Universal Fluctuations in 2D Anderson Localization

Jiangbin Gong

Department of Physics & Centre for Quantum Technologies

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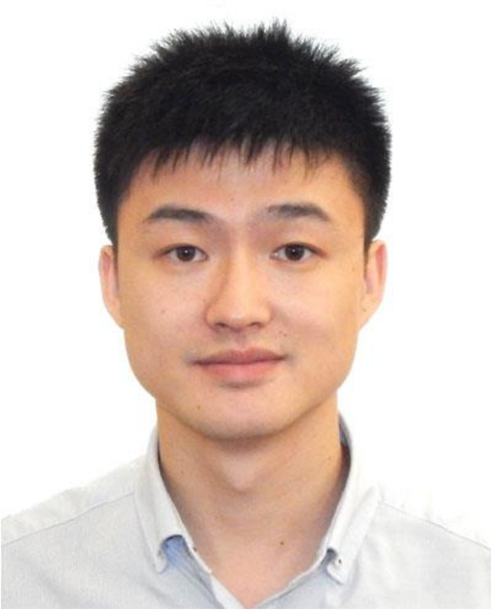
Main reference

Sen Mu, Jiangbin Gong, Gabriel Lemarié, Phys. Rev. Lett. 132, 046301 (2024)



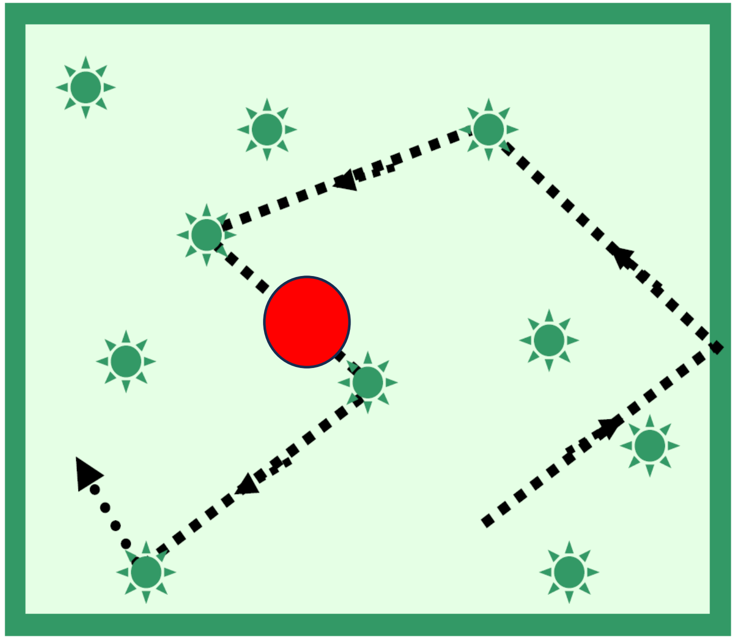
Sen Mu (NUS)

Gabriel Lemarié (CNRS & NUS)

Outline

- Warm up on Anderson localization
- Kardar-Parisi-Zhang (KPZ) & Directed-Polymer Physics
- Fluctuation growth exponent of 1/3
- Correction to exponential localization profile in 2D
- Optimal paths and the wandering exponent of 2/3

A particle moving in a in random medium



Diffusion for a classical particle,

$$\langle r^2(t)\rangle \sim t$$

Localization can happen for a quantum particle,

$$\langle r^2(t)\rangle \to \text{constant. as } t \to \infty.$$

Question: features in the fluctuations of localized quantum wavepackets?

Anderson Localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

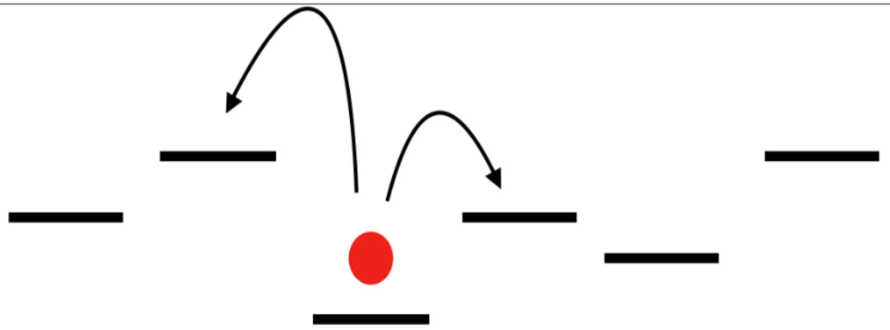
This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

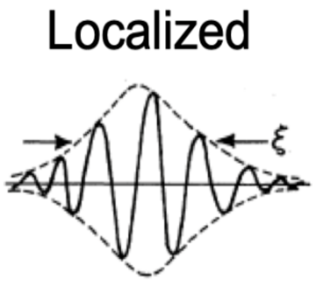
A simple model

- hopping from site to site on a lattice
- random on-site potentials

***Nobel Prize in Physics 1977**, "for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems"*

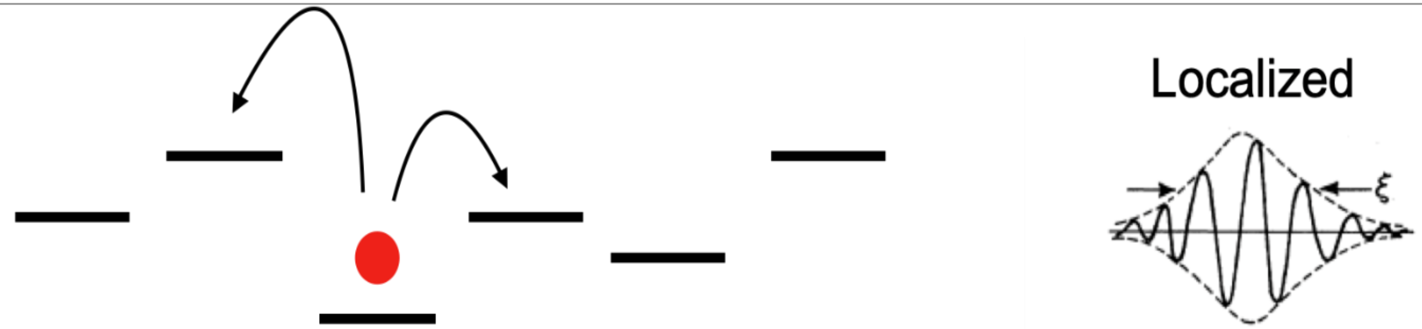
Anderson Localization





$$H = \sum_{\langle i,j \rangle} t\,|i\rangle\langle j| + \sum_{i} \epsilon_i\,|i\rangle\langle i| \qquad \epsilon_i \in [-W, W]$$

Anderson Localization

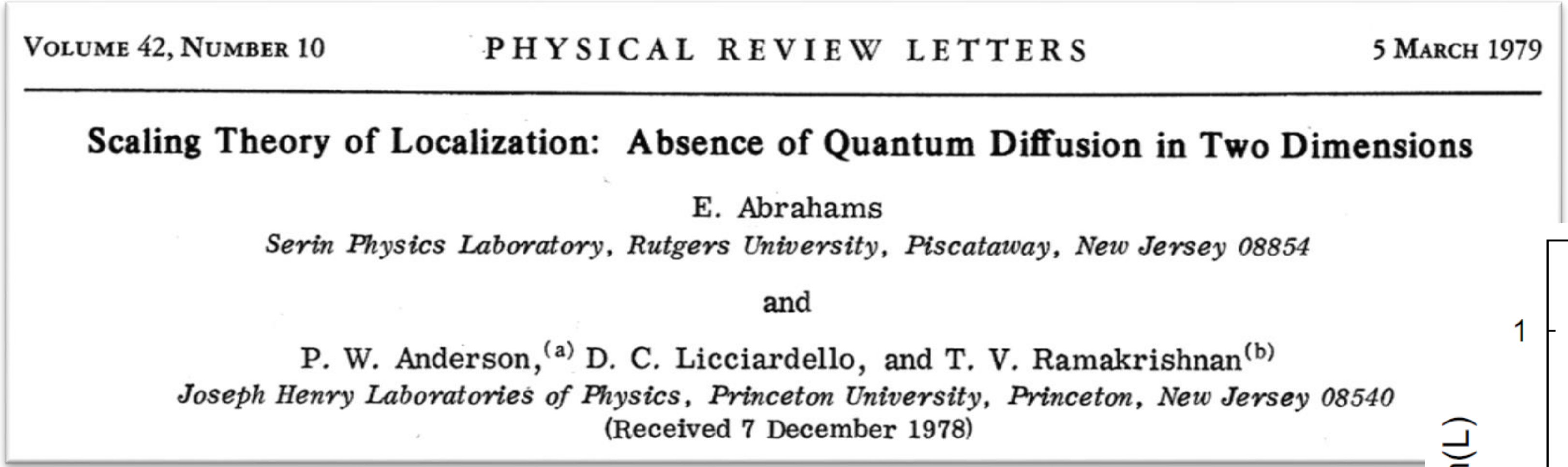


$$H = \sum_{\langle i,j \rangle} t\,|i\rangle\langle j| + \sum_{i} \epsilon_i\,|i\rangle\langle i| \qquad \epsilon_i \in [-W, W]$$

$\frac{W}{K}$: typical spacing of random energies of sites directly connected to a given site. $t \ll \frac{W}{K}$, hybridization suppressed, thus localization

$$\boldsymbol{\psi(r, time \to \infty) \sim e^{-(r-r_0)/\xi}}$$

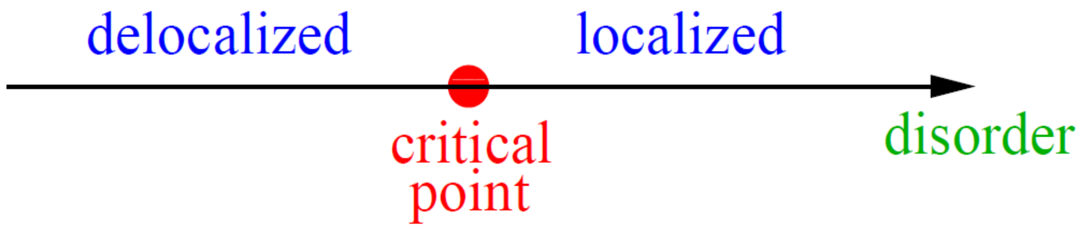
Role of dimensionality on Anderson localization

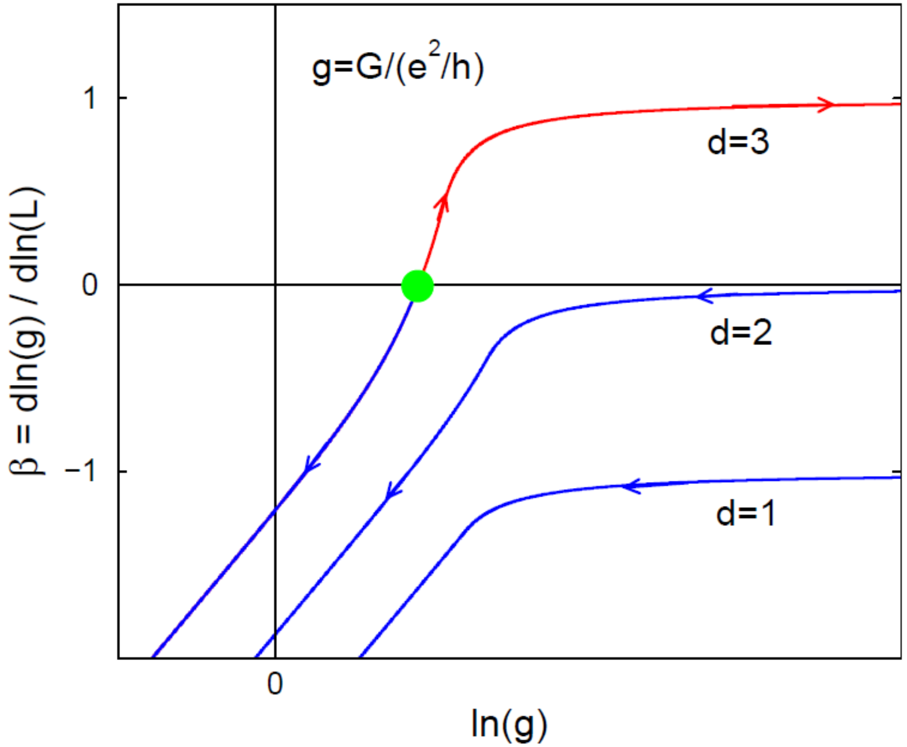

VOLUME 42, NUMBER 10 PHYSICAL REVIEW LETTERS 5 MARCH 1979

Scaling Theory of Localization: Absence of Quantum Diffusion in Two Dimensions

E. Abrahams
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08854
and
P. W. Anderson,[a] D. C. Licciardello, and T. V. Ramakrishnan[b]
Joseph Henry Laboratories of Physics, Princeton University, Princeton, New Jersey 08540
(Received 7 December 1978)

$d > 2$: Anderson metal-insulator transition





Rev. Mod. Phys. 80, 1355 (2008)
50 years of Anderson localization, World Scientific, 2010

Experiments on Anderson localization

❑Spin diffusion

- ❖ Feher, G., Phys. Rev. 114, 1219 (1959);
- ❖ Feher, G. & Gere, E. A., Phys. Rev. 114, 1245 (1959).

❑Light

- Wiersma, D.S., Bartolini, P., Lagendijk, A. & Righini R., *Nature* 390, 671-673 (1997).
- Scheffold, F., Lenke, R., Tweer, R. & Maret, G., *Nature* 398,206-270 (1999).
- Schwartz, T., Bartal, G., Fishman, S. & Segev, M., *Nature* 446, 52-55 (2007).

