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Dynamics Days Asia Pacific 13

Dynamics of particles in turbulent flow

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PLASMA SIMULATOR

大阪大学

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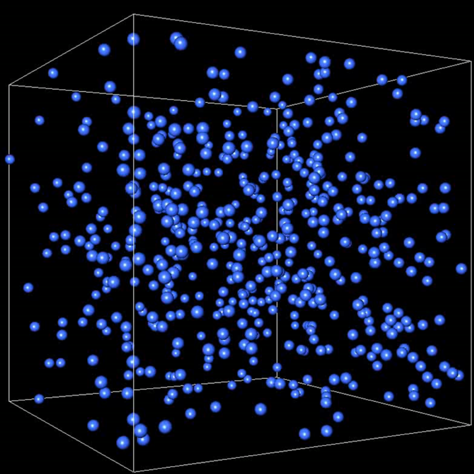
Introduction



Turbulence can be modulated by particles.

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Turbulence modulation by solid particles

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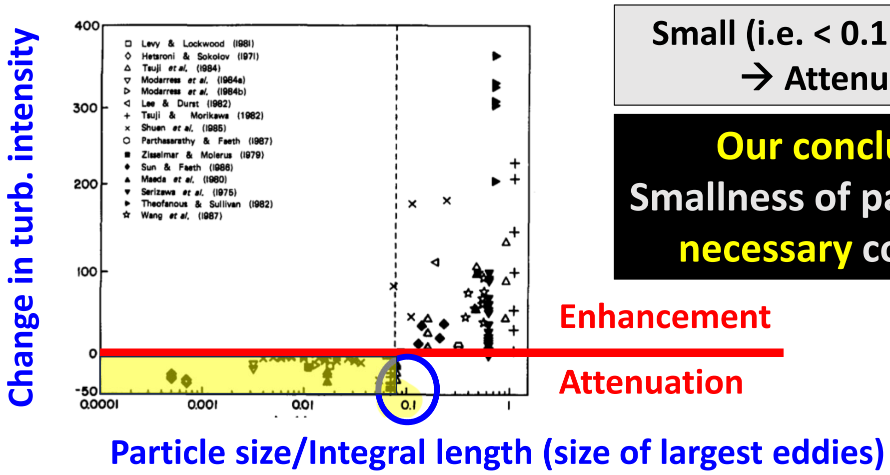
Gore & Crowe (1989) **"Effect of particle size on modulating turbulent intensity"**

Compiled experimental data of pipe flow and jets.



Small (i.e. < 0.1L) particles
→ Attenuation

Our conclusion:
Smallness of particles is a necessary condition

Two key notions

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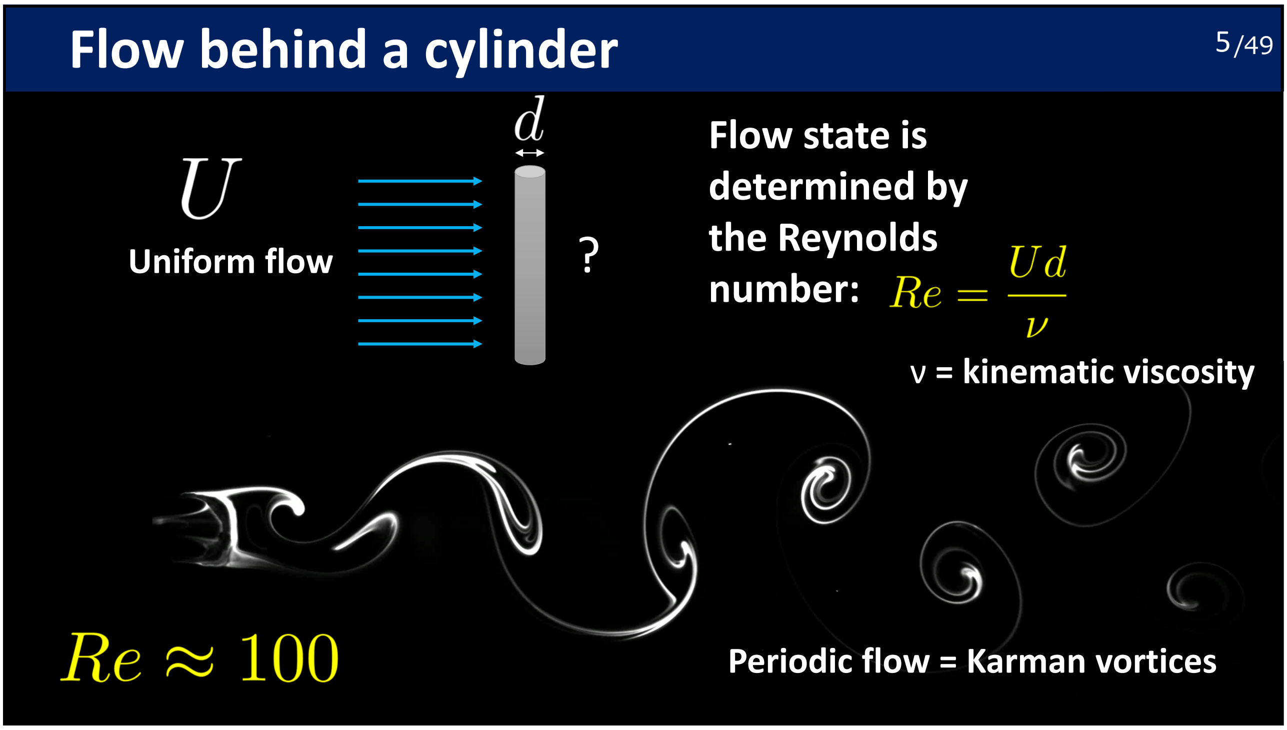
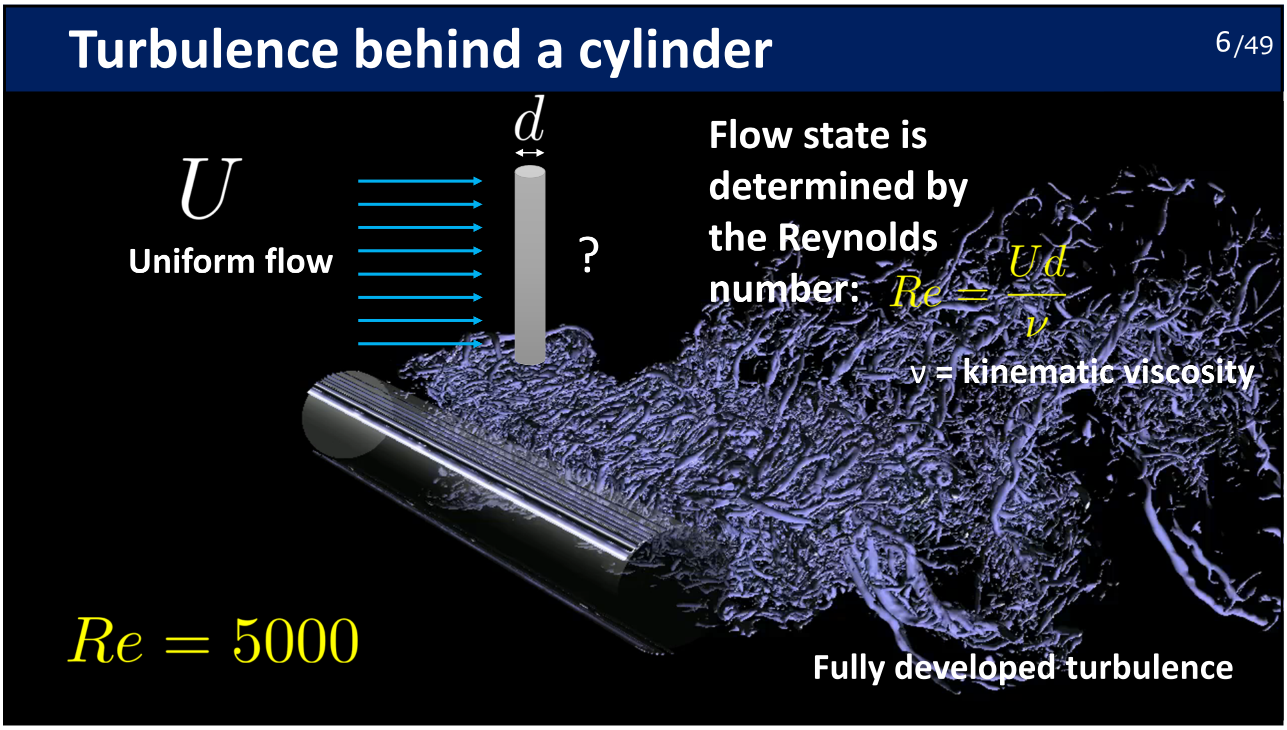
① Coherent structures in turbulence
→ Energy cascade

