Frontiers in Non-equilibrium Physics 2024
Dynamics Days Asia Pacific 13

YKIS2024

Wave-packet and entanglement dynamics in a non-Hermitian system

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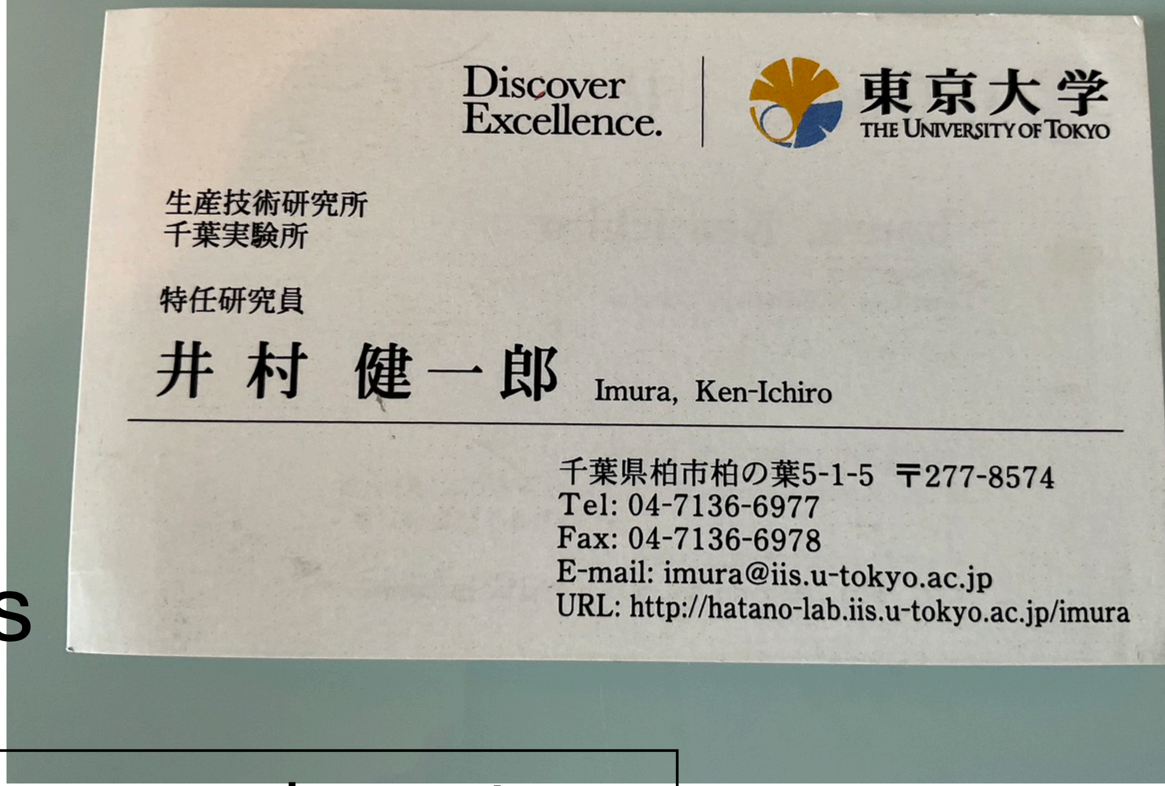

Since I am not really from this community…

<u>Let me first introduce myself:</u>

I have been working on

✅ quantum Hall edge states

✅ quantum dots/wires

mesoscopic systems

✅ graphene

✅ topological insulators

focusing mainly on

transport properties

Recently,

✅ non-Hermitian quantum systems

Maybe, in the near future,

✅ machine/deep learning, etc.

✅ Dynamics Days Asia Pacific 13 / YKIS2024

Yukawa Institute for Theoretical Physics, Kyoto University, Japan
July 1st - 5th, 2024

Home | Invited Speakers | Program | Important Dates | Registration
Banquet | Venue | Access | Accommodation | Sponsors | Visa

✅ dynamics

✅ non-equilibrium
✅ quantum
✅ CM & photonic

Important Dates

July 1st — August 2nd, 2024: Workshop

March 31st, 2023: Website open

YITP
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Dynamics Days Asia Pacific 13 will be held as YKIS2024
University.

Scope

Dynamics Days meetings are international meetings which launched in 1980 with focus on the nonlinear dynamics. Dynamics Days Asia Pacific started in 1999 in Hong Kong, and are moved to Hangzhou (2002), Singapore (2004), Pohang (2006), Nara (2008), Sydney (2010), Taipei (2012), Chennai (2014), Hong Kong (2016), Xiamen (2018), Singapore (2020) and Daejong (2022). Now, DDAP13 will be held in Kyoto, Japan on July 1st-5th as a part of the long-term workshop. Topics include

- Dynamics of complex systems
- Dynamics of nonequilibrium systems ✅
- Dynamics of quantum systems ✅
- Dynamics of condensed matter and photonics ✅
- Dynamics of active and biological systems
- Dynamics of earth climate
- Dynamics of machine learning

The meeting will offer invited talks without any parallel sessions. The meeting is supposed to be hybrid, but participants are strongly encouraged to come to the YITP which is the meeting place.

Since 1987, Yukawa Institute for Theoretical Physics (YITP) has hosted an international research meeting known as the Yukawa International Seminar (YKIS). This is a five-day international symposium held jointly with the Yukawa Memorial Foundation.
Past YKISes

Frontiers in Non-equilibrium Physics 2024

Yukawa Institute for Theoretical Physics, Kyoto University, Japan
July 1st - August 2nd, 2024

✅ (emergent) gaussian fluctuation/ diffusion
✅ non-reciprocal (hopping)

Important Dates

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Scope

In thermal equilibrium, Gaussian fluctuations play a decisive role, and their behavior is well understood. However, the statistical mechanics in non-equilibrium systems is characterized by non-Gaussian fluctuations. One of the milestones is to establish the framework of stochastic thermodynamics, which is relatively new. Such a framework is relevant to describe fluctuating motion in small systems. Nevertheless, we still do not know how relaxation processes are affected by non-equilibrium fluctuations, topological constraints, and quantum effects. Thus, we invite researchers in this field from all over the world. Moreover, non-Gaussian features in non-equilibrium systems require a new framework of statistical physics. Indeed, there is no reciprocal theorem for strongly non-equilibrium systems associated with a non-reciprocal phase transition. Such non-reciprocal relations can be observed in active matter systems easily. Classical densely packed systems exhibit different behavior from usual systems: thermal fluctuations do not significantly affect the motion of individual elements like colloids, powders, and bubbles when these constituents are random and large. To describe their dynamics, we require a distinct logic. Thus, a new branch of non-equilibrium statistical physics is essential to understand and describe these dynamic, self-driven systems.

There is a worldwide momentum for discussion on these topics, and week of the workshop is dedicated to a relatively large international (DDAP13), the Yukawa International Seminar (YKIS2024). The second related to stochastic thermodynamics. The fourth week will focus on active matter, non-reciprocal transitions, and the Mpemba effect. The fifth week of the conference will focus on jamming and rheology of dense

✅ (non-)gaussian
✅ (non-)reciprocal

As the 3rd speaker of the workshop,

Conjecture:
This talk merges the ideas of the 1st and 2nd speakers

cf. in the Hermitian case: ETH vs. MBL

- ETH: eigenstates thermalizes themselves in the presence of interaction at sufficiently weak disorder

  The interaction also tends to delocalize the wave function

- MBL: a counter example

What about the non-Hermitian case?

In this talk, we aim at addressing the questions such as:

- Do eigenstates still <u>thermalize</u> also in the non-Hermitian delocalized regime?

Possibly, a related question:

- What is the (analogue of) <u>ground state</u> in the non-Hermitian delocalized regime?

Non-Hermitian quantum mechanics

In quantum mechanics, textbooks say

Hamiltonian must be (?) Hermitian in a *closed* system

- real eigenvalues
- probability conservation

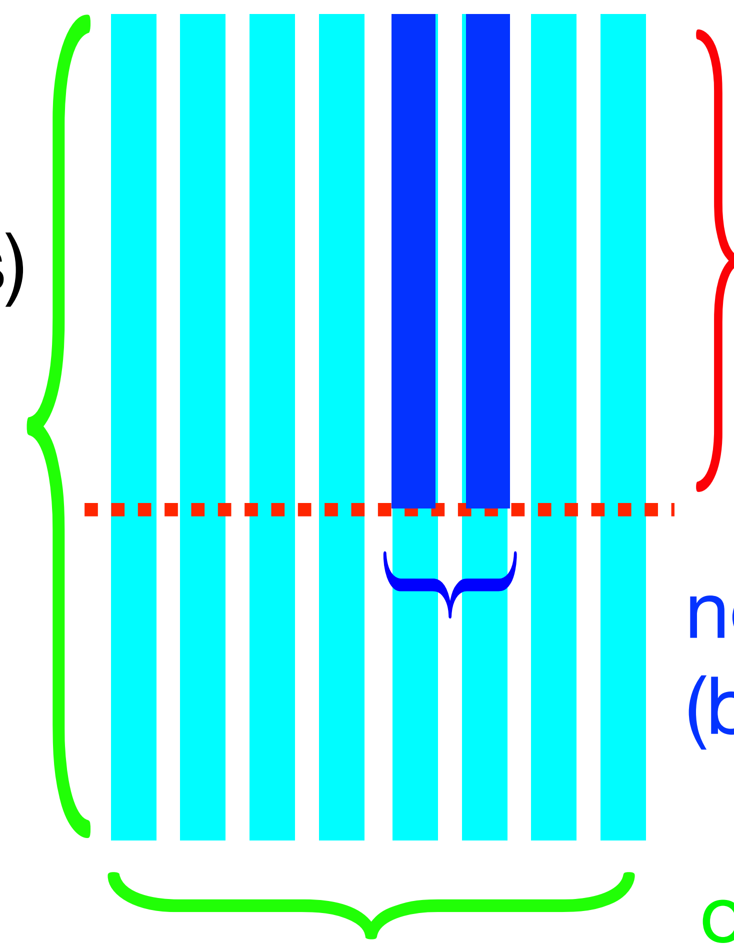
cf. von Neumann, C. Bender,…

Here, we consider *open* quantum systems

= system + environment

reservoirs/leads

Eigenstates:
(eigenvectors)



system

$$H|n_R\rangle = E_n|n_R\rangle$$
$$\langle n_L|H = E_n\langle n_L|$$
$$\langle n_L| \neq |n_R\rangle^\dagger \quad \langle m_R|n_R\rangle \neq \delta_{mn}$$
$$\langle m_L|n_R\rangle = \delta_{mn}$$

system
+ environment
= Hermitian

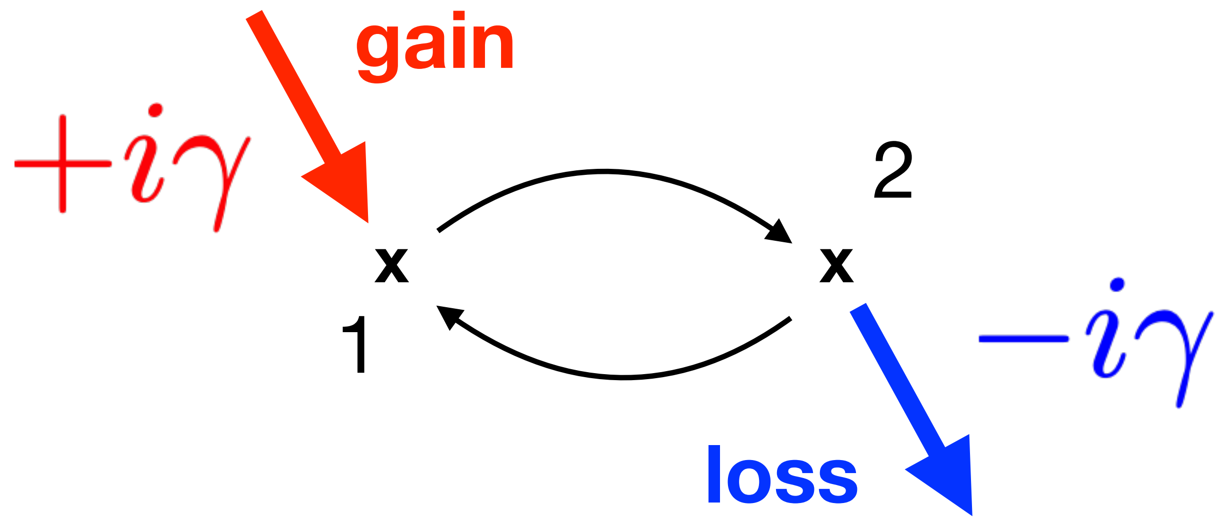
no longer orthogonal
(bi-orthogonal: left/right eigenstates)

orthogonal (unitary)

Typical examples of a non-Hermitian Hamiltonian

1) Gain vs. loss type : *(sometimes) PT symmetric* → next speaker

$$H = \begin{bmatrix} i\gamma & t \\ t & -i\gamma \end{bmatrix}$$

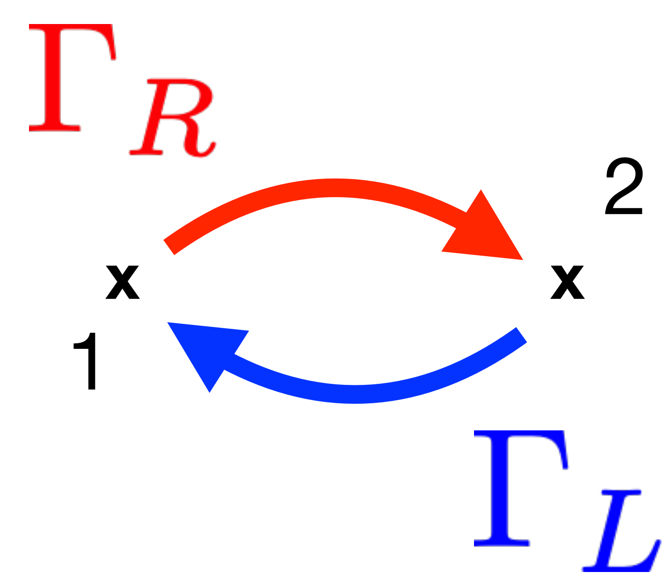


Bender & Boettcher, PRL1998;
Guo et al. PRL2009

2) Non-reciprocal hopping type
(asymmetric)

The Hatano-Nelson type

$$H = \begin{bmatrix} 0 & \Gamma_L \\ \Gamma_R & 0 \end{bmatrix}$$



Hatano & Nelson, PRL1996; PRB1998
Yao & Wang, PRL2018;

$$\Gamma_L \neq \Gamma_R$$

The Hatano-Nelson model:

Hatano & Nelson, PRL1996; PRB1998

A 1D tight-binding model with asymmetric/non-reciprocal hopping:

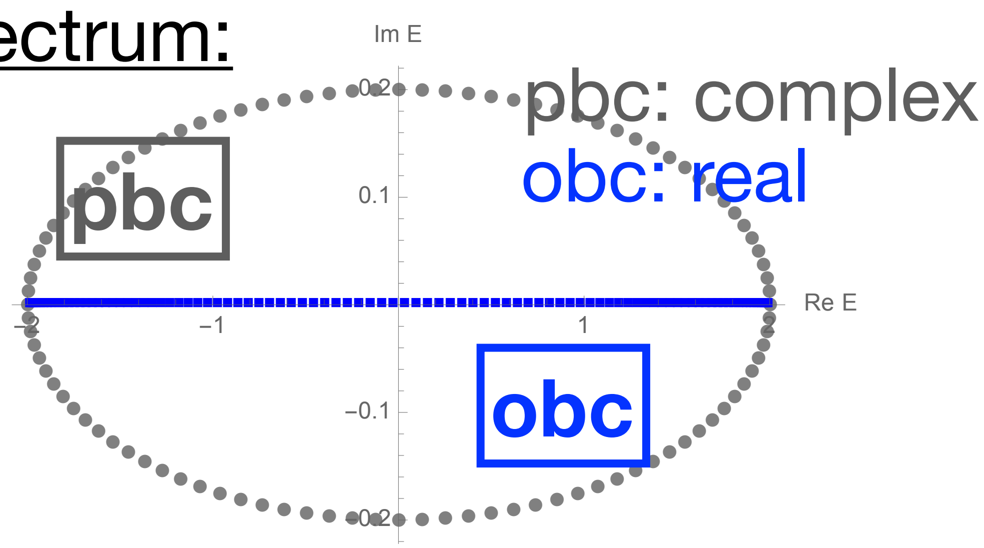
$$H_{\mathrm{HN}} = \sum_j \left(\Gamma_R |j+1\rangle\langle j| + \Gamma_L |j\rangle\langle j+1| + W_j |j\rangle\langle j| \right)$$

$$\Gamma_R = e^{-g}\Gamma_0, \Gamma_L = e^{g}\Gamma_0$$

$g \neq 0$ → asymmetry/non-reciprocity in hopping

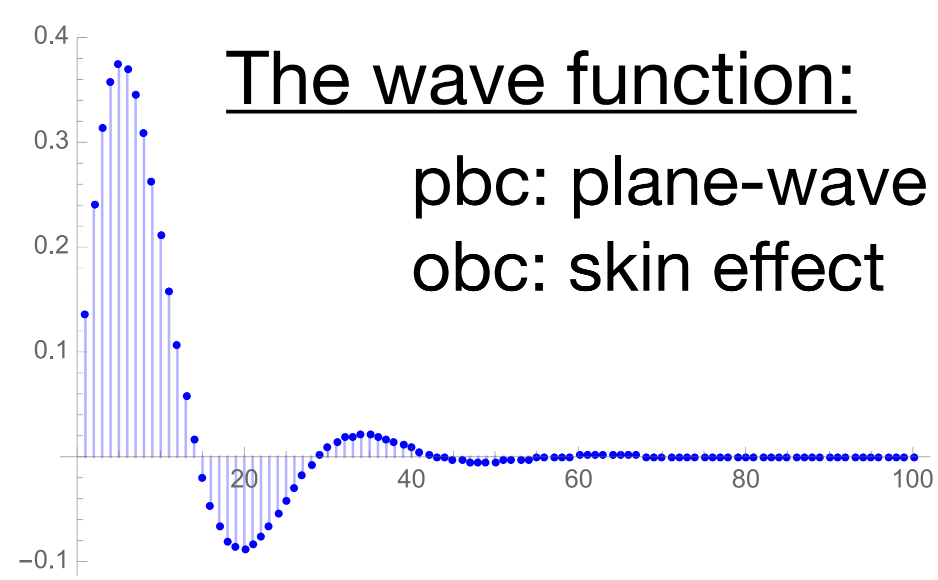
Basic (static) properties:
In the clean limit: complex spectrum (pbc), skin effect (obc), and the sensitivity to the boundary condition

