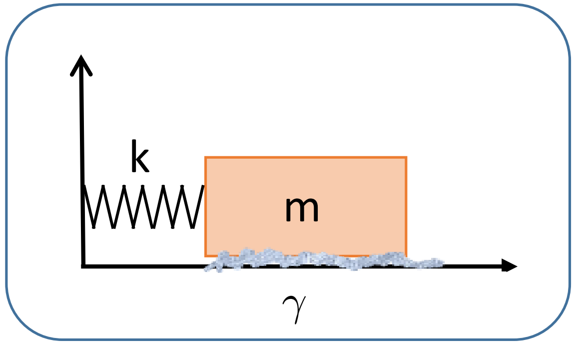
UNIVERSITÉ DE GENÈVE

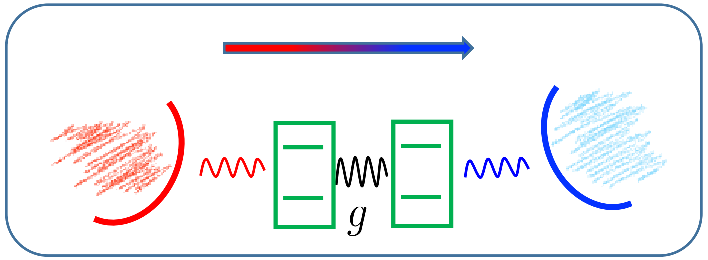
Geneva Quantum Centre

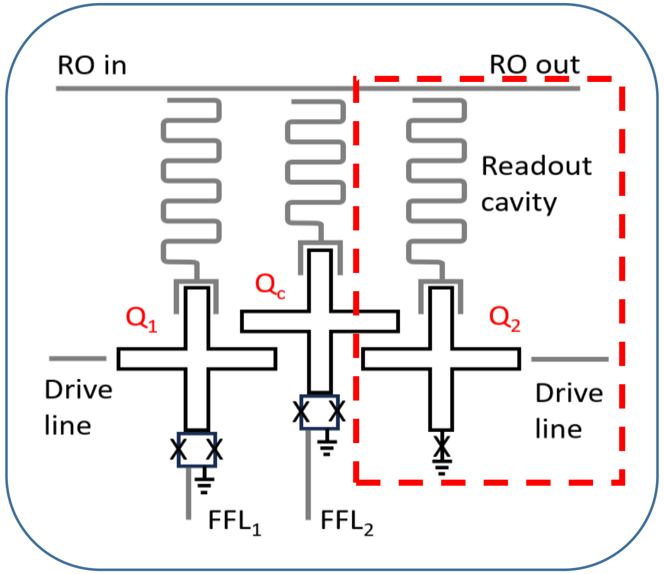
Manipulating entanglement at exceptional points In dissipative quantum systems

Géraldine Haack
University of Geneva, Switzerland

Khandelwal, Brunner, Haack, PRX Quantum 2 (2021)
Khandelwal, Chen, Murch, Haack, arXiv:2310.11381 (2023), PRL





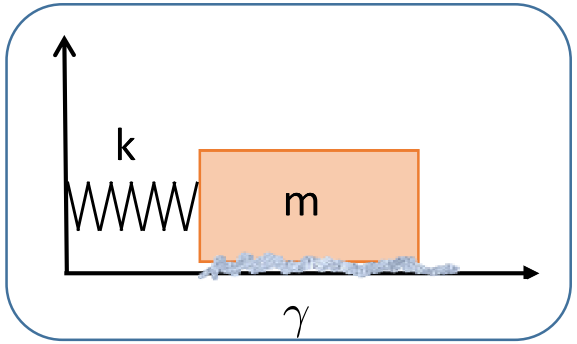


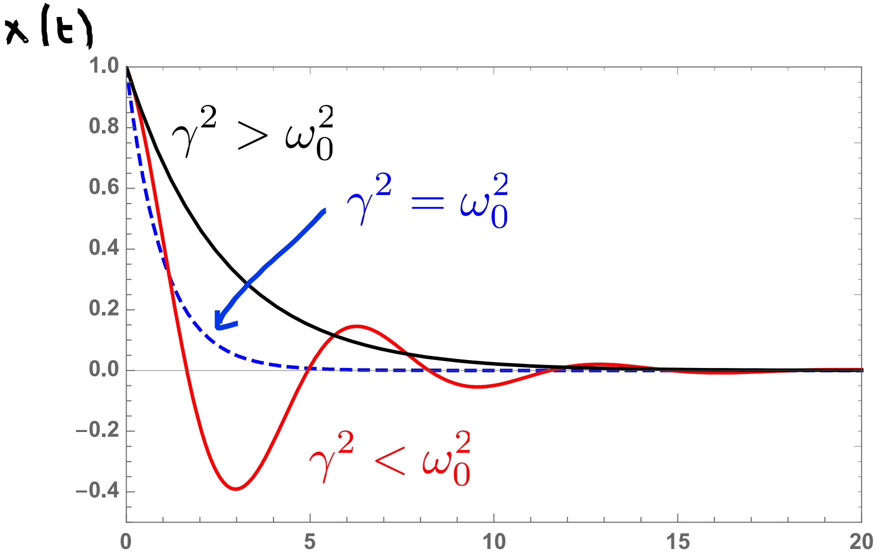
@ Weijian Chen, Kater Murch

Frontiers in Non-equilibrium Physics 2024
Yukawa Institute for Theoretical Physics, Kyoto University, 11.07.2024

Non-Hermitian physics in classical physics

- Damped harmonic oscillator





$$m\ddot{x} + f\dot{x} + kx = 0$$

$$\omega_0 = \sqrt{k/m}$$

$$\gamma = f/(2m)$$

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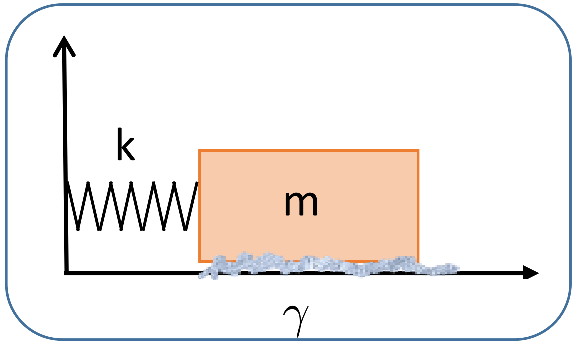

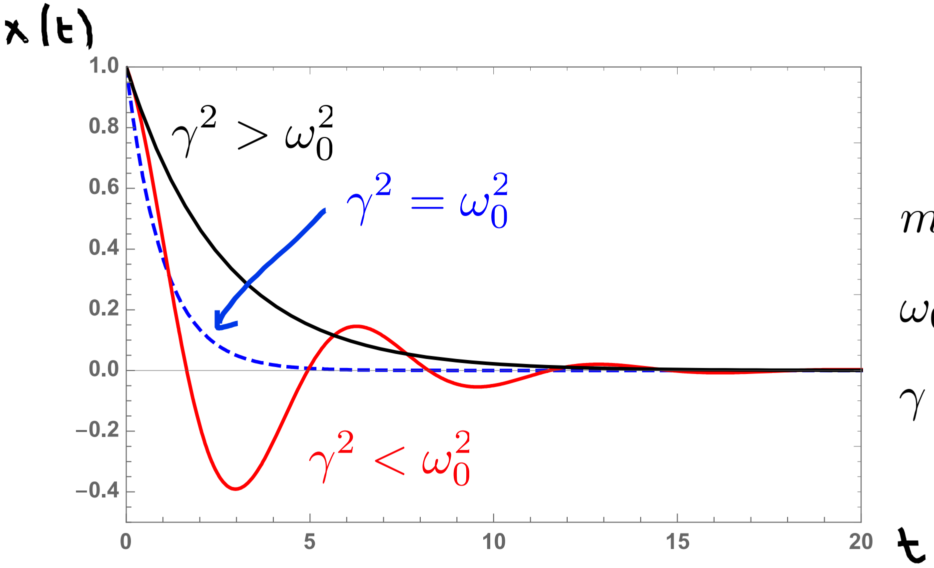

$$m\ddot{x} + f\dot{x} + kx = 0$$

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Evolution equations

$$\begin{pmatrix} \dot{x} \\ \dot{p} \end{pmatrix} = \begin{pmatrix} 0 & 1/m \\ -k & -2\gamma \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$$

$$\lambda_\pm = -\gamma \pm \sqrt{\gamma^2 - k/m}$$

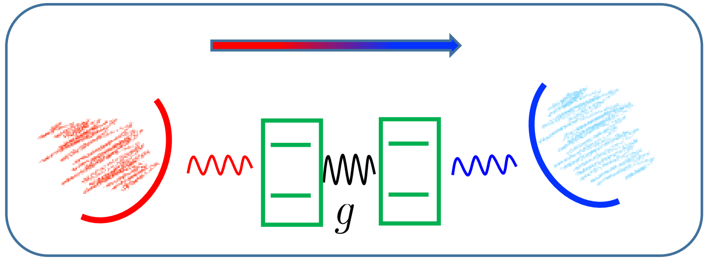
$$\lambda_+ = \lambda_- \qquad |v_+\rangle = |v_-\rangle$$

degenerescence of two eigenvalues
AND
coalescence of their respective eigenvectors.

