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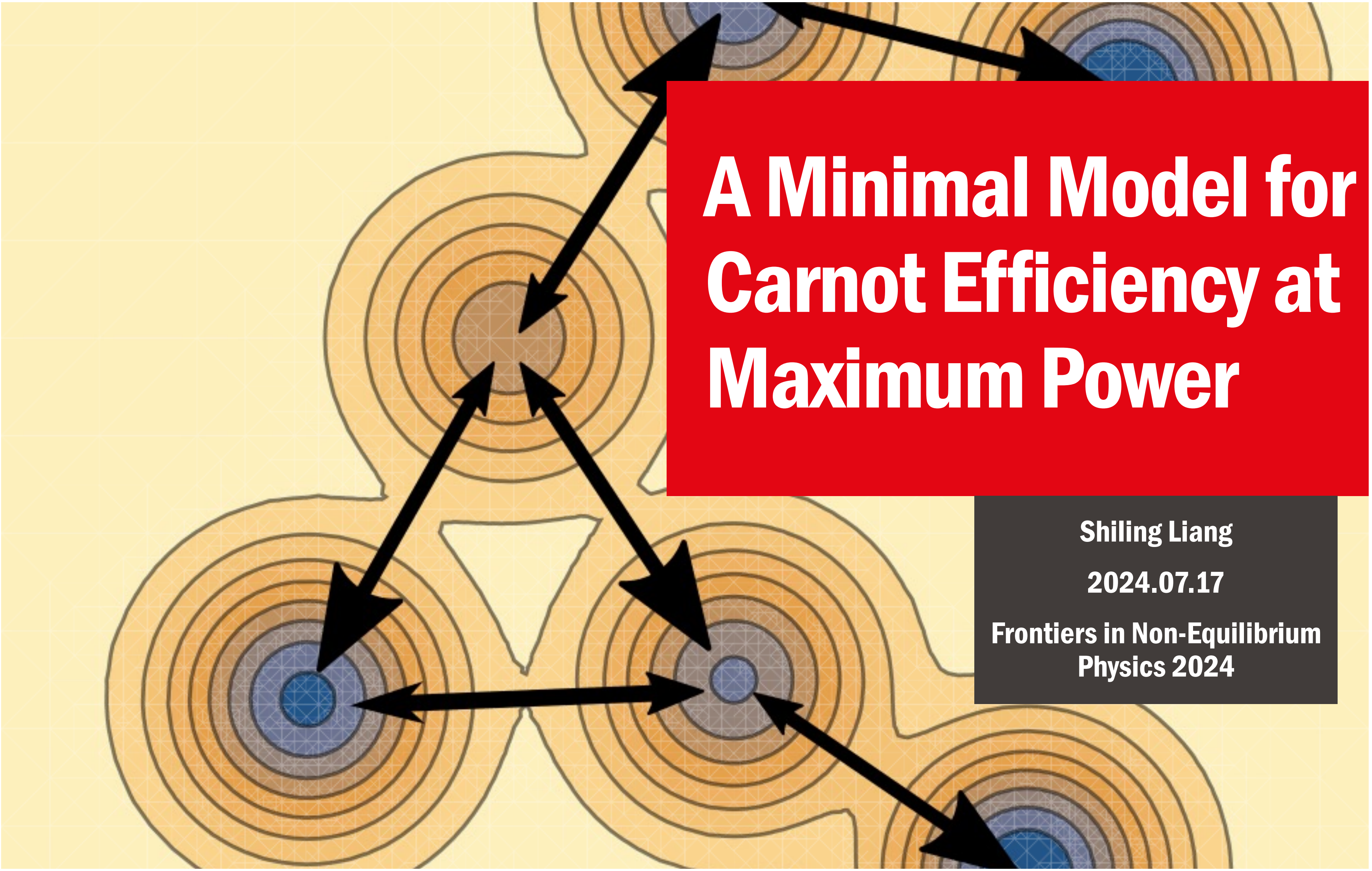


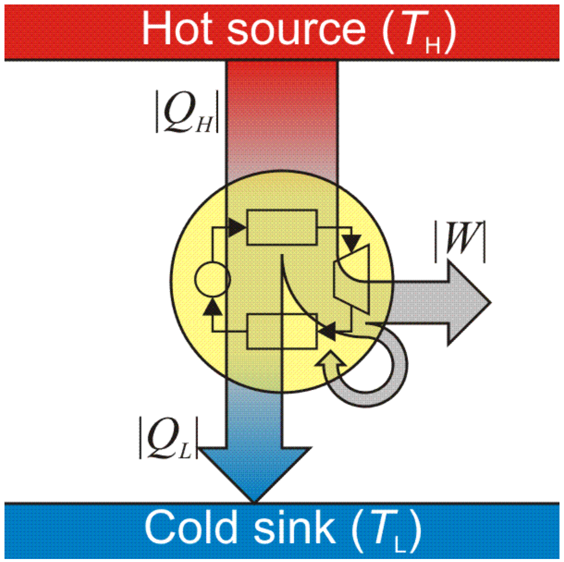
A Minimal Model for Carnot Efficiency at Maximum Power

Shiling Liang

2024.07.17

Frontiers in Non-Equilibrium Physics 2024

Quasi-static limit ⇔ zero power

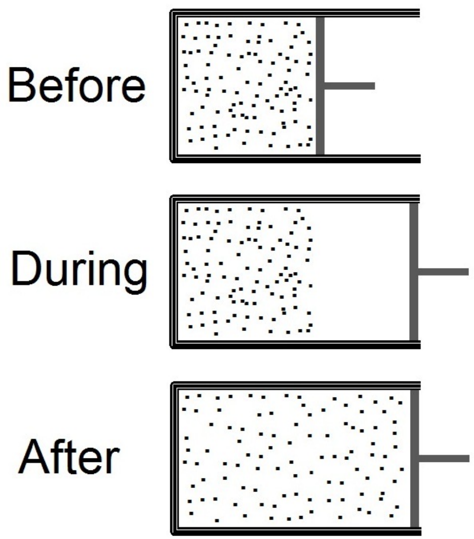


$$\eta_c = \max \frac{W}{Q_h} = 1 - \frac{T_c}{T_h}$$

- For any heat engine, the exact Carnot efficiency is achieved at the **quasi-static limit** ($\tau_{cycle} \to \infty$)
- The power at quasi-static limit is 0.
- What's the **efficiency at finite power**?

Why finite-time engine cannot achieve Carnot efficiency?

- The intrinsic time-scale leads to inevitable dissipation at **finite time operation**



- The deviation from Carnot efficiency is due to **irreversible dissipation**

$$\Delta S_{\text{irr}} = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} = \underbrace{(\eta_c - \eta)}_{\Delta\eta} \frac{Q_h}{T_c}$$

$$\Rightarrow \boxed{\Delta\eta = \frac{\Delta S_{\text{irr}}}{Q_h/T_c}}$$

Efficiency at maximum power (EMP)

- An model specific relation[1]:

$$\eta_{CA} = 1 - \sqrt{\frac{T_c}{T_h}} = 1 - \sqrt{1 - \eta_c}$$

  - Proved by C. Van den Broeck using **linear irreversible thermodynamics**[2]

