Weak-coupling bound states in semi-infinite topological waveguide QED

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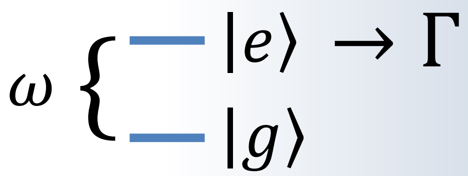
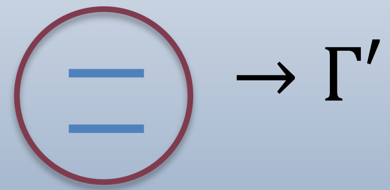
Background: environmental engineering

Cavity QED: the emission properties of an atom can be modified by placing it inside a cavity or a waveguide





Dynamical properties of a system are modified by the environment into which a system decays

Background: environmental engineering

We can generalize from this picture in a variety of ways.

One can consider the interaction of qubits and atom-like states with a variety of artificial environments (photonic lattices, etc.)

- atom-photon bound states

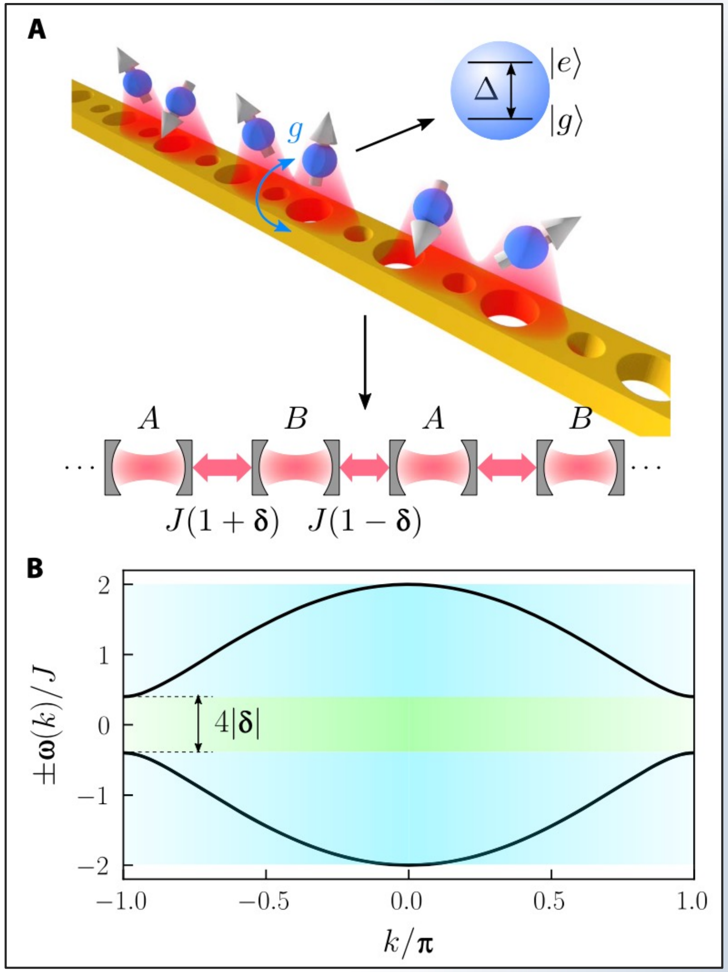
- environments that behave as a topological insulator with topologically-protected surface states

E. Kim, et al, PRX **11**, 011015 (2021)

S. G. and K. Noba, PRA **104**, 062215 (2021)

H. Zhang, et al, PRA **105**, 053703 (2022)

Quantum optics with topological baths

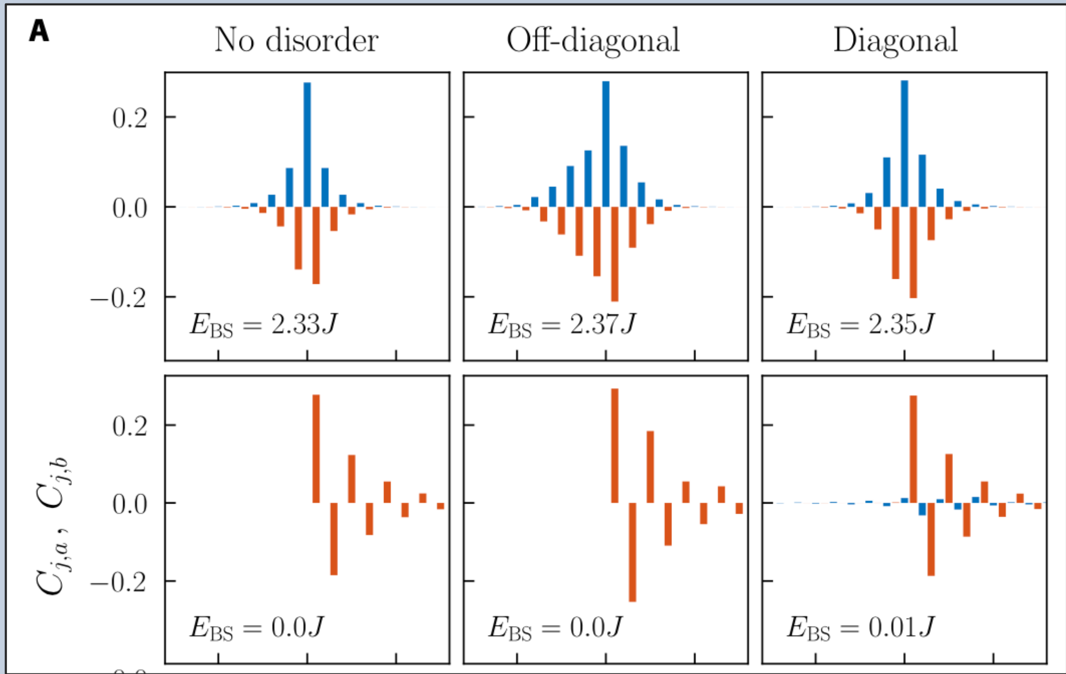


SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Unconventional quantum optics in topological waveguide QED

M. Bello[1], G. Platero[1], J. I. Cirac[2], A. González-Tudela[2,3]*



Bound states in a 2-D topological ring resonator array with quantum emitter

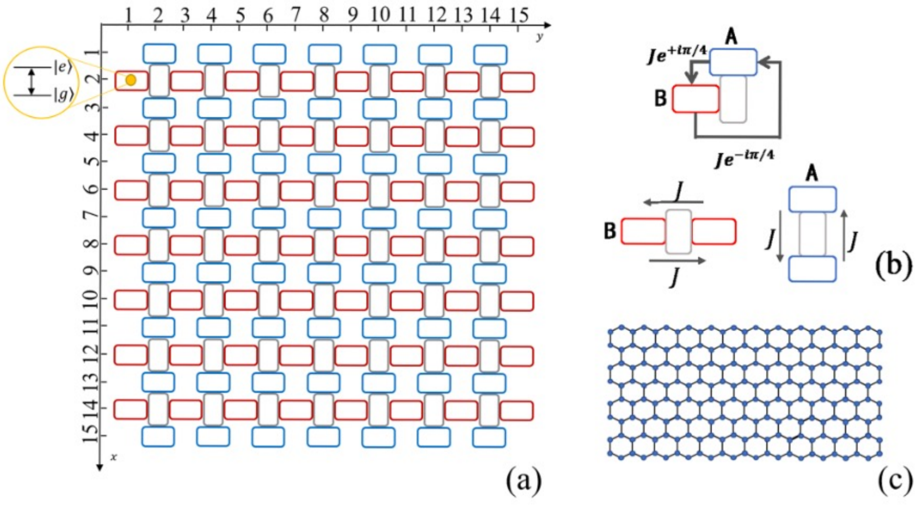
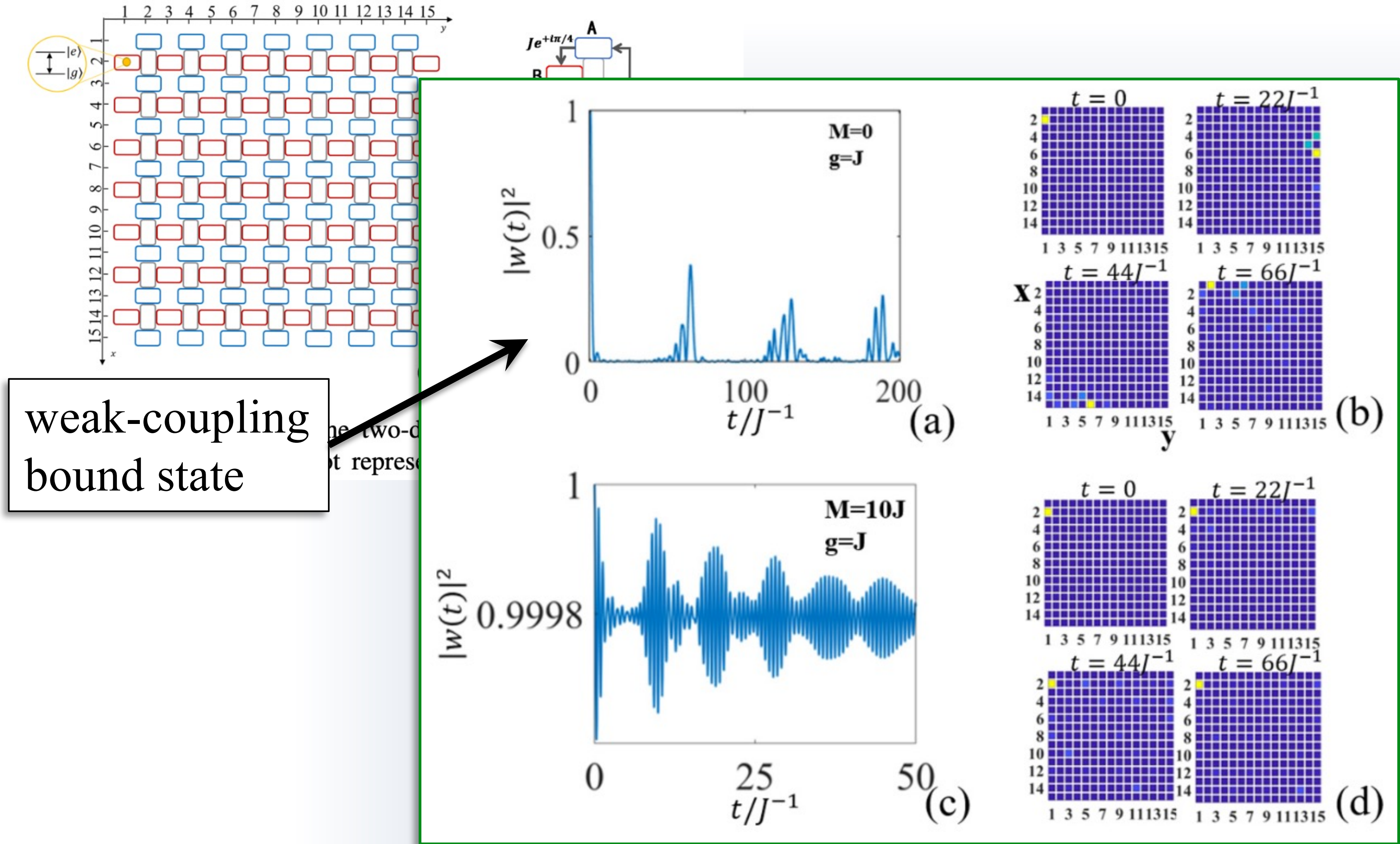


