Dynamics of a quantum system with non-Gaussian baths, collective interactions

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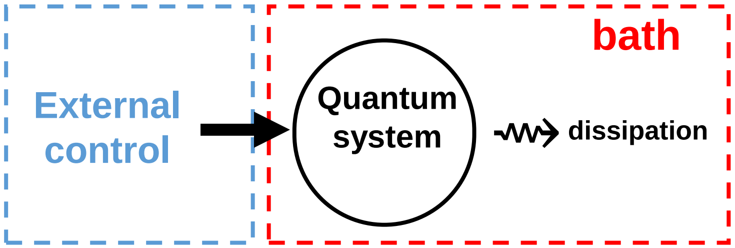
ERATO Sagawa Information-to-Energy Interconversion Project

Contents

■ Open quantum systems

Investigate the impact of ambient degrees of freedom on quantum systems

- *decoherence, dissipation, etc.*



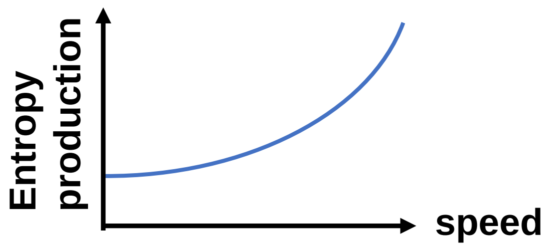
How to formulate the open quantum system dynamics that could possibly be non-Gaussian and non-Markovian?

→ *first part of this talk*

■ Stochastic thermodynamics

Quantification and optimization of the energetic costs in finite-time and nonequilibrium processes

- *quantum effects*



How would quantum effects change the speed limit set by finite-time thermodynamic relations?

→ *second part of this talk*

Dynamics of a quantum system interacting with non-Gaussian baths: Poisson noise master equation

Collaborator: Akihito Ishizaki
(Institute for Molecular Science)

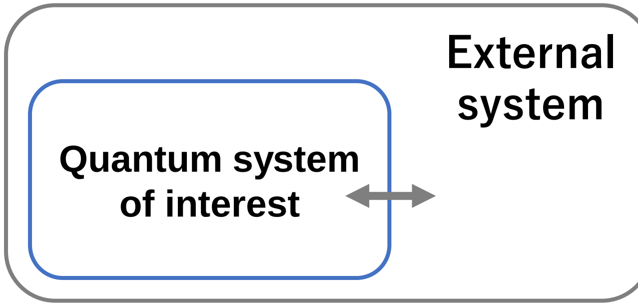
Based on: K. Funo & A. Ishizaki, PRL **132**, 170402 (2024)

Theory of open quantum systems

Quantum systems are inevitably in contact with the "outside world"

⇒ Theory of **open quantum systems**

- decoherence, dissipation

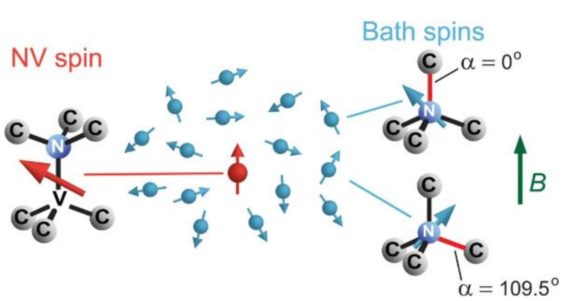


■ Analysis based on **harmonic oscillator baths** [Feynman-Vernon (1963), Caldeira-Leggett (1983)]

