Out-of-Equilibrium Fluctuation-Dissipation Bounds

Ludovico Tesser

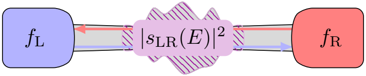
Phys. Rev. Lett. 132, 186304 (2024)

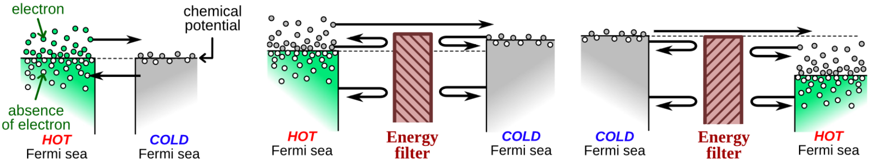
CHALMERS
UNIVERSITY OF TECHNOLOGY

July 12th 2024, Kyoto

Thermal engines at the nanoscale

Steady-state thermoelectric heat engines:





(a) Direct contact – no energy filter. (b) Energy-filter as heat-engine. (c) Energy-filter as refrigerator.

$$\Delta T \neq 0, \Delta\mu = 0$$

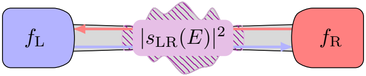
Electron-hole symmetry

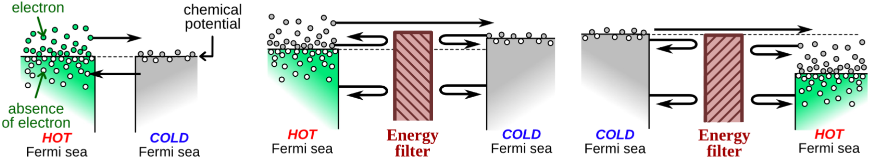
$$\Downarrow$$

No average current

G. Benenti, G. Casati, K. Saito, R. S. Whitney: Phys. Rep. **694**, 1 (2017)

Steady-state thermoelectric heat engines:





(a) Direct contact — no energy filter. (b) Energy-filter as heat-engine. (c) Energy-filter as refrigerator.

$\Delta T \neq 0, \Delta\mu = 0$

Electron-hole symmetry

$\Downarrow$

No average current

$\Delta T \neq 0, \Delta\mu \neq 0$

Current driven *against* $\Delta\mu$

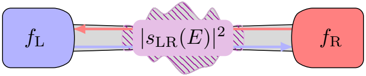
$\Downarrow$

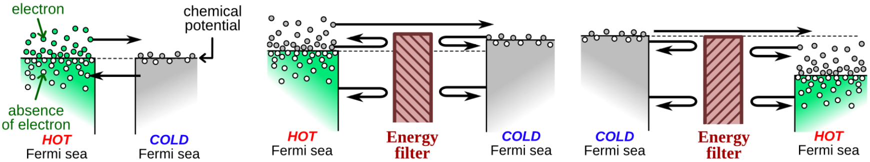
Power production

G. Benenti, G. Casati, K. Saito, R. S. Whitney: Phys. Rep. **694**, 1 (2017)

Thermal engines at the nanoscale

Steady-state thermoelectric heat engines:





(a) Direct contact – no energy filter.

(b) Energy-filter as heat-engine.

(c) Energy-filter as refrigerator.