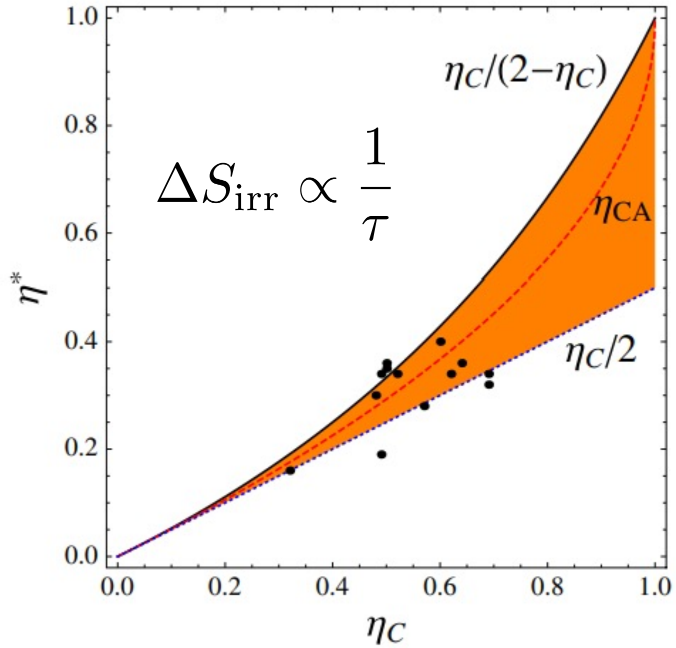
- But how about **further away from equilibrium?**

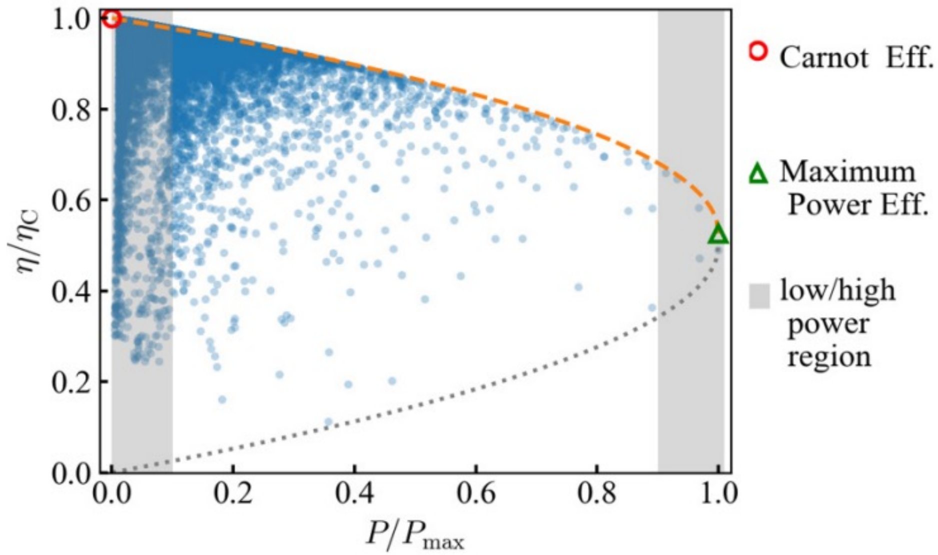
  - The difficulty: **optimization of nonlinear functions**

[1] Curzon & Ahlborn, AJP (1975)
[2] Van den Broeck, PRL (2005)

EPFL

Low-dissipation Heat Engine





- Low-dissipation heat engine has the lower and upper bound of EMP

Esposito et al. PRL (2010)

- A power-efficiency trade-off curve of low-dissipation heat engine

Yu-Han Ma et al. PRE (2018)

Universal power-efficiency trade-off (2016)

- For **finite-size** heat engines, there is a **universal power-efficiency trade-off**

$$P \leq \bar{\Theta}\beta_c \eta(\eta_C - \eta)$$

Shiraishi et al. PRL (2016)

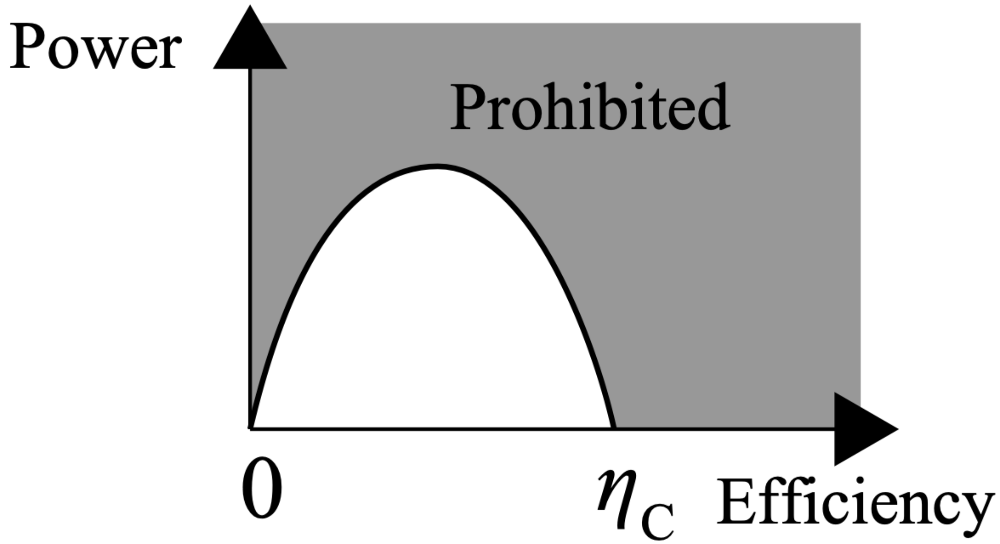


Figure from Shiraishi, Springer (2023)

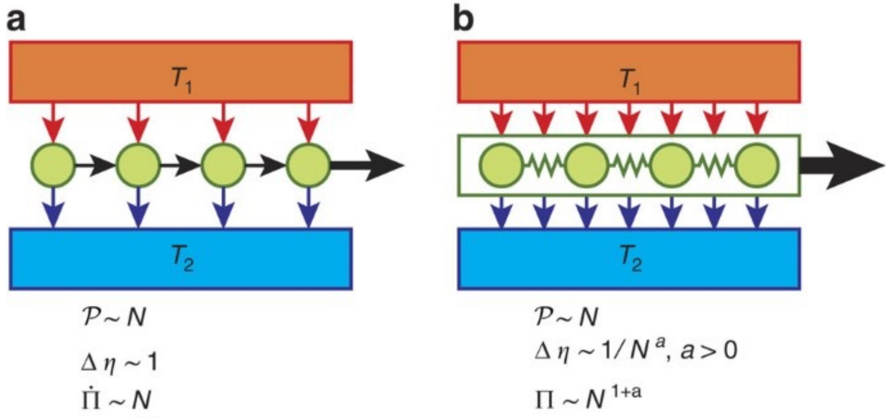
Attainability of Carnot efficiency with finite power (2016)

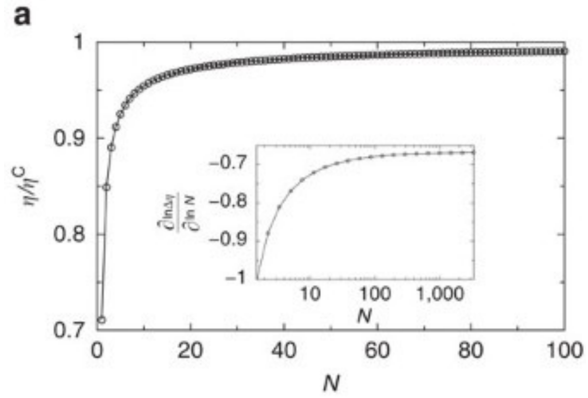
Open Access | Published: 20 June 2016

The power of a critical heat engine

Michele Campisi & Rosario Fazio

Nature Communications 7, Article number: 11895 (2016) | Cite this article





- Critical heat engines can **approach Carnot efficiency at finite power**.