-> Exceptional point

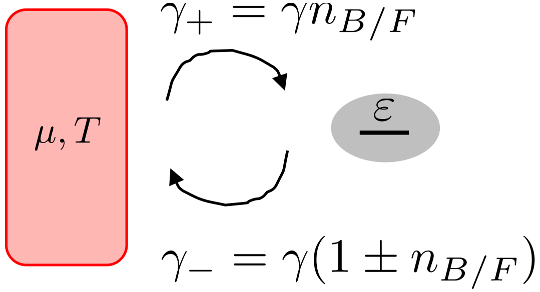
Critical damping happens at the exceptional point of the evolution matrix of the damped h.o.

In an open quantum system ?



$$H_s = E\big(|1\rangle\langle 1|_1 + |1\rangle\langle 1|_2\big)$$

$$H_{int} = g\,(|01\rangle\langle 10| + h.c.)$$



$$\gamma_+ = \gamma n_{B/F}$$

$$\gamma_- = \gamma(1 \pm n_{B/F})$$

$$n_F = \frac{1}{e^{(E-\mu)/(k_B T)} + 1}$$

- **Lindblad master equation:** $\dot{\rho}_S = -\frac{i}{\hbar}[H_S, \rho_S] + \sum_i \left(\gamma_+^{(i)} \mathcal{D}[\sigma_+^{(i)}]\rho_S + \gamma_-^{(i)} \mathcal{D}[\sigma_-^{(i)}]\rho_S\right)$

  → $\dot{\rho}_S = \mathcal{L}\rho_S$ with $\mathcal{L}$ a non-Hermitian Liouvillian

- **General solution:** $\rho_S(t) = e^{\mathcal{L}t}\rho_S(0)$

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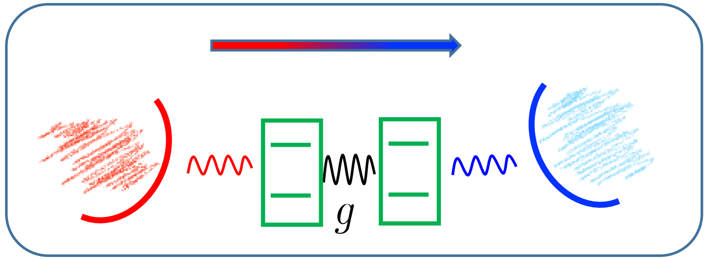

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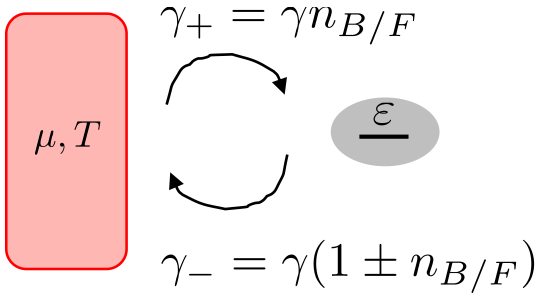

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$$\longrightarrow \quad \dot{\rho}_S = \mathcal{L}\rho_S \qquad \text{with } \mathcal{L} \text{ a non-Hermitian Liouvillian}$$

- <u>General solution:</u> $\rho_S(t) = e^{\mathcal{L}t}\rho_S(0)$

- If the Liouvillian is diagonalizable with eigenvalues λ_i, $\mathcal{L} = SLS^{-1}$, the solution is simply:

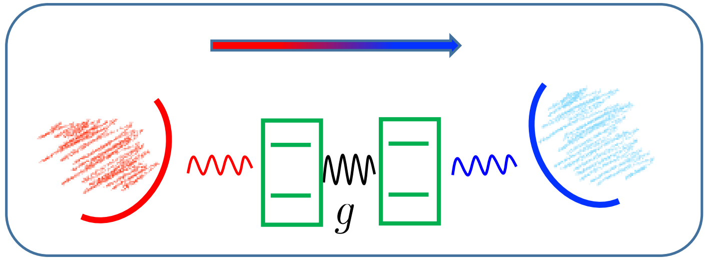
$$\rho_S(t) = S \begin{pmatrix} e^{\lambda_1 t} & 0 & \dots & 0 \\ 0 & e^{\lambda_2 t} & \dots & 0 \\ \vdots & 0 & \dots & 0 \\ 0 & 0 & \dots & e^{\lambda_{N^2} t} \end{pmatrix} S^{-1}\rho_S(0) \longrightarrow \rho_S(t) = \sum_i c_i e^{\lambda_i t} v_i^{(R)} = \rho_{ss} + \sum_{i \neq 1} c_i e^{\lambda_i t} v_i^{(R)}$$

Steady state

Decay modes

The spectrum of the Liouvillian sets the decay modes of the quantum system towards its steady state.

In an open quantum system ?



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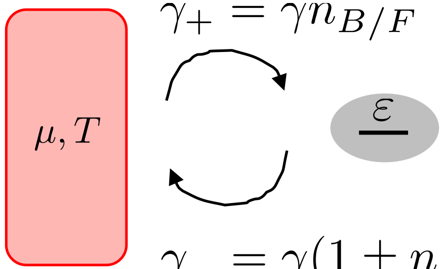

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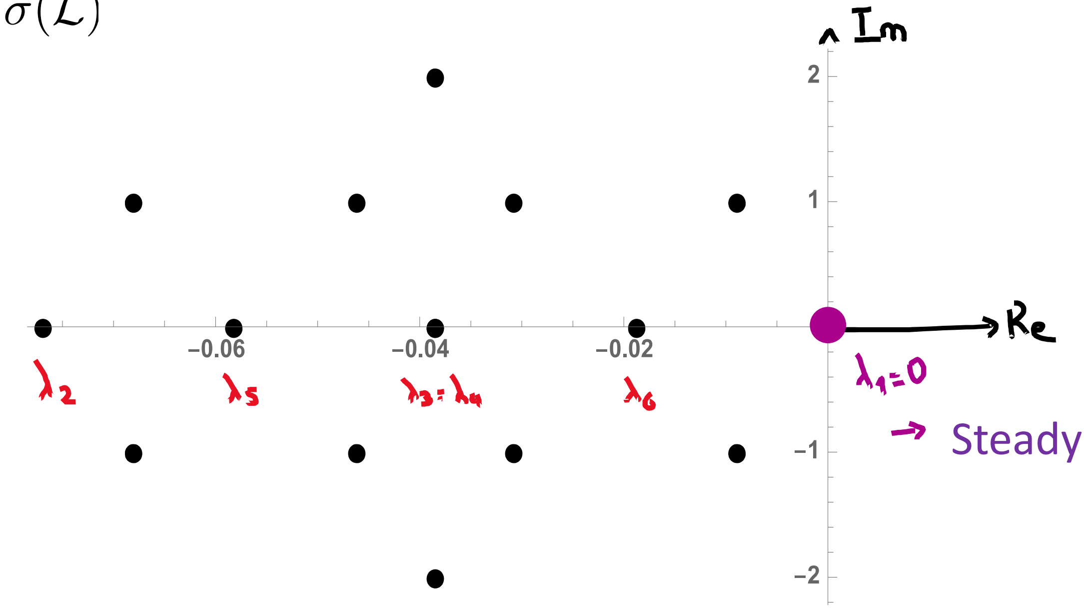
$\sigma(\mathcal{L})$



→ Steady-state

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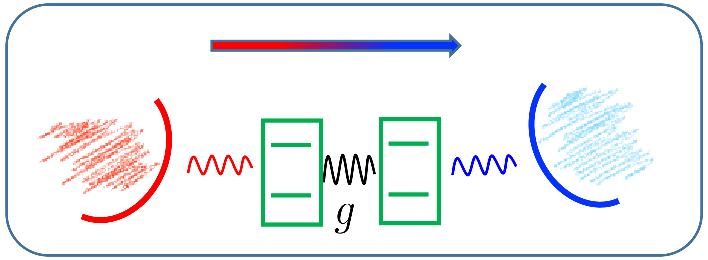

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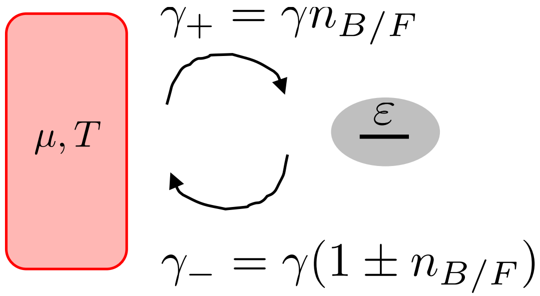

