Measurement-induced Spectral Transition

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arXiv:2406.18234.

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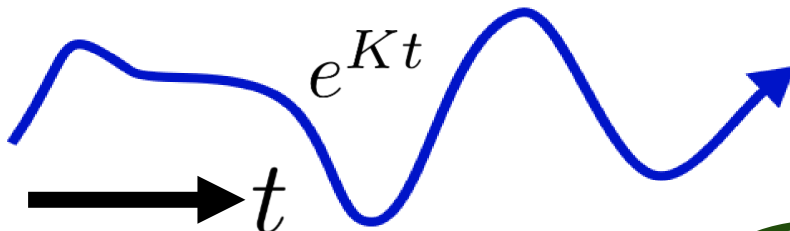
科研費 KAKENHI

Spectrum in quantum systems

systems described by time-independent generators

$$\frac{d}{dt}\left|\psi(t)\right\rangle = -iH\left|\psi(t)\right\rangle, \quad \frac{d}{dt}\hat{\rho}(t) = \mathcal{L}\hat{\rho}(t), \cdots$$

$\left|\psi(0)\right\rangle, \hat{\rho}(0)$

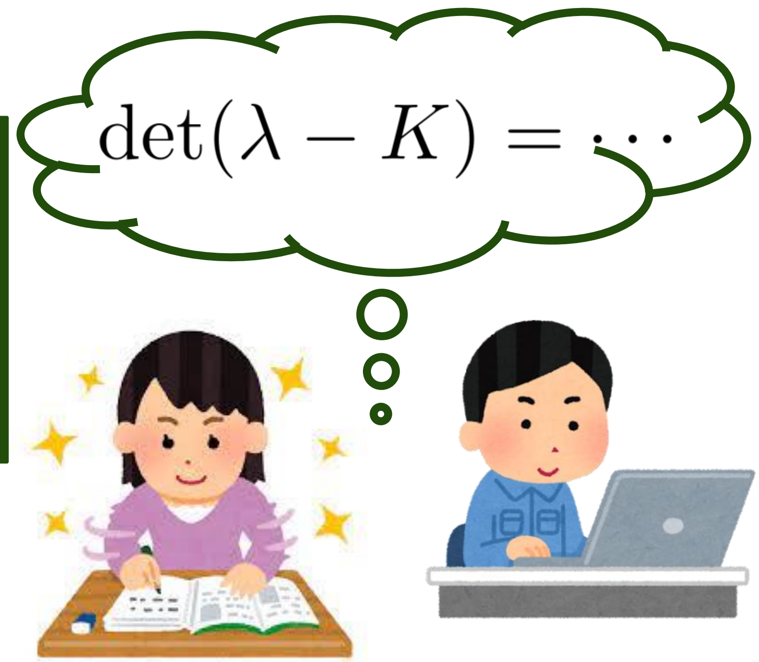


$\left|\psi(t)\right\rangle, \hat{\rho}(t)$

the spectrum of $K = -iH, \mathcal{L}$

↓

essential information of the system



Spectral gap in open quantum systems

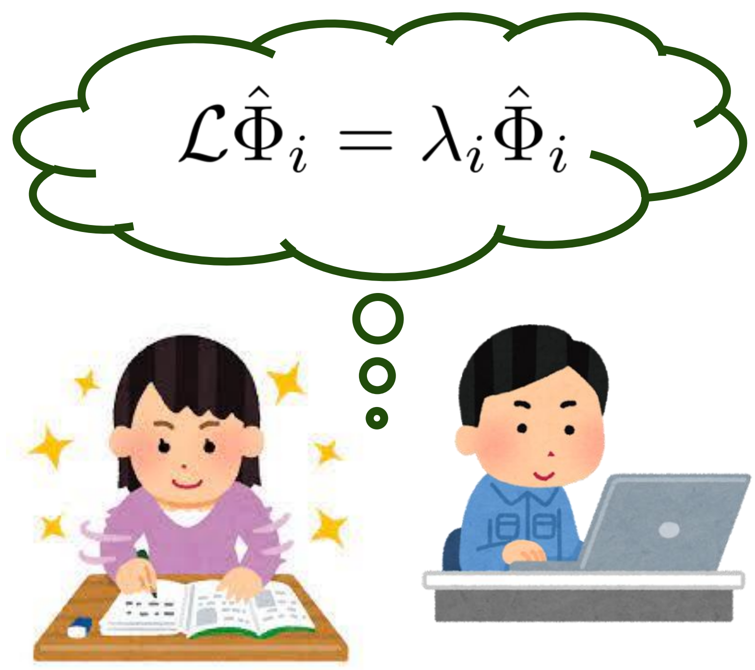
open quantum systems

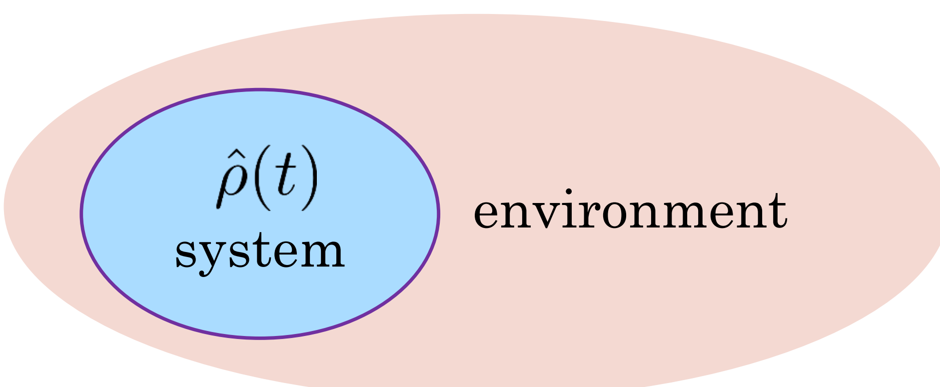
$$\frac{d}{dt}\hat{\rho}(t) = \mathcal{L}\hat{\rho}(t) \qquad \hat{\rho}(t) = e^{\mathcal{L}t}\hat{\rho}(0) = \sum_i c_i e^{\lambda_i t}\hat{\Phi}_i$$

the spectral gap $\Delta = |\mathrm{Re}(\lambda_2) - \mathrm{Re}(\lambda_1)|, \quad \mathrm{Re}(\lambda_j) \geq \mathrm{Re}(\lambda_{j+1})$