❑Microwaves

- o Dalichaouch, R., Armstrong, J.P., Schultz, S.,Platzman, P.M. & McCall, S.L., *Nature* 354, 53, (1991).
- o Chabanov, A.A., Stoytchev, M. & Genack, A.Z., *Nature* 404, 850, (2000).
- o Pradhan, P., Sridar, S, PRL 85, (2000)

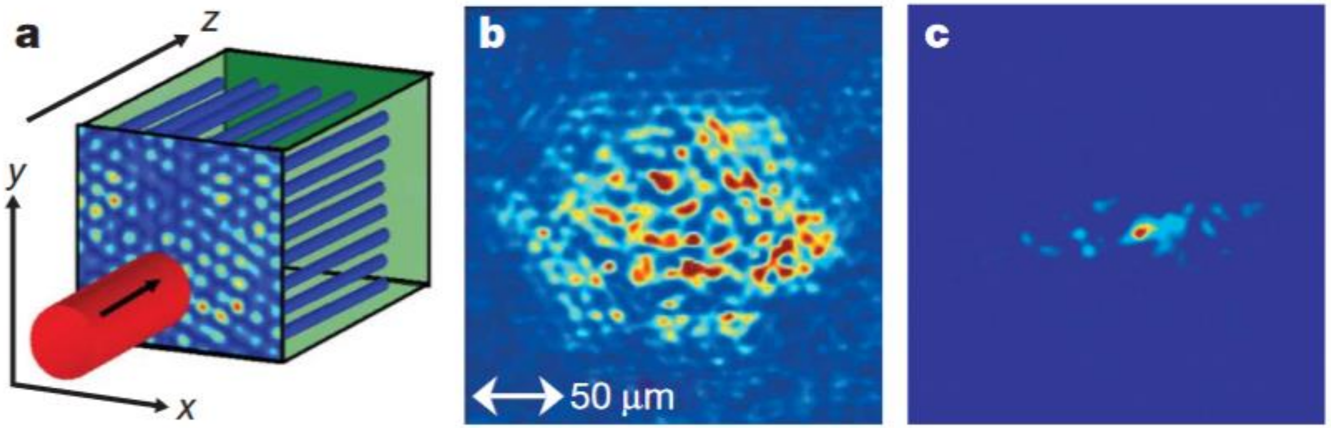
❑Cold atoms

- ➢ F. L. Moore, J. C. Robinson, C. F. Bharucha, Bala Sundaram, and M. G. Raizen, *PRL* 75, 4598
- ➢ Roati, G., D’Errico, C., Fallani, L. et al. *Nature* 453,895 (2008)
- ➢ Billy, J., Josse, V., Zuo, Z. et al. *Nature* 453, 891 (2008)

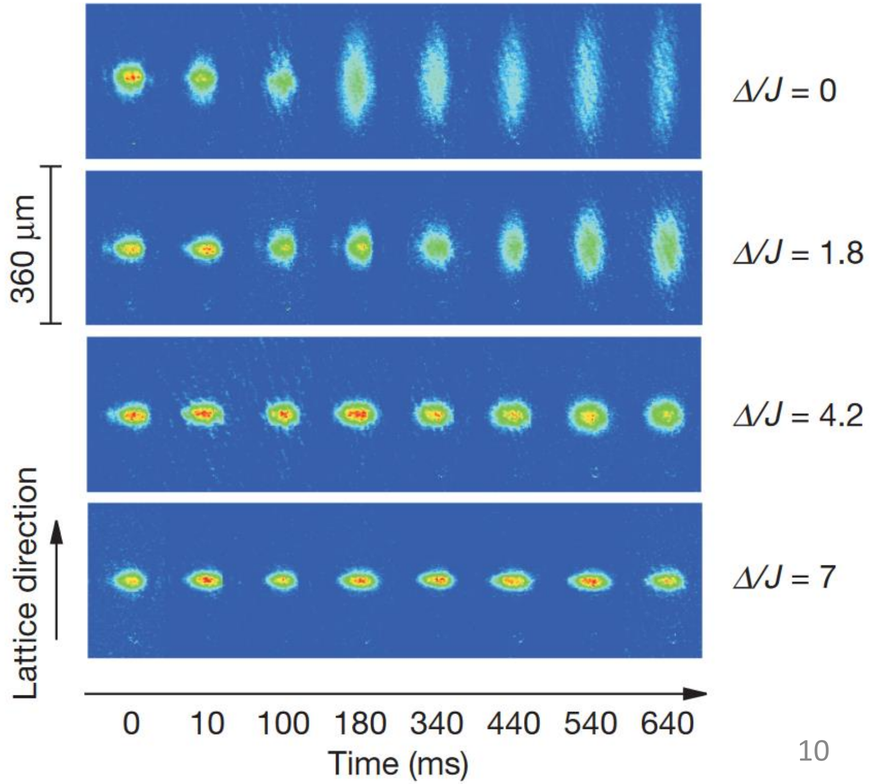
Anderson localization on physical simulators

Simulator platforms, such as **cold atoms, light waves or ultrasounds** offers an in-situ and dynamical depiction of localization.

Nature, 446, 52 (2007)



Nature, 453, 895 (2008)



Kardar-Parisi-Zhang (KPZ) Equation [(1+1)D]

VOLUME 56, NUMBER 9 PHYSICAL REVIEW LETTERS 3 MARCH 1986

Dynamic Scaling of Growing Interfaces

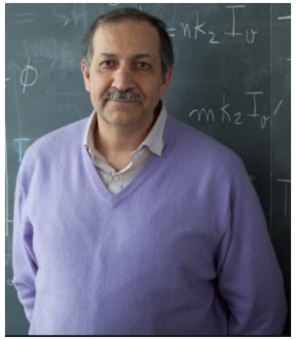
Mehran Kardar
Physics Department, Harvard University, Cambridge, Massachusetts 02138

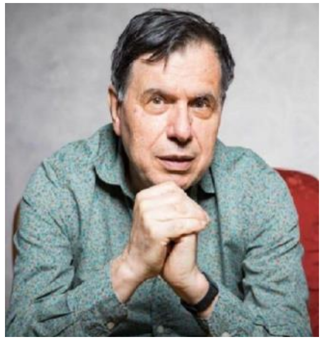
Giorgio Parisi
Physics Department, University of Rome, I-00173 Rome, Italy

and

Yi-Cheng Zhang
Physics Department, Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1985)

A model is proposed for the evolution of the profile of a growing interface. The deterministic growth is solved exactly, and exhibits nontrivial relaxation patterns. The stochastic version is studied by dynamic renormalization-group techniques and by mappings to Burgers's equation and to a random directed-polymer problem. The exact dynamic scaling form obtained for a one-dimensional interface is in excellent agreement with previous numerical simulations. Predictions are made for more dimensions.





$$\frac{\partial}{\partial t}h(x,t)=\frac{1}{2}\lambda\left(\frac{\partial}{\partial x}h(x,t)\right)^2+\nu\frac{\partial^2}{\partial x^2}h(x,t)+\sqrt{D}\eta(x,t)$$

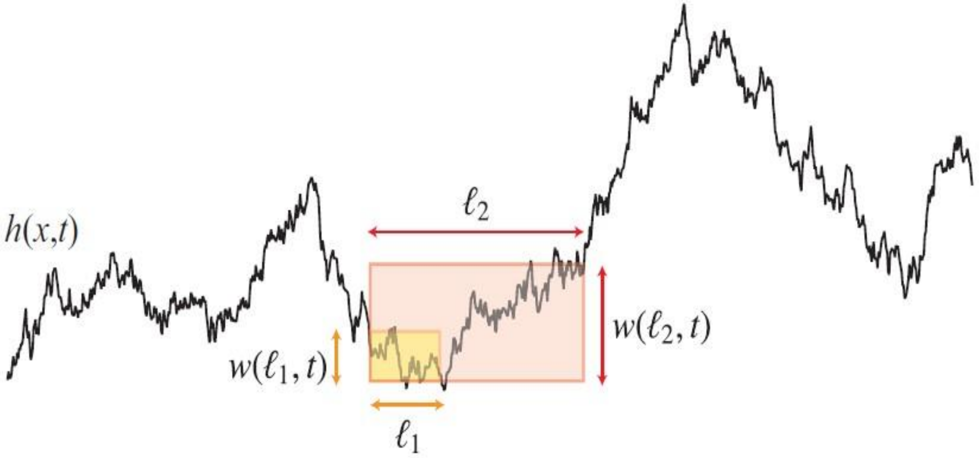
KPZ Universality Class

VOLUME 52, NUMBER 19 PHYSICAL REVIEW LETTERS 7 MAY 1984

Dynamic Scaling for Aggregation of Clusters

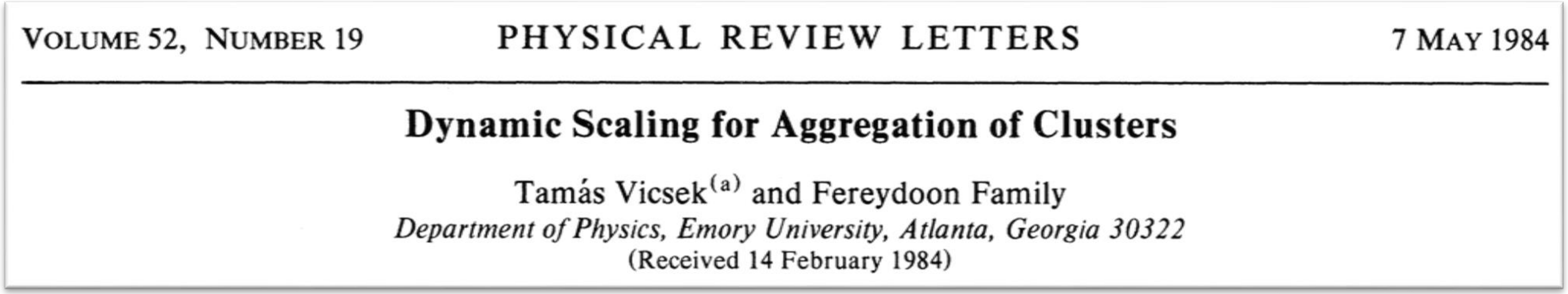
Tamás Vicsek[a] and Fereydoon Family
Department of Physics, Emory University, Atlanta, Georgia 30322
(Received 14 February 1984)

growing interface



Physica A 504, 77-105 (2018)

KPZ Universality Class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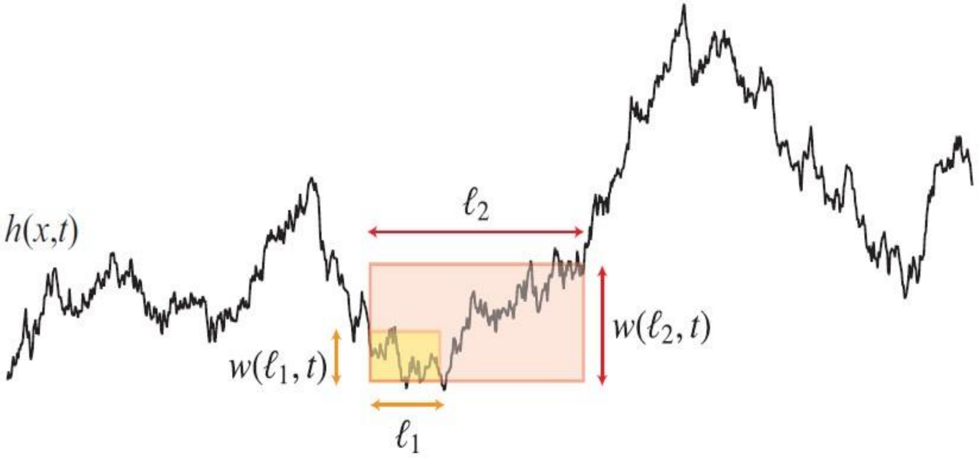

Family-Vicsek Scaling

$$w(\ell, t) \sim t^{\beta} F_w(\ell t^{-1/z}) \sim \begin{cases} \ell^{\alpha} & \text{for } \ell \ll \xi(t), \\ t^{\beta} & \text{for } \ell \gg \xi(t), \end{cases}$$

$$\xi(t) \sim t^{1/z} \; (z \equiv \alpha/\beta)$$

Physica A 504, 77-105 (2018)

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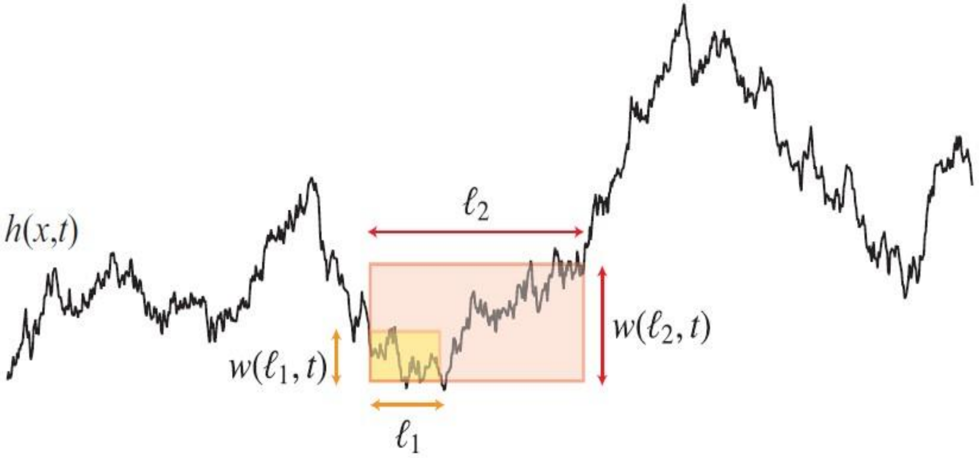

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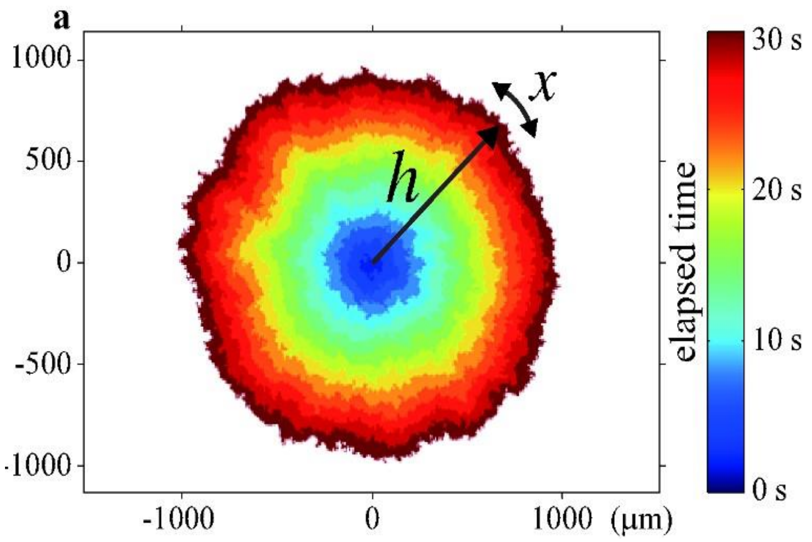
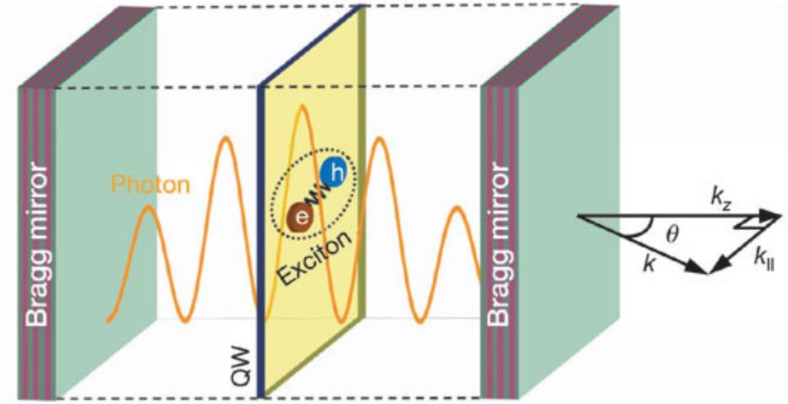
$$\xi(t) \sim t^{1/z} \quad (z \equiv \alpha/\beta)$$

(1+1)D KPZ class: $\alpha = 1/2, \quad \beta = 1/3, \quad z = 3/2$

KPZ Universality Class

A class of systems showing the same statistical properties on large scales, i.e. fluctuations, characterized by **universal scaling exponents** and **probability distribution**.