$\Delta T \neq 0, \Delta\mu = 0$

Electron-hole symmetry

$\Downarrow$

No average current

$\Delta T \neq 0, \Delta\mu \neq 0$

Current driven *against* $\Delta\mu$

$\Downarrow$

Power production

$\Delta T \neq 0, \Delta\mu \neq 0$

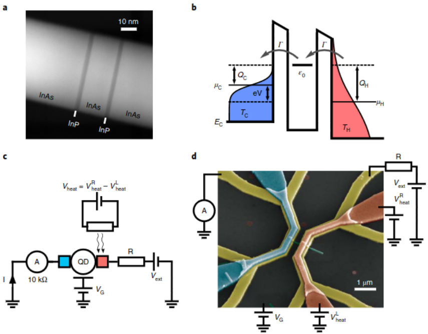
Heat flowing from cold to hot

$\Downarrow$

Refrigeration

G. Benenti, G. Casati, K. Saito, R. S. Whitney: Phys. Rep. **694**, 1 (2017)

Thermal engines at the nanoscale



Power output: $P \sim 1$ fW

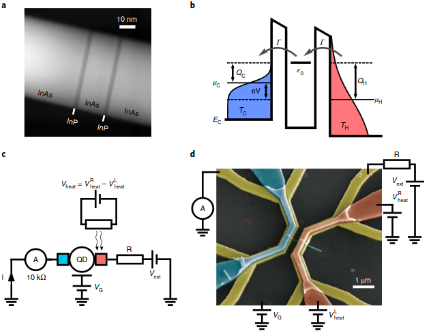
Efficiency: $\eta \sim 0.5\eta_{\text{Carnot}}$

Temperatures: $T \sim 0.5 - 2$ K

Josefsson, Svilans, Burke, Hoffmann, Fahlvik, Thelander, Leijnse, Linke:
Nat. Nanotechnol. **13**, 920 (2018)

It is possible to make nanoscale heat engines!

Thermal engines at the nanoscale



Power output: $P \sim 1$ fW

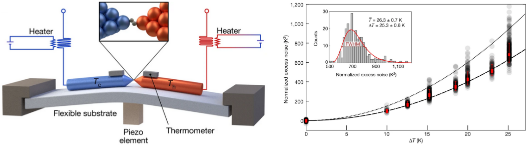
Efficiency: $\eta \sim 0.5\eta_{\text{Carnot}}$

Temperatures: $T \sim 0.5 - 2$ K

Josefsson, Svilans, Burke, Hoffmann, Fahlvik, Thelander, Leijnse, Linke:
Nat. Nanotechnol. **13**, 920 (2018)

It is possible to make nanoscale heat engines!

It is also possible to measure the noise!



O. S. Lumbroso, L. Simine, A. Nitzan, D. Segal, O. Tal: Nature **562**, 240 (2018)

Noise at the nanoscale

Thermodynamic Uncertainty Relation (TUR)

$$\frac{I^2}{S^I} \leq \frac{\dot{\sigma}}{2k_\mathrm{B}}$$

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

Noise at the nanoscale

Thermodynamic Uncertainty Relation (TUR)

$$\frac{I^2}{S^I} \leq \frac{\dot{\sigma}}{2k_\mathrm{B}}$$

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

↓

Does not hold for *coherent scatterers*

K. Brandner, T. Hanazato, K. Saito: Phys. Rev. Lett. 120, 090601 (2018)

M. Gerry, D. Segal: Phys. Rev. B 105, 155401 (2022)

Noise at the nanoscale

Thermodynamic Uncertainty Relation (TUR)

$$\frac{I^2}{S^I} \leq \frac{\dot{\sigma}}{2k_\mathrm{B}}$$

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

↓

Does not hold for *coherent scatterers*

K. Brandner, T. Hanazato, K. Saito: Phys. Rev. Lett. 120, 090601 (2018)

M. Gerry, D. Segal: Phys. Rev. B 105, 155401 (2022)

Can we still find a trade-off relation between *noise* and *current* for coherent scatterers?

Noise at the nanoscale

Thermodynamic Uncertainty Relation (TUR)

$$\frac{I^2}{S^I} \leq \frac{\dot{\sigma}}{2k_\mathrm{B}}$$

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

↓

Does not hold for *coherent scatterers*

K. Brandner, T. Hanazato, K. Saito: Phys. Rev. Lett. 120, 090601 (2018)

M. Gerry, D. Segal: Phys. Rev. B 105, 155401 (2022)

Can we still find a trade-off relation between *noise* and *current* for coherent scatterers?

Yes, **but** not a TUR

Outline

Scattering theory

We consider *coherent* transport with *negligible* electron-electron interactions

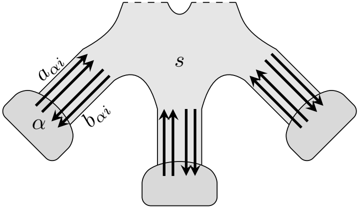
$\rightarrow$ Scattering approach!

Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000)

Scattering theory

We consider *coherent* transport with *negligible* electron-electron interactions

$\rightarrow$ Scattering approach!



Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000)

The scattering matrix s determines how incoming states $a_{\alpha i}$ propagate into outgoing states $b_{\alpha i}$.