② Time-scale competition
→ Stokes number

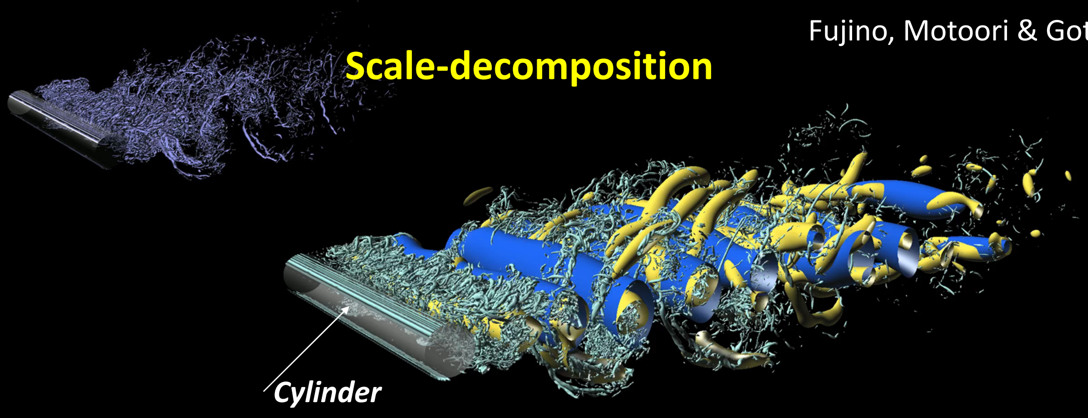

Flow behind a cylinder
5/49
d
U
Uniform flow
?
Flow state is determined by the Reynolds number: $Re = \frac{Ud}{\nu}$
ν = kinematic viscosity
$Re \approx 100$
Periodic flow = Karman vortices


Turbulence behind a cylinder
6/49
d
U
Uniform flow
?
Flow state is determined by the Reynolds number: $Re = \frac{Ud}{\nu}$
ν = kinematic viscosity
$Re = 5000$
Fully developed turbulence

Scale decomposition

Turbulence=Hierarchy of coherent vortices

Fujino, Motoori & Goto (2023)



Blue roller (shedding) vortices: Corresponding to the Karman vortices
→ Yellow rib vortices are created around the rollers.
→ Further smaller vortices are created around the ribs.

Turbulence=Hierarchy of coherent vortices

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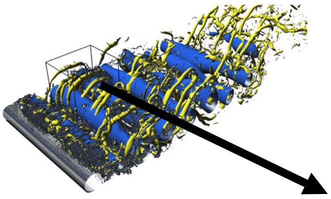
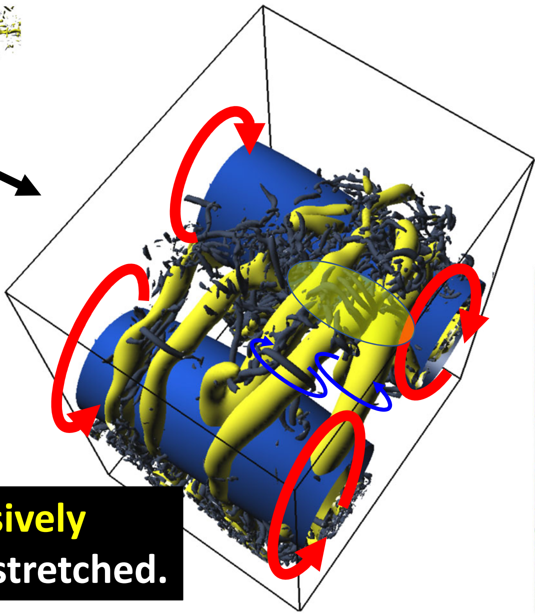
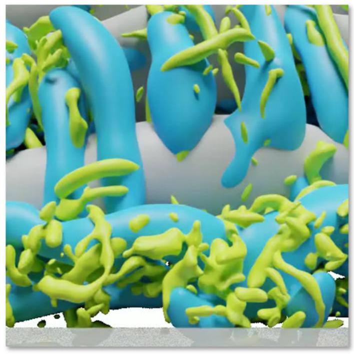




Vortices are **successively** amplified by being stretched.