Campisi & Fazio, Nat. Commun (2016)

Collective advantage (2023)

PHYSICAL REVIEW LETTERS **131,** 210401 (2023)

Collective Advantages in Finite-Time Thermodynamics

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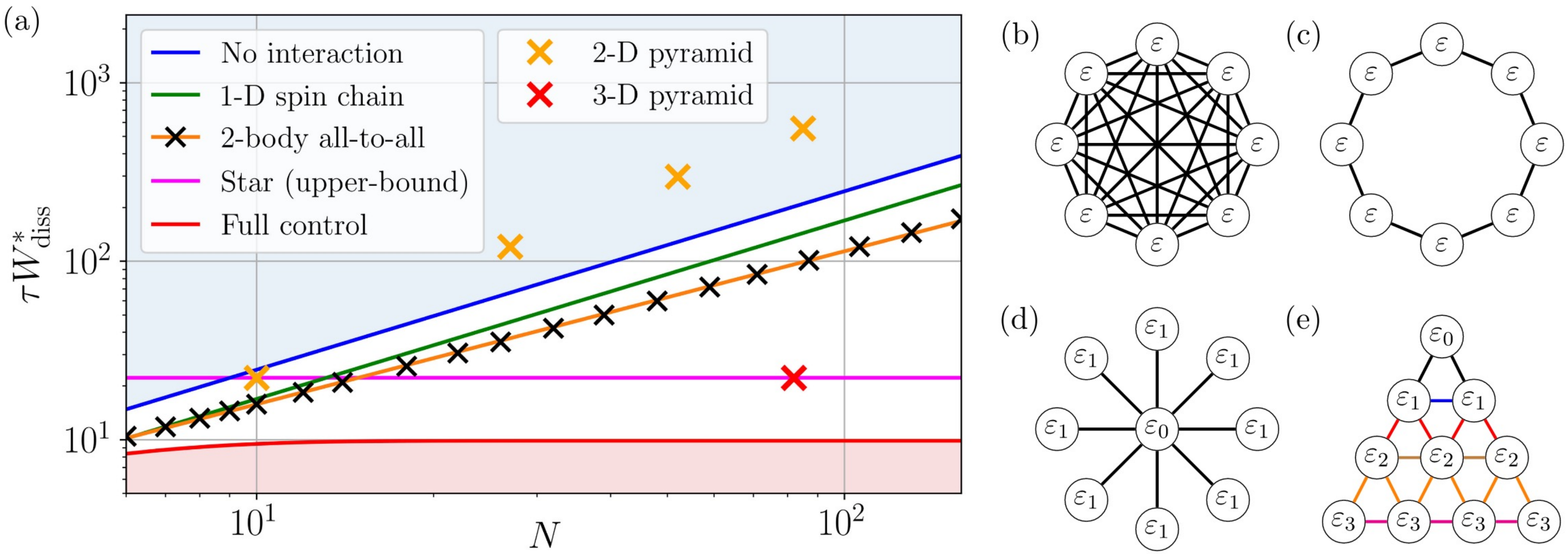
$$W = \Delta F + W_{\text{diss}}$$

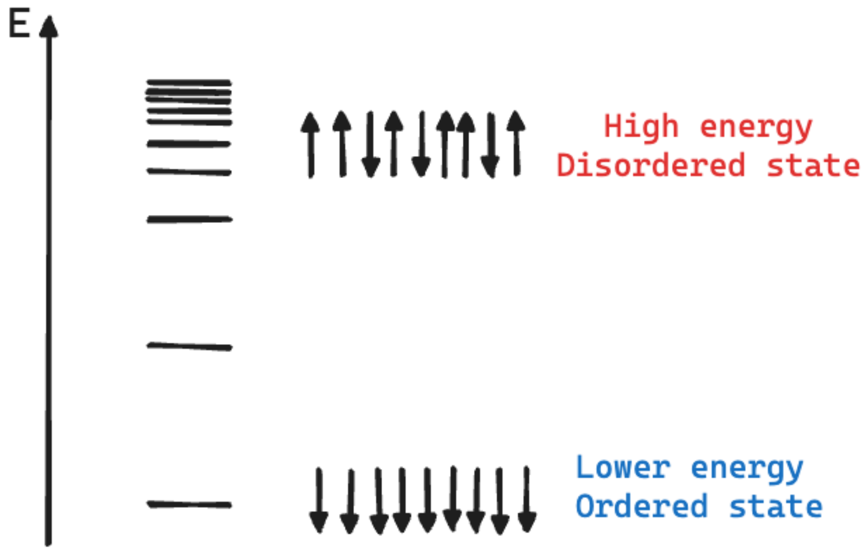
$$W_{\text{diss}} \propto N^x$$

$$W \propto N$$

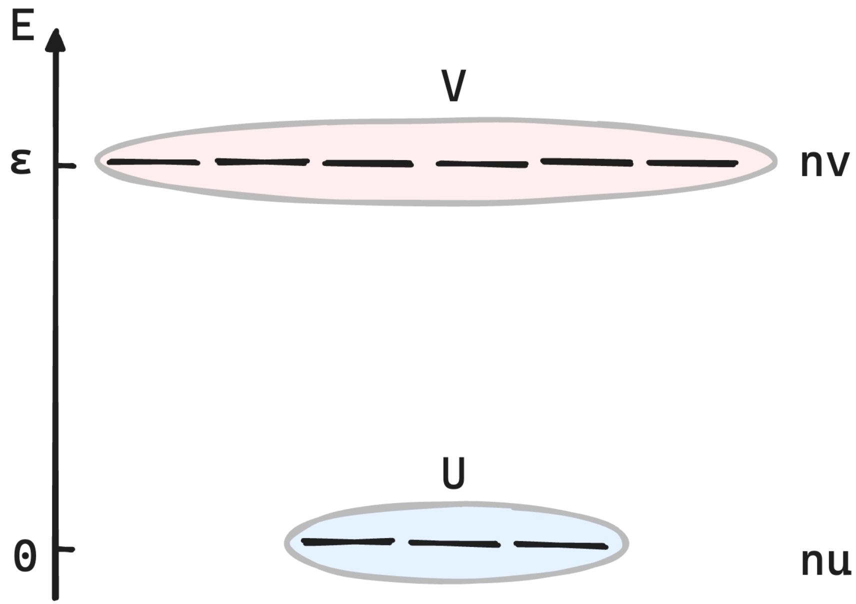


Collective advantage from energy level perspective

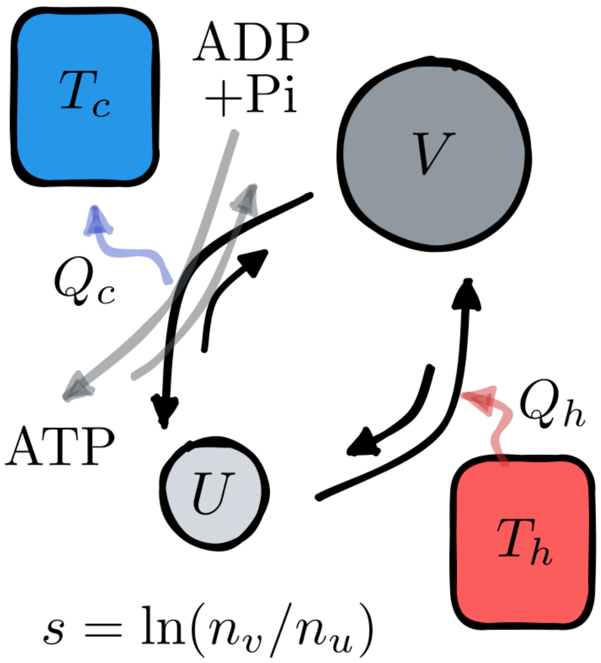
- Many-body interaction generates sets of energy levels