- Induces Gaussian fluctuations and dissipation **(Gaussian baths)**

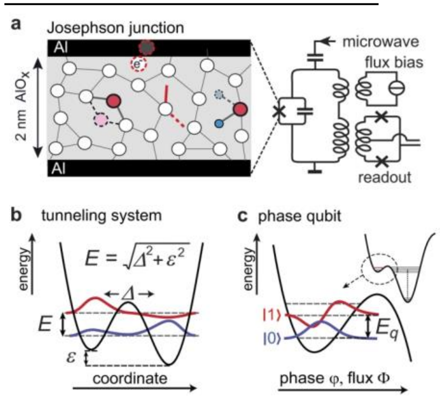
■ **Non-Gaussian baths**

Spin baths



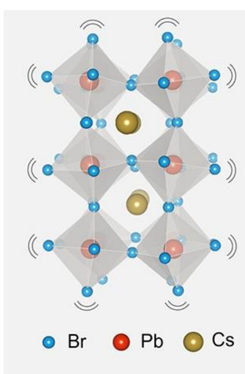
de Lange, *et al.*, *Scientific Reports* (2012)

Two-state fluctuators



Lisenfield, *et al.*, *Scientific Reports* (2016)

Anharmonicity in lattice vibration modes



perovskite

Cannelli, *et al.*, *J. Phys. Chem. Lett.* (2022)

General theory in the white noise regime

<u>Non-Gaussian bath theory</u>

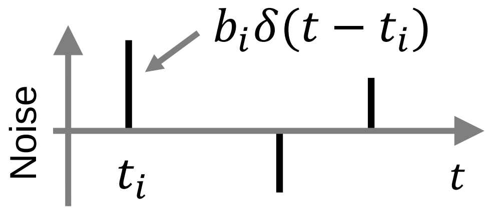
- not well established yet!
- As a first step, we consider the white noise regime

<u>Lévy-Itô decomposition theorem of stochastic processes</u>

white noise ＝ white Gaussian noise ＋ white Poisson noise

■ white Gaussian noise

■ white Poisson noise



➢ Describing the system dynamics under the influence of Poisson noise facilitates the investigation of the effect of ***any non-Gaussian environments*** in the white noise regime

Theoretical approach to describe open systems

■ Noise-based approach

- Generalized Langevin eq. (Gaussian noise)

$$\frac{dP_t}{dt} = -V'(X_t) - \int_0^t ds \partial_s K(t-s) X_t + \xi(t)$$

■ System-bath approach

- Projection operator method, etc

$$\partial_t \rho_{SB} = \mathcal{L}\rho_{SB}$$

↓

Eq. of motion for $\rho_S(t) = \mathrm{Tr}_B[\rho_{SB}(t)]$

↔

- A connection is known for the Gaussian case by using the Harmonic oscillator bath model

[Zwanzig (1973), Caldeira-Leggett (1983)]

Theoretical approach to describe open systems

Noise-based approach

- Generalized Langevin eq. (Gaussian noise)

$$\frac{dP_t}{dt} = -V'(X_t) - \int_0^t ds \partial_s K(t-s) X_t + \xi(t)$$

- Non-Markovian jump process (non-Gaussian noise)

$$\frac{dv_t}{dt} = \xi^{CP}_{\lambda(y|\{v_s\}_{s\le t})}$$

K. Kanazawa and D. Sornette (2023)

System-bath approach

- Projection operator method, etc

$$\partial_t \rho_{SB} = \mathcal{L} \rho_{SB}$$

↓

Eq. of motion for $\rho_S(t) = \mathrm{Tr}_B[\rho_{SB}(t)]$

?

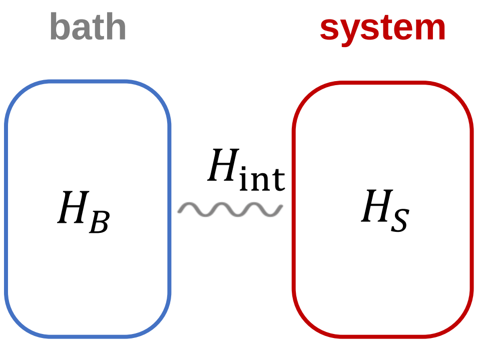
Non-Gaussian baths ?