Generation mechanism = Energy cascade

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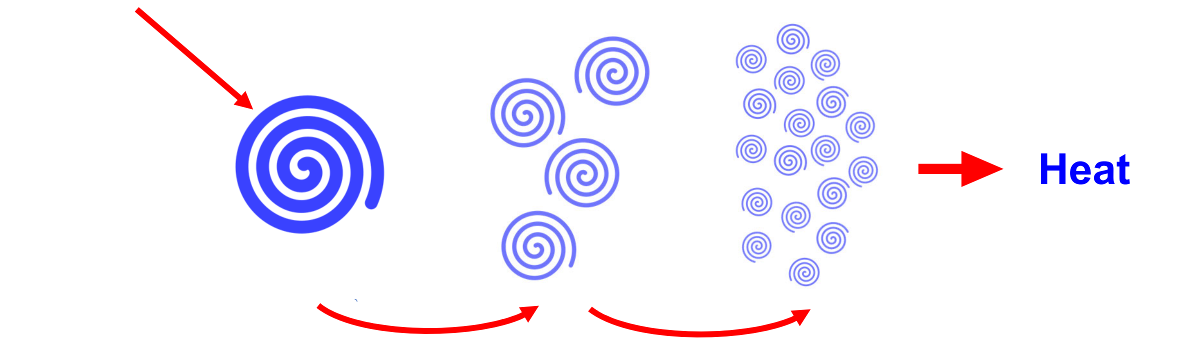


Fujino, Motoori, Goto (2023)

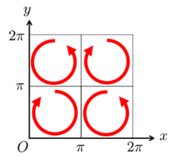
Turbulence far from solid walls

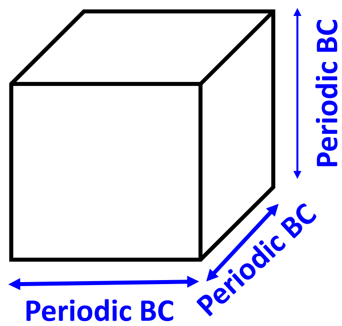
Turbulence in a periodic cube

Velocity: $\boldsymbol{u}(\boldsymbol{x}, t)$

Pressure: $p(\boldsymbol{x}, t)$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} +\cos(x)\sin(y) \\ -\sin(x)\cos(y) \\ 0 \end{bmatrix}$$





Continuity equation

$$\nabla \cdot \boldsymbol{u} = 0$$

Need forcing

Navier-Stokes equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}$$

Kinematic viscosity (ν)

Mass density (ρ)

2048×2048×2048 Fourier modes → $R_\lambda \simeq 700$ ($Re \simeq 10^5$)

are numerically integrated by Fourier spectral method.

Turbulence in a periodic cube

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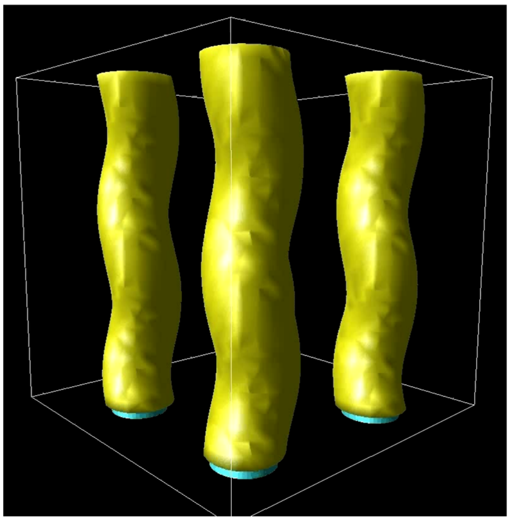
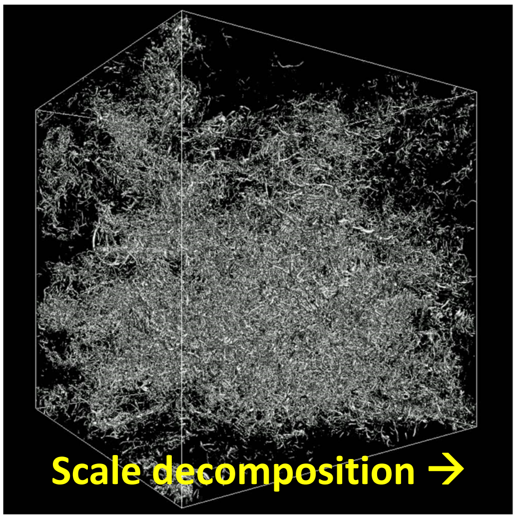


Low Re → Periodic flow



High Re → Turbulence

Turbulence in a periodic cube

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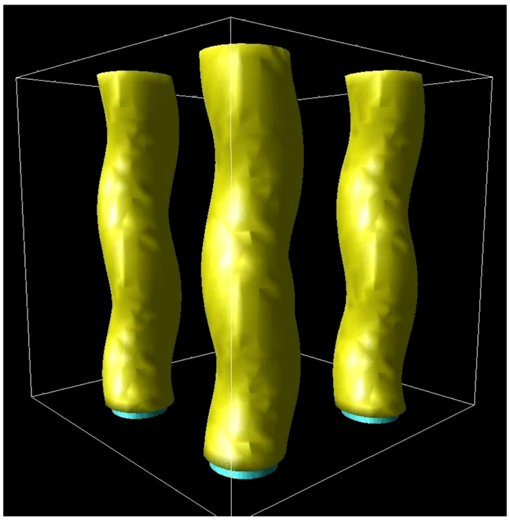
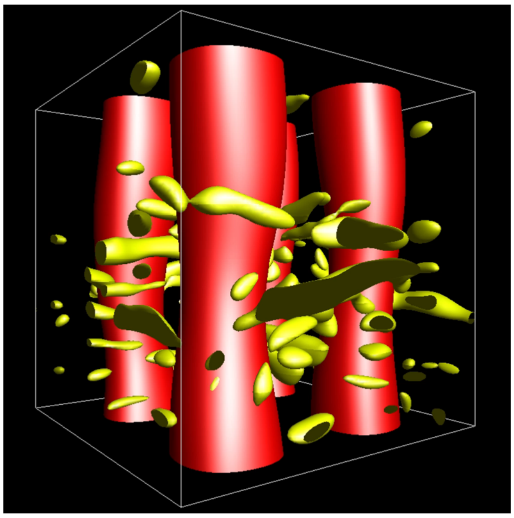