FIG. 1. (a) Scheme of the two-dimensional coupled ring resonator array. The orange dot represents the QE, which is placed

H. Zhang, et al, PRA **105**, 053703 (2022)

Bound states in a 2-D topological ring resonator array with quantum emitter



H. Zhang, et al, PRA **105**, 053703 (2022)

Weak-coupling bound states - 1-D topological model

In this study, we consider a quantum emitter coupled to a topological 1-D reservoir (the SSH model)

The SSH model is often used to illustrate the concept of bulk-boundary correspondence…

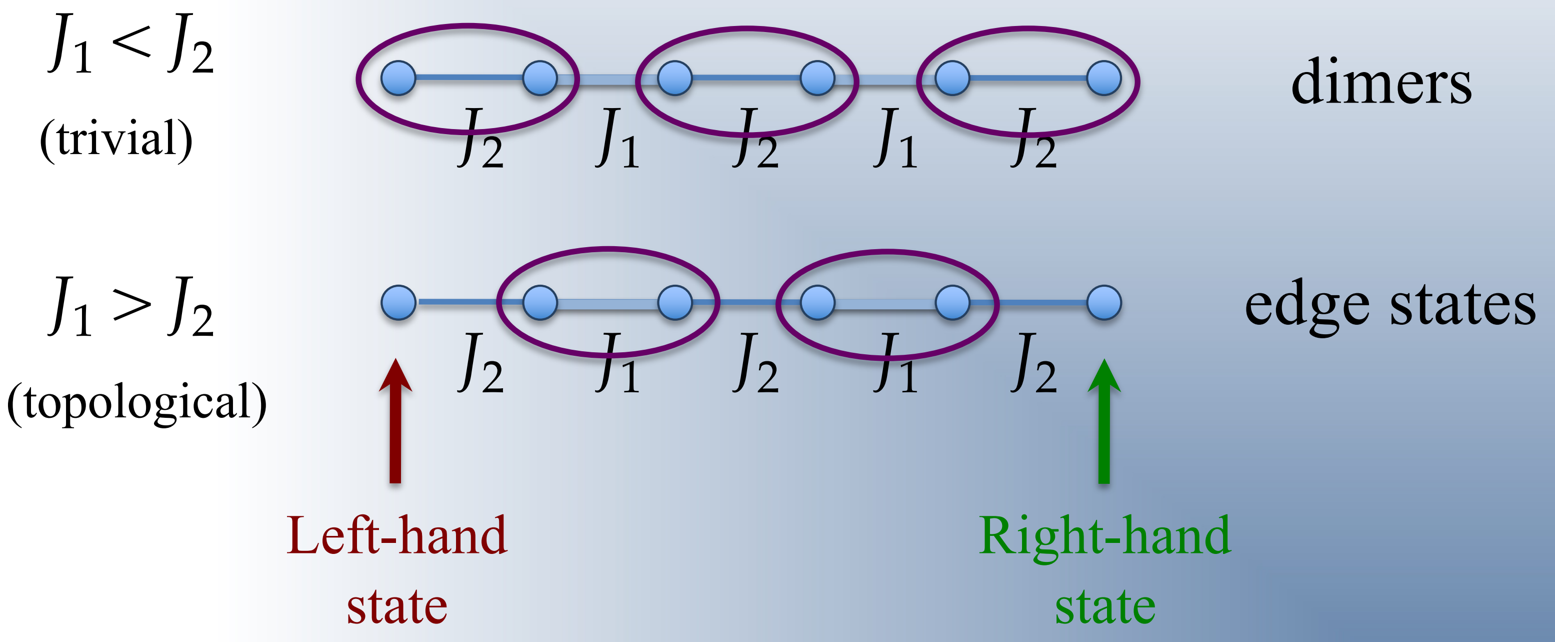
Bulk-boundary correspondence

A key property of topological insulator models is the so-called bulk-boundary correspondence

Hamiltonian H (periodic boundary conditions)	winding number $w \neq 0$
Hamiltonian H (open boundary conditions)	number of surface states $N_e = w$

Su-Schrieffer-Heeger (SSH) model

SSH gives a simple prototype of a 1-D topological insulator



These are zero-energy modes $E \approx 0$

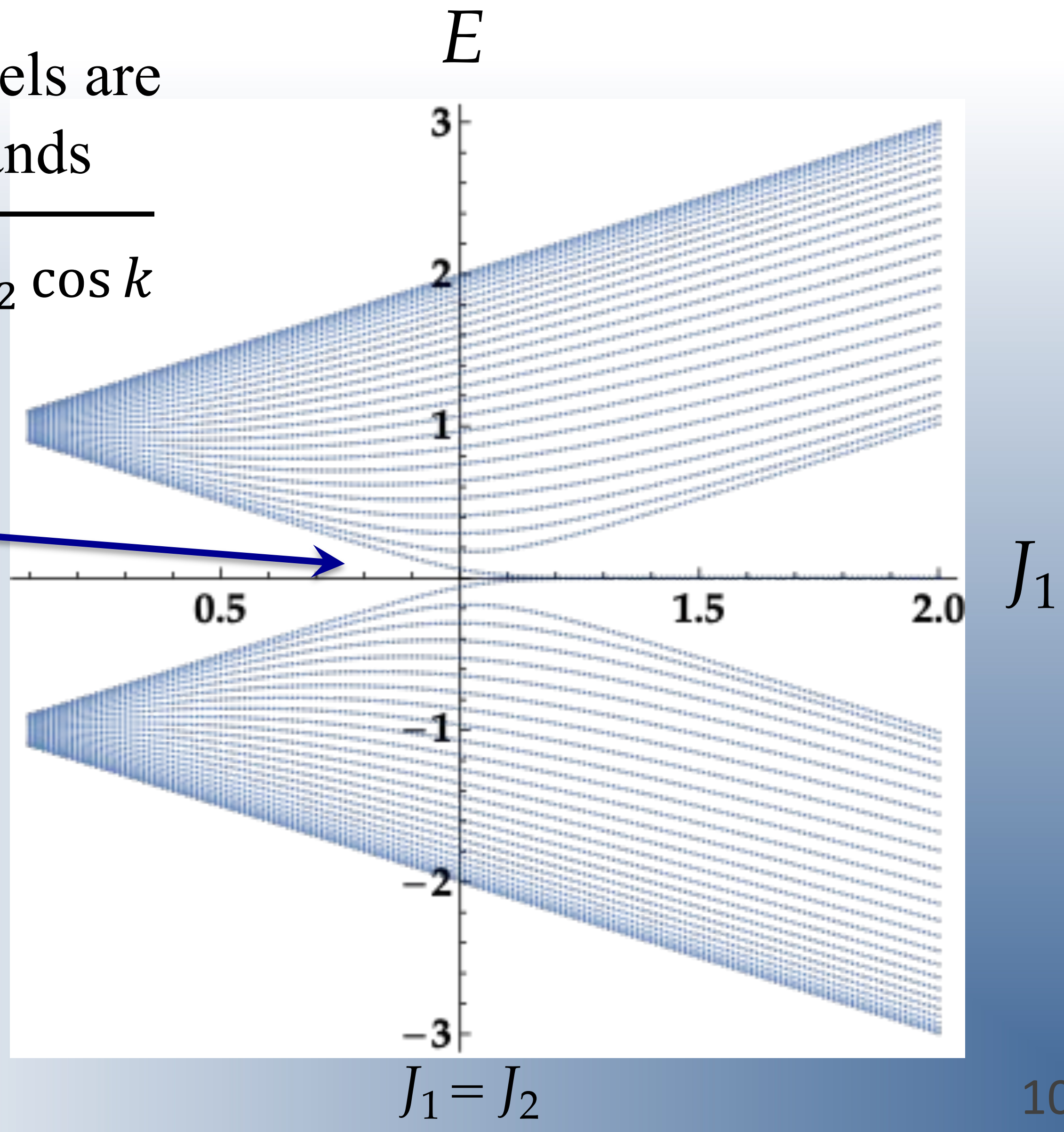
J. K. Asbóth, L. Oroszlány, and A. Pályi, Lecture Notes in Physics **919** (Springer International Publishing, Switzerland 2016).