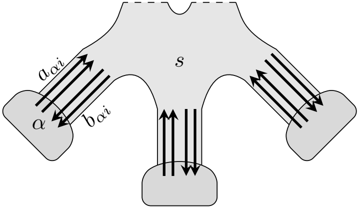
The average charge current reads

$$I_\alpha = \langle \hat{I}_\alpha \rangle = \frac{q}{h} \int dE \sum_{\beta,i} |s_{\alpha\beta,i}(E)|^2 (f_\beta - f_\alpha)$$

Scattering theory

We consider *coherent* transport with *negligible* electron-electron interactions

→ Scattering approach!



Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000)

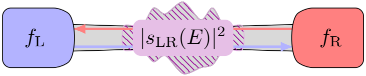
The scattering matrix s determines how incoming states $a_{\alpha i}$ propagate into outgoing states $b_{\alpha i}$.

The average charge current reads

$$I_\alpha = \langle \hat{I}_\alpha \rangle = \frac{q}{h} \int dE \sum_{\beta,i} |s_{\alpha\beta,i}(E)|^2 (f_\beta - f_\alpha)$$

Can we find relations that do not rely on explicit knowledge of the scattering matrix s?

Charge current and fluctuation-dissipation theorem



The current is

$$I = \langle \hat{I} \rangle = \frac{q}{h} \int dE |s_{\mathrm{LR}}(E)|^2 (f_{\mathrm{L}} - f_{\mathrm{R}})$$

The zero-frequency noise is

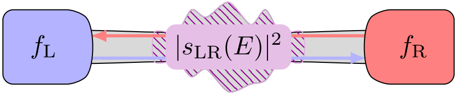
$$S^I = \int \langle \delta \hat{I}(t) \delta \hat{I}(0) \rangle dt$$

At equilibrium the noise (Johnson-Nyquist) is given by

$$S^I = 2qk_{\mathrm{B}}T \left. \frac{\partial I}{\partial \Delta\mu} \right|_{\Delta\mu=0}$$

→ **Fluctuation-dissipation theorem**

Fluctuation-dissipation relations



$\Gamma_{\rightarrow} = \frac{1}{h}\int dE |s_{\mathrm{LR}}|^2 \left[f_{\mathrm{L}}(1 - f_{\mathrm{R}})\right]$
$\Gamma_{\leftarrow} = \frac{1}{h}\int dE |s_{\mathrm{LR}}|^2 \left[f_{\mathrm{R}}(1 - f_{\mathrm{L}})\right]$

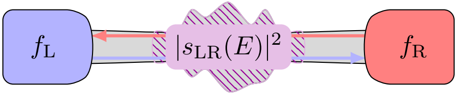
$\rightarrow$Tunneling rates

The current satisfies

$$I = q\left(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}\right)$$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

Fluctuation-dissipation relations



$\Gamma_\rightarrow = \frac{1}{h}\int dE |s_\mathrm{LR}|^2 \left[f_\mathrm{L}(1-f_\mathrm{R})\right]$

$\Gamma_\leftarrow = \frac{1}{h}\int dE |s_\mathrm{LR}|^2 \left[f_\mathrm{R}(1-f_\mathrm{L})\right]$

$\rightarrow$Tunneling rates

The current satisfies

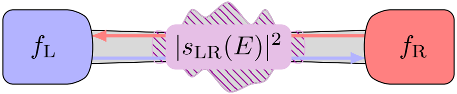
$$I = q\left(\Gamma_\rightarrow - \Gamma_\leftarrow\right)$$

In the **tunneling regime** ($|s_\mathrm{LR}|^2 \ll 1$) the noise satisfies

$$S^I = q^2(\Gamma_\rightarrow + \Gamma_\leftarrow)$$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

Fluctuation-dissipation relations



$\Gamma_{\rightarrow} = \frac{1}{h}\int dE |s_{\rm LR}|^2 \left[f_{\rm L}(1-f_{\rm R}) \right]$
$\Gamma_{\leftarrow} = \frac{1}{h}\int dE |s_{\rm LR}|^2 \left[f_{\rm R}(1-f_{\rm L}) \right]$ $\rightarrow$Tunneling rates

The current satisfies

$$I = q\left(\Gamma_{\rightarrow} - \Gamma_{\leftarrow}\right)$$

In the **tunneling regime** ($|s_{\rm LR}|^2 \ll 1$) the noise satisfies

$$S^I = q^2(\Gamma_{\rightarrow} + \Gamma_{\leftarrow})$$

When $\Delta T = 0$: Generalization of fluctuation-dissipation theorem

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_{\rm B}T}\right)$$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

Generalization of the Fluctuation-dissipation theorem at finite $\Delta\mu$.

