Coherence costs and effects in dynamics of thermodynamics

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Frontiers in non-equilibrium Physics

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Topics of this talk

Coherence costs of operations in quantum thermodynamics

・・・**and coherence enhancement on heat engines**

Remark on wording in this talk:

・quantum thermodynamics=thermodynamics in quantum systems

・Resource theory of thermodynamics
=resource theoretic approach to thermodynamics

Topics of this talk

Coherence costs of operations in thermodynamics of quantum systems

Related works:

HT and R. Takagi,
arXiv:2404.03479 (2024)

HT, N. Shiraishi, K. Saito,
PRL **121**, 110403 (2018)

HT, N. Shiraishi, K. Saito,
PRR **2**,043374 (2020)

HT, R. Takagi, Y. Kuramochi,
arXiv:2206.11086, (2022)

➡First part of this talk

・・・and coherence enhancement on heat engines

Related work:

HT and K. Funo,
PRL. **127**, 190604 (2021)
(Editor's suggestion +Featured in Physics)

➡Second part of this talk

Part I:
Coherence costs of operations in quantum thermodynamics

➡Gibbs-preserving maps requiring infinite amount of coherence

➡Lower and upper bounds of coherence cost of approximate implementation of the cost-diverging Gibbs preserving maps

➡trade-off between generalized entropy production and coherence cost of arbitrary quantum operations

Outline of Part I

Backgrounds and brief summery of results

Details of Results:

- Setup and quantities
- Gibbs-preserving maps requiring infinite amount of coherence
- Lower and upper bounds of coherence cost of approximate implementation of Gibbs preserving maps
- trade-off between generalized entropy production and coherence cost of arbitrary quantum operations

Techniques:
Trade-off between symmetry, irreversibility and quantum coherence

Gibbs-preserving maps

In quantum thermodynamics, the Gibbs preserving map is an important class of operations

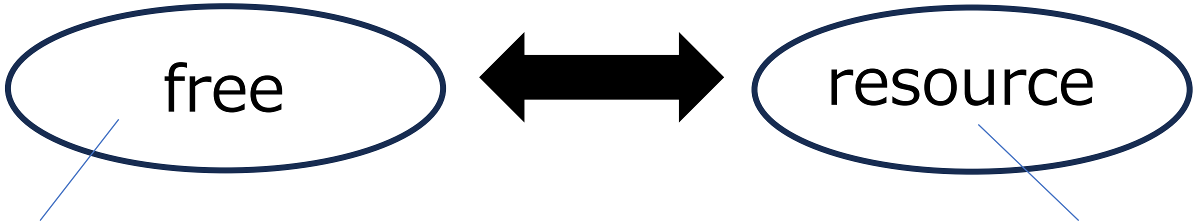
・$\mathcal{E}(\rho_{\beta\|H}) = \rho_{\beta\|H'}$ ・$\mathcal{E}$ is CPTP	⟺	$\mathcal{E}$ is a Gibbs preserving map

・It corresponds to the "isothermal process":
a common class of operations in quantum thermodynamics

・It also plays a central role in resource theory of thermodynamics
➡one of two possible "free operations"

Gibbs-preserving maps

In resource theory, we divide states and operations into two classes, respectively



We consider "free states" and "free operations" easy to realize.

➡We assume that "free states" and "free operations" can be used freely.

All states and operations that are not achievable with combining free states and operations are resource.

Good references for resource theory:

Review for various resource theories:
E. Chitambar and G. Gour, Rev. Mod. Phys. 91, 025001 (2019).

Resource theory of thermodynamics:
T. Sagawa, SpringerBriefs in Mathematical Physics 16 (2022) (textbook)

Resource theory of asymmetry:
I. Marvian, PhD thesis (2012).

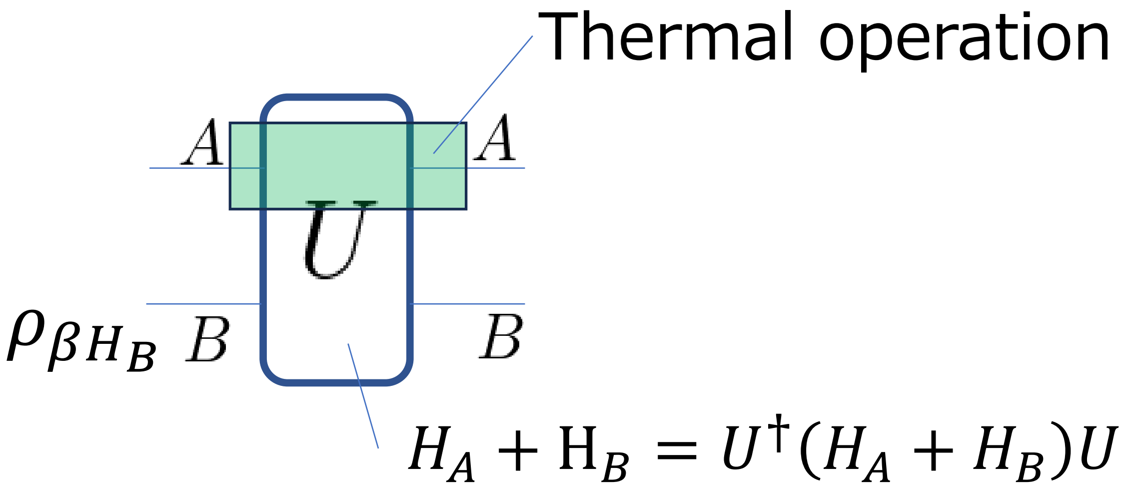
The structure of the resource theory depends on the choice of free operations.

And resource theory of thermodynamics has two candidates of free operations:

Gibbs-preserving operations and thermal operations

Gibbs-preserving operation v.s. thermal operation

Thermal operation:



Gibbs preserving operation:

$$\mathcal{E}(\rho_{\beta|H}) = \rho_{\beta|H}$$

Thermal operation: implementable, but difficult to treat

Gibbs preserving operation: easy to treat.