Edge states in the SSH model: spectrum

Most SSH energy levels are organized into two bands

$$E_k = \pm\sqrt{J_1^2 + J_2^2 + 2J_1J_2\cos k}$$

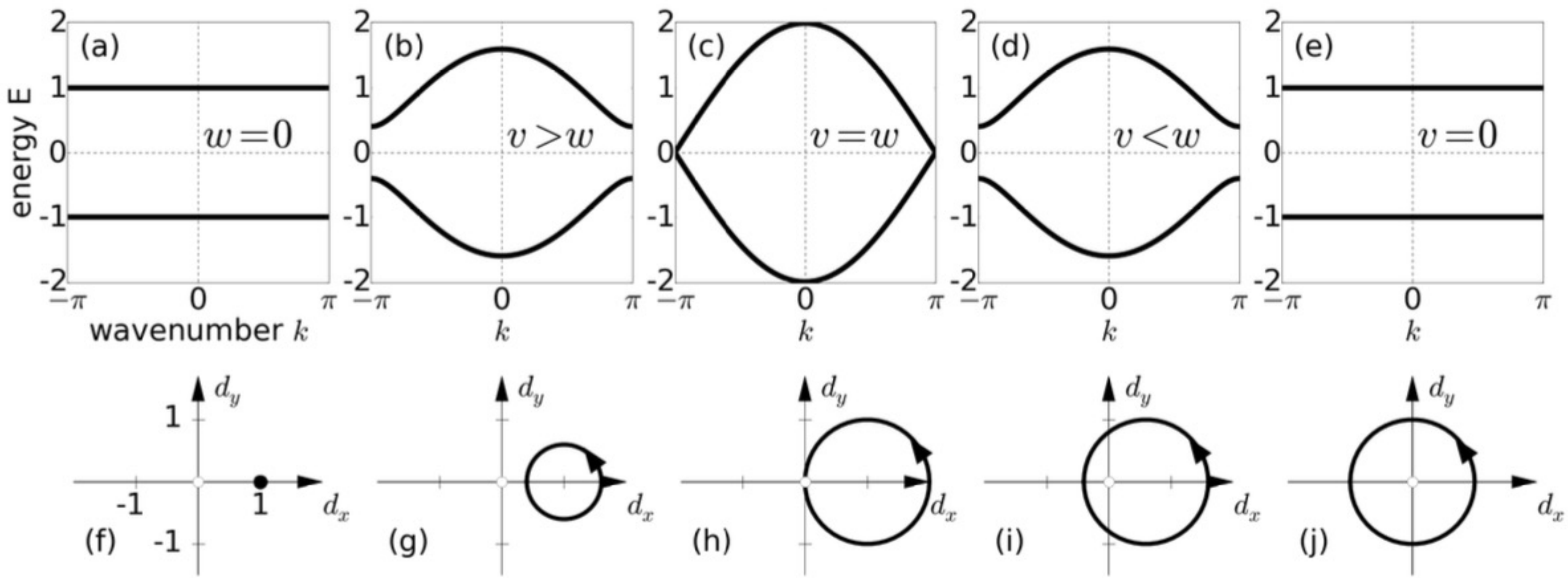
edge states with $E \approx 0$ split off from SSH bands for $J_1 > J_2$



Bulk-boundary correspondence in SSH model

$$E(k) = \left|v + e^{-ik} w\right| = \sqrt{v^2 + w^2 + 2vw\cos k}. \qquad (1.15)$$

We show this dispersion relation for five choices of the parameters in Fig. 1.2.



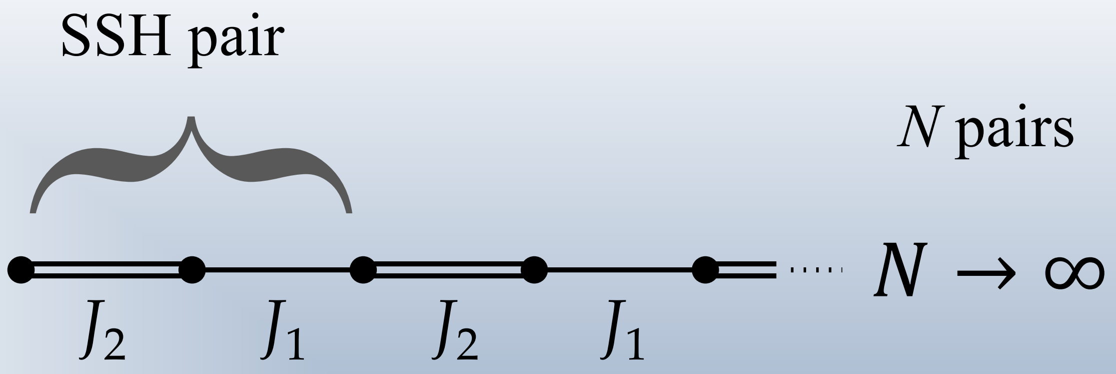
$w = J_1$

$v = J_2$

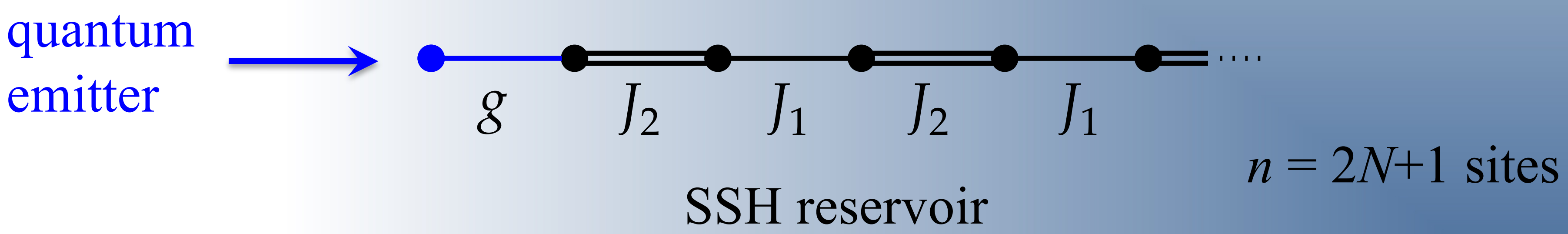
figure borrowed from: J. K. Asbóth, L. Oroszlány, and A. Pályi, Lecture Notes in Physics **919** (Springer International Publishing, Switzerland 2016).

SSH model as topological structured reservoir

We first consider SSH under semi-infinite extension:

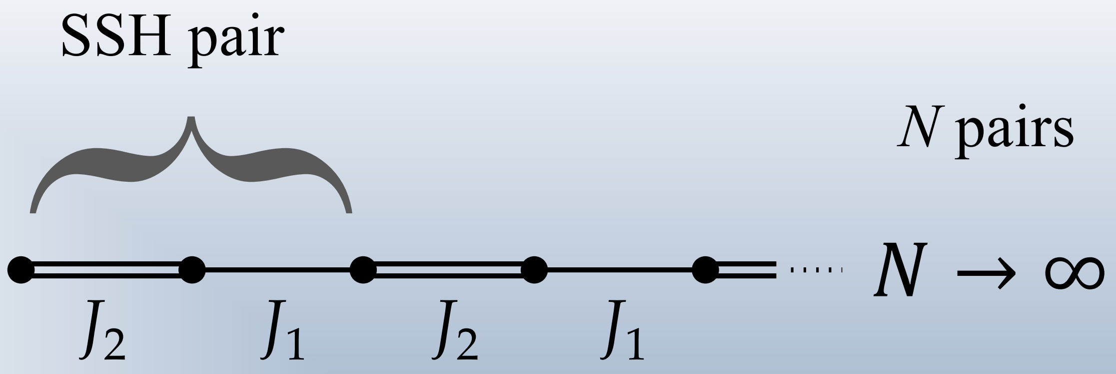


We then attach a quantum emitter to the semi-inf chain:

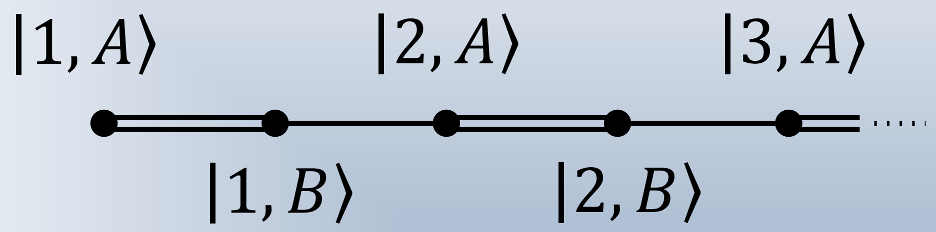


S. G. and K. Noba, Phys. Rev A **104**, 062215 (2021)

Standard SSH notation: *A* and *B* sites



Standard SSH notation: A and B sites



First step: infinite SSH chain

We first diagonalize an infinite SSH chain, and then build solutions to the semi-infinite chain.