The spectrum:



The wave function:

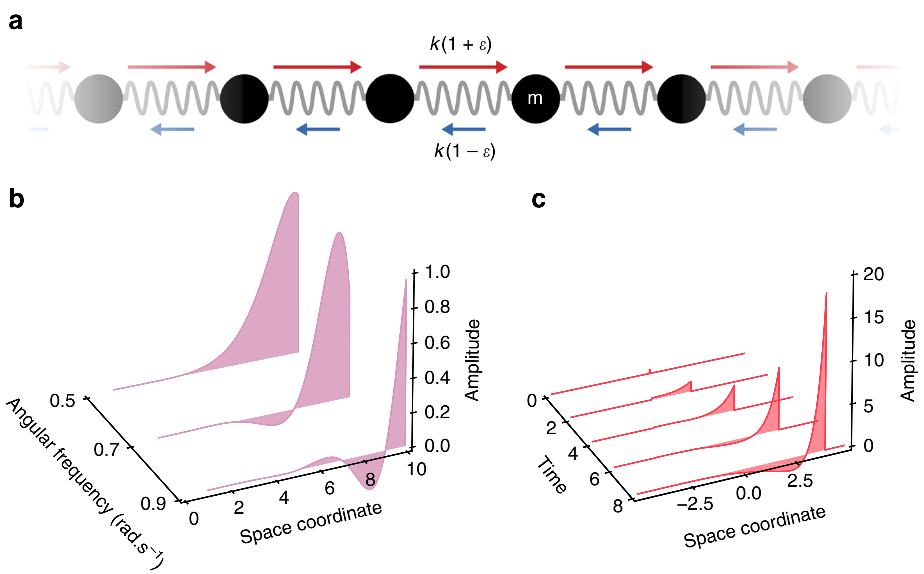
pbc: plane-wave like
obc: skin effect



Experimental realizations

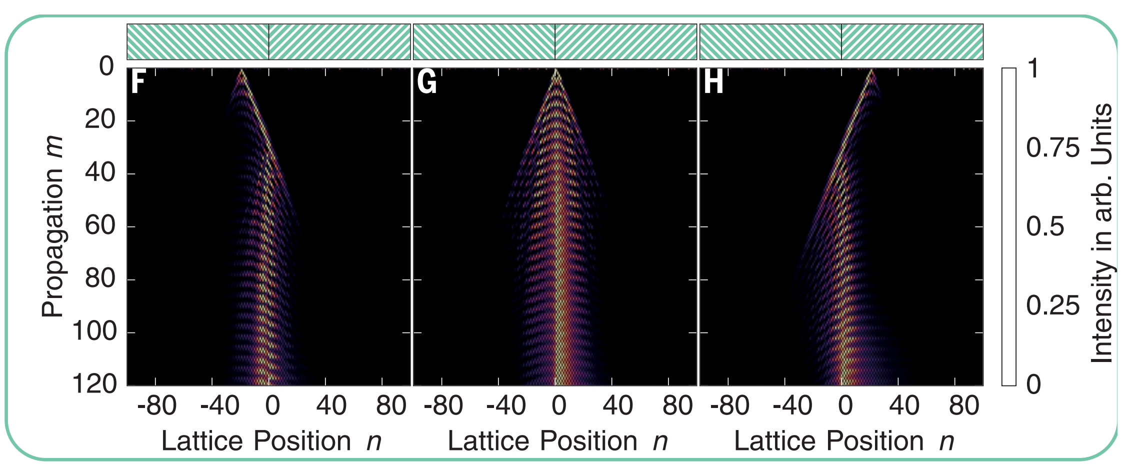
of the "effective" Hatano-Nelson models

- mechanical material



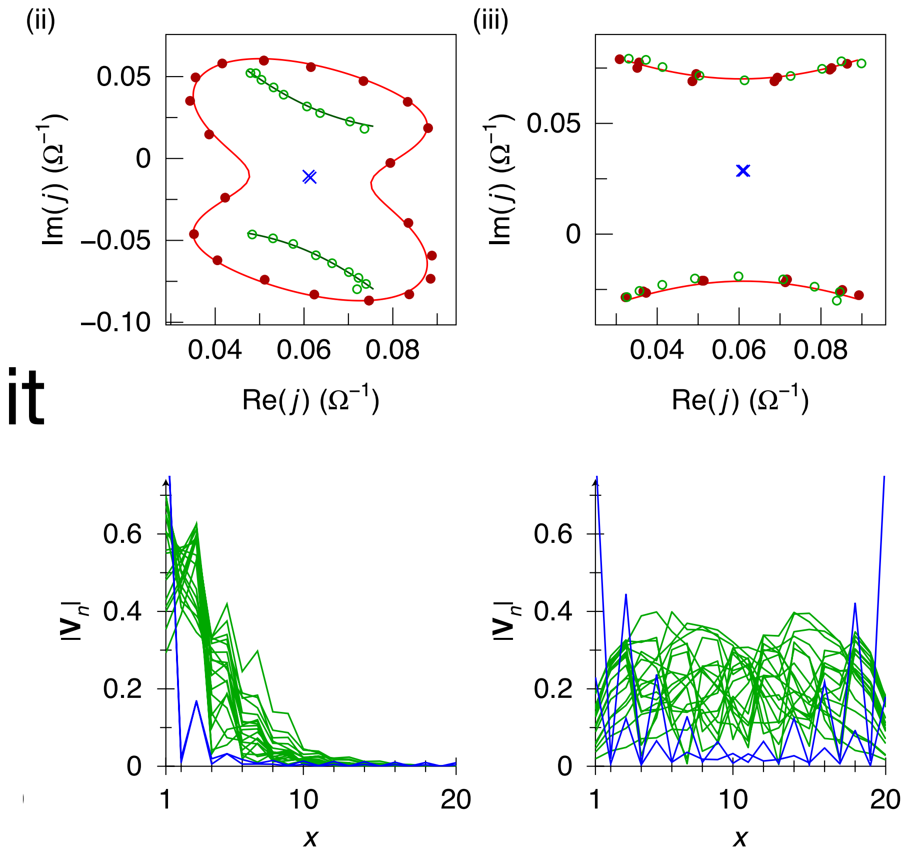
Brandenbourger et al., Nat. Commun. 2019

- photonic lattice



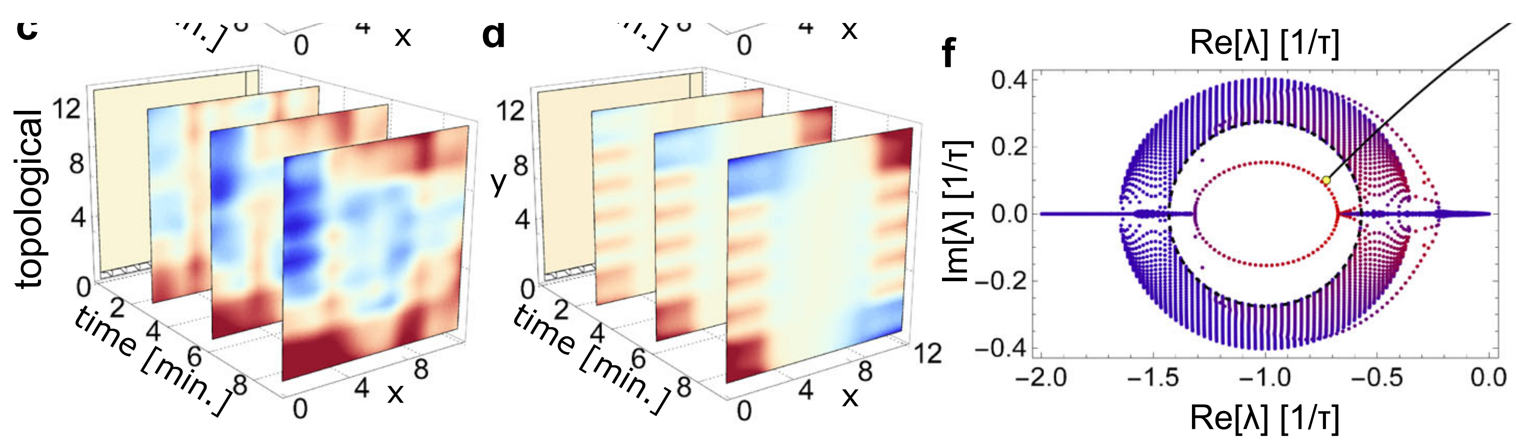
Weidemann et al., Science 2020

- electric circuit



Helbig et al., Nat. Phys. 2020

- active matter



Palacios et al., Nat. Commun. 2021

skin effect/open boundary

- Disordered case: localization-delocalization transition

Non-Hermiticity tends to delocalize the wave function

The (on-site) disorder potential: $W_j \in [-W/2, W/2]$

(in the original work of Hatano & Nelson) (uncorrelated disorder)

Here, we consider the case of quasi-periodic disorder (Aubry-Andre model) $W_j = W\cos(2\pi\theta j + \theta_0),$ *Aubry & Andre, AIPS '80*

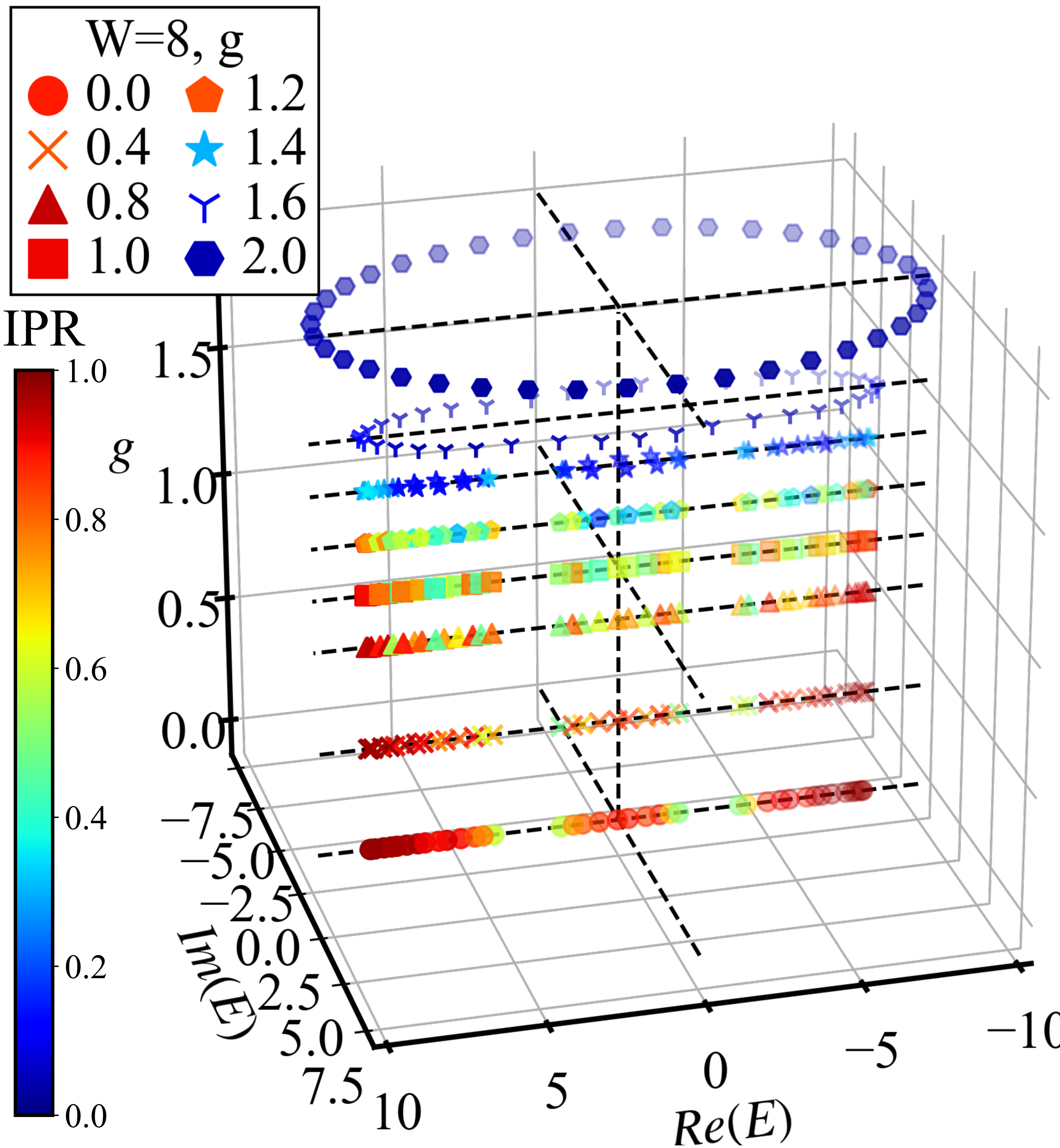
θ : an irrational constant e.g., (chosen typically to be) $\theta = \frac{\sqrt{5}-1}{2}$

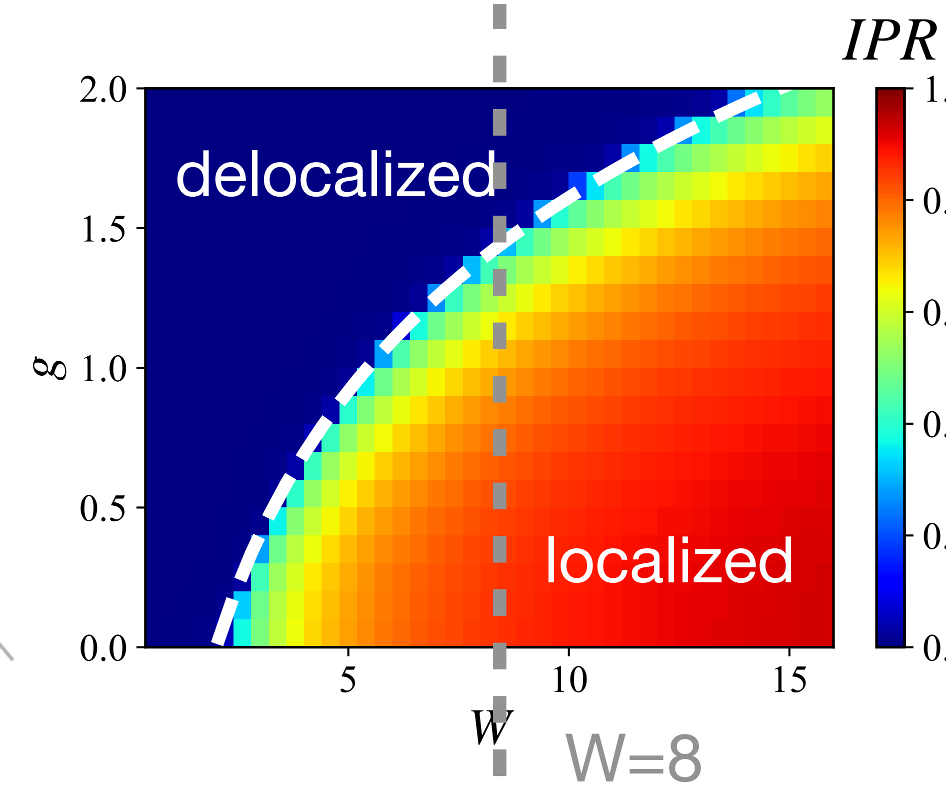
θ_0 : disorder configuration ⟶ sample average

wave function:		spectrum (eigenvalues):
delocalized	⟷	complex
localized	⟷	real

Hatano & Nelson, PRL '96

cf. in 1D all the states are localized in a Hermitian disordered system (Anderson '58)





