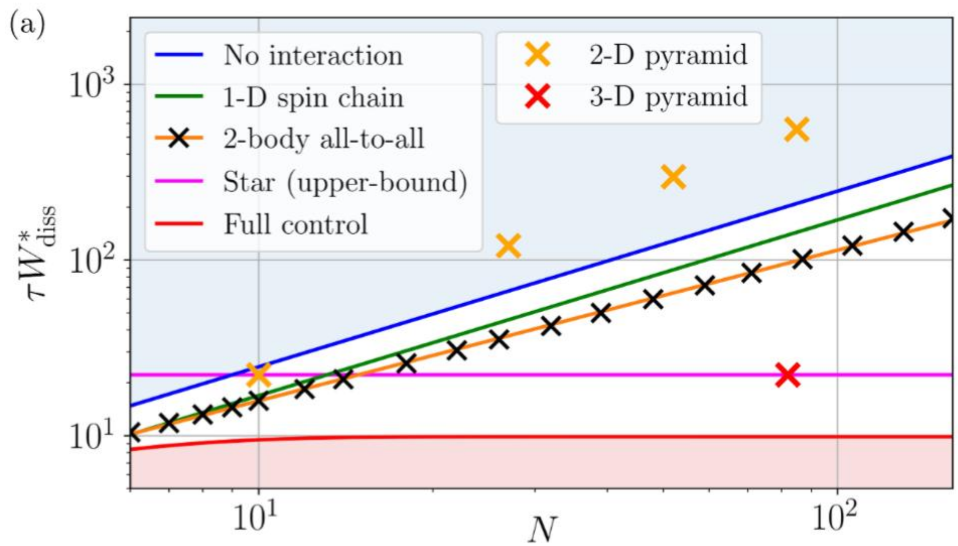


D. Rossini, et al., Phys. Rev. Lett. 125, 236402 (2020)



J.-Y. Gyhm, D.ˇSafránek, and D. Rosa, Phys. Rev. Lett. 128, 140501 (2022)

Collective effect for qubit reset



$$\beta W^*_{\mathrm{qubit}} = \ln 2 + \frac{\pi^2}{\tau N} + \mathcal{O}(\tau^{-2}).$$

minimizing thermal dissipation by thermodynamic geometry

and find a faster convergence to Landauer bound

A. Rolandi, P. Abiuso, and M. Perarnau-Llobet, Phys. Rev. Lett. 131, 210401 (2023).

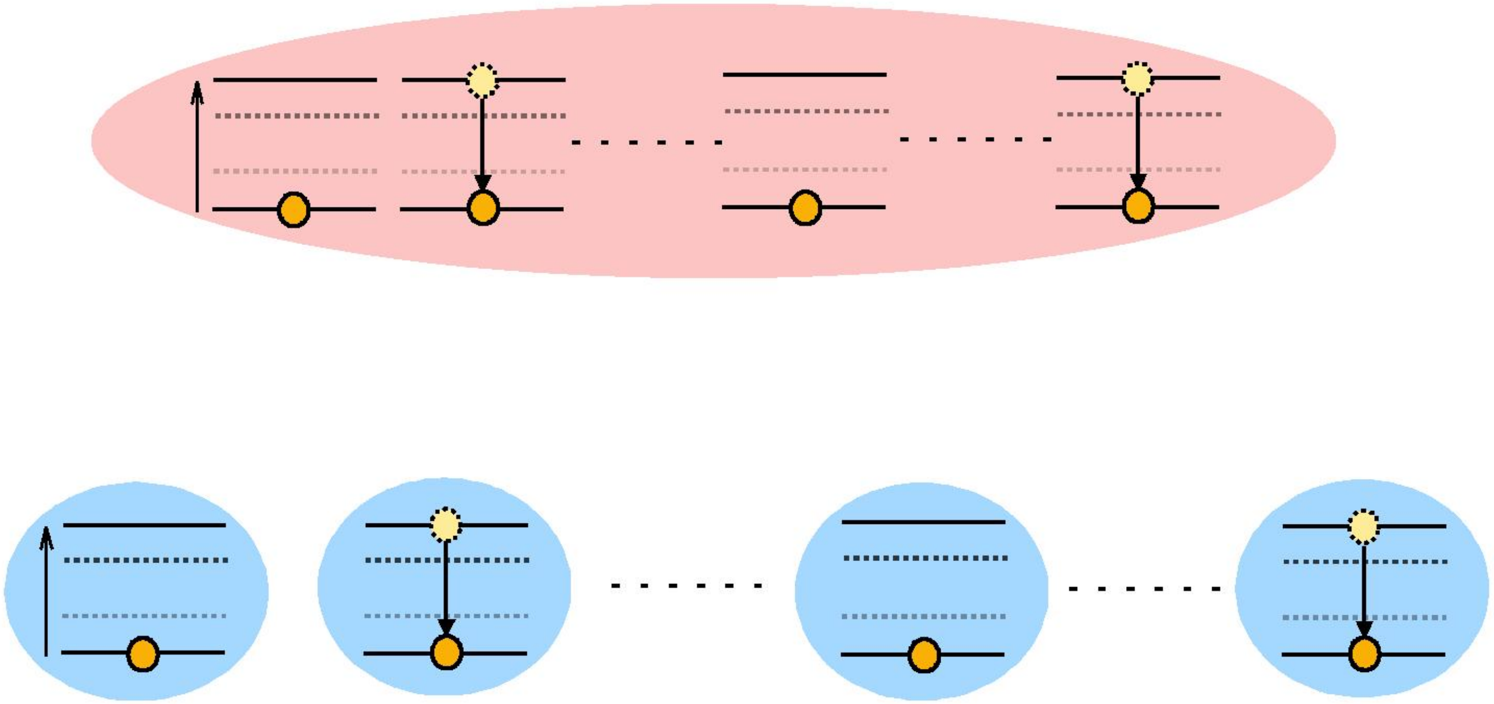
due to the interplay between interactions and dissipation to thermal bath