➡ Recent important works in resource theory of thermodynamics employ Gibbs preserving maps:

P. Faist and R. Renner, PRX 8, 021011 (2018).
P. Faist, M. Berta, and F. Brandao, PRL122, 200601 (2019).
F. Buscemi, D. Sutter, and M. Tomamichel, Quantum 3, 209 (2019).
N. Shiraishi and T. Sagawa, PRL 126, 150502 (2021).

Therefore, it is important to clarify the relation between the thermal operations and the Gibbs preserving maps.

Gibbs-preserving map v.s. thermal operation

It is easily obtained:

Thermal operation $\subset$ **Gibbs preserving map**

But, the reverse is not true. Namely,

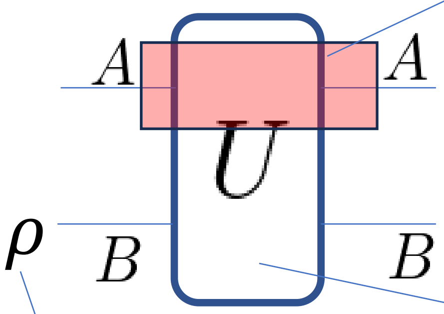
Gibbs preserving map

Operations belonging to here actually exist:

P. Faist, J. Oppenheim, and R. Renner, New J. Phys. 17, 043003 (2015)

The article gives an example of Gibbs-preserving map Λ such that

To realize Λ,



$H_A + \mathrm{H}_B = U^\dagger (H_A + H_B) U$

we need to prepare a state satisfying $[\rho, H_B] \neq 0$ here.

Since Gibbs state satisfies

$$[\rho_{\beta|H_B}, H_B] = 0,$$

the GPM Λ cannot be a thermal operation.

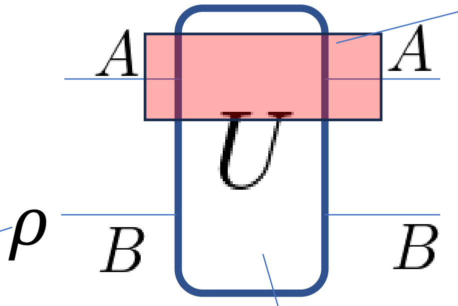
Cost of Gibbs-preserving map

Some Gibbs-preserving maps require extra states satisfying $[\rho, H_B] \neq 0$.

➡some Gibbs preserving maps requires extra energetic coherence (i.e. quantum fluctuation of energy).

Question: What is a general upper limit of required energetic coherence to realize an arbitrary Gibbs-preserving map?

It is important, because Gibbs-preserving maps are considered to be free, and thus they should be easy to realize.



To realize an arbitrary GPO Λ

how much resource should ρ have?

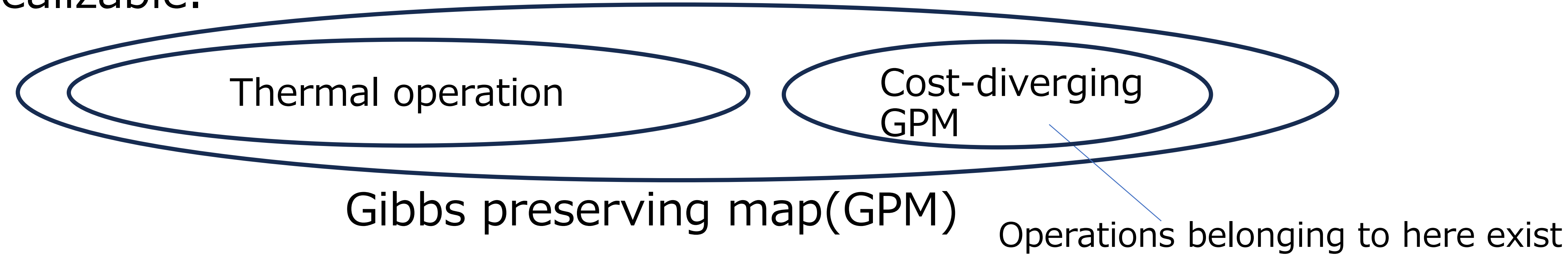
$$H_A + \mathrm{H}_B = U^\dagger (H_A + H_B) U$$

This problem is in a formal list of open problems in quantum information theory:
https://oqp.iqoqi.oeaw.ac.at/thermodynamic-implementation-of-gibbs-preserving-maps

Gibbs-preserving maps requiring infinite amount of coherence

HT and R. Takagi, arXiv:2404.03479 (2024)

Result 1-A: some Gibbs-preserving maps require infinite amount of energetic coherence (quantum fluctuation of energy), and thus they are not realizable.



Result 1-B: There are sets of (ρ, σ) such that

$$\rho \longrightarrow \sigma$$

possible by cost-diverging GPM, but impossible by finite-cost GPM

These results are given by general tradeoff between coherence cost and irreversibility: **Results 2 and 3**

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- ・Setup and quantities
- ・Gibbs-preserving maps requiring infinite amount of coherence
- ・Lower and upper bounds of coherence cost of approximate implementation of Gibbs preserving maps
- ・trade-off between generalized entropy production and coherence cost of arbitrary quantum operations

Techniques:
Trade-off between symmetry, irreversibility and quantum coherence

Preparation 1: measure of energetic coherence

Energetic coherence in ρ is measured by the SLD-quantum Fisher information for state family $\{e^{-iHt}\rho e^{iHt}\}$:

$$\mathcal{F}_\rho(H) \coloneqq \sum_{ij} \frac{2(p_i - p_j)^2}{p_i + p_j} |\langle i|H|j\rangle|^2$$

$\{p_i, |i\rangle\}$ are the eigenvalues and eigenvectors of ρ.