Takeuchi, Sano PRL 104, 230601(2010)

Takeuchi, Sano, Sasamoto and Spohn, Scientific Reports, 1,34 (2011)

Liquid crystals

Fontaine, Q., Squizzato, D., Baboux, F. et al. Nature 608, 687 (2022)

Excitons

M. Ljubotina, M. Žnidarič, and T. Prosen, PRL. 122, 210602 (2019)

Spin chains

Cole-Hopf mapping

KPZ equation:

$$\partial_t h = \nu \partial_x^2 h + \frac{\lambda_0}{2} (\partial_x h)^2 + \eta(x,t)$$

define:

$$Z(x,t) = e^{\frac{\lambda_0}{2\nu} h(x,t)}$$

$$\lambda_0 h(x,t) = T \ln Z(x,t)$$

$$T = 2\nu$$

it satisfies:

$$\partial_t Z = \frac{T}{2} \partial_x^2 Z - \frac{V(x,t)}{T} Z$$

$$\lambda_0 \eta(x,t) = -V(x,t)$$

Directed polymer (DP)

$$Z(x,t) = \int_{(0,0)}^{(x,t)} Dt' e^{-\int_0^t dt' [\frac{\nu}{2}(\frac{dx'}{dt'})^2 - \eta(x',t')]}$$ in continuum

Directed polymer (DP)

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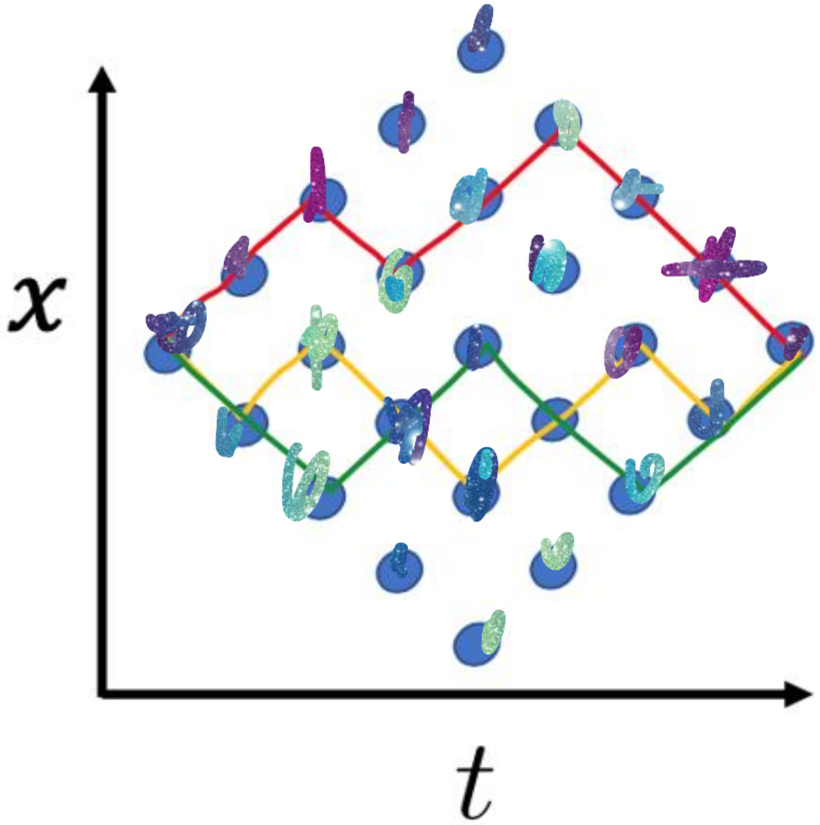
Discretized version in the lattice setting



$$Z = \sum_{D\mathcal{P}} \prod_{\boldsymbol{r}_j \in D\mathcal{P}} e^{-\beta W(\boldsymbol{r}_j)}$$ on lattice

χ is Tracy-widom distribution

$$F = -\ln Z,\ F = vt + \Gamma t^{1/3}\chi$$

As exact results from the replica methods and Bethe ansatz.

Calabrese, Le Doussal, A. Rosso EPL 90 20002 (2010)
Calabrese, Le Doussal, PRL 106 250603 (2011) and J. Stat. Mech. P06001 (2012) Gueudre, Le Doussal, EPL 100 26006 (2012), etc.

Anderson Localization in discretized time evolution

Continuous time evolution (Anderson model):

$$|\psi_t\rangle = \hat{U}|\psi_{t=0}\rangle = e^{-iHt}|\psi_{t=0}\rangle$$

Discrete time evolution:

$$|\psi_{t+1}\rangle = \hat{U}|\psi_t\rangle = e^{-iKV(\hat{\boldsymbol{k}})}e^{-iW(\hat{\boldsymbol{r}})}|\psi_t\rangle$$

- The kinetic operator is parameterized by the hopping strength K,

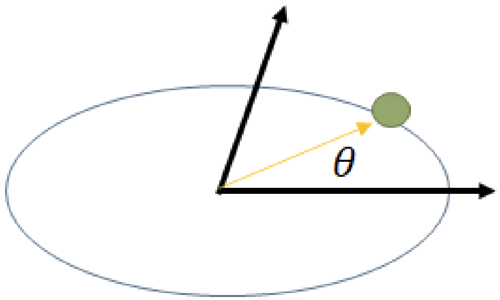
$$V(\hat{\boldsymbol{k}}) = \sum_i \cos k_i$$

- The on-site phases are independent, identically distributed uniformly in the interval

$$W(\hat{\boldsymbol{r}}) \in [-\pi, \pi]$$

Quantum Kicked Rotor (QKR)

Fishman, Grempel, and Prange
PRL 49, 509 (1982)



$$H = \frac{p^2}{2} + k \cos x \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

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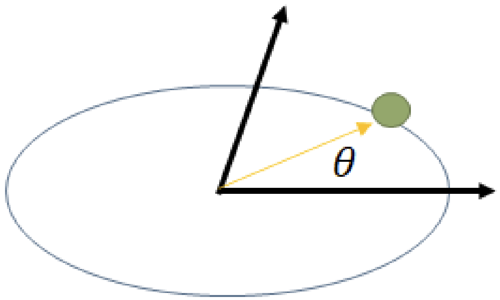

Momentum: $p = -i\partial_\theta$ and the evolution (Floquet) operator:

$$U = U(T,0) = \exp\left(-\frac{i}{\hbar}\frac{p^2 T}{2}\right)\exp\left(-\frac{i}{\hbar}\,k\cos x\right)$$

$$H = \frac{p^2}{2} + k\cos x \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

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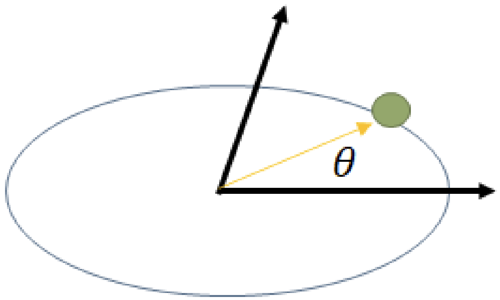

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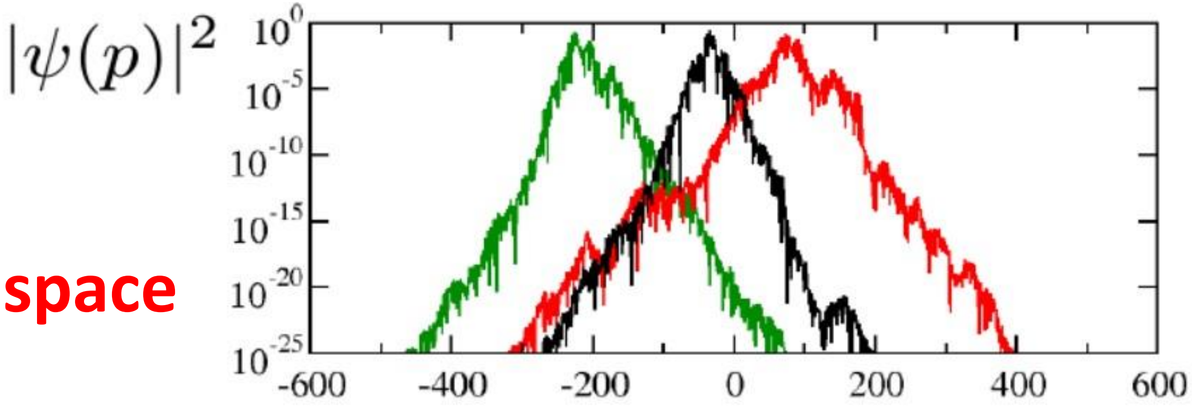
$$H = \frac{p^2}{2} + k\cos x \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$U|\phi_j\rangle = \exp\left(-\frac{iE_jT}{\hbar}\right)|\phi_j\rangle$$



Dynamical localization, i.e,

Anderson localization in momentum space

KPZ universality from unitary dynamics

Discrete time evolution on a lattice,

$$|\psi_{t+1}\rangle = \hat{U}|\psi_t\rangle = e^{-iJV(\hat{\boldsymbol{k}})} e^{-iW(\hat{\boldsymbol{r}})}|\psi_t\rangle$$

$V(\hat{\boldsymbol{k}}) = \sum_i \cos k_i \qquad W(\hat{\boldsymbol{r}}) \in [-\pi, \pi]$

KPZ universality from unitary dynamics

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$V(\hat{\boldsymbol{k}}) = \sum_i \cos k_i \qquad W(\hat{\boldsymbol{r}}) \in [-\pi, \pi]$

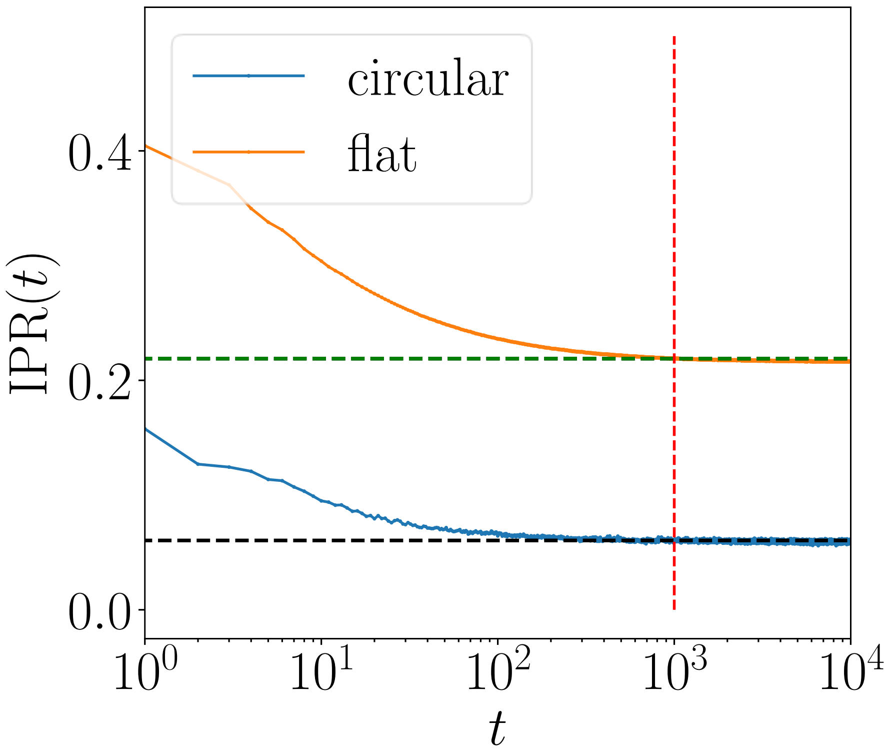
Initial Conditions ($t = 0$):

$$\psi_0(x, y) = \delta_{x,0}\delta_{y,0}, \quad \text{circular;}$$
$$\psi_0(x, k_y) = \frac{1}{\sqrt{N}}\delta_{x,0}\delta_{k_y,0}, \quad \text{flat.}$$

Localized wave packets in two dimensions

Let us estimate the localization time scale with inverse participation ratio (IPR),

$$\mathrm{IPR}(t) = \sum_r |\psi_t(r)|^4$$



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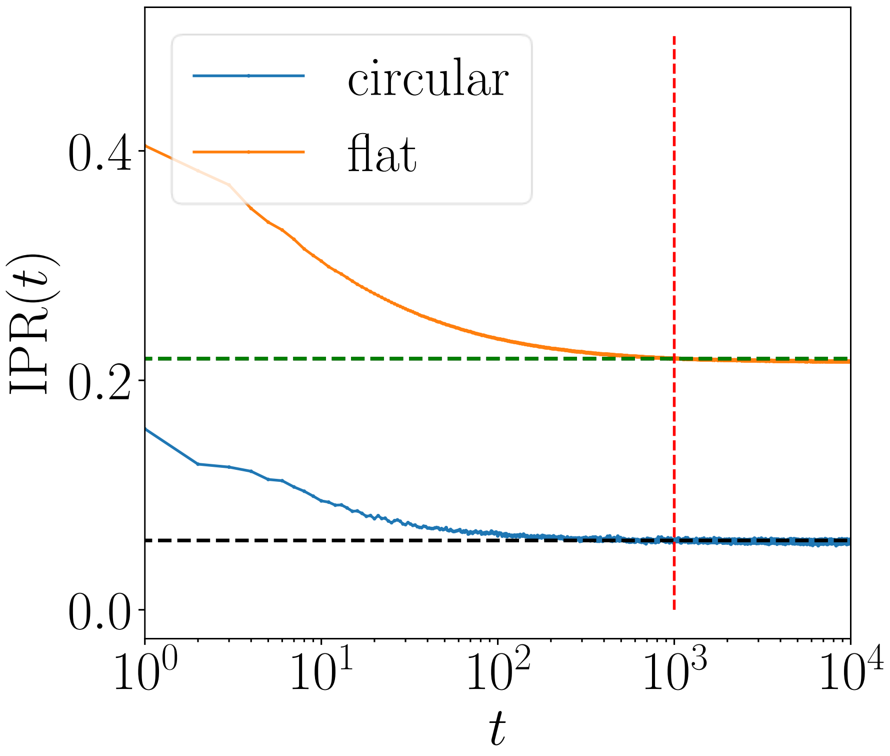
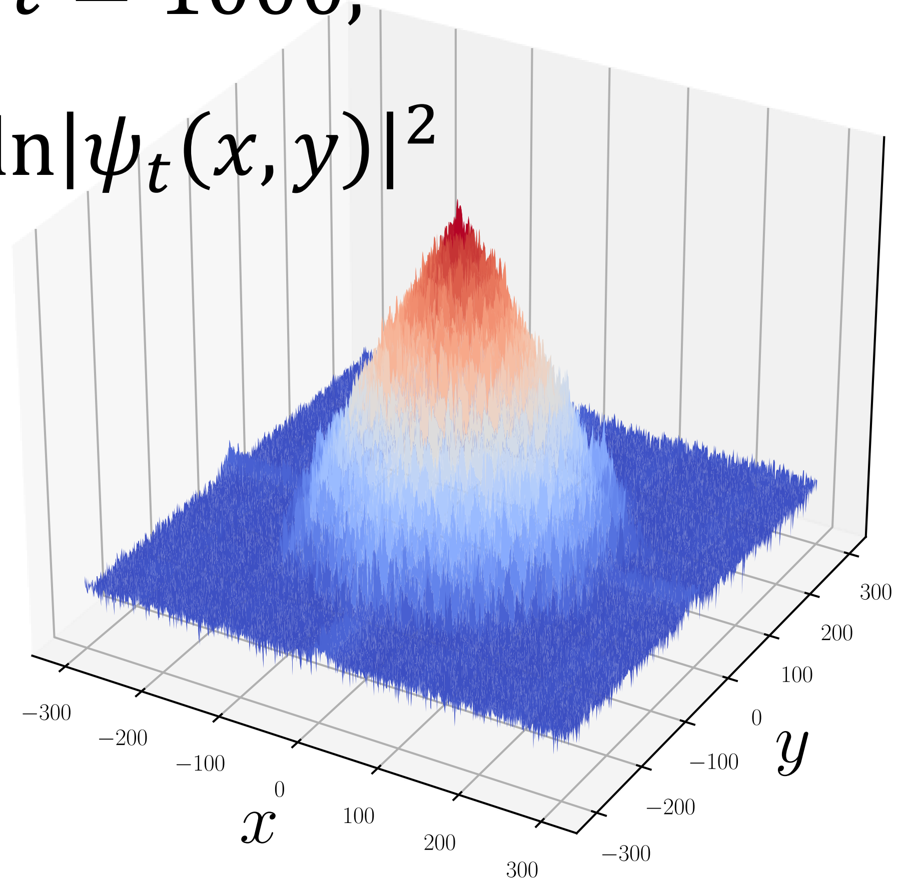