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)

L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)

B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

Generalization of the Fluctuation-dissipation theorem at finite $\Delta\mu$.

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)

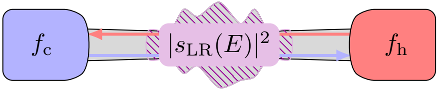
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)

B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

What happens to the noise far from the tunneling regime or if ΔT is not negligible?

Out-of-equilibrium fluctuation-dissipation bounds

Out-of-equilibrium

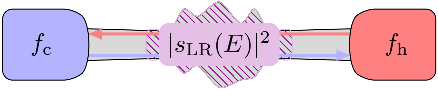


- Average current: $I \neq 0$
- Out-of-equilibrium noise: S^I

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

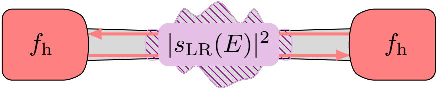
Out-of-equilibrium fluctuation-dissipation bounds

Out-of-equilibrium



- Average current: $I \neq 0$
- Out-of-equilibrium noise: S^I

Hot equilibrium

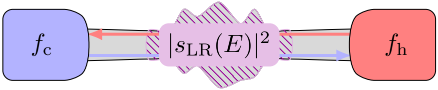


- Average current: $I = 0$
- Equilibrium noise: S^I_{eq}
  $\rightarrow$ Thermal noise

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

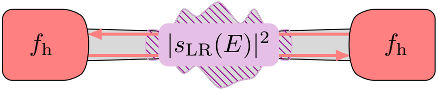
Out-of-equilibrium fluctuation-dissipation bounds

Out-of-equilibrium



- Average current: $I \neq 0$
- Out-of-equilibrium noise: S^I

Hot equilibrium



- Average current: $I = 0$
- Equilibrium noise: $S^I_{\rm eq}$
  $\rightarrow$ Thermal noise

$\tilde{\Gamma}_{\rightarrow} = \frac{1}{h} \int dE |s_{\rm LR}|^2 \left[f_{\rm c}(1-f_{\rm h}) - f_{\rm h}(1-f_{\rm h})\right]$
$\tilde{\Gamma}_{\leftarrow} = \frac{1}{h} \int dE |s_{\rm LR}|^2 \left[f_{\rm h}(1-f_{\rm c}) - f_{\rm h}(1-f_{\rm h})\right]$

$\rightarrow$*Excess* tunneling rates

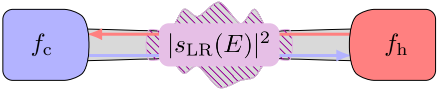
The current satisfies

$$I = q\left(\tilde{\Gamma}_{\rightarrow} - \tilde{\Gamma}_{\leftarrow}\right)$$

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

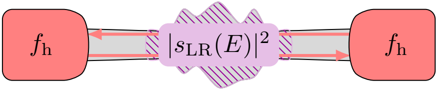
Out-of-equilibrium fluctuation-dissipation bounds

Out-of-equilibrium



- Average current: $I \neq 0$
- Out-of-equilibrium noise: S^I

Hot equilibrium



- Average current: $I = 0$
- Equilibrium noise: S^I_{eq}
  $\rightarrow$ Thermal noise

$$\tilde{\Gamma}_{\rightarrow} = \frac{1}{h}\int dE |s_{LR}|^2 \left[f_c(1-f_h) - f_h(1-f_h)\right]$$
$$\tilde{\Gamma}_{\leftarrow} = \frac{1}{h}\int dE |s_{LR}|^2 \left[f_h(1-f_c) - f_h(1-f_h)\right]$$

$\rightarrow$ *Excess* tunneling rates

The current satisfies

$$I = q\left(\tilde{\Gamma}_{\rightarrow} - \tilde{\Gamma}_{\leftarrow}\right)$$

and the *excess* noise obeys

$$S^I - S^I_{eq} \leq q^2(\tilde{\Gamma}_{\rightarrow} + \tilde{\Gamma}_{\leftarrow}) \leq -qI \tanh\left(\frac{\Delta\mu}{2k_B \Delta T}\right)$$

$\rightarrow$ It can be generalized to the multi-terminal case!