Property 1: $\mathcal{F}_\rho(H)$ is a standard measure of quantum fluctuation of energy:

$$\mathcal{F}_\rho(H) = 4 \min_{\{q_j, |\phi_j\rangle\}: \rho = \sum_j q_j \phi_j} \sum_j q_j V_{\phi_j}(H)$$

S. Yu, arXiv:1302.5311 (2013).

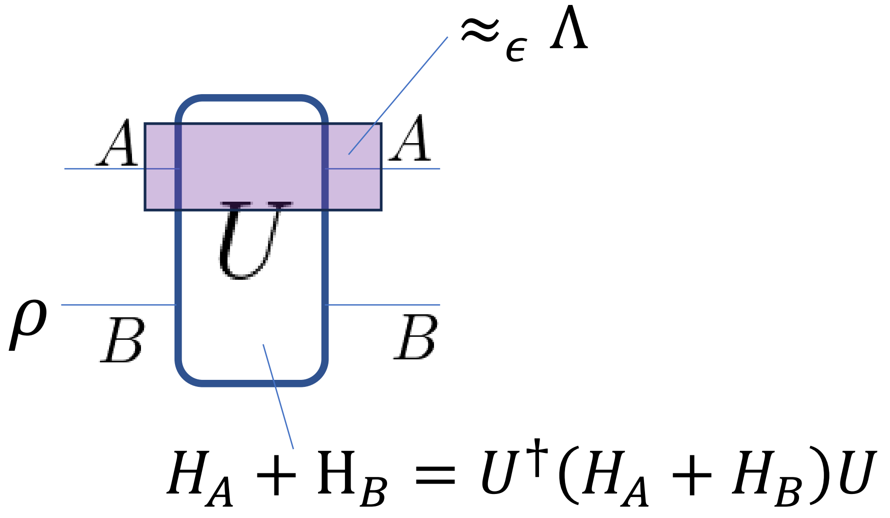
Property 2: SLD-Quantum Fisher information is a standard measure of resource in the resource theory of asymmetry.

C. Zhang, et al., PRA **96**, 042327 (2017).
R. Takagi, Scientific Reports **9**, 14562 (2019).
I. Marvian, Nat. Comm. **11**, 25 (2020).
I. Marvian, PRL **129** 190502 (2022).

Preparation 2: coherence cost of quantum operations

(U,ρ,H_B) realizes Λ within error ϵ

$\overset{\text{def}}{\Longleftrightarrow}$ $\max_{\xi} D_F\left(\Lambda(\xi),\mathrm{Tr}_B\left[U\xi\otimes\rho U^\dagger\right]\right)\le\epsilon$ & $H_A+\mathrm{H}_B=U^\dagger(H_A+H_B)U$



$\mathcal{F}_c^\epsilon(\Lambda):=\inf\{\mathcal{F}_\rho(H_B)|(U,\rho,H_B)\text{ realizes }\Lambda\text{ within error }\epsilon\}$

Result 1: Cost-diverging Gibbs-preserving operations

HT and R. Takagi, arXiv:2404.03479 (2024)

We prove the following theorem:

Theorem 1:

If systems S and S′ satisfy the following ordering condition

$$\exists i, i', j,\ \ \tau_{S,i} < \tau_{S',i'} < \tau_{S,j},$$

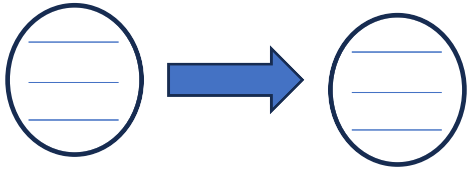
$\tau_{S,i}$: i-th eigenvalue of the Gibbs state on S

there exists a cost-diverging Gibbs-preserving operation $\Lambda: S \to S'$ such that

$$\mathcal{F}_c^{\epsilon=0}(\Lambda) = \infty$$

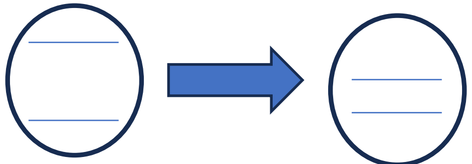
Therefore, there are cost-diverging GPO between almost arbitrary systems.

Ex: S and S′ are three level systems, and H_S=H_{S′} are the same non-degenerate Hamiltonian



Then, $\tau_{S,3} < \tau_{S',2} < \tau_{S,1}$ holds, and thus there is a cost-diverging Gibbs-preserving map from S to S′.

Ex 2: S and S′ are two level systems and the energy gap of H_{S′} is smaller than H_S.



Then, $\tau_{S,2} < \tau_{S',2} < \tau_{S,1}$ holds, and thus there is a cost-diverging Gibbs-preserving map from S to S′.

Result 1: Cost-diverging Gibbs-preserving operations

HT and R. Takagi, arXiv:2404.03479 (2024)

We prove the following theorem:

Theorem 2:

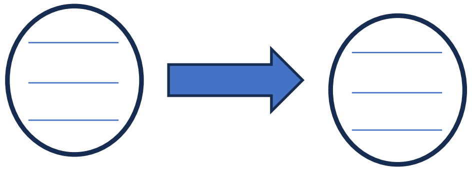
When S and S′ satisfy the same condition as Theorem 1,

there exist ρ on S and σ on S′ such that

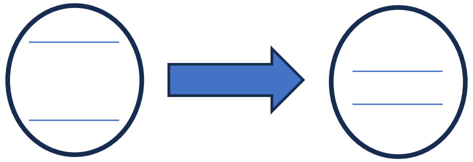
- $\rho \to \sigma$ can be realized by a Gibbs-preserving map
- $\rho \to \sigma$ cannot be realized by a finite-cost Gibbs-preserving map

Therefore, there are infeasible state transition between almost arbitrary S and S′

Ex1:



Ex 2:



Result 2: Upper and lower bound for approximate implementation

HT and R. Takagi, arXiv:2404.03479 (2024)

We also find upper and lower bounds of cost for approximate implementation of the cost-diverging GPMs

$$\frac{\mathcal{C}}{\epsilon} - a \leq \sqrt{\mathcal{F}^{\epsilon}_{\mathrm{cost}}(\Lambda)} \leq \frac{2\mathcal{C}}{\epsilon} + a$$

$\mathcal{C}$: coefficient corresponding to the change of energy by Λ. We define it later.

a : a constant

From this result, we can see the $\mathcal{F}^{\epsilon}_{\mathrm{cost}} \approx \frac{N}{\epsilon^2}$, where N is the system size.