at $t = 1000$,

$-\ln|\psi_t(x, y)|^2$



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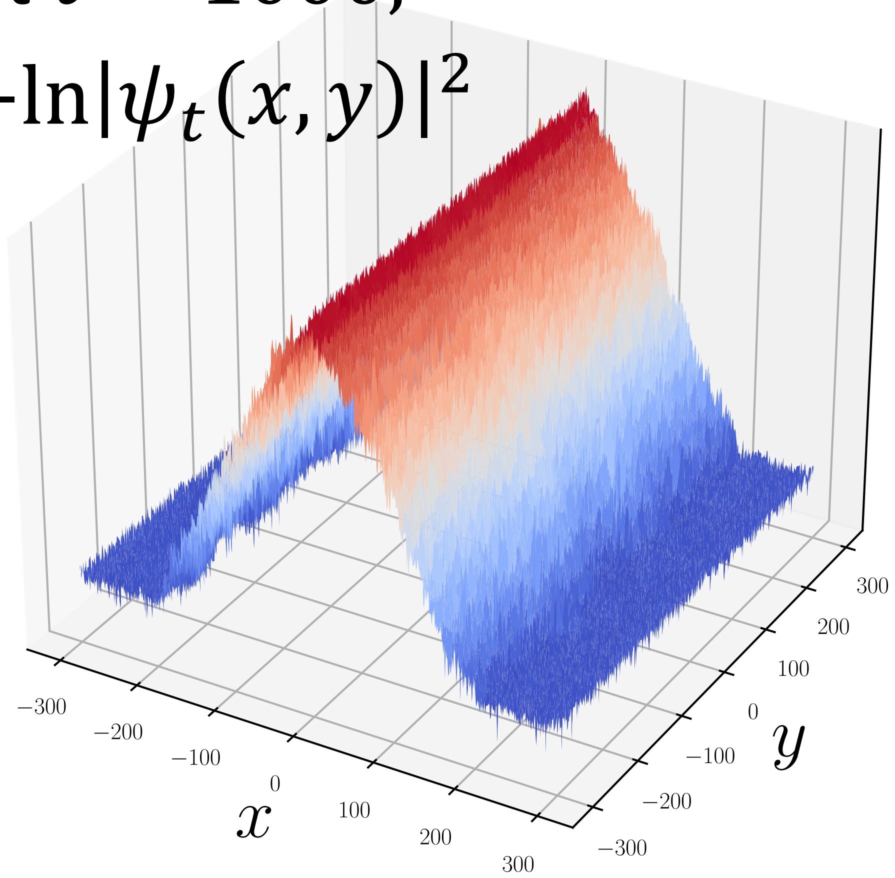

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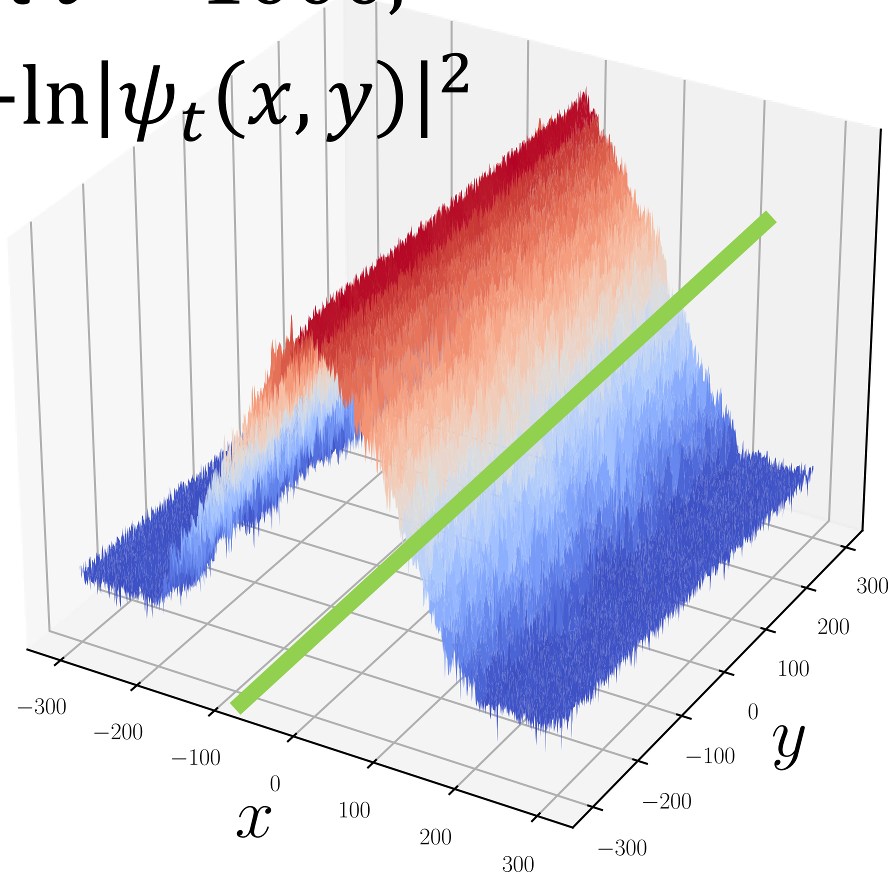

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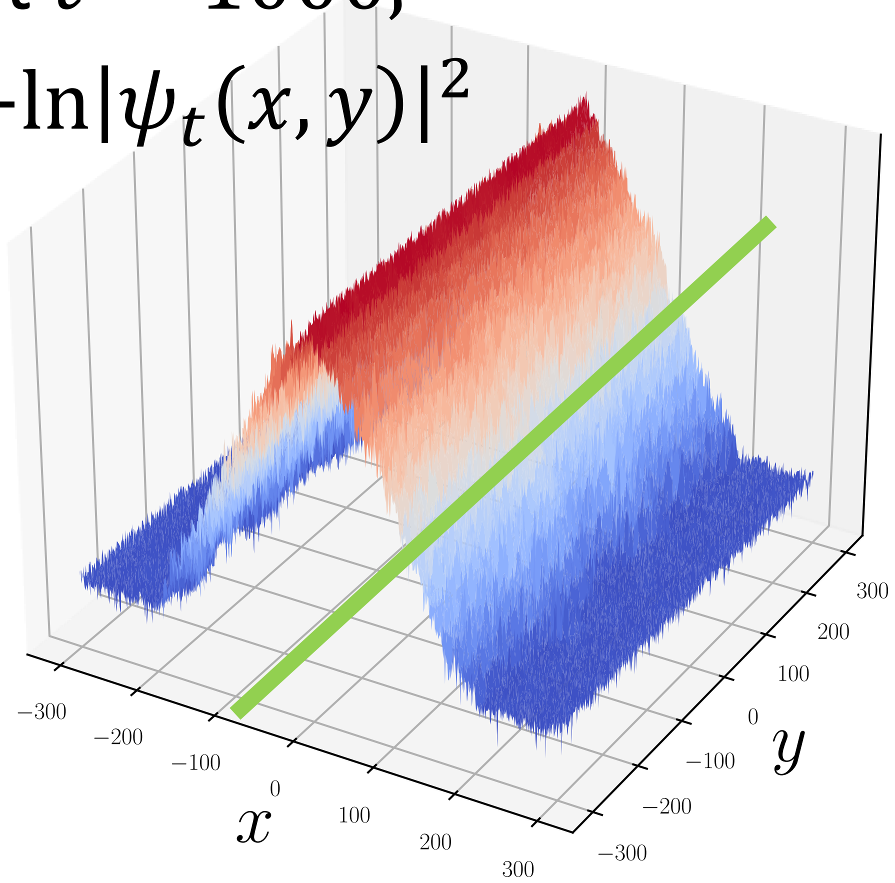
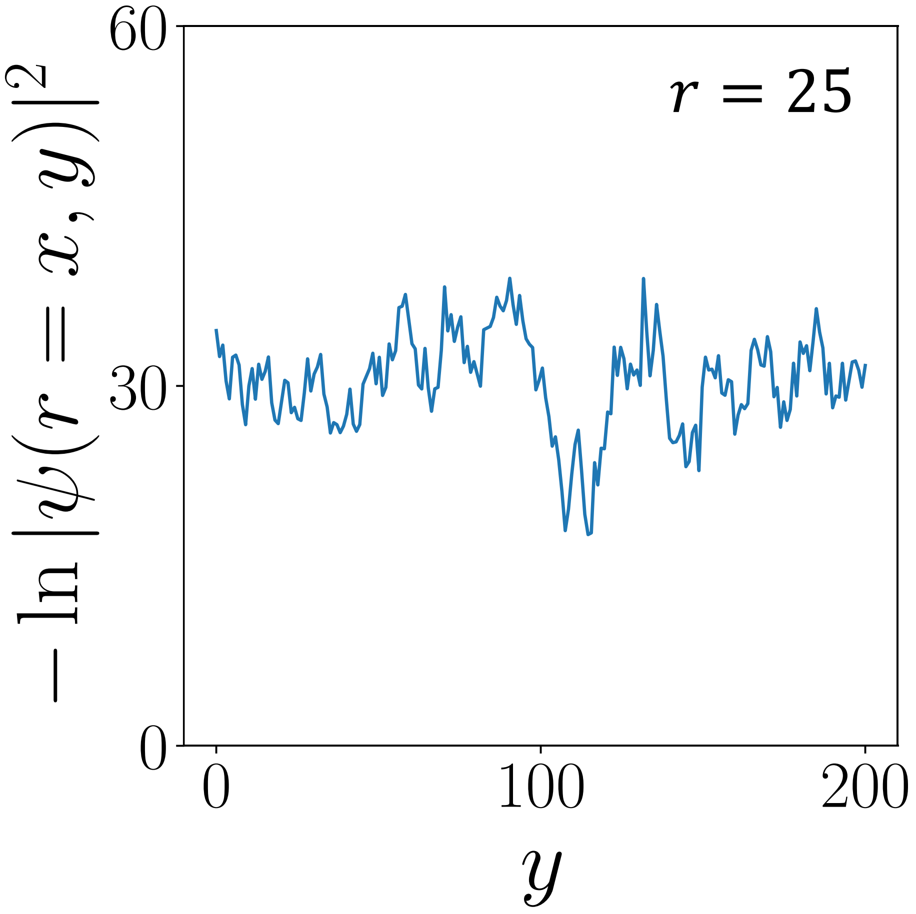
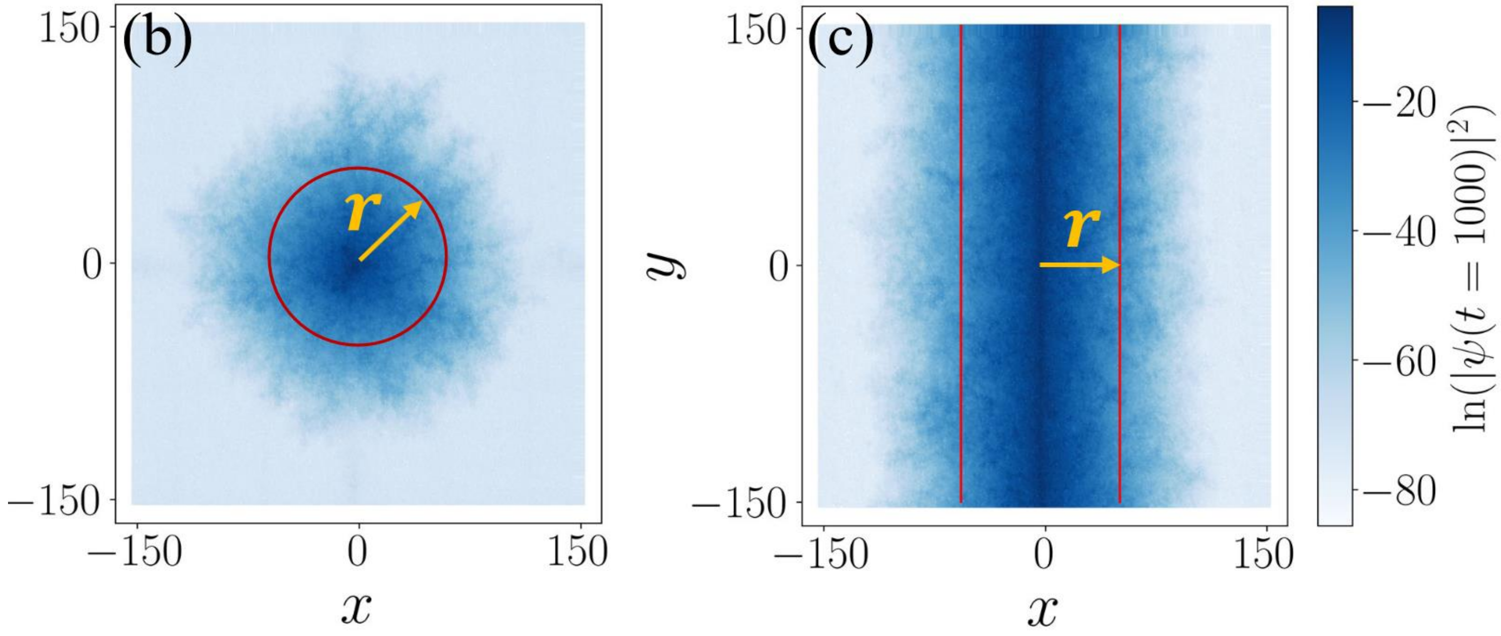
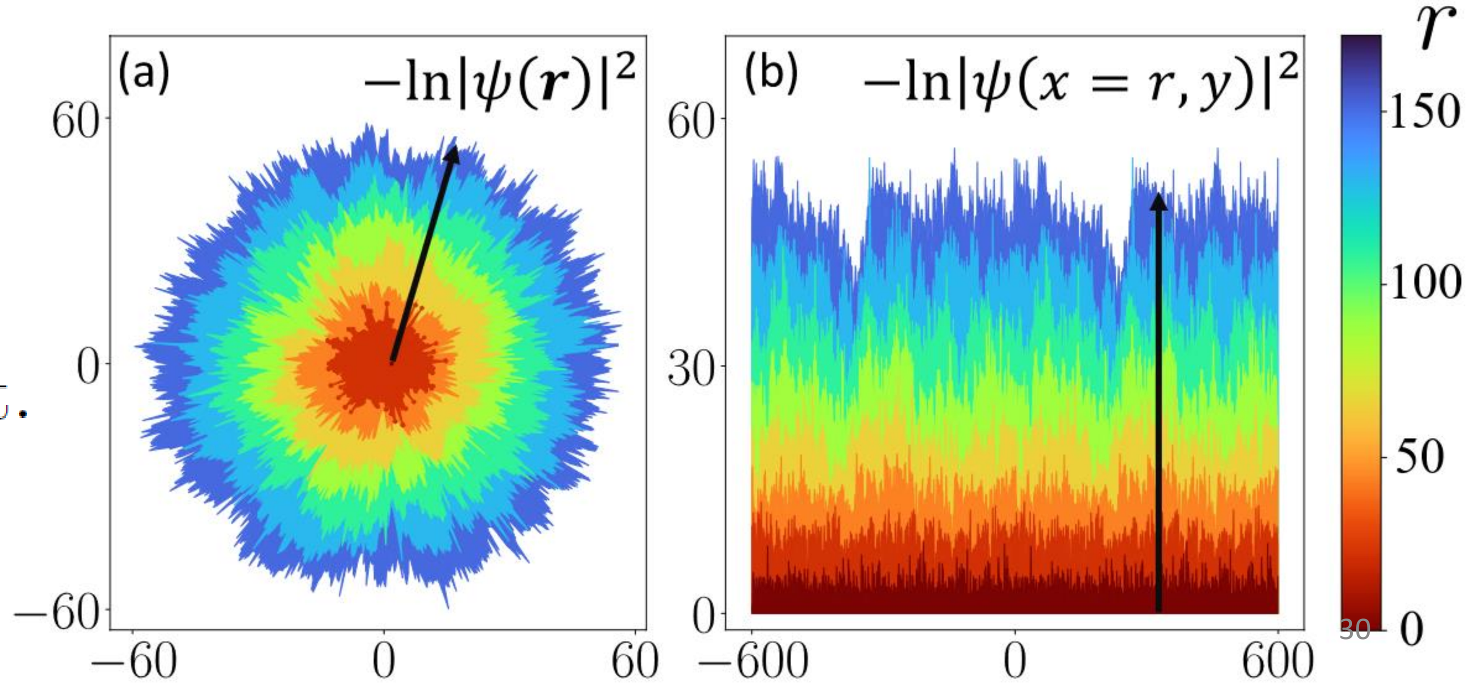