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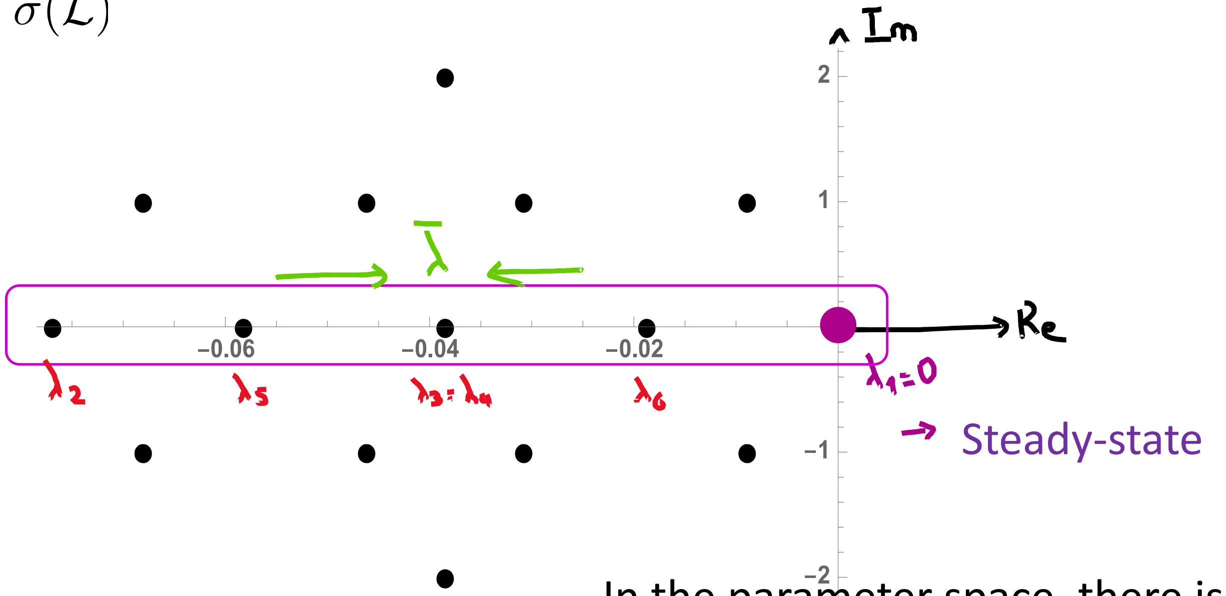
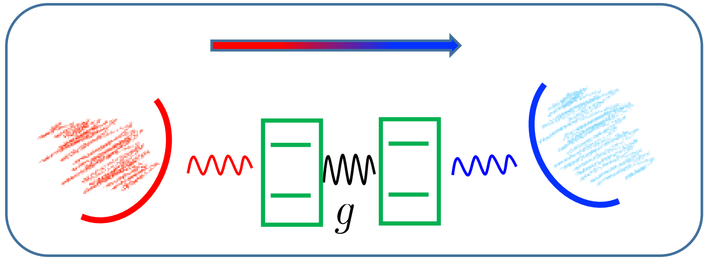


3rd-order exceptional point

$$g = (\Gamma_1 - \Gamma_2)/2 \qquad \Gamma_i = \gamma_{i,+} + \gamma_{i,-}$$

In the parameter space, there is a solution s.t. $\lambda_4 = \lambda_5 = \lambda_6$ & $|v_4\rangle = |v_5\rangle = |v_6\rangle$

In an open quantum system ?



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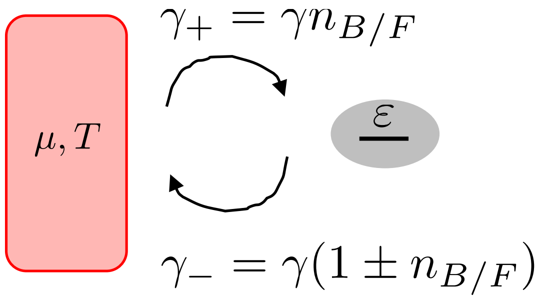

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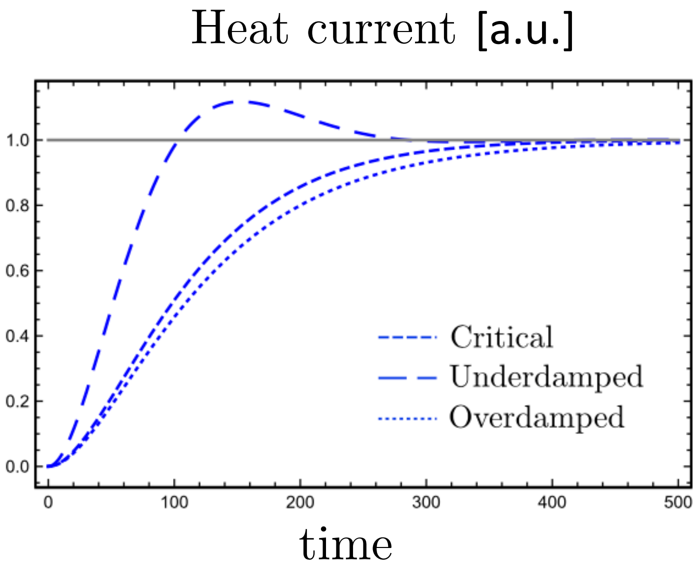

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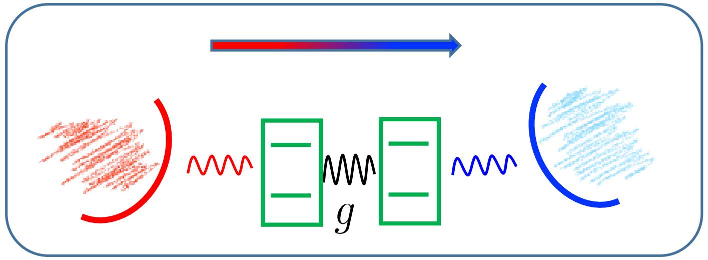

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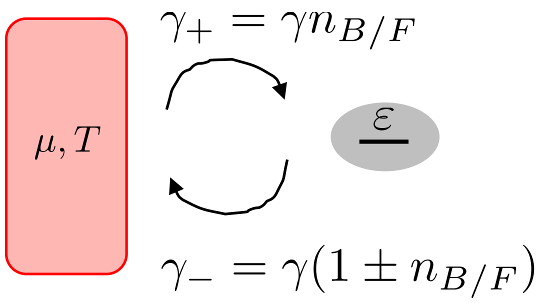

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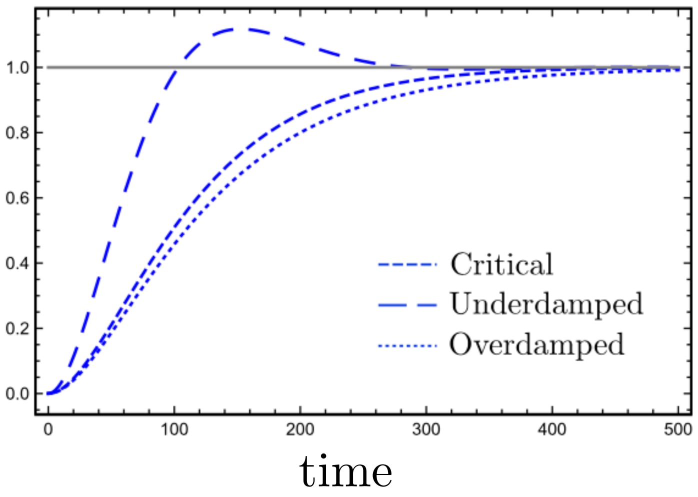
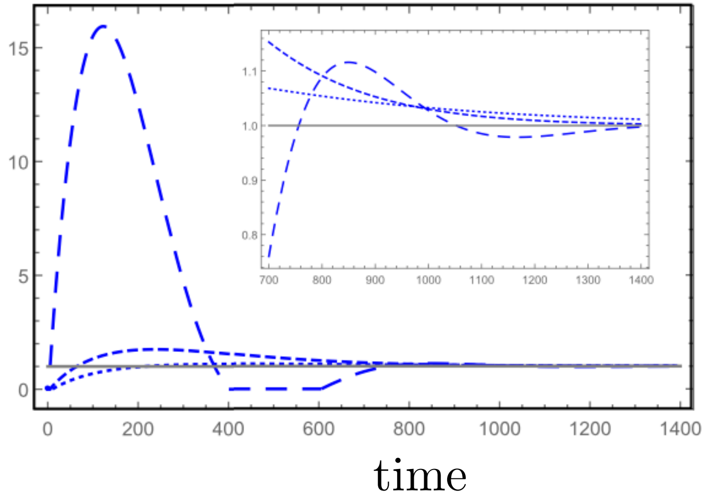
Heat current [a.u.]



Concurrence [a.u.]



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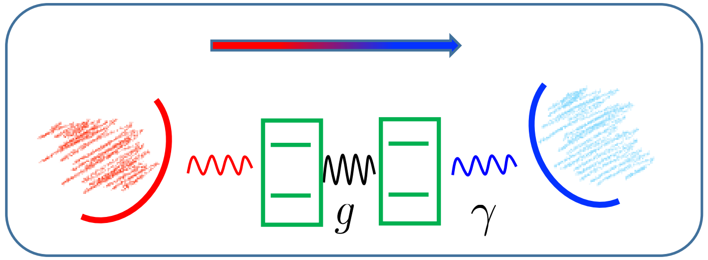
Manipulation of entangled states



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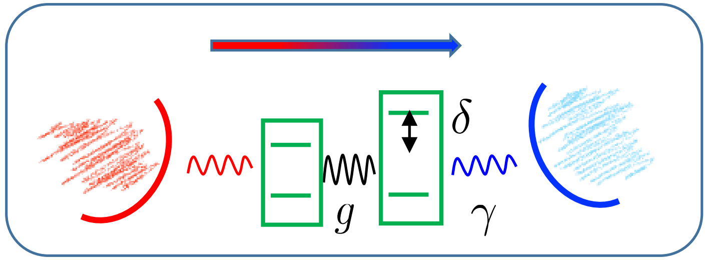

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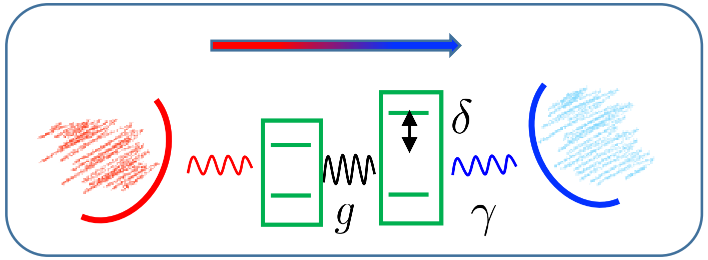