Low Re → Periodic flow



High Re → Turbulence

Turbulence = Hierarchy of coherent vortices

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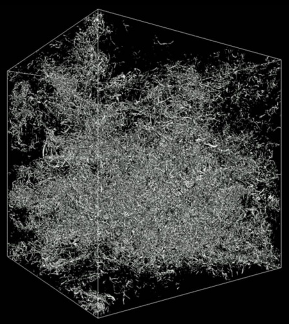
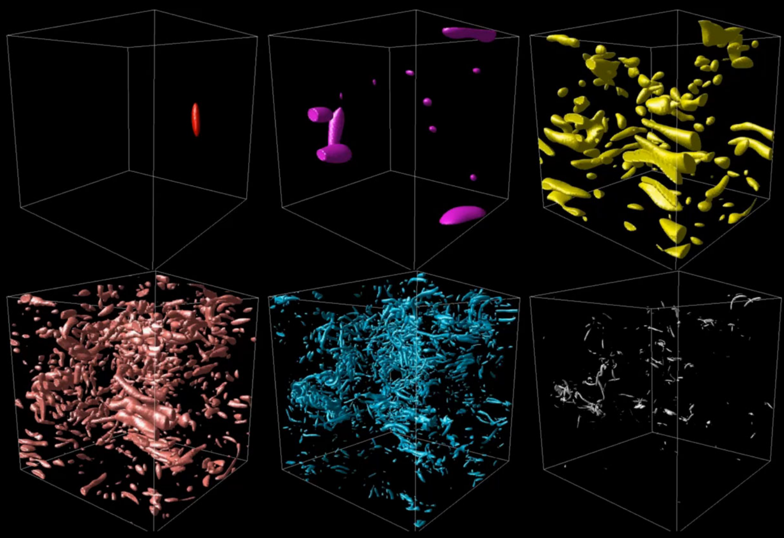


Scale decomposition →



Generation mechanism = Energy cascade

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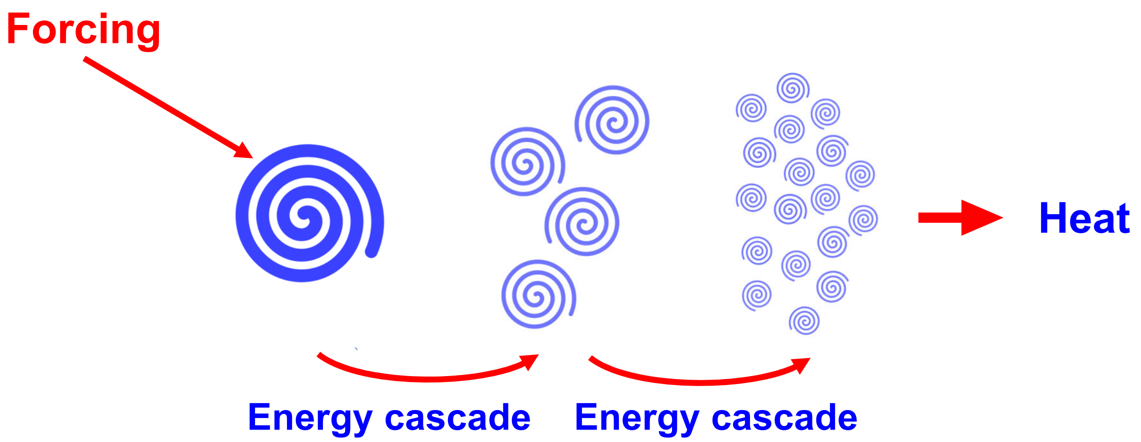


Q: How do particles modify the energy cascade process?

Two key notions

① Coherent structures in turbulence

→ Energy cascade

② Time-scale competition

→ Stokes number

Stokes number

Equation of motion for a Stokes spherical particle:

(density ρ_p, diameter D)

$$m\frac{d\boldsymbol{u}_p}{dt} = -C(\boldsymbol{u}_p - \boldsymbol{u}_f)$$

$m = \rho_p \times \frac{\pi D^3}{6}$, $C = 3\pi\mu D$

($\boldsymbol{u}_p$ = particle velocity)

($\boldsymbol{u}_f$ = fluid velocity)

$$\frac{d\boldsymbol{u}_p}{dt} = -\frac{1}{\tau_p}(\boldsymbol{u}_p - \boldsymbol{u}_f)$$

With the velocity relaxation time

$$\tau_p = \frac{\rho_p D^2}{18\mu}$$

$$St = \frac{\tau_p}{T}$$

T: Turnover time of the largest eddies

Stokes number

$$\frac{d\boldsymbol{u}_p}{dt} = -\frac{1}{\tau_p}\left(\boldsymbol{u}_p - \boldsymbol{u}_f\right)$$

With the velocity relaxation time

$$\tau_p = \frac{\rho_p D^2}{18\mu}$$

$$St = \frac{\tau_p}{T}$$

Turnover time of the largest eddies

Light particles

If $St \ll 1$ (i.e. $\tau_p \ll T$),

particles can follow flow.
→ Nothing happens.

Heavy particles

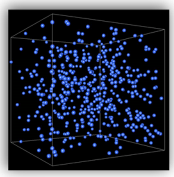
If $St \gg 1$ (i.e. $\tau_p \gg T$),

particles **cannot** follow flow.
→ Flow modulation.

Summary so far

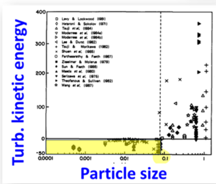
Target

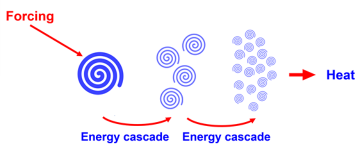
Modulation of turbulence by solid particles.



Classical view

Small particles → Attenuation
Large particles → Enhancement





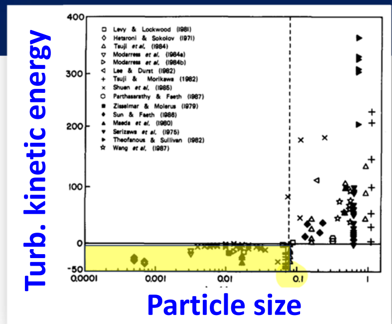
Key notions

Turbulence is not random, but composed of coherent eddies, which are sustained by the energy cascade.
Time-scale competition: Stokes number $St = \frac{\tau_p}{T}$

Particle time-scale
Time scale of the largest eddies

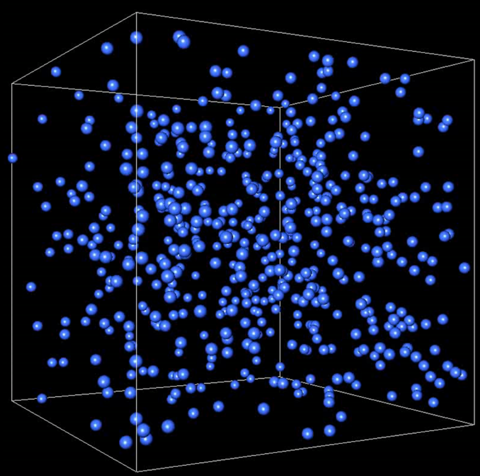
Will show following conclusions

Classical view (Gore & Crowe 1989):
particles smaller than L → attenuation
particles larger than L → enhancement