→ **the relaxation time to the stationary state** $1/\Delta$

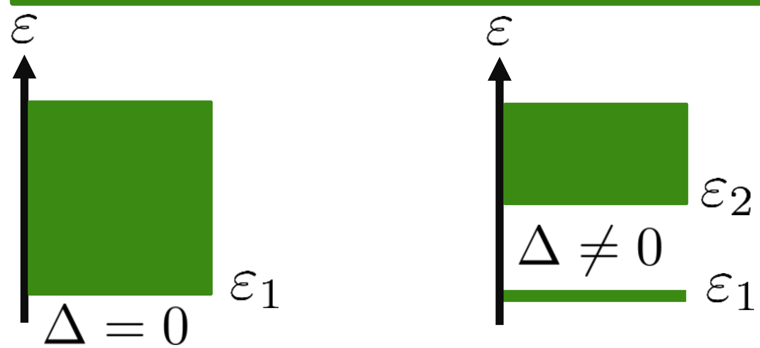
$$t \gg \frac{1}{\Delta} \to \hat{\rho}(t) \simeq \hat{\Phi}_1$$

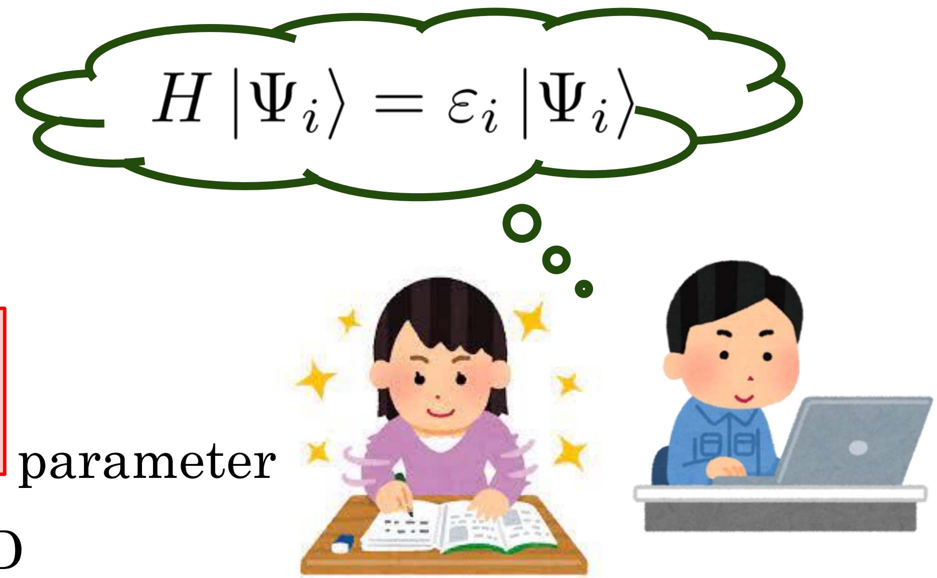


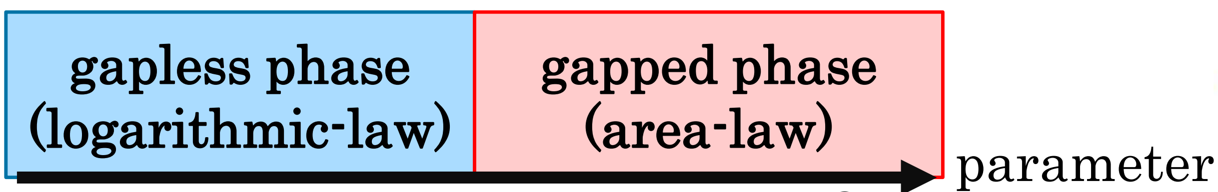


Spectral gap in isolated quantum systems

the spectral gap $\Delta = \varepsilon_2 - \varepsilon_1, \quad \varepsilon_i \leq \varepsilon_{i+1}$
transition of the gap ⟷ ground-state phase transition



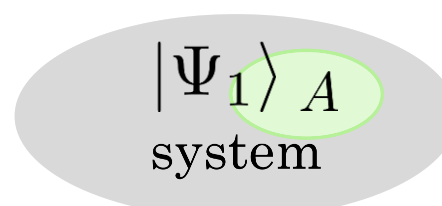




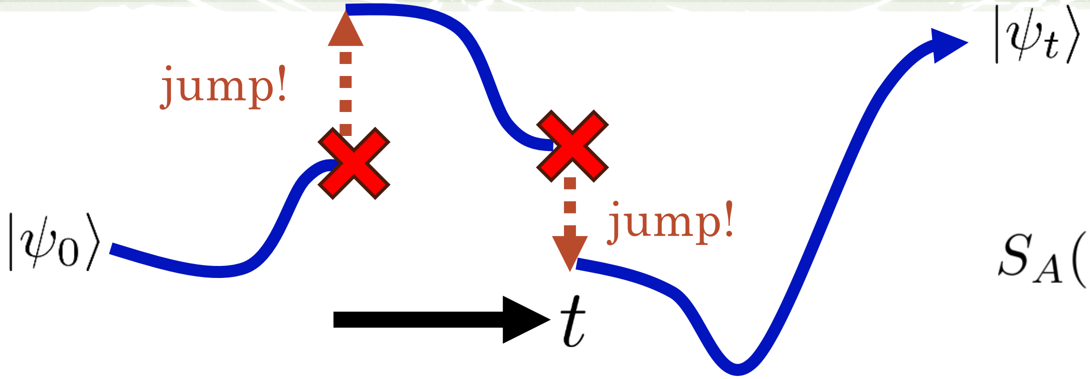
$S_A(\Psi_1) \propto \log(|A|)$ $\quad S_A(\Psi_1) \propto |A|^0$ *1D