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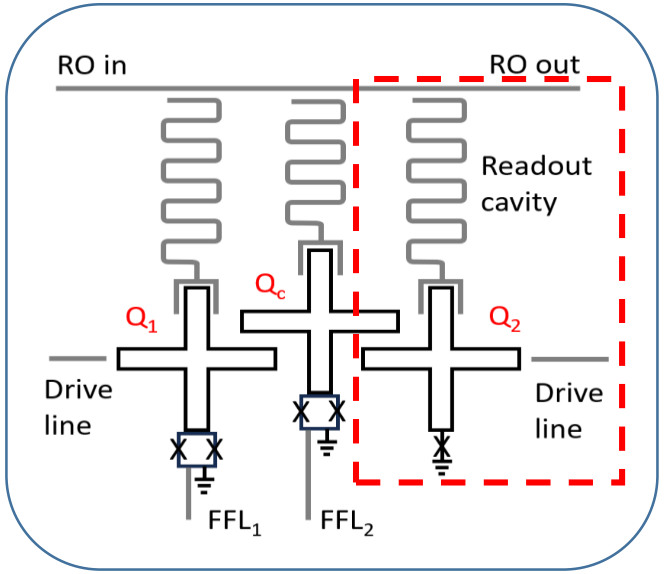

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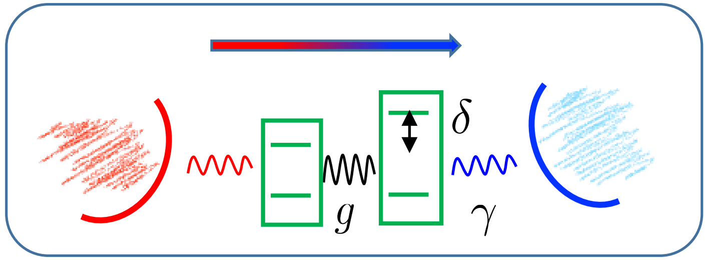

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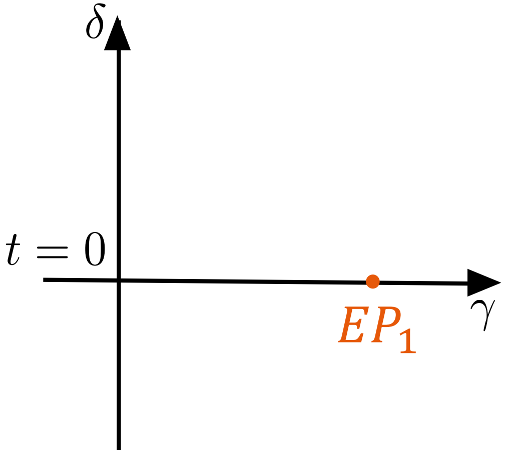

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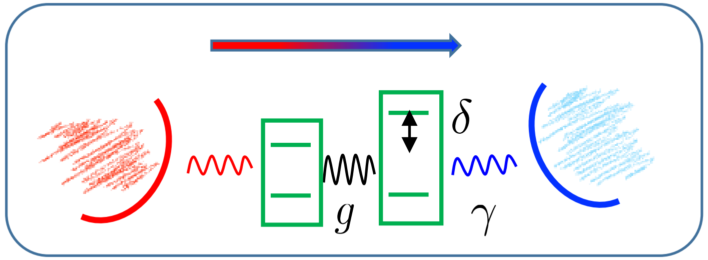

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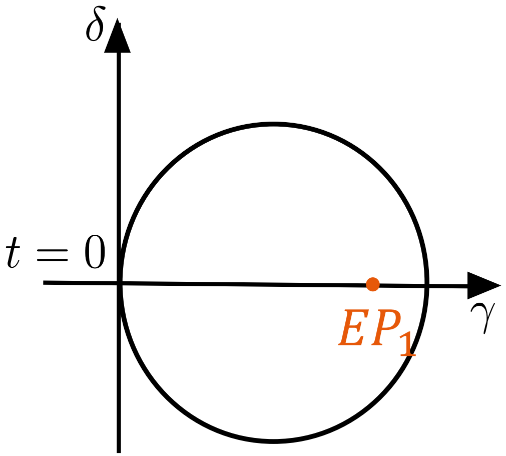

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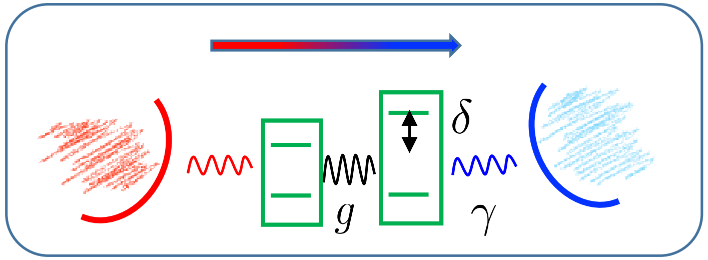

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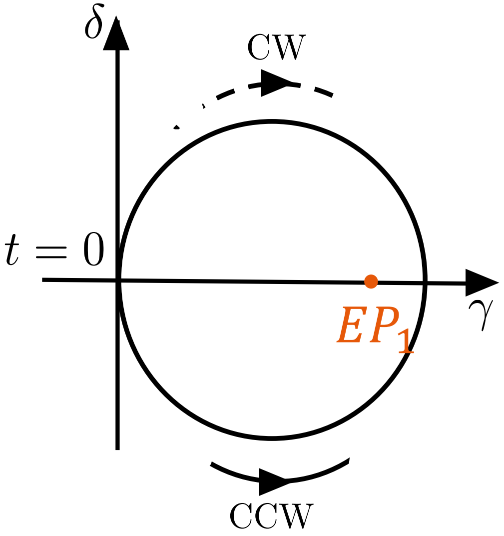

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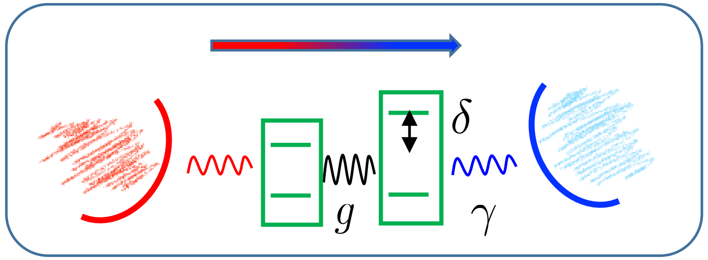

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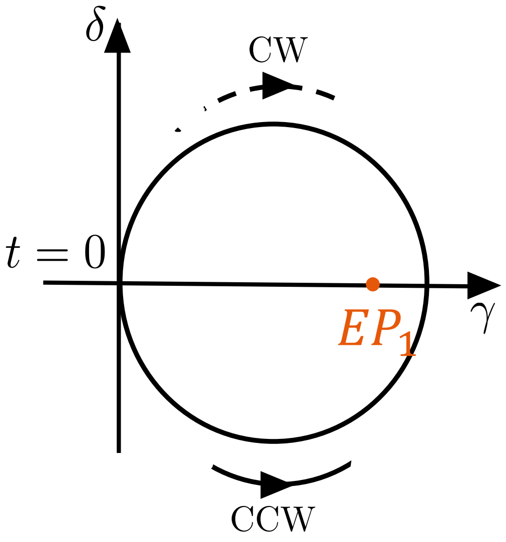

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Dynamics along this contour: $\rho(t) = \mathcal{T} e^{\int_0^t \mathcal{L}_{[q]}(t')dt'} \rho(0)$

At each time, we compute the fidelity with the singlet Bell state (maximally entangled state):

$$\mathcal{F}_{|\Psi^-\rangle}(t) = \mathrm{Tr}\{|\Psi^-\rangle\langle\Psi^-|\rho(t)\}$$

First, we start with the qubits in an orthogonal Bell state with respect to the singlet (overlap 0).

$$\rho(t=0) = |\Psi^+\rangle\langle\Psi^+|$$

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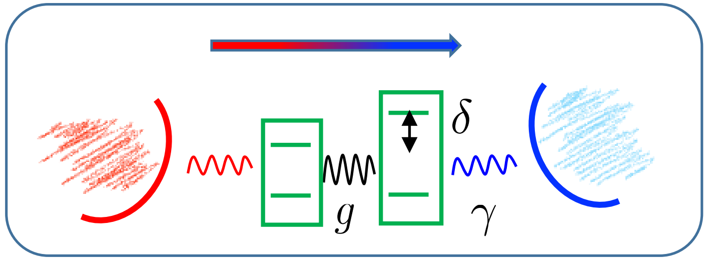

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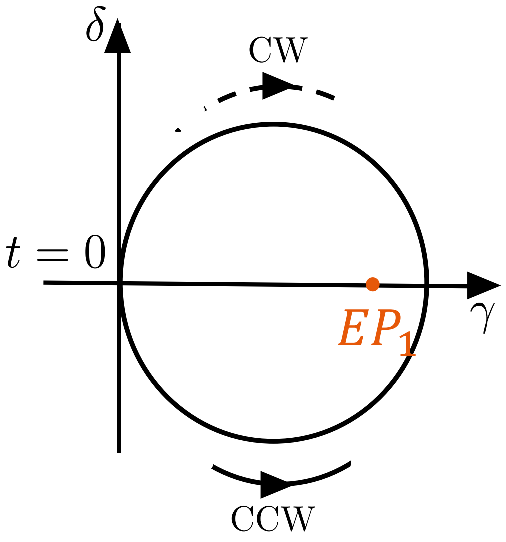