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

$$S^I - S^I_\mathrm{eq} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

- ✗ Requires *tunneling regime*
- ✗ Requires $\Delta T = 0$

$$S^I - S^I_\mathrm{eq} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

- ✓ Satisfied for any scattering matrix
- ✓ Satisfied at any $\Delta T, \Delta\mu$

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_B T}\right)$$

- ✗ Requires *tunneling regime*
- ✗ Requires $\Delta T = 0$
- ✓ Satisfied in the presence of interactions
- ✓ In the limit $\Delta\mu \ll k_B T$ recovers Fluctuation-dissipation theorem

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

$$S^I - S^I_{\text{eq}} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_B \Delta T}\right)$$

- ✓ Satisfied for any scattering matrix
- ✓ Satisfied at any $\Delta T, \Delta\mu$
- ✗ Requires negligible interactions

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

✗ Requires *tunneling regime*

✗ Requires $\Delta T = 0$

✓ Satisfied in the presence of interactions

✓ In the limit $\Delta\mu \ll k_\mathrm{B}T$ recovers Fluctuation-dissipation theorem

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

$$S^I - S^I_\mathrm{eq} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

✓ Satisfied for any scattering matrix

✓ Satisfied at any $\Delta T, \Delta\mu$

✗ Requires negligible interactions

- Approaches equality in tunneling regime and for ΔT *largest* energy scale

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_\mathrm{B}T}\right)$$

- ✗ Requires *tunneling regime*
- ✗ Requires $\Delta T = 0$
- ✓ Satisfied in the presence of interactions
- ✓ In the limit $\Delta\mu \ll k_\mathrm{B}T$ recovers Fluctuation-dissipation theorem

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

$$S^I - S^I_\mathrm{eq} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

- ✓ Satisfied for any scattering matrix
- ✓ Satisfied at any $\Delta T, \Delta\mu$
- ✗ Requires negligible interactions

- Approaches equality in tunneling regime and for ΔT *largest* energy scale

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I = -qI \coth\left(\frac{\Delta\mu}{2k_B T}\right)$$

- ✗ Requires *tunneling regime*
- ✗ Requires $\Delta T = 0$
- ✓ Satisfied in the presence of interactions
- ✓ In the limit $\Delta\mu \ll k_B T$ recovers Fluctuation-dissipation theorem

D. Rogovin, D. J. Scalapino: Ann. Phys. **86**, 1 (1974)
L. S. Levitov, M. Reznikov: Phys. Rev. B **70**, 115305 (2004)
B. Roussel, P. Degiovanni, I. Safi: Phys. Rev. B **93**, 045102 (2016)

$$S^I - S^I_{\mathrm{eq}} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_B \Delta T}\right)$$

- ✓ Satisfied for any scattering matrix
- ✓ Satisfied at any $\Delta T, \Delta\mu$
- ✗ Requires negligible interactions
- Approaches equality in tunneling regime and for ΔT *largest* energy scale

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

ΔT →

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I - S^I_{\text{eq}} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\text{B}\Delta T}\right)$$

Connection with power output $P = I\frac{\Delta\mu}{q}$:
For $k_\text{B}\Delta T \gg |\Delta\mu|$

$$S^I - S^I_{\text{eq}} \leq -\frac{q^2}{2k_\text{B}}\frac{P}{\Delta T}$$

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Out-of-equilibrium fluctuation-dissipation bounds

$$S^I - S^I_{\text{eq}} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\text{B}\Delta T}\right)$$

Connection with power output $P = I\frac{\Delta\mu}{q}$:
For $k_\text{B}\Delta T \gg |\Delta\mu|$

$$S^I - S^I_{\text{eq}} \leq -\frac{q^2}{2k_\text{B}}\frac{P}{\Delta T}$$

Power is *produced*:
$P > 0$
$\Downarrow$
Out-of-equilibrium noise is *smaller* than equilibrium noise