IPR: inverse participation ratio

$$\mathrm{IPR} = \int |\psi|^4 dx$$

- in the localized regime:

$$\psi(x) \simeq \delta(x) \quad \mathrm{IPR} \simeq 1$$

- in the delocalized regime: $\psi(x) \simeq \frac{1}{\sqrt{L}} e^{ikx}$

$$\mathrm{IPR} \simeq \frac{1}{L^2} \times L = \frac{1}{L} \to 0$$

Wave-packet dynamics

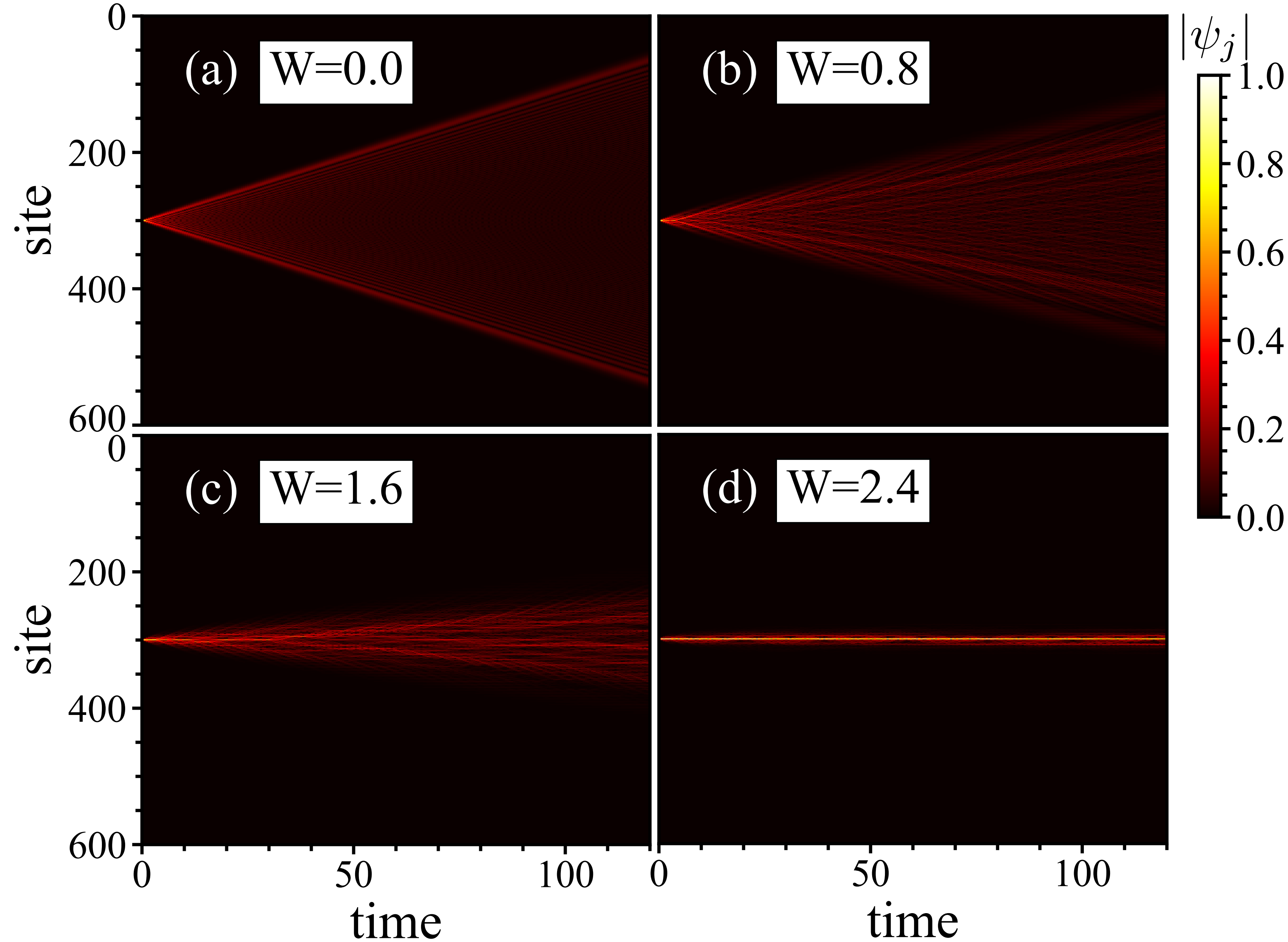
<u>Hermitian case</u> : disorder *suppresses* spreading of the wave packet

$$|\psi(t)\rangle = \sum_j \psi_j(t)|j\rangle$$
$$= \sum_n c_n e^{-i\epsilon_n t}|n\rangle,$$

- initial wave packet:

$$|\psi(t=0)\rangle = |j_0\rangle$$

localized at a single site



- Mechanism underlying the spreading of wave packet:

$$|\psi(t)\rangle = \sum_k e^{-i\epsilon_k t}|k\rangle\langle k|j_0\rangle \left(\equiv \sum_k \psi_k(t)|k\rangle\right)$$
$$= \frac{1}{\sqrt{L}} \sum_j \sum_k e^{-i\epsilon_k t + ik(j_0 - j)}|j\rangle,$$

<u>The stationary phase condition</u>:

$$2\Gamma_0 \sin \bar{k}\ t = j - j_0.$$

→ location of wave front: “light cone”

cf. Lieb-Robinson bound

Wave-packet dynamics

(on-Hermitian case

A caveat:

Specificity of the non-unitary time evolution:

Hamiltonian: non-Hermitian
time evolution: non-unitary

$\langle\Psi(t)|\Psi(t)\rangle$ is not conserved

- in the expansion:

complex: $\mathrm{Im}\ E_\alpha \neq 0$

$$|\Psi(t)\rangle = \sum_\alpha c_\alpha(t)|\alpha\rangle, \quad c_\alpha(t) = c_\alpha(0)e^{-i\underline{E_\alpha} t}$$

eigenstates

$|c_\alpha(t)|^2 \neq |c_\alpha(0)|^2$ are no longer constants

→ we focus on the relative importance of $c_\alpha(t)$

by rescaling/(re)normalizing the wave function as:

$$|\Psi(t)\rangle \to |\tilde{\Psi}(t)\rangle = \frac{|\Psi(t)\rangle}{\sqrt{\langle\Psi(t)|\Psi(t)\rangle}}$$

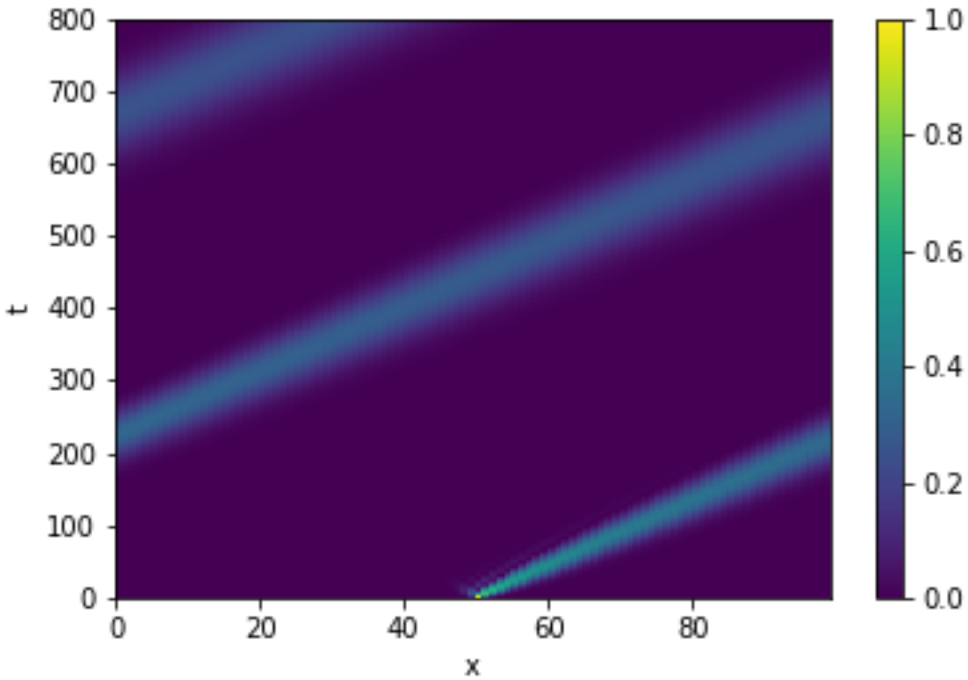
Simulation of the wave-packet dynamics:
(b) non-Hermitian case

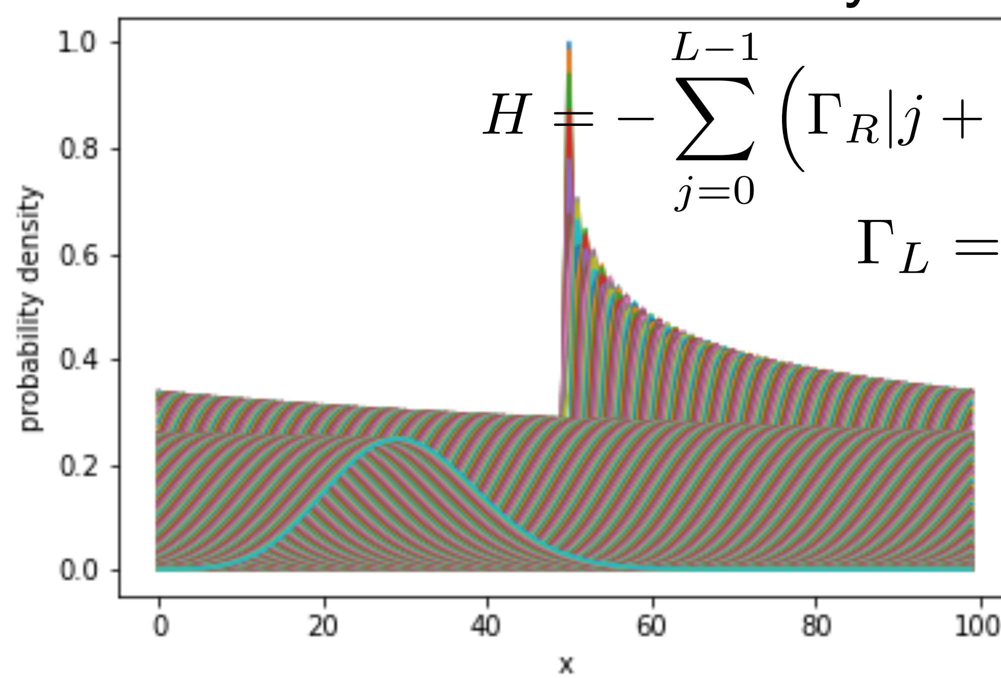
- clean limit: W=0

model: Hatano-Nelson × Aubry-Andre model;

$$H = -\sum_{j=0}^{L-1}\left(\Gamma_R|j+1\rangle\langle j| + \Gamma_L|j\rangle\langle j+1|\right)$$

$$\Gamma_L = e^{g}\Gamma_0,\ \Gamma_R = e^{-g}\Gamma_0$$



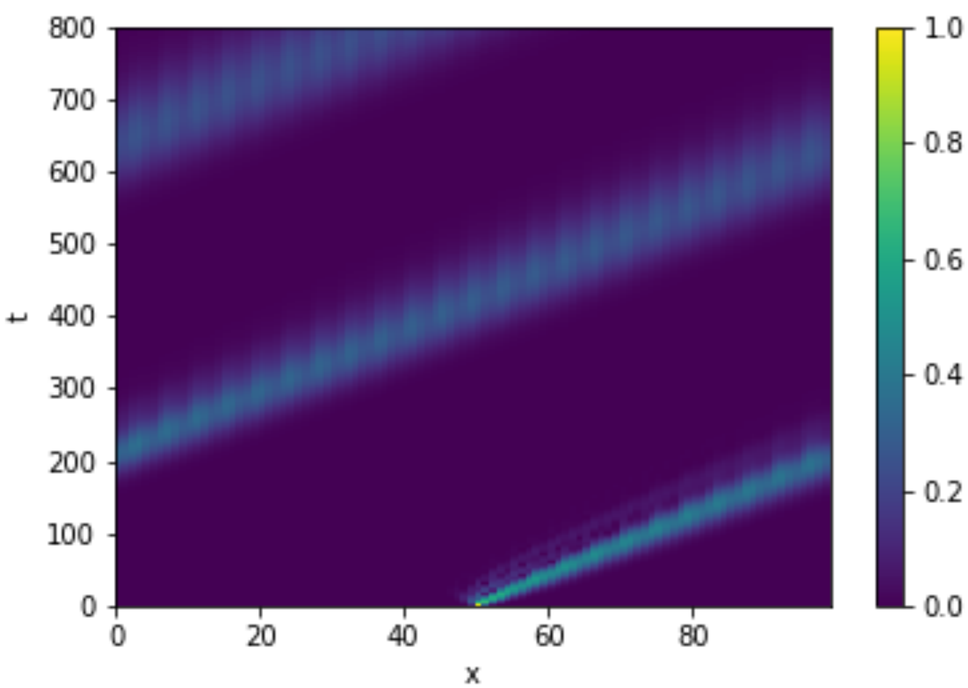


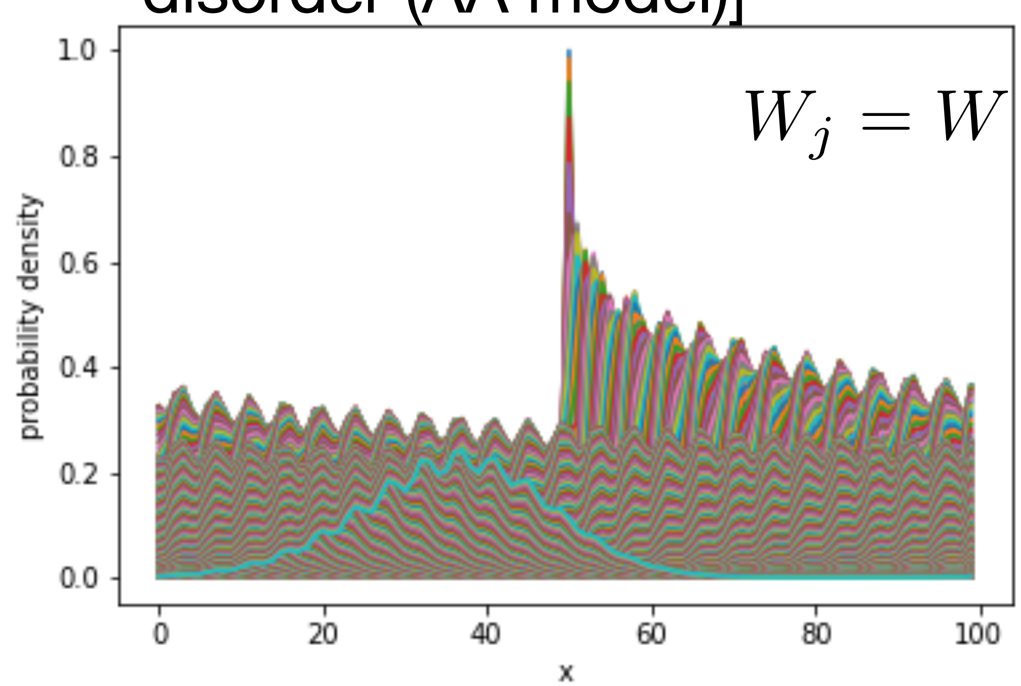
- (weakly) disordered case: W=1.0

[type of disorder: quasi-periodic potential disorder (AA model)]