take-home figure

$$\psi_0(x,y) = \delta_{x,0}\delta_{y,0}, \quad \text{circular};$$
$$\psi_0(x,k_y) = \frac{1}{\sqrt{N}}\delta_{x,0}\delta_{k_y,0}, \quad \text{flat}.$$

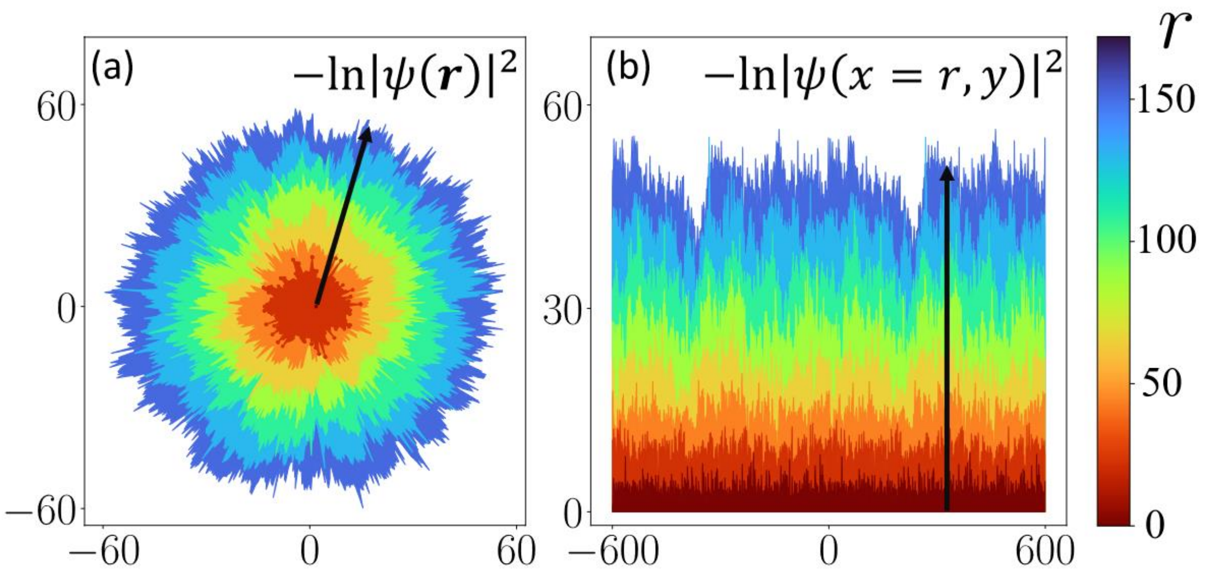


Emergent height function of a rough interface

$$-\ln|\psi(r)|^2 \ \rightarrow \ h(t)$$

with r acting as t

KPZ universality ?



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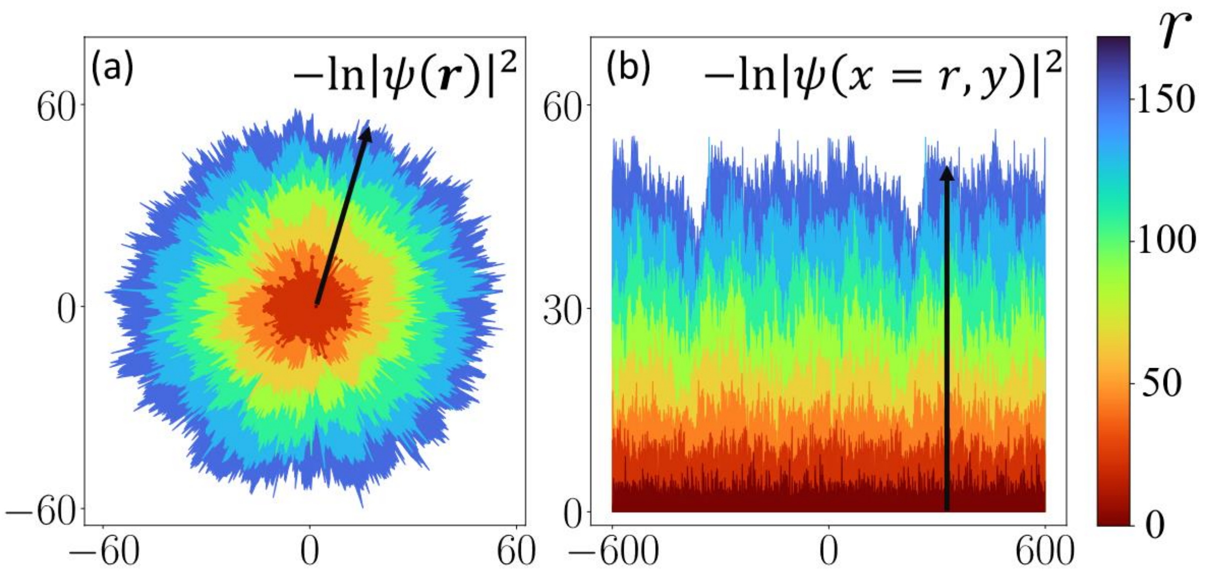
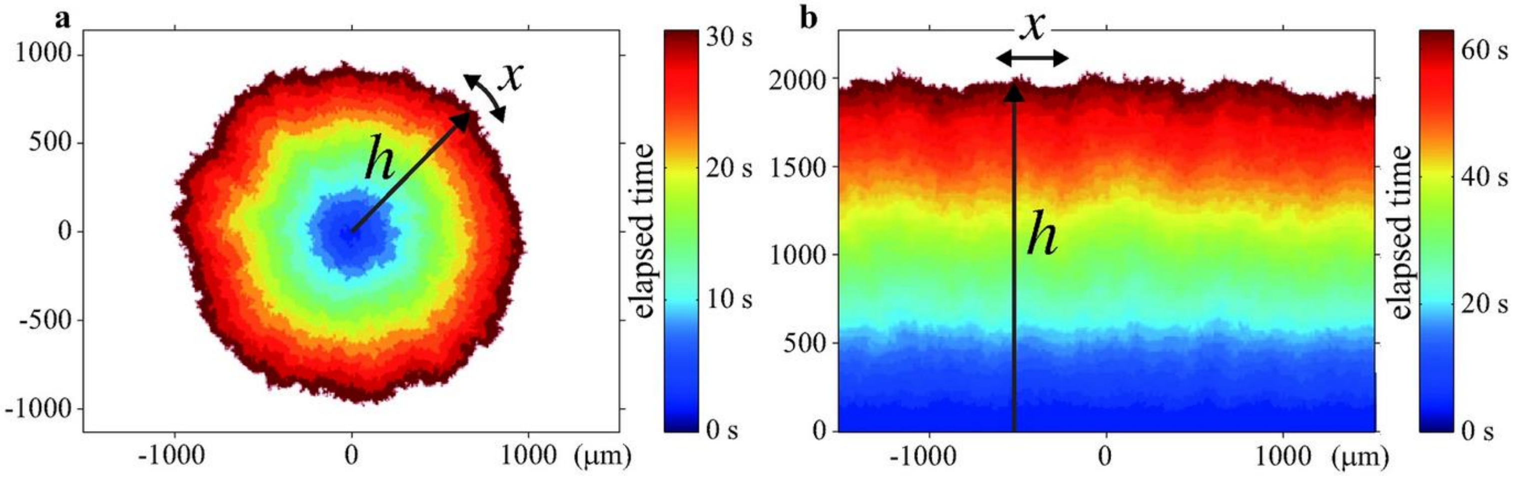




Growing DSM2 cluster with a circular (a) and flat (b) interface.

Scientific Reports, 1,34 (2011)

Wave packet dynamics

Consider: $|\psi_{t+1}\rangle = \hat{U}|\psi_t\rangle = e^{-iKV(\hat{\boldsymbol{k}})}e^{-iW(\hat{\boldsymbol{r}})}|\psi_t\rangle$

$$\langle x', y'|U|x, y\rangle = J(x, y, x', y')e^{-iW(x,y)}$$

Amplitude of the wave function

Consider: $|\psi_{t+1}\rangle = \hat{U}|\psi_t\rangle = e^{-iKV(\hat{\boldsymbol{k}})}e^{-iW(\hat{\boldsymbol{r}})}|\psi_t\rangle$

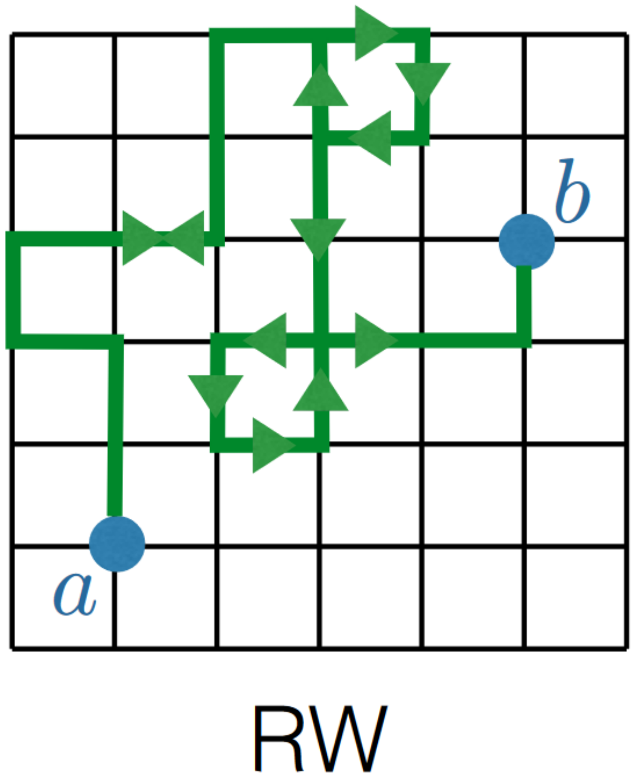
$$\langle x', y'|U|x, y\rangle = J(x, y, x', y')e^{-iW(x,y)}$$

Path integral representation:

$$\psi_t(\boldsymbol{r}) = \langle \boldsymbol{r}|\hat{U}^t|\mathbf{0}\rangle \; = \sum_{\boldsymbol{r}_{t-1}} \dots \sum_{\boldsymbol{r}_1} \langle \boldsymbol{r}|\hat{U}|\boldsymbol{r}_{t-1}\rangle \dots \langle \boldsymbol{r}_1|\hat{U}|\mathbf{0}\rangle$$

Amplitude of the wave function

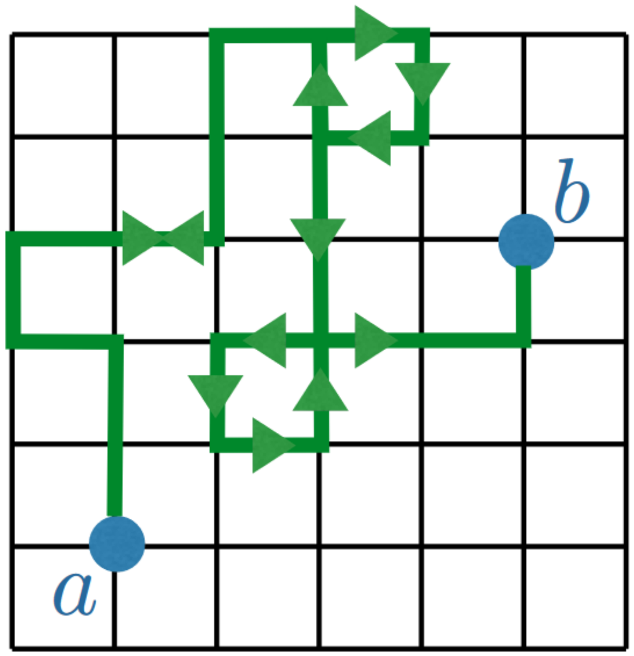
$$\psi_t(\boldsymbol{r}) = \langle \boldsymbol{r} | \hat{U}^t | \mathbf{0} \rangle \;=\; \sum_{\boldsymbol{r}_{t-1}} \ldots \sum_{\boldsymbol{r}_1} \langle \boldsymbol{r} | \hat{U} | \boldsymbol{r}_{t-1} \rangle \ldots \langle \boldsymbol{r}_1 | \hat{U} | \mathbf{0} \rangle$$



arxiv:1710.01234

Amplitude of the wave function

$$\psi_t(\boldsymbol{r}) = \langle \boldsymbol{r}|\hat{U}^t|\mathbf{0}\rangle = \sum_{\boldsymbol{r}_{t-1}} \dots \sum_{\boldsymbol{r}_1} \langle \boldsymbol{r}|\hat{U}|\boldsymbol{r}_{t-1}\rangle \dots \langle \boldsymbol{r}_1|\hat{U}|\mathbf{0}\rangle$$