$S_A(\Psi_1)$: bipartite entanglement entropy of the ground state for the subsystem A

an indicator of how complex the state is

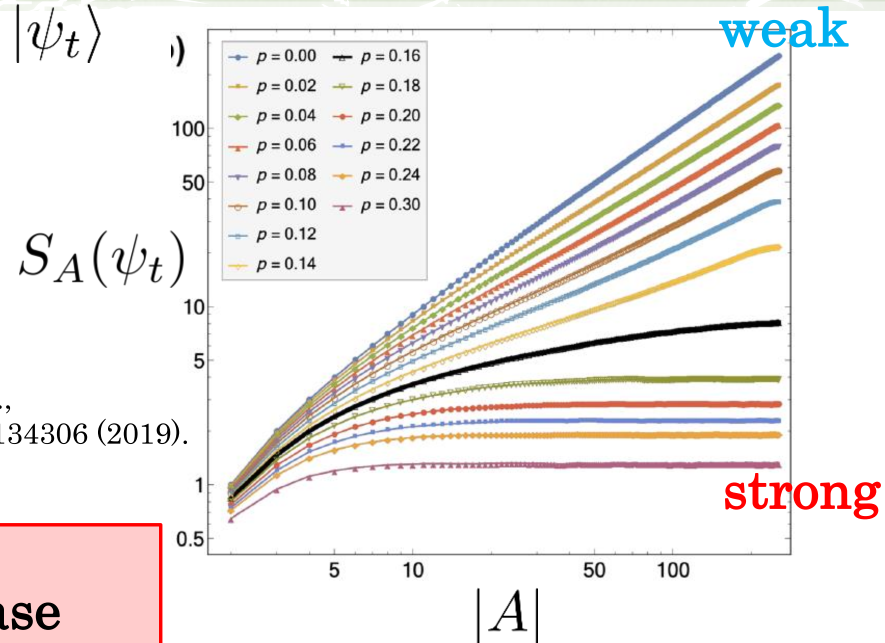


Entanglement transition in monitored dynamics



❌ quantum measurement

Y. Li et. al.,
PRB **100**, 134306 (2019).



volume-law phase	**area-law phase**
$S_A(\psi_t) \propto |A|$	$S_A(\psi_t) \propto |A|^0$

→ strength of measurement

temporal randomness due to measurement
➡ **no static generator** $|\psi_t\rangle \neq e^{Kt}|\psi_0\rangle$

Objective

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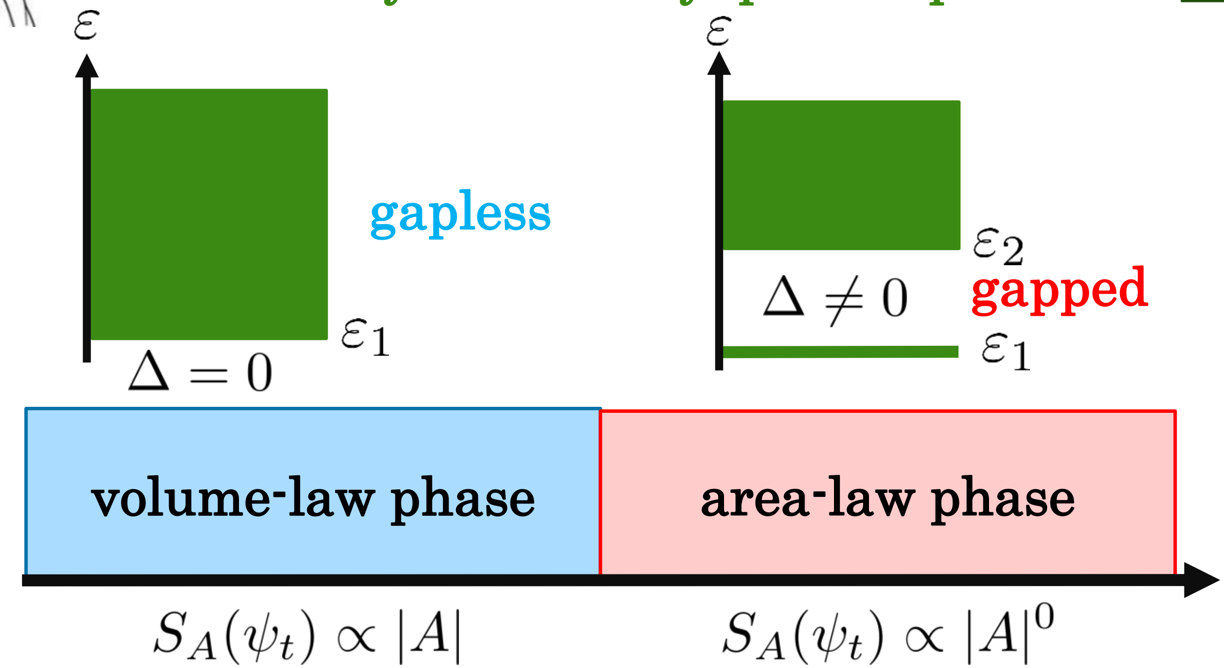
Measurement-induced entanglement transitions are related to some spectral features??

analysis of the Lyapunov spectrum → **YES**



quantum measurements
⇒ entanglement transitions without static generator $|\psi_t\rangle \neq e^{Kt}|\psi_0\rangle$