$$W_j = W\cos(2\pi\theta j + \theta_0),$$

$$\theta = \frac{\sqrt{5}-1}{2}$$





pbc

periodic boundary condition

Non-Hermitian case

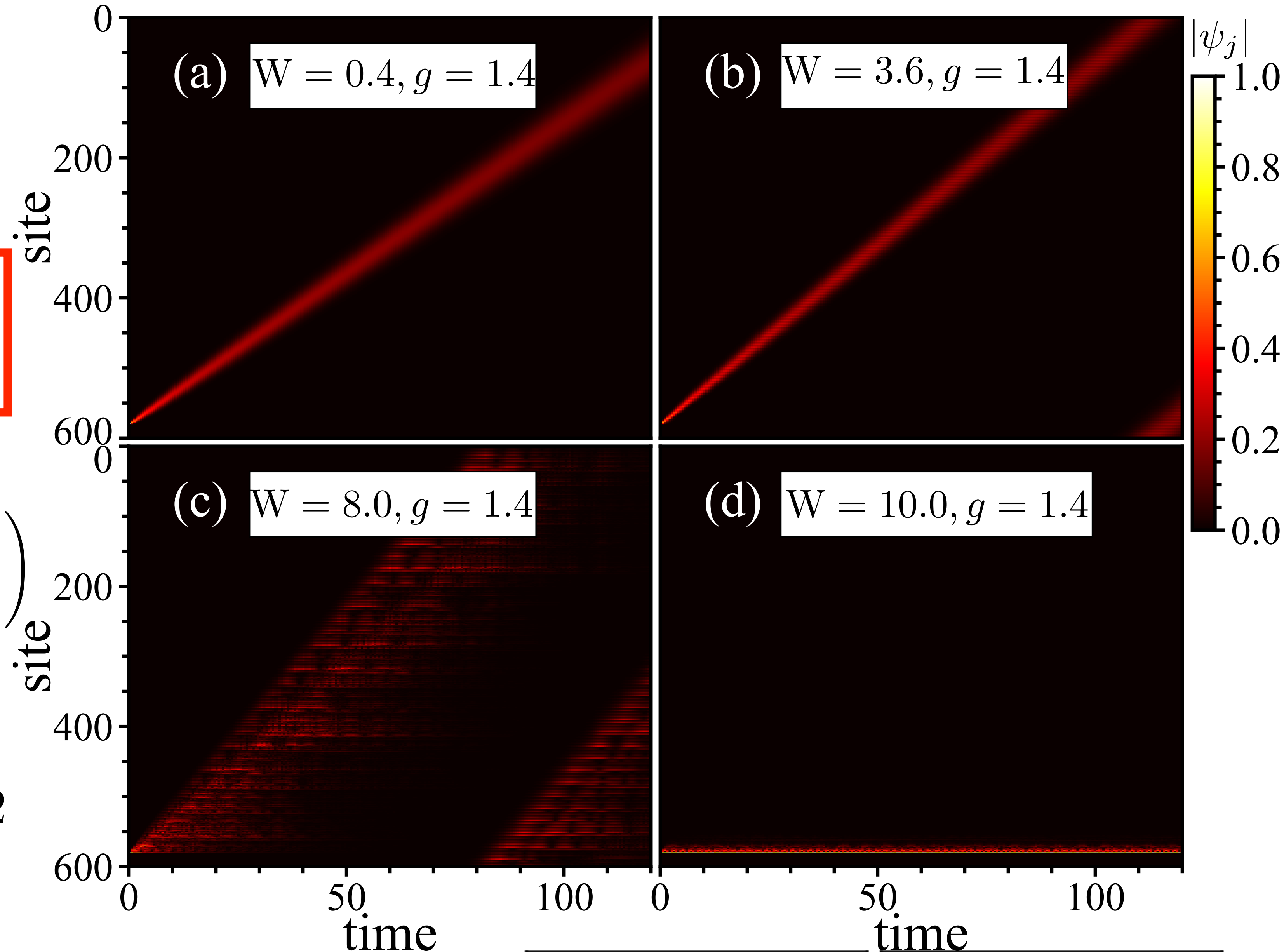
wave-packet dynamics

- Disorder *enhances* spreading of the wave packet

- in the superposition:

$$|\psi(t)\rangle = \sum_k e^{-i\epsilon_k t}|k\rangle\langle k|j_0\rangle \left(\equiv \sum_k \psi_k(t)|k\rangle\right)$$
$$= \frac{1}{\sqrt{L}}\sum_j \sum_k e^{-i\epsilon_k t + ik(j_0-j)}|j\rangle,$$

contribution from predominates: $k = k_0 = -\pi/2$ $\quad \mathrm{Max}\ \mathrm{Im}\ \epsilon_k$

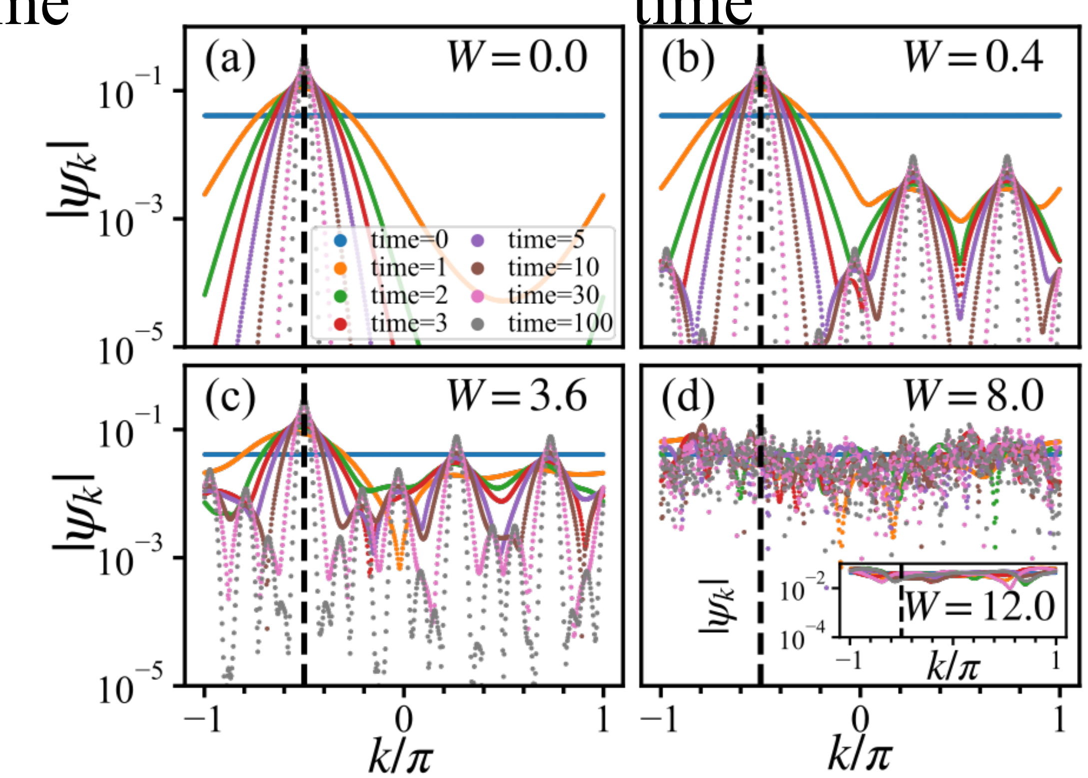


As a result, (in the clean limit)

$$|\psi(t)\rangle \simeq \sum_j |j\rangle \exp\left(-\frac{((j_0-j)+2(\cosh g)t)^2}{4(\sinh g)t}\right)$$
$$\times e^{2(\sinh g)t}/\sqrt{4(\sinh g)t},$$

Emergent gaussian diffusion!

Orito & Imura, Phys. Rev. B ***105****, 024303 (2022)*



Finally, statement of the problem!

cf. in the Hermitian case: ETH vs. MBL

- ETH: eigenstates thermalizes themselves in the presence of interaction at sufficiently weak disorder

  The interaction also tends to delocalize the wave function

- MBL: a counter example

What about the non-Hermitian case?

In this talk, we aim at addressing the questions such as:

- Do eigenstates still thermalize also in the non-Hermitian delocalized regime?

Possibly, a related question:

- What is the (analogue of) ground state in the non-Hermitian delocalized regime?

Hermitian case: ETH vs. MBL

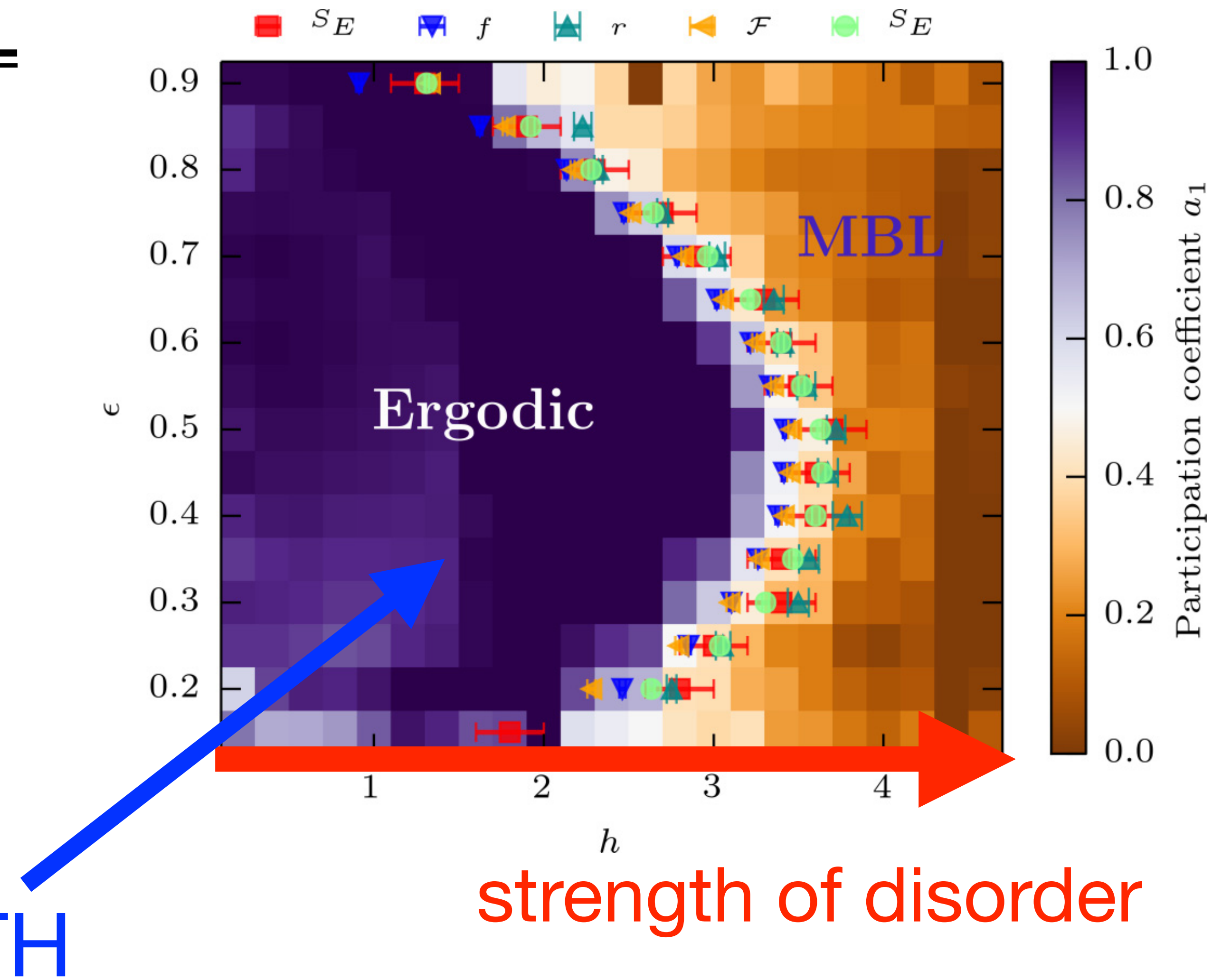
In the regime of sufficiently weak disorder,

interactions tend to mediate eigenstates to thermalize

→ Eigenstate thermalization

cf. ETH: eigenstate thermalization hypothesis

Luitz et al, PRB '15



Still many open issues on the ETH-MBL transition/crossover;
KT-like?

Thiery et al., PRL '18; Goremykina et al., PRB '19; Morningstar & Huse, PRB '19

Oganeysyan & Huse, PRB '07; Pal & Huse, PRB '10; Luitz et al, PRB '15

Remarks:

- inter-particle interactions included
- focus on the scaling of entanglement entropy

volume vs. area law

The interaction (Hermitian case)

- delocalize the wave function
- thermalizes the eigenstates

Non-Hermiticity also?

In the wave-packet and entanglement dynamics of a non-Hermitian system

we will see an emergence of non-equilibrium steady state in the asymptotic long time regime

Is this non-equilibrium steady state an analogue of the ground state in a Hermitian quantum mechanics?

A non-Hermitian version of Fermi sea?

The entanglement dynamics:

(Hermitian case, g=0)

Definitions:

- The bipartite entanglement entropy:

$$S_A(t) = -\mathrm{Tr}\ \Omega_A(t) \log \Omega_A(t),$$
$$= -\sum_\alpha \lambda_\alpha(t) \log \lambda_\alpha(t).$$

- The reduced/full density matrix:

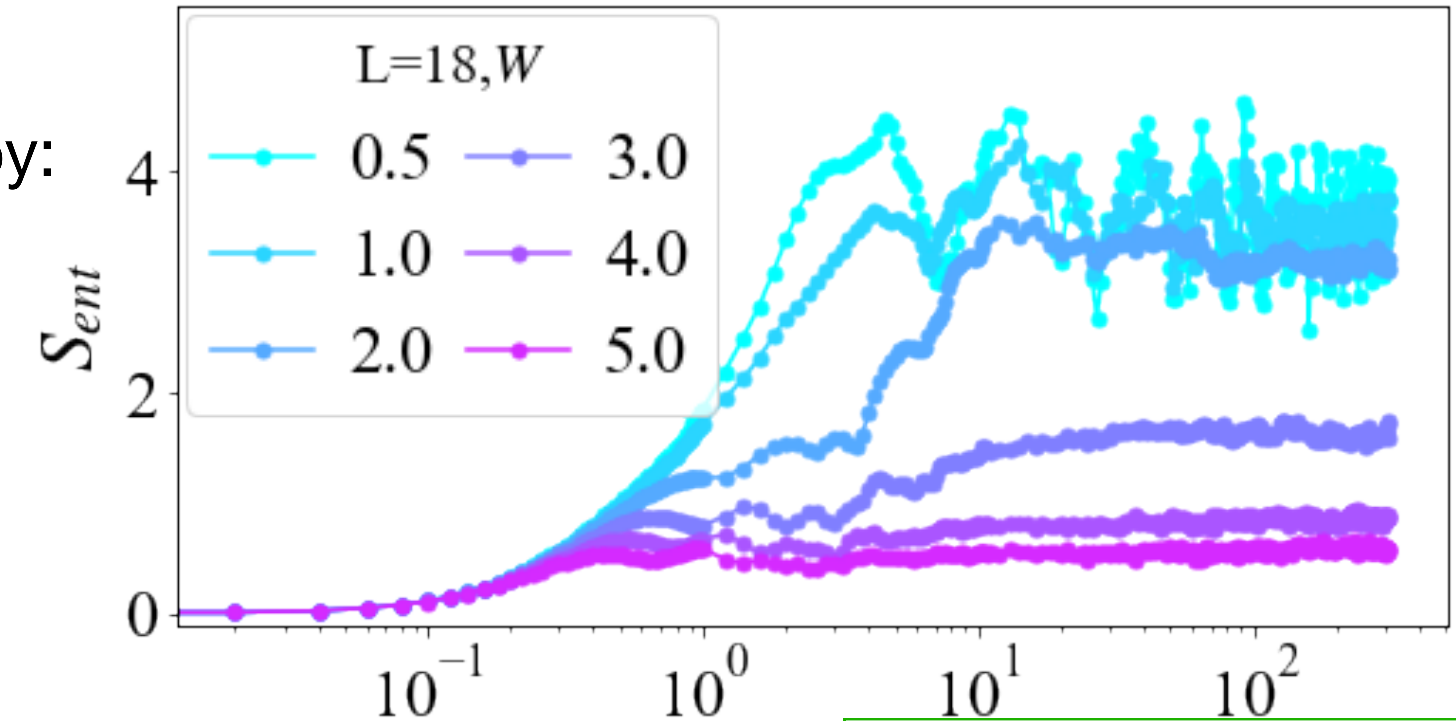
$$\Omega_A(t) = \mathrm{Tr_B}\ \Omega(\mathrm{t}),$$
$$\Omega(t) = |\Psi(t)\rangle\langle\Psi(t)|.$$