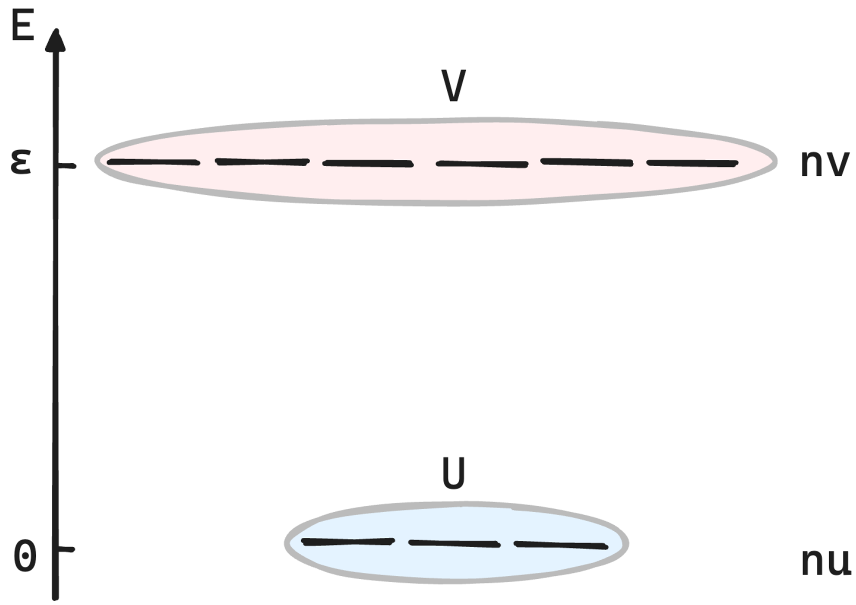


- We can directly optimize over energy level configurations



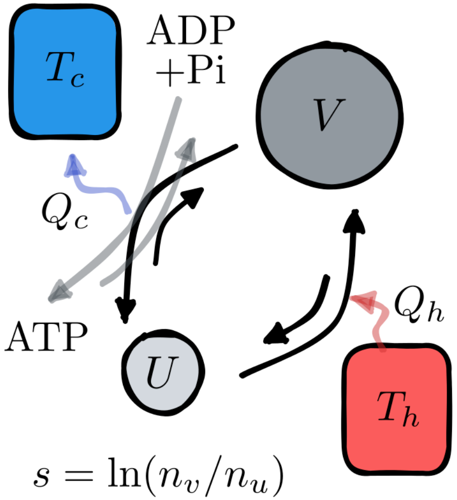
Two-(coarse-grained)-state system as a heat engine

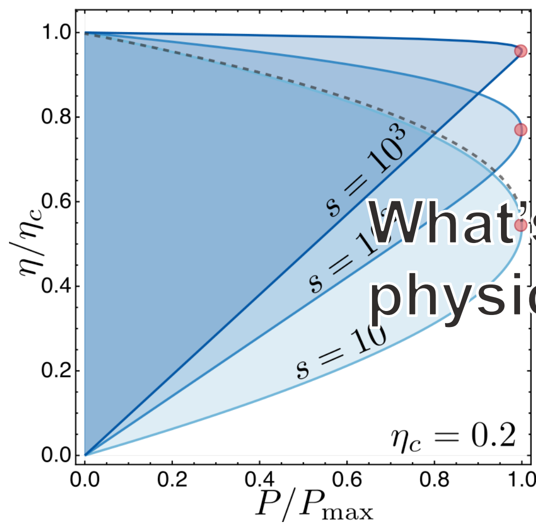


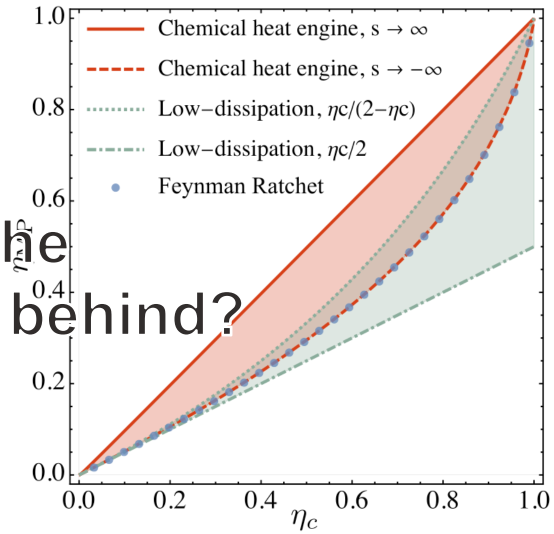


The power-efficiency trade-off and EMP

- Two-state heat engine



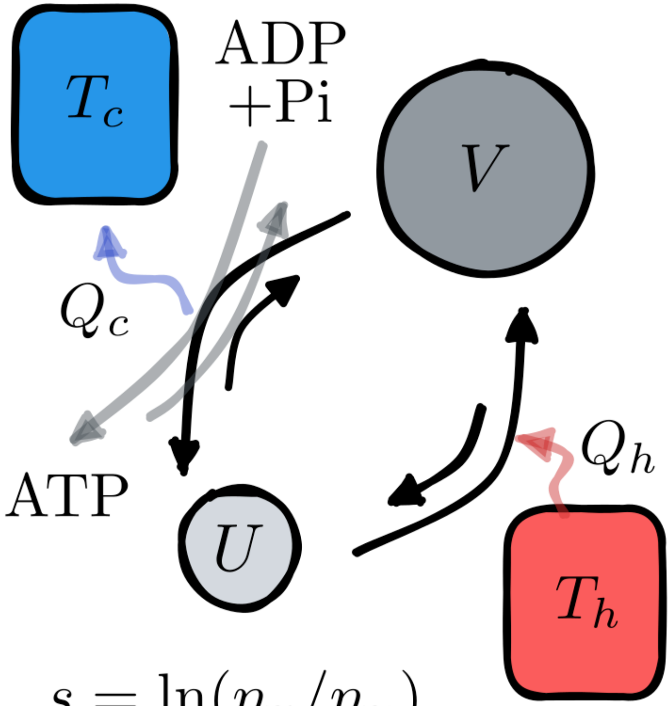




What's the physics behind?

Increase degeneracy, EMP shift to η_c

2-state heat engine



$$Q_c = \epsilon_c, \quad Q_h = \epsilon_h$$
$$W = Q_h - Q_c = \epsilon_h - \epsilon_c = \Delta\mu$$

- Master equation

$$\frac{d}{dt}p_u = (k_{h-} + k_{c-})p_v - (k_{h+} + k_{c+})p_u,$$
$$\frac{d}{dt}p_v = (k_{h+} + k_{c+})p_u - (k_{h-} + k_{c-})p_v.$$

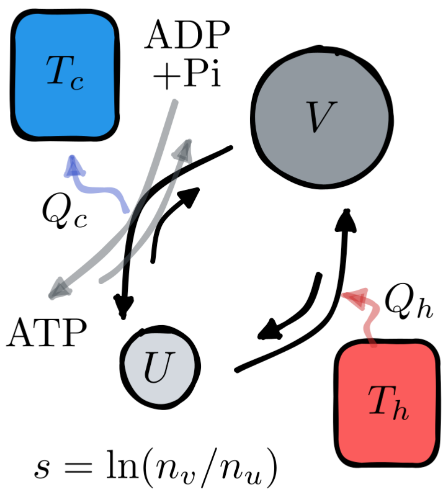
- NESS flux

$$J^{ss} = \frac{1}{\tau_h + \tau_c}\left(\frac{1}{1 + e^{\beta_h \epsilon_h - s}} - \frac{1}{1 + e^{\beta_c \epsilon_c - s}}\right)$$

- Thermodynamic efficiency

$$\eta \equiv \frac{Q_h - Q_c}{Q_h} = \frac{\Delta\mu}{\epsilon_h}$$

How to find the EMP



- An analytic expression of the **power** can be obtained

$$P = J^{\mathrm{ss}}(\epsilon_h - \epsilon_c) = \frac{(\pi_v^h - \pi_v^c)\Delta\mu}{\tau_h + \tau_c}.$$