We also show that this scaling is the worst in all of approximate implementations of the CPTP maps.

GPMs include the “most costly” class in all CPTP maps.

Result 3: Tradeoff between coherence and entropy production for arbitrary Gibbs preserving map

・When Λ is Gibbs-preserving, the entropy production can be expressed as

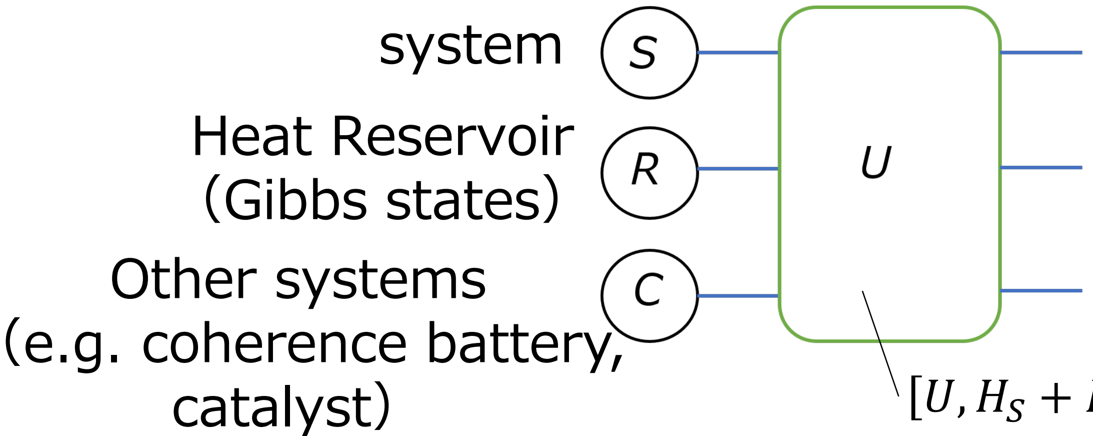
$$\Sigma(\rho) := \Delta S(\rho) - \beta Q$$

$$= D\left(\rho | \rho_{\beta|H}\right) - D(\mathcal{E}(\rho) | \mathcal{E}(\rho_{\beta|H'}))$$

・Then, the coherence cost of Λ is bounded by the entropy production as follows:

$$\mathcal{F}_c^{\epsilon=0}(\Lambda) \geq \max_{\rho} \frac{\mathcal{C}^2}{\sqrt{\Sigma(\rho)}} - \Delta^2$$

Example of application:



$[U, H_S + H_R + H_C] = 0$

Gibbs states do not have coherence of energy.
➡we can evaluate the amount of coherence in C.

Result 3B: Tradeoff between coherence and entropy production for arbitrary Gibbs preserving map

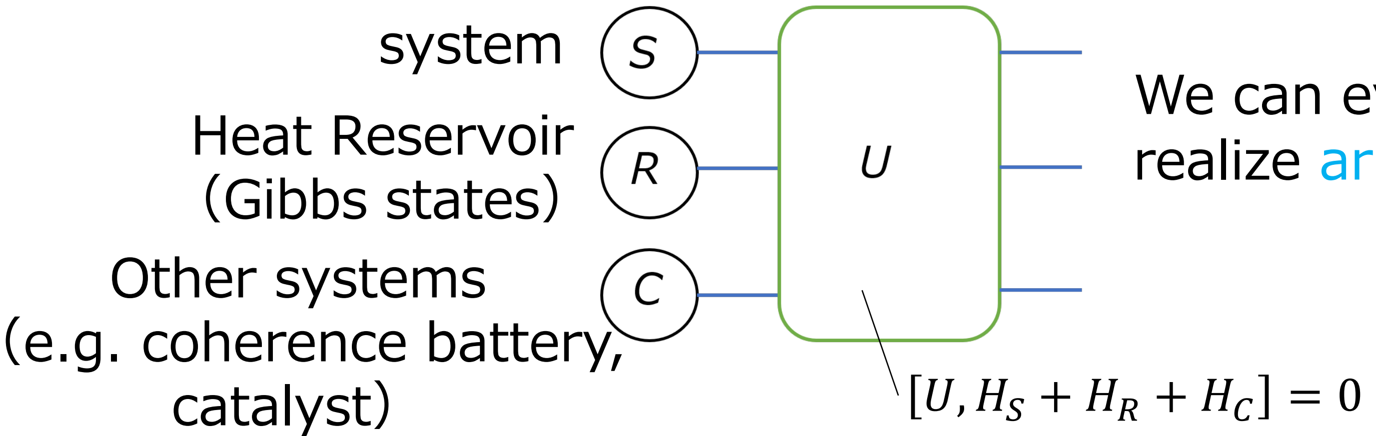
When Λ is not Gibbs-preserving, the generalized entropy production is defined as

$$\Sigma(\rho|\sigma) := D(\rho|\sigma) - D(\mathcal{E}(\rho)|\mathcal{E}(\sigma))$$

Then, the coherence cost of Λ is bounded by the entropy production as follows:

$$\mathcal{F}_c^{\epsilon=0}(\Lambda) \geq \max_{\rho} \frac{\mathcal{C}^2}{\sqrt{\Sigma(\rho|\sigma)}} - \Delta^2$$

Example of application:



We can evaluate the amount of coherence in C to realize arbitrary channel Λ.