$$H_\infty = \sum_{n=-\infty}^{\infty} [J_1(|n,B\rangle\langle n+1,A| + |n+1,A\rangle\langle n,B|) + J_2(|n,B\rangle\langle n,A| + |n,A\rangle\langle n,B|)]$$

Eigenstate ansatz: $|k,\pm\rangle = \sum_{n=-\infty}^{\infty} \left[C_{A,\pm}(k)|n,A\rangle + C_{B,\pm}(k)|n,B\rangle\right]e^{ikn}$

Then we find effective eigenvalue equation:

$$\begin{pmatrix} 0 & J_2 + J_1 e^{-ik} \\ J_2 + J_1 e^{ik} & 0 \end{pmatrix} \begin{pmatrix} C_{A,\pm} \\ C_{B,\pm} \end{pmatrix} = \pm E_k \begin{pmatrix} C_{A,\pm} \\ C_{B,\pm} \end{pmatrix}$$

Infinite SSH chain: continuum eigenstates

We find solutions for the infinite chain of the form

$$|k,\pm\rangle = \frac{1}{\sqrt{2}} \sum_{n=-\infty}^{\infty} [f_k|n,A\rangle \pm |n,B\rangle] e^{ikn}$$

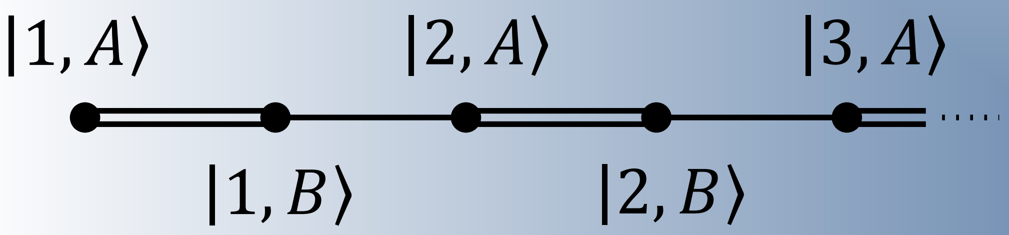
with $f_k = \sqrt{\frac{J_2 + J_1 e^{-ik}}{J_2 + J_1 e^{ik}}}$

And $E_{k,\pm} = \pm\sqrt{(J_2 + J_1 e^{ik})(J_2 + J_1 e^{-ik})}$

Semi-infinite SSH chain: Hamiltonian

We turn now to our original goal: semi-infinite SSH chain

$$H = \sum_{n=1}^{\infty} [J_1(|n,B\rangle\langle n+1,A| + |n+1,A\rangle\langle n,B|) + J_2(|n,B\rangle\langle n,A| + |n,A\rangle\langle n,B|)]$$



Semi-infinite SSH chain: Solution

Eigenstates of semi-infinite SSH chain: written as a linear combination of solutions from the infinite case, satisfying appropriate boundary conditions.

$$|\phi_k, \pm\rangle = \frac{1}{i\sqrt{2}}\left(|k, \pm\rangle - |-k, \pm\rangle\right)$$

$$= \sum_{n=1}^{\infty}\left(\frac{f_k\ e^{ikn} - f_k^* e^{-ikn}}{2i}|n, A\rangle \pm \sin k\, n\, |n, B\rangle\right)$$

with the key property:

$$f_k = \sqrt{\frac{J_2 + J_1 e^{-ik}}{J_2 + J_1 e^{ik}}}$$

$$\langle 0, B|\phi_k, \pm\rangle = \sin 0 = 0$$

(decouples from sites to the left of $|1, A\rangle$)

Diagonalized SSH semi-infinite model

Our eigenstates indeed diagonalize H as expected

$$H = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sum_{s=\pm} E_{k,s} |\phi_k, s\rangle\langle\phi_k, s|$$

$$E_{k,\pm} = \pm\sqrt{(J_2 + J_1 e^{ik})(J_2 + J_1 e^{-ik})}$$

$$|\phi_k, \pm\rangle = \sum_{n=1}^{\infty} \left(\frac{f_k \ e^{ikn} - f_k^* e^{ikn}}{2i} |n, A\rangle \pm \sin k\, n\, |n, B\rangle \right)$$

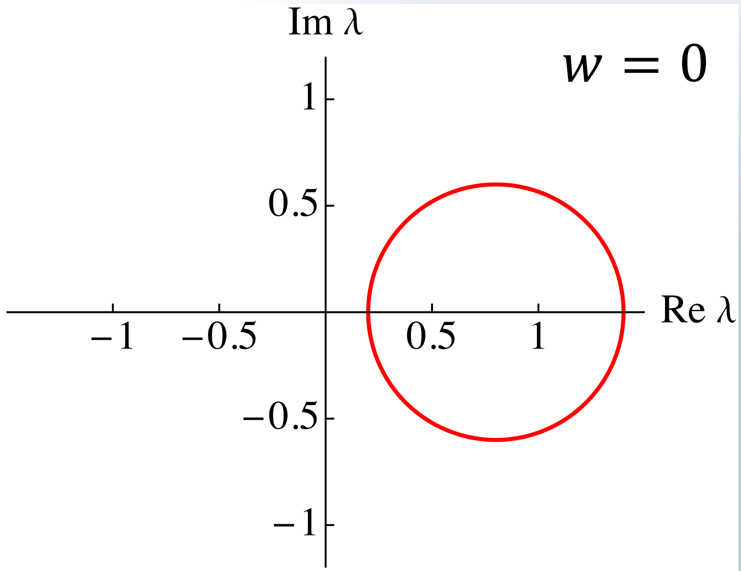
$$f_k = \sqrt{\frac{J_2 + J_1 e^{-ik}}{J_2 + J_1 e^{ik}}}$$

factor f_k introduces edge state **winding number**

Winding number properties

$J_1 < J_2$

Winding number is zero

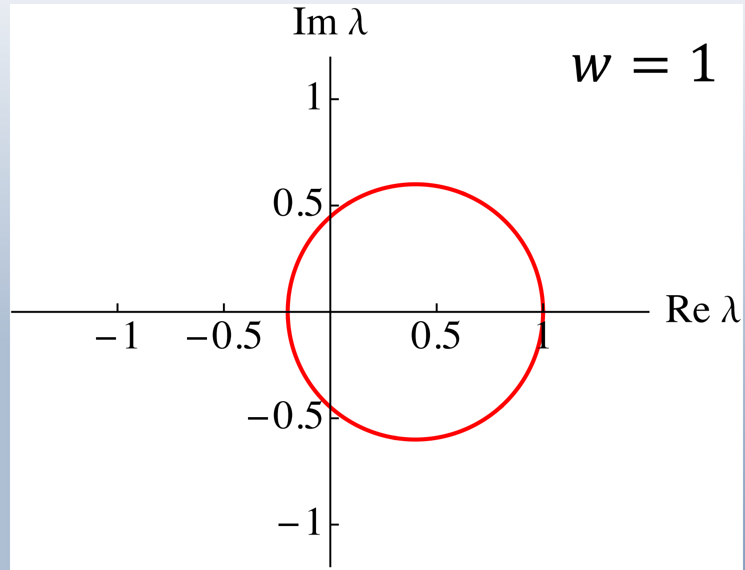


$J_1 = 0.6,\ J_2 = 0.8$

"trivial case"

$J_1 > J_2$

Non-vanishing winding number



$J_1 = 0.6,\ J_2 = 0.4$

"topological case"

$$f_k = \sqrt{\frac{J_2 + J_1 e^{-ik}}{J_2 + J_1 e^{ik}}}$$

parametrize according to $\lambda = e^{ik}$

Edge state revealed in the resolution of unity

If we assume the resolution of unity is given by

$$1 = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sum_{s=\pm} |\phi_k, s\rangle\langle\phi_k, s|$$

We find

$$\langle n, A|n', A\rangle = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left[\frac{f_k e^{ikn} - f_k^* e^{-ikn}}{2i}\right]\left[\frac{f_k^* e^{-ikn'} - f_k \ e^{ikn'}}{-2i}\right]$$

$$= \delta_{n,n'} - \int_{-\pi}^{\pi} \frac{dk}{2\pi} \frac{J_2 + J_1 e^{-ik}}{J_2 + J_1 e^{ik}} e^{ik(n+n')}$$

missing state is revealed in "topological" case

Resolution of unity in the topological case

For $J_1 > J_2$ we must include the (left) edge state

$$1 = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sum_{s=\pm} |\phi_k, s\rangle\langle\phi_k, s| + |\phi_e\rangle\langle\phi_e|$$

$$|\phi_e\rangle = \frac{\sqrt{J_1^2 - J_2^2}}{J_1} \sum_{n=1}^{\infty} |n, A\rangle \left(-\frac{J_2}{J_1}\right)^{n-1}$$