in the slow-driving regime and perfect reset

Collective effect for qubit reset

(a)

(b)

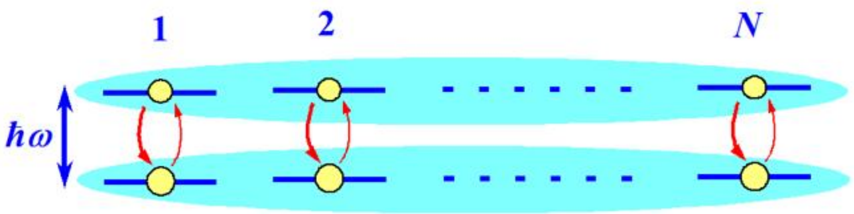


Resetting entangled qubits, which is also a collective process, play a pivotal role in high-precision quantum measurements and computations

However, how the entanglement affects the qubits reset is still unclear.

Dicke state

N-qubit system



Hamiltonian $$H(t) = \hbar\omega(t)\sum_{j=1}^{N} |e,j\rangle\langle e,j|$$ energy-level spacing $\omega(t)$

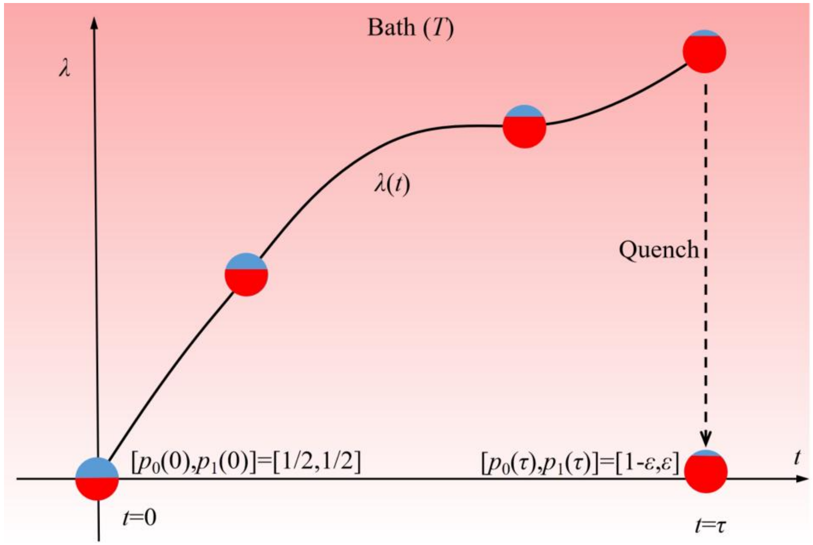
⇩

$$H(t) = \hbar\omega(t)\sum_{\mathcal{C}} |eg..g\rangle\langle eg..g| + 2\hbar\omega(t)\sum_{\mathcal{C}} |ee..g\rangle\langle ee..g|$$
$$+ \ldots + N\hbar\omega(t)|e\ldots e\rangle\langle e\ldots e|,$$

$\mathcal{C}$ for combination

$$\sum_{\mathcal{C}} |eg..g\rangle\langle eg..g| = |eg\ldots g\rangle\langle eg..g| + |ge\ldots g\rangle\langle ge..g| + \ldots + |gg\ldots e\rangle\langle gg\ldots e|$$

Steps of qubit reset



The purpose of qubit reset is to restore each qubit to its ground state.