Model

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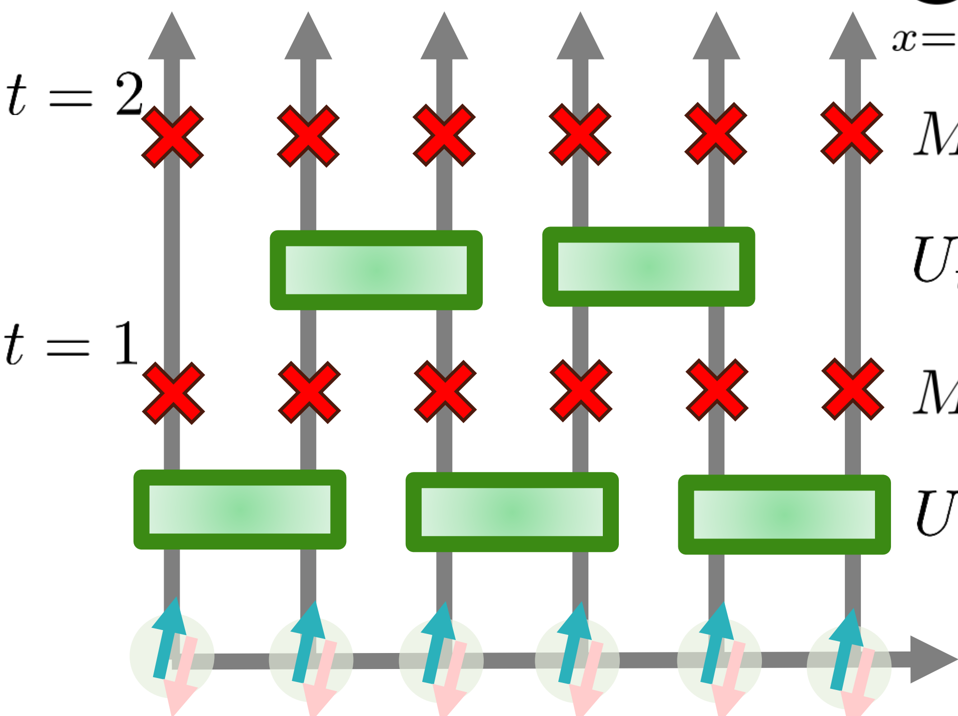
temporally random dynamics conditioned on measurement outcomes

$$|\psi_t(\eta)\rangle = \frac{1}{N_t(\eta)} M_t(\eta) U_t M_{t-1}(\eta) U_{t-1} \cdots M_1(\eta) U_1 |\psi_0\rangle$$

$N_t(\eta)$: normalization

$$M_t(\eta) = \bigotimes_{x=1}^{L} \mathsf{M}_{\omega_{t,x}}(\eta), \quad \omega_{t,x} = \pm \text{ (Born rule)}$$



 generalized measurement

$$\mathsf{M}_{\pm}(\eta) = \frac{\sigma_0 \pm \eta\sigma_3}{\sqrt{2(1+\eta^2)}}$$

η : strength of measurement

 random unitary gate

 1/2 spin

Lyapunov analysis

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$$|\psi_t(\eta)\rangle \propto V_t(\eta)\,|\psi_0\rangle\,, \quad V_t(\eta) = M_t(\eta)U_t M_{t-1}(\eta)U_{t-1}\cdots M_1(\eta)U_1$$

effective "Hamiltonian": $$K_t(\eta) = -\frac{1}{2t}\log\left[V_t(\eta)V_t^\dagger(\eta)\right]$$

intuitive picture: **imaginary time evolution:** $|\psi_t(\eta)\rangle \sim \exp[-K_t(\eta)t]\,|\psi_0\rangle$

$$K_t(\eta)\,|\Psi_{t,i}(\eta)\rangle = \varepsilon_{t,L,i}(\eta)\,|\Psi_{t,i}(\eta)\rangle\,, \quad \varepsilon_{t,L,i}(\eta) \le \varepsilon_{t,L,i+1}(\eta)$$

spectral gap: $\Delta_L(\eta) = \lim_{t\to\infty}\Delta_{t,L}(\eta), \quad \Delta_{t,L}(\eta) = \varepsilon_{t,L,2}(\eta) - \varepsilon_{t,L,1}(\eta)$

ground state of the effective Hamiltonian (dominant Lyapunov vector): $|\Psi_{t,1}(\eta)\rangle$

analogy to equilibrium phase transitions

relaxation time:

$$t \gg \tau_{\delta,L}(\eta) = \left|\frac{\log(\delta)}{\Delta_L(\eta)}\right| \to |\psi_t(\eta)\rangle \simeq |\Psi_{t,1}(\eta)\rangle \text{ within the precision } \delta$$