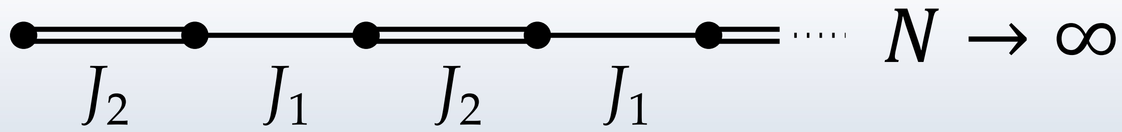
eigenvalue: $E_e = 0$

$$= \frac{\sqrt{J_1^2 - J_2^2}}{J_1} \sum_{n=1}^{\infty} (-1)^{n-1} |n, A\rangle\, e^{-(n-1)/\xi}$$

$$\xi \equiv \frac{1}{\log(J_1/J_2)}$$

exponentially localized and sublattice localized

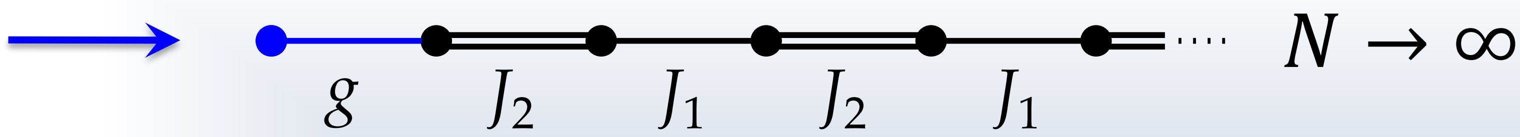
Energy spectrum and quantum emitter



Edge state is exact, but isolated:

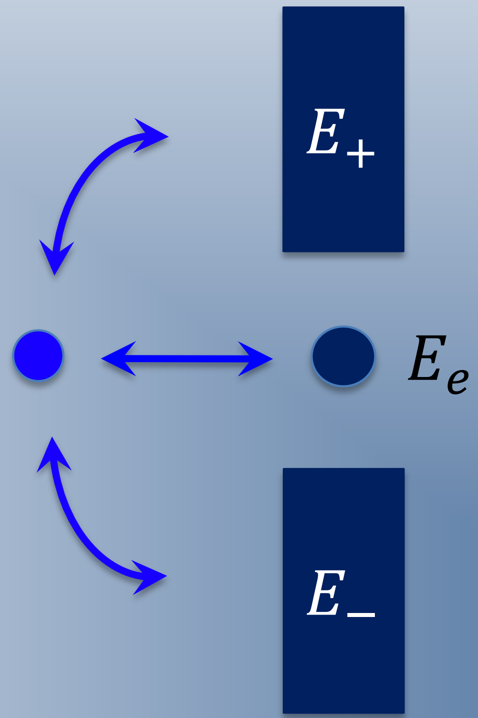
Energy spectrum and quantum emitter

quantum emitter

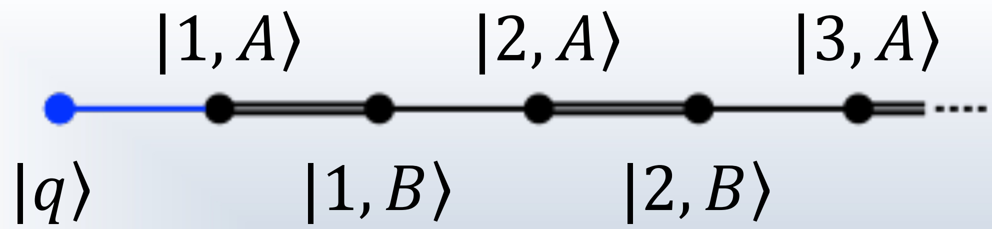


Edge state is exact, but isolated:

Emitter acts as a probe of the topological properties of the system



Chiral sublattice symmetry



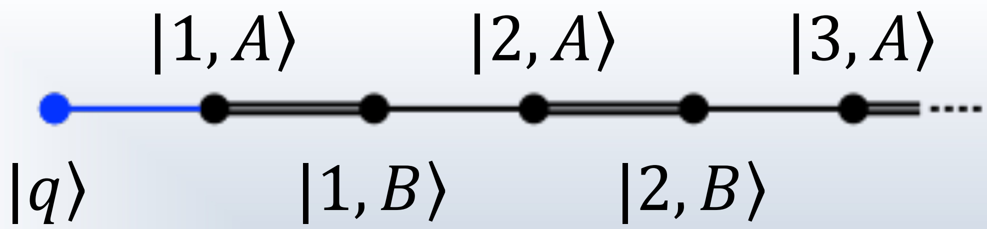
Sublattice operator:

$$\Sigma_z = P_A - P_B$$

Sublattice projectors:

$$P_A = \sum_{j=1}^{N} |j, A\rangle\langle j, A| \qquad P_B = |q\rangle\langle q| + \sum_{j=1}^{\infty} |j, B\rangle\langle j, B|$$

Chiral sublattice symmetry



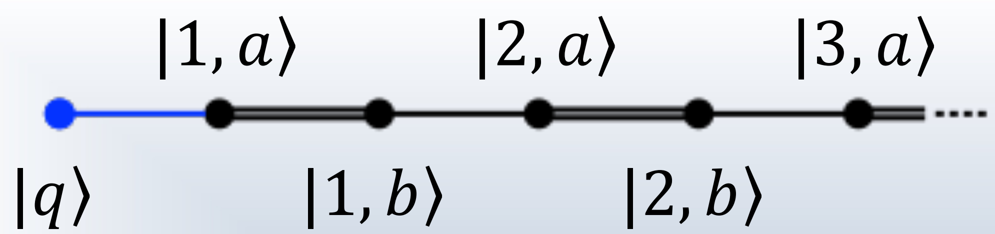
Sublattice operator:

$$\Sigma_z = P_A - P_B$$

Chiral symmetry:

$$\Sigma_z H \, \Sigma_z = -H$$

Chiral sublattice symmetry: zero energy state



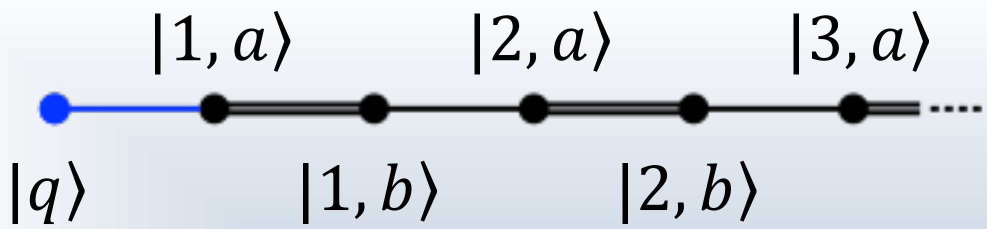
$\det H = (-)^n \det H$ implies $\det H = 0$

setting $E = 0 = \sqrt{J_1^2 + J_2^2 + 2J_1J_2\cos k}$

yields the condition $e^{ik} = -\dfrac{J_1}{J_2}$ $\begin{cases} \text{localized} & J_1 < J_2 \\ \text{anti-localized} & J_1 > J_2 \end{cases}$

$\psi_{n,B} \sim e^{ikn}$

Green's function: ordinary discrete eigenstates

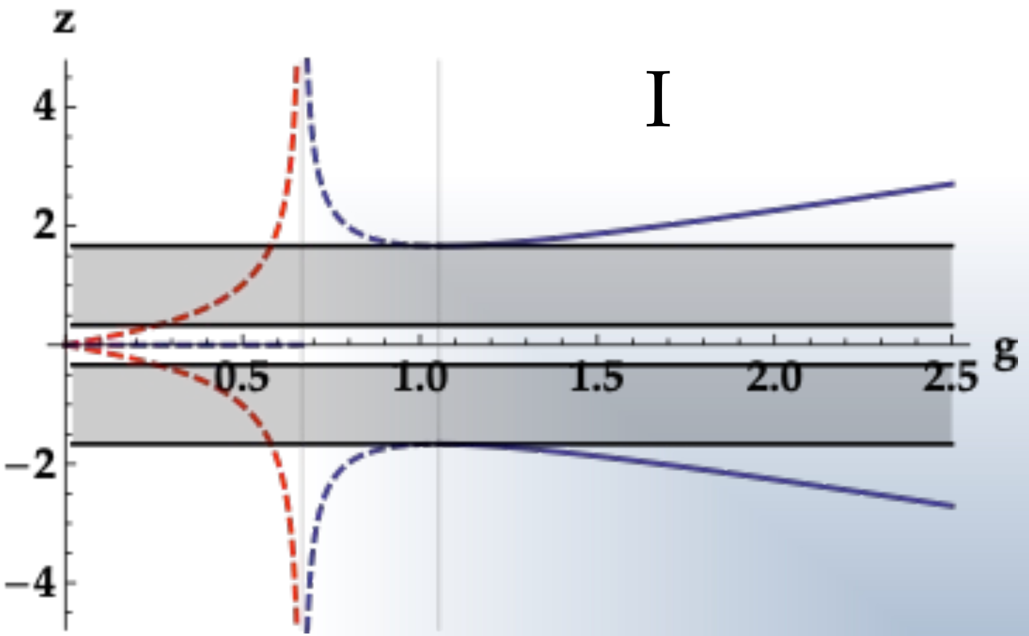


We can obtain the remaining discrete energy solutions from the Green's function at the quantum emitter

$$\left\langle q \left| \frac{1}{z-H} \right| q \right\rangle = \frac{1}{z-\Sigma(z)} \quad = \frac{2z}{z^2 - J_1^2 + J_2^2 \pm \sqrt{z^4 - 2z^2(J_1^2+J_2^2) + (J_1^2-J_2^2)^2}}$$

$$\boxed{z_\pm = \pm g \sqrt{\frac{g^2-(J_1^2-J_2^2)}{g^2-J_1^2}}} \qquad k_+ = k_- = -i \log \frac{J_1 J_2}{g^2 - J_1^2}$$