The benchmark when we discuss the advantages

Step I
Initial

$\hbar\omega(0) = 0$

the probability of finding the ground state and the excited state is equal for each qubit

Step II
Reducing population

$\hbar\omega$ is controlled by a protocol

the Lindblad equation

$$\dot{\rho} = \mathcal{L}[\rho] = -i[H,\rho] + \Gamma_{\downarrow}[L\rho L^{\dagger} - \frac{1}{2}\{L^{\dagger}L,\rho\}] + \Gamma_{\uparrow}[L^{\dagger}\rho L - \frac{1}{2}\{LL^{\dagger},\rho\}]$$

$$Q_N = \int_0^{\tau} \mathrm{Tr}(\dot{\rho}H)\mathrm{d}t$$

Step III
Quench

$\hbar\omega$ is quenched to zero

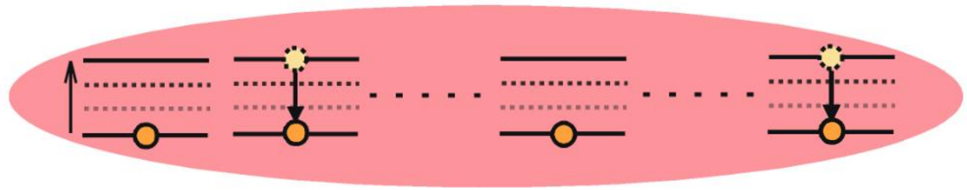
$$W_N = Q_N$$

$$\rho(\tau) \approx |gg..g\rangle\langle gg..g|$$

with an error probability

Collective reset

The interatomic coherence effects occur if qubits are distributed at very short spacings



"Quantum lubrication"

H. Tajima and K. Funo, Phys. Rev. Lett. 127, 190604 (2021)

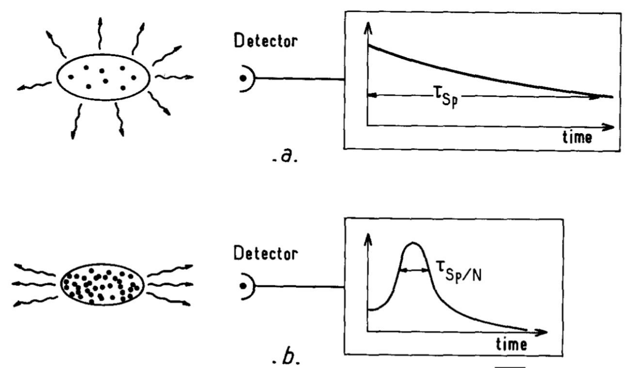
Dicke state

$$|D_N^n\rangle = \frac{1}{\sqrt{C_N^n}} \sum_{\mathcal{C}} |\underbrace{e \ldots e}_{n} g \ldots g\rangle$$

N-qubit Dicke state with *n* excitations

$$\rho = \sum_n p_n |D_N^n\rangle\langle D_N^n|$$

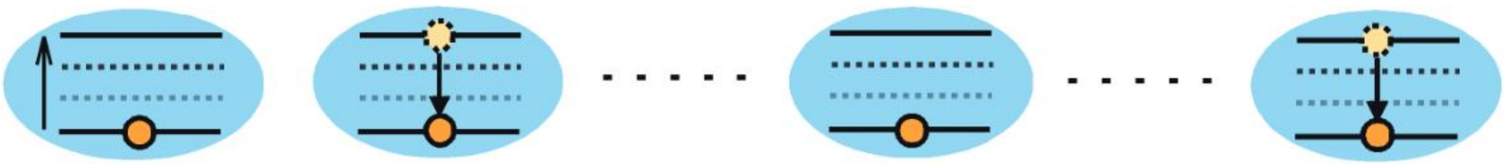
the form of block diagonal matrix for energy eigenstates



superradiance

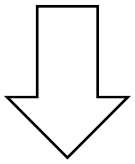
Parallel reset

the coherence between qubits can be easily broken by environment and noise



each qubit is reset independently of each other

The density matrix is the strictly diagonal form



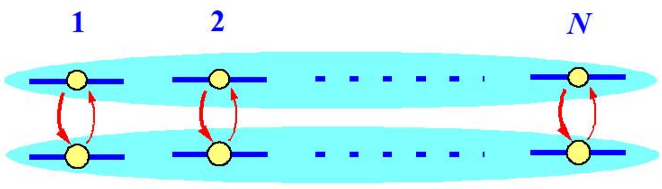
the total thermodynamic cost of qubit reset is N times that of the single qubit

