2 min. preview talk

P23 Quantum chaos and Bifurcation in billiard systems

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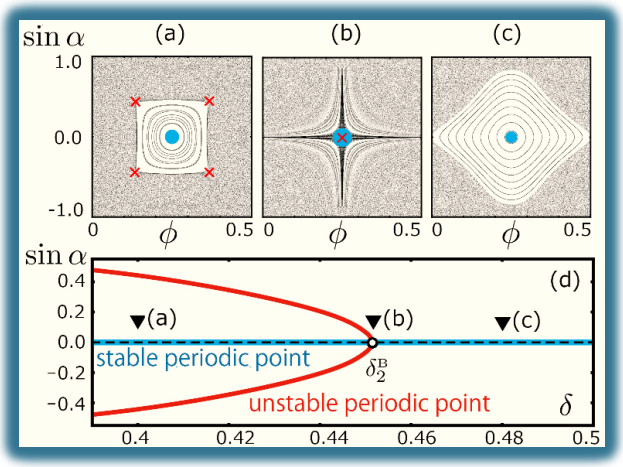

Fig.1 Bifurcation diagram of the Lemon Billiard system.

Is it possible to detect bifurcation points of classical dynamical system from quantum mechanical data?

→ Yes, it is!

They can be detected by analysing the energy levels and energy eigenfunction of the corresponding quantum system.

- The energy levels at the bifurcation points show a strong accumulation on the energy axis with a certain period P_2, and this period is well predicted by the Gutzwiller's semiclassical theory (see Fig.2).

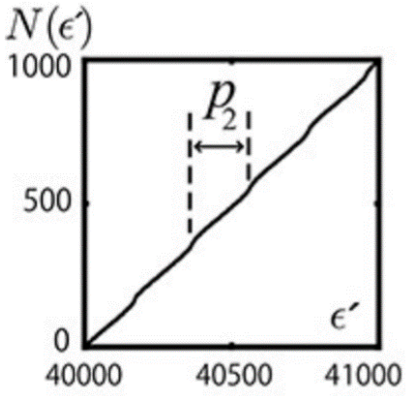

Fig.2 Integrated density of states. P_2 is the accumulation period.

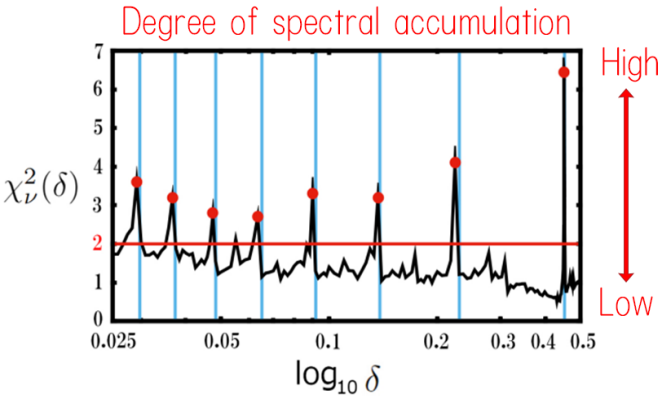

Fig.3 Degree of energy level accumulation χ^2 vs the bifurcation parameters δ (Blue lines). Is's quantum mechanical predictions are plotted by the red marks●.

- By observing this accumulation quantitatively, we can obtain predicted values of the bifurcation parameters (red marks ● in Fig. 3), which are in good agreement with the bifurcation points of the classical dynamical system (blue lines in Fig. 3).

2 min. preview talk

We also analyse the energy eigenfunctions at the bifurcation points of Fig.3(at the red marks), and observed the eigenfunction scarring.

- The energy eigenfunctions at each accumulation point are strongly amplified along the bifurcating orbits (Fig.4), and always be the eigenfunction scarring. T This phenomenon also occurs periodically along the energy axis with the period P_2 (see Fig.5).
- By observing this phenomenon quantitatively, we attempt to estimate the position of a fixed point on the Poincaré surface of section.

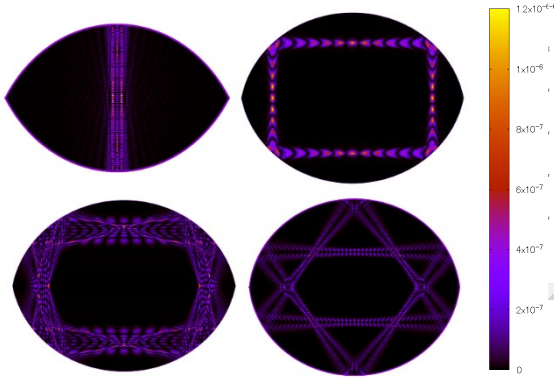

Fig.4 Typical eigenfunction scarrings at the bifurcation points, strongly amplified along the bifurcating orbits.

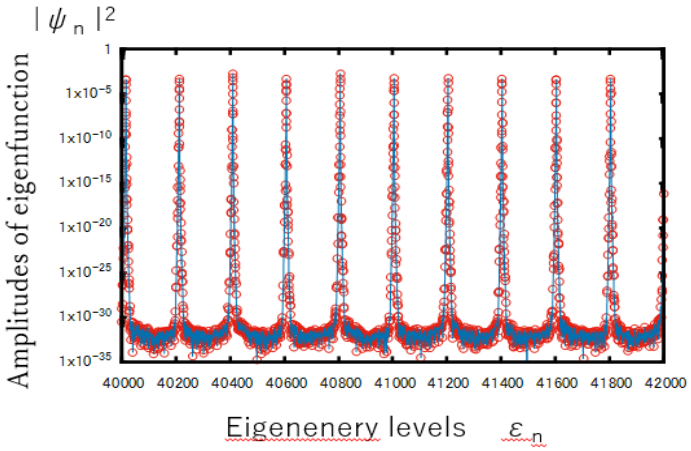

Fig.5 Amplitudes $|\psi_n|^2$ of energy eigenfunctions on bifurcated orbits.

Summary

I. By observing the accumulation of quantal energy levels, one can predict the bifurcation parameters of classical dynamical system.

II. The eigenfunction scarring is induced frequently at the bifurcation points and it is associated with the periodic accumulation of quantal levels. However, it's mechanism is not obvious.