- Disorder *suppresses* the entanglement growth

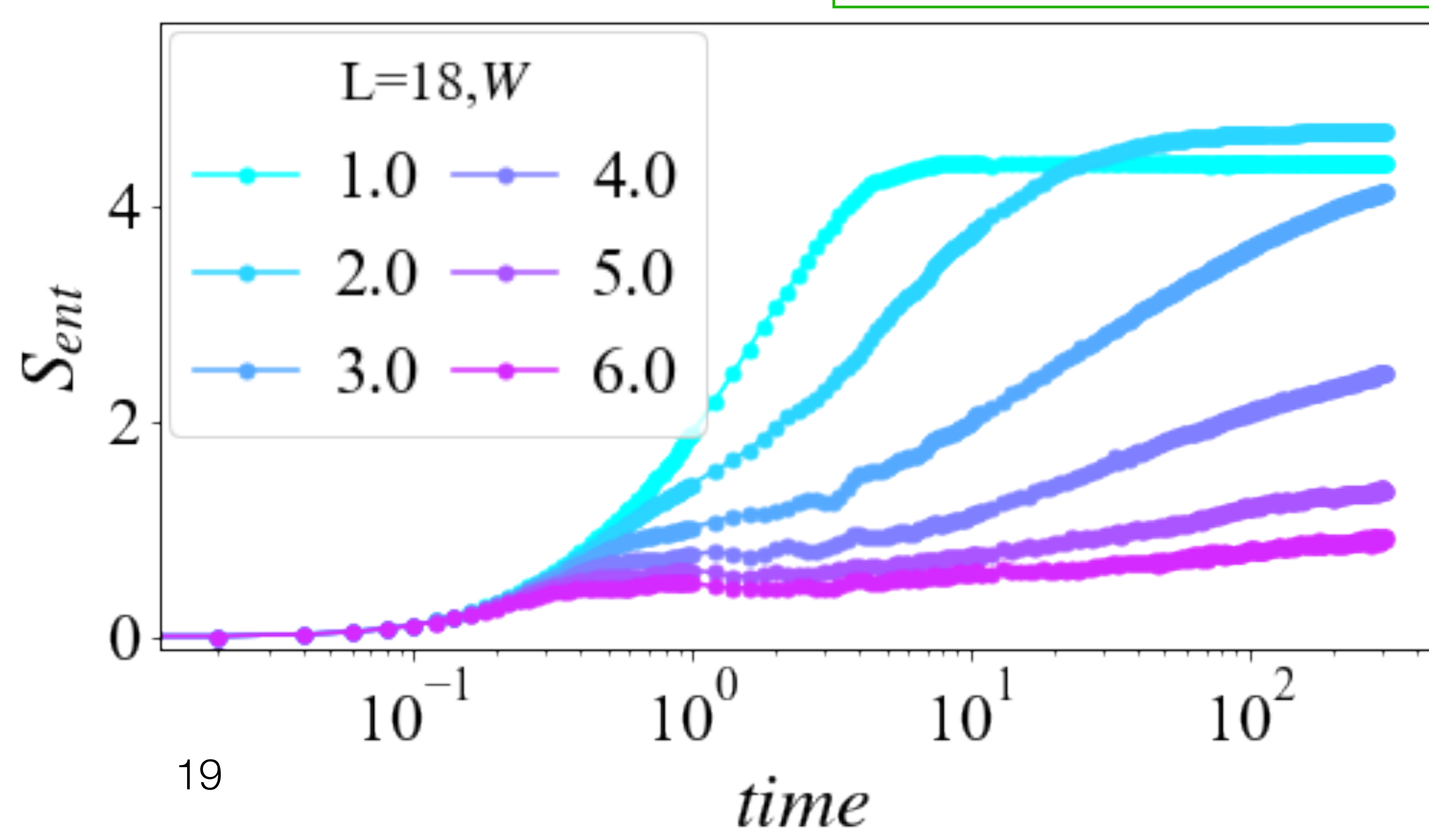
- Interacting case: *logarithmic* growth in the localized (MBL) regime

Znidaric, et al. PRB '08; Bardarson et al., PRL '12; Serbyn et al., PRL '13

Non-interacting case: V=0

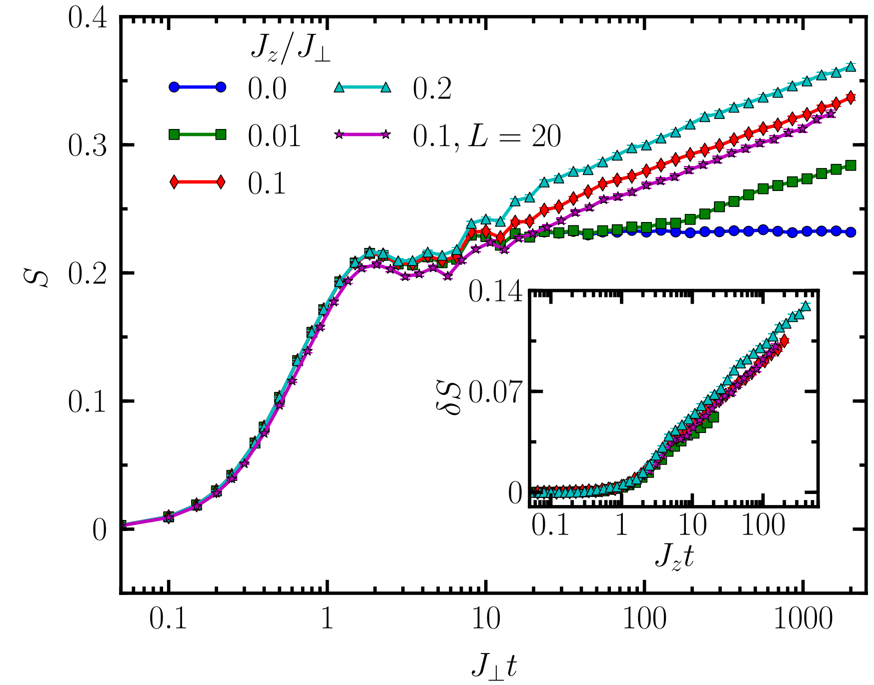


Interacting case: V=2



Nature of MBL (many-body localization)

Gornyi et al., PRL '05; Basko et al., AP '06



- LIOM: quasi-local integrals of motion

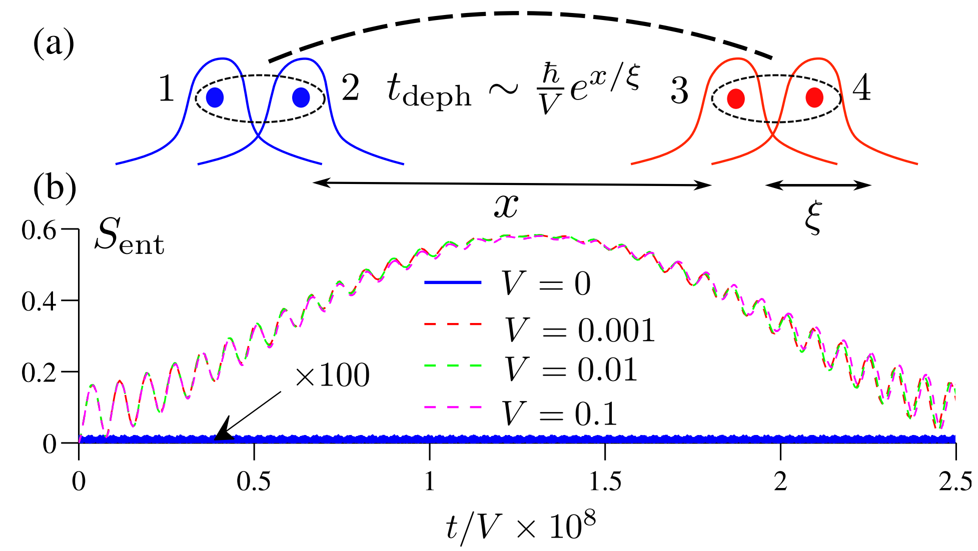
$$H = \sum_i h_i \tau_i^z + \sum_{i,j} J_{ij} \tau_i^z \tau_j^z + \sum_{n=1}^{\infty} \sum_{i,j,\{k\}} K_{i\{k\}j}^{(n)} \tau_i^z \tau_{k_1}^z \cdots \tau_{k_n}^z \tau_j^z,$$

Bardarson et al., PRL '12

Ros & Mueller, NPB '12; Serbyn, et al, PRL, '13; Huse et al., PRB '14; Imbrie et al., AdP '17

- Logarithmic growth of the entanglement entropy in the MBL phase:
  - dephasing
  - manifestation of the many-body nature

  Znidaric, et al. PRB '08; Bardarson et al., PRL '12; Serbyn et al., PRL '13



Serbyn et al., PRL '13

The entanglement dynamics:

(non-Hermitian case, g=0.5)

Two very characteristic features!

1) As opposed to the Hermitian case

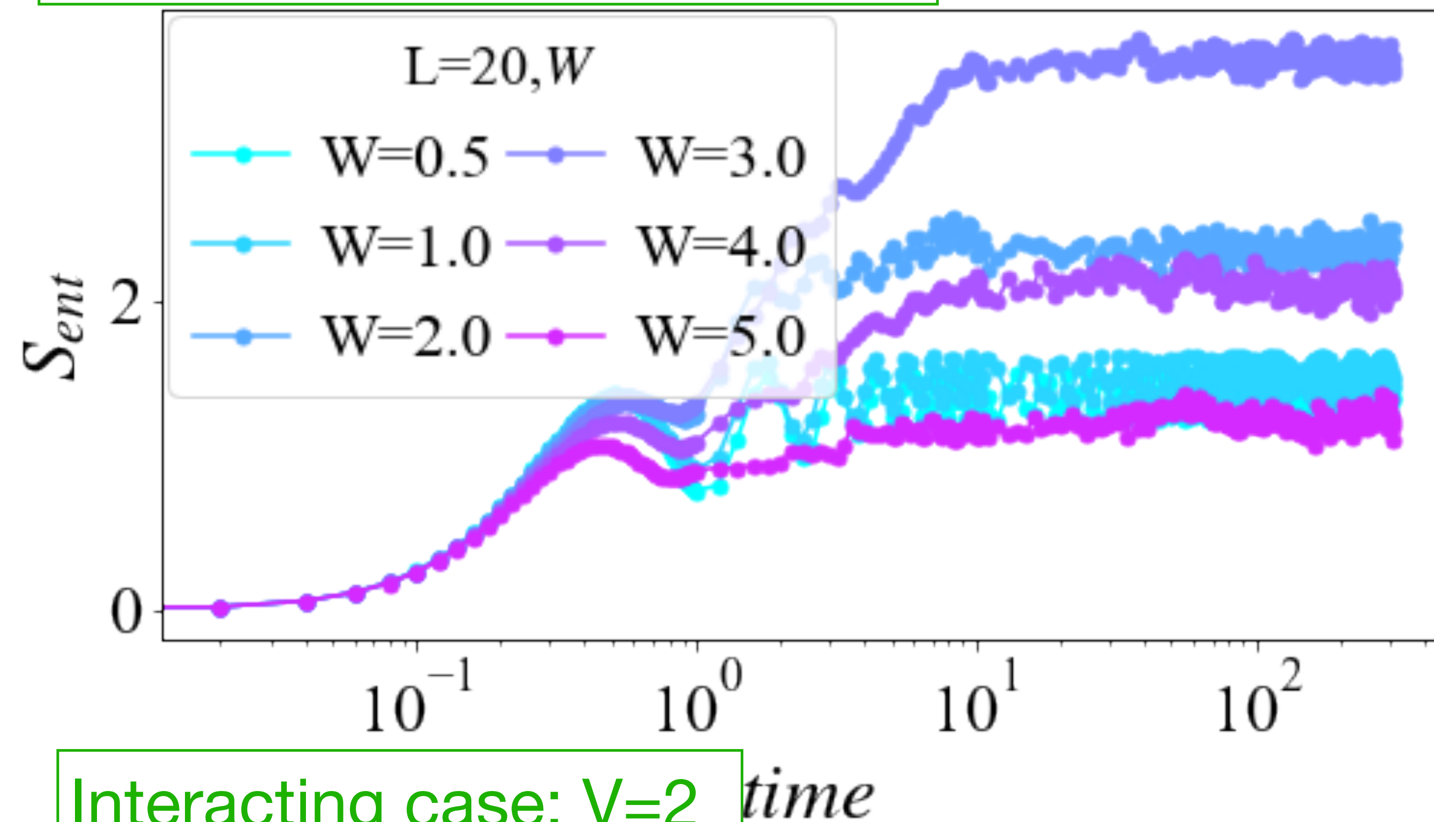
disorder *enhances* the entanglement growth

in the delocalized regime (small W<W_c), then it suppresses the entanglement entropy (W>W_c)

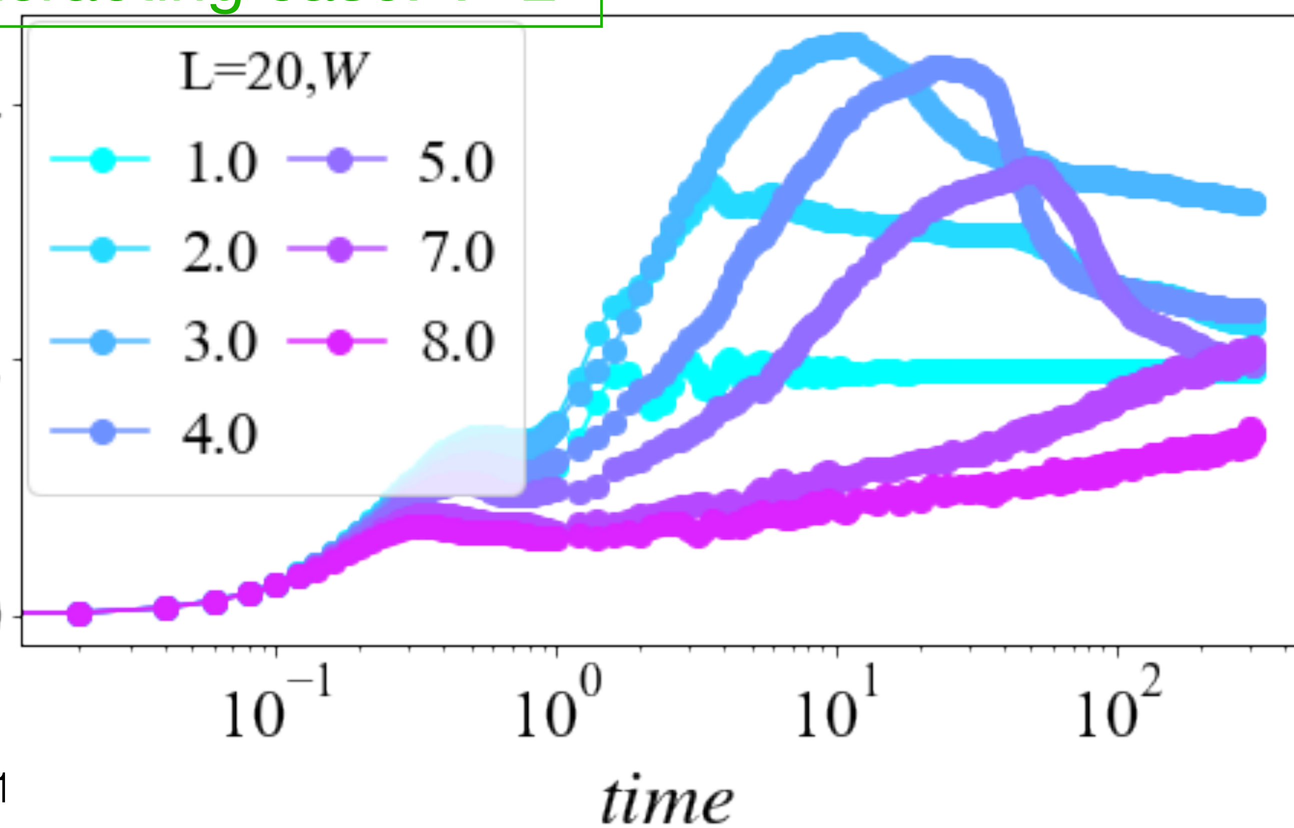
2) *Non-monotonic time evolution* of the entanglement entropy in the regime of intermediate disorder

interacting case
$V \neq 0$

Non-interacting case: V=0



Interacting case: V=2



Hamiltonian: non-Hermitian
time evolution: non-unitary

Nature of the non-monotonic time evolution:

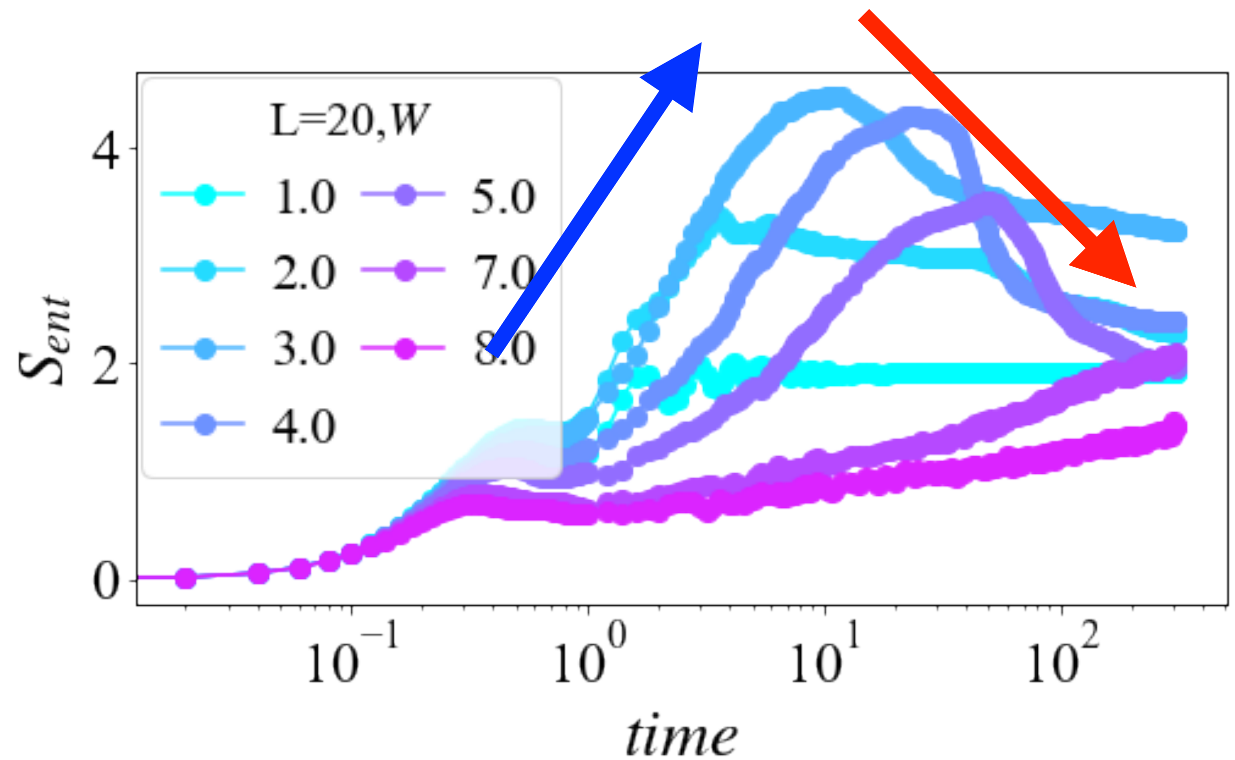
collapse of the superposition *in the initial state in the course of non-unitary time evolution*