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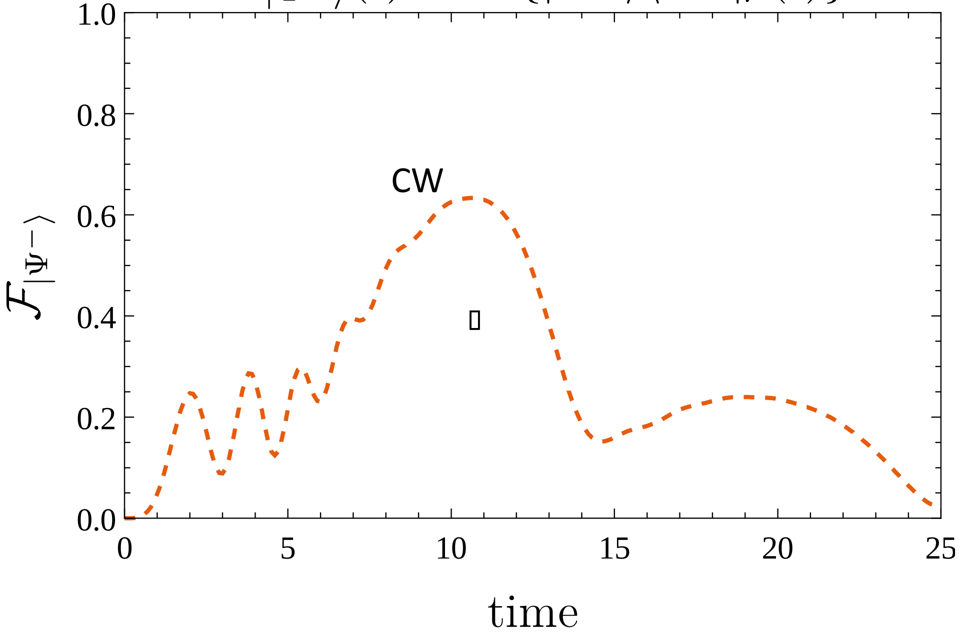

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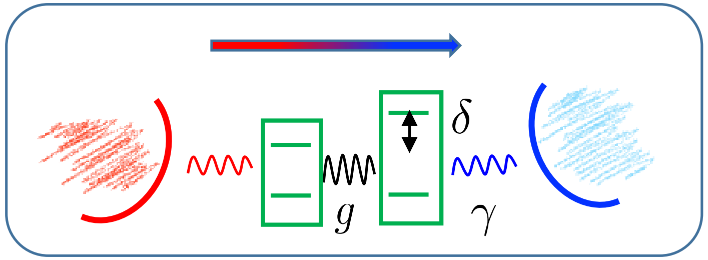

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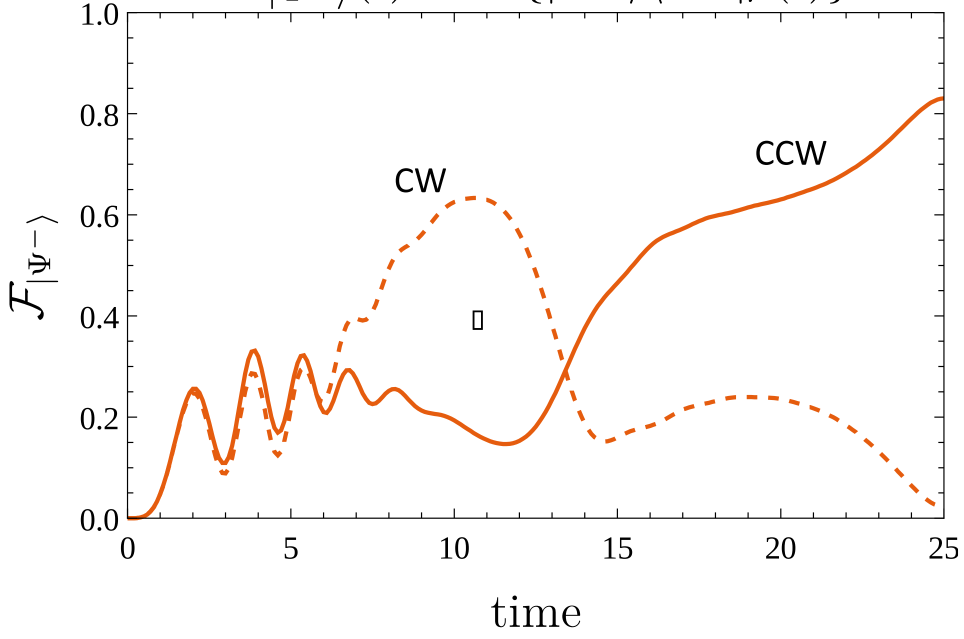
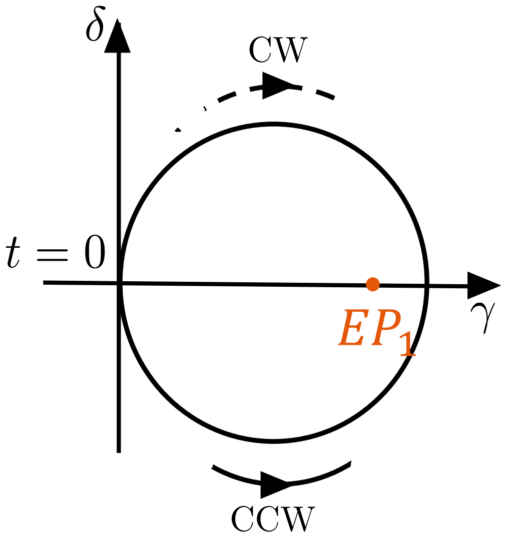


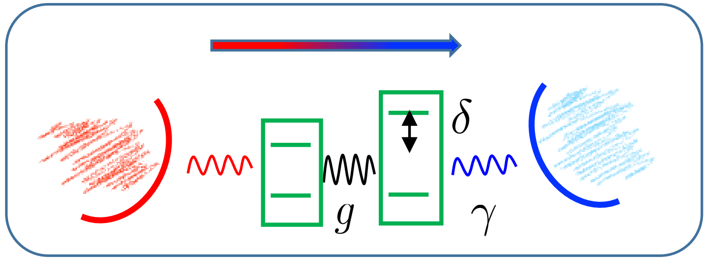


$$\delta(t) = \pm \Delta\delta \sin\left(\frac{2\pi t}{T}\right)$$

$$\gamma(t) = \gamma_0 + \Delta\gamma \sin^2\left(\frac{\pi t}{T}\right)$$

Khandelwal et al., arXiv:2310.11381 (2023), accepted in PRL

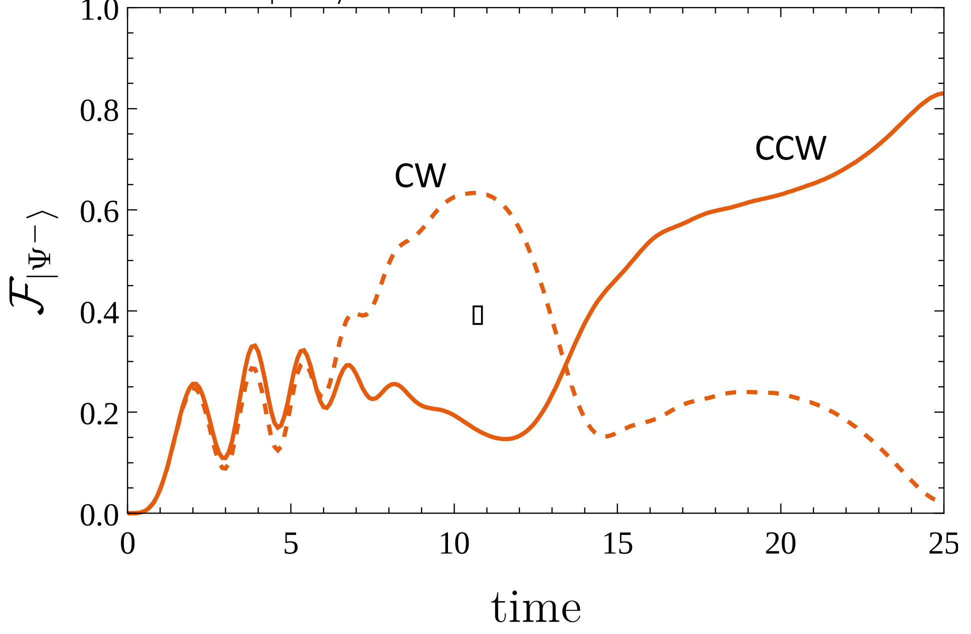
Manipulation of entangled states

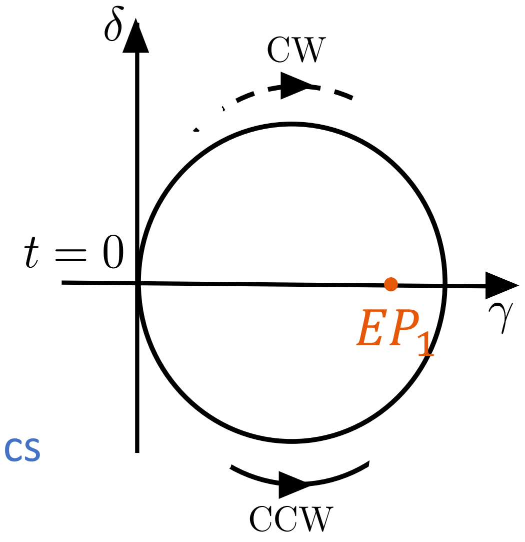


$$\dot{\rho} = -i[H_S, \rho_S] + \sum_{j=1,2} \gamma_j^+ \left(\cancel{\sigma_+^{(j)} \rho_S \sigma_-^{(j)}} - \frac{1}{2}(\sigma_-^{(j)}\sigma_+^{(j)}\rho_S + \rho_S \sigma_-^{(j)}\sigma_+^{(j)}) \right)$$