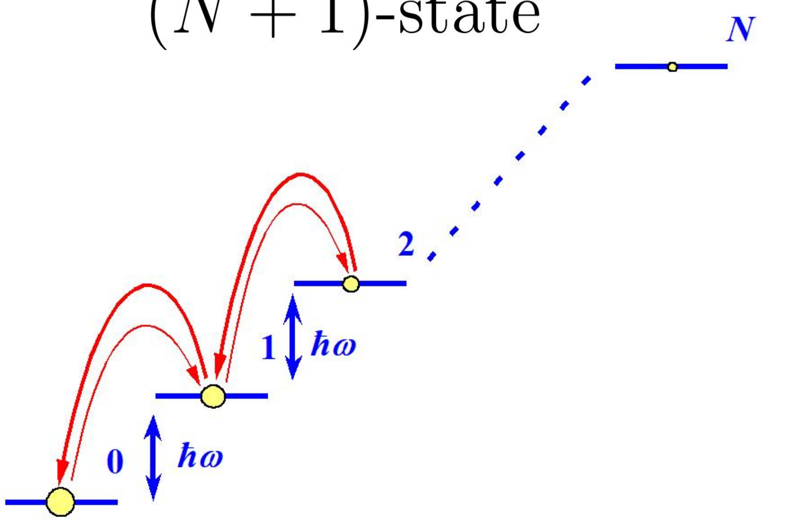
the overall error probability is the same as each qubit

(N+1)-state birth and death process

N-qubit

$(N+1)$-state





$$\dot{p}_n = (N-n+1)n\Gamma_\uparrow p_{n-1} - [(N-n+1)n\Gamma_\downarrow + (N-n)(n+1)\Gamma_\uparrow]p_n + (N-n)(n+1)\Gamma_\downarrow p_{n+1}$$

$$p_n(0) = \frac{1}{N+1}$$

(N+1)-state with special details

$$Q_N = \beta^{-1}\ln(N+1) \le N\beta^{-1}\ln 2$$

corresponding Landauer bound

entanglement as the "lubrication"

The properties of finite-time collective reset are related to the special details.

Main Results

First main result—the thermodynamic limit

The definition of the error probablility

Requirements:

1. each qubit has an equal error probability for the parallel-reset case, and the overall error probability has to be equal to the error probability of each qubit.

2. equal to 1/2 for the initial state.

To meet the requirements, we define the error probability as the proportion of qubits in the excited state

$$\varepsilon = \frac{N_e}{N}$$

totally basing on the understanding on the *N*-qubit model

for the *N*-qubit Dicke state, the error probability can be written as

$$\varepsilon = \frac{1}{N}\sum_{n=0}^{N} n p_n$$

First main result—the thermodynamic limit

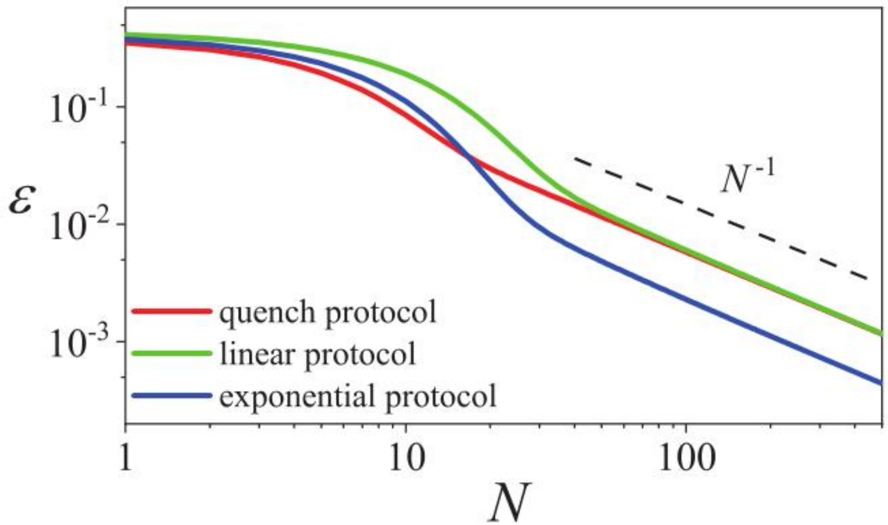
We prove that:

for any monotonically increasing protocol in an arbitrary operating time τ>0, the error probability satisfies a universal scaling behavior for the same protocol

$$\varepsilon = O(N^{-1}) \quad \text{for} \quad N \to \infty$$

which implies that the error probability vanishes in the thermodynamic limit.