Spectral gap

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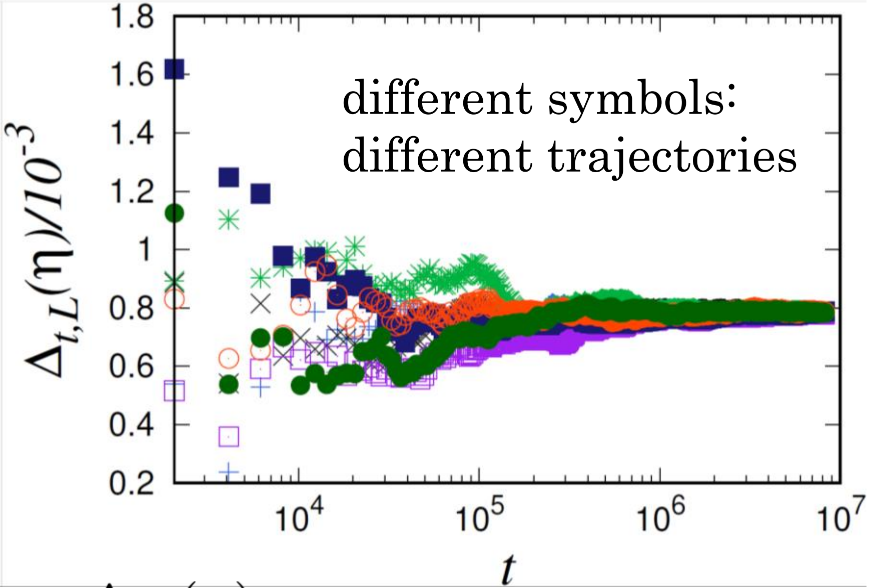
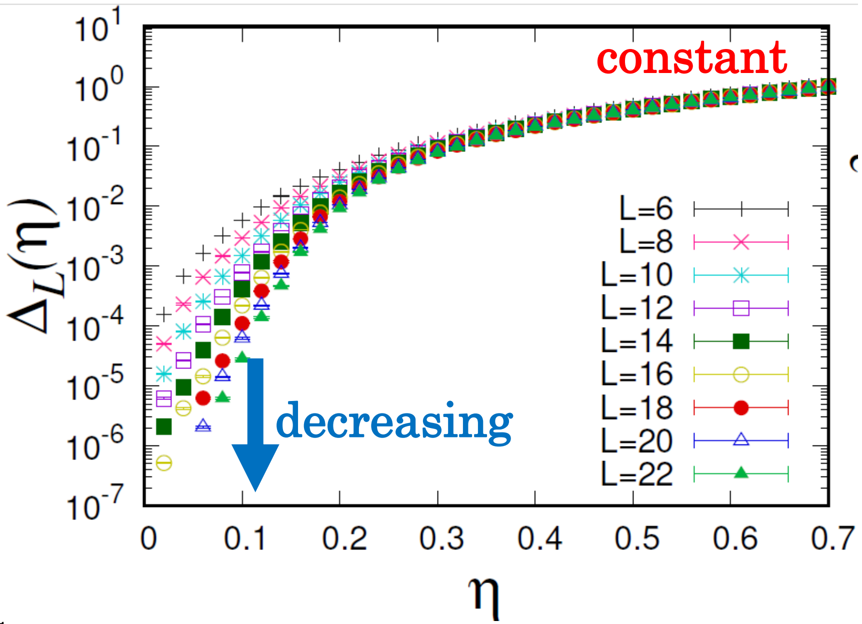
$$K_t(\eta)\,|\Psi_{t,i}(\eta)\rangle = \varepsilon_{t,L,i}(\eta)\,|\Psi_{t,i}(\eta)\rangle$$

spectral gap: $\Delta_L(\eta) = \lim_{t\to\infty} \Delta_{t,L}(\eta), \quad \Delta_{t,L}(\eta) = \varepsilon_{t,L,2}(\eta) - \varepsilon_{t,L,1}(\eta)$





$\Delta_L(\eta)$: independent of trajectories

global quantity!

η : measurement strength

L : system size

small η : $\lim_{L\to\infty} \Delta_L(\eta) = 0$

large η : $\lim_{L\to\infty} \Delta_L(\eta) \neq 0$

Spectral transition

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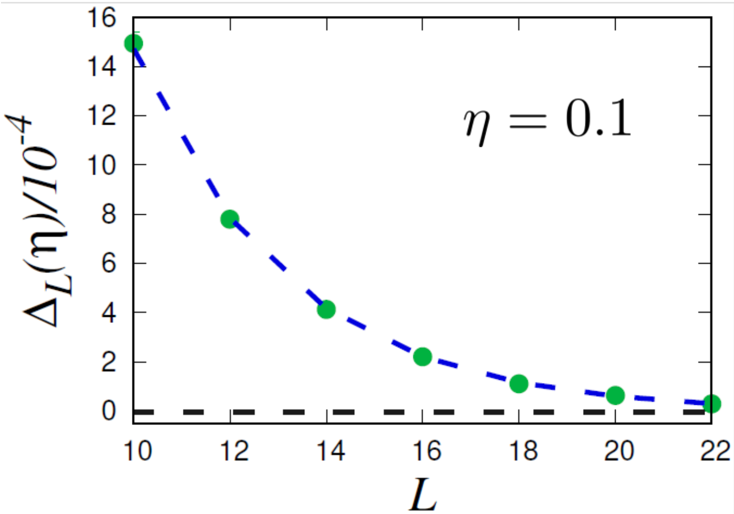
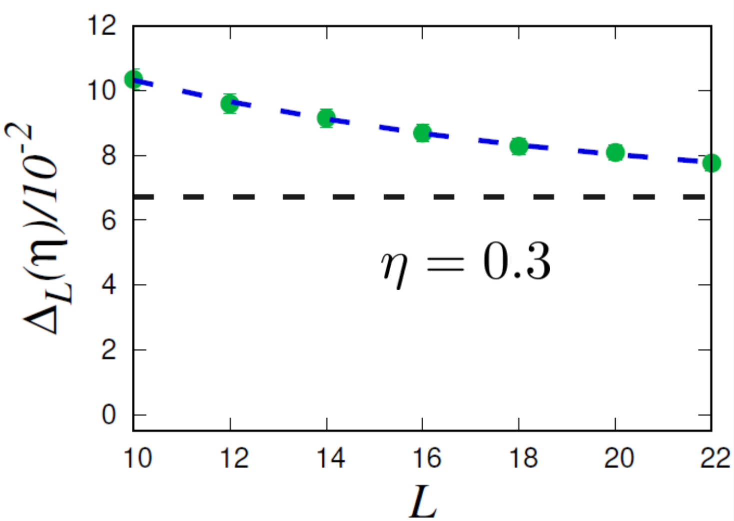
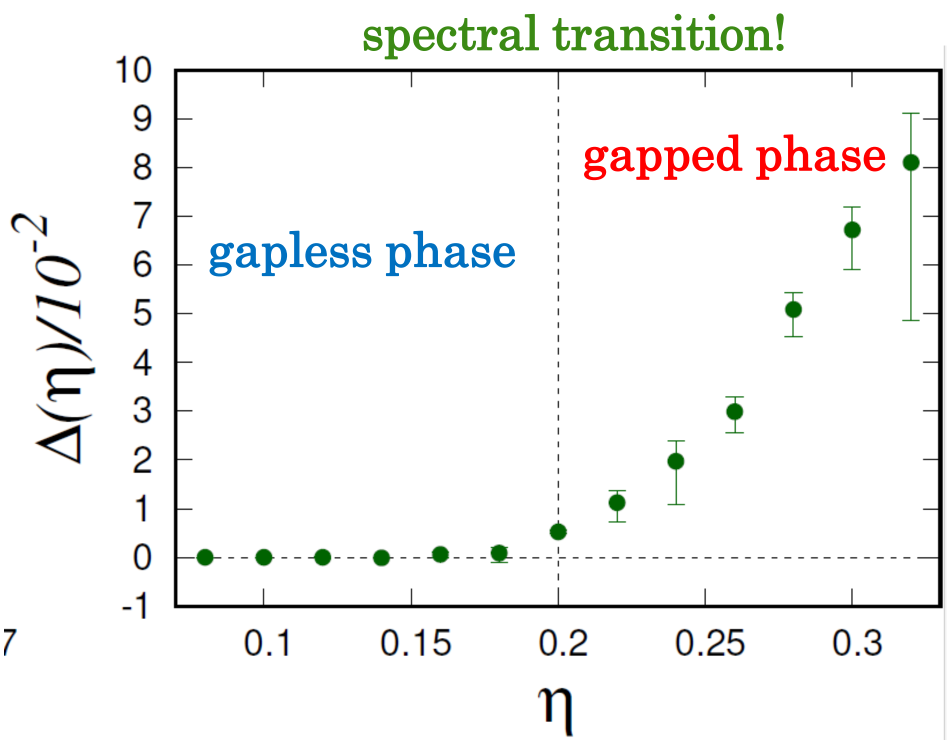