1. **The smallness of particles is a necessary condition.**
2. **Large-Stokes-number particles take energy form the largest eddies and bypass the cascade.**
3. **The attenuated kinetic energy of turbulence is determined by the additional energy dissipation rate.**

2 Method



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Method

- Forced turbulence in a periodic cube
- Finite difference + Immersed boundary method

(Uhlmann 2005, Lucci et al. 2010)

- **Two kinds of forcing**

① Taylor-Green forcing

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} +\cos(x)\sin(y) \\ -\sin(x)\cos(y) \\ 0 \end{bmatrix}$$

With mean flow $R_\lambda = 48$

② Homogeneous isotropic forcing to keep energy input rate

No mean flow $R_\lambda = 94$

Parameters

- Volume fraction

$\Lambda_p = 8.2 \times 10^{-3}$ Dilute (fixed)

- (Particle diameter)/(Integral length=size of largest eddies)

$0.17 \leq D/\mathcal{L} \leq 1.3$ (Forcing ①)

$0.15 \leq D/\mathcal{L} \leq 1.2$ (Forcing ②)

- Stokes number

(Velocity relaxation time)/(Turnover time of the largest eddies)

$0.51 < \mathrm{St} = \tau_p/\mathcal{T} < 2100$ (Forcing ①)

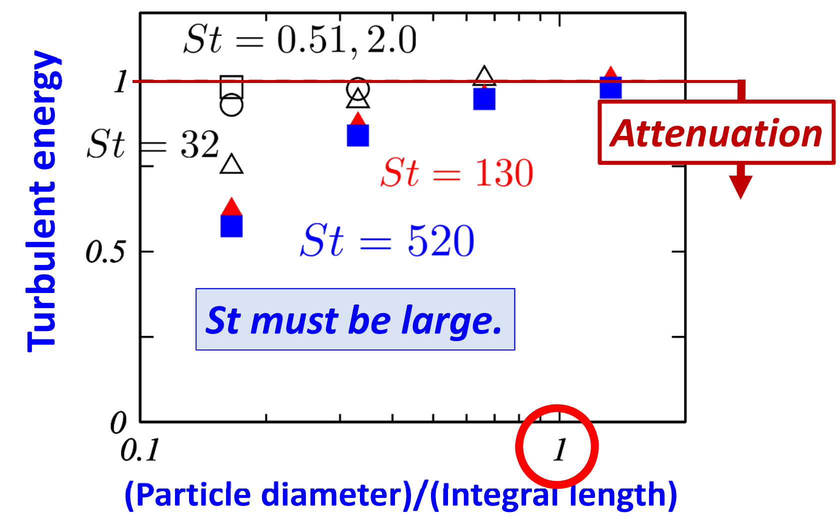
$0.64 < \mathrm{St} = \tau_p/\mathcal{T} < 2700$ (Forcing ②)

$$\tau_p = \frac{\gamma D^2}{18\nu} \qquad (\gamma = \rho_p/\rho_f)$$

3.1 Main results

Turbulence kinetic energy

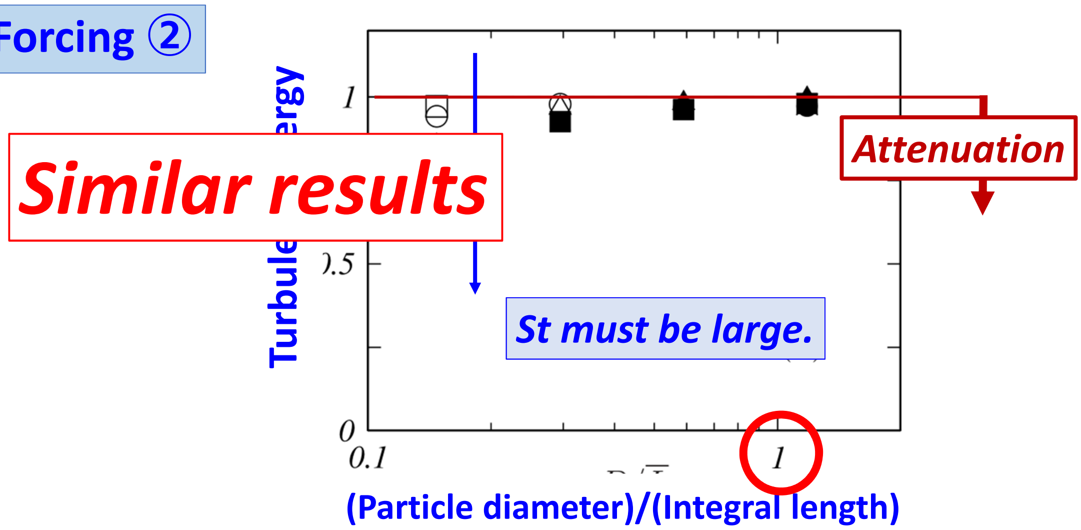
Forcing ①



Smallness of the particle size is a necessary condition.

Turbulence kinetic energy

Forcing ②

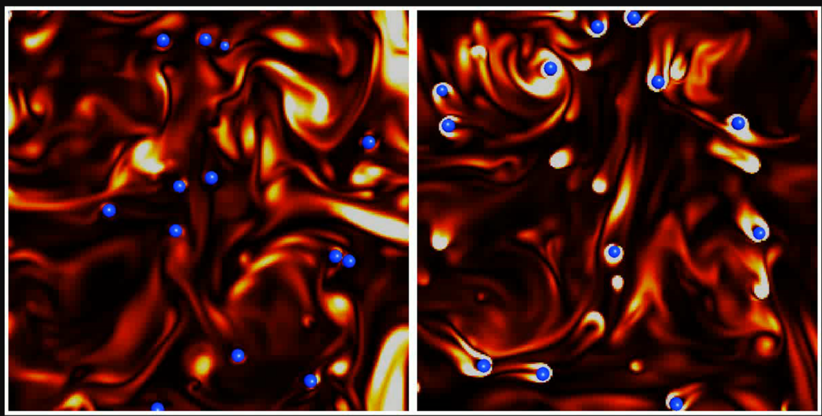


Smallness of the particle size is a necessary condition.