a collective advantage



The quench protocol: $\hbar\omega(t) = \beta^{-1}$

The linear protocol: $\hbar\omega(t) = \beta^{-1}\Gamma_0 t$

The exp. protocol: $\hbar\omega(t) = \beta^{-1}[e^{\Gamma_0 t} - 1]$

First main result—the thermodynamic limit

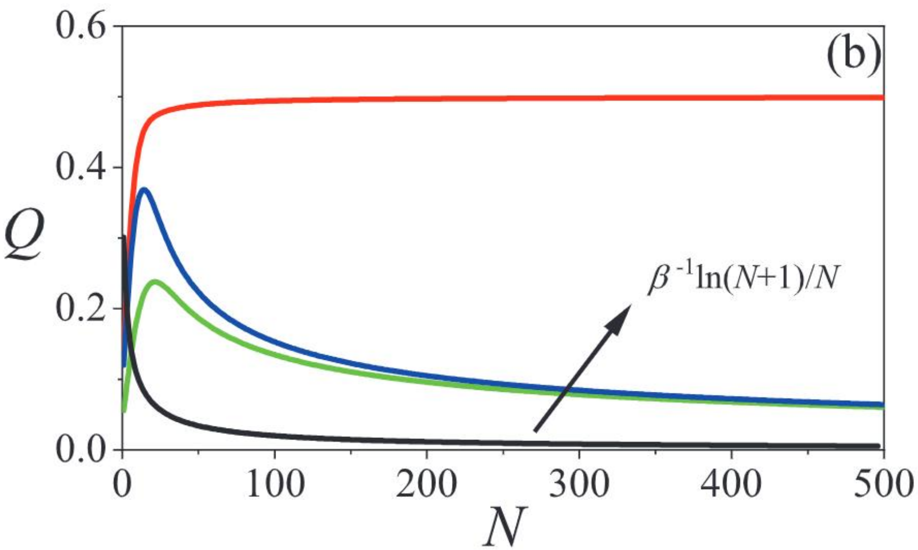
We prove that:

the per-qubit heat production in the thermodynamic limit for any protocol is given by

$$\lim_{N\to\infty} Q = \frac{\hbar\omega(0^+)}{2}$$

the per-qubit heat production is related to the initial continuity of protocols and tends to zero for protocols that are continuous initially

approach Landauer bound in finite-time

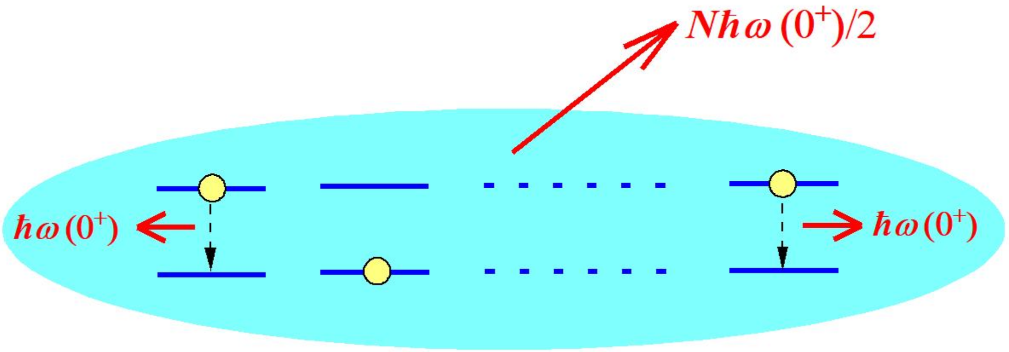


The quench protocol: $\hbar\omega(t) = \beta^{-1}$

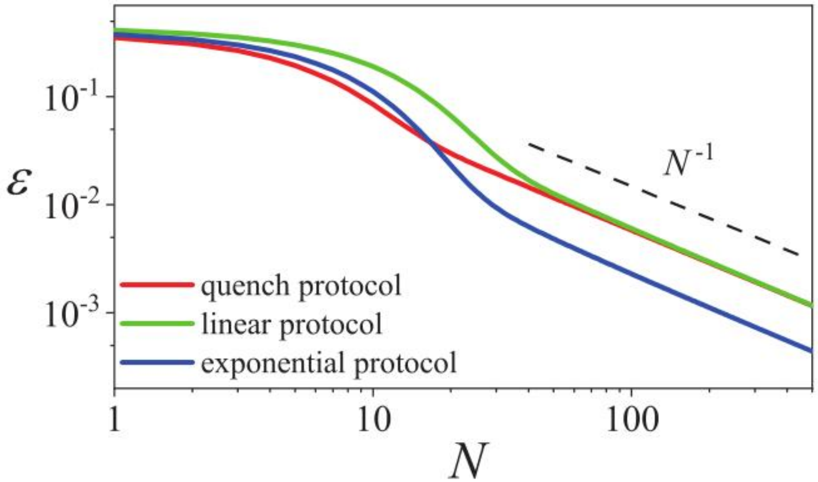
The linear protocol: $\hbar\omega(t) = \beta^{-1}\Gamma_0 t$