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Techniques:
Trade-off between symmetry, irreversibility and quantum coherence

Technique: symmetry-irreversibility-quantumness tradeoff

Theorem :

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}} + \Delta} \leq \delta \text{ or } \sqrt{\delta}$$

H. Tajima, R. Takagi, Y. Kumorachi, arXiv:2206.11086 (2022).

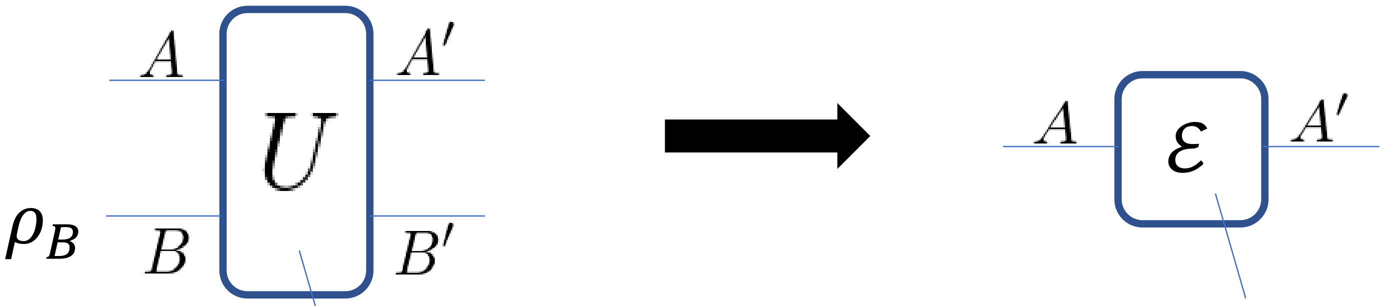
Examples of applications :

Quantum computation, error corrections, measurement theory,
OTOC, black hole physics,⋯

Let’s see the details.

Technique: symmetry-irreversibility-quantumness tradeoff

setup



$$X_{A'} + X_{B'} = U^\dagger (X_A + X_B) U$$

$$\mathcal{E}(\dots) := \mathrm{Tr}_{B'}[U(\dots \otimes \rho_B)U^\dagger]$$

Result

Change of local conserved charge X_A by $\mathcal{E}$

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}} + \Delta} \le \delta \text{ or } \sqrt{\delta}$$

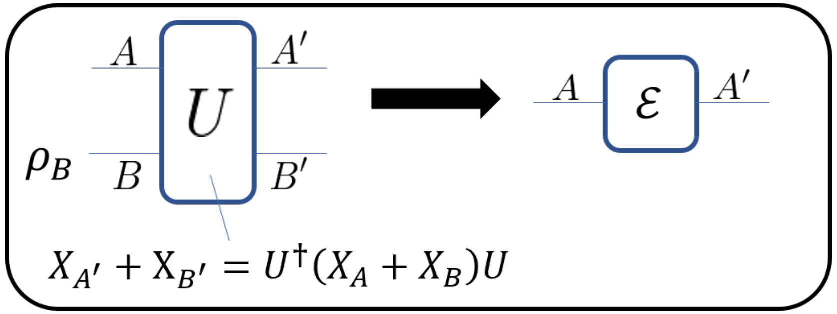
Irreversibility of $\mathcal{E}$

Quatnum coherence of X_B in B = Fisher information

Positive constant

Properties of key quantities

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$



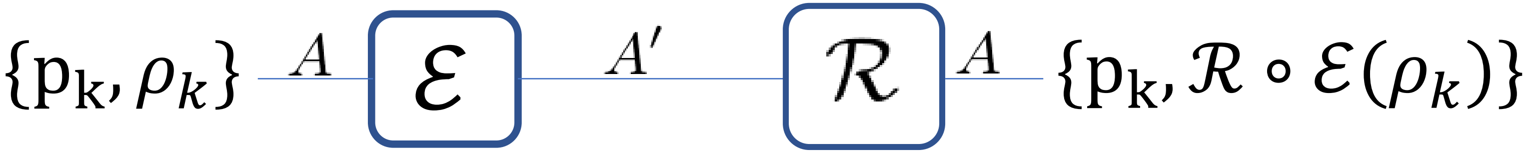
Irreversibility δ : function of $\mathcal{E}$ and a "test ensemble"$\{\mathrm{p_k}, \rho_k\}_{k\in\mathcal{K}}$

$$\delta := \min_{\mathcal{R}:A'\to A} \sqrt{\sum_k p_k \delta_k^2}$$

$$\delta_k := D_F(\rho_k, \mathcal{R}\circ\mathcal{E}(\rho_k))$$

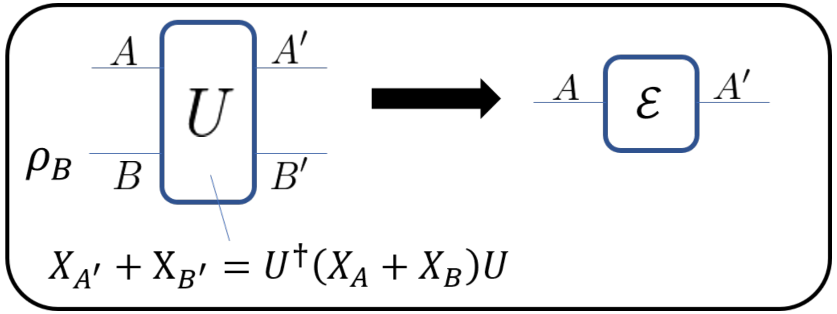
recovery error for ρ_k

$$D_F\,(\rho,\ \sigma) := \sqrt{1 - F^2(\rho,\sigma)}$$



Properties of key quantities

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$



Irreversibility δ : function of $\mathcal{E}$ and a "test ensemble" $\{\mathrm{p_k}, \rho_k\}_{k\in\mathcal{K}}$

$$\delta := \min_{\mathcal{R}:A'\to A} \sqrt{\sum_k p_k \delta_k^2}$$

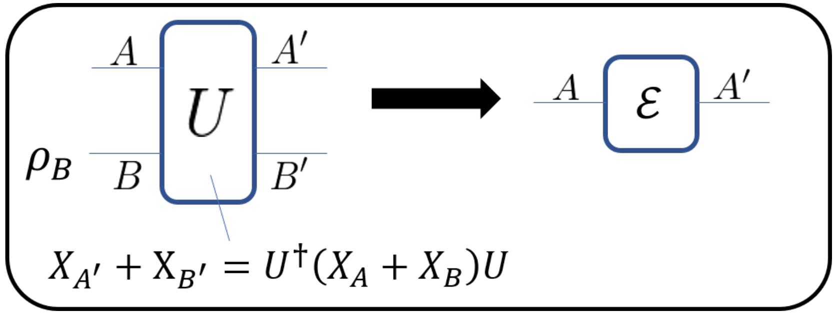
Property: δ gives lower bounds for various irreversibility measures.