- in the expansion:

$$|\Psi(t)\rangle = \sum_\alpha c_\alpha(t)|\alpha\rangle,$$ eigenstates

$$c_\alpha(t) = c_\alpha(0)e^{-iE_\alpha t}$$

complex: $\mathrm{Im}\ E_\alpha \neq 0$



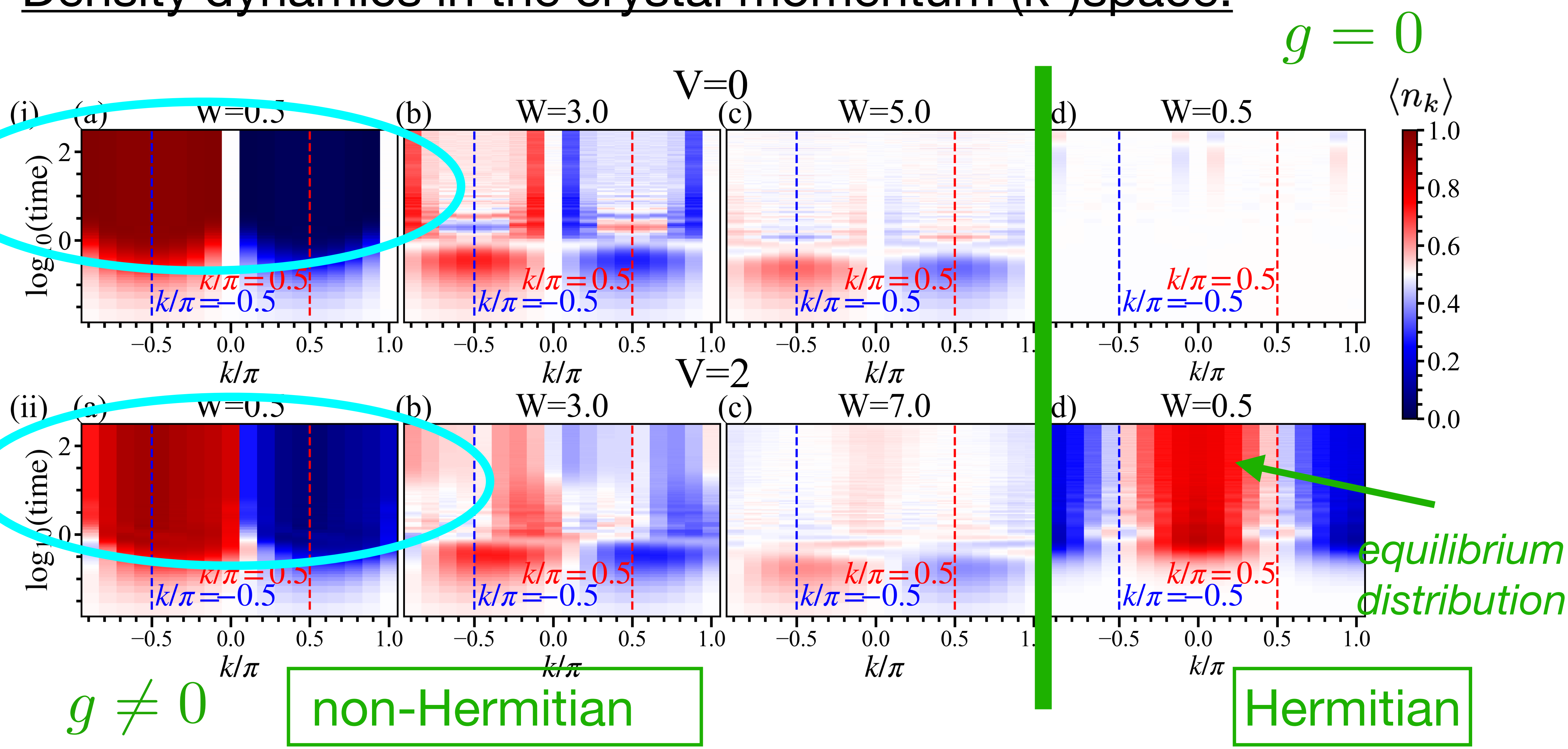
If $\mathrm{Im}(E_{\alpha_1}) > \mathrm{Im}(E_{\alpha_2}) > \cdots$

$$|c_{\alpha_1}(t)|^2 \gg |c_{\alpha_2}(t)|^2 \gg \cdots$$

→ collapse of the superposition, evolution to a single eigenstate: $\lim_{t\to\infty} |\tilde{\Psi}(t)\rangle \sim |\alpha_1\rangle$

Competition between
spreading of the density/information vs. collapse of the superposition
→ non-monotonic time evolution of entanglement entropy

Density dynamics in the crystal momentum (k-)space:

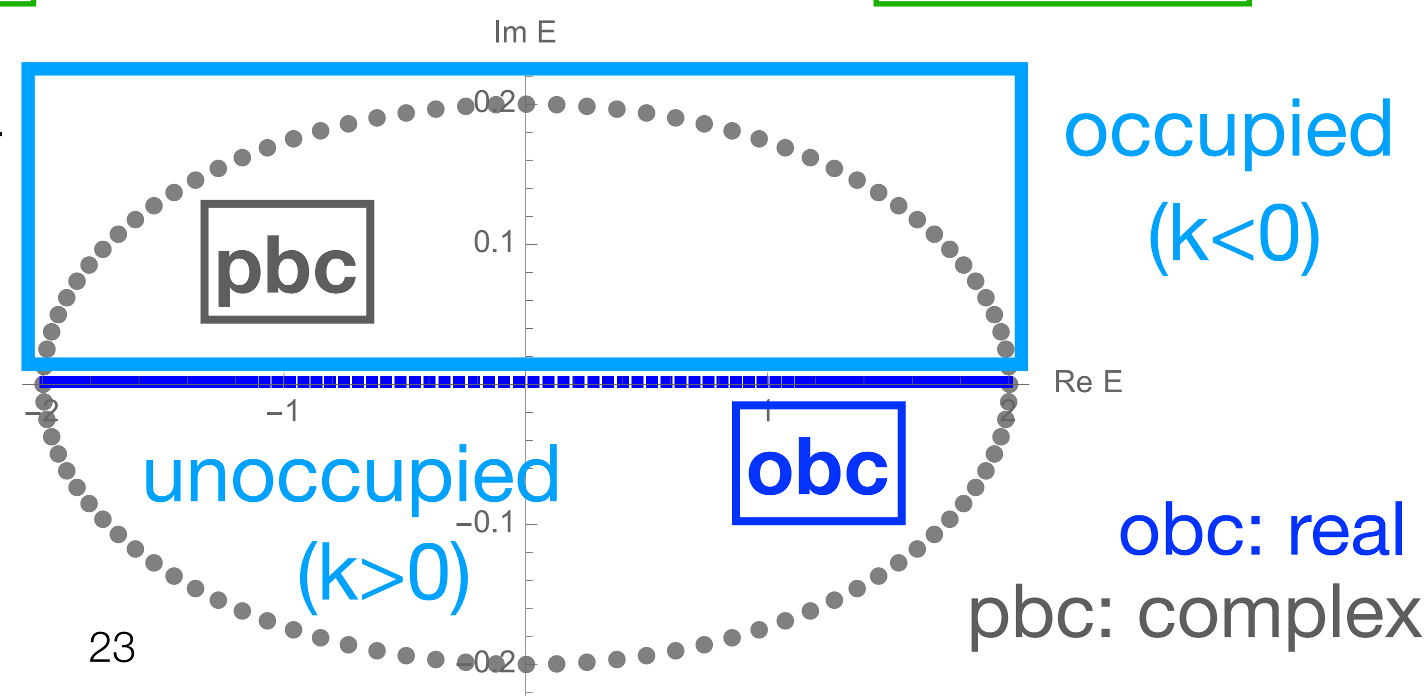


Recall the (one-body) spectrum:

$$\epsilon_k = -2\Gamma_0 \cos(k - ig),$$

$$\left(\frac{\mathrm{Re}\ \epsilon_k}{\Gamma_0 \cosh g}\right)^2 + \left(\frac{\mathrm{Im}\ \epsilon_k}{\Gamma_0 \sinh g}\right)^2 = 1.$$

(periodic boundary condition)



Logarithmic scaling of the entanglement entropy

- in the asymptotic regime: $t \to \infty$

in the asymptotic regime

$$|\Psi(t \to \infty)\rangle \sim e^{-iE_{\alpha_0}t} \left(\prod_{k<0} c_k^\dagger \right) |0\rangle \equiv e^{-iE_{\alpha_0}t} |\alpha_0\rangle$$

(non-interacting case, clean limit: V=0, W=0)

Analogy with the Fermi-sea ground state:

$$|\Psi_G\rangle = \left(\prod_{k s.t. |k|<k_F} c_k^\dagger \right) |0\rangle$$

$$\rho_G(k) = \theta(\epsilon_F - \epsilon_k) = \begin{cases} 1 & (\text{for } \epsilon_k - \epsilon_F) \\ 0 & (\text{for } \epsilon_k - \epsilon_F) \end{cases}$$

→ *Logarithmic scaling of the entanglement entropy*

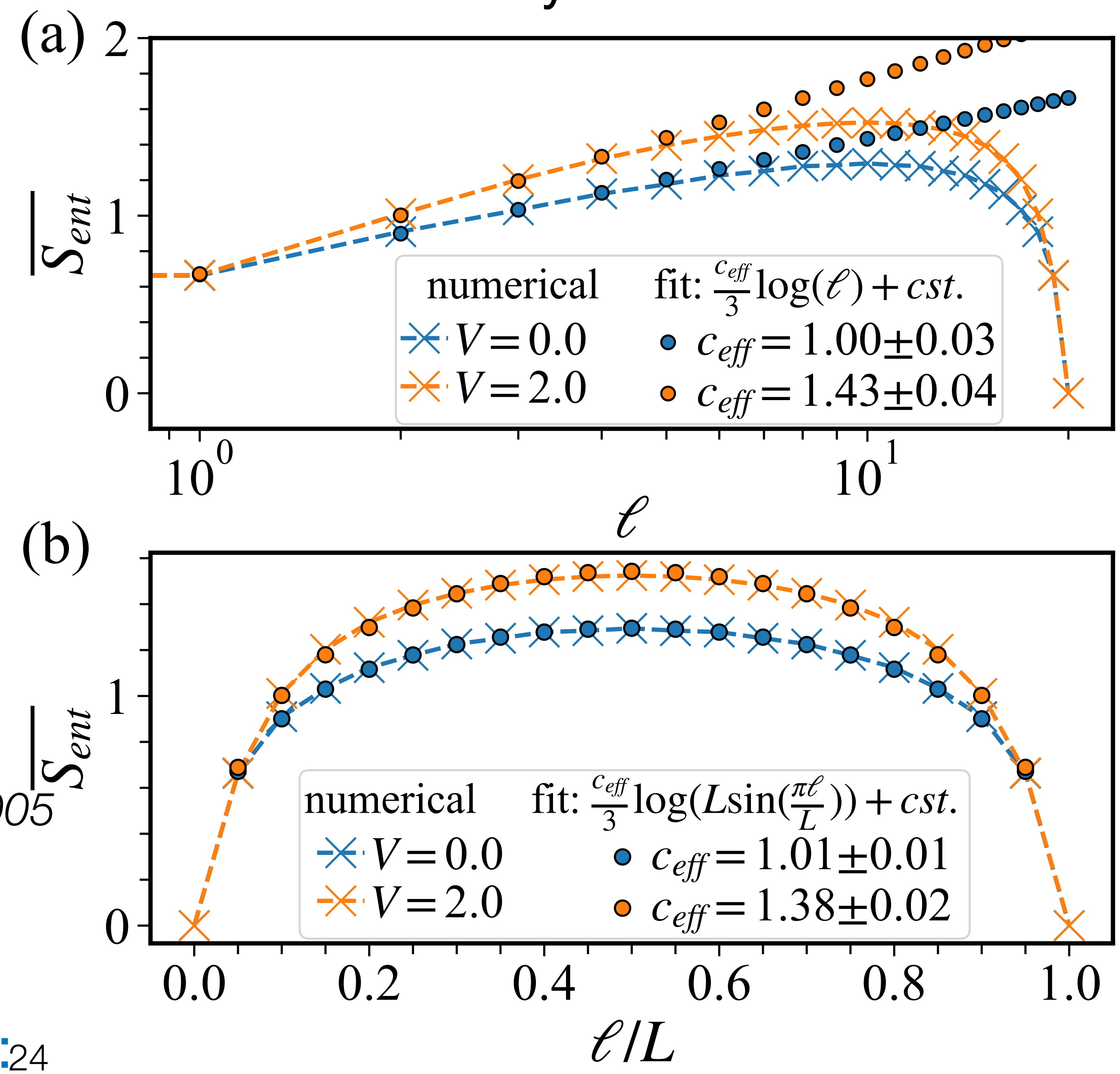
$$S = \frac{c}{3} \log x + cst.$$

cf. Carabrese & Cardy, 2005

$$S = \frac{1}{3} \log \left(2L \sin \frac{\pi x}{L} \right) + cst.$$

chord distance:

l: subsystem size



Conclusions

1) (Non-Hermitian) wave-packet dynamics

- robust uni-directional dynamics
- (Unlike in the Hermitian case) disorder *enhances* spreading of the wave packet

2) Entanglement dynamics

- Non-monotonic time evolution of entanglement entropy

spreading of information vs. collapse of the superposition

evolution to a single eigenstate with maximal Im E

→ quasiparticle picture $\lim_{t\to\infty} |\tilde{\Psi}(t)\rangle \sim |\alpha_1\rangle$

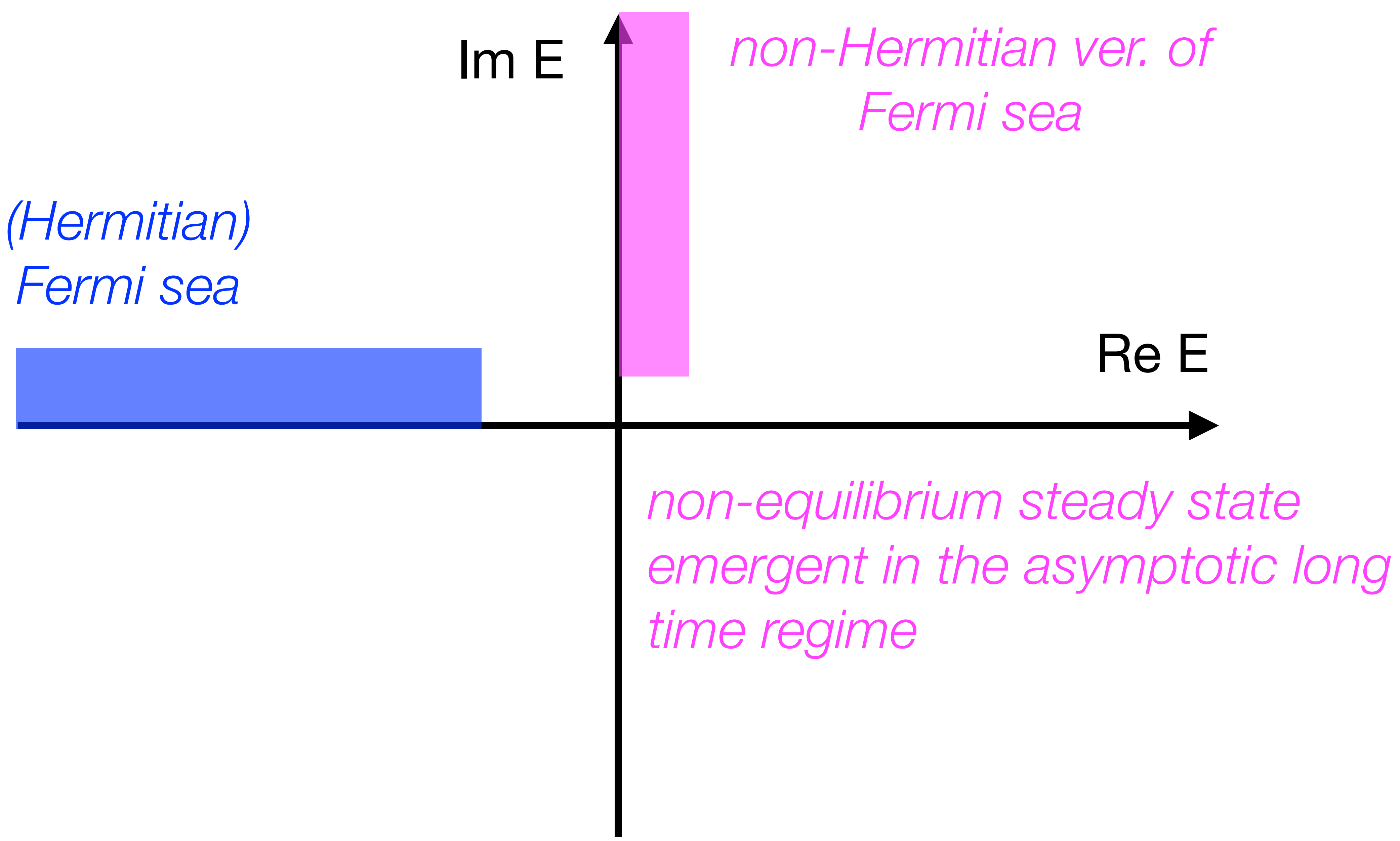
- *Logarithmic scaling* in the asymptotic regime: $t \to \infty$