3.2 Mechanism

Attenuation is due to shedding vortices

Particles with a common size   Color=enstrophy



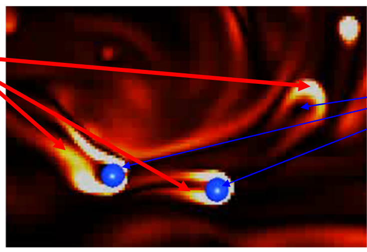
No modulation $St = 0.51$ Significant attenuation $St = 130$

Mechanism

When St≫1   *Particles cannot follow flow on the largest scale.*

→ Velocity difference b/w fluid and particle.

→ Energy dissipation in the wake.



→ Energy cascade is by-passed.

[Verification 1] St≫1 → large ΔU

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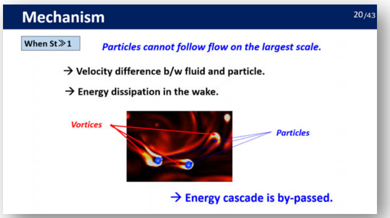

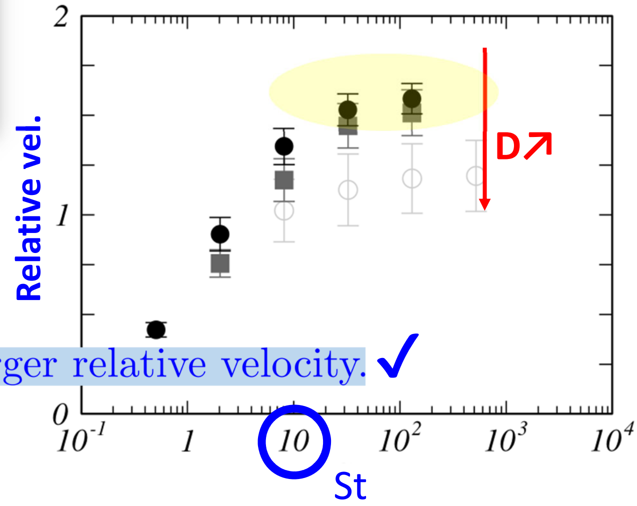

$St \gg 1 \;\rightarrow$ larger relative velocity. ✓

[Verification 2] Dissipation ∝ attenuation rate

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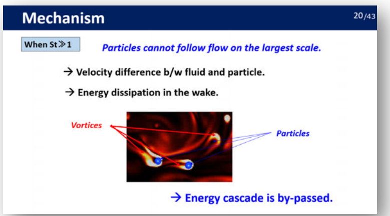
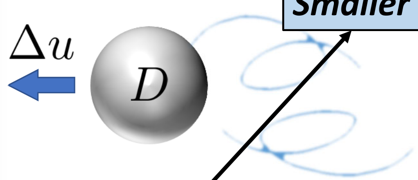




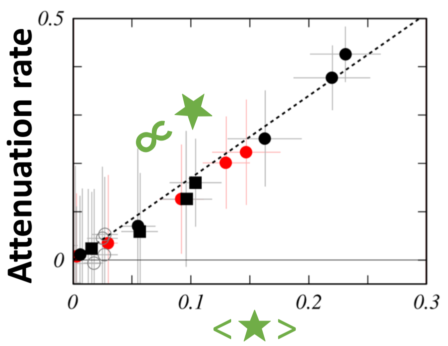
Smaller D → Larger ε_p

Dissipation in the wake:

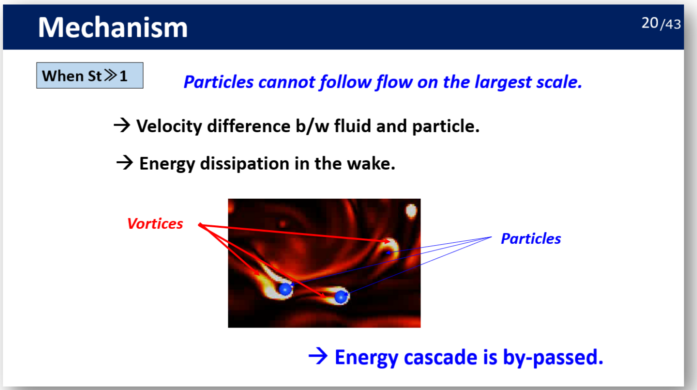
$$\epsilon_p \sim \Lambda_p \frac{|\Delta u|^3}{D} \quad ★$$

Relative velocity (numerically estimated)

Volume fraction



On the basis of

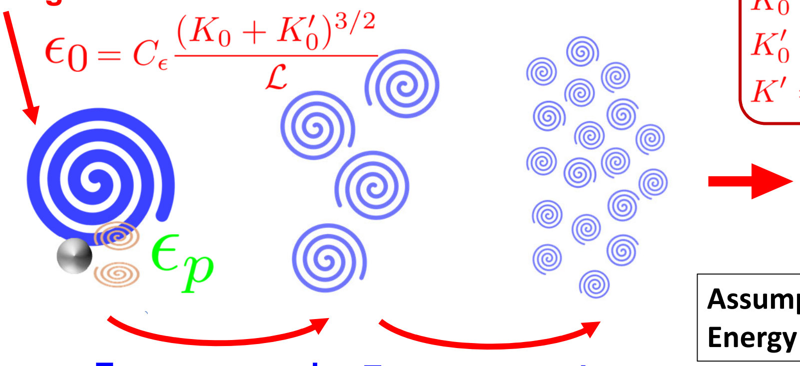


.. we estimate the attenuation **rate.**

Degree of attenuation

Forcing

$$\epsilon_0 = C_\epsilon \frac{(K_0 + K_0')^{3/2}}{\mathcal{L}}$$



K_0 = Mean-flow energy
K_0' = TKE w/o particles
K' = TKE with particles

Assumption:
Energy input rate is not altered.

$$\epsilon_c = C_\epsilon \frac{(K_0 + K')^{3/2}}{\mathcal{L}}$$

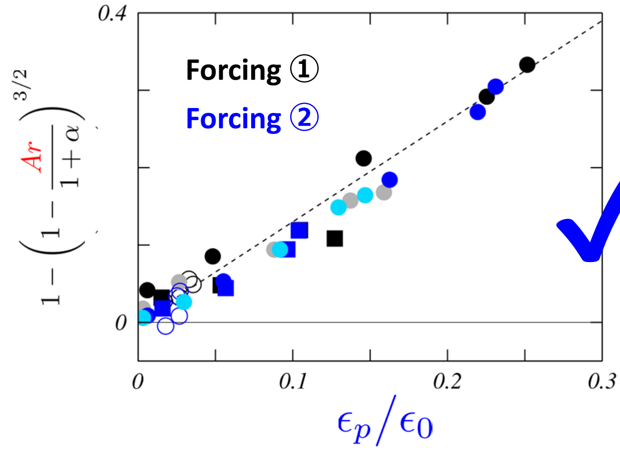
$$\epsilon_0 = \epsilon_c + \epsilon_p \quad \rightarrow \quad 1 - \frac{\epsilon_p}{\epsilon_0} = \left(\frac{K_0 + K'}{K_0 + K_0'} \right)^{\frac{3}{2}} \quad \rightarrow$$