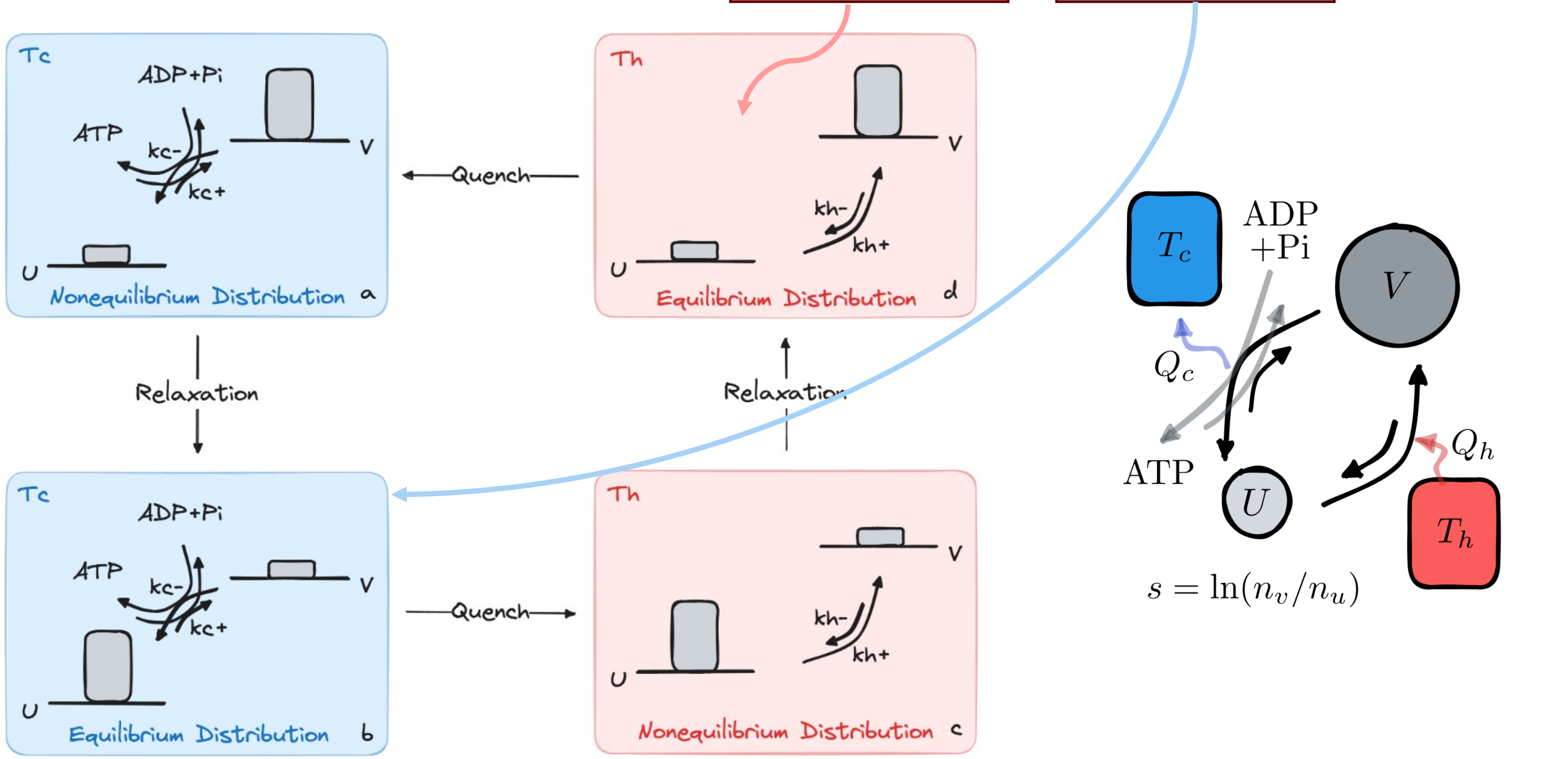
  - Where $\tau_{h/c} = (k_{h/c}^{+} + k_{h/c}^{-})^{-1}$

$$P = \frac{1}{\tau}\left(\frac{1}{1+e^{\beta_h\epsilon_h - s}} - \frac{1}{1+e^{\beta_c\epsilon_c - s}}\right)(\epsilon_h - \epsilon_c)$$

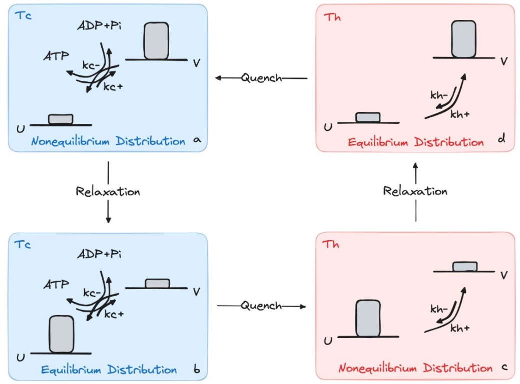
Thermodynamic part

Thermal cycle and bi-partite system

$$P = \frac{1}{\tau}\left(\frac{1}{1+e^{\beta_h \epsilon_h - s}} - \frac{1}{1+e^{\beta_c \epsilon_c - s}}\right)(\epsilon_h - \epsilon_c)$$