↔ analogy with a Fermi sea (many-body) ground state

- (particle-hole/bosonic) excitations: conformal field theory at c=1
- interacting case: *effective central charge?*

*Orito & Imura, Phys. Rev. B **108**, 214308 (2023)*

Take-home message



Localization-delocalization transition
in the Hatano-Nelson×Aubry-Andre model

wave function:		spectrum (eigenvalues):
delocalized	⟷	complex
localized	⟷	real

Hatano & Nelson, PRL '96

- Localization length in the Aubry-Andre model: $\xi^{-1} \simeq \log \frac{W}{2\Gamma}$

*in the Hermitian case: $\xi \to \infty$ at $W = W_c$ ⟶ $\frac{W_c}{2\Gamma_0} = 1.$

- in the Non-Hermitian case:

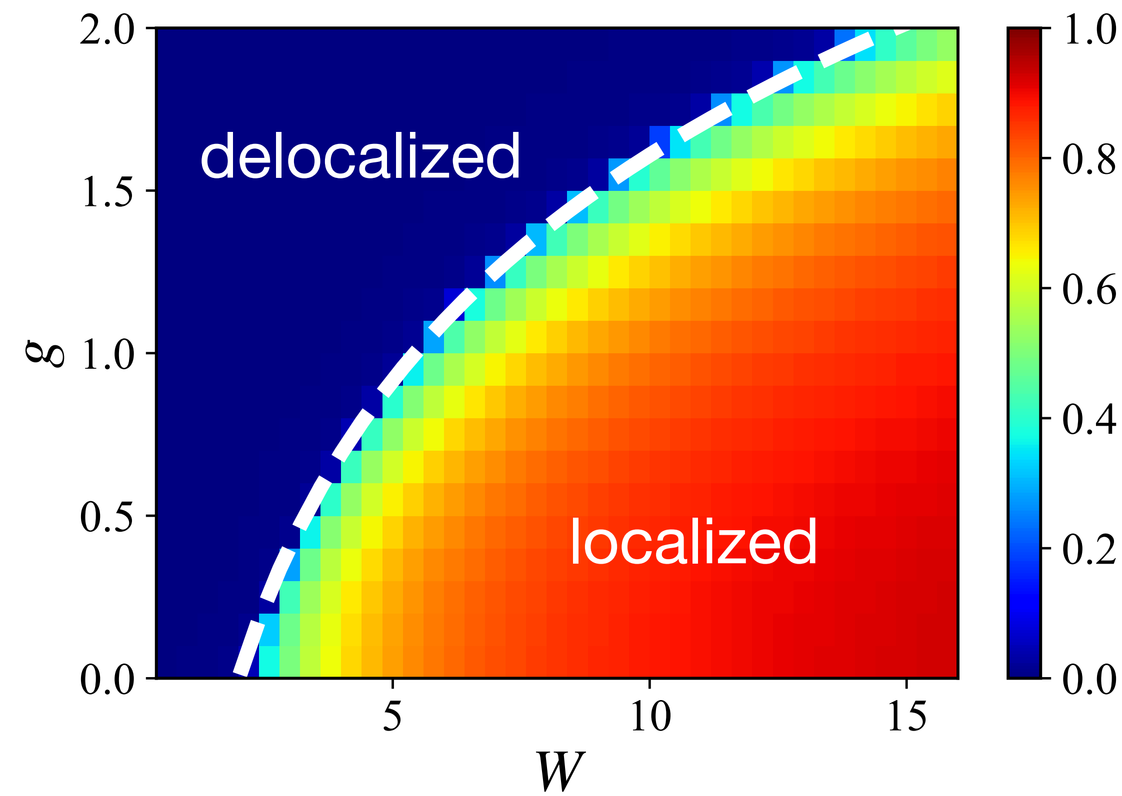
$$\psi^{L,R}(x) \sim \exp\left(-\frac{|x - x_c|}{\xi} \mp g(x - x_c)\right),$$

delocalization point: $\xi^{-1} = g > 0.$

⟶ $W = W_c = 2\Gamma_0\, e^{g} = 2\Gamma_L$

$$g = \log \frac{W}{2}$$

- phase diagram



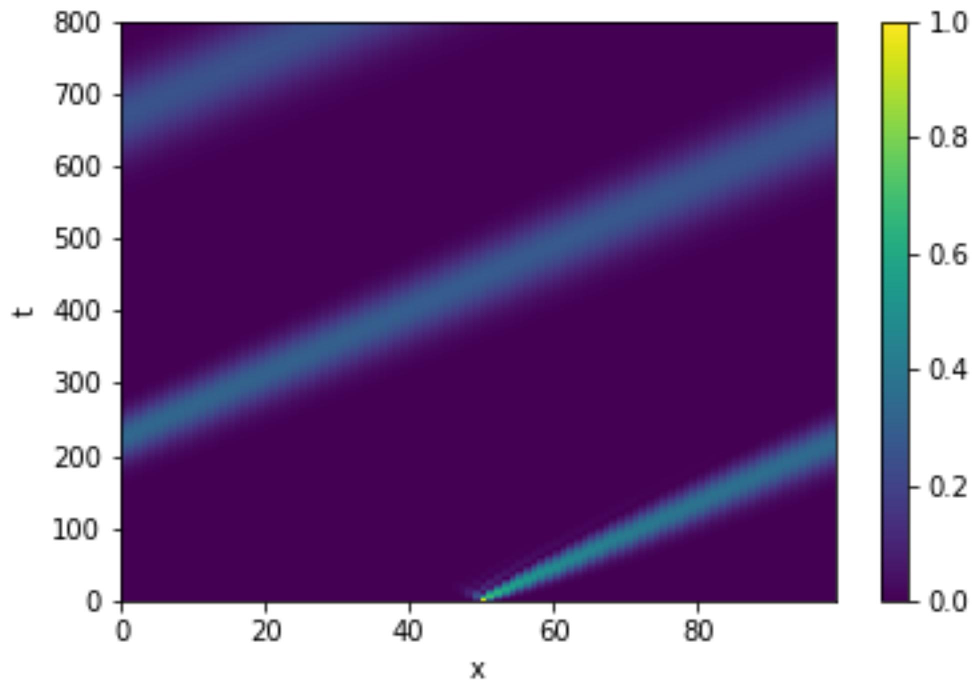
IPR: inverse participation ratio

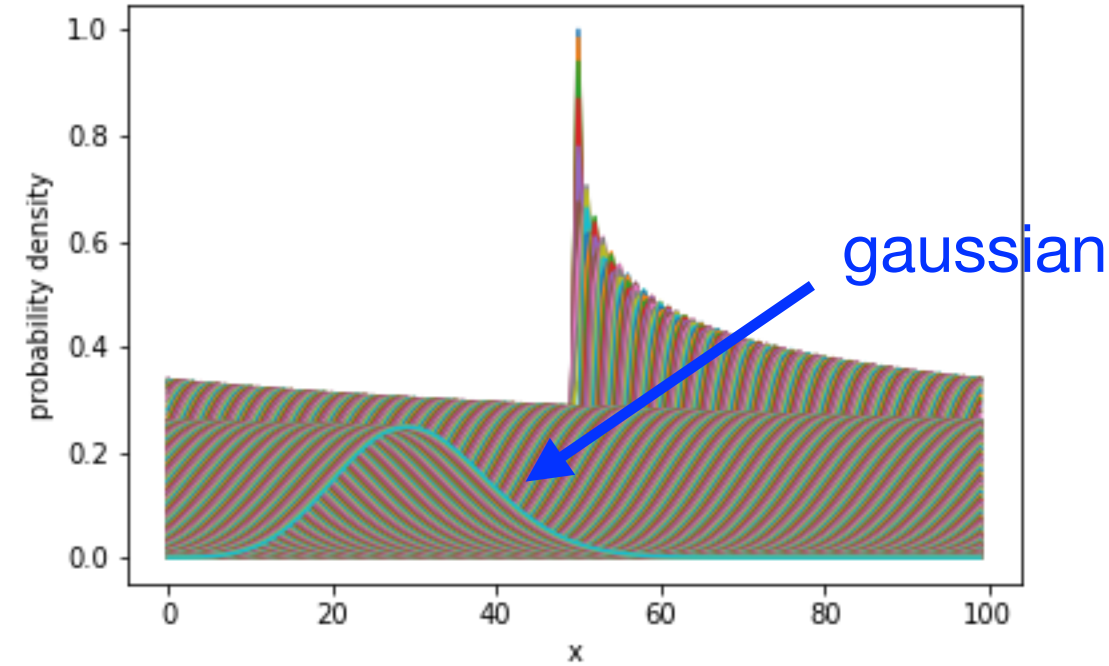
Comparison of periodic vs. open boundary conditions

case of pbc:
the *periodic* boundary conditions

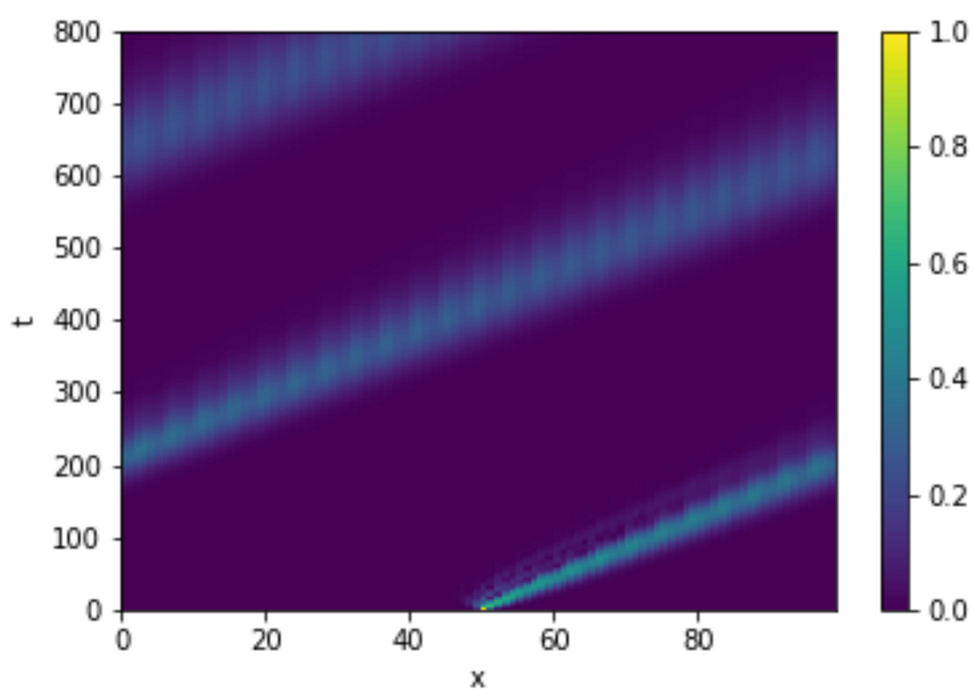
Simulation of the wave-packet dynamics:
(b) non-Hermitian case

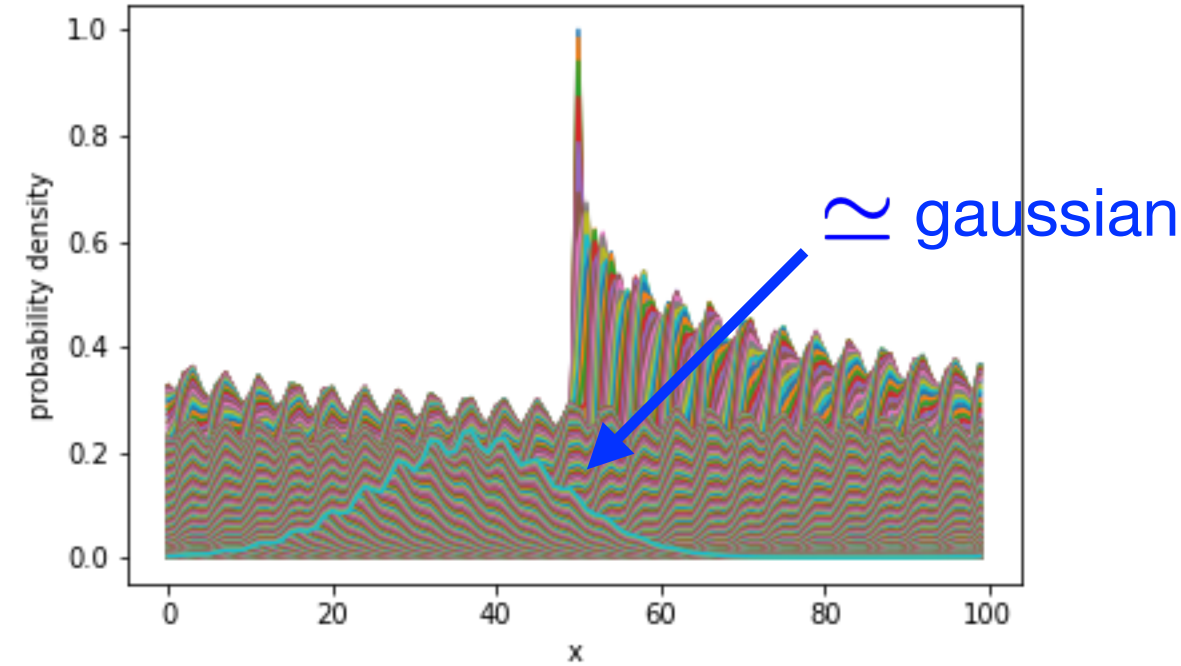
- clean limit: W=0





- case of weak disorder: W=1.0



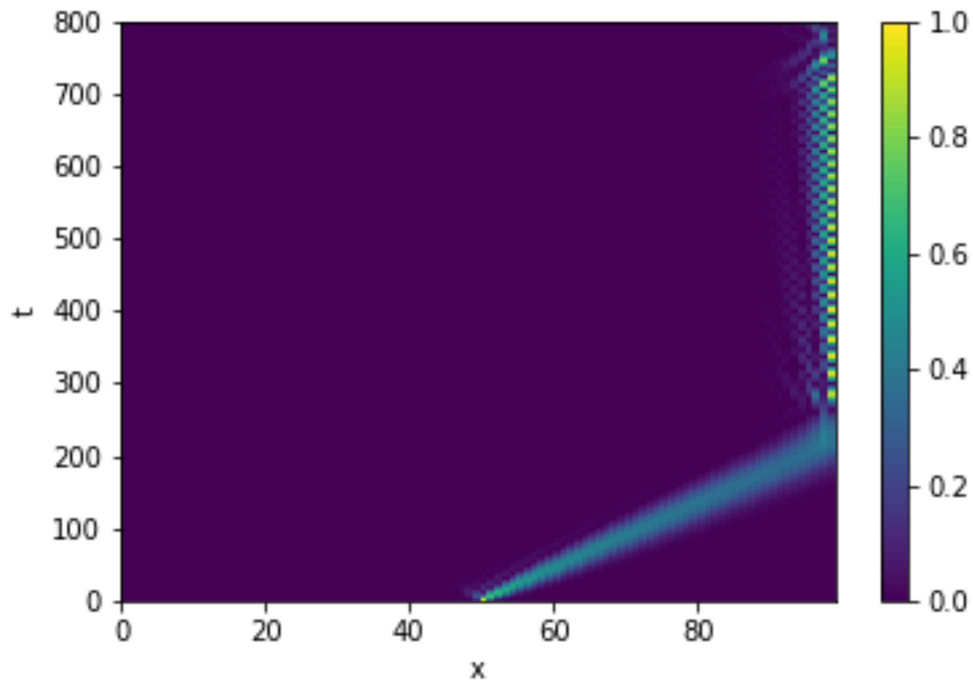


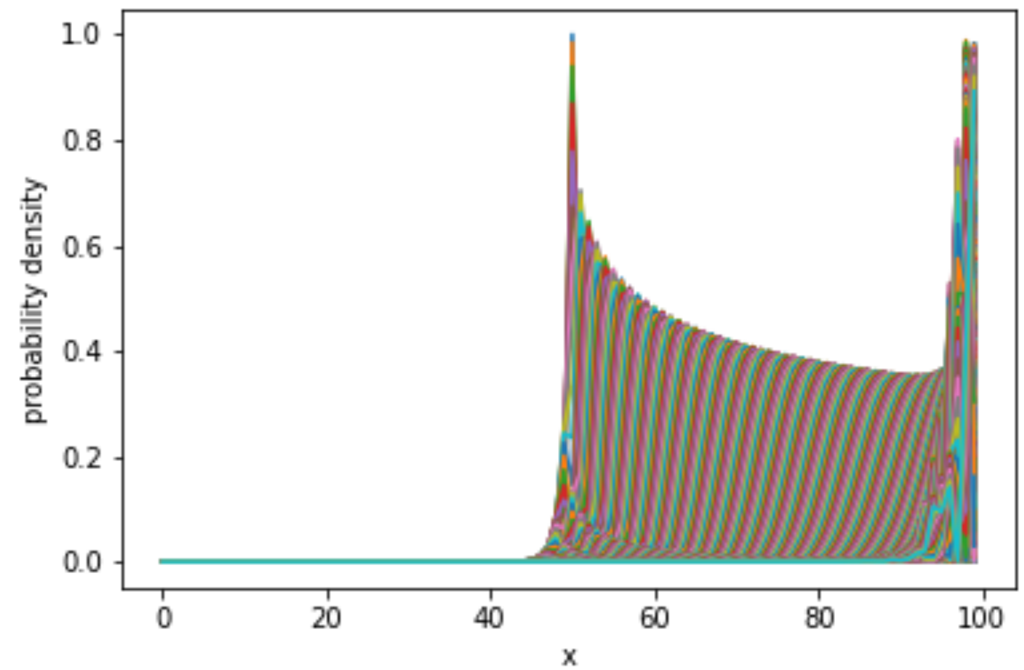
Comparison of periodic vs. open boundary conditions