- We consider the system-bath approach and consider the quantum mechanical modeling of the Poisson noise bath

Noise correlation functions ⟷ Bath correlation functions

Derivation of the reduced system dynamics

System-bath time-evolution



- Liouville eq. $\partial_t \rho_{SB} = \mathcal{L}\rho_{SB}$

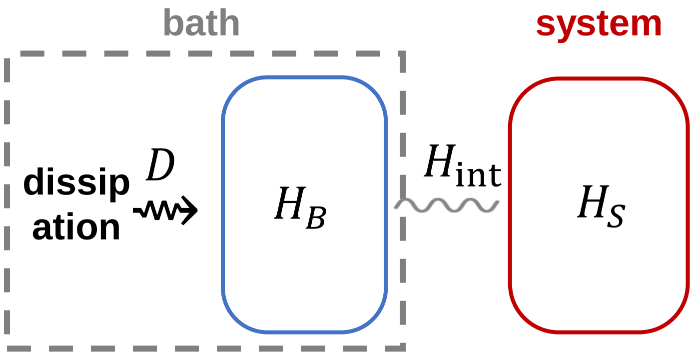
$$\mathcal{L}\rho_{SB} = -i[H_S + H_{int} + H_B, \rho_{SB}]$$

remarks

- Usually, we consider *unitary dynamics* of S+B space (dimension of H_B is typically large)

Derivation of the reduced system dynamics

System-bath time-evolution



- Liouville eq. $\partial_t \rho_{SB} = \mathcal{L}\rho_{SB} = (\mathcal{L}_S + \mathcal{L}_B + \mathcal{L}_{int})\rho_{SB}$

  $\mathcal{L}_S\rho = -i[H_S, \rho]$, $\mathcal{L}_{int}\rho = -i[H_{int}, \rho]$

  $\mathcal{L}_B\rho = -i[H_B, \rho] + D[\rho]$ **(GKLS equation)**

remarks

- Usually, we consider *unitary dynamics* of S+B space (dimension of H_B is typically large)
- Here, we generalize the setting and consider *dissipative dynamics* of S+B (dimension of H_B can be **small**)

  → *minimal model* to describe the Poisson noise bath

Nakajima-Zwanzig projection operator method

- Projection operator: $P = \rho_B^{eq} Tr_B$, $Q = 1 - P$
- Thermal state $\rho_B^{eq} = e^{-\beta H_B}/Z_B$, $(\mathcal{L}_B \rho_B^{eq} = 0)$

→ Derive equation of motion for the reduced system

Derivation of the reduced system dynamics

Generalized ME $D=0$ case: Yoon, Deutch, Freed J. Chem. Phys. (1975)

$$\partial_t \rho_S^I(t) = \sum_{n=1} (-i)^{n+1} \int_0^t du_1 \cdots \int_0^{u_{n-1}} du_n \sum_{\vec{l},\vec{k}} (-i)^l \chi_{n+1}^{\vec{l},\vec{k}}(u_1,\cdots,u_n,0)\, L_{l_1}^{k_1}(t) L_{l_2}^{k_2}(t-u_1+u_2)\cdots L_{l_{n+1}}^{k_{n+1}}(t-u_1)\rho_S^I(t-u)$$

- interaction: $\mathcal{L}_{int} = \sum_{l,k}(-i)^l L_l^k B_l^k$ Superoperator notation: $A_+\rho \equiv A\rho, A_-\rho \equiv \rho A$
- multi-time bath correlation function

$$\chi_n^{\vec{l},\vec{k}}(t_1,\cdots,t_n) \equiv Tr_B\left[B_{l_1}^{k_1} e^{\mathcal{L}_B(t_1-t_2)} Q B_{l_2}^{k_2} \cdots e^{\mathcal{L}_B(t_{n-1}-t_n)} Q B_{l_n}^{k_n} \rho_B^{eq}\right]$$

- **Influence of the bath is fully captured by $\chi_n^{\vec{l},\vec{k}}$**

The systems of two different system-bath models with the same n-point bath correlation functions $\chi_n^{\vec{l},\vec{k}}$ exhibit **equivalent reduced dynamics**