Optimization:
Phase-space representation

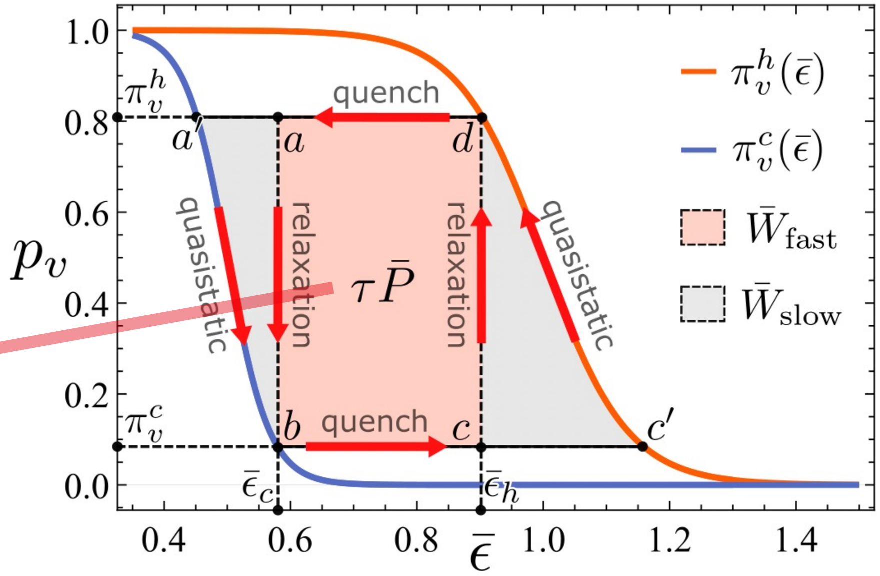


- The equilibrium curves

$$\pi_v = \frac{1}{1+\exp[s(\bar{\epsilon}/T-1)]}$$

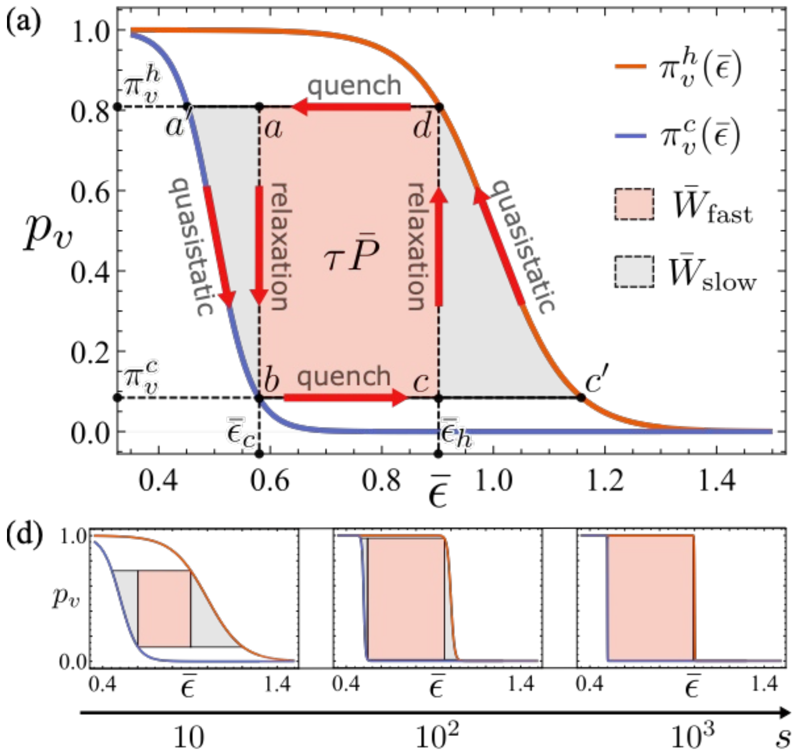
- The power

$$P = \frac{s}{\tau}\underbrace{\left(\frac{1}{1+e^{s\left(\frac{\bar{\epsilon}_h}{T_h}-1\right)}} - \frac{1}{1+e^{s\left(\frac{\bar{\epsilon}_c}{T_c}-1\right)}}\right)}_{\pi_v^h-\pi_v^c}(\bar{\epsilon}_h-\bar{\epsilon}_c)$$



where $\bar{\epsilon} \equiv \epsilon/s$

Optimization:
Otto cycle to Carnot cycle

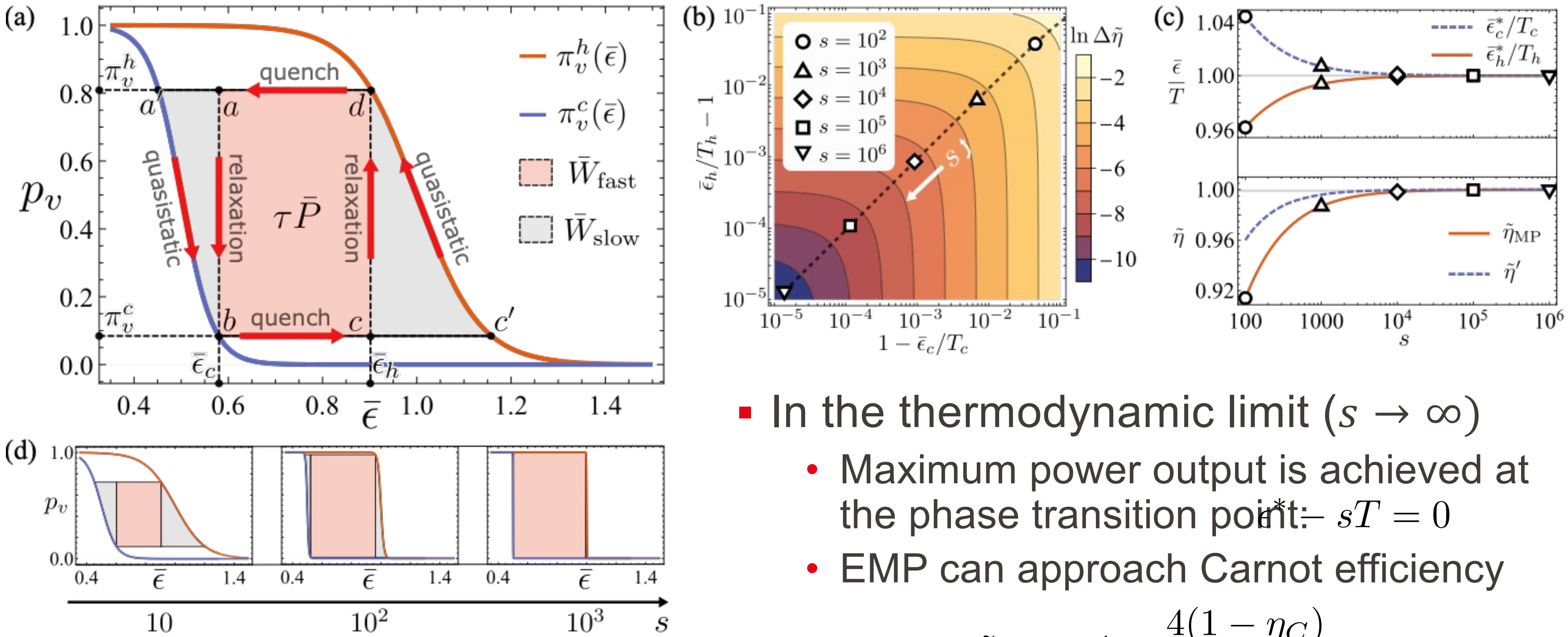


- In the thermodynamic limit ($s \to \infty$)
  - The sensitivity of equilibrium response curves diverges

$$\left.\frac{d\pi_v^{h/c}(\bar{\epsilon})}{d\bar{\epsilon}}\right|_{\bar{\epsilon}=T_{h/c}} = -\frac{s}{4T_{h/c}}$$

  - The Otto cycle approaches the Carnot cycle in large-s limit

Optimization:
Optimal parameters and phase transition

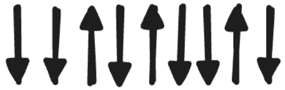


- In the thermodynamic limit ($s \to \infty$)
  - Maximum power output is achieved at the phase transition point: $\epsilon^* - sT = 0$
  - EMP can approach Carnot efficiency

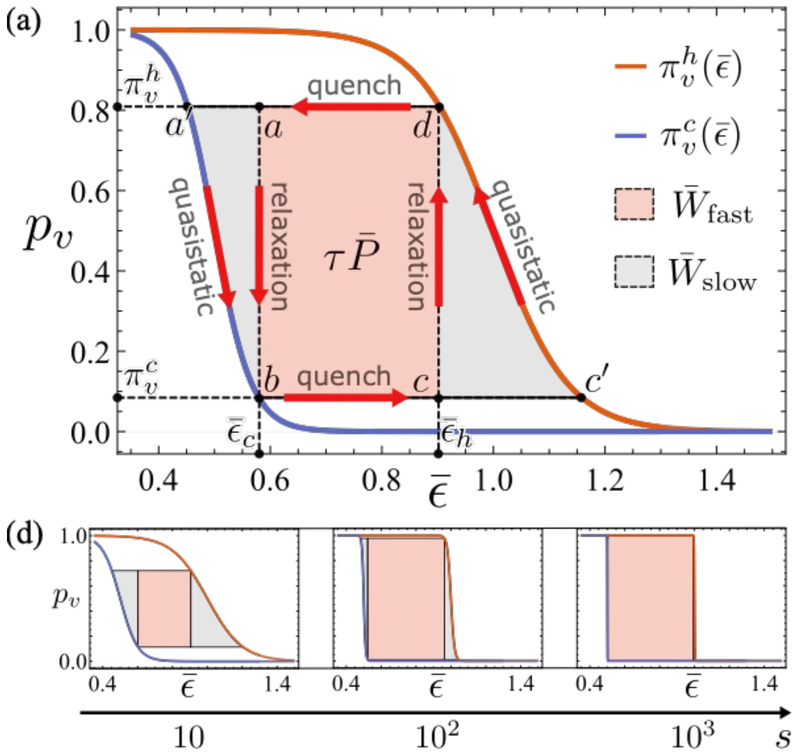
$$\tilde{\eta}_{\text{MP}} \simeq 1 - \frac{4(1-\eta_C)}{s\eta_C},$$