Post-selection – no quantum jump – effective Hamiltonian dynamics

$$\mathcal{F}_{|\Psi^-\rangle}(t) = \mathrm{Tr}\{|\Psi^-\rangle\langle\Psi^-|\rho(t)\}$$





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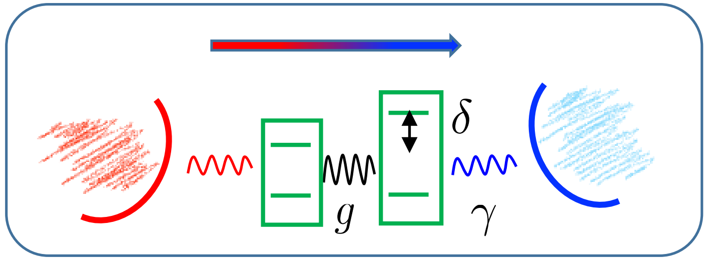

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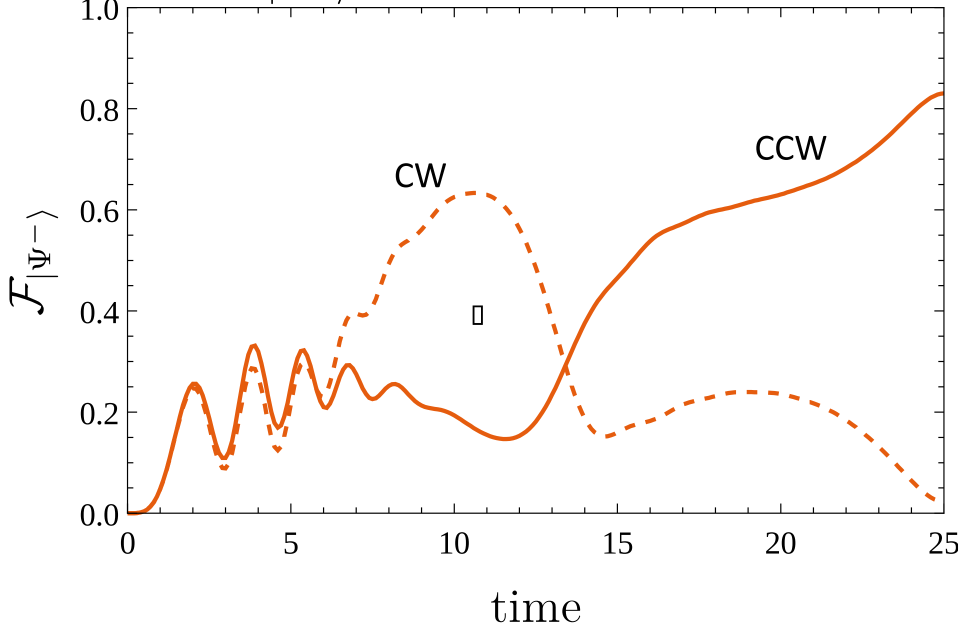

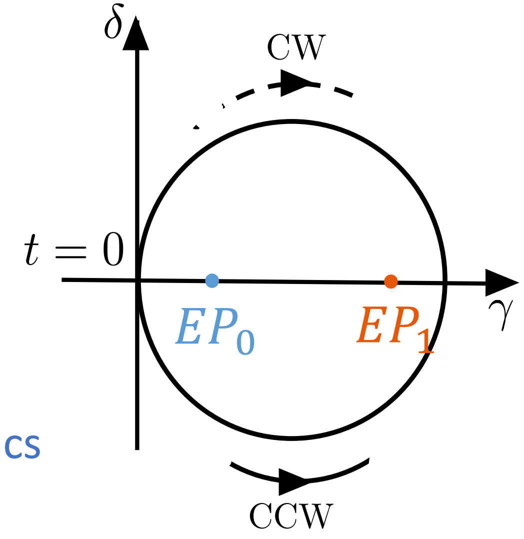

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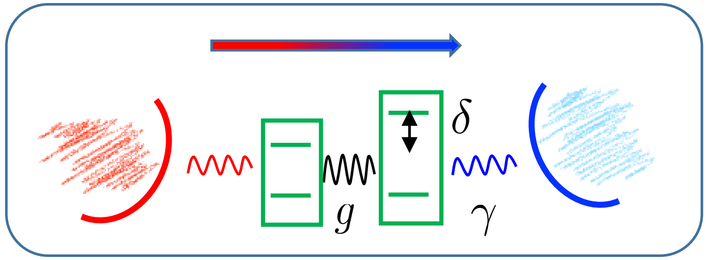

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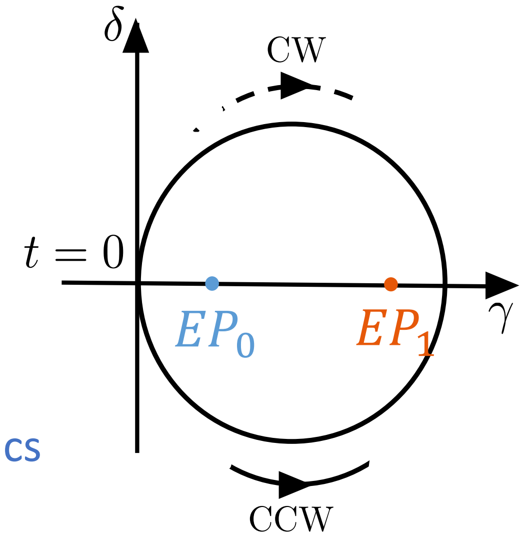
Post-selection – no quantum jump – effective Hamiltonian dynamics



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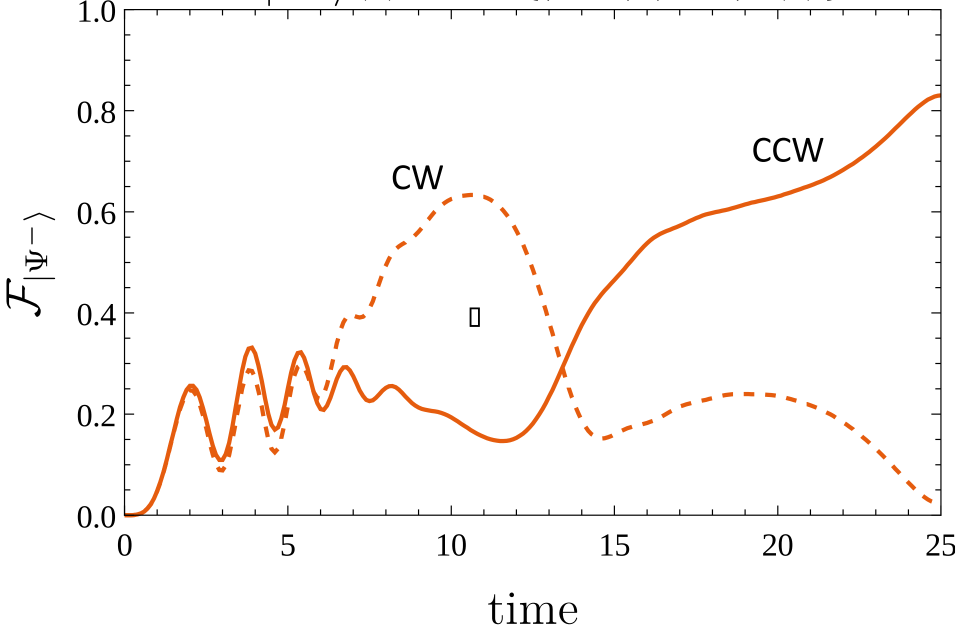