$$\delta \leq \sqrt{\Sigma}, \quad \delta \leq \delta_Q, \quad \text{and } \delta \leq \frac{\delta_P}{\sqrt{2}},$$

e.g., it bounds the entropy production, the entanglement-fidelity recovery error, and the error of Petz map recovery, etc.

Properties of key quantities

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$



$\mathcal{C}$: Degree of change of local charge

$$\mathcal{C} := \sqrt{\sum_{k\neq k'} p_k p_{k'} \mathrm{Tr}[(\rho_k - \rho_{k'})_+ Y_A (\rho_k - \rho_{k'})_- Y_A]}$$

$(O)_\pm$:=Positive (negative) part of the Hermitian operator O

$Y_A := X_A - \mathcal{E}^\dagger(X_{A'})$ **Operator of change of local charge**

Property: ・For the test states are orthogonal pure states $\{|\psi_k\rangle\}$,

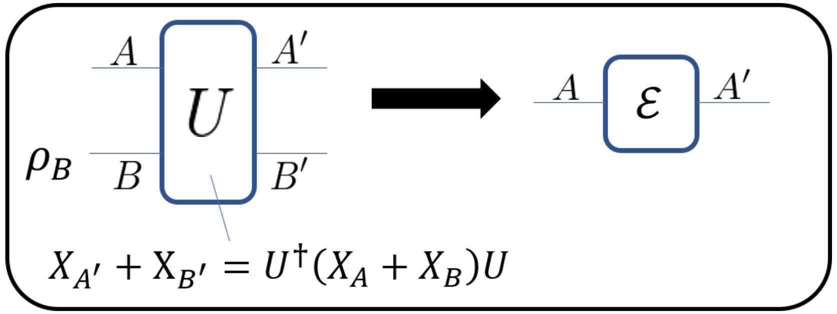
$$\mathcal{C} = \sum_{k\neq k'} p_k p_{k'} |\langle\psi_k|Y_A|\psi_{k'}\rangle|^2$$

Sum of absolute values non-diagonal parts

・$\mathcal{E}$ changes X_A non-trivially (i.e.$Y_A \not\propto 1_A$)⇒ $\mathcal{C} > 0$

SIQ tradeoff

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \le \delta \text{ or } \sqrt{\delta}$$

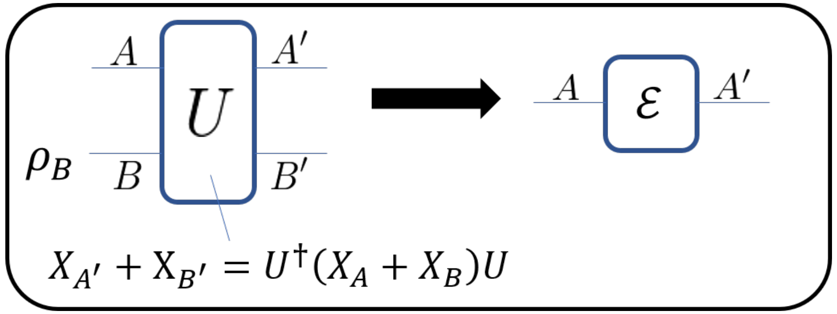


For an arbitrary test ensemble $\{p_k, \rho_k\}$

For the test states $\{\rho_k\}$ are orthogonal each other,
i.e., $\mathrm{F}(\rho_k, \rho_{k'}) = 0$ for $\mathrm{k} \neq k'$,

Messages of SIQ tradeoff

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta}$$



1. $\mathcal{C}>0 \Rightarrow \delta>0$

→ When $\mathcal{E}$ changes the local charge nontrivially, $\mathcal{E}$ must be irreversible.

2. The coherence in B ($= \mathcal{F}$) can mitigate the irreversibility.

Take home message:

Under global *symmetry*, local charge cannot be changed without *irreversibility*.
But the irreversibility can be mitigated by *quantum coherence*.

Derivation of result 1: Finding cost-diverging Gibbs-preserving maps

HT and R. Takagi, arXiv:2404.03479 (2024)

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta} \quad \Rightarrow \quad \frac{\mathcal{C}}{\delta} - \Delta \leq \mathcal{F}_c^{\epsilon=0}(\Lambda)$$

HT, R. Takagi and Y. Kuramochi, arXiv:2206.03479 (2022)

Therefore, if we can find a GPM satisfying $\mathcal{C} > 0$ and $\delta = 0$, the GPM nees infinite coherence!

And we find a systematic way to construct such GPMs.

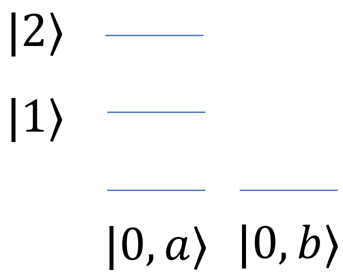
Derivation of result 1: Finding cost-diverging Gibbs-preserving operations

<u>HT</u> and R. Takagi, arXiv:2404.03479 (2024)

Our strategy is to construct a "measurement-and-prepare" channel satisfying $\mathcal{C} > 0$ and $\delta = 0$.

(for simplicity, I give a special example for this talk. A more general one is in our paper.)