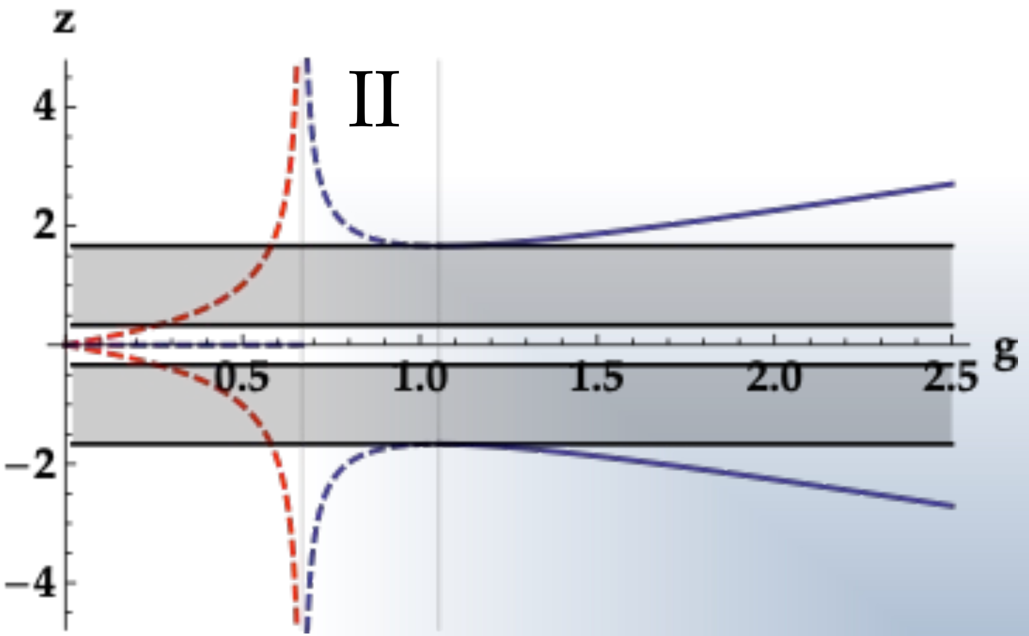
Spectrum: $J_2 > J_1$ case



two bound states

and (localized)
zero-energy mode

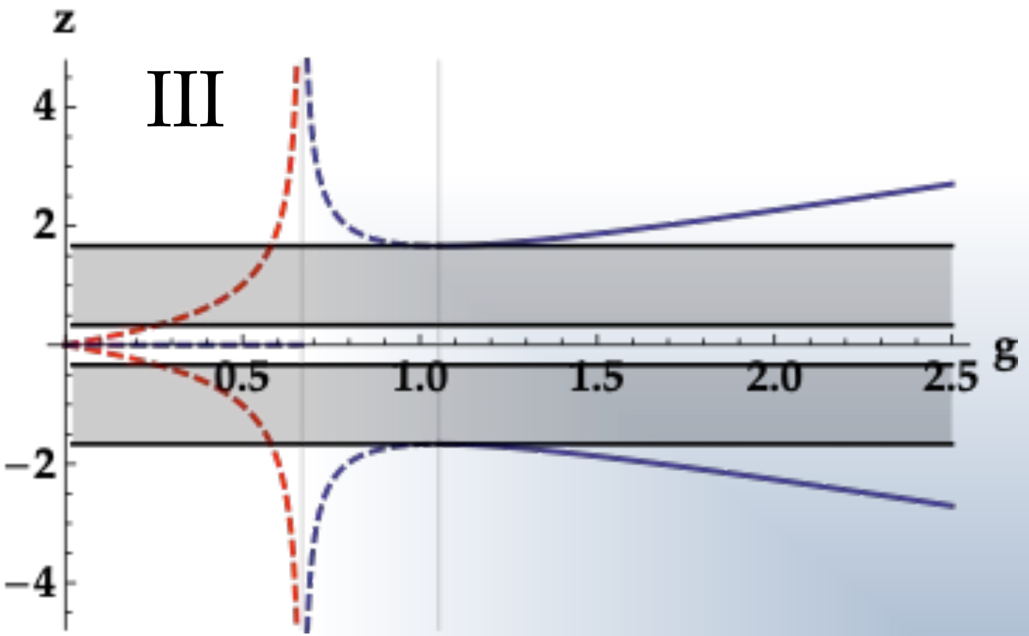
Spectrum: $J_2 > J_1$ case



two virtual states

and (localized)
zero-energy mode

Spectrum: $J_2 > J_1$ case

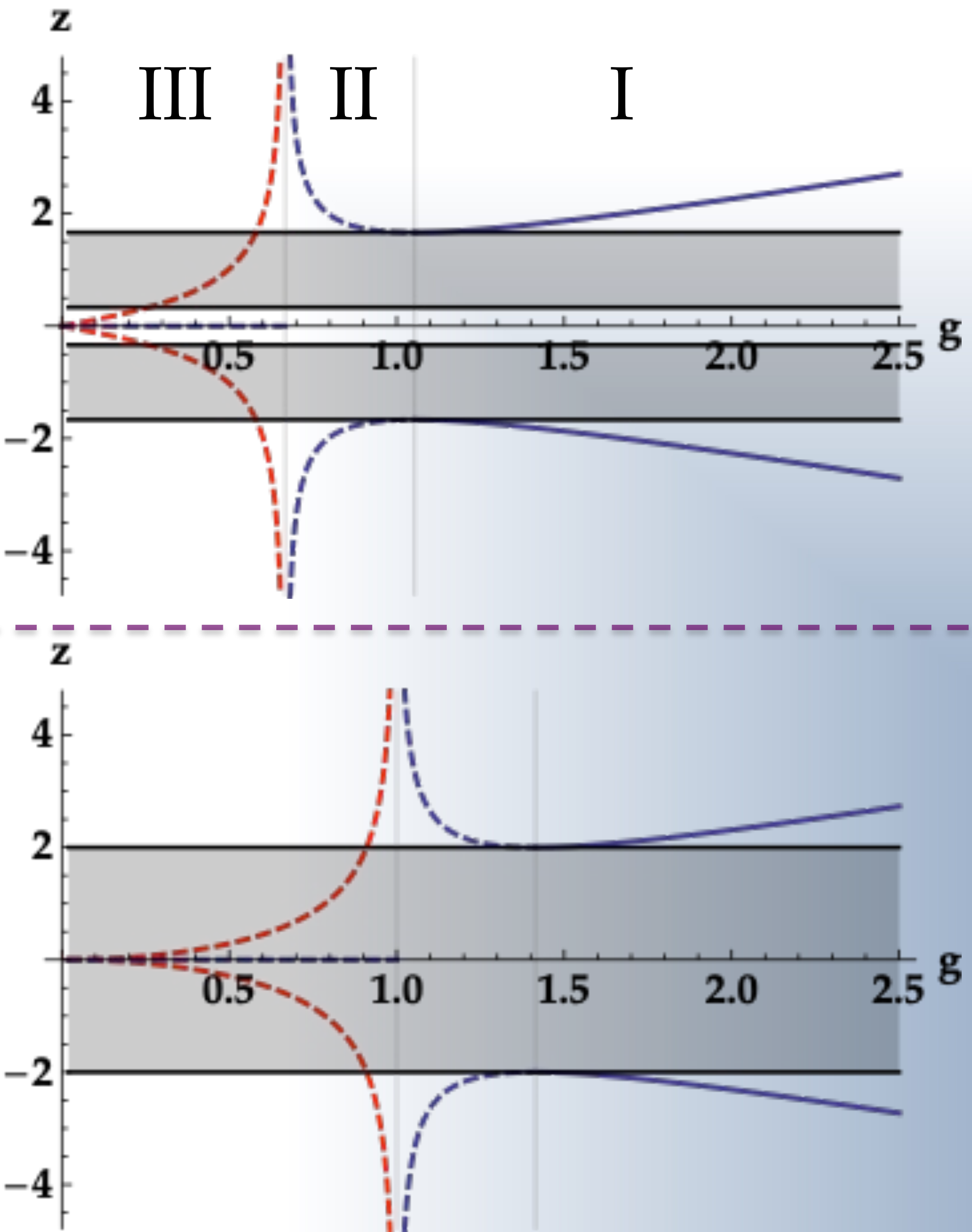


$z = E \pm i\,\Gamma$

resonance/anti-res pair

and (localized)
zero-energy mode

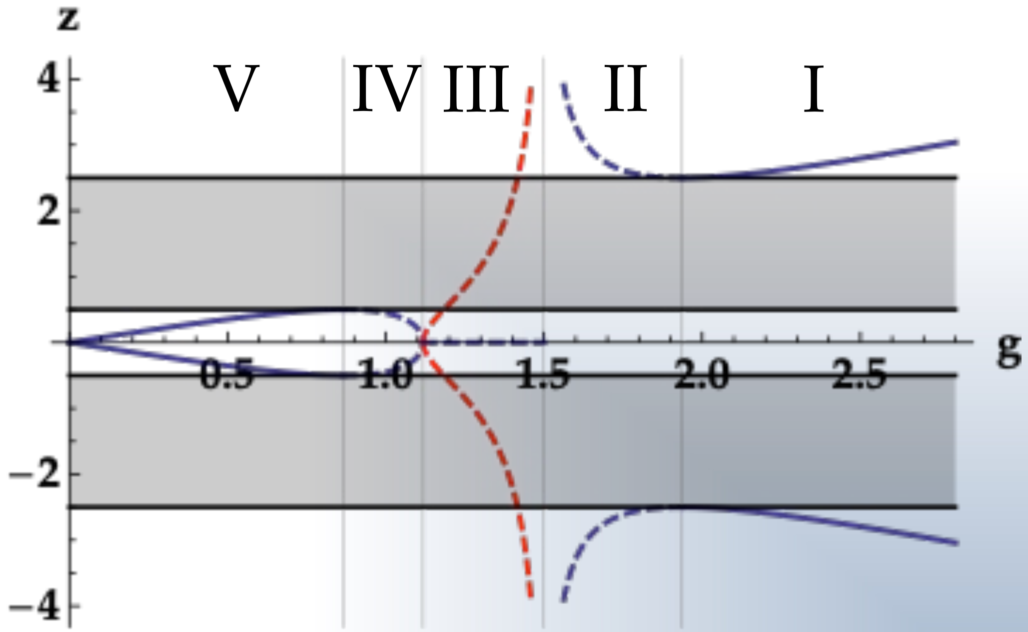
Spectrum: $J_2 > J_1$ case



and (localized) zero-energy mode

for J_2 —> J_1, this reduces to the uniform chain in an obvious way

Spectrum: $J_1 > J_2$ case

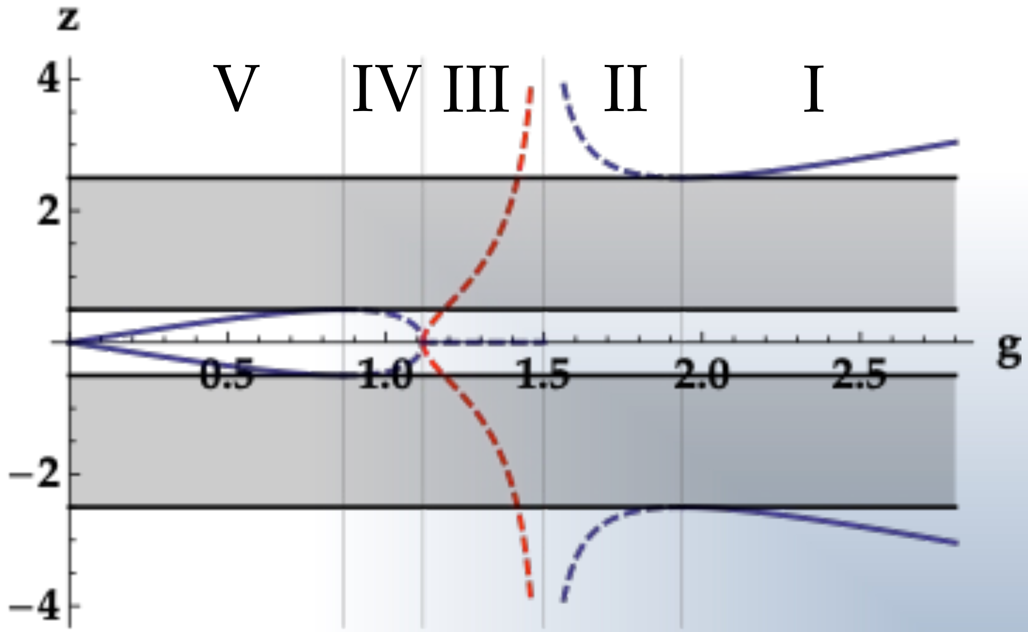