RW

$$\langle x', y'|U|x, y\rangle = J(x, y, x', y')e^{-iW(x,y)}$$

$$\Downarrow$$

$$J(x, y, x', y') = i^{(x+y)-(x'+y')} J_{x-x'}(K) J_{y-y'}(K)$$

arxiv:1710.01234

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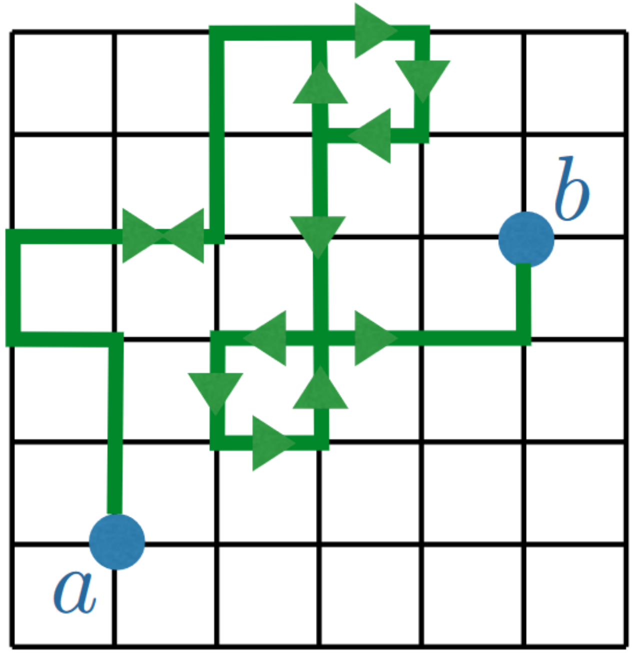

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arxiv:1710.01234

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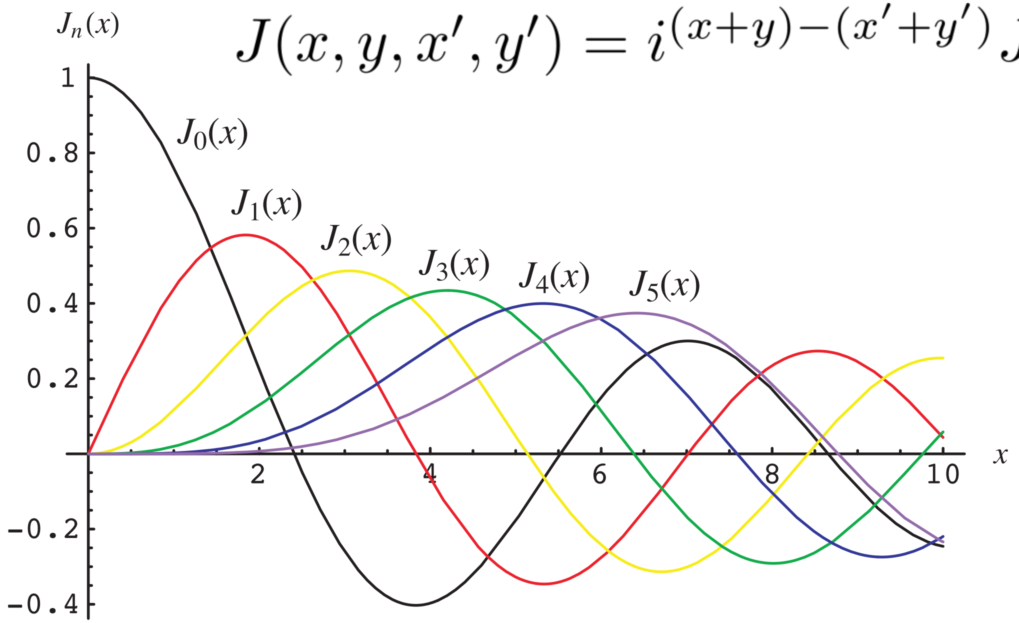

Amplitude of the wave function

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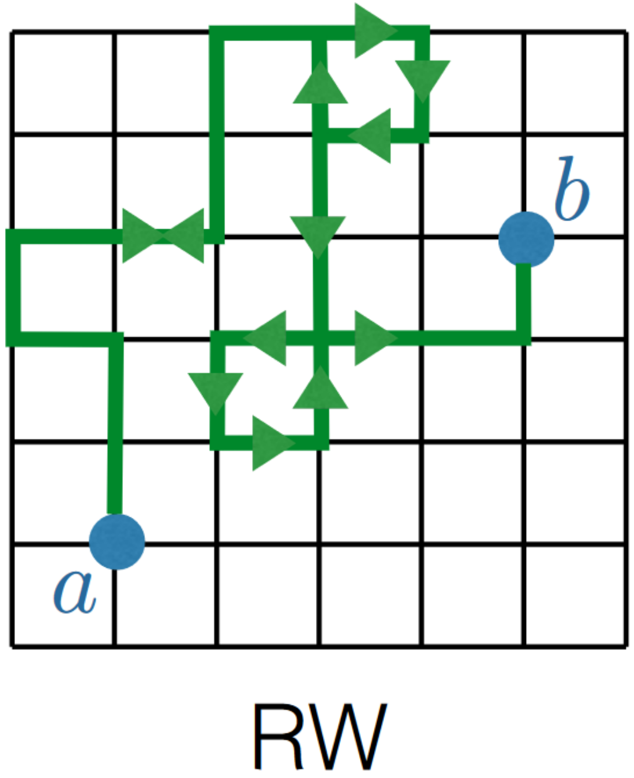

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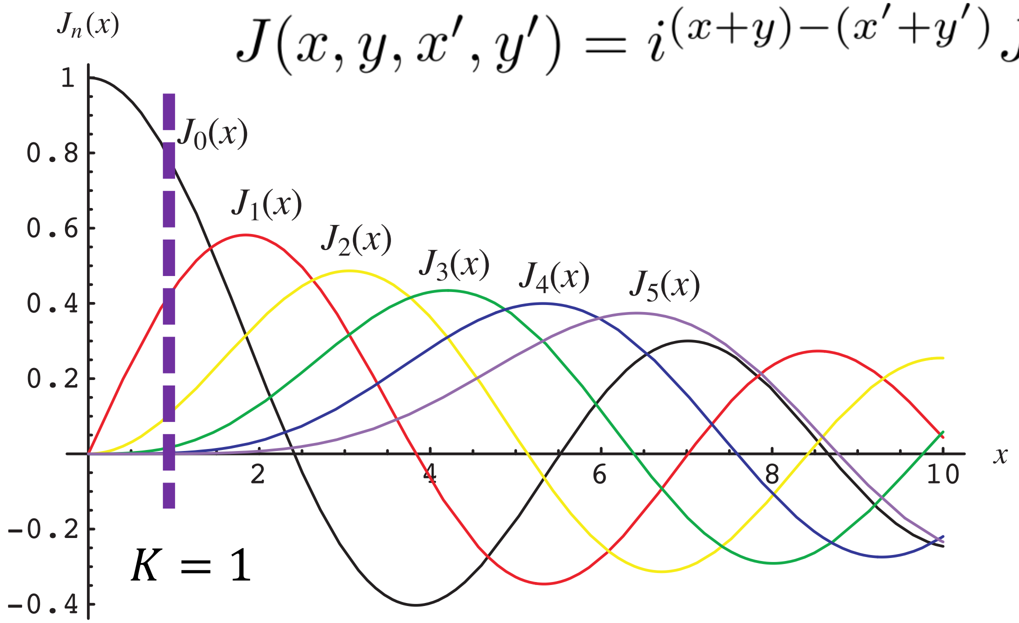


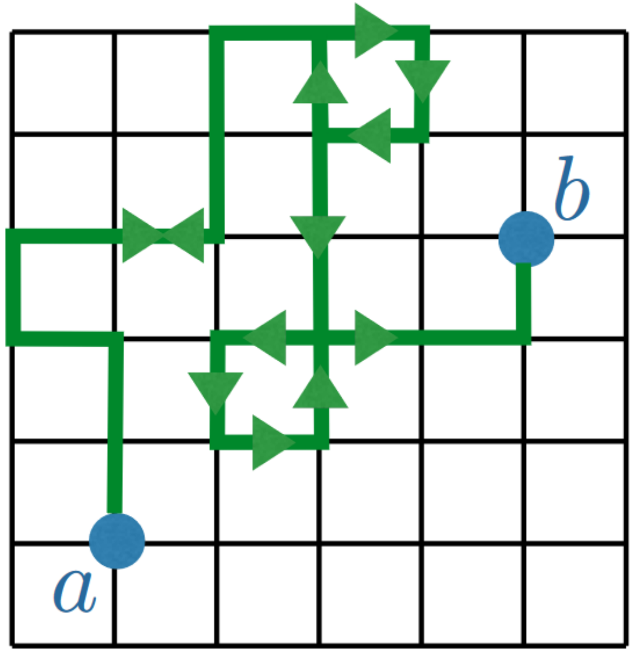
$$|J_1| \ll 1$$

for nearest neighbours

Amplitude of the wave function

$$\psi_t(\boldsymbol{r}) = \langle \boldsymbol{r}|\hat{U}^t|\boldsymbol{0}\rangle = \sum_{\boldsymbol{r}_{t-1}} \dots \sum_{\boldsymbol{r}_1} \langle \boldsymbol{r}|\hat{U}|\boldsymbol{r}_{t-1}\rangle \dots \langle \boldsymbol{r}_1|\hat{U}|\boldsymbol{0}\rangle$$

dominant contributions



RW

$$\langle x', y'|U|x, y\rangle = J(x, y, x', y')e^{-iW(x,y)}$$

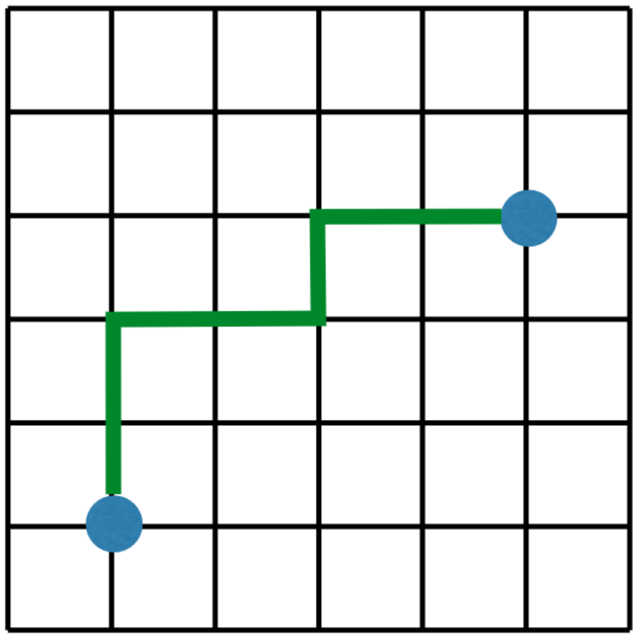
$$\wr\wr$$

$$|J_1| \ll 1$$

for nearest neighbours.

This leads strong localization, then

Forward Scattering Approximation (FWA)



DP

arxiv:1710.01234

Analogy to directed polymers (DP)

$$\psi_t(\boldsymbol{r}) = |J_1|^r \sum_{D\mathcal{P}} \prod_{\boldsymbol{r}_j \in D\mathcal{P}} e^{-\tilde{W}(\boldsymbol{r}_j)} \qquad |J_1| \ll 1$$

Analogy to directed polymers (DP)

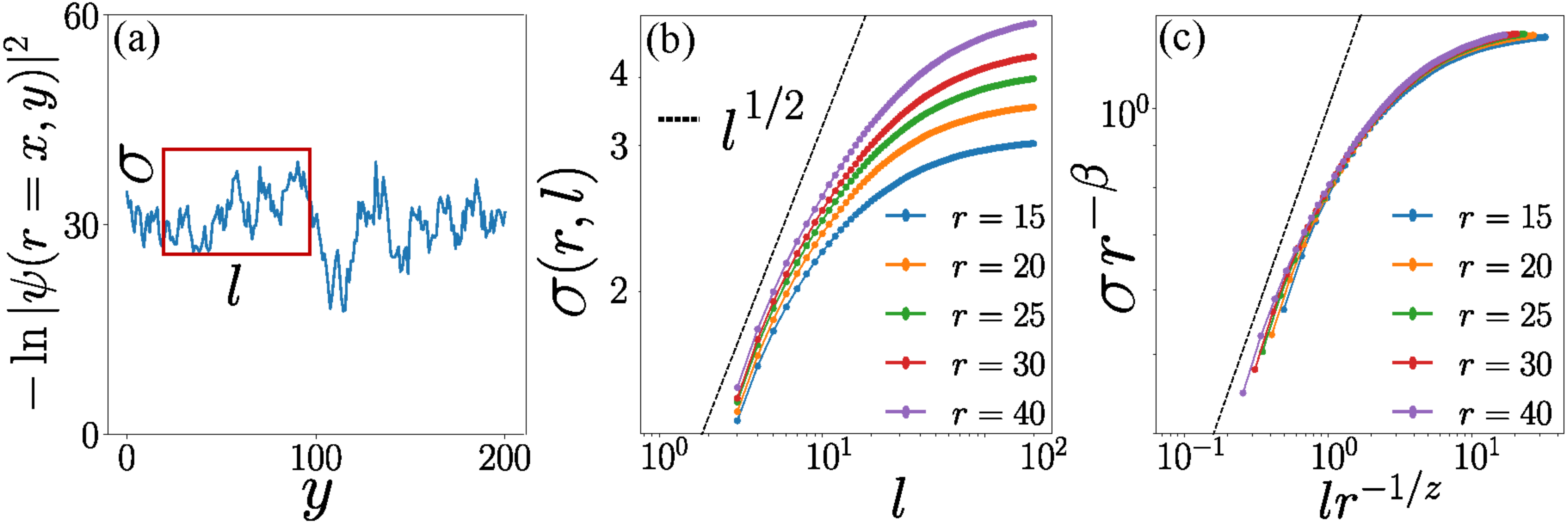
$$\psi_t(\boldsymbol{r}) = |J_1|^r \sum_{\mathcal{DP}} \prod_{\boldsymbol{r}_j \in \mathcal{DP}} e^{-\tilde{W}(\boldsymbol{r}_j)} \qquad |J_1| \ll 1$$

Partition function for DP with complex onsite disorder,

$$Z = \sum_{\mathcal{DP}} \prod_{\boldsymbol{r}_j \in \mathcal{DP}} e^{-\tilde{W}(\boldsymbol{r}_j)}$$

$F = \ln|Z|^2 \leftrightarrow$ $h(t)$ height function of a growing rough surface described by the KPZ equation.