We consider a four level system and a channel on it as follows:



$$\Lambda(\rho) \coloneqq \mathrm{Tr}[\rho|+_{1,2}\rangle\langle+_{1,2}|]|0,a\rangle\langle 0,\mathrm{a}| + \mathrm{Tr}[\rho|-_{1,2}\rangle\langle-_{1,2}|]|0,b\rangle\langle 0,\mathrm{b}| + \mathrm{Tr}[\rho I_0]\xi$$

$$\left|\pm_{1,2}\right\rangle \coloneqq \frac{|1\rangle \pm |2\rangle}{\sqrt{2}} \qquad I_0 \coloneqq |0,a\rangle\langle 0,\mathrm{a}| + |0,b\rangle\langle 0,\mathrm{b}|$$

$$\xi \coloneqq \frac{\rho_{\beta|H} - (\mathrm{Tr}[\rho_{\beta|H}|+_{1,2}\rangle\langle+_{1,2}|]|0,a\rangle\langle 0,\mathrm{a}| + \mathrm{Tr}[\rho_{\beta|H}|-_{1,2}\rangle\langle-_{1,2}|]|0,b\rangle\langle 0,\mathrm{b}|)}{\mathrm{Tr}[\rho_{\beta|H} - (\mathrm{Tr}[\rho_{\beta|H}|+_{1,2}\rangle\langle+_{1,2}|]|0,a\rangle\langle 0,\mathrm{a}| + \mathrm{Tr}[\rho_{\beta|H}|-_{1,2}\rangle\langle-_{1,2}|]|0,b\rangle\langle 0,\mathrm{b}|)]}$$

This channel satisfies: $\left|+_{1,2}\right\rangle \to |0,a\rangle \quad \left|-_{1,2}\right\rangle \to |0,b\rangle$ $\quad \rho_{\beta|H} \to \rho_{\beta|H}$ $\quad \xi \geq 0$

- $\mathcal{C} > 0$ and $\delta = 0$ for the ensemble $\{(1/2,1/2),(\left|-_{1,2}\right\rangle, \left|-_{1,2}\right\rangle)\}$
- Gibbs-preserving
- CPTP

This channel satisfies $\mathcal{C} > 0$ and $\delta = 0$, and thus it needs infinite coherence!

Derivation of Result 2

HT and R. Takagi, arXiv:2404.03479 (2024)

We also find upper and lower bounds of cost for approximate implementation of the cost-diverging GPOs

$$\frac{\mathcal{C}}{\epsilon} - a \leq \sqrt{\mathcal{F}_{\mathrm{cost},\epsilon}(\Lambda)} \leq \frac{2\mathcal{C}}{\epsilon} + a$$

obtained from SIQ tradeoff

obtained from upper bound of coherence cost for unitary implementation

HT, N. Shiraishi and K. Saito, PRL 2018 and PRR 2020

From this result, we can see the $\mathcal{F}_{\mathrm{cost},\epsilon} \approx \frac{N}{\epsilon^2}$, where N is the system size.

This scaling is the worst in all of approximate implementations of the CPTP maps.

➡GPMs include the “most costly” class in all CPTP maps.

Derivation of Result 3

Result 3 is easily obtained from SIQ.

First, when the test ensemble is $\{(1/2,1/2),(\rho,\sigma)\}$, we can obtain

$$\delta \leq \sqrt{\Sigma(\rho|\sigma)}$$

Generalized entropy production

$$\Sigma(\rho|\sigma) \coloneqq D(\rho|\sigma) - D(\Lambda(\rho)|\Lambda(\sigma))$$

When σ is Gibbs state, and Λ is Gibbs-preserving, $\Sigma(\rho|\sigma) = \Sigma(\rho)$, and thus

$$\frac{\mathcal{C}^2}{\left(\sqrt{\mathcal{F}} + \Delta\right)^2} \leq \delta \leq \sqrt{\Sigma(\rho)}$$

Part II:
Coherence enhancement of heat engines

➡general relation between heat current, coherence and entropy production

➡approximate achieving Carnot efficiency with finite power with collective jump

Implication on trade-offs in stochastic thermodynamics?

SIQ tradeoff:

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}}+\Delta} \leq \delta \text{ or } \sqrt{\delta} \quad \Longleftrightarrow \quad \frac{\text{change of local charge}}{\text{coherence} + \text{const.}} \leq \text{irreversiility}$$

Trade-offs in stochastic thermodynamics:

$$\text{Entropy production rate} \times A \geq \text{some current}^{\,2}$$

$$\frac{\text{change of local charge}^2}{A} \leq \text{irreversiility}$$

SIQ tradeoff structure in stochastic thermodynamics⋯?

Coherence effect on tradeoff between heat current and entropy production rate

HT and K. Funo, Phys. Rev. Lett. **127**, 190604 (2021)
Editor's suggestion +Featured in Physics

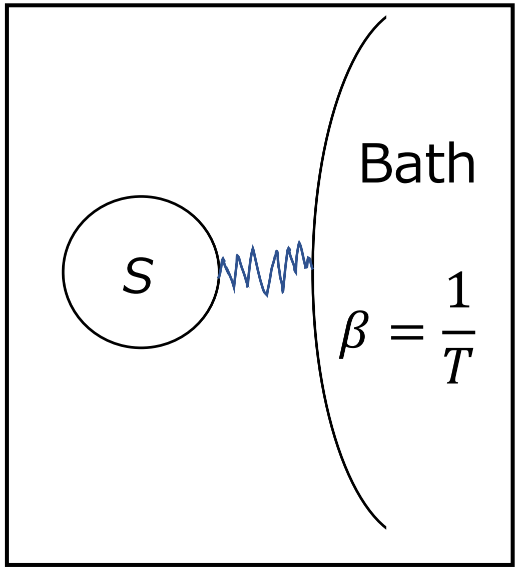
Result:

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \leq \frac{A_{\mathrm{cl}} + A_{\mathrm{qm}}}{2}$$