EPFL

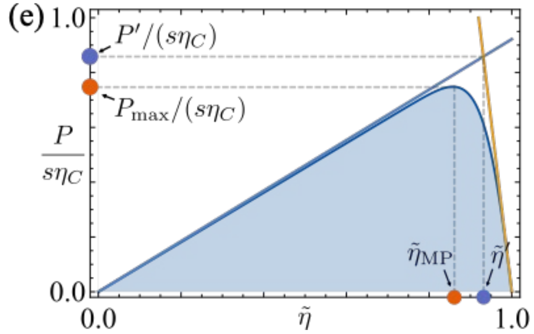
Optimization:
Power-efficiency tradeoff

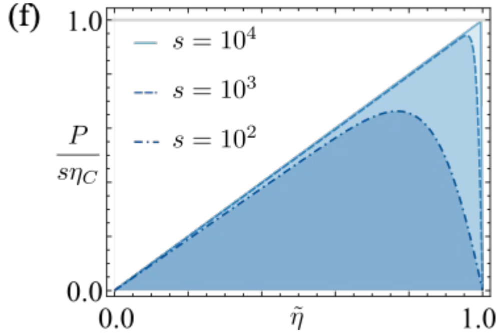


$$n \simeq 2^N \propto e^N$$
$$s = \ln n_u / n_s \sim N$$

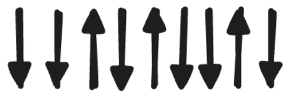


- In the thermodynamic limit ($s \to \infty$)
  - The power-efficiency tradeoff becomes a right triangle
  - The maximum power scales with $s\,\eta_c \sim N\,\Delta T \Rightarrow$ power per unit ~ ΔT





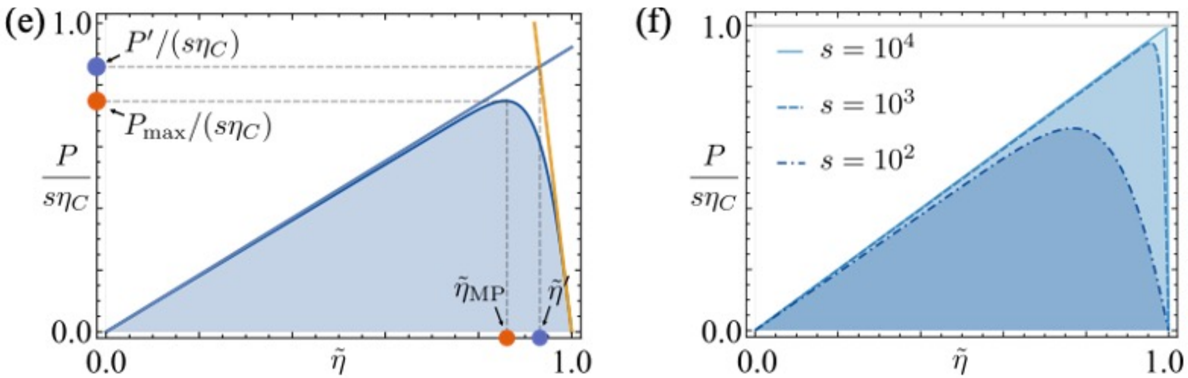
Universal trade-off with divergent quantities



$n \simeq 2^N \propto e^N$

$s = \ln n_u/n_s \sim N$

- With divergent quantities, the universal trade-off does not prohibit Carnot efficiency at maximum power



$$P \le \bar{\Theta}\beta_c\eta(\eta_C - \eta)$$

Shiraishi et al. PRL (2016)

$$\begin{cases} \lim_{\tilde{\eta}\to 0} P \simeq (s - \ln s)\eta_C\tilde{\eta}, \\ \lim_{\tilde{\eta}\to 1} P \simeq \frac{s^2\eta_C^2}{4(1-\eta_C)}(1-\tilde{\eta}). \end{cases} \Rightarrow \begin{cases} \Theta_0 \equiv \lim_{\tilde{\eta}\to 0}\Theta \simeq \frac{(s-\ln s)}{\eta_C\beta_c} \\ \Theta_1 \equiv \lim_{\tilde{\eta}\to 1}\Theta \simeq \frac{s^2}{4(1-\eta_C)\beta_c} \end{cases} \Rightarrow \frac{\Theta_1}{\Theta_0} \sim s \sim N$$

Close-to-equilibrium condition?

- The universality of efficiency at maximum power

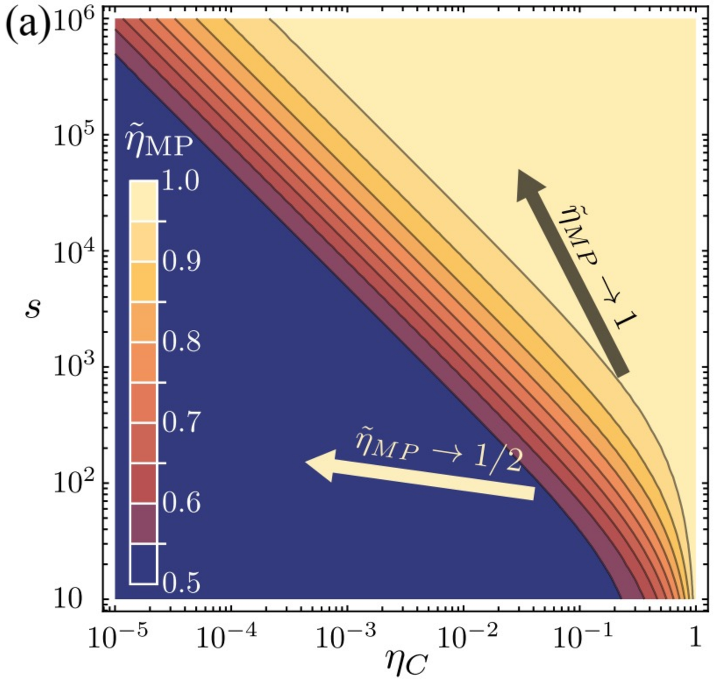
$$\eta_{\mathrm{MP}} = \boxed{\frac{1}{2}}\eta_c + \boxed{\frac{1}{8}}\eta_c^2 + \mathcal{O}(\eta_c^2)$$

linear response — left-right symmetry

Esposito et al. PRL 2009

- Is small η_c a **sufficient condition** for linear response approximation?

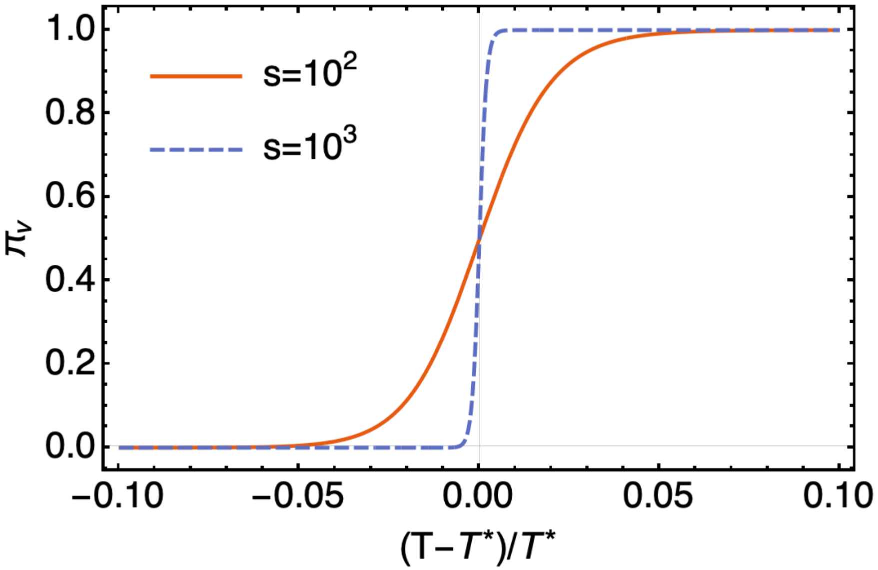
$\tilde{\eta}_{\mathrm{MP}} \equiv \eta_{\mathrm{MP}}/\eta_c$