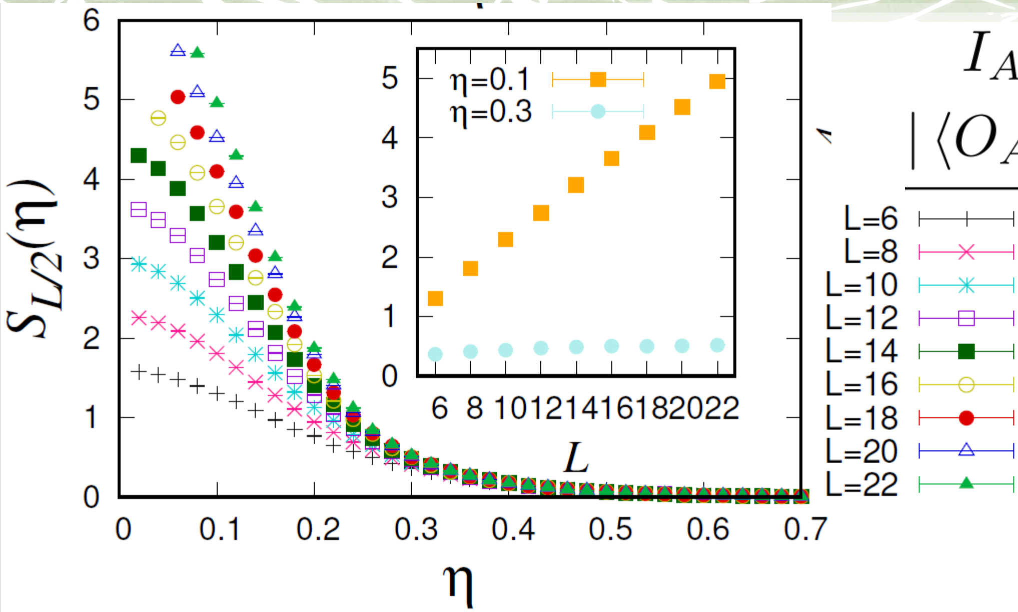
● numerical data

$$\Delta_L^{\mathrm{fit}}(\eta) = \Delta(\eta) + \alpha(\eta)[\beta(\eta)]^{-L}$$

$$\Delta(\eta) = \lim_{L\to\infty} \Delta_L^{\mathrm{fit}}(\eta)$$



Transition of the entanglement entropy of the ground state (dominant Lyapunov vector)



$$I_{AB}(\psi) = S_A(\psi) + S_B(\psi) - S_{AB}(\psi)$$

$$\frac{|\langle O_A O_B\rangle_\psi - \langle O_A\rangle_\psi \langle O_B\rangle_\psi|^2}{2|O_A|^2|O_B|^2} \leq I_{AB}(\psi)$$

M. M. Wolf et.al., PRL **100**, 070502 (2008).

$S_{L/2}(\eta) = \mathbb{E}_t\left(S_{L/2}\left[\Psi_{t,1}(\eta)\right]\right)$

$\mathbb{E}_t$:time average $\quad \rho_\psi = \mathrm{tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$

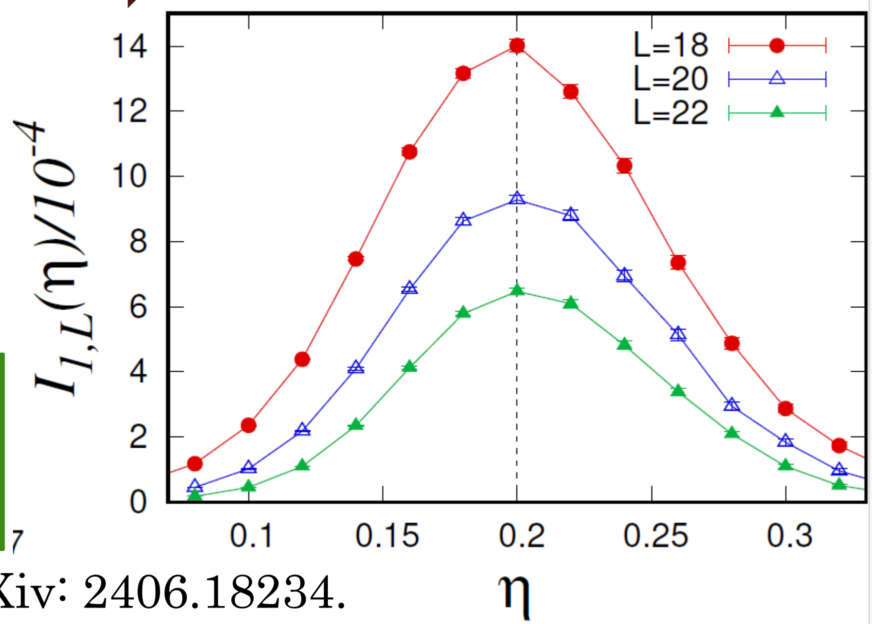
$S_A(\psi) = -\mathrm{tr}_A[\rho_\psi \log(\rho_\psi)]$