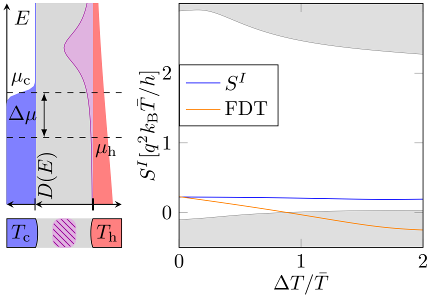
Power is *dissipated*:
$P < 0$
$\Downarrow$
Out-of-equilibrium noise can be *larger* than equilibrium noise

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Constraints on the noise

The fluctuation-dissipation bounds implies constraints on the out-of-equilibrium noise S^I:

$$qI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right) \leq S^I$$

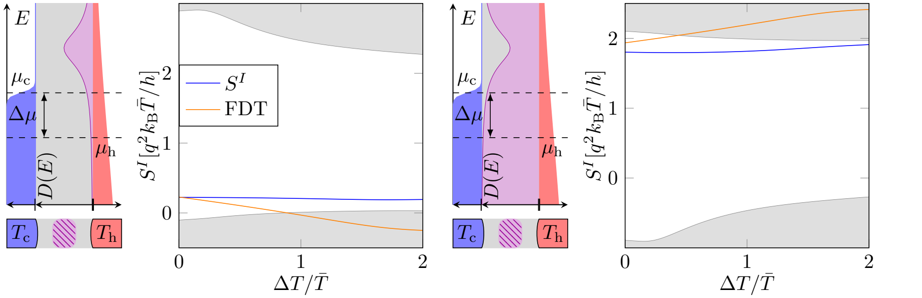


L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Constraints on the noise

The fluctuation-dissipation bounds implies constraints on the out-of-equilibrium noise S^I:

$$qI \tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right) \leq S^I \leq \frac{q^2}{h}k_B(T_c+T_h)+\left(qI+\frac{q^2\Delta\mu}{h}\right)\tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right)$$



L. **Tesser**, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Constraints between output power and noise

Output power $P = I\frac{\Delta\mu}{q}$. Noise of the power $S^P = \left(\frac{\Delta\mu}{q}\right)^2 S^I$

Thermodynamic uncertainty relation (TUR)

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

$$S^P \geq S^P_{\text{TUR}} \equiv 2\frac{k_{\text{B}}P^2}{\dot{\sigma}}$$

Does not hold in coherent scatterers!

Constraints between output power and noise

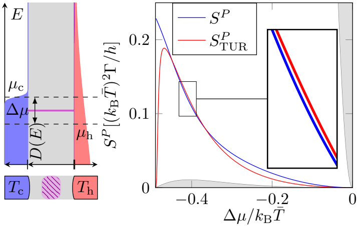
Output power $P = I\frac{\Delta\mu}{q}$.   Noise of the power $S^P = \left(\frac{\Delta\mu}{q}\right)^2 S^I$

Thermodynamic uncertainty relation (TUR)

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

$$S^P \geq S^P_{\mathrm{TUR}} \equiv 2\frac{k_\mathrm{B}P^2}{\dot{\sigma}}$$

Does not hold in coherent scatterers!



L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Constraints between output power and noise

Output power $P = I\frac{\Delta\mu}{q}$.   Noise of the power $S^P = \left(\frac{\Delta\mu}{q}\right)^2 S^I$

Thermodynamic uncertainty relation (TUR)

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

$$S^P \geq S^P_{\text{TUR}} \equiv 2\frac{k_B P^2}{\dot{\sigma}}$$

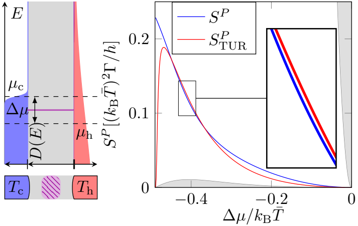
Does not hold in coherent scatterers!

Fluctuation-dissipation bounds

$\Downarrow$

$$S^P \geq P\Delta\mu \tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right)$$

Holds for arbitrary $\Delta\mu, \Delta T$, scattering matrix $s(E)$



L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Constraints between output power and noise

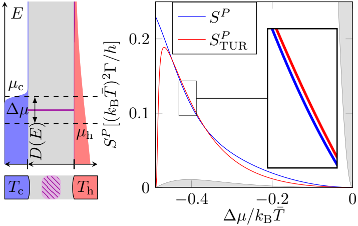
Output power $P = I\frac{\Delta\mu}{q}$. Noise of the power $S^P = \left(\frac{\Delta\mu}{q}\right)^2 S^I$

Thermodynamic uncertainty relation (TUR)

A. C. Barato, U. Seifert: Phys. Rev. Lett. **114**, 158101 (2015)

$$S^P \geq S^P_{\mathrm{TUR}} \equiv 2\frac{k_B P^2}{\dot{\sigma}}$$

Does not hold in coherent scatterers!