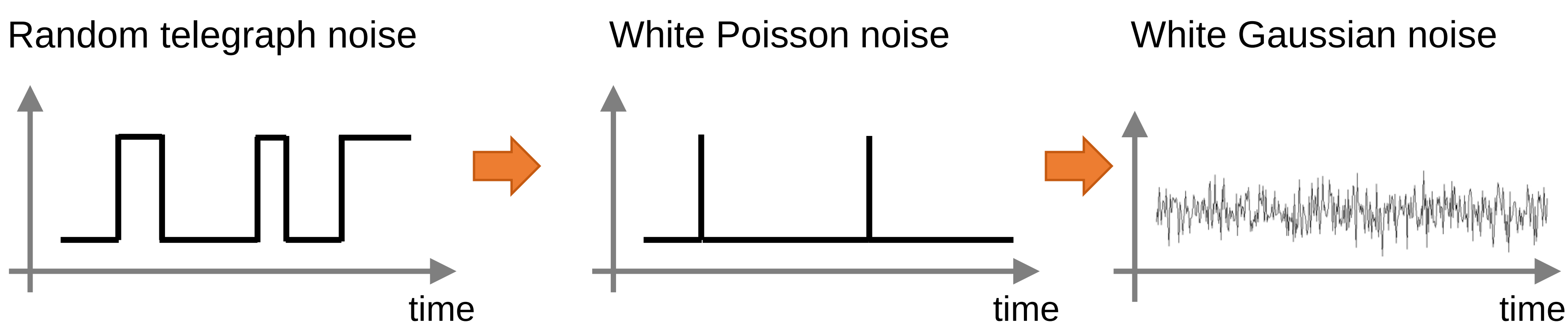
- The mathematical structures of the generalized ME and $\chi_n^{\vec{l},\vec{k}}$ serve as a starting point to analyze and classify the influence of non-Gaussian baths on the system.

Non-Gaussian noise examples

<u>Bath properties, equation of motion, and noise examples</u>

	Gaussian bath	Non-Gaussian bath
Markov	• White Gaussian noise Gauss noise master eq.	**• White Poisson noise** **Poisson noise master eq.**
Non-Markov	• Colored Gaussian noise Feynman-Vernon theory, …	**• Random telegraph noise** **Generalized master eq.**

■ c.f. analysis of the classical noise model and master equation Van Den Broeck (1982)



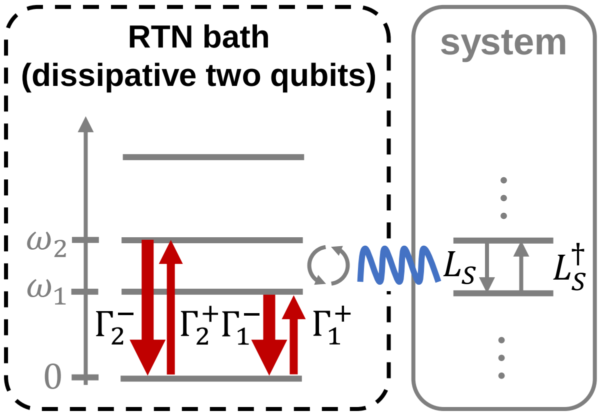
Quantum mechanical modeling of Poisson noise

- **Quantum mechanical modeling** of the random telegraph noise (RTN) and Poisson noise bath

➢ **We use dissipative two qubit systems**

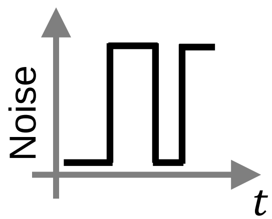
$$\mathcal{L}_B^j \rho_B = -i\left[\frac{\omega_j}{2}\sigma_j^z, \rho_B\right] + \Gamma_j^- D_{\sigma_j^-}[\rho_B] + \Gamma_j^+ D_{\sigma_j^+}[\rho_B] \quad j = 1, 2$$