small η : volume law $S_{L/2}(\eta) \propto L$

large η: area law $S_{L/2}(\eta) \propto L^0$

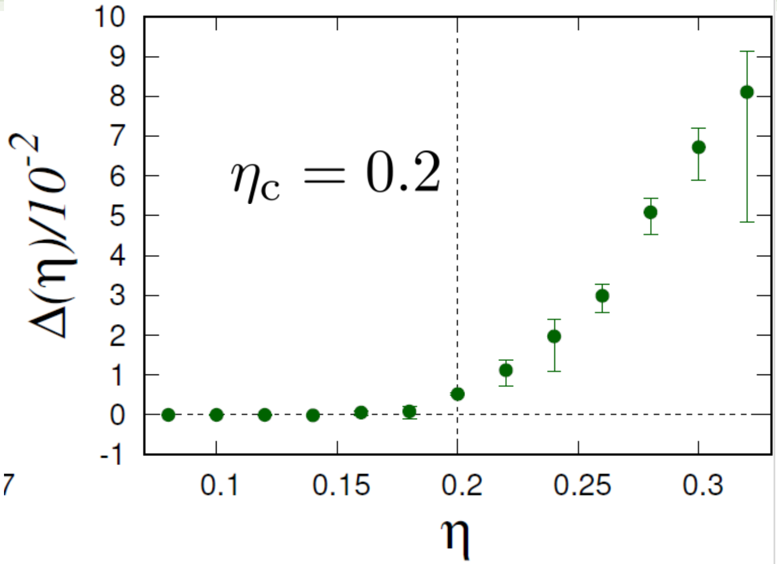
peak of $I_{1,L}(\eta) = \mathbb{E}_t\left(I_{1,L}\left[\Psi_{t,1}(\eta)\right]\right)$

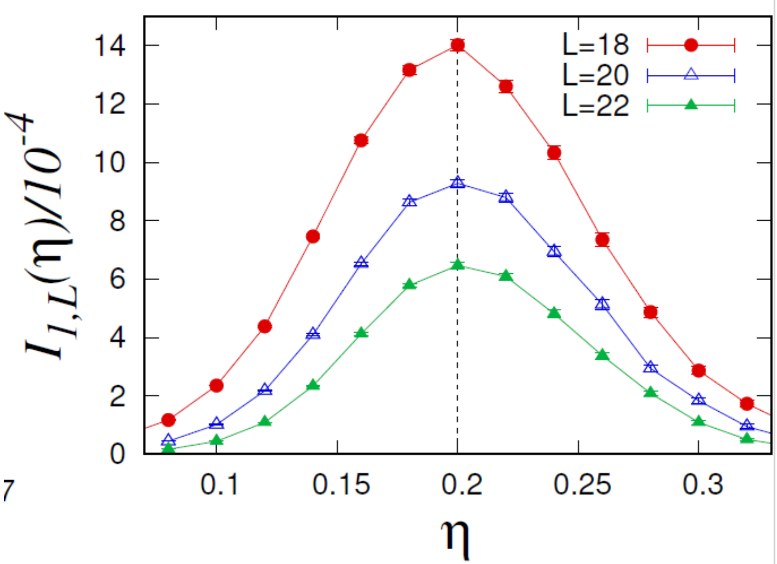
→ phase transition!



Ken Mochizuki and Ryusuke Hamazaki arXiv: 2406.18234.

Coincidence of the thresholds





$$K_t(\eta)\,|\Psi_{t,i}(\eta)\rangle = \varepsilon_{t,L,i}(\eta)\,|\Psi_{t,i}(\eta)\rangle$$

gapless phase = volume-law phase

$\Delta(\eta) = 0$ $\quad S_{L/2}(\eta) \propto L$

gapped phase = area-law phase

$\Delta(\eta) \neq 0$ $\quad S_{L/2}(\eta) \propto L^0$

analogous to ground-state phase transitions

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Comparison of ground-state transitions and measurement-induced transitions

*1D

	Measurement-induced phase transition in noisy dynamics	Ground-state phase transitions in equilibrium
gapped phase	entanglement entropy: $O(L^0)$ spectral gap: $O(L^0)$	entanglement entropy: $O(L^0)$ spectral gap: $O(L^0)$
gapless phase	entanglement entropy: $O(L)$ spectral gap: $O(e^{-L})$	entanglement entropy: $O[\log(L)]$ spectral gap: $O[1/\mathrm{poly}(L)]$

B. Zeng, X. Chen, D.-L. Zhou, and X.-G. Wen,
Quantum information meets quantum matter (Springer, 2019).

Qualitative similarity in gapped phases
Distinct scalings in gapless phases

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Ryusuke Hamazaki
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Transition of the memory-loss time

Ken Mochizuki, Ryusuke Hamazaki arXiv: 2406.18234.

$$t \gg \tau_{\delta,L}(\eta) = \left|\frac{\log(\delta)}{\Delta_L(\eta)}\right| \to |\psi_t(\eta)\rangle \propto \underline{V_t(\eta)}\, |\psi_0\rangle \simeq |\Psi_{t,1}(\eta)\rangle \text{ within } \delta$$