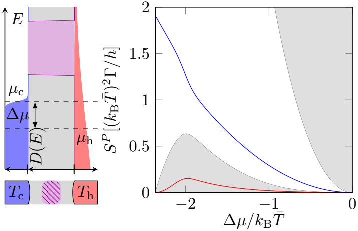


Fluctuation-dissipation bounds

$\Downarrow$

$$S^P \geq P\Delta\mu \tanh\left(\frac{\Delta\mu}{2k_B\Delta T}\right)$$

Holds for arbitrary $\Delta\mu, \Delta T$, scattering matrix $s(E)$



L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Conclusions

Out-of-equilibrium fluctuation-dissipation bound

$$S^I - S^I_{\mathrm{eq}} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_\mathrm{B}\Delta T}\right)$$

→ Complements fluctuation-dissipation theorem

→ Provides constraint between *power* and its fluctuations in nanoscale thermoelectric heat engines

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

Conclusions

Out-of-equilibrium fluctuation-dissipation bound

$$S^I - S^I_{\text{eq}} \leq -qI \tanh\left(\frac{\Delta\mu}{2k_{\text{B}}\Delta T}\right)$$

→ Complements fluctuation-dissipation theorem

→ Provides constraint between *power* and its fluctuations in nanoscale thermoelectric heat engines

L. Tesser, J. Splettstoesser: Phys. Rev. Lett. 132, 186304 (2024)

——— • Questions are welcome • ———

EXTRA: Charge noise

- Classical noise contribution
  $S^I_{\text{cl}} \equiv \frac{q^2}{h} \int dE \, |s_{\text{LR}}|^2 [f_{\text{L}}(1 - f_{\text{R}}) + f_{\text{R}}(1 - f_{\text{L}})]$
  - $\rightarrow$ Single-electron process contribution
  - $\rightarrow$ Finite at equilibrium
  - $\rightarrow$ Satisfies Fluctuation-dissipation theorem (Johnson-Nyquist noise)

Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000)

EXTRA: Charge noise

- Classical noise contribution
  $S^I_{\mathrm{cl}} \equiv \frac{q^2}{h} \int dE \, |s_{\mathrm{LR}}|^2 [f_{\mathrm{L}}(1 - f_{\mathrm{R}}) + f_{\mathrm{R}}(1 - f_{\mathrm{L}})]$
  - $\rightarrow$ Single-electron process contribution
  - $\rightarrow$ Finite at equilibrium
  - $\rightarrow$ Satisfies Fluctuation-dissipation theorem (Johnson-Nyquist noise)

- Quantum noise contribution
  $S^I_{\mathrm{qu}} \equiv -\frac{q^2}{h} \int dE \left[|s_{\mathrm{LR}}|^2 (f_{\mathrm{L}} - f_{\mathrm{R}}) \right]^2$
  - $\rightarrow$ Two-electron process contribution
  - $\rightarrow$ At equilibrium vanishes!
  - $\rightarrow$ Always negative
  - $\rightarrow$ Negligible in the tunneling regime ($|s_{\mathrm{LR}}|^2 \ll 1$)

Ya. M. Blanter, M. Büttiker: Phys. Rep. **336**, 1 (2000)

EXTRA:

Calling h the *hottest* reservoir, we have

$$S^I_{\mathrm{hh}} - S^I_{\mathrm{hh,eq}} \leq -\frac{q^2}{h}\sum_{\alpha\neq \mathrm{h}} \tanh\left(\frac{1}{2}\frac{\mu_\mathrm{h}-\mu_\alpha}{k_\mathrm{B}(T_\mathrm{h}-T_\alpha)}\right)\int dE \sum_{i=1}^{N_\mathrm{h}} D_{\mathrm{h}\alpha,i}\left[f_\alpha - f_\mathrm{h}\right],$$