Degree of attenuation

$$Ar = \frac{K' - K_0'}{K_0'}$$

$$1 - \left(1 - \frac{Ar}{1+\alpha}\right)^{3/2} = \frac{\epsilon_p}{\epsilon_0}$$

$$\alpha = \frac{K_0}{K_0'}$$



K_0 = Mean-flow energy
K_0' = TKE w/o particles
K' = TKE with particles

Oka & Goto (2022, JFM)

3.3 Estimation of ϵ_p

Additional energy dissipation rate around particles

☺ ϵ_p **explains the phenomenon**

☹ **We need data of** ϵ_p

$$\epsilon_p = \Lambda_p C_{p(\Delta u)} \frac{\Delta u^3}{D}$$

Δu = Relative velocity b/w particle and fluid

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Estimation of Δu

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Estimation of relative velocity (1/2)

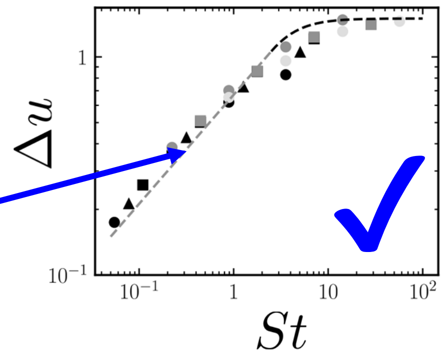
$$\frac{d\boldsymbol{u}_p}{dt} = -\frac{1}{\tau_p}\left(\boldsymbol{u}_p - \boldsymbol{u}_f\right)$$

Balachandar (2009)

$$\frac{d\boldsymbol{u}_p}{dt} \approx -\frac{1}{\tau_p}\left(\boldsymbol{u}_p - \boldsymbol{u}_f^{>\tau_p}\right)$$

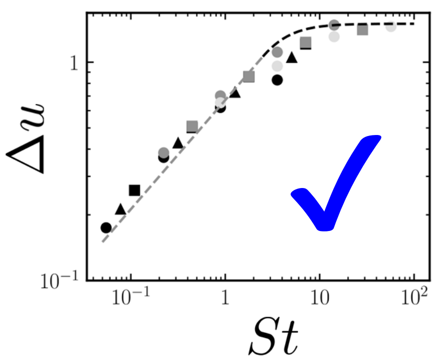
$$\Delta u = \begin{cases} u_L\sqrt{\dfrac{St}{2}} & (St < 1) \\ u_L\dfrac{St}{\sqrt{1+St^2}} & (St > 1) \end{cases}$$

Data of turbulent channel flow

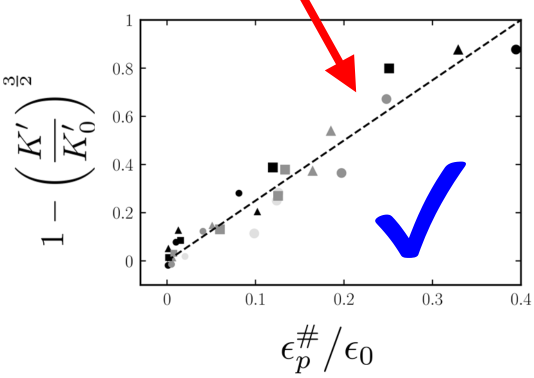


Estimation of relative velocity (2/2)

$$\Delta u = \begin{cases} u_L\sqrt{\dfrac{St}{2}} & (St < 1) \\ u_L\dfrac{St}{\sqrt{1+St^2}} & (St > 1) \end{cases}$$



$$\epsilon_p^{\#} = \Lambda_p C_{p(\Delta u)} \frac{\Delta u^3}{D}$$



3.4 Particle shape effects

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Anisotropic particles (1/4)

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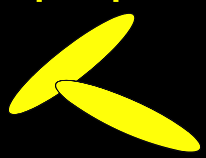
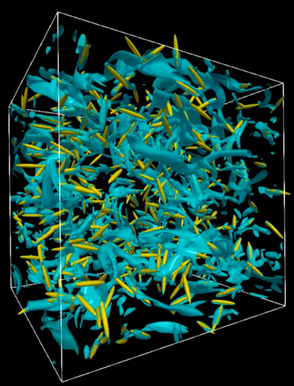
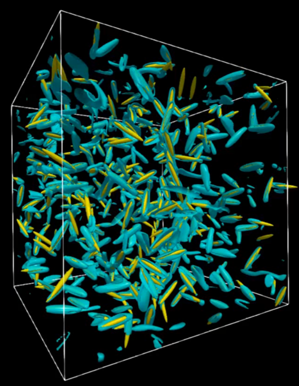
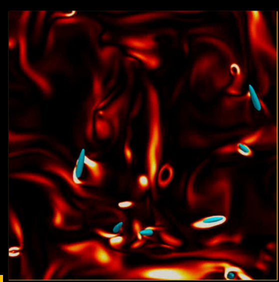
Elliptic particles in a periodic cube



With vortices



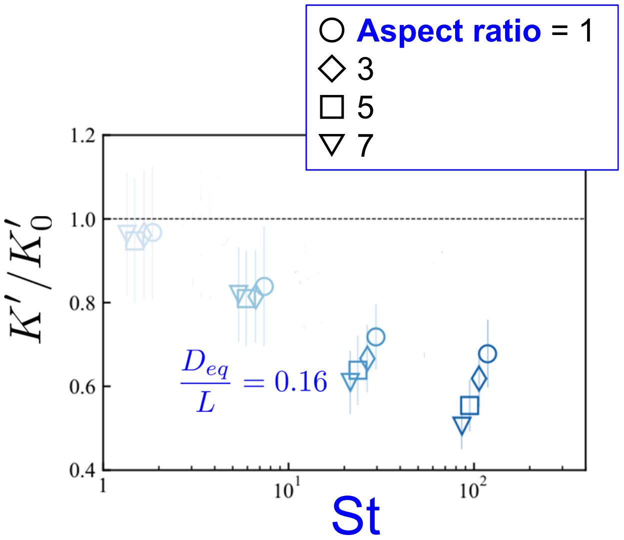




St≈1 → No modulation

St ≫ 1 → Shedding vortices

Anisotropic particles (2/4)