Dissipator: $D_L \rho_B = L \rho_B L^\dagger - \frac{1}{2}\{L^\dagger L, \rho_B\}$



➢ **Interaction:**

$$H_{int} = \lambda(L_S \sigma_1^+ \sigma_2^- + L_S^\dagger \sigma_1^- \sigma_2^+)$$

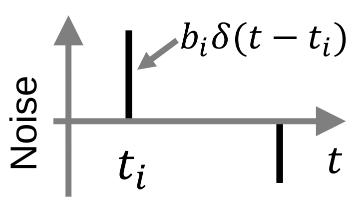


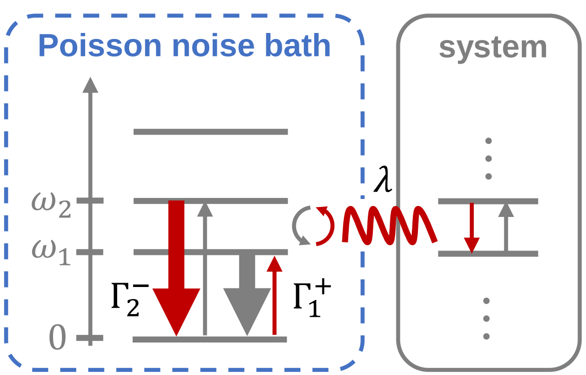
Poisson white noise limit

$$\Gamma_1^- = \Gamma_2^- \gg \Gamma_i^+ , \quad \lambda \gg \Gamma_i^+$$

$$(\lambda/\Gamma_1^- = \mu)$$

Discrete, strong, and short time (δ-function type) interaction with the system





n-point bath correlation functions

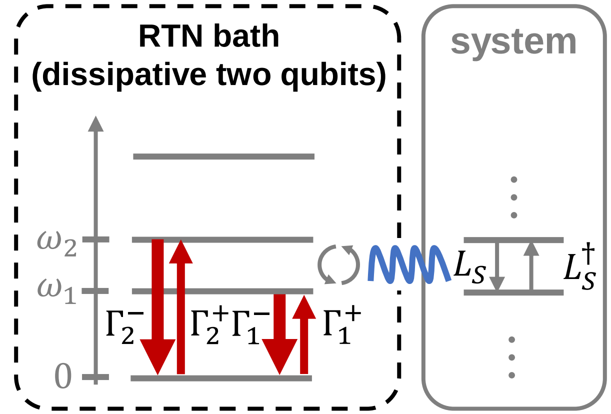
■ Random telegraph noise bath (non-Markovian)

$\chi_n^{\vec{l},\vec{k}}$ can be decomposed into a product of 2-point bath correlation-like functions

$$\chi_{2n}^{\vec{l},\vec{k}}(t_1,\ldots,t_{2n}) = \prod_{j=1}^{n-1} \mathrm{Tr_B}[B_{l_{2j-1}}^{k_{2j-1}} e^{\mathcal{L}_B(t_{2j-1}-t_{2j})} B_{l_{2j}}^{k_{2j}} e^{\mathcal{L}_B(t_{2j}-t_{2j+1})} \mathcal{Q}\rho_B^{l_{2j+1}k_{2j+1}}]\,\mathrm{Tr_B}[B_{l_{2n-1}}^{k_{2n-1}} e^{\mathcal{L}_B(t_{2n-1}-t_{2n})} B_{l_{2n}}^{k_{2n}} \rho_B^{\mathrm{eq}}],$$

2-point bath correlation function with initial bath states $|g_1, e_2\rangle_B$ or $|e_1, g_2\rangle_B$