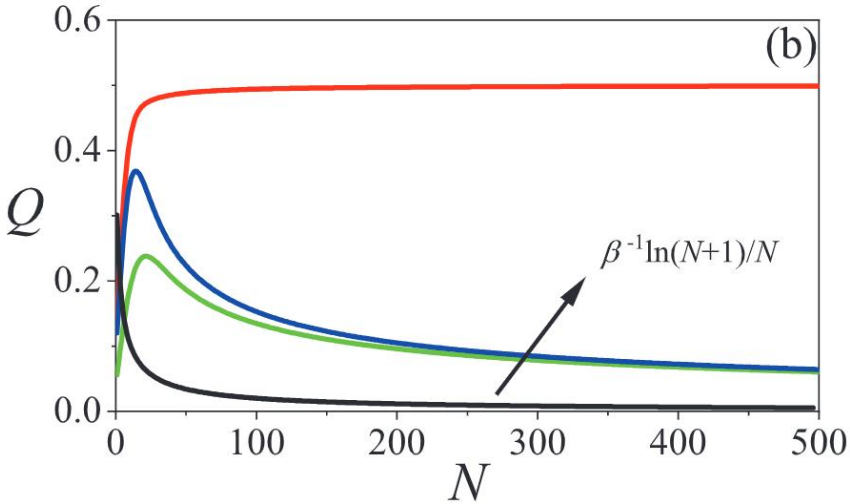
The exp. protocol: $\hbar\omega(t) = \beta^{-1}[e^{\Gamma_0 t} - 1]$

First main result—the thermodynamic limit



the superradiance effect





Second main result—reset factor

Accroding to the speed limit proposed in PRL 127, 190602 (2021), we obtain

$$\frac{Q\tau}{(1-2\varepsilon)^2} \geq \frac{1}{\beta\Gamma_0}$$

which motivates us to define a reset factor

$$F := \frac{Q\tau}{(1-2\varepsilon)^2} \in (0, +\infty)$$

the modified version of the "action" in Quantum 7, 961 (2023)

$$A = Q\tau$$

smaller means better reset performance

For quasistatic protocol

$$\tau \to \infty$$

For trivial protocol

$$\hbar\omega(t) = 0 \quad \Rightarrow \quad \varepsilon = \frac{1}{2}$$

⇩

$$F \to +\infty$$

the worst performance

The best case we hope for is that the reset factor tends to zero

$$F \to 0$$

However, the speed limit sets a lower bound on the reset factor, making this case is unattainable.

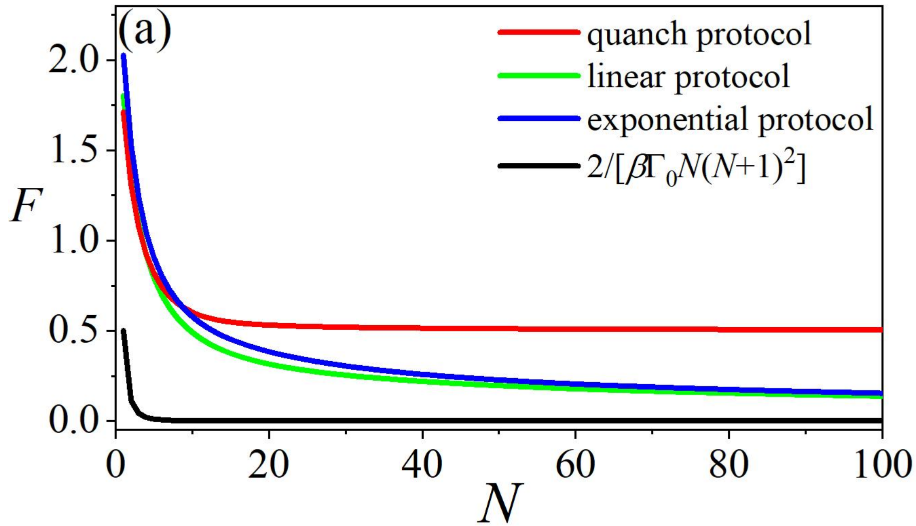
Second main result—reset factor

Based on the speed limit proposed in PRL 121, 070601 (2018), we prove

$$F = \frac{Q\tau}{(1-2\varepsilon)^2} \geq \frac{2}{\beta\Gamma_0 N(N+1)^2}$$

for collective reset. This can be considered as the trade-off between heat production of per qubit reset, the duration time, the error probability and the number of qubits.

the performance of qubit reset can be enhanced by increasing the number of qubits



a collective advantage

Summary

Summary

We discuss the collective advantages of qubit reset

1. For the quasistatic process, quantum entanglement make the thermodynamic cost required to reset per qubit for collective reset is less than parallel reset.

2. By defining the error probability, we prove that the error probability vanishes in the thermodynamic limit.

3. The per-qubit heat production can be given by $\hbar\omega(0^{+})/2$ in the thermodynamic limit.

4. By define the reset factor, we prove that the performance of qubit reset can be enhanced by increasing the number of qubits.

Limitation

1. The optimal protocol for the given time, the set error probability and the number of qubit.

2. The definition of the error probability and the reset factor is not unique.

Advertisement

I am looking for a one-year or two-year post-doctor position.