Region IV: two inner gap virtual states

Region V: two inner gap bound states

(anti-localized) zero-energy mode

Spectrum: $J_1 > J_2$ case



Region IV: two inner gap virtual states

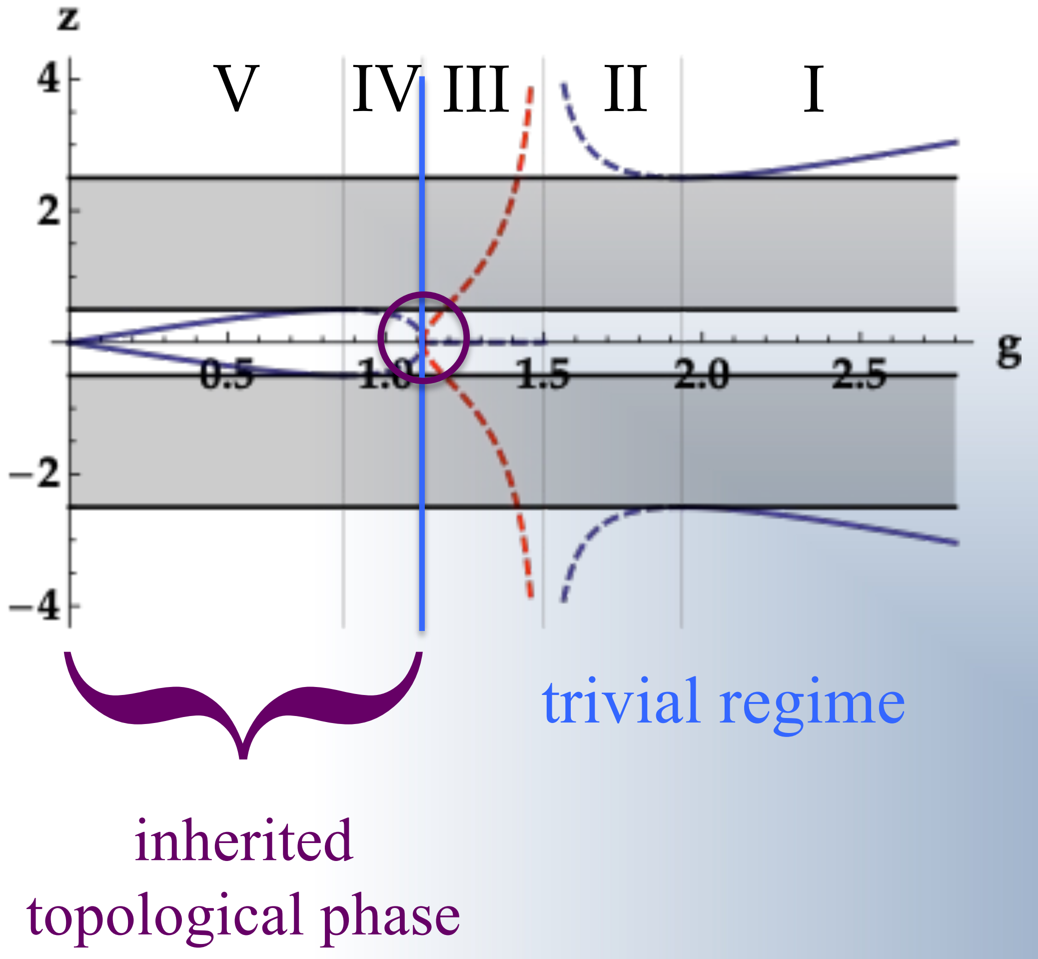
Region V: two inner gap bound states

(anti-localized) zero-energy mode

Reg. V bound states:

$$\left|\psi_\pm\right\rangle \approx \frac{1}{\sqrt{2}}\left(|q\rangle \pm |\phi_e\rangle\right)$$

Spectrum: $J_1 > J_2$ case



Region IV: two inner gap virtual states

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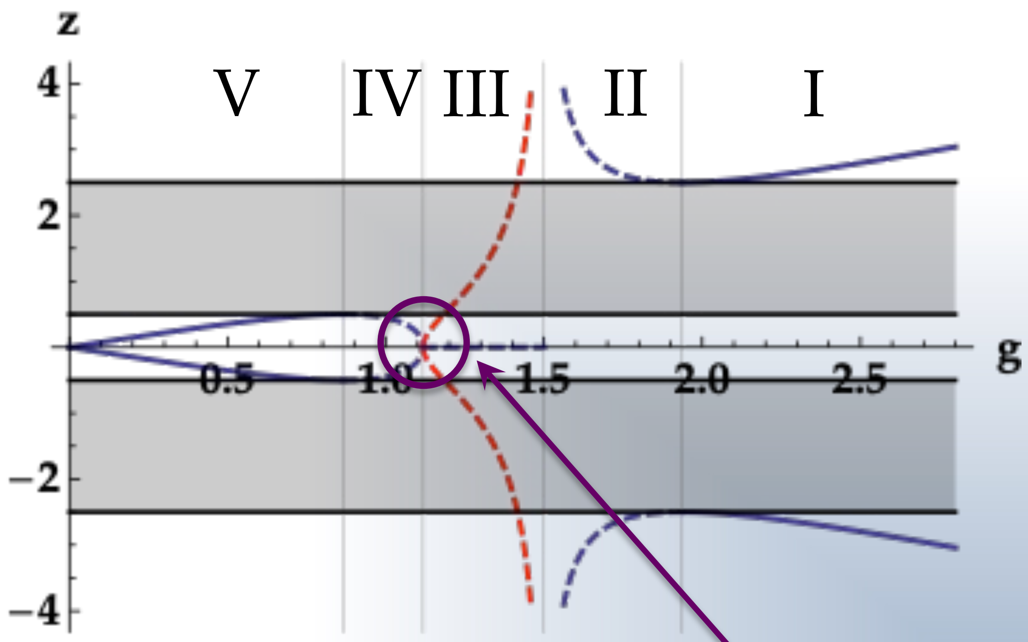
(anti-localized) zero-energy mode