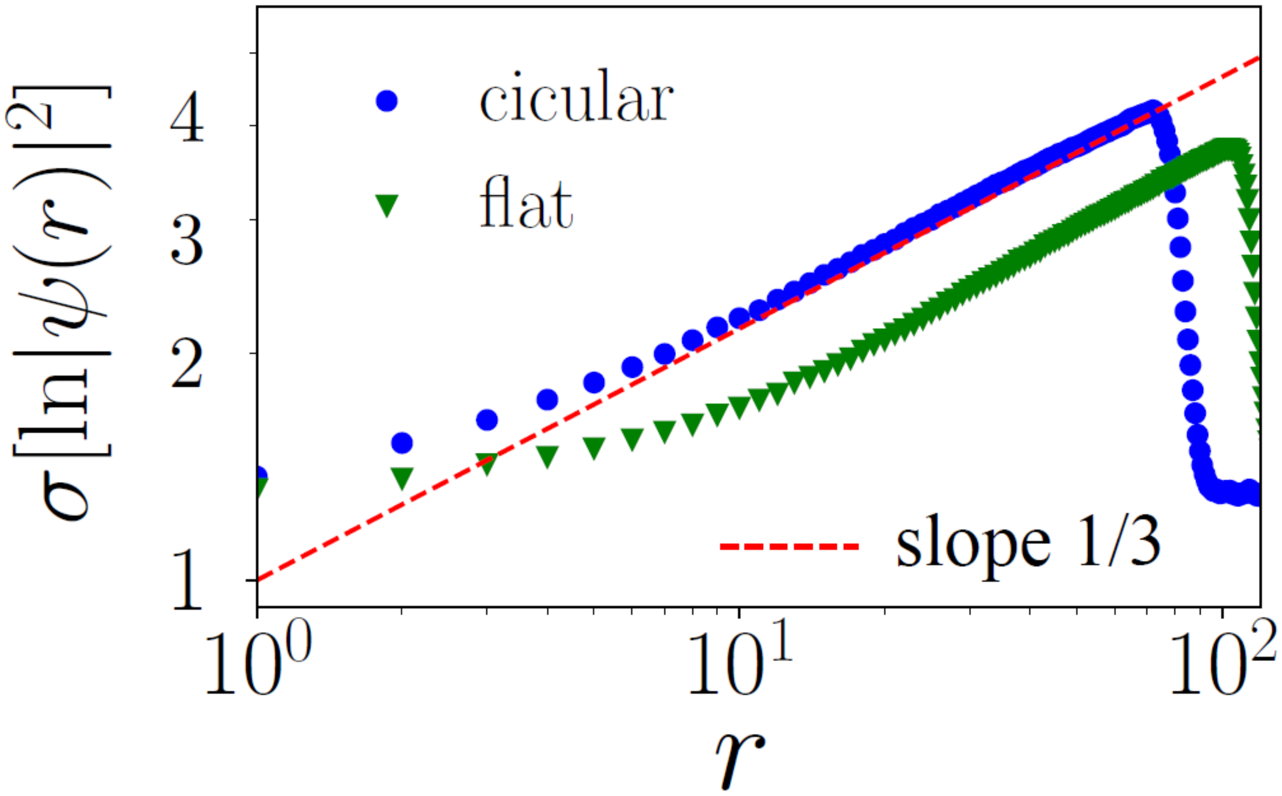
FV scaling in the localized wave packets



(1+1)D KPZ class: $\alpha = 1/2, \quad \beta = 1/3, \quad z = 3/2$

Universal fluctuation growth exponent

$$\sigma[\ln|\psi(r)|^2] = \sqrt{\langle(\ln|\psi(r)|^2)^2\rangle - \langle\ln|\psi(r)|^2\rangle^2}$$



$$\ln|\psi(\boldsymbol{r})|^2 \approx -\frac{2r}{\xi} + \left(\frac{r}{\xi}\right)^{\beta} \Gamma\,\chi(\boldsymbol{r})$$

with $\beta \approx 1/3$

Universal distribution function

$$\ln|\psi(\boldsymbol{r})|^2 \approx -\frac{2r}{\xi} + \left(\frac{r}{\xi}\right)^{\beta} \Gamma\,\chi(\boldsymbol{r})$$

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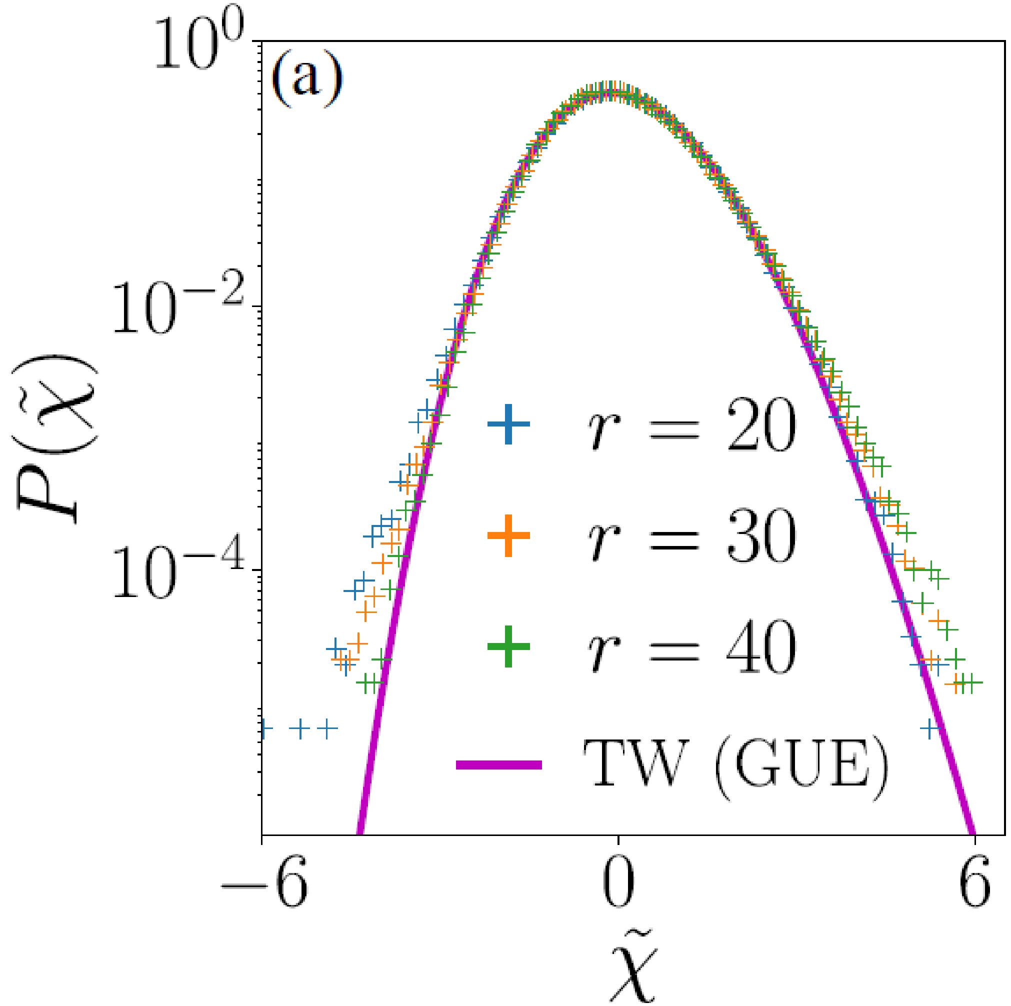

follows Tracy-Widom distribution (GUE)
(under circular condition)

$$\tilde{\chi} = (\ln|\psi(r)|^2 - \langle \ln|\psi(r)|^2\rangle)/\sigma[\ln|\psi(r)|^2]$$

Shape of localized wave packets in 2D

Typical wave density:

$$\langle \ln|\psi(\boldsymbol{r})|^2\rangle \underset{r\gg\xi}{\approx} -\frac{2r}{\xi} + \left(\frac{r}{\xi}\right)^{1/3} \Gamma\,\mu$$

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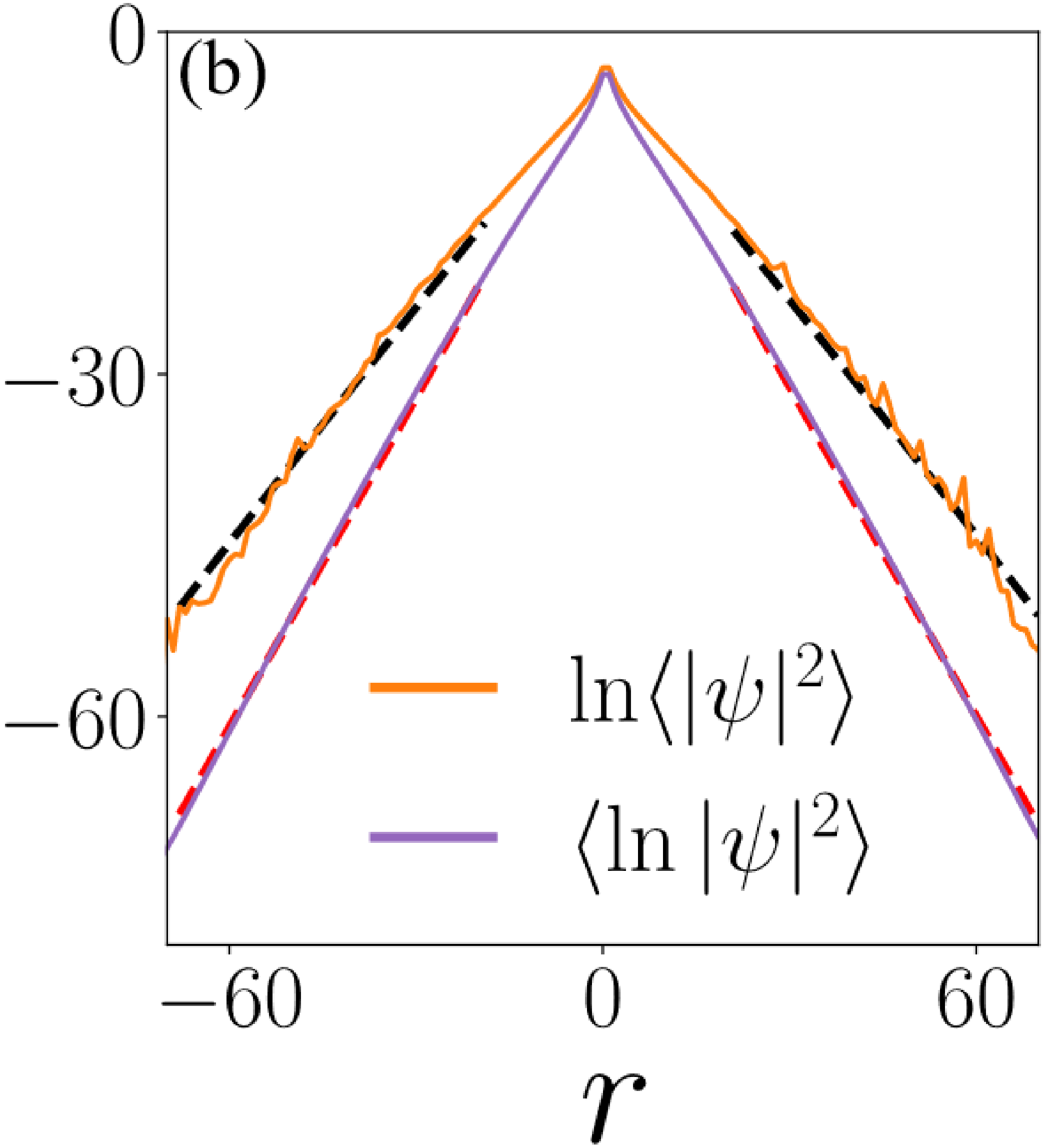
Average wave density:

$$\langle |\psi(\boldsymbol{r})|^2 \rangle = e^{\langle | \ln \psi(\boldsymbol{r})|^2 \rangle} \int_{-\infty}^{\infty} e^{\sigma(r) x} P_{\mathrm{TW}}(x) \mathrm{d}x$$

$$\underset{r \gg \xi}{\approx} e^{-2r/\xi + \Gamma'(r/\xi)^{1/3} + \Gamma''(r/\xi)^{2/3} + \Gamma'''}$$

Shape of localized wave packets in 2D



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$$\underset{r\gg\xi}{\approx} e^{-2r/\xi+\Gamma'(r/\xi)^{1/3}+\Gamma''(r/\xi)^{2/3}+\Gamma'''}$$

Stretched exponential function, not a simple exponential profile !!

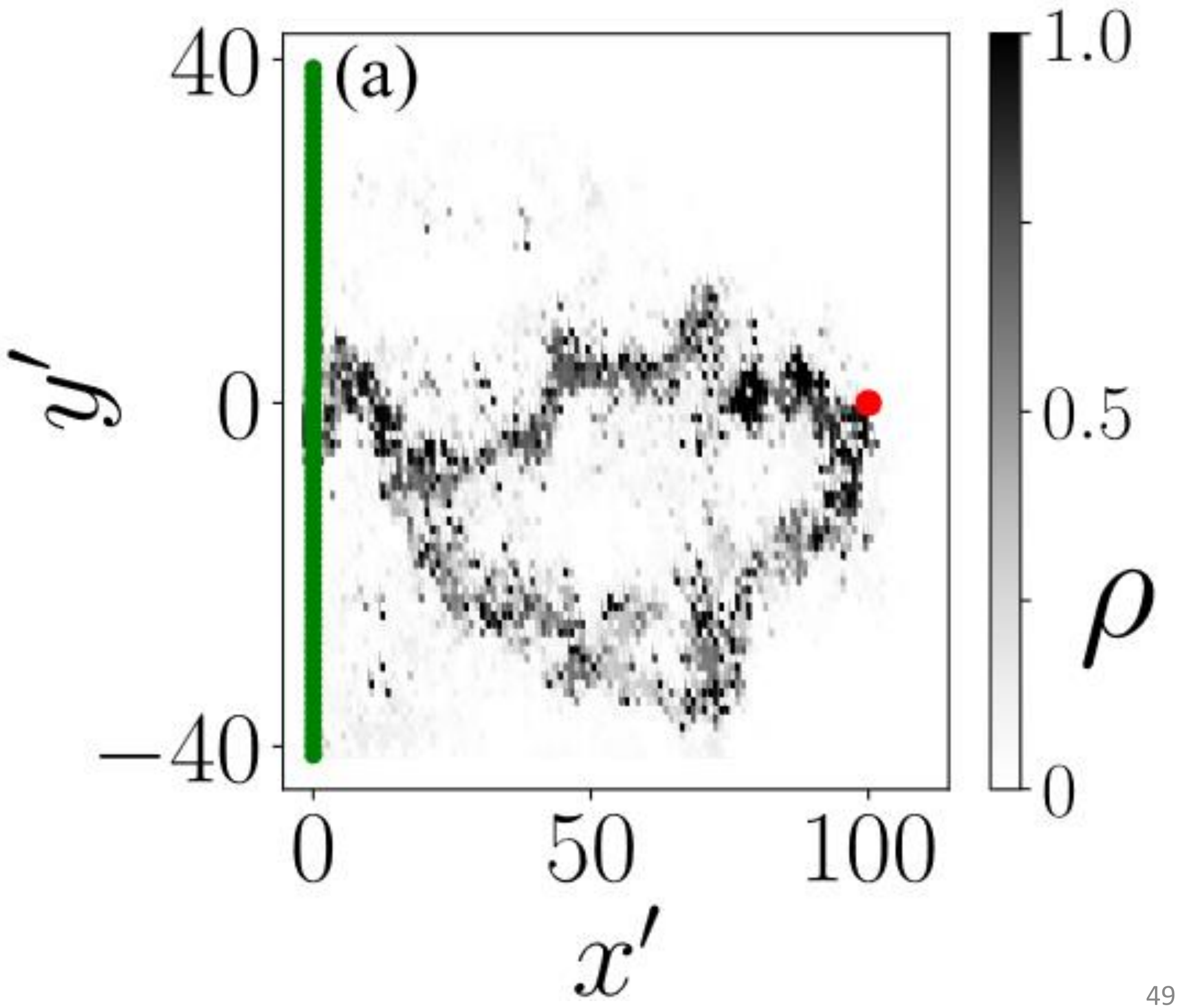
Most sensitive or optimal (directed) paths