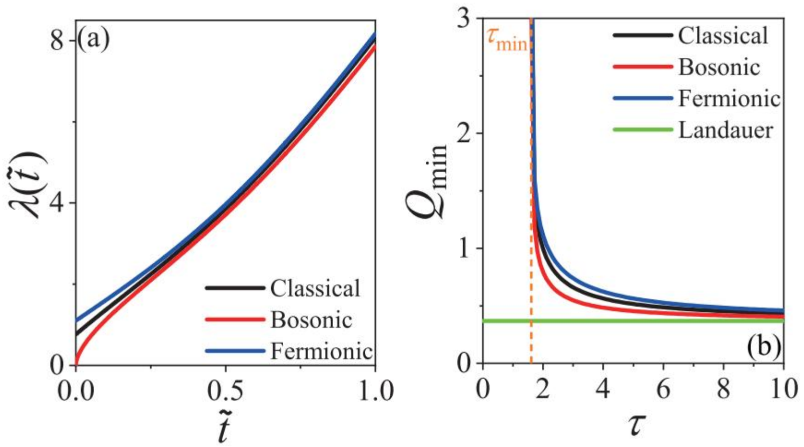
Reserch interests:

- **Stochastic thermodynamics**
- **Quantum thermodynamics**
- **Nonequilibrium statistical mechanics**
- **Classical thermalization**

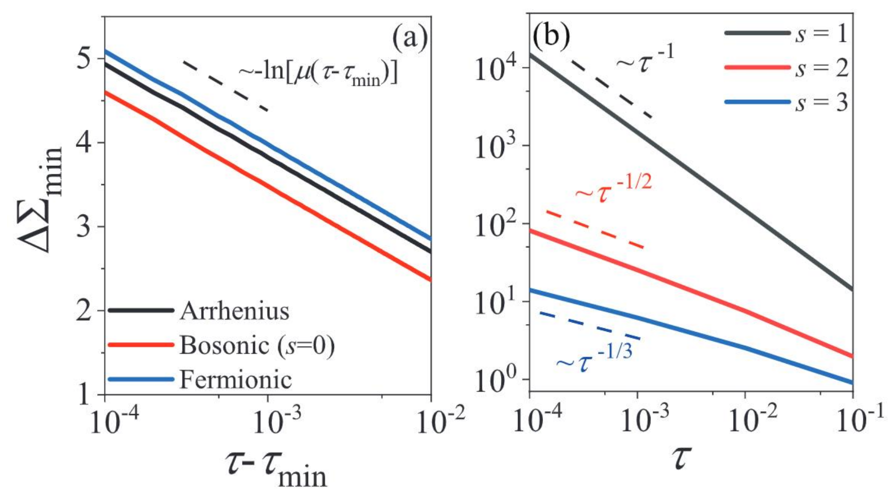
Email: yueliu@xmu.edu.cn

General form for optimal protocol

our recent work



behavior in the fast-driving regime



Y. Liu, C. Huang, X, Zhang and D. He. submiteed

First main result—the quasistatic process

thermal equilibrium state $\dot{p}_n = 0$ $\Rightarrow$ $p_n^{\rm eq} = \frac{e^{-n\beta\hbar\omega}}{\sum_{n=0}^{N} e^{-n\beta\hbar\omega}}$ The detailed balance condition

power $\dot{W} = {\rm Tr}(\rho\dot{H})$ $\Rightarrow$ $\dot{W} = \hbar\dot{\omega}\frac{\sum_{n=0}^{N} n e^{-n\beta\hbar\omega}}{\sum_{n=0}^{N} e^{-n\beta\hbar\omega}}$

$\Rightarrow$ $Q_N = k_B T \ln{(N+1)}$

heat production for per qubit reset $Q = \frac{Q_N}{N} = k_B T \frac{\ln{(N+1)}}{N} \leq k_B T \ln 2$ a collective advantage

not violate the second law

Second main result—the thermodynamic limit

We prove that the overall error probability has to be equal to the error probability of each qubit for the paralle-reset case

$$\frac{1}{N}\sum_{n=0}^{N} nC_N^n \varepsilon^n (1-\varepsilon)^{N-n} = \varepsilon$$

The traditional fidelity is not applicable here to define the error probability

$$F(\rho,\sigma) = \mathrm{Tr}[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}]^2$$

which only represents the case that all qubits are in the ground state and do not satisfy the two requirements.