Quantum part

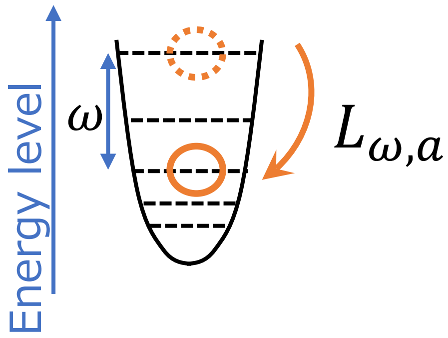
A_{qm} is proportional to the amount of coherence between degeneracies

Setup:



$$\frac{\partial \rho}{\partial t} = -i[H,\rho] + \mathcal{D}[\rho].$$

$$\mathcal{D}[\rho] = \sum_{\omega,a} \gamma_a(\omega)\left(L_{\omega,a}\rho L_{\omega,a}^\dagger - \frac{1}{2}\{L_{\omega,a}^\dagger L_{\omega,a}, \rho\}\right).$$



$$J(\rho) := [H\mathcal{D}[\rho]],$$

$$\dot{\sigma}(\rho) := \dot{S}(\rho) - \beta[H\mathcal{D}[\rho]].$$

Coherence effect on tradeoff between heat current and entropy production rate

HT and K. Funo, Phys. Rev. Lett. **127**, 190604 (2021)
Editor's suggestion +Featured in Physics

Result:

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \le \frac{A_{\rm cl} + A_{\rm qm}}{2}$$

Quantum part

$A_{\rm qm}$ is proportional to the amount of coherence between degeneracies

$$\frac{2J(\rho)^2}{A_{\rm qm} + A_{\rm cl}} \le \dot{\sigma}(\rho)$$

⟺

$$\frac{(\text{change of local charge})^2}{\text{coherence} + \text{const.}} \le \text{irreversiility}$$

S-I-Q structure in stochastic thermodynamics!

Dissipation-less current

HT and K. Funo, Phys. Rev. Lett. **127**, 190604 (2021)

$$\frac{J(\rho)^2}{\dot{\sigma}(\rho)} \leq \frac{A_{\mathrm{cl}} + A_{\mathrm{qm}}}{2}$$

Quantum part

A_{qm} is proportional to the amount of coherence between degeneracies

When $A_{\mathrm{qm}} = O(N^2)$, the scaling $J = O(N)$ and $\dot{\sigma} = O(1)$ are formally possible.

➡**Coherence can cause macroscopic heat current without entropy increase**

We can construct an concrete example satisfying

$$A_{\mathrm{qm}} = O(N^2) \quad \& \quad J = O(N), \quad \dot{\sigma} = O(1)$$

➡**dissipation-less current**

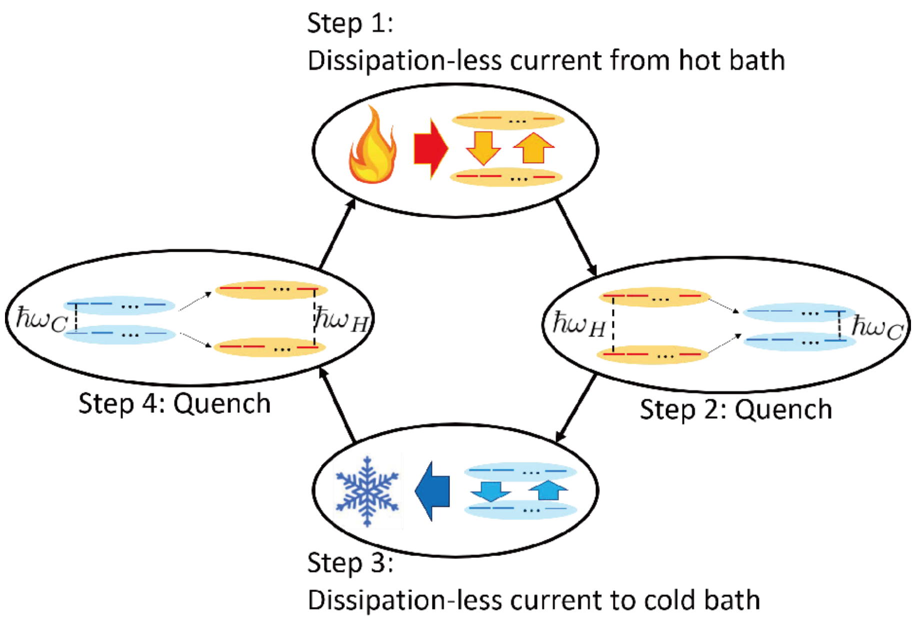
Application：effective realization finite power with Carnot efficiency

With using dissipation-less current, we can achieve the Carnot efficiency with finite power *effectively*.

$$J = O(N)$$
$$\dot{\sigma} = O(1)$$

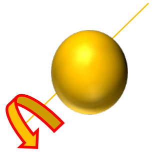
$\Rightarrow$

$$\eta = \eta_{Car} - O(1/N)$$
$$W/\tau = O(N)$$



Summary

Symmetry



→

Quantum Coherence

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

suppresses →

irreversibility

$$\frac{\mathcal{C}}{\sqrt{\mathcal{F}} + \Delta} \le \delta \text{ or } \sqrt{\delta}$$

$$\frac{2J(\rho)^2}{A_{\rm qm} + A_{\rm cl}} \le \dot{\sigma}(\rho)$$

HT, R. Takagi, Y. Kuramochi, arXiv:2206.11086, (2022)

・Gibbs-preserving maps requiring infinite amount of coherence

・trade-off between generalized entropy production and coherence cost of arbitrary quantum operations

HT and R. Takagi, arXiv:2404.03479 (2024)

・general relation between heat current, coherence and entropy production

・approximate achieving Carnot efficiency with finite power with collective jump

HT and K. Funo, Phys. Rev. Lett. **127**, 190604 (2021)

Thank you for attention!