Perturbation at a single site,

$$W(r') \rightarrow W(r') + \pi$$

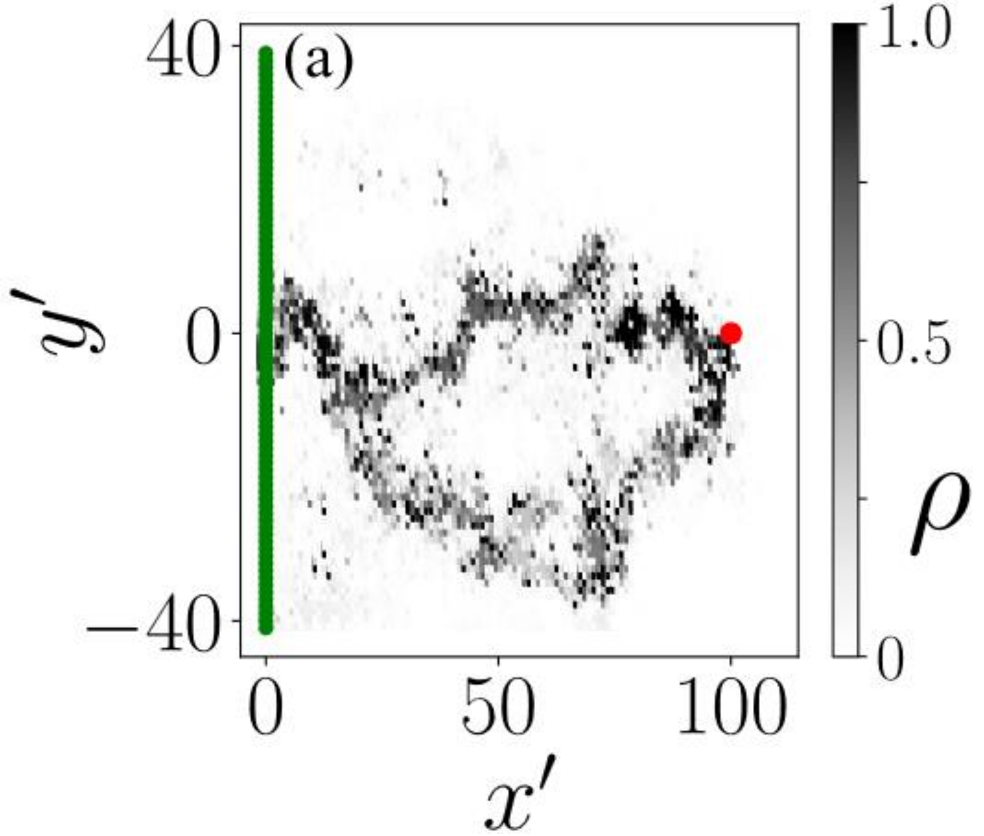
then repeat the evolution.

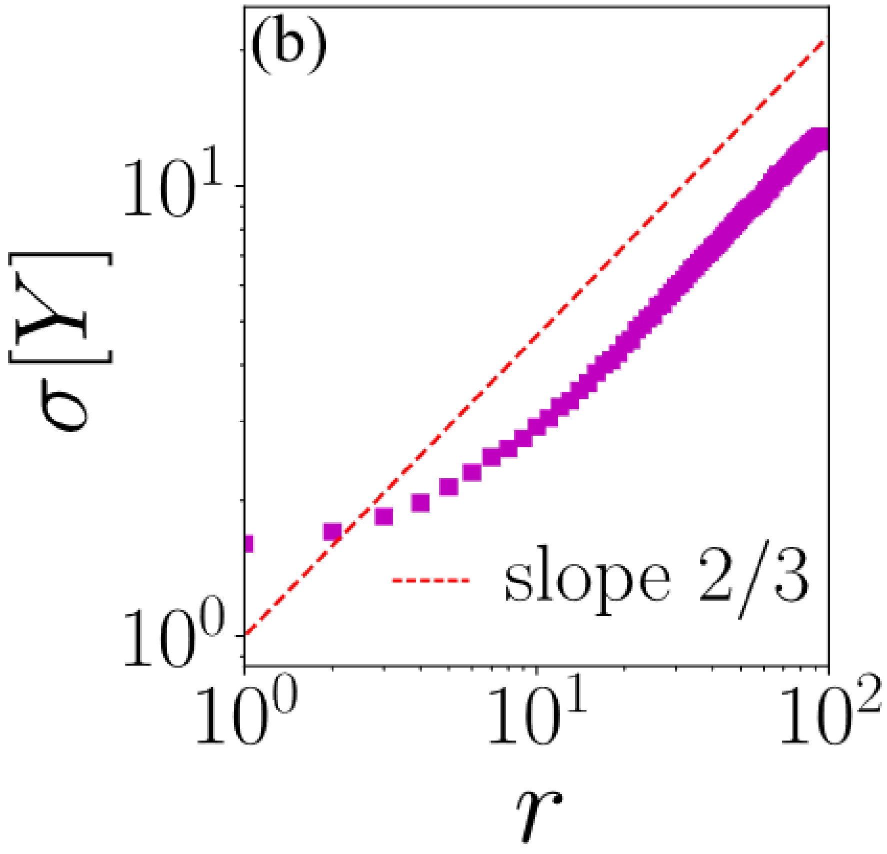
$$\rho_{\boldsymbol{r}}(\boldsymbol{r}') \equiv \frac{\left| |\tilde{\psi}_{\boldsymbol{r}'}(\boldsymbol{r})|^2 - |\psi(\boldsymbol{r})|^2 \right|}{|\psi(\boldsymbol{r})|^2}$$



Wandering exponent 2/3

$$\rho_{\boldsymbol{r}}(\boldsymbol{r}') \equiv \frac{\left| |\tilde{\psi}_{\boldsymbol{r}'}(\boldsymbol{r})|^2 - |\psi(\boldsymbol{r})|^2 \right|}{|\psi(\boldsymbol{r})|^2}$$



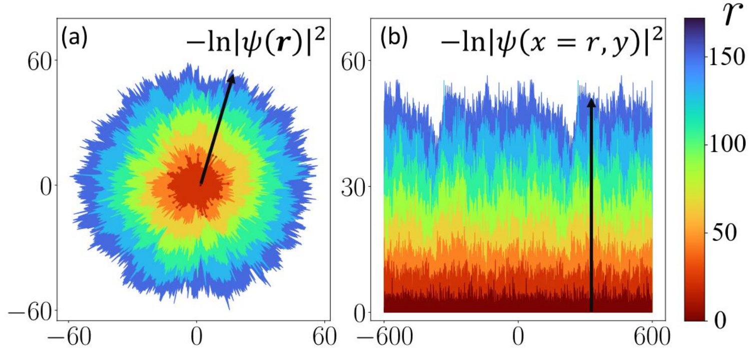


$$Y = \frac{\sum_{y'} y' \rho_{\boldsymbol{r}}(0, y')}{\sum_{y'} \rho_{\boldsymbol{r}}(0, y')}$$

Conclusions

- Localized wave packets in 2D and its analogy with DP problem.

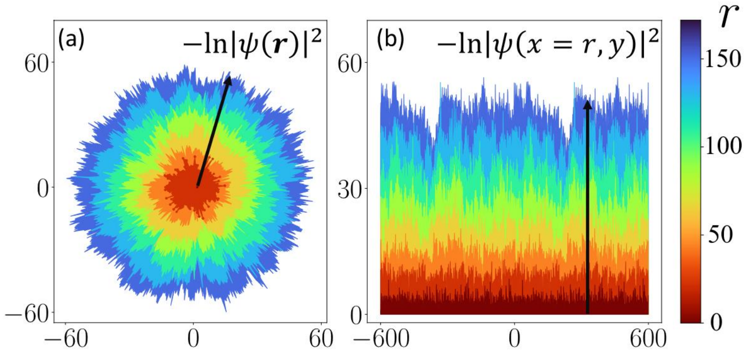
$$\psi_t(\boldsymbol{r}) = |J_1|^r \sum_{D\mathcal{P}} \prod_{\boldsymbol{r}_j \in D\mathcal{P}} e^{-\tilde{W}(\boldsymbol{r}_j)}$$



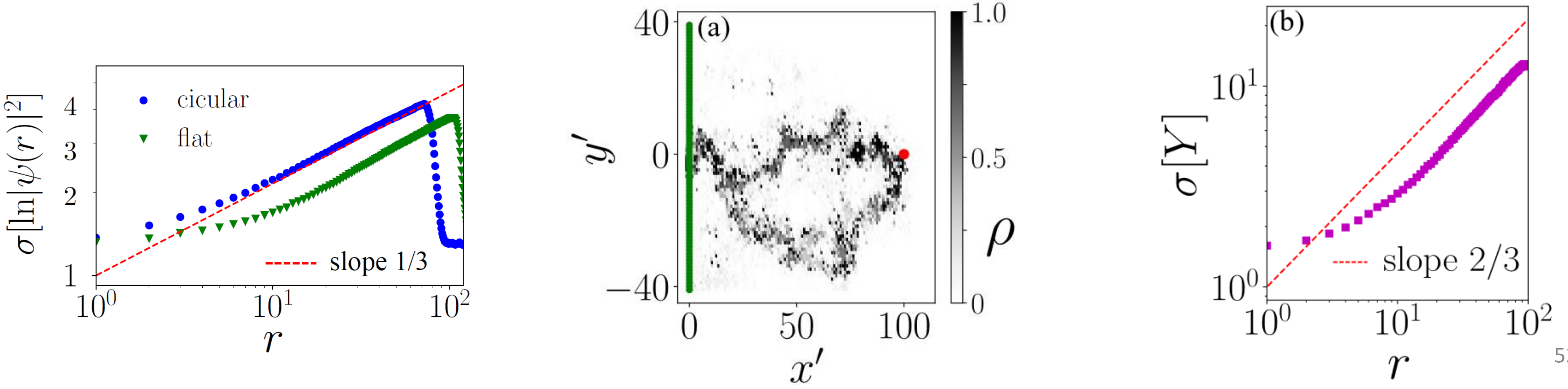
Conclusions

- Localized wave packets in 2D and its analogy with DP problem.

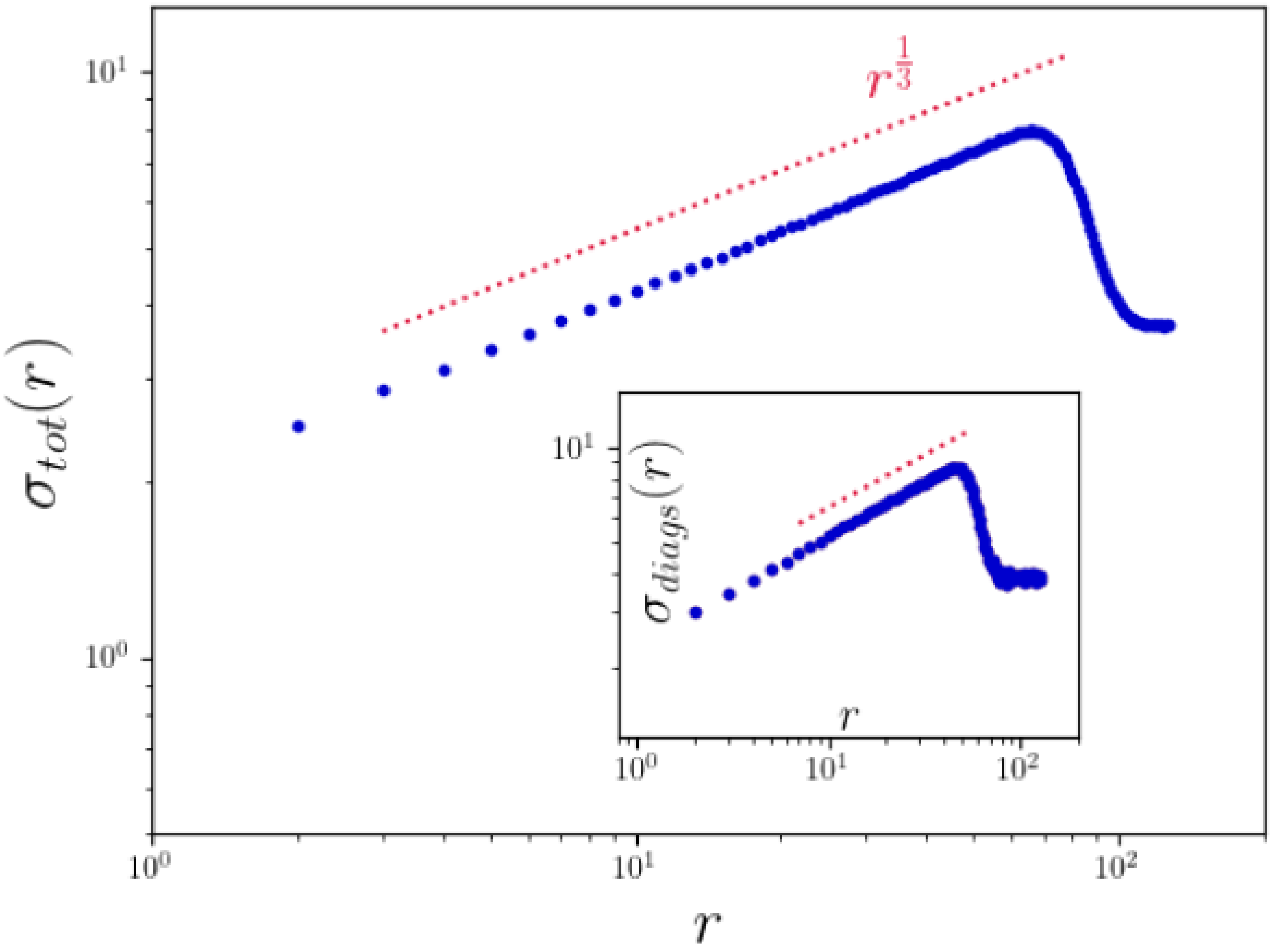
$$\psi_t(\boldsymbol{r}) = |J_1|^r \sum_{DP} \prod_{\boldsymbol{r}_j \in DP} e^{-\tilde{W}(\boldsymbol{r}_j)}$$



- Fluctuation exponent 1/3 and wandering exponent 2/3.



More Results: Universal fluctuations in the statistics of eigenvectors of 2D Anderson model



Fluctuation growth exponent is also 1/3 even when we directly look at eigenvectors !

Outlook

- In higher dimensions or with interactions, e.g., Anderson metal insulator transition in 3D, mapped to the complex-weight directed polymer in (2+1)D? – what physics can we possibly explore?

- Impact of an external field or Kerr-type self interaction on the fluctuations growth exponent (work ongoing) – new perspectives to investigate variants of Anderson localization physics.

Other work by "Gong-Lemarie collaboration" on Anderson Transition

- **Quantum logarithmic multifractality, arXiv:2312.17481** (accepted by PR Research as a letter)

- **Describing the critical behavior of the Anderson transition in infinite dimension by random-matrix ensembles: logarithmic multifractality and critical localization, arXiv:2405.10975** (under review by PRB).

- **Critical dynamics of long-range quantum disordered systems, arXiv:2307.00999** (PRE2023)

Thank you for your attention!