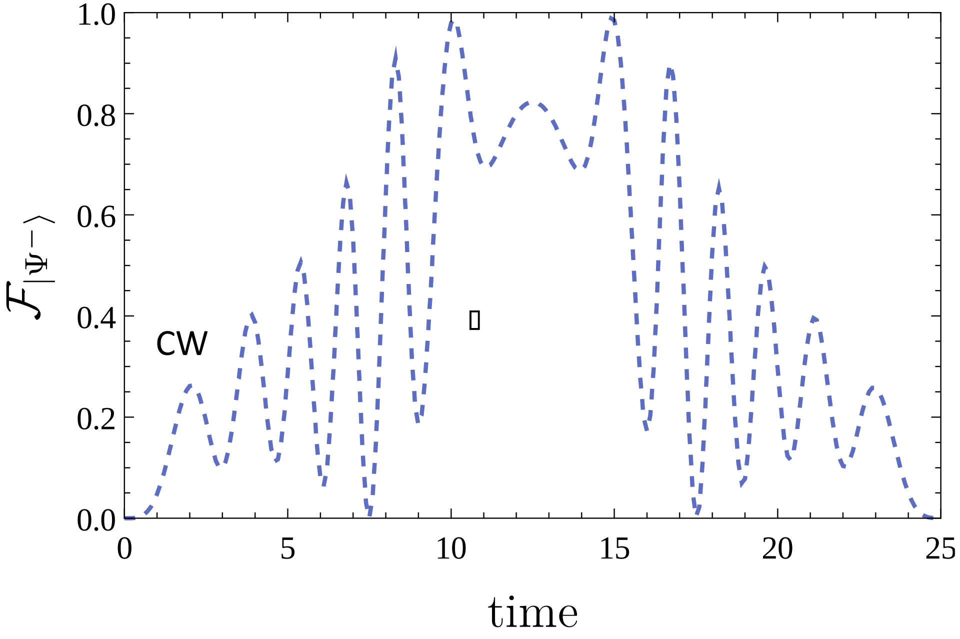

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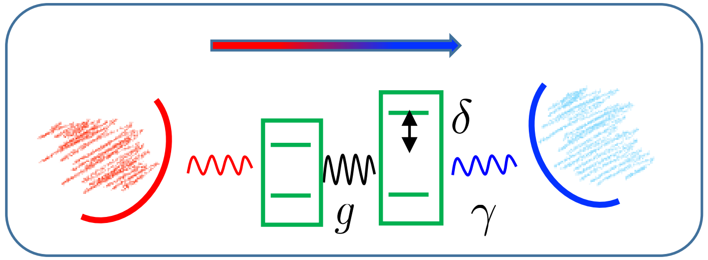
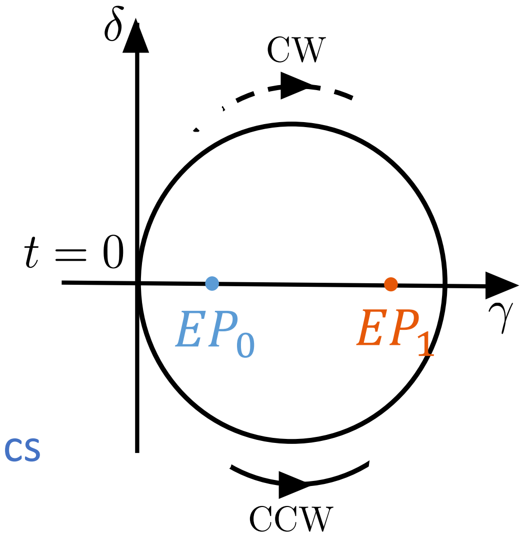
Manipulation of entangled states





$$\delta(t) = \pm\Delta\delta\sin\left(\frac{2\pi t}{T}\right)$$

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Post-selection – no quantum jump – effective Hamiltonian dynamics

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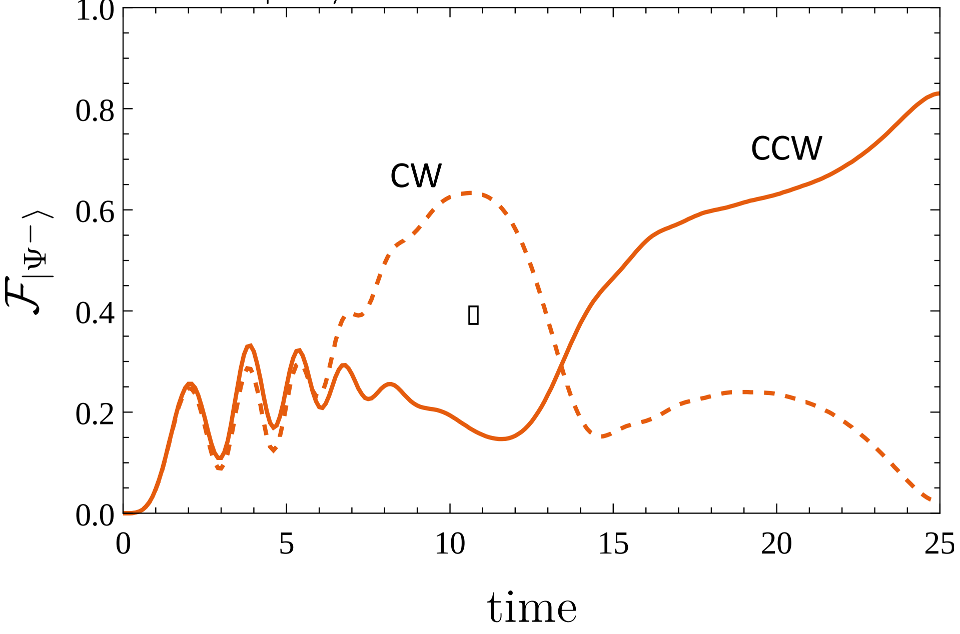

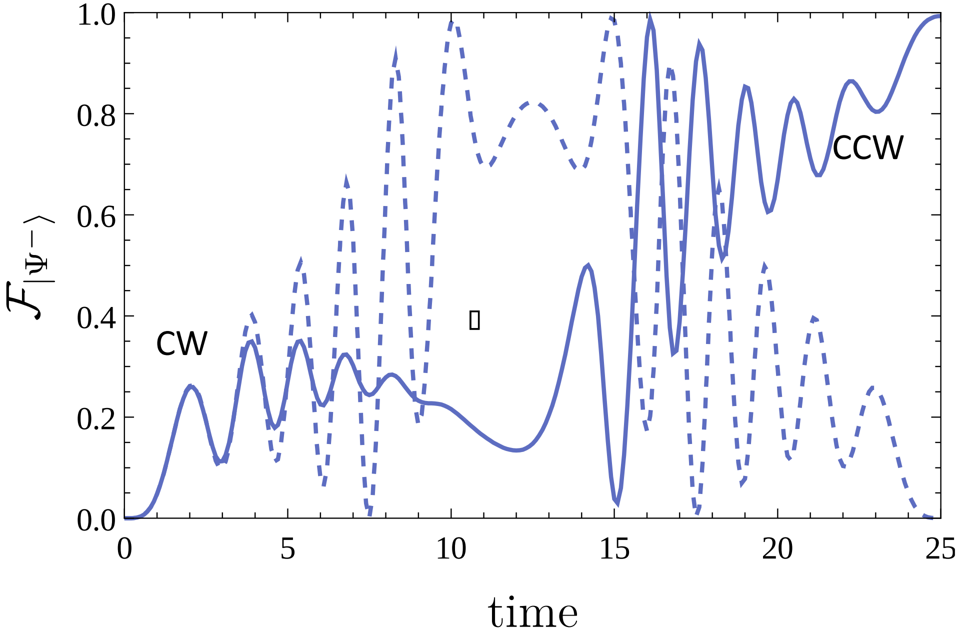

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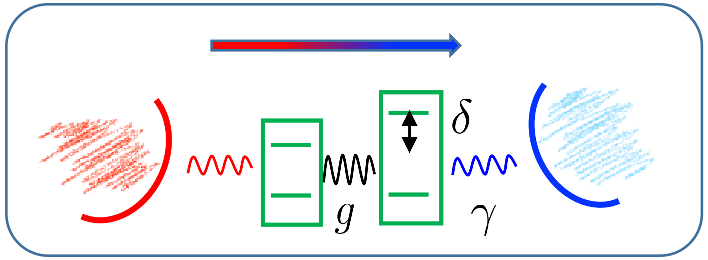
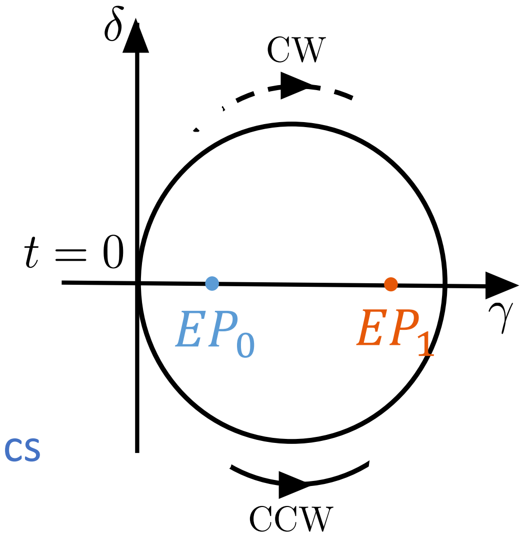
Manipulation of entangled states





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$$\delta(t) = \pm\Delta\delta \sin\left(\frac{2\pi t}{T}\right)$$