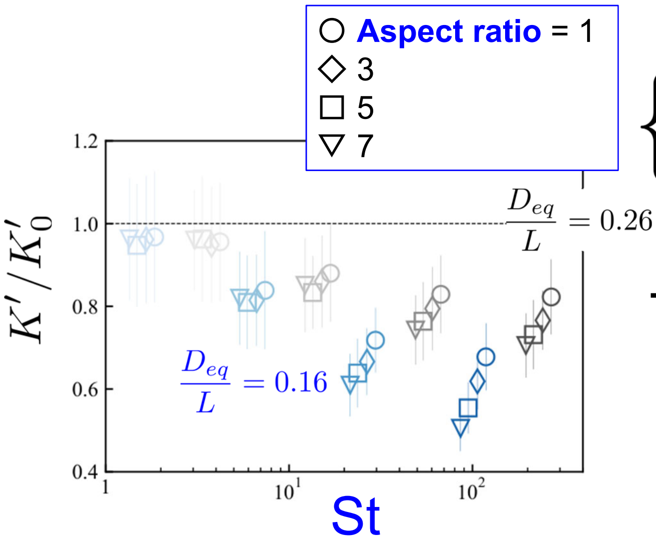


Particle volume is common.
Volume fraction is common.

Turbulence is more attenuated:

for larger St,
for larger aspect ratio.

Anisotropic particles (3/4)



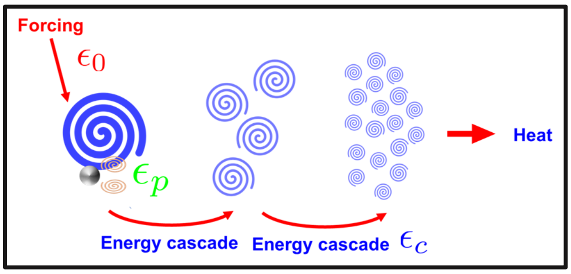
Particle volume is common.
Volume fraction is common.

Turbulence is more attenuated:

for larger St,
for larger aspect ratio,
for smaller size.

→ Larger dissipation rate ϵ_p

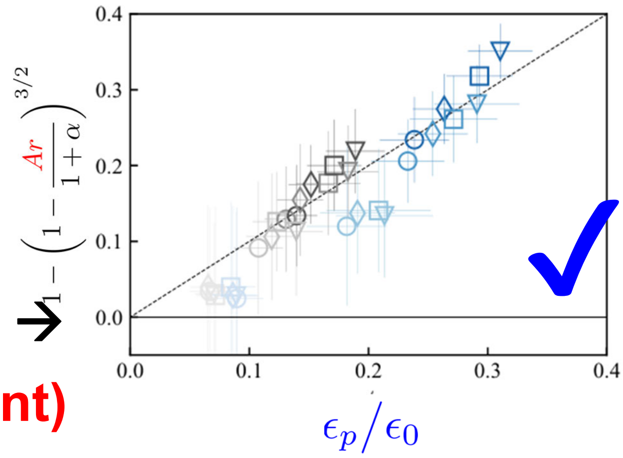
Anisotropic particles (4/4)



$\epsilon_0 = \epsilon_c + \epsilon_p$

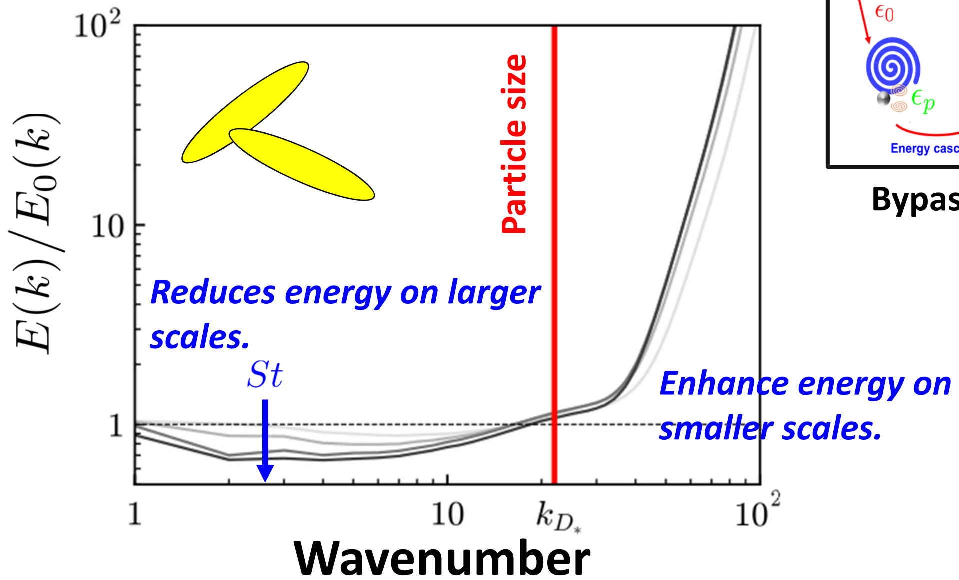
All the data collapse →

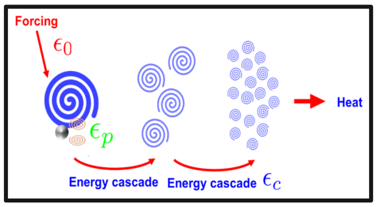
(Rotation is unimportant)



3.5 Bypassing cascade

Energy spectrum





Bypassing cascade

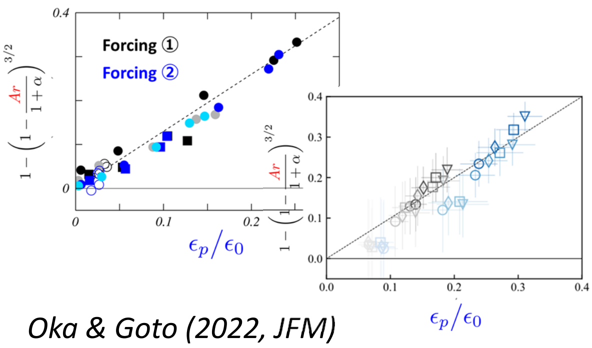
4

Conclusions

Conclusions

Large-St particles can attenuate turbulence because the additional energy dissipation ε_p reduces the flux.

We may estimate the attenuation rate by using ε_p, which may be estimated by



$$\epsilon_p \sim \Lambda_p \frac{|\Delta u|^3}{D}$$

Volume fraction

Relative velocity

Oka & Goto (2022, JFM)
Awai, Motoori & Goto (2024 submitted)
Motoori & Goto (to be submitted)

This also explains why smaller particles are more effective.