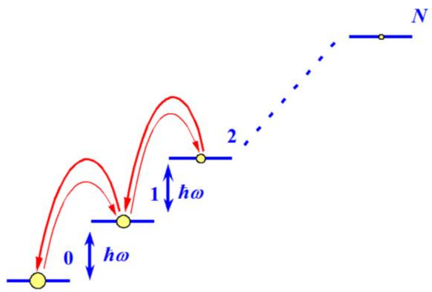
Sketch of proof

Sketch of proof——second main result

For any monotonically increasing protocol

$$p_n(t) > p_{n+1}(t)$$

intuitive from the physical point of view. We prove it by the comparison theorem



Define the fluctuation of the error probability

$$\zeta = \sum_{n=0}^{N} \frac{n^2}{N^2} p_n$$

$$0 < \zeta < \left(\frac{2}{3} + \frac{1}{3N}\right)\varepsilon$$

Sketch proof——second main result

master equation

$$\dot{p}_n = (N-n+1)n\Gamma_{\uparrow}p_{n-1} - [(N-n+1)n\Gamma_{\downarrow} + (N-n)(n+1)\Gamma_{\uparrow}]p_n + (N-n)(n+1)\Gamma_{\downarrow}p_{n+1}$$

⇩

$$\dot{\varepsilon} = \frac{1-\Delta}{2} - \varepsilon - N\Delta(\varepsilon - \zeta)$$

the evolution equation of the error probability

$$\Delta = \Gamma_{\downarrow} - \Gamma_{\uparrow}$$

$0 < \zeta < \left(\frac{2}{3} + \frac{1}{3N}\right)\varepsilon$ ⇩

$$\frac{1-\Delta}{2} - (1+N\Delta)\varepsilon \le \dot{\varepsilon} \le \frac{1-\Delta}{2} - \left(1 - \frac{\Delta}{3} + N\frac{\Delta}{3}\right)\varepsilon$$

Integrating ⇩ Laplace's method $\int_a^b \varphi(x)e^{nh(x)}dx = -(1+o(1))\frac{\varphi(a)}{nh'(a)}e^{nh(a)}.$

$$\frac{1-\Delta}{2N\Delta} < \varepsilon < \frac{3(1-\Delta)}{2N\Delta} \quad \text{for } N \to \infty$$

Sketch proof——second main result

per-qubit heat $$Q = \frac{1}{N}\sum_{n=0}^{N}\int_0^\tau n\hbar\omega(t)\dot{p}_n(t)\mathrm{d}t = \hbar\int_0^\tau \omega^*(t)\dot{\varepsilon}(t)\mathrm{d}t,$$ $$\omega^*(t) = \begin{cases}\omega(0^+), & t=0,\\ \omega(t), & 0<t<\tau,\\ \omega(\tau^-), & t=\tau,\end{cases}$$

error probability $$\varepsilon = O(N^{-1}) \quad \text{for} \quad N\to\infty$$

⇩

for a given $\eta\in(0,\tau)$ and $\epsilon>0$, there exist $\bar{N}$, s.t. when $N>\bar{N}$, $\varepsilon(\tau)<\varepsilon(\eta)<\epsilon$

⇩

mean value theorem of integrals

$$Q = \hbar\int_0^\eta \omega^*(t)\dot{\varepsilon}(t)\mathrm{d}t + \hbar\int_\eta^\tau \omega^*(t)\dot{\varepsilon}(t)\mathrm{d}t$$
$$= \hbar\omega^*(\xi)\left(\frac{1}{2}-\varepsilon(\eta)\right) + \hbar\omega^*(\bar{\xi})(\varepsilon(\eta)-\varepsilon(\tau)),$$

$\xi\in[0,\eta]$, $\bar{\xi}\in[\eta,\tau]$

let $\epsilon = \frac{\eta}{2\tau}$ and $\eta\to 0$

⇩

$$Q = \hbar\omega^*(\xi)\left(\frac{1}{2}-\varepsilon(\eta)\right) + \hbar\omega^*(\bar{\xi})(\varepsilon(\eta)-\varepsilon(\tau)) \to \frac{\hbar\omega^*(0)}{2} = \frac{\hbar\omega(0^+)}{2}.$$

Sketch of proof——third main result

the speed limit $\frac{D^2}{2\Sigma\langle A\rangle_\tau \tau} \le \frac{1}{k_\mathrm{B}}$ for Markov process $\dot{p}_n = \sum_m k_{nm} p_m$

N. Shiraishi *et al.*, Phys. Rev. Lett 121, 070601 (2018).

1-norm distance $D := \sum |p_i(\tau) - p_i(0)|$

time-averaged dynamical activity $\langle A\rangle_\tau = \frac{1}{\tau}\int_0^\tau \sum_{n\neq m} k_{nm} p_m \mathrm{d}t$

---

we prove

$$D \ge 1 - 2\varepsilon$$

a lower bound of the 1-norm distance

$$\langle A\rangle_\tau \le \frac{\Gamma_0 (N+1)^2}{4}$$

a upper bound of the time-averaged dynamical activity

$\Sigma \le \frac{NQ}{T}$ ⇩

$$\frac{Q\tau}{(1-2\varepsilon)^2} \ge \frac{2}{\beta\Gamma_0 N(N+1)^2}$$