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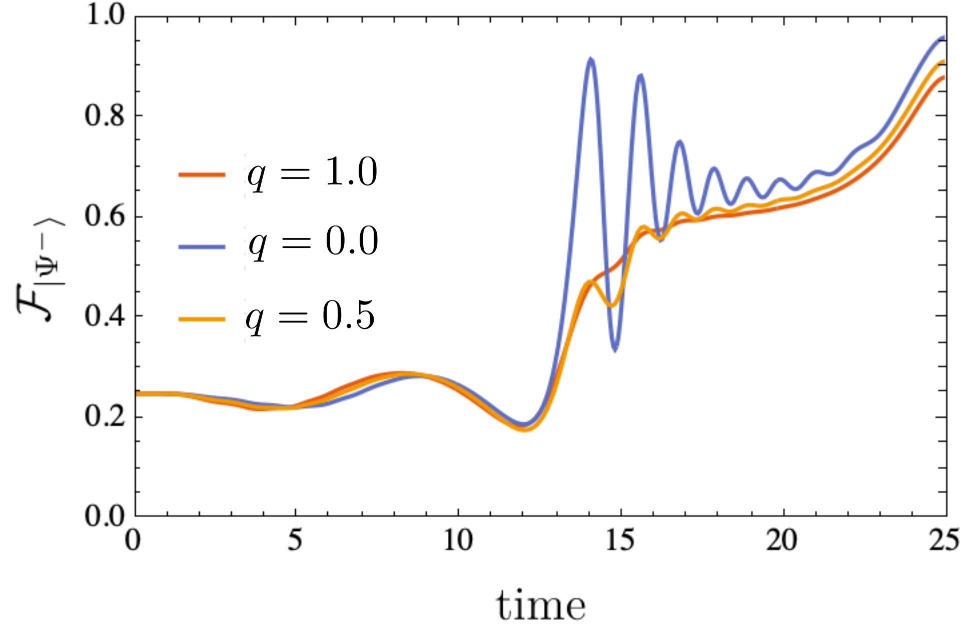
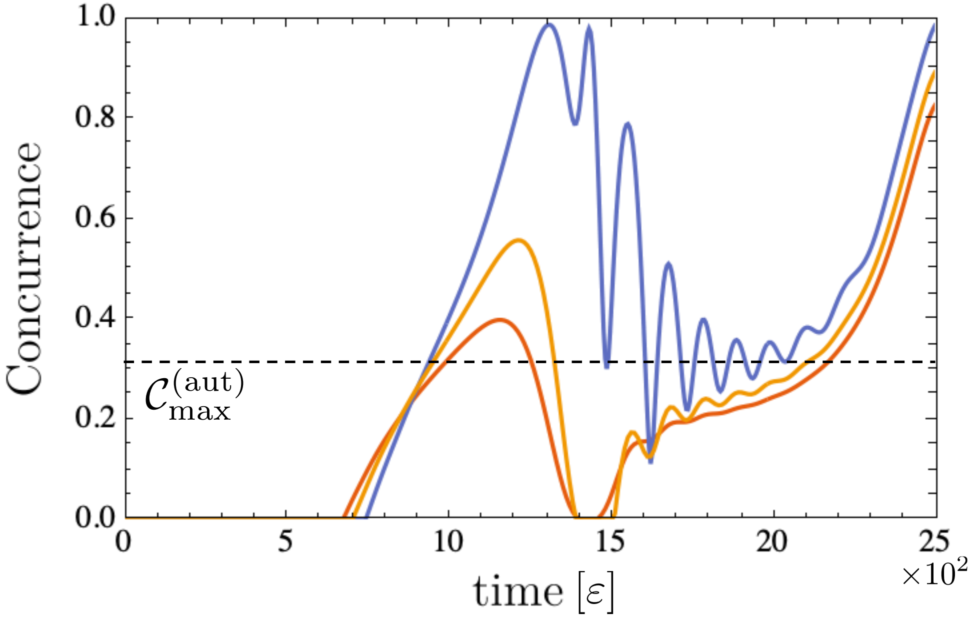
$\rho(0) = \mathbf{1}/4$ (separable state)





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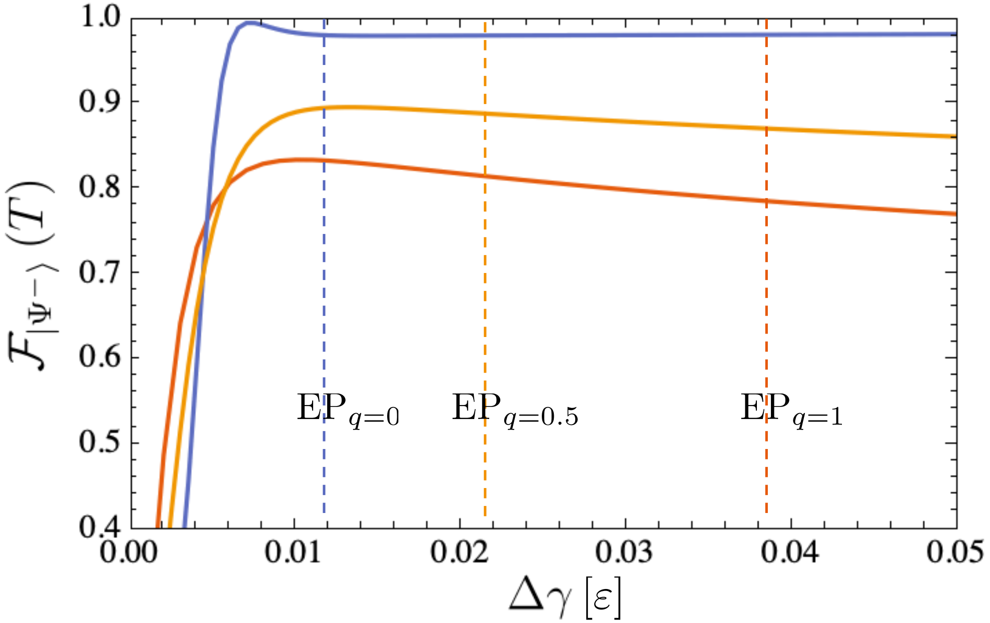
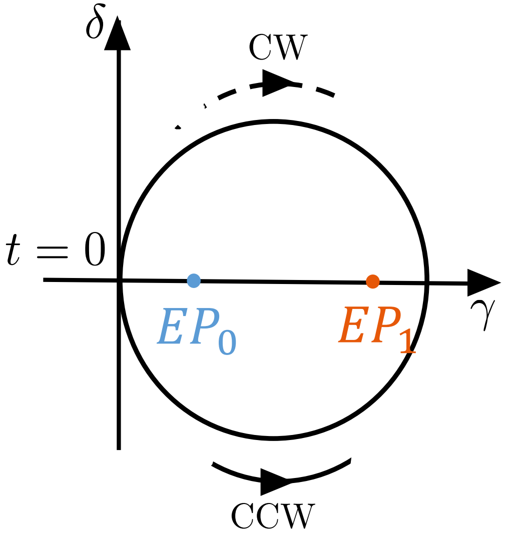
Manipulation of entangled states

It also works when the trajectory is only close to the EP !





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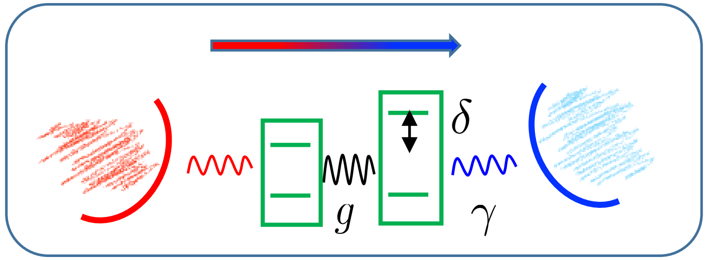
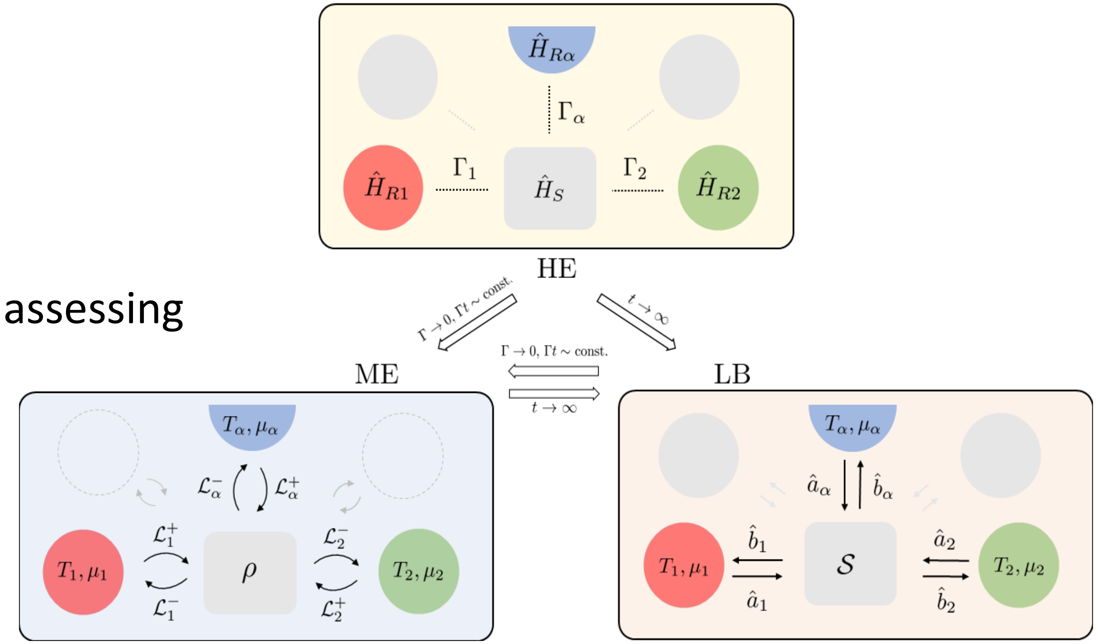
Summary & Perspectives

Uncontrolled dissipation as a resource to generate and manipulate entanglement



Bohr Brask, Haack, Brunner, Huber, NJP 17 (2015)
Khandelwal, Brunner, Haack, PRX Quantum 2 (2021)
Khandelwal, Chen, Murch, Haack, arXiv:2310.11381 (2023), PRL

Connections between theoretical frameworks for assessing the dynamics of open quantum systems



Blasi, Khandelwal, Haack, arXiv:2312.15065 (2023)
Bourgeois, Blasi, Khandelwal, Haack, Entropy 26 (2024)

Non-Hermitian and topological properties of Lindbladians towards quantum sensing