2-point (equilibrium) bath correlation function

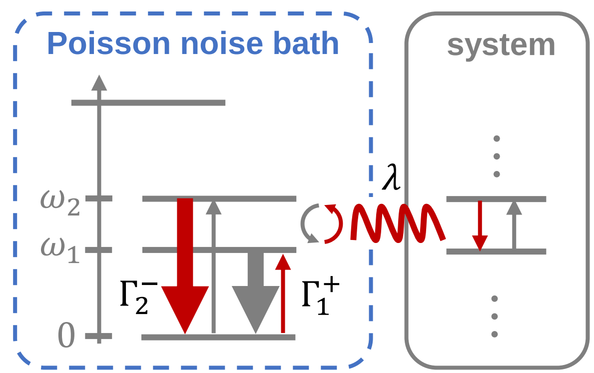


✓ explicit analytical expression can be obtained

■ White Poisson noise bath

$$\Gamma_1^- = \Gamma_2^- \gg \Gamma_i^+, \quad \lambda \gg \Gamma_i^+ \qquad (\lambda/\Gamma_1^- = \mu)$$

$$\chi_{2n}^{\vec{l},\vec{k}} = \begin{cases} \frac{\Gamma_1^+}{2}(2\mu)^{2n}\delta(t_1-t_2)\cdots\delta(t_{2n-1}-t_{2n}) \\ \frac{\Gamma_2^+}{2}(2\mu)^{2n}\delta(t_1-t_2)\cdots\delta(t_{2n-1}-t_{2n}) \end{cases}$$



no Memory effect

Generalized ME ⇒ Markovian ME

Poisson noise GKLS master equation

K. Funo & A. Ishizaki, PRL 2024

- Poisson noise GKLS master equation

$$\partial_t \rho_S = -i[H_S, \rho_S] + \int_0^\infty \frac{da}{\mu} e^{-\frac{a}{\mu}} \left[\Gamma_2^+ \left(D_{L_a}[\rho] + D_{M_a}[\rho]\right) + \Gamma_1^+ \left(D_{L_a^\dagger}[\rho] + D_{N_a}[\rho]\right)\right]$$

- 2μ: noise strength, $\Gamma_1^+/2, \Gamma_2^+/2$: noise rate
- Distribution of the time-duration: $p(\tau_B) \propto e^{-a/\mu}$ Dimensionless parameter ($a \equiv \lambda\tau_B$)
- Multiple jumps during the time-duration τ_B

- $L_a = \sum_{n=0} \frac{(-ia)^{2n+1}}{(2n+1)!} L_S \left(L_S^\dagger L_S\right)^n$
- $M_a = \sum_{n=1} \frac{(-ia)^{2n}}{(2n)!} \left(L_S^\dagger L_S\right)^n$
- $N_a = \sum_{n=1} \frac{(-ia)^{2n}}{(2n)!} \left(L_S L_S^\dagger\right)^n$

White Poisson noise limit and jump operators

■ Energy relaxation

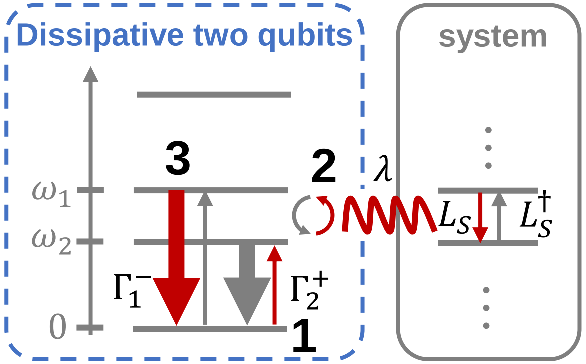
1. Qubit excitation $|g_1, g_2\rangle_B \rightarrow |g_1, e_2\rangle_B$

2. Interaction (time-duration: τ_B)

$$L_a \equiv \langle e_1, g_2 | e^{-i\tau_B H_{int}} | g_1, e_2 \rangle_B = \sum_{n=0} \frac{(-ia)^{2n+1}}{(2n+1)!} L_S \left(L_S^\dagger L_S\right)^n$$