Spectrum: $J_1 > J_2$ case



Region IV: two inner gap virtual states

Region V: two inner gap bound states

(anti-localized) zero-energy mode

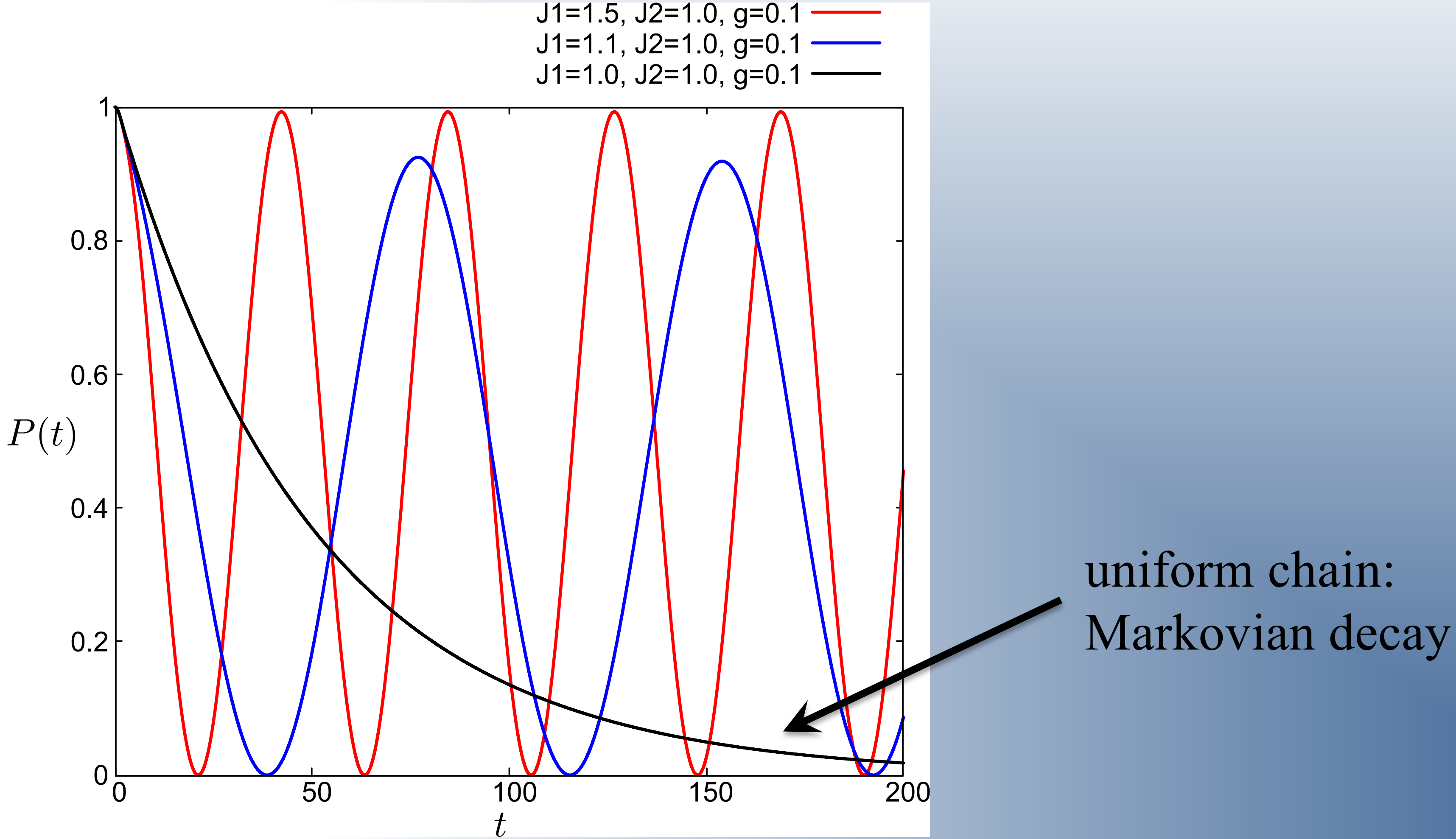
inherited topological phase

topological exceptional point

$z_{\mathrm{EP}} = 0$ $\qquad g = g_{\mathrm{EP}} \equiv \sqrt{J_1^2 - J_2^2}$

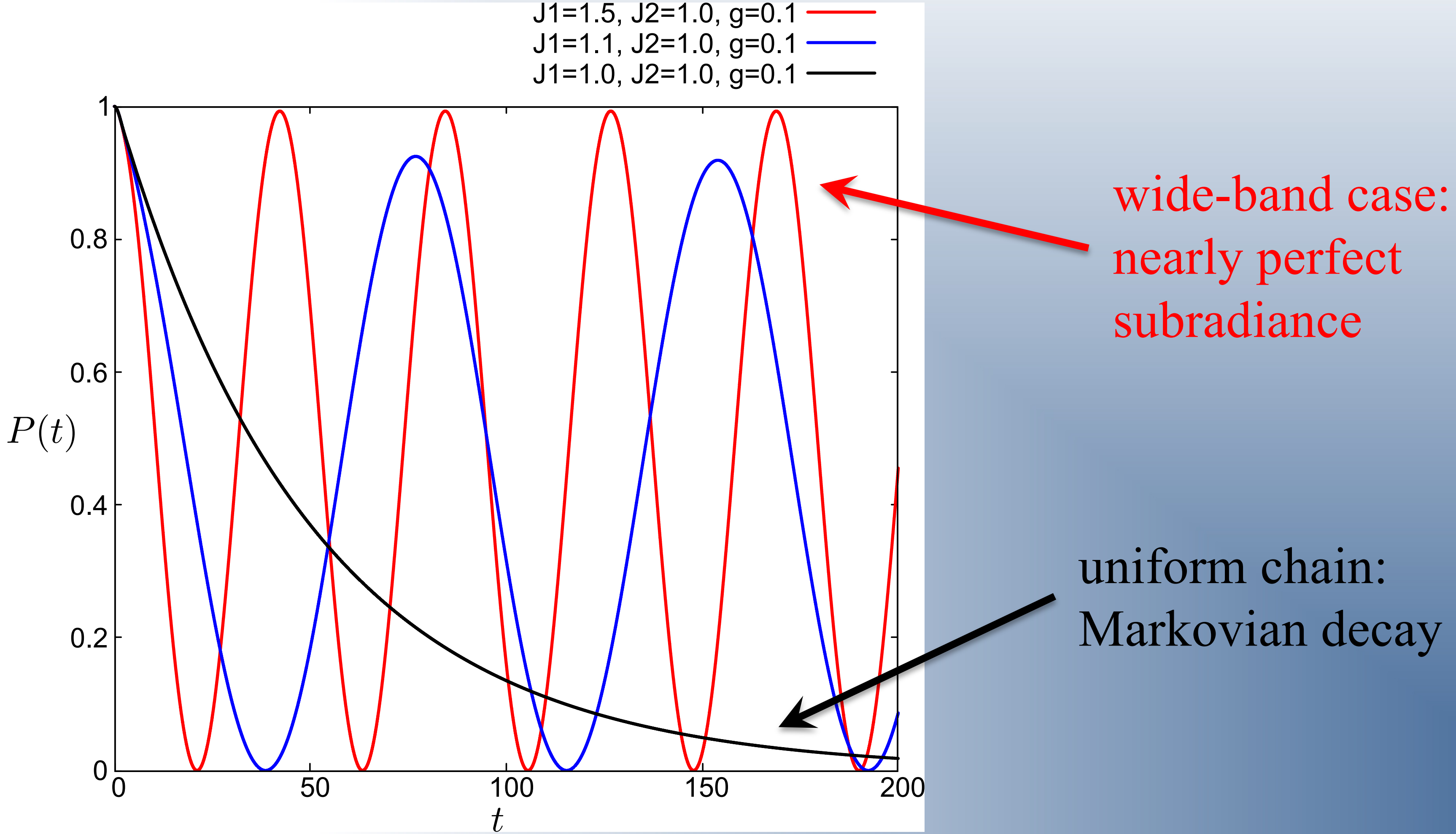
Region V dynamics

Region V dynamics (non-Markovian, non-dissipative) are totally distinct from uniform chain (Markovian decay)



Region V dynamics

Region V dynamics (non-Markovian, non-dissipative) are totally distinct from uniform chain (Markovian decay)



Conclusions (1/2)

Semi-infinite extension of the SSH model:

- left edge state becomes exact zero mode (right state vanishes)
- "in-house bulk-boundary correspondence"
  - continuum winding number predicts edge state under the *same* B.C.'s

Semi-infinite SSH model with attached quantum emitter

- non-trivial Regions IV and V appear in the topological case of the bare chain
- weak-coupling bound states in Region V are hybridizations between the quantum emitter and edge state

Numerical study (2-D model): H. Zhang, et al, PRA **105**, 053703 (2022)

Conclusions (2/2)

Non-Markovian Dynamics at the EP:

- early time behavior: $P(t) \sim (1 - C_2 t + C_3 t^2)^2$
- late time: $P(t) \sim 1/t^3$

previous paper: S. G. and K. Noba, PRA **104**, 062215 (2021)

(Reveals that the edge state can influence PT-symmetry breaking in a model with a local PT-symmetric potential)