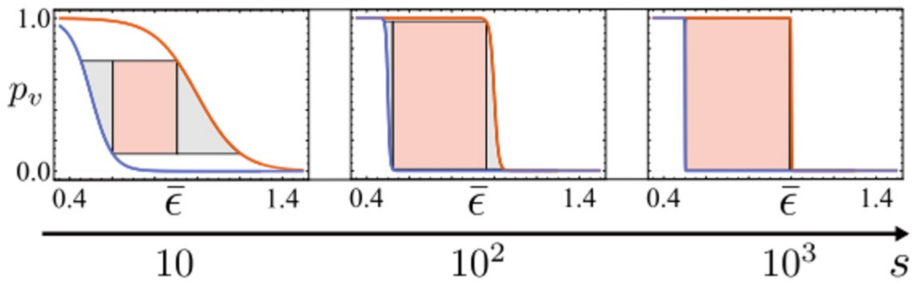


$$\begin{cases} \lim_{\eta_C \to 0} \lim_{s \to \infty} \eta_{\mathrm{MP}} = \eta_C/2 \\ \lim_{s \to \infty} \lim_{\eta_C \to 0} \eta_{\mathrm{MP}} = \eta_C. \end{cases}$$

On close-to-equilibrium condition

$$\pi_v = \frac{1}{1+\exp[\epsilon/T - s]}$$





- The phase transition point

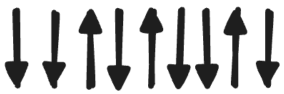
$$\epsilon/T^* - s = 0 \Rightarrow T^* = \epsilon/s$$

- Linear expansion around T^*

$$\left.\frac{d\pi_v}{d(T/T^*)}\right|_{T=T^*} = \frac{s}{4}$$

$$\pi_v(T) = \pi_v(T^*) + \frac{s}{4}\frac{T-T^*}{T^*} + O^2(\frac{T-T^*}{T^*})$$

- Linear-response regime: $s\eta_c \ll 1$



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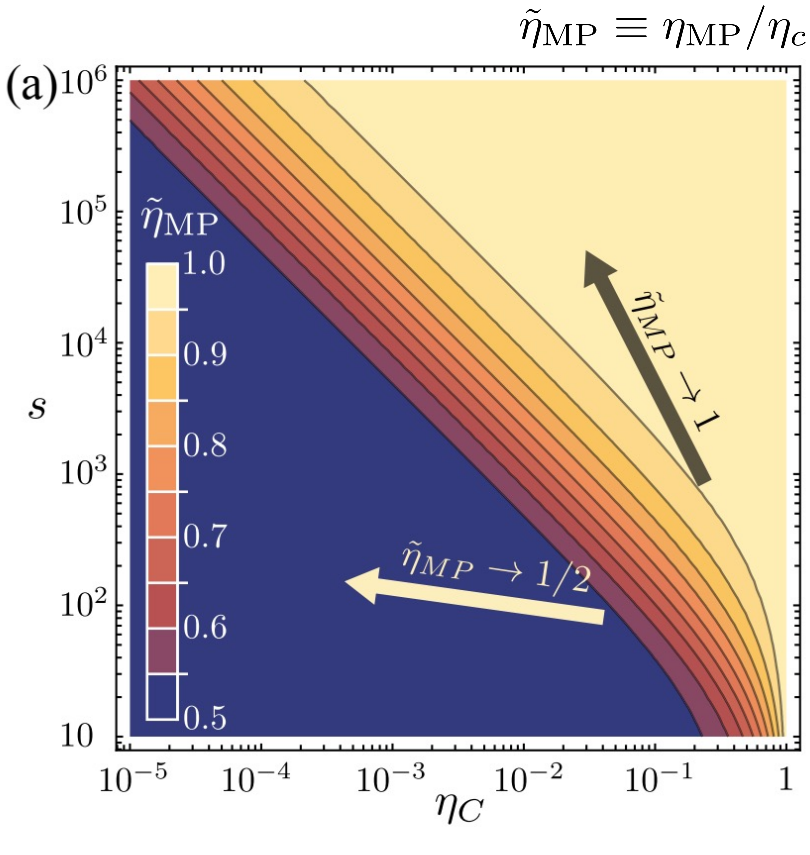

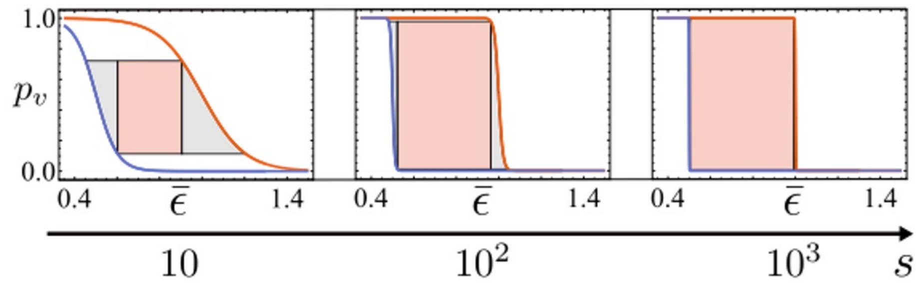

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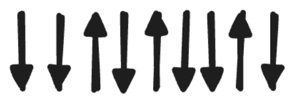

$$n \simeq 2^N \propto e^N$$
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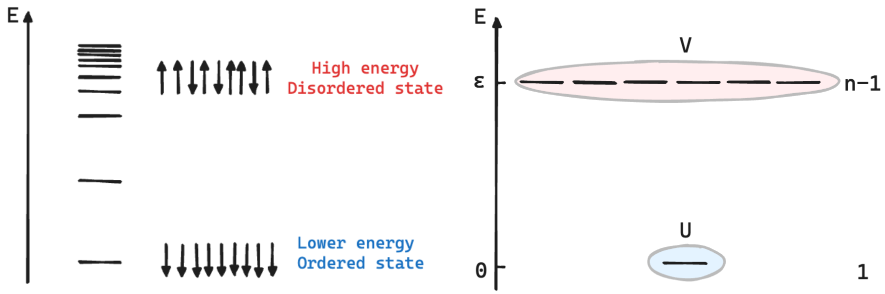
Conclusion

S. Liang, Y.-H. Ma, D. M. Busiello, and P. De Los Rios. (2023). **A Minimal Model for Carnot Efficiency at Maximum Power**. *arXiv preprint* arXiv:2312.02323.

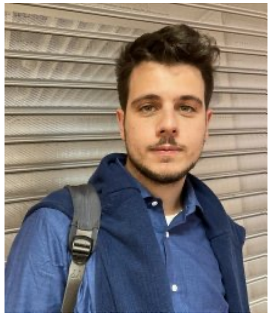
- An **energy-level-optimized** heat engine can achieve **Carnot efficiency at maximum power.**



- There may exists a **universal power-efficiency-size tradeoff.**

- A **criterion for linear-response approximation** for heat engines: $s\eta_C \ll 1 \quad (N\eta_C \ll 1)$

Yu-Han Ma



Daniel Maria Busiello

Paolo De Los Rios

A Minimal Model for Carnot Efficiency at Maximum Power

Shiling Liang, Yu-Han Ma, Daniel Maria Busiello, Paolo De Los Rios

Carnot efficiency sets a fundamental upper bound on the heat engine efficiency, attainable in the quasi-static limit, albeit at the cost of completely sacrificing power output. In this Letter, we present a minimal heat engine model that can attain Carnot efficiency while achieving maximum power output. We unveil the potential of intrinsic divergent physical quantities within the working substance, such as degeneracy, as promising thermodynamic resources to break through the universal power-efficiency trade-off imposed by nonequilibrium thermodynamics for conventional heat engines. Our findings provide novel insights into the collective advantage in harnessing energy of many-body interacting systems.

Comments: 6+6 pages, 4 figures
Subjects: **Statistical Mechanics (cond-mat.stat-mech)**
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