case of obc:
the *open* boundary conditions

Simulation of the wave-packet dynamics:
(b) non-Hermitian case

- clean limit: W=0





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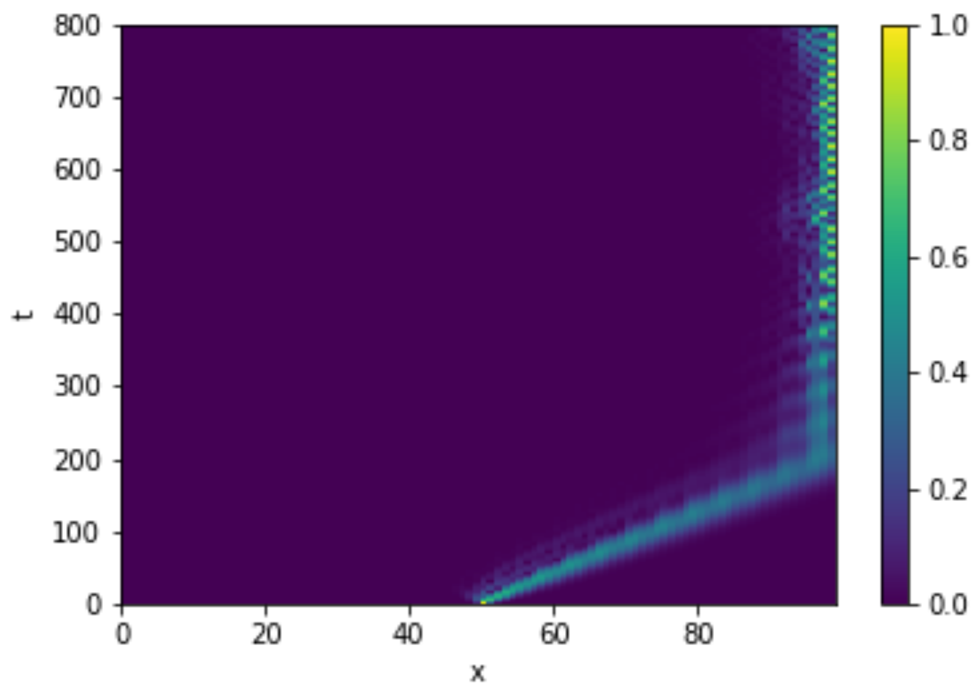
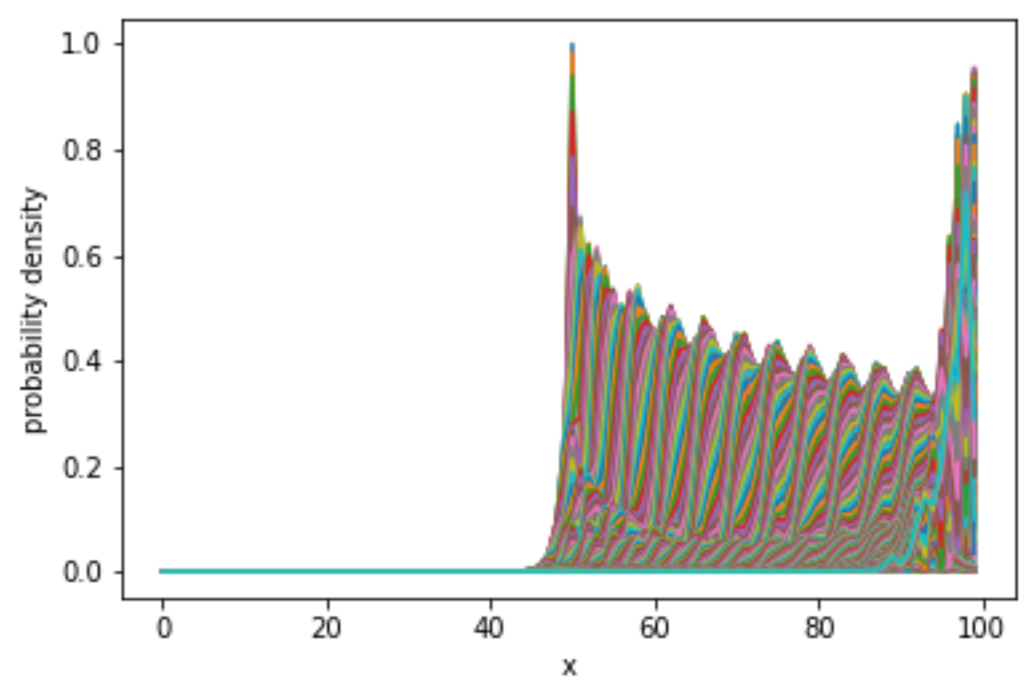


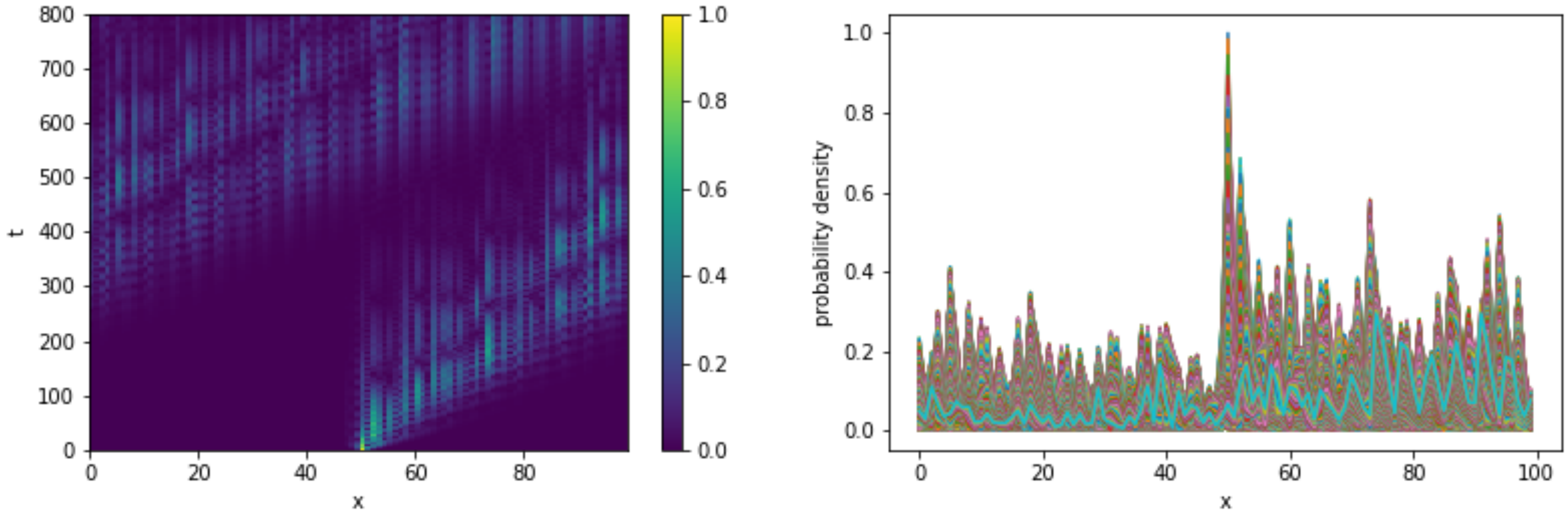


Simulation of the wave-packet dynamics:
(b) non-Hermitian case

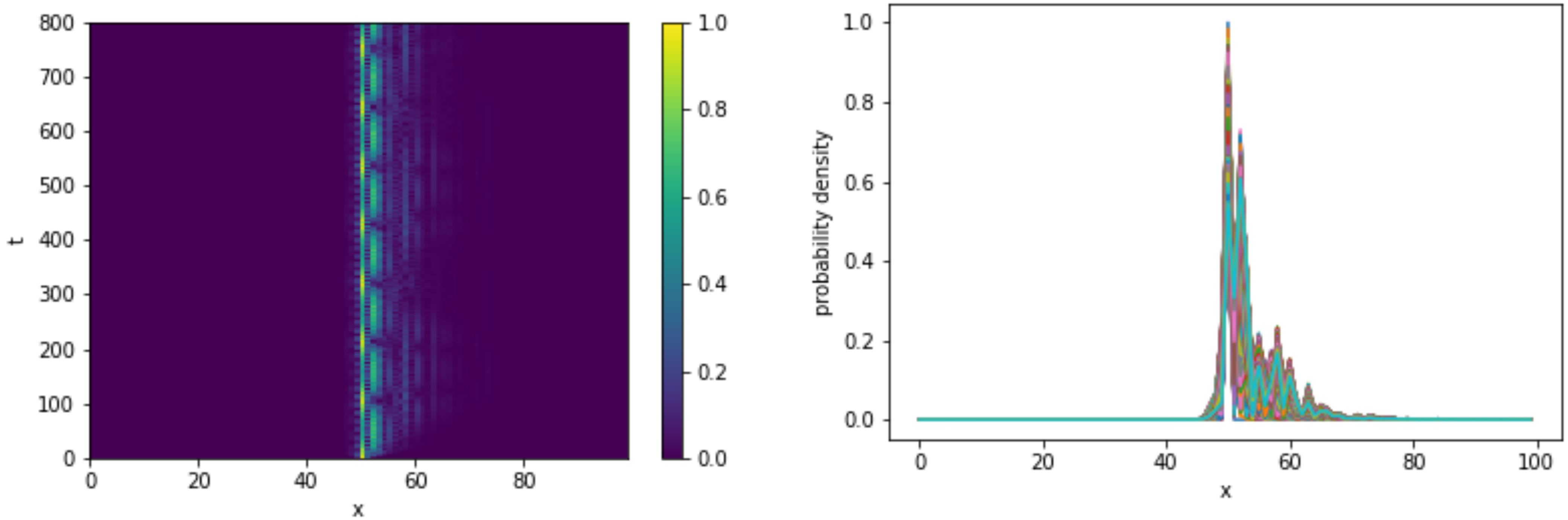
<u>pbc vs. obc</u>
(continued)

case of pbc:
the *periodic* boundary conditions

- Case of stronger disorder: W=3.0 (still in the extended phase)



- Case of even stronger disorder: W=4.0 (localized phase)

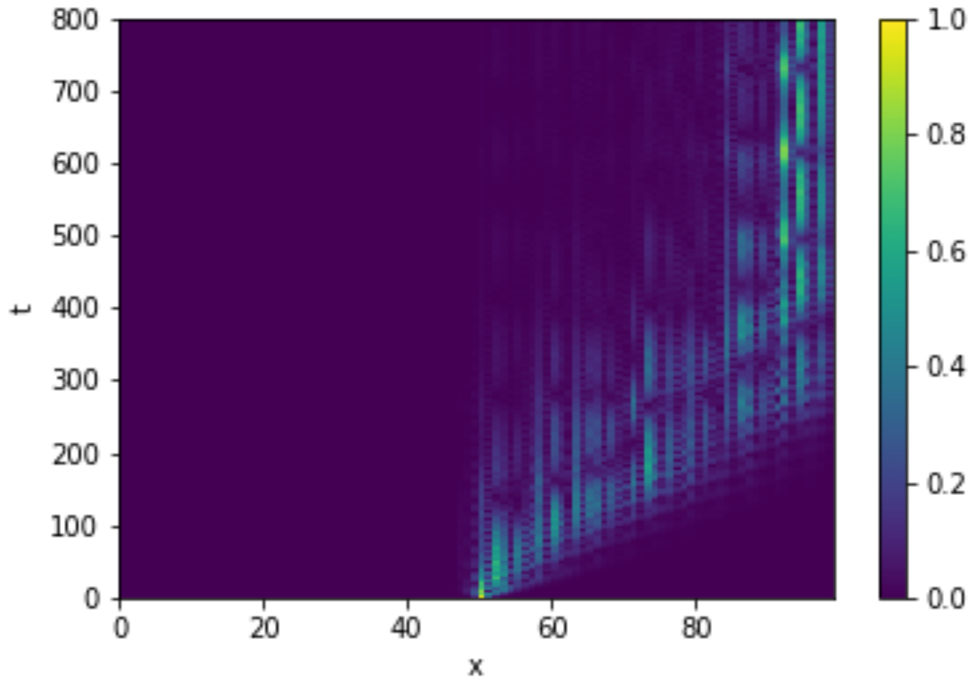


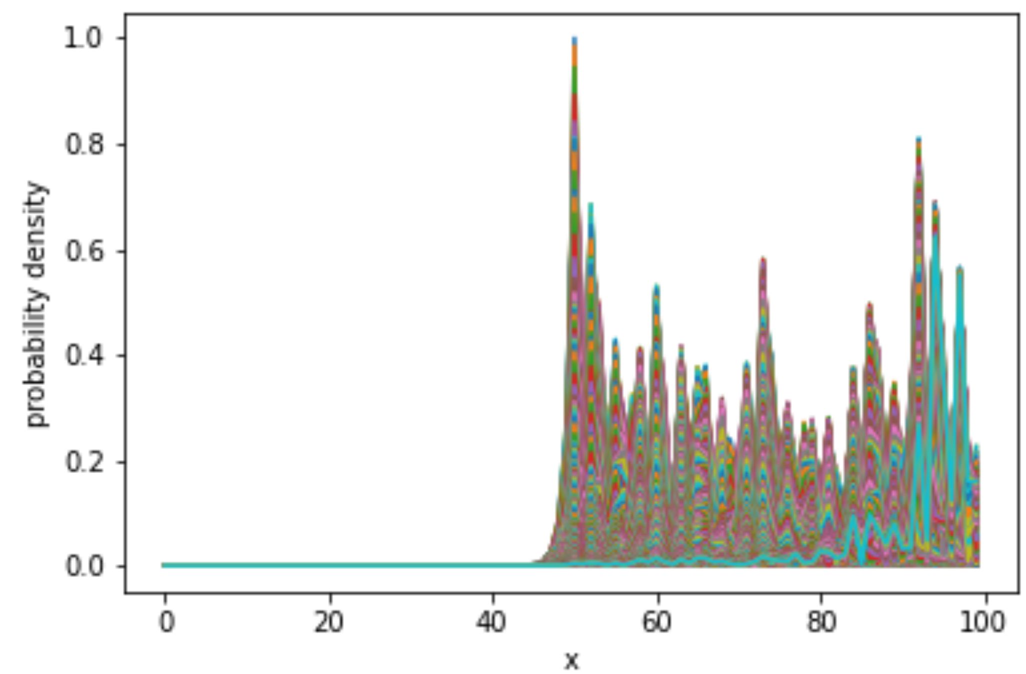
Simulation of the wave-packet dynamics:
(b) non-Hermitian case

pbc vs. obc
(continued)

case of obc:
the *open* boundary conditions

- Case of stronger disorder: W=3.0 (still in the extended phase)





- Case of even stronger disorder: W=4.0 (localized phase)