$a \equiv \lambda \tau_B$

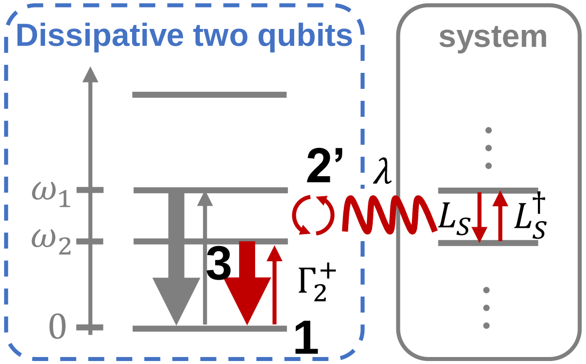
3. Qubit relaxation $[p(\tau_B) \propto e^{-\Gamma_1^- \tau_B}]$



■ Dephasing

2'. Interaction (time-duration: τ_B)

$$M_a \equiv \langle g_1, e_2 | e^{-i\tau_B H_{int}} | g_1, e_2 \rangle_B - 1 = \sum_{n=1} \frac{(-ia)^{2n}}{(2n)!} \left(L_S^\dagger L_S\right)^n$$



Jump operators depend on higher order of the system-bath coupling strength λ

Poisson noise GKLS master equation

K. Funo & A. Ishizaki, PRL 2024

■ Poisson noise GKLS master equation

$$\partial_t \rho_S = -i[H_S, \rho_S] + \int_0^\infty \frac{da}{\mu} e^{-\frac{a}{\mu}} \left[\Gamma_2^+ \left(D_{L_a}[\rho] + D_{M_a}[\rho] \right) + \Gamma_1^+ \left(D_{L_a^\dagger}[\rho] + D_{N_a}[\rho] \right) \right]$$

- ➢ 2μ: noise strength, $\Gamma_1^+/2, \Gamma_2^+/2$: noise rate
- ➢ Distribution of the time-duration: $p(\tau_B) \propto e^{-a/\mu}$ Dimensionless parameter ($a \equiv \lambda \tau_B$)
- ➢ Multiple jumps during the time-duration τ_B

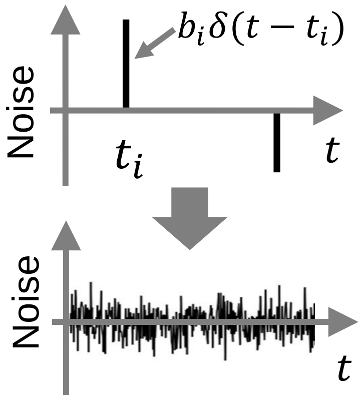
- $L_a = \sum_{n=0} \frac{(-ia)^{2n+1}}{(2n+1)!} L_S \left(L_S^\dagger L_S \right)^n$
- $M_a = \sum_{n=1} \frac{(-ia)^{2n}}{(2n)!} \left(L_S^\dagger L_S \right)^n$
- $N_a = \sum_{n=1} \frac{(-ia)^{2n}}{(2n)!} \left(L_S L_S^\dagger \right)^n$

■ Gauss noise limit $\mu \to 0$, $\Gamma_j^+ \to \infty$ for fixed $\mu^2 \Gamma_j^+$

(weak noise strength and high noise rate limit)

Reproduces the weak coupling GKLS master eq

$$\partial_t \rho_S = -i[H_S, \rho_S] + 2\mu^2 \Gamma_2^+ D_{L_S}[\rho_S] + 2\mu^2 \Gamma_1^+ D_{L_S^\dagger}[\rho_S]$$