$\simeq$ rank-1 matrix

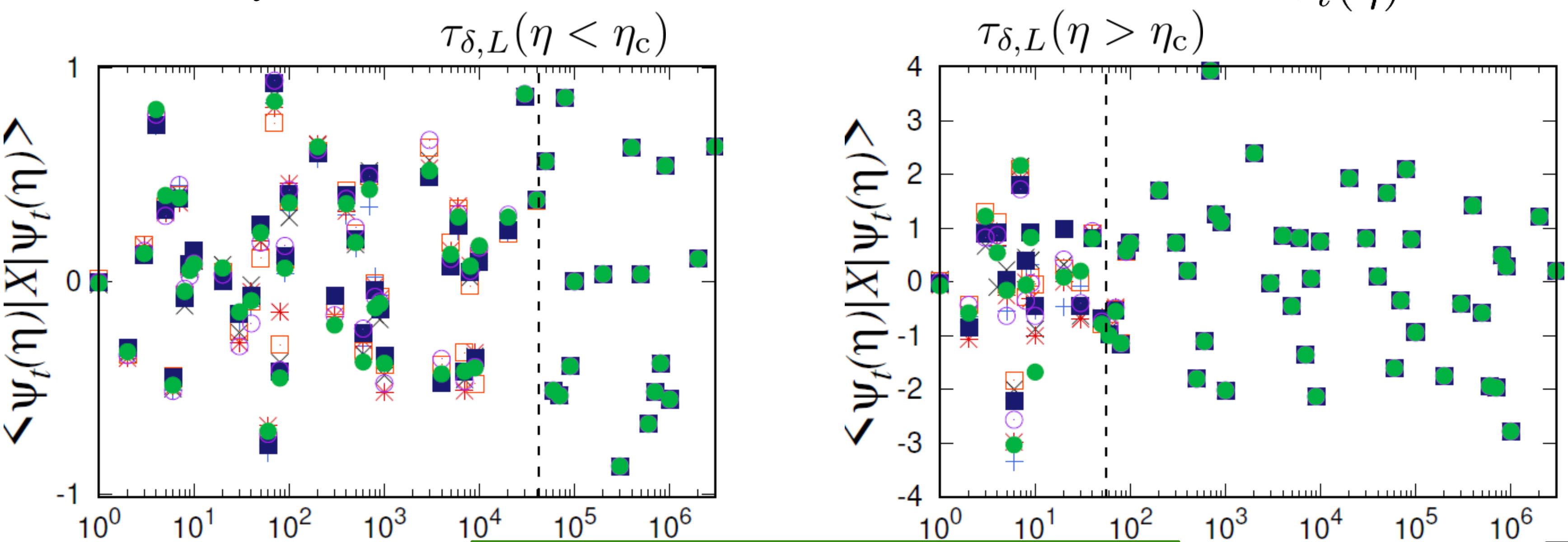
gapless phase:

$$\Delta_L(\eta) = O(e^{-L}) \to \tau_{\delta,L}(\eta) = O(e^L)$$

gapped phase:

$$\Delta_L(\eta) = O(L^0) \to \tau_{\delta,L}(\eta) = O(L^0)$$

various trajectories from different initial states with the same $V_t(\eta)$



$\delta = 10^{-2}$

$$\tau_{\delta,L}(\eta < \eta_c) \gg \tau_{\delta,L}(\eta > \eta_c)$$

$$X = \sum_x \sigma_1^x$$

Summary

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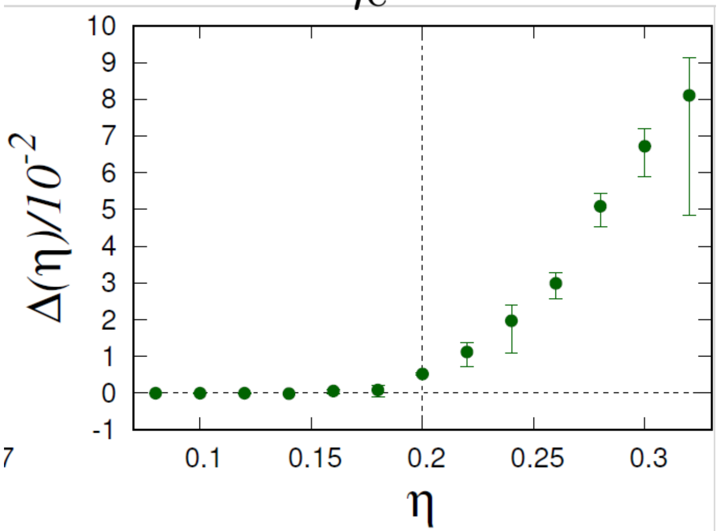
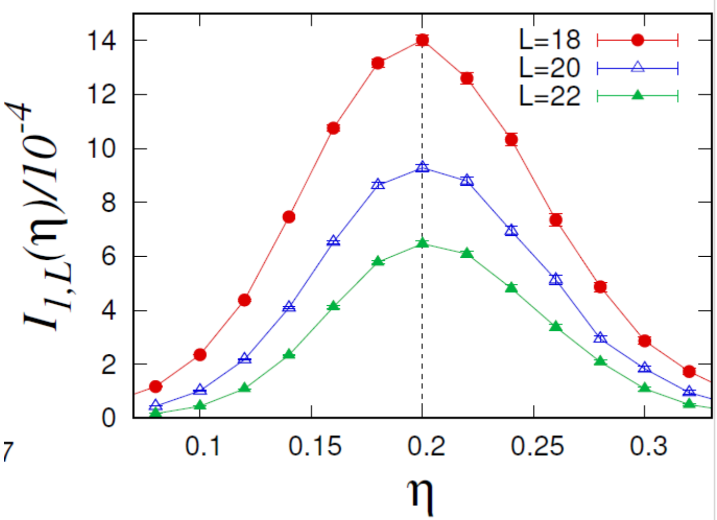
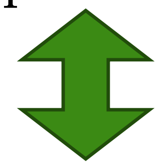
$\eta_{\mathrm{c}} = 0.2$





Spectral transition of the Lyapunov spectrum



analogous to the ground-state phase transitions in equilibrium

Entanglement transition of the dominant Lyapunov vector

$$K_t(\eta)\,|\Psi_{t,i}(\eta)\rangle = \varepsilon_{t,L,i}(\eta)\,|\Psi_{t,i}(\eta)\rangle$$