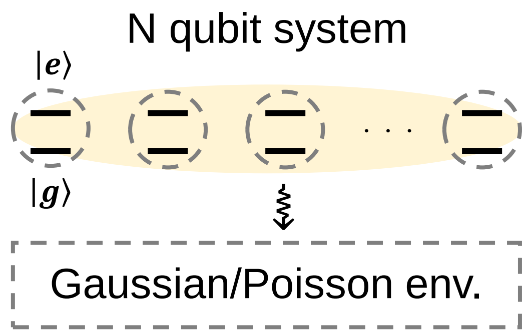


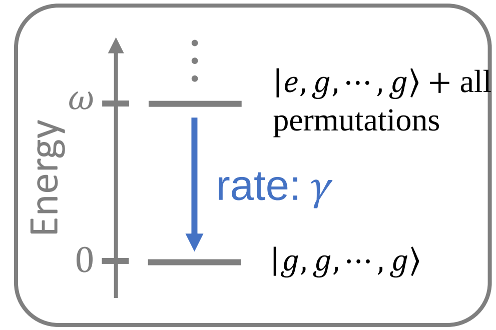
Application to identical qubit systems

Ex: N qubit system

- Hamiltonian $H_S = \frac{\omega}{2}\sum_i \sigma_i^z$
- Interaction op. $L_S = \sum_i \sigma_i^-$

➢ Consider the energy relaxation rate from the first excited state to the ground state





■ Gauss noise bath

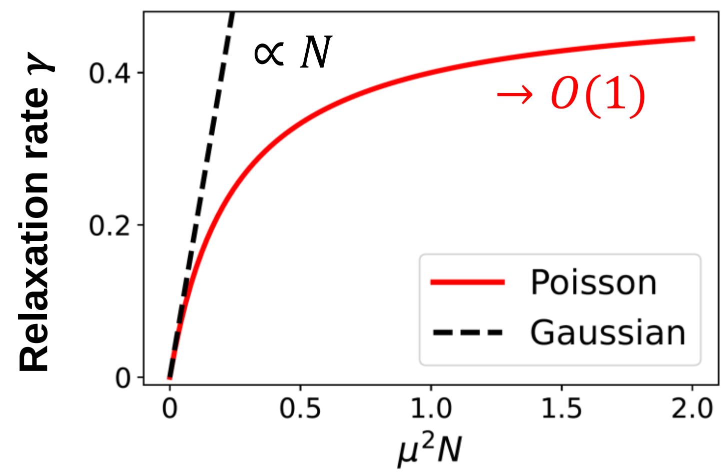
rate $\propto N$

(N-qubit system collectively interacts with the bath, known as super-radiance)

■ Poisson noise bath

$$\text{rate} = \frac{2\Gamma_2^+\mu^2 N}{1+4\mu^2 N} = \begin{cases} 2\Gamma_2^+\mu^2 N = O(N) & \mu^2 N \ll 1 \\ \frac{\Gamma_2^+}{2} = O(1) & \mu^2 N \gg 1 \end{cases}$$

2μ: noise strength, $\Gamma_2^+/2$: noise rate



Summary of the first part

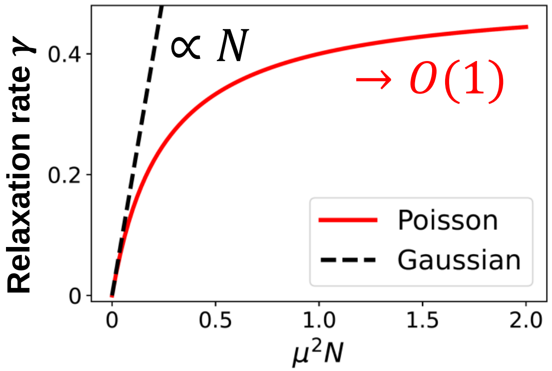
General bath = Gauss bath + Poisson bath

In the white noise regime

	Gaussian bath	Non-Gaussian bath
Markov	• White Gaussian noise Gauss noise master eq.	• **White Poisson noise** **Poisson noise master eq.**
Non-Markov	• Colored Gaussian noise Feynman-Vernon theory, …	• **Random telegraph noise** **Generalized master eq.**

We have derived a Poisson noise master equation

- different scaling behavior of the relaxation rate
- The obtained results suggest that bath statistical differences may significantly alter the dissipative property



K. Funo & A. Ishizaki, PRL **132**, 170402 (2024)